

# EE353 - Processamento de Sinais

## Lista 1

(Z.2)  $h[n] \neq 0 \wedge N_0 \leq n \leq N_1$

$$x[n] \neq 0 \wedge N_2 \leq n \leq N_3 \therefore y[n] = \sum_{i=N_0}^{N_1} h[i] \cdot x[n-i]$$

$\Downarrow$

$N_4 \leq n \leq N_5 \quad N_2 \leq n-i \leq N_3$

Os limites para  $x[n-i] \neq 0$  são

$$\begin{cases} N_2 = n - N_0 \Rightarrow n = N_4 = N_2 + N_0 \\ N_3 = n - N_1 \Rightarrow n = N_5 = N_3 + N_1 \end{cases}$$

b) Utilizando os valores da lista (a), tem-se que:

$$N_4 \leq n \leq N_5$$

$$N_2 + N_0 \leq n \leq N_3 + N_1 \Rightarrow \begin{cases} N_3 = N_2 + N - 1 \\ N_1 = N_0 + M - 1 \end{cases}$$

$$\Downarrow$$

$$N_2 + N_0 \leq n \leq N_2 + N_0 + N + M - 2$$

Então, o número máximo de pontos consecutivos em que  $y[n] \neq 0$  é de  $N_2 + N_0$ , inclusive, até  $N_2 + N_0 + N + M - 2$ .

Por ser inclusivo, o total de pontos é  $[N + M - 1]$

(Z.3)  $h[n] = a^{-n} \cdot M[-n], 0 < a < 1 \Rightarrow y[n] = \sum_{i=-\infty}^n a^{-i} \cdot M[-i] \cdot M[n-i]$

$$n > 0:$$

$$y[n] = \sum_{i=0}^{\infty} a^i \Rightarrow \text{para } 0 < a < 1 \Rightarrow y[n] = \frac{1}{1-a}$$

$$n \leq 0$$

$$y[n] = \sum_{i=-n}^{\infty} a^i \Rightarrow \text{para } 0 < a < 1$$

$$y[n] = \frac{a^{-n}}{1-a}$$

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(2.4)  $y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = z \cdot x[n-1]$

of

$$Y(e^{jw}) - \frac{3}{4}Y(e^{jw})e^{-jw} + \frac{1}{8}Y(e^{jw})e^{-2jw} = z \cdot X(e^{jw})e^{-jw}$$

$$Y(e^{jw})(1 - \frac{3}{4}e^{-jw} + \frac{1}{8}e^{-2jw}) = X(e^{jw})(z \cdot e^{-jw})$$

$$\frac{Y(e^{jw})}{X(e^{jw})} = H(e^{jw}) = \frac{z \cdot e^{-jw}}{1 - \frac{3}{4}e^{-jw} + \frac{1}{8}e^{-2jw}} = Y(e^{jw}) \text{ para } x[n] = \delta[n]$$

↓ of -1

$$y[n] = -8\left(\frac{1}{4}\right)^n u[n] + 8\left(\frac{1}{2}\right)^n u[n]$$

(2.7)  $\frac{2\pi}{\omega_0} = \frac{N}{K}$  a)  $2\pi K = \pi n/6 \Rightarrow K = n/12 \rightarrow \frac{n}{K} = \frac{12}{1} \rightarrow K=1, n=12$

b)  $2\pi K = 3\pi n/4 \Rightarrow K = 3n/8 \rightarrow \frac{n}{K} = \frac{8}{3} \rightarrow K=3, n=8$

c) Não periódico. O n no denominador faz com que a amplitude decaia com o aumento de n.

