A model of optimal extraction and site reclamation*

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Abstract

Environmental issues during and after extraction are a major issue in contemporary exhaustible resource production. Production operation deteriorates the state of the environment and is a source of possibly harmful emissions. After the extraction has ceased, the site is in need of reclamation and clean-up. This paper analyses the last two stages of exhaustible resource production: extraction and site reclamation decisions. The socially optimal regulation is investigated, and it is found that a pollution tax, a shut-down date and a requirement for the firm to deposit funds for costly reclamation can be used to incentivize socially optimal extraction of the resource. It is also found that the firm can be required to pay the monies to a reclamation trust at the beginning of the extraction operation, which protects the tax payers from the possible insolvency of the firm who tries to avoid paying for the reclamation.

Keywords: Dynamic optimization; Emission tax; Environmental policy; Exhaustible resources; Reclamation; Stock pollution; Waste

JEL codes: Q30, Q38, Q50, Q58.

1 Introduction

Exhaustible resource producers are in principle required to pay the costs related to site reclamation that occurs after the production has shut-down. The purpose of site reclamation operations is to clean-up the site and return it to alternative uses. The current problem is that the required payment is often set too low, and therefore the reclamation operation might be underfunded and sub-optimal from the social point of view. This has not gone unnoticed in newspapers around the world, including the United Kingdom (Monbiot, 2015), Australia (Seccombe, 2014), the United States (Preston, 2017)

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and Canada (Hoekstra, 2017).[1] Regarding the low bonding requirements in Canada, the Energy and Mines Minister of British Columbia Bill Bennett stated in 2017 that

> *"We are going to have to establish some concrete, specific, measurable objectives and principles or parameters - whatever you want to call it - on how do we actually assess the amount of financial security we need to have full assurance for the taxpayer."*

The objective of this paper is to formalize this need as an optimization model with resource extraction decision followed by site reclamation and clean-up and to investigate the needed regulation in the form of a pollution tax and a reclamation payment.

Site reclamation and externalities related to the environment are relevant for almost any exhaustible resource production. Extracting an exhaustible resource such as oil, gas or minerals from the ground requires moving large quantities of material, and the refinement of the end product produces wastes. In Canada, most of the oil reserves are in tar sands (oil sands) where the oil is often obtained by extracting and refining the bitumen found in sandstone. The resource is extracted by surface mining, which causes loss of land and biodiversity. In addition, refining oil generates wastes, which are stored in the constructed tailings ponds. After the extraction site has been shut-down, these tailings ponds remain unless suitable reclamation operations are conducted at the site. The problem is that the tailings ponds contain contaminants and represent a hazard to the environment and to the public health (Heyes *et al.*, 2018).

In the U.S, shale oil and gas (tight oil and gas) extraction using hydraulic fracturing and horizontal drilling is a source of contaminants such as uranium, lead, salt and methane. The environmental problems remain after shut-down unless reclamation is conducted. This is also true for hard rock mining, in which the main environmental problem is acid mine drainage (AMD), that is, the flow of toxic substances from the site that have been released by acidic waters (Dold, 2014). In modern mining, large quantities of waste rocks and tailings are produced, and this is a costly challenge for eventual mine

[1] Different types of instruments, such as bonds and trust payments, are used in an attempt to incentivize reclamation. The required payments are often insufficient to cover the estimated costs. For example in British Columbia, the cost estimate is over one billion dollars higher than the collected securities (Hoekstra, 2017).

area reclamation (Mudd, 2010). A particularly harmful method of surface mining is the so-called mountaintop mining: Coal seams are extracted by removing the top of a mountain or a hill and by dumping of spoils to adjacent streams and rivers, which results in environmental impacts and possible health issues (Palmer *et al.*, 2010).

Research questions and the model. In some of the above examples the environmental problems occur during and after the extraction of the resource. In others, as with AMD, the environmental problems manifest mainly after the extraction operation has been shut-down (Dold, 2014). The purpose of this study is to model the socially optimal extraction of the resource and the costly reclamation and clean-up of the site, and to analyse the regulation needed to internalize the externality. The research questions are: 1. If the reclamation operation is financed using a reclamation trust, how large is the optimal deposit by the producer to the reclamation trust? 2. What is the optimal tax on pollution generation? 3. When should the extraction operation be shut-down? 4. Should there be any regulation regarding the producer's payments, that is, does it matter when the producer pays the monies?

These questions are investigated in a two-stage model. During the extraction stage, the socially optimal extraction rate and the shut-down date are chosen. At this stage, the model is analysed under multiple cost structures, which all have been applied to describe the extraction technology. The extraction model itself is used to describe AMD, in which the pollution stock is built during the extraction stage, but which causes damages only after shut-down. The extraction model is also analysed in the more general setting with a more general pollution stock dynamics including natural cleaning processes during the extraction stage and with a stock that causes damages during the extraction stage. After the extraction stage, the reclamation stage begins and the site is optimally reclaimed. The pollution stock causes damages during this stage and the reclamation is costly. To implement the socially optimal allocation, the regulator sets a time-dependent pollution tax defined from the beginning of the extraction operation until the optimal shut-down date and a requirement that the producer pays the eventual reclamation costs by depositing sufficient amount of money to a reclamation trust.[2] These monies the producer

[2]As the tax is defined on the socially optimal extraction interval and the reclamation payment is lump-sum, it can also be said that there is only one instrument: a time-dependent tax with a lump-sum

is assumed to transfer to the trust from its operational profits.

Contribution to the literature. The study contributes to the literature of polluting exhaustible resources extraction by analysing optimal regulation in a two-stage model. The model is "a full description" of the last two stages of contemporary exhaustible resource exploitation, namely of extraction and reclamation.[3] The study is related to the literature on optimal use of exhaustible resources and to the literature on optimal environmental policy applied to polluting exhaustible resources with a specific focus on mining. In contemporary exhaustible resource extraction environmental externalities play a major role.[4] In general, extraction produces a pollution stock as a side-product, which causes damages during and after extraction.

Recently, the literature has focused on the reclamation of the extraction site (or on the clean-up of the pollution stock) after the extraction operation has been shutdown.[5] Sullivan and Amacher (2009) analyse the socially optimal reclamation decision between forestry and grassland options and Lappi (2018) argues that it may be socially optimal to delay the reclamation of the extraction site, but neither paper considers the pollution generation or the optimal extraction of the resource that ultimately cause the reclamation problem. This important contribution is made by Yang and Davis (2018), who analyse the optimal regulation of a polluting exhaustible resource producer who must be given incentives to reclaim the site after the extraction has ended. The pollution stock is taken

component.

[3]Exploration and discovery are the first stages of the exploitation process, but as environmental issues and site reclamation are currently important, and because of the logical process of thinking optimality from the final stage backwards, this study aims to partly complete the picture of optimal use of polluting exhaustible resources by investigating the last two stages of exploitation. The theoretical literature on exhaustible resource extraction is vast, and it includes the early studies on the optimal use of exhaustible resources (Dasgupta and Heal, 1974; Hartwick, 1978), on exploration (Pindyck, 1978) and on imperfect competition (Stiglitz, 1976; Salant, 1976; Pindyck, 1987). The literature has expanded along these lines and in many others. For example the literature on mining has investigated the optimal capacity choice (Campbell, 1980; Lozada, 1993; Cairns, 2001; Holland, 2003).

[4]One strand of literature along this line includes studies investigating the optimal carbon tax on a polluting exhaustible resource (Ulph and Ulph, 1994; Hoel and Kverndokk, 1996; Tahvonen, 1997).

[5]Earlier studies related specifically to mining and environmental policy include Stollery (1985), Roan and Martin (1996), Cairns (2004), Farzin (1996), White *et al.* (2012) and Lappi and Ollikainen (2018). For example Roan and Martin (1996) analyse the reclamation and extraction decision of a mine under command and control regulation with a model, where the reclamation is conducted during the production process by reclaiming some of the waste. In a recent working paper, Aghakazemjourabbaf and Insley (2018) compare environmental bond and strict liability rule, when the extracting firm must clean-up the extraction site, but they do not consider the social optimum like done here.

in their model to be the amount of mine damaged land, and they show that a tax on the pollution stock can be used to reach the social optimum, if the firm is required to pay the marginal damage for every increment in the pollution stock.[6] The model applied by Yang and Davis (2018) has two important simplifications: First, they assume that the pollution stock damage and the reclamation cost are linear functions with respect to the stock, and second, they assume that there is no natural decay of the pollution stock (in fact, in their model the pollution stock can never decrease unless reclaimed). These assumptions imply that the reclamation is of "now or never"-type, that is, the reclamation is either performed when the extraction operation is shutdown or never. Also Lappi (2018) arrives at this conclusion in a special case of his model when the pollution damages and the reclamation cost are linear and when the pollution stock decays exponentially. It is not possible in Yang and Davis (2018) to delay the reclamation or the clean-up for some time. The authors mention that the "now or never"-type reclamation response matches empirical observation, but this does not mean that it is socially optimal.

