#### Geometry and Physics Modeling with Python

#### A. DiCarlo $^{\dagger}$ A. Paoluzzi $^{\dagger}$ G. Scorzelli $^{\ddagger}$

<sup>†</sup>University "Roma Tre", Italy <sup>‡</sup>University of Utah, USA

July 3, 2010

э

 $\begin{array}{l} \mbox{Pyplasm: Plasm} \rightarrow \mbox{Python} \\ \mbox{Modeling with Chain Complexes} \\ \mbox{Chompy: Python} \rightarrow \mbox{Python} \cup \mbox{Erlang} \\ \mbox{Towards Complex Systems Simulations} \end{array}$ 

## Outline

#### 1 Pyplasm: Plasm $\rightarrow$ Python

- Geometric Computing with a functional language
- Python Embedding
- Examples
- 2 Modeling with Chain Complexes
  - Cell complexes vs Chain complexes
  - The Hasse Matrix Representation
- 3 Chompy: Python  $\rightarrow$  Python  $\cup$  Erlang
  - Dataflow streaming of geometry
  - Distributed Computing via Message Passing
- 4 Towards Complex Systems Simulations
  - The ProtoPlasm framework

・ロト ・ 同ト ・ ヨト ・ ヨト

Geometric Computing with a functional language Python Embedding Examples

イロト イポト イヨト イヨト

э

#### Motivations for a new entry

 Python: multi-paradigm language with efficient built-in data structures and simple/effective approach to OO programming.

Geometric Computing with a functional language Python Embedding Examples

#### Motivations for a new entry

- Python: multi-paradigm language with efficient built-in data structures and simple/effective approach to OO programming.
- Python's elegant syntax and dynamic typing, and its interpreted nature, make it ideal for scripting and RAD

Geometric Computing with a functional language Python Embedding Examples

#### Motivations for a new entry

- Python: multi-paradigm language with efficient built-in data structures and simple/effective approach to OO programming.
- Python's elegant syntax and dynamic typing, and its interpreted nature, make it ideal for scripting and RAD
- We wished for easy access to Biopython, NumPy, SciPy, Femhub, and the geometry libraries already interfaced with Python

Geometric Computing with a functional language Python Embedding Examples

(D) (A) (A)

#### Motivations for a new entry

- Python: multi-paradigm language with efficient built-in data structures and simple/effective approach to OO programming.
- Python's elegant syntax and dynamic typing, and its interpreted nature, make it ideal for scripting and RAD
- We wished for easy access to Biopython, NumPy, SciPy, Femhub, and the geometry libraries already interfaced with Python

#### The easiest solution?

```
Pyplasm: Plasm \rightarrow Python
```

Geometric Computing with a functional language Python Embedding Examples

(D) (A) (A) (A) (A)

#### PLaSM (Programming Language for Solid Modeling) Geometric extension of Backus' FL (IBM Yorktown)

(Multidimensional) Geometric Programming at Function Level

 Points, curves, surfaces, solids and higher-dim manifolds

Geometric Computing with a functional language Python Embedding Examples

#### PLaSM (Programming Language for Solid Modeling) Geometric extension of Backus' FL (IBM Yorktown)

- Points, curves, surfaces, solids and higher-dim manifolds
- Domain: Hierarchical polyhedral complexes

Geometric Computing with a functional language Python Embedding Examples

#### PLaSM (Programming Language for Solid Modeling) Geometric extension of Backus' FL (IBM Yorktown)

- Points, curves, surfaces, solids and higher-dim manifolds
- Domain: Hierarchical polyhedral complexes
- Representations:

Geometric Computing with a functional language Python Embedding Examples

(ロ) (同) (三) (三)

#### PLaSM (Programming Language for Solid Modeling) Geometric extension of Backus' FL (IBM Yorktown)

- Points, curves, surfaces, solids and higher-dim manifolds
- Domain: Hierarchical polyhedral complexes
- Representations:
  - BSP trees

Geometric Computing with a functional language Python Embedding Examples

(ロ) (同) (三) (三)