d)  $2\pi K = \pi n/\sqrt{2} \Rightarrow \frac{n}{K} = 2\sqrt{2} \Rightarrow$  Não periódico. Não há valores de n e K que satisfaça  $\frac{n}{K} = 2\sqrt{2}$

$$\textcircled{8} \quad h[n] = 5\left(-\frac{1}{2}\right)^n u[n] \Rightarrow y[n] = h[n] * x[n]$$

$$x[n] = \left(\frac{1}{3}\right)^n u[n] \quad Y(e^{jw}) = H(e^{jw}) \cdot X(e^{jw})$$

$$Y(e^{jw}) = \frac{5}{1 + \frac{1}{2}e^{-jw}} \cdot \frac{1}{1 - \frac{1}{3}e^{-jw}} = \frac{3}{1 + \frac{1}{2}e^{-jw}} + \frac{2}{1 - \frac{1}{3}e^{-jw}}$$

$\Downarrow \mathcal{Z}^{-1}$

$$y[n] = 2\left(\frac{1}{3}\right)^n u(n) + 3\left(-\frac{1}{2}\right)^n u(n)$$

$$\textcircled{2.18} \quad a) \quad y[n] = \sum_{i=-\infty}^{\infty} a^i \cdot u[-i-1] u[n-i]$$

$$n \leq -1 \Rightarrow \sum_{i=-\infty}^{-1} a^i = \frac{a^n}{1 - \frac{1}{a}}$$

$$n > -1 \Rightarrow \sum_{i=-\infty}^{-1} a^i = \frac{1/a}{1 - 1/a}$$

$$b) \quad \text{Sabendo que } l[n] = z^n u[-n-1] \text{ e que}$$

$$t[n] = l[n] * u[n] = \begin{cases} 1 & \text{para } n > -1 \\ z^{n+3} & \text{para } n \leq -1 \end{cases}$$

Então:

$$y[n] = u[n-4] * l[n] = t[n-4] = \begin{cases} 1 & \text{para } n \geq 3 \\ z^{n-3} & \text{para } n \leq 3 \end{cases}$$

c) Usando  $l[n]$  e  $t[n]$ :

$$h[n] = z^{n-1} u[-(n-1)-1] = l[n-1]$$

$$y[n] = X[n] * l[n-1] = t[n-1] = \begin{cases} 1 & \text{para } n > 0 \\ z^n & \text{para } n \leq 0 \end{cases}$$

d) Usando  $l[n]$  e  $t[n]$

$$Y[n] = (u[n] - u[n-10]) * l[n] = t[n] - t[n-10]$$

$$Y[n] = z^{n-1} u[-(n+1)] + u[n] - (z^{n-9} u[-(n-9)]) + u[n-10]$$

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$$y[n] = \begin{cases} 0 & \text{para } n \geq 9 \\ 1 - z^{n-9} & \text{para } -1 \leq n \leq 8 \\ z^{n+1} - z^{n-9} & \text{para } n \leq -2 \end{cases}$$

$$Z.1) H(e^{jw}) = \frac{1 - e^{-jw}}{1 + \frac{1}{2} e^{-jw}} \quad \text{para } -\pi < w \leq \pi$$

$$x[n] = \sin\left(\frac{\pi n}{4}\right) \quad \text{para } \forall n \in \mathbb{Z}$$

Da fórmula de Euler  $\Rightarrow \sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$

$$x[n] = \frac{e^{jn\pi/4} - e^{-jn\pi/4}}{2j}$$

Usando a propriedade de auto funções para exponenciais complexas

$$y[n] = H(e^{j\pi/4}) e^{jn\pi/4} - H(e^{-j\pi/4}) e^{-jn\pi/4}$$

$$\text{Para } w = \pm \frac{\pi}{4} \quad \frac{1 - e^{-j\pi/2}}{1 + 1/2 e^{-j\pi}} \Rightarrow H(e^{j\pi/4}) = 2(1-j) = 2\sqrt{2} e^{-j\pi/4}$$

$$\frac{1 - e^{j\pi/2}}{1 + 1/2 e^{j\pi}} \Rightarrow H(e^{-j\pi/4}) = 2(1+j) = 2\sqrt{2} e^{j\pi/4}$$

$$y[n] = 2\sqrt{2} e^{-j\pi/4} \cdot e^{jn\pi/4} - 2\sqrt{2} e^{j\pi/4} \cdot e^{-jn\pi/4} = 2\sqrt{2} \cdot \sin\left(n\frac{\pi}{4} - \frac{\pi}{4}\right)$$