The model here allows to delay or wait with the reclamation if it is socially optimal to do so. To allow this possibility (and others), three models are applied for the description of the reclamation stage: first, the clean-up model of Lappi (2018), which allows waiting with the clean-up; and second, the clean-up model of Caputo and Wilen (1995) in which the pollution or waste stock is cleaned continuously as time goes on; and third, a model in which the level of reclamation effort is chosen to balance the marginal costs and benefits.[7] The above mentioned simplifications of Yang and Davis (2018) are dropped, and therefore the pollution damage and reclamation cost functions are allowed to be non-linear and the pollution stock can decay through natural processes such as dilution, microbial degradation and immobilization. But contrary to their model, it is assumed in this paper that there are no abatement possibilities for the firm during the extraction operation. This modelling choice is made to simplify the exposition, but adding an abatement option for the firm (as an additional control variable as for example in Lappi and Ollikainen (2018)) is a relatively straightforward extension to the model.

[6] This tax scheme, in which the polluter pays for the losses of the other agents (here pollution damages), is in fact based on the same principles as the ones used for example by Loeb and Magat (1979) (regulation of a monopoly) and Kim and Chang (1993) (regulation of a polluting oligopoly).

[7] The last two models are presented in sections 5 and 6 of Supplementary material.

Policy implications. Optimal regulation of a polluting exhaustible resource firm must guide the producer to exploit the resource optimally, to shut-down the site at the optimal date, to leave optimal amount of pollution to the site after production shut-down and to deposit a sufficient amount of money to a reclamation trust. In principle, the results give a formula to calculate the socially optimal deposit to the reclamation trust. Mitchell and Casman (2011) state that the payment to the so-called trust account in Pennsylvania's mining sector is based on the present value reclamation costs, and it is unclear how the damages from the pollution stock are taken into account.[8] Similarly, in Canada and Australia the bond payment by the coal producer is based on the reclamation costs (Cheng and Skousen, 2017). Social optimality, however, calls for reclamation trust payments that are also based on the pollution damages and not just on the reclamation costs.

More importantly, the socially optimal reclamation payment must take into account the socially optimal extraction stage choices that yield the pollution stock or the state of the environment which is to be reclaimed. It is not sufficient from the social optimum point of view to simply use a pollution tax during the extraction stage, since then the site is not reclaimed (also Yang and Davis (2018) arrive at this conclusion). Neither is it sufficient to require only a reclamation payment from the firm, since then extraction rate and pollution stock level are distorted relative to the social optimum. Therefore both a tax with an optimal shut-down date and a reclamation payment are needed to internalize the externality. The results show that the calculation of the optimal trust payment must take into account information on both the extraction and reclamation stages including extraction technology, prices, natural decay of the pollution stock, pollution damages, reclamation costs and the applied interest rate. If successful, this information can be used to design a tax and reclamation payment policy, which together with a requirement that the firm must pay the monies at the beginning of the extraction operation, yields the social optimum. As the monies are paid at the beginning of the extraction operation, the risk that the firm folds to avoid the reclamation payment is diminished.

The study is organized as follows. Section 2 develops the notation and the main

[8] According to Mitchell and Casman (2011), at the beginning of the mining operation, the producer pays the present value of the reclamation cost multiplied with a volatility premium.

assumptions. In Section 3 the reclamation problem and in particular the properties of the problem's value function is investigated. The optimal extraction decision is analysed in Section 4. Section 5 analyses the producer's incentives when it is confronted with the optimal regulation and a profit tax. Section 6 concludes the study. Most of the proofs and additional analysis are relocated in the appendices or in Supplementary material.

2 Notation and assumptions

This section introduces the notation and assumptions for the extraction stage model and for the reclamation stage model that allows waiting with the reclamation decision. Figure 1 depicts the time-line of the model.

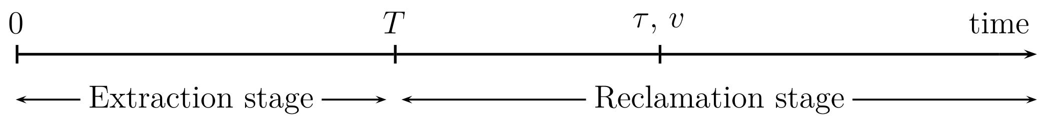

Figure 1: An illustration of the model's time-line. The extraction stage is followed by the reclamation stage, which begins at time T. The reclamation date is τ and the size is v.

Extraction begins at time zero and continues until the optimally chosen shut-down date T. After that the reclamation stage commences. During the extraction stage, the extraction rate $q(t)$ and the shut-down date T are chosen to maximize the net benefits while taking into account that the extraction depletes the resource stock $X(t)$ and creates a stock externality, say pollution, which is essentially captured by the state variable $N(t)$ and by the damage function. In contrary to Yang and Davis (2018), this stock is allowed to decay, and it is interpreted as the amount of pollution at time t, but other interpretations are of course possible.[9] For example, variable N can be interpreted simply as an

[9]Yang and Davis (2018) write without references on page 285 that ”Given that the pollutants from mining are mainly disturbed lands and other effects with no natural attenuation, we do not allow for natural attenuation of the stock of pollution”. But natural attenuation is relevant for mining as the following examples show. According to Lottermoser (2010, page 265) the amount of cyanide in gold mining diminishes naturally. Wilkin (2007) writes on page 8 related to acid mine drainage that ”...at nearly all mining sites, natural processes are contributing to varying degrees and in some cases may contribute significantly to site remedial goals”. In oil sand extraction natural attenuation of the pollution stock occurs via biodegradation (Quagraine *et al.*, 2005; Clemente and Fedorak, 2005; Whitby, 2010; Foght *et al.*, 2017). Although the current paper allows the pollution stock to decay, the description of the dynamics using a single differential equation is admittedly a crude simplification of the very complex real phenomena.

index for the state of the environment. A higher index value means lower environmental state. At the beginning of the extraction stage $N(0) = 0$, and the environmental is at its best condition. As extraction commences, the index begins to increase and the state to worsen. At the shut-down date T the index obtains a relatively high value, and as the extraction ceases the state of the environment begins to improve by natural processes such as re-vegetation and pollutant decay and immobilization. In what follows the stock variable N is referred to as the (size of the) pollution stock. The price of the resource is assumed to be a constant p. The extraction costs are captured by cost function G, whose value depends on the extraction rate and possibly on the amount of resource left in the ground. In Section 4 the extraction stage is analysed under different extraction cost function specifications, and the respective assumptions are developed there.

When choosing the extraction rate and the shut-down date of the extraction stage, the regulator must take into account the effect of these choices on the reclamation stage. These effects are conveyed through the discounted scrap value, which is denoted with $S(N(T), T)$. This is included in the extraction stage objective, and it is the central target for investigation in Section 3 where the optimal reclamation is analysed. Function S gives the discounted reclamation stage value. The initial value of the pollution stock at the reclamation stage, denoted with n_T, is the shut-down date value of the pollution stock at the extraction stage, $N(T)$. That is, $n_T = N(T)$. The reclamation cost function is C. This cost depends on the size of the reclamation (the amount of pollution stock cleaned at the reclamation date τ), $v \in [0, n_T]$, and it satisfies properties $C' > 0$ and $C'' \geq 0$. Hence the reclamation cost is strictly increasing and convex in the reclamation size, and there are no fixed costs related to reclamation process. The pollution stock causes damages and the damage function is denoted with D. This function satisfies properties $D(0) = 0$, $D' > 0$ and $D'' \geq 0$. In Yang and Davis (2018) the reclamation cost and the pollution damage are linear in the stock, but here strictly convex functions are allowed.