#### PLaSM (Programming Language for Solid Modeling) Geometric extension of Backus' FL (IBM Yorktown)

- Points, curves, surfaces, solids and higher-dim manifolds
- Domain: Hierarchical polyhedral complexes
- Representations:
  - BSP trees
  - Polytopes

Geometric Computing with a functional language Python Embedding Examples

(ロ) (同) (三) (三)

#### PLaSM (Programming Language for Solid Modeling) Geometric extension of Backus' FL (IBM Yorktown)

- Points, curves, surfaces, solids and higher-dim manifolds
- Domain: Hierarchical polyhedral complexes
- Representations:
  - BSP trees
  - Polytopes
  - Hasse graphs

Geometric Computing with a functional language Python Embedding Examples

#### PLaSM (Programming Language for Solid Modeling) Geometric extension of Backus' FL (IBM Yorktown)

(Multidimensional) Geometric Programming at Function Level

- Points, curves, surfaces, solids and higher-dim manifolds
- Domain: Hierarchical polyhedral complexes
- Representations:
  - BSP trees
  - Polytopes
  - Hasse graphs

• Geometric operators:

(ロ) (同) (三) (三)

Geometric Computing with a functional language Python Embedding Examples

#### PLaSM (Programming Language for Solid Modeling) Geometric extension of Backus' FL (IBM Yorktown)

(Multidimensional) Geometric Programming at Function Level

- Points, curves, surfaces, solids and higher-dim manifolds
- Domain: Hierarchical polyhedral complexes
- Representations:
  - BSP trees
  - Polytopes
  - Hasse graphs

- Geometric operators:
  - Affine maps

(ロ) (同) (三) (三)

Geometric Computing with a functional language Python Embedding Examples

#### PLaSM (Programming Language for Solid Modeling) Geometric extension of Backus' FL (IBM Yorktown)

(Multidimensional) Geometric Programming at Function Level

- Points, curves, surfaces, solids and higher-dim manifolds
- Domain: Hierarchical polyhedral complexes
- Representations:
  - BSP trees
  - Polytopes
  - Hasse graphs

- Geometric operators:
  - Affine maps
  - Hierarchical structures

(ロ) (同) (三) (三)

Geometric Computing with a functional language Python Embedding Examples

#### PLaSM (Programming Language for Solid Modeling) Geometric extension of Backus' FL (IBM Yorktown)

(Multidimensional) Geometric Programming at Function Level

- Points, curves, surfaces, solids and higher-dim manifolds
- Domain: Hierarchical polyhedral complexes
- Representations:
  - BSP trees
  - Polytopes
  - Hasse graphs

- Geometric operators:
  - Affine maps
  - Hierarchical structures

Boolean Ops

Geometric Computing with a functional language Python Embedding Examples

#### PLaSM (Programming Language for Solid Modeling) Geometric extension of Backus' FL (IBM Yorktown)

(Multidimensional) Geometric Programming at Function Level

- Points, curves, surfaces, solids and higher-dim manifolds
- Domain: Hierarchical polyhedral complexes
- Representations:
  - BSP trees
  - Polytopes
  - Hasse graphs

- Geometric operators:
  - Affine maps
  - Hierarchical structures
  - Boolean Ops
  - Cartesian products

Geometric Computing with a functional language Python Embedding Examples

#### PLaSM (Programming Language for Solid Modeling) Geometric extension of Backus' FL (IBM Yorktown)

(Multidimensional) Geometric Programming at Function Level

- Points, curves, surfaces, solids and higher-dim manifolds
- Domain: Hierarchical polyhedral complexes
- Representations:
  - BSP trees
  - Polytopes
  - Hasse graphs

- Geometric operators:
  - Affine maps
  - Hierarchical structures
  - Boolean Ops
  - Cartesian products

Minkowski sums

Geometric Computing with a functional language Python Embedding Examples

#### PLaSM (Programming Language for Solid Modeling) Geometric extension of Backus' FL (IBM Yorktown)

(Multidimensional) Geometric Programming at Function Level

- Points, curves, surfaces, solids and higher-dim manifolds
- Domain: Hierarchical polyhedral complexes
- Representations:
  - BSP trees
  - Polytopes
  - Hasse graphs

- Geometric operators:
  - Affine maps
  - Hierarchical structures
  - Boolean Ops
  - Cartesian products
  - Minkowski sums
  - Charts and atlases