- (2.13) a) É autofunção de LTI, pois possui forma a  $\lambda^n$

b) É autofunção de LTI, pois possui forma a  $\lambda^n$

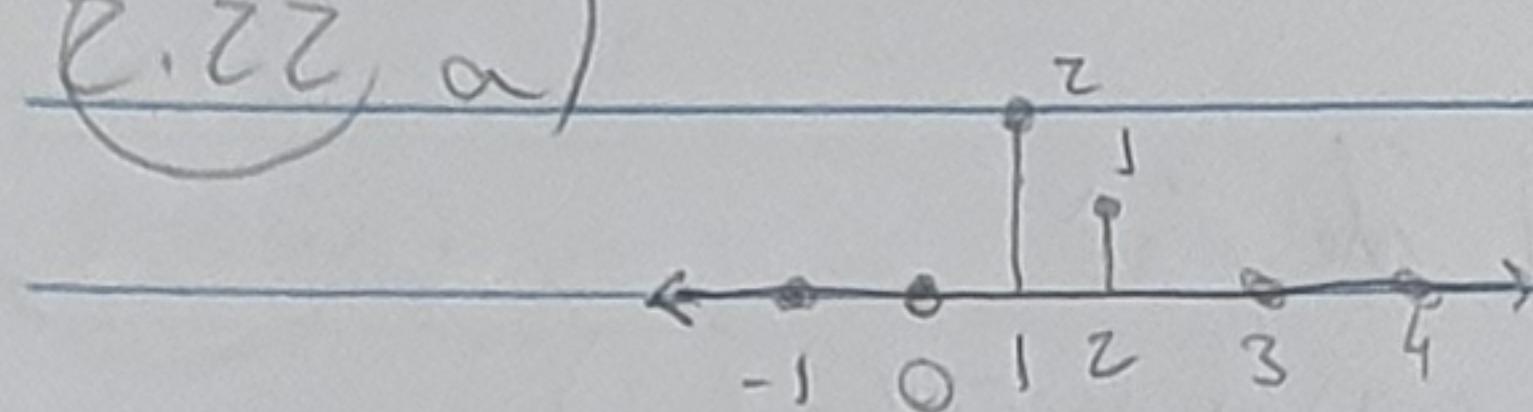
c) Não " " " " " " não possui forma a  $\lambda^n$

d) " " " " " " " " " "