The pollution stock dynamics during the extraction stage are given by the following initial value problem:

$$\dot{N}(t) = \alpha q(t) - f(N(t)), \quad N(0) = 0. \tag{1}$$

Here, parameter $\alpha > 0$ describes the accumulation of the pollution stock as the resource is extracted. Function f, which describes the decrease of the stock through natural processes, is explained shortly. The pollution stock dynamics during the reclamation stage are given by the following initial value problem:

$$\dot{N}(t) = -f(N(t)), \quad N(T^-) = n_T > 0, \tag{2}$$

where symbols $N(T^-)$ denote the left-sided limit of $N(T)$, that is, $N(T^-) = \lim_{t \to T^-} N(t)$. The extraction rate and hence the pollution accumulation is zero during the reclamation stage.

Two important assumptions are made regarding the stock dynamics. First, it is assumed that f is twice continuously differentiable, $f > 0$ for $N > 0$, $f(0) = 0$ and that there exists a constant B such that $|f'(N)| \leq B$. This guarantees that there exists a unique global solution to Equation (2) (and to Equation (1) for a given $q(t)$). This solution is denoted with $N(t; n_T, T)$. Second, it is supposed that the solution satisfies inequality $N(t; n_T, T) > 0$ for all t, that is, the stock never disappears other than through reclamation. For future reference, denote with $N_n(t; n_T, T)$ the partial derivative with respect to the second variable (evaluated at the point (n_T, T)). Similar notation will be used in what follows without further explanations. For example, notation $N_T(t; n_T, T)$ means the partial derivative of $N(t; n_T, T)$ with respect to third variable.

3 Reclamation stage

The model is analysed starting from the reclamation stage after which the optimal extraction stage decision and its properties are investigated. The main link between the stages is the reclamation stage value function, which acts as the scrap value in the extraction stage decision.[10] The regulator takes as given the inherited pollution stock from the extraction stage and minimizes the sum of the total discounted damages from the pollution stock and the discounted reclamation costs by choosing the date and the size of

[10]Two alternative reclamation models are presented in Supplementary material. One model is the continuous clean-up model of Caputo and Wilen (1995) and the other is a simple reclamation model, in which reclamation operation is done at the shut-down date to reclaim a fraction of the pollution stock. It is shown that the value function has similar properties as in the model presented here (some minor additional assumptions are applied in the continuous clean-up model, though). This means that the extraction stage analysis of Section 4 carries over to these alternative specifications.

the reclamation operation. To simplify the analysis, it is assumed that all of the remaining pollution stock is reclaimed, which means that $v = N(\tau; n_T, T)$. Regulator's problem becomes then

$$\max_{\tau \in [T, \infty]} \left\{ \int_T^\tau -D(N(t; n_T, T))e^{-r(t-T)} \, \mathrm{d}t - C(N(\tau; n_T, T))e^{-r(\tau-T)} \right\}, \tag{3}$$

since after τ the damages are zero by the assumption $D(0) = 0$. This problem with $T = 0$ has been analysed in Lappi (2018).[11] The Lagrangian related to problem (3) is

$$L(\cdot) = \lambda_0 \left(\int_T^\tau -D(N(t; n_T, T))e^{-r(t-T)} \, \mathrm{d}t - C(N(\tau; n_T, T))e^{-r(\tau-T)} \right) - \lambda(T - \tau), \tag{4}$$

in which λ_0 and λ are multipliers (constants). If the problem has a solution, then Fritz John Theorem tells that there exists multipliers $(\lambda_0, \lambda) \neq (0, 0)$ such that $\lambda_0 \in \{0, 1\}$, and that the following conditions hold at the optimal reclamation date τ^*:

$$\begin{aligned} \lambda_0 \Big(&- D(N(\tau; n_T, T))e^{-r(\tau-T)} + rC(N(\tau; n_T, T))e^{-r(\tau-T)} \\ &- C'(N(\tau; n_T, T))\dot{N}(\tau; n_T, T)e^{-r(\tau-T)} \Big) + \lambda = 0, \end{aligned} \tag{5}$$

$$\lambda \geq 0, \quad \tau - T \geq 0, \quad \lambda(\tau - T) = 0. \tag{6}$$

Clearly $\lambda_0 = 1$. Hence the optimal reclamation date satisfies the following condition:

$$\begin{aligned} &D(N(\tau^*; n_T, T)) - rC(N(\tau^*; n_T, T)) \\ &\qquad - C'(N(\tau^*; n_T, T))f(N(\tau^*; n_T, T)) \begin{cases} \geq 0, & \text{if } \tau^* = T, \\ = 0, & \text{if } \tau^* > T. \end{cases} \end{aligned} \tag{7}$$

In words, in an interior optimum, the reclamation is postponed until the cost of waiting one more unit of time equals the benefit of waiting. The cost is the additional damage and the benefit is the sum of the interest on the unused reclamation funds and of the decrease in the reclamation costs due to decrease in the pollution stock through natural processes. This condition is essentially the optimality condition presented in Lappi (2018) for the clean-up of a polluted site. What is important here, is that the optimal reclamation date depends on the decisions made in the extraction stage. Namely, the date depends on the pollution stock at the shut-down date of the extraction operation. Note also, that if there

[11] Some of the presented results are replications. Namely Lemma A.1, and Part (i) of Lemma A.2 and Part (i) of Proposition 1 are replications of the results in Lappi (2018).

are no natural cleaning processes (that is, if $f = 0$), the benefit of waiting consists only of the interest on the unused reclamation funds. If, in addition, the pollution stock damage and reclamation cost functions are linear, the site is reclaimed either at the shut-down date or never as in Lappi (2018, Example 1) and in Yang and Davis (2018). The related optimal reclamation size is

$$v^* = N(\tau^*; n_T, T). \tag{8}$$

Analysis is continued by investigating the properties of the reclamation stage program's value function and of the discounted reclamation cost. It is assumed throughout that for an optimal interior reclamation date the following inequality holds:

$$\begin{aligned} D'(N(\tau^*; n_T, T)) - C'(N(\tau^*; n_T, T))r - C''(N(\tau^*; n_T, T))f(N(\tau^*; n_T, T)) \\ - C'(N(\tau^*; n_T, T))f'(N(\tau^*; n_T, T)) < 0. \end{aligned} \tag{9}$$

It can be calculated that if the second derivative of the objective function in (3) is strictly negative (which implies that the function is strictly concave), then the inequality in (9) holds for an optimal interior reclamation date.[12]

The optimal reclamation date τ^* and the optimal reclamation size v^* depend on the parameters n_T, T and r. Since only n_T and T are endogenous variables in the whole model, the optimal reclamation date and size are denoted with $\tau^* = \tau(n_T, T)$ and $v^* = v(n_T, T)$, and the explicit dependency of the optimal values on r is left out of the notation. The value function of the maximization problem (3) is defined as

$$V(n_T, T) := \int_T^{\tau(n_T,T)} -D(N(t; n_T, T))e^{-r(t-T)}\, \mathrm{d}t - C(v(n_T, T))e^{-r(\tau(n_T,T)-T)}. \tag{10}$$

The necessary amount of money needed for the reclamation at the end of the extraction stage is given by $C(v(n_T, T))e^{-r(\tau(n_T,T)-T)}$, since this sum will increase to $C(v(n_T, T))$ as time progresses from T to τ^*. To simplify the notation define two variable functions K and S with

$$K(n, T) := C(v(n, T))e^{-r(\tau(n,T)-T)} \quad \text{and} \quad S(n, T) := V(n, T)e^{-rT}. \tag{11}$$

Function K measures the necessary amount of money in the reclamation trust at the end of the extraction stage (the cost of reclamation, which has been discounted to the shut-down date T). This function is needed to define the amount of money the firm is required

[12]This is shown in Section 1 of Supplementary material.

to deposit to the trust by the end of the extraction stage. Function S measures the reclamation stage value, which is discounted to time $t = 0$, when the extraction operation begins. This function acts as the discounted scrap value function in the extraction stage problem.

Dependency of S and K on the shut-down date pollution stock level. Before analysing the properties of S and K, the dependency of the reclamation date and size on the shut-down date pollution stock is investigated. Lemma A.2 in Appendix A.1 provides the mathematical details of the dependency of τ^* and v^* on this endogenous parameter. This lemma is a part of the proof of Proposition 1.

