Gibbs Sampling to sample multiparameters

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We saw in our work earlier, that we have two parameters, namely μ and σ^2.

The full conditional update for the mean μ is

$$N\left(\mu \middle| \frac{\frac{n\bar{y}}{\sigma^2} + \frac{\mu_0}{\sigma_0^2}}{\frac{n}{\sigma^2} + \frac{1}{\sigma_0^2}}, \frac{1}{\frac{n}{\sigma^2} + \frac{1}{\sigma^2}}\right)$$

The full conditional update for the mean σ^2 is

$$IG\left(\sigma^2 | \nu_0 + \frac{n}{2}, \beta_0 + \frac{\sum_{i=1}^{n}(y_i - \mu)^2}{2}\right)$$

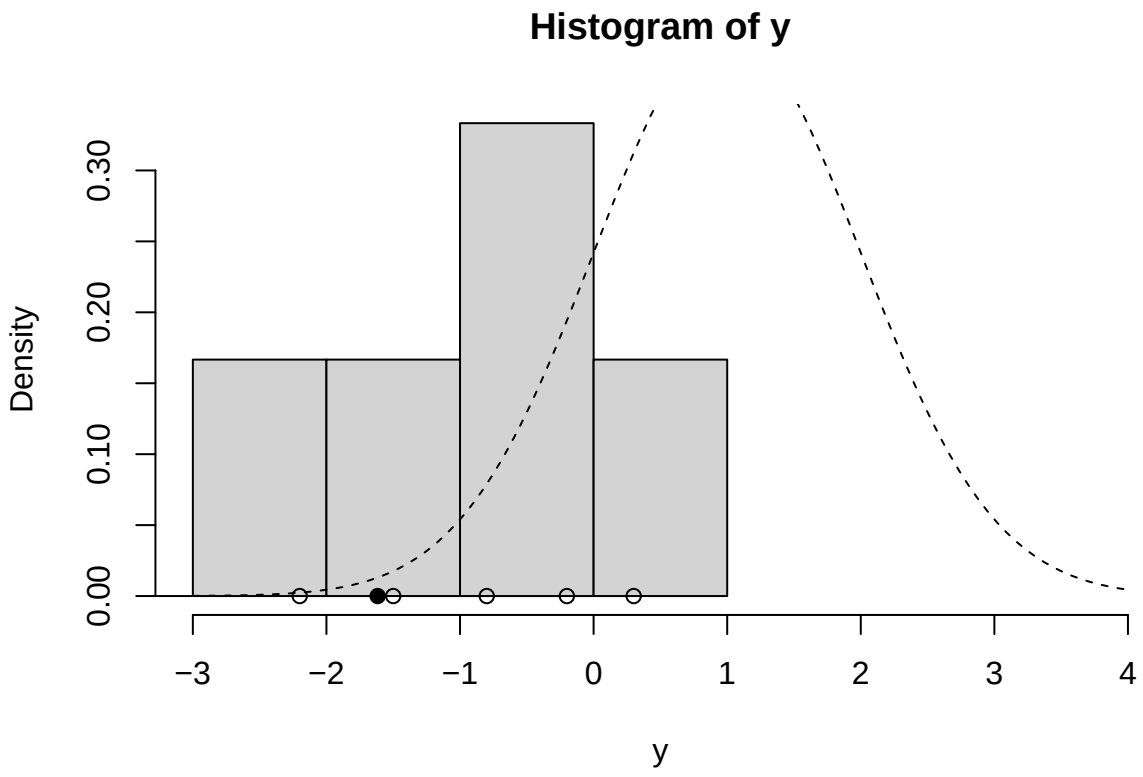
Here's the code

```
update_mu = function(n, ybar, sig2, mu_0, sig2_0) {
  sig2_1 = 1.0 / (n / sig2 + 1.0 / sig2_0)
  mu_1 = sig2_1 * (n * ybar / sig2 + mu_0 / sig2_0)
  rnorm(n=1, mean=mu_1, sd=sqrt(sig2_1))
}

update_sig2 = function(n, y, mu, nu_0, beta_0) {
  nu_1 = nu_0 + n / 2.0
  sumsq = sum( (y - mu)^2 ) # vectorized
  beta_1 = beta_0 + sumsq / 2.0
  out_gamma = rgamma(n=1, shape=nu_1, rate=beta_1) # rate for gamma is shape for inv-gamma
  1.0 / out_gamma # reciprocal of a gamma random variable is distributed inv-gamma
}

gibbs = function(y, n_iter, init, prior) {
  ybar = mean(y)
  n = length(y)

  ## initialize
  mu_out = numeric(n_iter)
  sig2_out = numeric(n_iter)

  mu_now = init$mu

  ## Gibbs sampler
  for (i in 1:n_iter) {
    sig2_now = update_sig2(n=n, y=y, mu=mu_now, nu_0=prior$nu_0, beta_0=prior$beta_0)
    mu_now = update_mu(n=n, ybar=ybar, sig2=sig2_now, mu_0=prior$mu_0, sig2_0=prior$sig2_0)
```

```
    sig2_out[i] = sig2_now
    mu_out[i] = mu_now
  }

  cbind(mu=mu_out, sig2=sig2_out)
}
```

Now we are ready to set up the problem in R.

```
y = c(-0.2, -1.5, -5.3, 0.3, -0.8, -2.2)
ybar = mean(y)
n = length(y)

## prior
prior = list()
prior$mu_0 = 1.0
prior$sig2_0 = 1.0
prior$n_0 = 2.0 # prior effective sample size for sig2
prior$s2_0 = 1.0 # prior point estimate for sig2
prior$nu_0 = prior$n_0 / 2.0 # prior parameter for inverse-gamma
prior$beta_0 = prior$n_0 * prior$s2_0 / 2.0 # prior parameter for inverse-gamma

hist(y, freq=FALSE, xlim=c(-3,4.0)) # histogram of the data
curve(dnorm(x=x, mean=prior$mu_0, sd=sqrt(prior$sig2_0)), lty=2, add=TRUE) # prior for mu
points(y, rep(0,n), pch=1) # individual data points
points(ybar, 0, pch=19) # sample mean
```



```r
set.seed(53)

init = list()
init$mu = 0.0

post = gibbs(y=y, n_iter=5000, init=init, prior=prior)
head(post)
```

```
##              mu     sig2
## [1,] -1.3369860 4.885669
## [2,] -1.6773224 2.365635
## [3,] -1.5837925 5.146892
## [4,] -0.2675581 3.124617
## [5,] -0.1823195 5.222408
## [6,]  0.2058328 5.220432
```

```r
library("coda")
plot(as.mcmc(post))
```