Geometric Computing with a functional language Python Embedding Examples

#### PLaSM (Programming Language for Solid Modeling) Geometric extension of Backus' FL (IBM Yorktown)

(Multidimensional) Geometric Programming at Function Level

- Points, curves, surfaces, solids and higher-dim manifolds
- Domain: Hierarchical polyhedral complexes
- Representations:
  - BSP trees
  - Polytopes
  - Hasse graphs

- Geometric operators:
  - Affine maps
  - Hierarchical structures
  - Boolean Ops
  - Cartesian products
  - Minkowski sums
  - Charts and atlases

• *d*-Skeletons,  $0 \le d \le n$ 

Geometric Computing with a functional language Python Embedding Examples

#### PLaSM (Programming Language for Solid Modeling) Geometric extension of Backus' FL (IBM Yorktown)

(Multidimensional) Geometric Programming at Function Level

- Points, curves, surfaces, solids and higher-dim manifolds
- Domain: Hierarchical polyhedral complexes
- Representations:
  - BSP trees
  - Polytopes
  - Hasse graphs

- Geometric operators:
  - Affine maps
  - Hierarchical structures
  - Boolean Ops
  - Cartesian products
  - Minkowski sums
  - Charts and atlases

- *d*-Skeletons,  $0 \le d \le n$
- Convex hulls

Geometric Computing with a functional language Python Embedding Examples

#### PLaSM (Programming Language for Solid Modeling) Geometric extension of Backus' FL (IBM Yorktown)

(Multidimensional) Geometric Programming at Function Level

- Points, curves, surfaces, solids and higher-dim manifolds
- Domain: Hierarchical polyhedral complexes
- Representations:
  - BSP trees
  - Polytopes
  - Hasse graphs

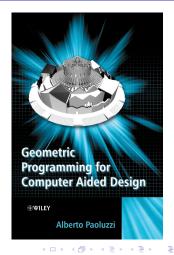
- Geometric operators:
  - Affine maps
  - Hierarchical structures
  - Boolean Ops
  - Cartesian products
  - Minkowski sums
  - Charts and atlases
  - *d*-Skeletons,  $0 \le d \le n$
  - Convex hulls
  - Domain integrals of polynomials

(D) (A) (A) (A) (A)

Geometric Computing with a functional language Python Embedding Examples

# PLaSM (Programming Language for Solid Modeling) Documentation

 Paoluzzi, A., Pascucci, V. & Vicentino, M. (1995).
 Geometric programming: a programming approach to geometric design.
 ACM Trans. Graph. 14 (3), 266–306.



Geometric Computing with a functional language Python Embedding Examples

#### Three general rules to write pyplasm code

**Plasm primitives** 

ALL CAPS

all capital letters

イロン イヨン イヨン イヨン

æ

Geometric Computing with a functional language Python Embedding Examples

#### Three general rules to write pyplasm code

P	lasm	primitives
		p

ALL CAPS

all capital letters

Application

is always unary

VIEW(CUBOID([1,4,9]))

・ロト ・回ト ・ヨト ・ヨト

Э

Geometric Computing with a functional language Python Embedding Examples

#### Three general rules to write pyplasm code

Plasm primitives	
ALL CAPS	all capital letters

ation

is always unary

VIEW(CUBOID([1,4,9]))

Higher-level functions

Unusual but legal in Python

COLOR(RED)(CUBE(1))

・ロト ・ 日 ・ ・ 日 ・ ・ 日 ・ ・

臣

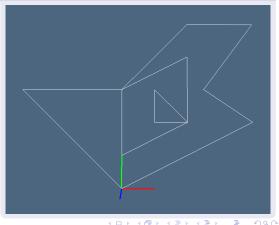
A. DiCarlo, A. Paoluzzi, G. Scorzelli Geometry and Physics Modeling with Python

Geometric Computing with a functional language Python Embedding Examples

# Boolean ops example: polygon filling

```
List = [
[[0,0],[4,2],[2.5,3],
 [4,5], [2,5], [0,3],
 [-3,3],[0,0]],
[[0,3],[0,1],[2,2],
 [2.4].[0.3]].
[[2,2],[1,3],[1,2],
 [2,2]]
```