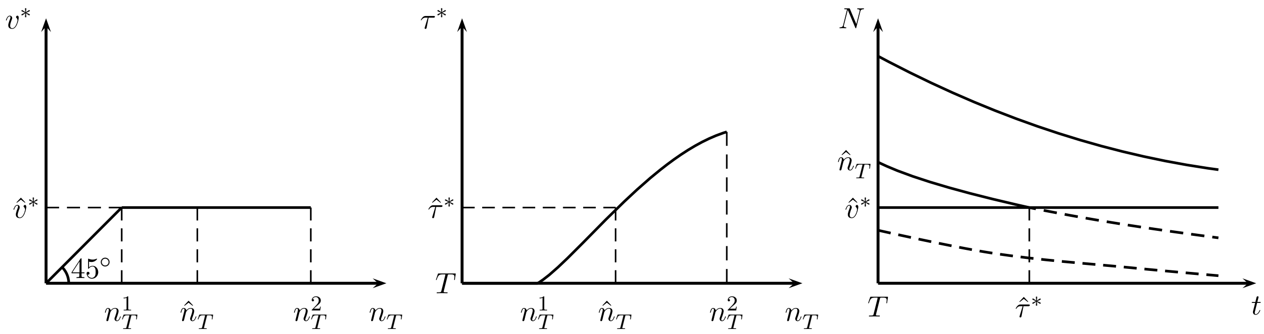

Figure 2: Illustration of the optimal reclamation decision for different shut-down date pollution stocks. Here (n_T^1, n_T^2) is the interval on which τ^* is an interior point. Figure on the left depicts the optimal reclamation size as function of the shut-down date pollution stock. In the middle, the optimal reclamation date is plotted as a function of the shut-down date pollution stock. The last figure contains pollution stock paths that start from different shut-down date pollution stocks and illustrates the different possibilities for the reclamation (the dashed curves represent to time paths of the pollution stock if reclamation is sub-optimally not conducted).

The optimal reclamation decision is illustrated in Figure 2, when τ^* is an interior point of $[T, \infty)$ on some open n-interval (n_T^1, n_T^2). The figure on the left depicts the optimal reclamation size as a function of the shut-down date pollution stock, and the figure on the middle depicts the optimal date as a function of the shut-down date pollution stock. These figures illustrate that as the pollution stock increases from zero towards n_T^1, the optimal reclamation date remains at the shut-down date T and consequently the optimal reclamation size increases. This is essentially the "now or never"-reclamation result of Yang and Davis (2018), where the reclamation cannot occur after the shut-down date and all of the stock is instantaneously cleaned.

The current model allows to wait with the reclamation operation. The pollution stock value n_T^1 is the largest value at which the optimal date is still T. After n_T^1 an increase in the pollution stock increases the optimal reclamation date, but keeps the size fixed. The figure on the right illustrates the evolution of the three possible paths of the pollution stock in time. The path starting from $\hat{n}_T$ decreases in time as the natural processes clean the stock until at time $\hat{\tau}^*$ the site is reclaimed. The amount of pollution cleaned is in this case $\hat{v}^*$. This figure also illustrates two other possibilities at which the reclamation is either instantaneous (the dashed line over the x-axis depicts the pollution stock path if the reclamation operation is sub-optimally not conducted at the shut-down date) or is delayed compared to $\hat{\tau}^*$.

Next proposition shows how a change in the shut-down date pollution stock affects the discounted reclamation cost and the discounted scrap value.

Proposition 1. *Let* (n_T^1, n_T^2) *be any interval on which* $\tau^* > T$. *Then*

(*i*) $K_n(n_T, T) < 0$ *for all* $n_T \in (n_T^1, n_T^2)$,

(*ii*) $K_n(n_T, T) > 0$ *for all* $n_T \in (0, n_T^1)$,

(*iii*) $S_n(n_T, T) < 0$ *for all* $n_T \in (0, \infty)$.

Proof. See Appendix A.1. □

The result regarding the the discounted reclamation cost in Part (i) of this proposition replicates the results of Lappi (2018). The discounted reclamation cost is either strictly increasing or strictly decreasing in the shut-down date pollution stock depending whether the reclamation date is $\tau^* = T$ or $\tau^* > T$. If it is optimal to delay the reclamation operation ($\tau^* > T$), the size of the reclamation is constant with respect to the shut-down date pollution stock and the reclamation date is increasing in the shut-down date pollution stock (see Part (i) of Lemma A.2 in Appendix A.1 and also Figure 2). This implies, that the discounted cost of reclamation decreases since the current reclamation cost is thus unaffected and the discount factor is decreased by the increase in the shut-down date pollution stock. When the reclamation operation is done at the shut-down date T as in Part (ii), a small increase in the shut-down date pollution stock causes no change in the reclamation date, but increases the size of the reclamation (see Part (*ii*) of Lemma A.2 in Appendix A.1).[13] Hence the reclamation cost and also the discounted reclamation cost increase. The result in Part (*ii*) is similar to the result in Yang and Davis (2018), where the reclamation cost increases linearly with the shut-down date pollution stock. As noted in Lappi (2018), non-linearities in the pollution damages and reclamation costs and the existence of natural decay of the pollution stock give rise to the possibility that the discounted reclamation cost decreases as the shut-down date pollution stock increases.

Part (*iii*) says that the discounted scrap value is decreasing in the shut-down date pollution stock.[14] This is very intuitive at least when the reclamation occurs at the shut-down date, since in that case the future pollution damages are zero and therefore an increase in the pollution stock to be reclaimed just increases the reclamation cost and therefore decreases the discounted scrap value. When it is optimal to delay the reclamation operation, two opposing effects exist. First, as Part (*i*) of Proposition 1 shows, the discounted reclamation cost is strictly decreasing in the shut-down date pollution stock. Second, as the shut-down date pollution stock increases, the path of the pollution damages over the reclamation stage shifts upwards, and therefore the overall damages increase. Part (*iii*) shows that the discounted scrap value is strictly decreasing in the shut-down date pollution stock, and therefore an increase in the overall damages is strictly greater than the decrease in the reclamation cost.

Dependency of S and K on the shut-down date. A change in the initial time of the reclamation stage has also an effect on the optimal reclamation date and size. Lemma A.3 presented in Appendix A.2 offers the mathematical details for these effects. When the reclamation occurs at the beginning of the reclamation stage, and the shut-down date T of the extraction stage increases by a small amount, the reclamation date also increases by approximately the same amount, but the size of the reclamation operations stays unchanged. Similarly, if the reclamation operation is delayed, then only the reclamation date adjusts upwards as the shut-down date T is increased.

[13]For this to hold, n_T must not equal to n_T^1, since at that stock level an increase in the stock causes a shift in the reclamation date from the shut-down date to some later date.

[14]In Yang and Davis (2018) the discounted scrap value is obtained from the reclamation cost by multiplying it with $-e^{-rT}$, since delay of the reclamation operation is not allowed. Therefore in their model the scrap value decreases linearly in the shut-down date pollution stock.

Next the dependency of the discounted reclamation costs and the discounted scrap value on the initial time of the reclamation stage T (the shut-down date of the extraction stage) is investigated.

Proposition 2. *Whether or not it is optimal to delay the reclamation operation, the following hold:*

$$K_T(n_T, T) = 0, \quad \textit{and} \quad S_T(n_T, T) = -rV(n_T, T)e^{-rT} > 0.$$

Proof. See Appendix A.2. □

This result means that an increase in the shut-down date has no effect on the discounted reclamation cost (recall that K measures the reclamation cost which is discounted to the shut-down date T). There are two reasons for this. First, the reclamation size and hence the reclamation cost remains unchanged, when the shut-down date increases by a small amount. Second, the discount factor remains the same because the reclamation date increases by one unit, when the shut-down date increases by one unit. The result regarding the discounted scrap value S (discounted to time $t = 0$) is central for the optimal shut-down date decision of the extraction site, which is analysed in Section 4. The result in the previous proposition shows that the discounted scrap value is strictly increasing in the shut-down date: a unit increase in the shut-down date increases the discounted scrap value approximately by the amount $-rV(n_T, T)e^{-rT}$. This is the discounted interest on the avoided monetary value of the reclamation stage program, and therefore, if one begins the reclamation stage one unit of time later, one receives a benefit which is in absolute value terms the interest on the discounted monetary value of the reclamation stage. It should be noted that also in Yang and Davis (2018) the reclamation cost (that is, the pollution stock multiplied by a constant) is unaffected by a change in the shut-down date and that the discounted scrap value increases as the shut-down date is increased.

This section concludes by noting that if its never optimal to reclaim, then similar calculations as in the proof of Proposition 2 yield equation $S_T(n_T, T) = -rV(n_T, T)e^{-rT}$, which has the clear economic intuition explained above.[15]

[15] These calculations are presented in Section 4 of Supplementary material.