e) É autofunção de LTI, pois possui forma a  $\lambda^n$

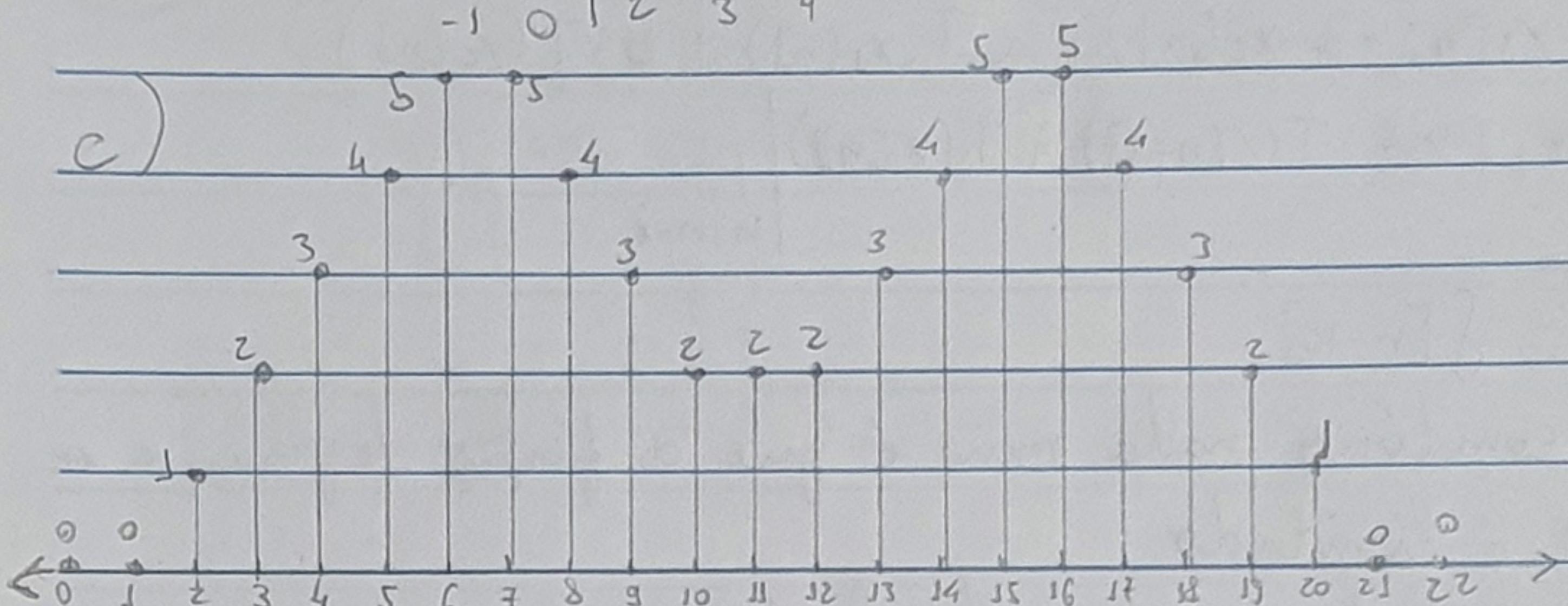
f) Não é autofunção de LTI, pois não possui forma a  $\lambda^n$

E.22, a)