4 Extraction stage

As explained above, the initial value of the pollution stock at the reclamation stage, n_T, is the size of the pollution stock at the end of the extraction stage, $N(T)$. When deciding the socially optimal extraction rate and the shut-down date, the regulator must take into account the effect of choices on the reclamation stage. These effects are conveyed through the discounted scrap value $S(N(T), T)$.

General case. Let the stock dynamics for the pollution stock be given by Equation (1) and suppose that this stock causes damages during the extraction stage. Suppose that the extraction cost depends on the amount of resource in the ground X and on the rate of extraction q. The cost function is denoted with G. A typical assumption about the cross-partial derivative G_{qX} is that it is strictly negative (as for example in Pindyck (1978), Caputo (1990), Pesaran (1990), Tahvonen (1997), Krautkraemer (1998) and Cairns (2014)), which reflects a situation where the marginal extraction cost increases as the resource stock diminishes. Fixed operating costs are allowed in the sense that $G(0, X) \geq 0$ for any resource stock level. In particular, if $G(0, X) = 0$, then there are no fixed costs and if $G(0, X) > 0$, then there are fixed costs. These costs include any operation costs that are borne even if the extraction rate is zero, but the site is not shut-down. After the extraction site is shut-down these costs are also zero. Then the regulator's maximization problem is given as

$$\max_{\{q(t),T\}} \int_0^T \Big(pq(t) - G(q(t), X(t)) - D(N(t))\Big)e^{-rt} \,\mathrm{d}t + S(N(T), T) \tag{12}$$

$$\text{s.t} \quad \dot{X}(t) = -q(t), \quad X(0) = x_0, \quad X(T) \geq 0, \tag{13}$$

$$\dot{N}(t) = \alpha q(t) - f(N(t)), \quad N(0) = 0, \quad N(T) \geq 0, \tag{14}$$

$$q(t) \geq 0, \tag{15}$$

where S is given by (11). Variable p is the constant price of the resource and x_0 is the initial amount of the resource in the ground. Next proposition presents the optimal shut-down rule for the extraction operation.[16]

Proposition 3. *Suppose that the extraction cost function G satisfies properties $G_q > 0$ and $G_{qq} > 0$ for all q and X.*

[16] Yang and Davis (2018) do not analyse the shut-down rule.

(i) If $q(T) > 0$, then

$$q(T)\left[\frac{G(q(T),X(T))}{q(T)} - G_q(q(T),X(T))\right] + D(N(T))$$
$$= -rV(N(T),T) - \gamma(T)e^{rT}f(N(T)). \tag{16}$$

(ii) If $q(T) = 0$, then

$$G(0,X(T)) + D(N(T)) = -rV(N(T),T) - \gamma(T)e^{rT}f(N(T)). \tag{17}$$

Proof. See Appendix A.3. □

On the left-side of these equations is the cost of waiting one more unit of time and on the right-side is the benefit of waiting one unit of time. At the optimal shut-down date these values must be the same. The cost of waiting consists of two parts. In the shut-down rule (16), the first is the difference between the average and the marginal costs multiplied by the terminal extraction amount, and the second is the damages from the pollution stock at the shut-down date. The benefit of waiting is also a sum of two parts. The first is the interest on the avoided reclamation stage value, and the second, namely the term $-\gamma(T)e^{rT}f(N(T))$, is the value of the removed pollution stock by the natural process during the unit of time spent on waiting, where the value is measured with the current shadow value of the pollution stock.

A typical assumption in the literature about the extraction cost function, found for example in Pindyck (1978), Pindyck (1987) and Tahvonen (1997), is $G(q,X) = qc(X)$. The function c is assumed to satisfy property $c' < 0$.[17] With this specification the shut-down rule becomes

$$D(N(T)) = -rV(N(T),T) - \gamma(T)e^{rT}f(N(T)), \tag{18}$$

since the marginal and average costs are equal. Hence the shut-down rule is characterized by the pollution stock dynamics and damages at the terminal pollution stock.

Acid mine drainage. The model can capture acid mine drainage (AMD). AMD takes a long time to develop, and it causes damages mainly after the extraction stage (Dold, 2014). As long as the tailings are kept water saturated the oxidization process is slow

[17]For this function $G_{qq} = 0$, but assumption $G_{qq} > 0$ was not used in the proof.

and AMD is limited. After shut-down the mine maintenance operations cease and the AMD is amplified and begins to cause problems. The amount of tailings grows as the resource is exploited. Hence, to capture AMD, it is now assumed that the stock variable N causes damages only during the reclamation stage. In this case the fixed extraction costs are important as the following result shows:

Proposition 4. *Suppose that the extraction cost function G satisfies properties $G_q > 0$ and $G_{qq} > 0$ for all q and X.*

(i) If $G(0, X) > 0$ for all X, then the extraction stage ends at the time instant $T > 0$, which satisfies either condition

$$q(T)\left[\frac{G(q(T), X(T))}{q(T)} - G_q(q(T), X(T))\right] = -rV(N(T), T) - \gamma(T)e^{rT} f(N(T)) \tag{19}$$

or conditions

$$q(T) = 0 \quad \textit{and} \quad G(0, X(T)) = -rV(N(T), T) - \gamma(T)e^{rT} f(N(T)). \tag{20}$$

(ii) However, if $G(0, X) = 0$ for all X, then no optimal solution exists with $T > 0$ and $q(t) > 0$ for any t.

Proof. See Appendix A.4. □

This result also holds if it is assumed, like Gaudet *et al.* (1995), Roan and Martin (1996) and Cairns (2004) do, that the cost function is independent of the resource left in the ground. When $q(T) > 0$, the average extraction cost is above the marginal extraction cost function at the end of the extraction operation. Therefore it is optimal during some interval at the end of the extraction stage for the operator to make a loss compared to the case in which the extraction is stopped when average cost equals the marginal cost. The amount of loss at the shut-down date is given by the left-side of (19), and it is balanced with the benefit of postponing the reclamation stage by one time unit.

The second part of this result says that if there are no fixed costs, then there is no economically interesting solution to the problem with acid mine drainage. Intuitively, zero fixed costs imply that the regulator can increase the value of the objective with any increase in the shut-down date, since this action postpones the (negative) reclamation stage value without causing any cost. However, from the practical point of view, fixed costs are positive in mining. For example, the tailings dams containing the wastes must be kept (and are kept in practice) water saturated in order to prevent AMD (Dold, 2014). This is costly even if extraction has stopped but the site is not shut-down. Another example is the compensation to the land owner as in oil production in Alberta (Muehlenbachs, 2015). However, note that no solution exists with the often applied cost function $G(q, X) = qc(X)$, since it does not involve fixed costs.

Of course, there are other possible assumptions about the extraction costs. For example, Eswaran *et al.* (1983), Mumy (1984) and Cairns (2008) assume a concave-convex extraction cost function (marginal cost is U-shaped) that does not depend on X. Recall that with this cost specification a standard exhaustible resource model without a reclamation stage produces as the optimal shut-down date an instant t, which satisfies equation $G(q(t))/q(t) = G_q(q(t))$. This does not hold when the optimal decision of the reclamation stage is taken into account. Instead, Equation (19) holds. The reason is that q cannot decrease below the value that minimizes G_q, since the net price increases at the rate of interest by (A.43) (cost is independent of X). Therefore $q(T) > 0$, and Equation (19) characterizes the shut-down date.

5 When should the monies be collected?

This section investigates producer's response to the optimal regulation, when the payments are collected using a profit tax. This profit tax is not part of the policy mix used to reach the social optimum, but it is analysed to show that the firm is indifferent between paying a unit of money to the trust now or at some later date. What this means for the policy, is that the firm can be required to pay the monies to the trust at the maximum rate from the beginning of the extraction operation onward, which has the benefit that the possible strategic bankruptcy before the monies have been paid is avoided.

The socially optimal values are denoted with the $*$-symbol. The regulation consists of the socially optimal shut-down date for the extraction stage, T^*, of the amount of money the producer must have deposited to the reclamation trust before the extraction ends,

$K(N^*(T^*), T^*)$, and of the pollution tax $\Gamma(t) := -\gamma(t)e^{rt}$ on the pollution generation αq. In addition, the producer is required to collect the monies to the reclamation trust from its profits. Does it matter from the social welfare point of view how the producer collects the funds from its profits, and is any additional regulation needed? Additional regulation, if any is needed, consists of rules that govern the payments. Suppose that for every time instant the producer has to decide a fraction of the net profit that goes to the trust. This fraction is denoted with $\theta(t)$, and the size of the reclamation trust at time t is denoted with $B(t)$. When extraction commences the trust is empty and the deposited monies grow at the rate of interest r. It is assumed that the producer's discount rate equals the regulator's discount rate r, but at the end of this section the other possibility that the producer's rate of time preference differs from r is investigated. For example, it can be greater than r, which seems often to be the case in practice.

The producer's problem is to choose the extraction rate and the fraction of profits going to the trust to maximize the total discounted net profit while taking into account the state constraints. Mathematically the problem is to

$$\max_{\{q(t),\theta(t)\}} \int_0^{T^*} \Big(pq(t) - G(q(t), X(t)) - \Gamma(t)\alpha q(t)\Big)\Big(1-\theta(t)\Big)e^{-rt}\ \mathrm{d}t \tag{21}$$

$$\text{s.t} \quad \dot{X}(t) = -q(t), \quad X(0) = x_0, \quad X(T) \geq 0, \tag{22}$$

$$\dot{B}(t) = rB(t) + \theta(t)\Big(pq(t) - G(q(t), X(t)) - \Gamma(t)\alpha q(t)\Big), \tag{23}$$

$$B(0) = 0, \quad B(T^*) = K(N^*(T^*), T^*), \tag{24}$$

$$q(t) \geq 0, \quad \theta(t) \in [0, 1]. \tag{25}$$

To simplify the notation, denote the instantaneous net profit with $Z(q) := pq - G(q, X) - \Gamma\alpha q$. Define the extraction project as profitable at time t, if it satisfies the inequality $Z(q(t))(1-\theta(t)) > 0$, and define the project as profitable on the extraction stage $[0, T^*]$, if

$$\int_0^{T^*} Z(q(t))\Big(1-\theta(t)\Big)\ \mathrm{d}t > 0. \tag{26}$$

That is, the project is profitable on the extraction stage, if the total discounted net profit is strictly positive. These definitions are applied to show that the shut-down date and the pollution tax together with the requirement that the reclamation funds are collected from the operative profits with a profit tax gives the producer incentives to extract the resource according to the social optimum.

Proposition 5. *Suppose that the producer's time preference matches regulator's preference. A profitable project that collects sufficient amount of money to cover the reclamation costs produces the socially optimal extraction and state variable paths, which is independent of the collection of the reclamation funds.*

Proof. See Appendix A.5. □

This result says, that the given regulation yields the social optimum independently of when the producer collects the monies for the reclamation. The producer is free from the social point of view to collect the monies for example during some interval at the beginning, or during some interval at the end of the extraction stage. This is essentially due to Lemma A.4 presented in Appendix A.5, which states that the shadow value of money in the reclamation trust decreases at the rate of interest. The producer is therefore indifferent between putting one unit of money to the trust and keeping it.

In practice the producer's rate of time preference is higher than the regulator's. To see what kind of tax rule is needed for social optimum with the profit tax, let $\sigma(t)$ be any time dependent rate of time preference for the producer and suppose that it differs from r. The above regulation is not sufficient to yield the socially optimal allocation. However, a slight adjustment to the profit tax rule recovers the social optimum.

Proposition 6. *Suppose that the producer's rate of time preference $\sigma(t)$ is different from the regulator's time preference r. Then a profitable project that collects sufficient amount of money to cover the reclamation costs produces the socially optimal extraction and state variable paths, if the fractional profit tax is replaced with a profit tax defined with*

$$\chi(t) := (1 - \theta(t))e^{(\sigma(t)-r)t}.$$

The social optimum is obtained independently of the collection of the reclamation funds.

Proof. The Hamiltonian in the producer's problem with time preference σ and tax rule $\chi(t) = (1-\theta(t))e^{(\sigma-r)t}$ equals the Hamiltonian H in (A.55)-(A.56). This implies that the results in Lemma A.4 and in Proposition 5 hold. □

This tax rule effectively changes the producer's rate of time preference to match the social rate of time preference and therefore the present value of profit tax for the producer is the same as for the regulator. Again, if this rule is used, the producer is free to make the payments at any time intervals it desires and the social optimum is achieved.

As stated in the beginning of this section, the indifference of the firm to pay money to the trust now or at some later date, means that the regulator can require the firm to pay the monies to the trust at the maximum rate at the beginning of the extraction operation (that is, at rate $\theta(t) = 1$ from $t = 0$ onward as long as it takes to satisfy the terminal condition (24)). This requirement has two positive qualities: first, the social optimum is achieved; second, as the monies are collected at the beginning of the extraction stage, the firm's incentives to declare bankruptcy before paying the reclamation bill are diminished.

6 Conclusions

This study analyses the final stages of polluting exhaustible resource production, namely, the extraction and reclamation stages. The socially optimal extraction and reclamation are characterized, and the regulation that induces the producer to behave in the socially optimal way is investigated. The key to analyse the model is to study the value function of the reclamation stage problem. Doing this enables to add this value function as the scrap value function for the extraction stage problem to find out what is the optimal regulation. The optimal regulation calls for a pollution tax (although other instruments may also be feasible), an optimal shut-down date and a requirement that the producer must deposit sufficient funds to cover the eventual reclamation costs.

The results dictate a formula for the size of the payment by the firm to enable the socially optimal reclamation operations. There is no need (in this model) to require the producer to pay the funds at the beginning of the extraction stage – indeed, from the social optimum point of view, the producer is free to choose when to pay the funds. In practice, though, it must be remembered that the firm may have incentives to postpone the deposits and try to drive the operation to bankruptcy before the socially optimal shut-down date in order to avoid paying the reclamation costs. This incentive is diminished, if the regulation is supplemented by a requirement that the firm must pay the monies to the reclamation trust at the beginning of extraction operation.

A Appendix

A.1 Proof of Proposition 1

First, a few helpful lemmas are presented.

Lemma A.1. $N_n(t; n_T, T) > 0$.

Proof. The proof is the same as in Lappi (2018) apart from the notation and is presented in Section 2 Supplementary material for completeness. □

Lemma A.2.

(*i*) *If* $\tau^* > T$ *on some open* n*-interval* (n_T^1, n_T^2)*, then* $\tau_n(n_T, T) > 0$ *and* $v_n(n_T, T) = 0$ *on that interval.*

(*ii*) *If* $\tau^* = T$ *on some open* n*-interval, then* $\tau_n(n_T, T) = 0$ *and* $v_n(n_T, T) = 1$ *on that interval.*

Proof. (*i*) The proof is the same as in Lappi (2018) apart from the notation and is presented in Section 3 of Supplementary material for completeness.

(*ii*) Suppose that $\tau^* = T$ on some open n-interval. Clearly $\tau_n(n_T, T) = 0$. Furthermore, since $v(n_T, T) = n_T$ for $\tau^* = T$, $v_n(n_T, T) = 1$ for $\tau^* = T$. □

Proof of Part (*i*):

Proof. The partial derivative of K with respect to n is

$$\begin{aligned} K_n(n_T, T) = {} & C'(v(n_T, T))v_n(n_T, T)e^{-r(\tau(n_T,T)-T)} \\ & - r\tau_n(n_T, T)C(v(n_T, T))e^{-r(\tau(n_T,T)-T)}. \end{aligned} \tag{A.1}$$

Note that for all $n_T \in (n_T^1, n_T^2)$, $\tau^* > T$, which implies by Part (*i*) of Lemma A.2 that $v_n(n_T, T) = 0$. Hence Equation (A.1) simplifies to

$$K_n(n_T, T) = -r\tau_n(n_T, T)C(v(n_T, T))e^{-r(\tau(n_T,T)-T)}. \tag{A.2}$$

This is strictly negative since $\tau_n(n_T, T) > 0$ by Part (*i*) of Lemma A.2. □

Proof of Part *(ii)*:

Proof. The reclamation date is T for all $n_T \in (0, n_T^1)$. This implies by Lemma A.2 that $\tau_n(n_T, T) = 0$ and $v_n(n_T, T) > 0$. Then

$$K_n(n_T, T) = C'(v(n_T, T))v_n(n_T, T)e^{-r(\tau(n_T,T)-T)} > 0 \tag{A.3}$$

by Equation (A.1). $\square$

Proof of Part *(iii)*:

Proof. An envelope theorem (Corollary 6.1.1 in Carter (2001); Milgrom and Segal (2002)) is applied to optimization problem (3). This gives

$$\begin{aligned} S_n(n_T, T) &= V_n(n_T, T)e^{-rT} && \text{(A.4)} \\ &= \left(\int_T^{\tau(n_T,T)} -D'(N(t; n_T, T))N_n(t; n_T, T)e^{-r(t-T)} \, \mathrm{d}t \right. && \text{(A.5)} \\ &\quad \left. - C'(N(t; n_T, T))N_n(t; n_T, T)e^{-r(\tau(n_T,T)-T)} \right) e^{-rT}. && \text{(A.6)} \end{aligned}$$

This is strictly negative by Lemma A.1. $\square$

A.2 Proof of Proposition 2

Note that it follows from the identity $v(n, T) = N(\tau(n, T); n, T)$ (recall Equation (8)) that

$$v_T(n, T) = \dot{N}(\tau^*; n, T)\tau_T(n, T) + N_T(\tau^*; n, T). \tag{A.7}$$

Lemma A.3.

$$\tau_T(n_T, T) = 1 \quad \textit{and} \quad v_T(n_T, T) = 0. \tag{A.8}$$

Proof. Suppose that $\tau^* = T$. Clearly $\tau_T(n_T, T) = 1$, since $\tau^* = \tau(n_T, T) = T$ by assumption. Furthermore, since the differential equation in Equation (2) is autonomous, it holds that

$$N(t; n_T, T) = N(t - T; n_T, 0). \tag{A.9}$$

Therefore

$$N_T(t; n_T, T) = \dot{N}(t - T; n_T, 0) \cdot (-1) = -\dot{N}(t; n_T, T) \tag{A.10}$$

for all $t \in [T, \infty)$. This, Equation (A.7) and $\tau_T(n_T, T) = 1$ imply that $v_T(n_T, T) = 0$.

Suppose that $\tau^* > T$. Condition (7) holds as an equality. Differentiating it with respect to variable T one obtains the following equation:

$$D'(N(\tau^*; n_T, T))\Big[\dot{N}(\tau^*; n_T, T)\tau_T(n_T, T) + N_T(\tau^*; n_T, T)\Big] \tag{A.11}$$
$$- rC'(N(\tau^*; n_T, T))\Big[\dot{N}(\tau^*; n_T, T)\tau_T(n_T, T) + N_T(\tau^*; n_T, T)\Big] \tag{A.12}$$
$$- C''(N(\tau^*; n_T, T))f(N(\tau^*; n_T, T))\Big[\dot{N}(\tau^*; n_T, T)\tau_T(n_T, T) + N_T(\tau^*; n_T, T)\Big] \tag{A.13}$$
$$- C'(N(\tau^*; n_T, T))f'(N(\tau^*; n_T, T))\Big[\dot{N}(\tau^*; n_T, T)\tau_T(n_T, T) \tag{A.14}$$
$$+ N_T(\tau^*; n_T, T)\Big] = 0. \tag{A.15}$$

The term $\dot{N}(\tau^*; n_T, T)\tau_T(n_T, T) + N_T(\tau^*; n_T, T)$ forms a common term, and the term, which multiplies it, is strictly negative by Equation (9). Hence,

$$\tau_T(n_T, T) = -\frac{N_T(\tau^*; n_T, T)}{\dot{N}(\tau^*; n_T, T)}, \tag{A.16}$$

which implies that $\tau_T(n_T, T) = 1$ since $N_T(t; n_T, T) = -\dot{N}(t; n_T, T)$ also at $t = \tau^*$. The proof that $v_T(n_T, T) = 0$ is when $\tau^* = T$. □

<u>Proof of Proposition 2:</u>

Proof. Note that by Equation (11) the formula for $K_T(n_T, T)$ is

$$K_T(n_T, T) = C'(v(n_T, T))e^{-r(\tau(n_T, T) - T)}v_T(n_T, T) \tag{A.17}$$
$$+ C(v(n_T, T))\Big(-r(\tau_T(n_T, T) - 1)\Big)e^{-r(\tau(n_T, T) - T)}. \tag{A.18}$$

Lemma A.3 says that $\tau_T(n_T, T) = 1$ and $v_T(n_T, T) = 0$. Applying these to Equation (A.17)-(A.18) implies that $K_T(n_T, T) = 0$.

Differentiating function S in (11) with respect to T gives

$$S_T(n,T) = V_T(n,T)e^{-rT} - rV(n,T)e^{-rT} = e^{-rT}\Big[V_T(n,T) - rV(n,T)\Big] \tag{A.19}$$

$$= e^{-rT}\Bigg[D(N(T;n,T)) - D(N(\tau(n,T);n,T))e^{-r(\tau(n,T)-T)}\tau_T(n,T) \tag{A.20}$$

$$+ \int_T^{\tau(n,T)} -D'(N(t;n,T))N_T(t;n,T)e^{-r(t-T)} - D(N(t;n,T))re^{-r(t-T)}\ \mathrm{d}t \tag{A.21}$$

$$- C'(v(n,T))e^{-r(\tau(n,T)-T)}v_T(n,T) \tag{A.22}$$

$$- C(v(n,T))\Big(-r(\tau_T(n,T)-1)\Big)e^{-r(\tau(n,T)-T)} \tag{A.23}$$

$$- r\left(\int_T^{\tau(n,T)} -D(N(t;n,T))e^{-r(t-T)}\ \mathrm{d}t - C(v(n,T))e^{-r(\tau(n,T)-T)}\right)\Bigg]. \tag{A.24}$$

After canceling terms, one obtains that

$$S_T(n,T) = e^{-rT}\Bigg(D(N(T;n,T)) - D(N(\tau(n,T);n,T))e^{-r(\tau(n,T)-T)}\tau_T(n,T) \tag{A.25}$$

$$+ \int_T^{\tau(n,T)} -D'(N(t;n,T))N_T(t;n,T)e^{-r(t-T)}\ \mathrm{d}t \tag{A.26}$$

$$- C'(v(n,T))e^{-r(\tau(n,T)-T)}v_T(n,T) \tag{A.27}$$

$$+ rC(v(n,T))e^{-r(\tau(n,T)-T)}\tau_T(n,T)\Bigg). \tag{A.28}$$

Let $\tau^* = T$. Then the formula for $S_T(n,T)$, (A.25)-(A.28), evaluated at (n_T,T) gives after using Lemma A.3

$$S_T(n_T,T) = e^{-rT}\Bigg(D(n_T) - D(n_T) \tag{A.29}$$

$$+ \int_T^T -D'(N(t;n_T,T))N_T(t;n_T,T)e^{-r(t-T)}\ \mathrm{d}t \tag{A.30}$$

$$+ rC(v(n_T,T))\Bigg). \tag{A.31}$$

Since $v(n_T,T) = n_T$,

$$S_T(n_T,T) = rC(n_T)e^{-rT} = -rV(n_T,T)e^{-rT}. \tag{A.32}$$

Let $\tau^* > T$. By Lemma A.3 $\tau_T(n_T,T) = 1$ and $v_T(n_T,T) = 0$. Using these, equation

$N_T(t; n_T, T) = -\dot{N}(t; n_T, T)$ and the formula for $S_T(n, T)$ give

$$S_T(n_T, T) = e^{-rT}\Bigg(D(n_T) - D(N(\tau(n_T, T); n, T))e^{-r(\tau(n_T,T)-T)} \tag{A.33}$$

$$+ \int_T^{\tau(n_T,T)} -D'(N(t; n_T, T))(-\dot{N}(t; n_T, T))e^{-r(t-T)}\ \mathrm{d}t \tag{A.34}$$

$$+ rC(v(n_T, T))e^{-r(\tau(n_T,T)-T)}\Bigg). \tag{A.35}$$

Integrating by parts gives

$$S_T(n_T, T) = e^{-rT}\Bigg(D(n_T) - D(N(\tau(n_T, T); n, T))e^{-r(\tau(n_T,T)-T)} \tag{A.36}$$

$$+ \Big/_{T}^{\tau(n_T,T)} D(N(t; n_T, T))e^{-r(t-T)} - \int_T^{\tau(n_T,T)} D(N(t; n_T, T))(-r)e^{-r(t-T)}\ \mathrm{d}t \tag{A.37}$$

$$+ rC(v(n_T, T))e^{-r(\tau(n_T,T)-T)}\Bigg) \tag{A.38}$$

$$= e^{-rT}\Bigg(r\int_T^{\tau(n_T,T)} D(N(t; n_T, T))e^{-r(t-T)}\ \mathrm{d}t \tag{A.39}$$

$$+ rC(v(n_T, T))e^{-r(\tau(n_T,T)-T)}\Bigg) \tag{A.40}$$

$$= -rV(n_T, T)e^{-rT}. \tag{A.41}$$

□

A.3 Proof of Proposition 3

The present value Hamiltonian related to this problem is

$$H(q, X, N, \mu, \gamma, t) = \mu_0\Big(pq - G(q, X) - D(N)\Big)e^{-rt} - \mu q + \gamma(\alpha q - f(N)). \tag{A.42}$$