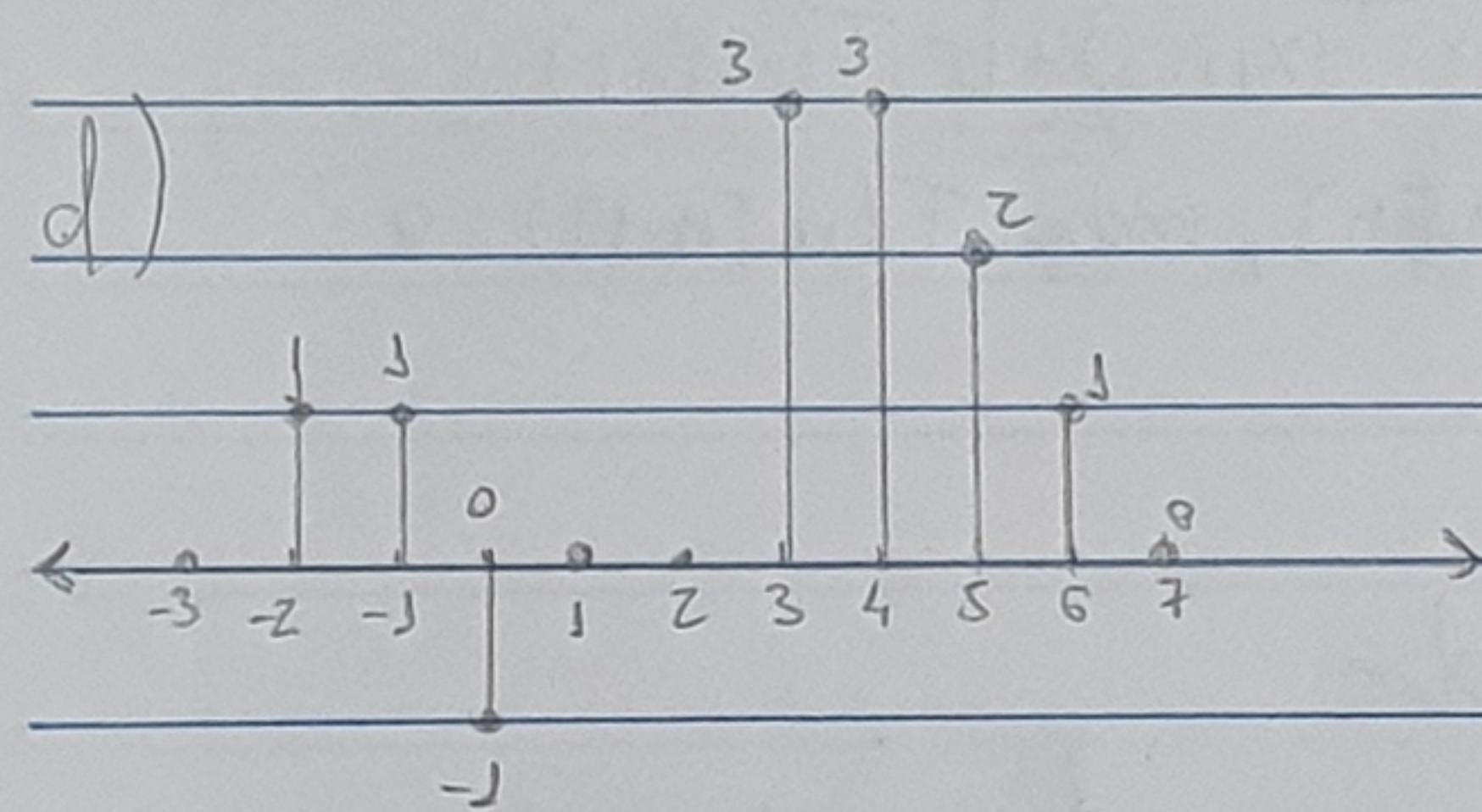


A hand-drawn number line for part b) showing integers from -7 to 5. The line is a horizontal arrow pointing to the right, with tick marks at every integer from -7 to 5. Above the line, the integers are labeled as follows: -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, and 5. The labels are written in a cursive or handwritten style.

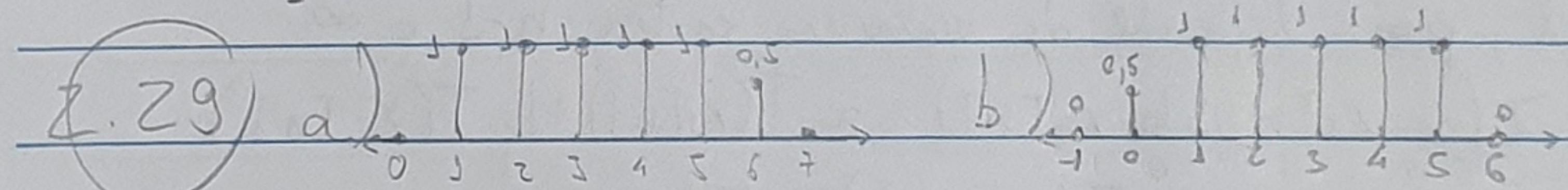
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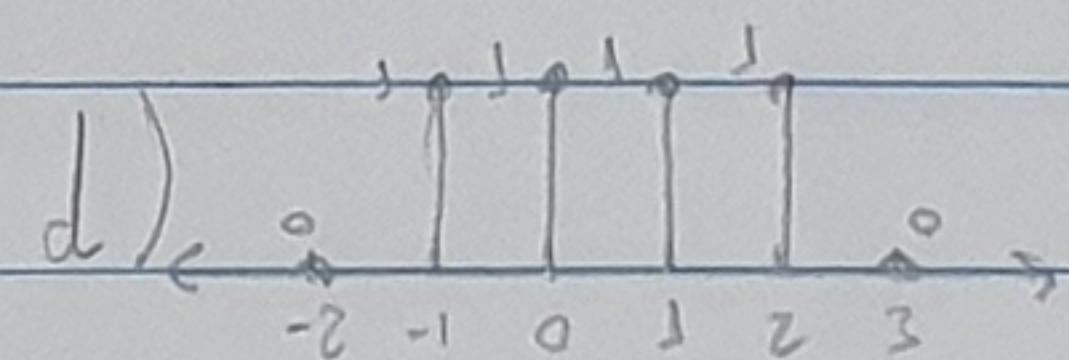
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8.29) a.)



A horizontal number line with arrows at both ends. Tick marks are placed at integer values: -1, 0, 1, 2, and 3. The interval between -1 and 2 is shaded with vertical hatching. The point 0.5 is marked on the line between 0 and 1.



A number line graph showing two open circles at -2 and 3. The region to the left of -2 is shaded with a gray background and labeled with a circled 'e'. The region to the right of 3 is shaded with a gray background and labeled with a circled 'o'.

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2.39 a)  $T(x[n]) = (\cos(\pi n))x[n]$

• Estável, pois o  $\cos(\pi n)$  apenas altera o sinal de  $x[n]$

• Causal, pois só depende do valor atual de "n"

• Linear, pois  $T(ax_1[n] + bx_2[n]) = a.T(x_1[n]) + b.T(x_2[n])$

• Variante no tempo, pois  $T(x[n-i]) \neq T(x[n])$

$$|_{n=h-i}$$

b)  $T(x[n]) = x[n^2]$

• Estável, pois  $x[n^2]$  é limitado

• Causal, pois só depende do valor atual de "n"

• Linear, pois  $T(a \cdot x_1[n] + b \cdot x_2[n]) = a.T(x_1[n]) + b.T(x_2[n])$

• Variante no tempo, pois  $T(x[n-i]) \neq T(x[n])$

$$|_{n=h-i}$$

c)  $T(x[n]) = y[n] \sum_{k=0}^{\infty} g[n-k]$

• Estável, pois o somatório nada mais é que a função degrau, e o produto  $x[n] \cdot u[n]$  é limitado

• Causal, pois só depende do valor atual de "n"