Theorem 16 in Chapter 6 of Seierstad and Sydsæter (1987) applied.[18] The necessary conditions include the following:

$$H_q = \Big(p - G_q(q, X)\Big)e^{-rt} - \mu + \gamma\alpha \leq 0, \quad q \geq 0, \quad qH_q = 0, \tag{A.43}$$

$$\dot{X} = -q, \tag{A.44}$$

$$\dot{N} = \alpha q - f(N), \tag{A.45}$$

$$\dot{\mu} = G_X(q, X)e^{-rt}, \tag{A.46}$$

$$\dot{\gamma} = D'(N)e^{-rt} + \gamma f'(N), \tag{A.47}$$

$$\mu(T) \geq 0, \quad X(T) \geq 0, \quad \mu(T)X(T) = 0, \tag{A.48}$$

$$\gamma(T) - S_n(N(T), T) \geq 0, \quad N(T) \geq 0, \quad N(T)[\gamma(T) - S_n(N(T), T)] = 0, \tag{A.49}$$

$$\begin{aligned} &H(q(T), X(T), N(T), \mu(T), \gamma(T), T) \\ &+ S_T(N(T), T) \begin{cases} \leq 0, & \text{if } T = 0, \\ = 0, & \text{if } T > 0. \end{cases} \end{aligned} \tag{A.50}$$

(i) Suppose that $q(T) > 0$. Then $\Big(p - G_q(q(T, X(T)))\Big)e^{-rT} - \mu(T) + \gamma(T)\alpha = 0$ by (A.43), and

$$\begin{aligned} &\Big(pq(T) - G(q(T), X(T)) - D(N(T))\Big)e^{-rT} - \mu(T)q(T) + \gamma(T)(\alpha q(T) - f(N(T))) \\ &\qquad = -S_T(N(T), T). \end{aligned} \tag{A.51}$$

by (A.50). Combining these equations and simplifying yields the following equation:

$$\begin{aligned} &q(T)\left[\frac{G(q(T), X(T))}{q(T)} - G_q(q(T), X(T))\right]e^{-rT} + e^{-rT}D(N(T)) + \gamma(T)f(N(T)) \\ &\qquad = -rV(N(T), T)e^{-rT}. \end{aligned} \tag{A.52}$$

The desired equation follows from this.

(ii) Suppose that $q(T) = 0$. The desired equation is obtained readily from condition (A.50).

A.4 Proof of Proposition 4

(i) Equations (19) and (20) follow from Part (i) of Proposition 3 since pollution damage is absent (set $D(N(T)) = 0$).

[18] It is assumed that $\mu_0 = 1$.

(ii) Suppose $G(0, X) = 0$ for all X. Assume $q(T) > 0$. Then

$$e^{-rT}\Big(G_q(q(T), X(T))q(T) - G(q(T)X(T))\Big) = \\ - S_T(N(T), T) + \gamma(T)f(N(T)) < 0, \qquad (A.53)$$

This is not possible with the assumed cost structure. Assume $q(T) = 0$. Then (A.50) implies that $0 = -S_T(N(T), T) + \gamma(T)f(N(T))$, since pollution damages are absent. But

$$- S_T(N(T), T) + \gamma(T)f(N(T)) = rV(N(T), T)e^{-rT} + \gamma(T)f(N(T)) \leq 0, \qquad (A.54)$$

since $V \leq 0$, $\gamma(T) < 0$ and $f \geq 0$. Therefore $f(N(T)) = 0$ and $V(N(T), T) = 0$. These equations are true only if $N(T) = 0$. This implies $q = 0$ for all t.

A.5 Proof of Proposition 5

The present value Hamiltonian related is

$$H(q, \theta, X, B, \mu, \eta, t) = \Big(pq - G(q, X) - \Gamma\alpha q\Big)(1 - \theta)e^{-rt} \qquad (A.55)$$

$$- \mu q + \eta\Big(rB + \theta(pq - G(q, X) - \Gamma\alpha q)\Big). \qquad (A.56)$$

The necessary conditions include the following:

$$H_q = \Big(p - G_q(q, X) - \Gamma\alpha\Big)(1 - \theta)e^{-rt} - \mu + \eta\theta(p - G_q(q, X) - \Gamma\alpha) \leq 0, \qquad (A.57)$$

$$q \geq 0, \quad qH_q = 0, \qquad (A.58)$$

$$\theta = \begin{cases} 0 & \text{if } - Z(q)e^{-rt} + \eta Z(q) < 0, \\ \text{any } \theta \in [0, 1] & \text{if } - Z(q)e^{-rt} + \eta Z(q) = 0, \\ 1 & \text{if } - Z(q)e^{-rt} + \eta Z(q) > 0, \end{cases} \qquad (A.59)$$

$$\dot{X} = -q, \qquad (A.60)$$

$$\dot{B} = rB + \theta\Big(pq - G(q, X) - \Gamma\alpha q\Big), \qquad (A.61)$$

$$\dot{\mu} = G_X(q, X)e^{-rt}, \qquad (A.62)$$

$$\dot{\eta} = -r\eta, \qquad (A.63)$$

$$\mu(T^*) \geq 0, \quad X(T^*) \geq 0, \quad \mu(T^*)X(T^*) = 0, \qquad (A.64)$$

$\eta(T^*)$ "has no condition". (A.65)

The following lemma is used to show that these conditions match the regulator's necessary conditions:

Lemma A.4. *Equation $\eta(t) = e^{-rt}$ holds for a profitable project that collects sufficient amount of money to cover the reclamation costs.*

Proof. Let the project be profitable and suppose that the terminal condition in (24) is met. Equation (A.63) implies that $\eta(t) = ke^{-rt}$ for some real number k. If $\theta \in (0, 1)$ maximizes the Hamiltonian, then $k = 1$ by (A.59). Suppose that the maximizer is on the boundary of $[0, 1]$. Since the project has to collect sufficient amount on money to the bank account, $\theta = 1$, $q > 0$ and $Z(q) > 0$ on some interval. Since $\theta = 1$, inequality $Z(q)(ke^{-rt} - e^{-rt}) \geq 0$ holds on that interval. This implies that $ke^{-rt} - e^{-rt} \geq 0$ since $Z(q) > 0$. Hence $k \geq 1$. Similarly, since the project is profitable, there exists an interval on which $\theta = 0$, $q > 0$ and $Z(q) > 0$. On that interval inequality $Z(q)(ke^{-rt} - e^{-rt}) \leq 0$ holds and therefore $k \leq 1$. Hence $k = 1$. $\square$

Lemma A.4 together with the optimal regulation T^*, $K(N^*(T^*), T^*)$ and $\Gamma(t) = -\gamma(t)e^{rt}$ implies that conditions (A.57), (A.58), (A.60), (A.62) and (A.64) become

$$H_q = \big(p - G_q(q, X)\big)e^{-rt} - \mu + \gamma\alpha \leq 0, \quad q \geq 0, \quad qH_q = 0, \tag{A.66}$$

$$\dot{X} = -q, \quad \dot{\mu} = G_X(q, X)e^{-rt}, \tag{A.67}$$

$$\mu(T^*) \geq 0, \quad X(T^*) \geq 0, \quad \mu(T^*)X(T^*) = 0. \tag{A.68}$$

These conditions match conditions (A.43), (A.44), (A.46) and (A.48) of the social optimum. These are independent of η and B. In addition, the regulation (Γ and T^*) has been chosen such that it satisfies conditions (A.45), (A.47), (A.49) and (A.50). Hence, the producer's choice of extraction coincides with the socially optimal extraction and also produces the socially optimal paths for the state variables X and N. Condition (A.59) guarantees that any path with $\theta \in [0, 1]$ such that the end-point constraint in (24) holds is sufficient for social optimum.

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