• Linear, pois  $T(a \cdot x_1[n] + b \cdot x_2[n]) = a.T(x_1[n]) + b.T(x_2[n])$

• Variante no tempo, pois  $T(u[n]) = u[n]$ , mas  $T(u[n+5]) = 0$

d)  $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$

• Instável para  $T(u[n])$ , por exemplo

• Não-causal, pois depende de valores entre  $n-1$  e  $\infty$

• Linear, pois  $T(a \cdot x_1[n] + b \cdot x_2[n]) = a.T(x_1[n]) + b.T(x_2[n])$

• Invariante no tempo, pois  $T(x[n-i]) = T(x[n])$

$$|_{n=h-i}$$

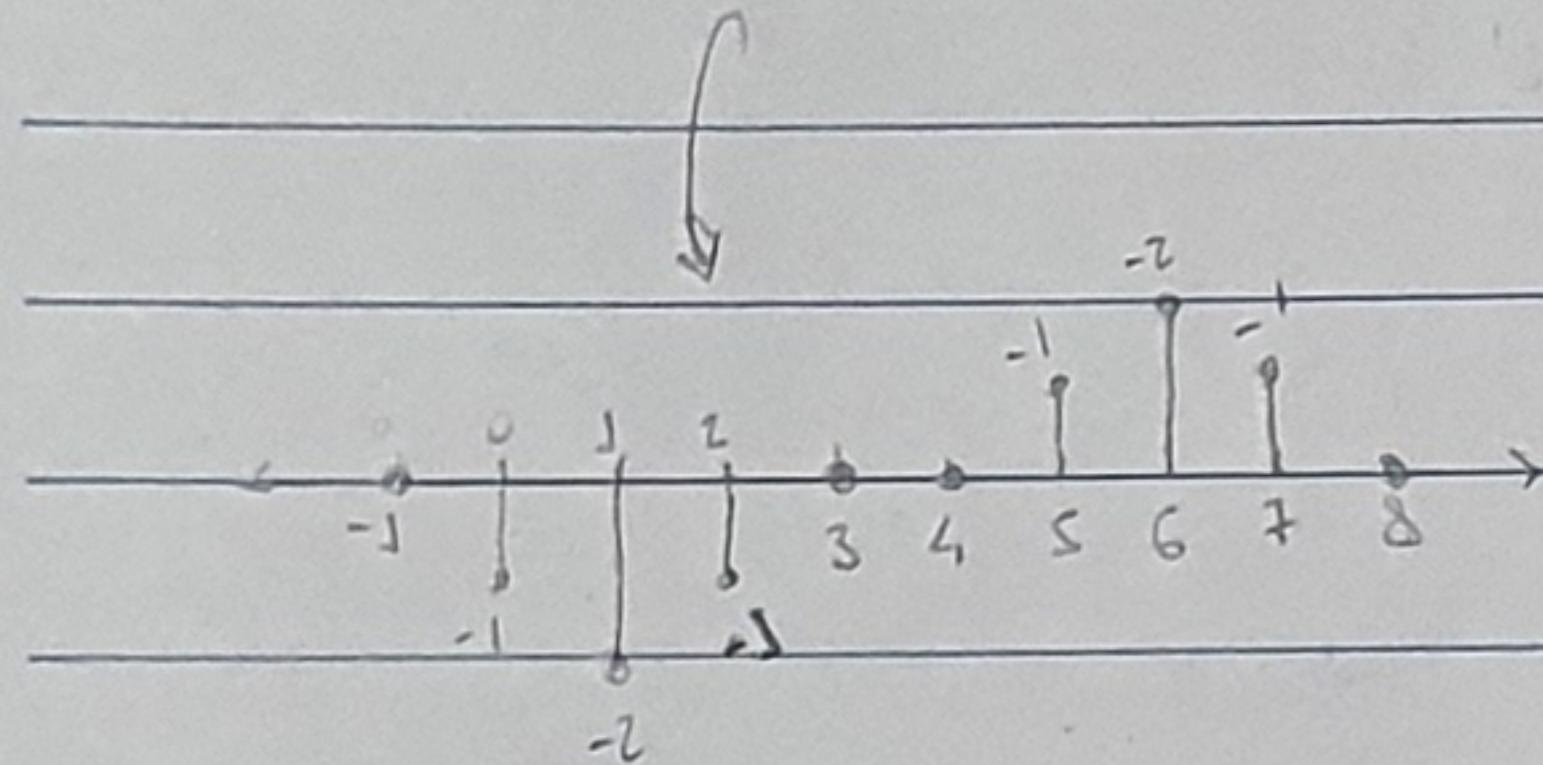
$$Z.33) \quad x[n] = \cos\left(\frac{3\pi n}{2} + \frac{\pi}{4}\right) \rightarrow \omega_0 = \frac{3\pi}{2} \quad H(e^{j\omega}) = |H(e^{j\omega})| \underbrace{\arg[H(e^{j\omega})]}$$

$$y[n] = \underbrace{|H(e^{j\pi/2})|}_{\downarrow} \cdot \cos\left(\frac{3\pi n}{2} + \frac{\pi}{4} + \underbrace{\arg[H(e^{j\pi/2})]}_{\text{Pelo gráfico e sabendo que } H(e^{j3\pi/2}) = H(e^{-j\pi/2}) \therefore \arg[H(e^{-j\pi/2})] = -\frac{\pi}{2}}\right)$$

$$y[n] = \cos\left(\frac{3\pi}{2}n + \frac{11\pi}{12}\right)$$

$$Z.34) \quad a) \quad x[n] = x_o[n-2] + 2 \cdot x_o[n-4] + x_o[n-6]$$

$$\rightarrow y[n] = y_o[n-2] + 2 \cdot y_o[n-4] + y_o[n-6]$$



$$b) \quad y_o[n] = -x_o[n-1] + x_o[n-3] = x_o[n] * (-\delta[n+1] + \delta[n-3])$$

$$h[n] = -\delta[n+1] + \delta[n-3]$$