#### polylines = STRUCT(AA(POLYLINE)(List))



Geometric Computing with a functional language Python Embedding Examples

# Boolean ops example: polygon filling

```
List = [
[[0,0],[4,2],[2.5,3],
[4,5],[2,5],[0,3],
[-3,3],[0,0]],
[[0,3],[0,1],[2,2],
[2,4],[0,3]],
[[2,2],[1,3],[1,2],
[2,2]]
```

#### polygon = SOLIDIFY(polylines)

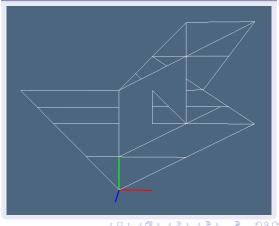


Geometric Computing with a functional language Python Embedding Examples

# Boolean ops example: polygon filling

```
List = [
[[0,0], [4,2], [2.5,3],
 [4,5], [2,5], [0,3],
 [-3,3],[0,0]],
[[0,3],[0,1],[2,2],
 [2.4].[0.3]].
[[2,2],[1,3],[1,2],
 [2,2]]
```

#### cells = SKELETON(1)(polygon)



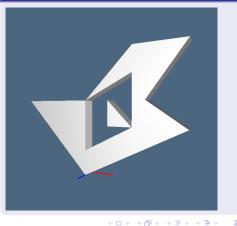
Pyplasm: Plasm  $\rightarrow$  Python Modeling with Chain Complexes Chompy: Python  $\rightarrow$  Python  $\cup$  Erlang Towards Complex Systems Simulations

Geometric Computing with a functional language Python Embedding Examples

# Boolean ops example: polygon filling

```
List = [
[[0,0],[4,2],[2.5,3],
 [4,5], [2,5], [0,3],
 [-3.3].[0.0]].
[[0,3],[0,1],[2.2].
 [2,4],[0,3]],
[[2,2],[1,3],[1.2]]
 [2,2]]
```

#### solid = PROD([polygon, Q(0.5)])



э

Geometric Computing with a functional language Python Embedding Examples

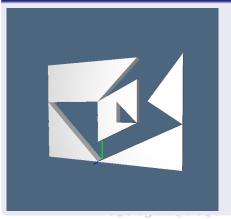
# Boolean ops example: polygon filling

```
List = [
[[0,0],[4,2],[2.5,3],
[4,5],[2,5],[0,3],
[-3,3],[0,0]],
```

```
[[0,3],[0,1],[2,2],
[2,4],[0,3]],
```

```
[[2,2],[1,3],[1,2],
[2,2]]
```

complement = DIFFERENCE([
BOX([1,2,3])(solid), solid ])



Geometric Computing with a functional language Python Embedding Examples

3

#### Coding a new pyplasm primitive

```
from pyplasm import *
def EXPLODE (params):
    sx,sy,sz = params
    def explode0 (scene):
        centers = AA(MED[[1,2,3]))(scene)
        scalings = N(len(centers))(S([1,2,3])([sx,sy,sz]))
        scaledCenters = AA(UK)(AA(APPLY)(TRANS([scalings,AA(MK)(centers)])))
        translVectors = AA(UECTDIFF)(TRANS([scaledCenters, centers]))
        translations = AA(T([1,2,3]))(translVectors)
        return STRUCT(AA(APPLY)(TRANS([translations, scene])))
        return explode0
```

Geometric Computing with a functional language Python Embedding Examples

3

#### The pyplasm EXPL in plain python

```
from pyplasm import *

def EXPLODE (dims):
    dims = [dims] if ISNUM(dims) else dims
    def EXPLODE0 (params):
        params = [params] if ISNUM(params) else params
        def EXPLODE1 (scene):
            centers = [MED(INTSTO(RN(obj)))(obj) for obj in scene]
            scalings = len(centers) * [S(dims)(params)]
            scaledCenters = [UK(APPLY(pair)) for pair in
                 zip(scalings, [MK(p) for p in centers])]
            translvectors = [ VECTDIFF((p,q)) for (p,q) in zip(scaledCenters, centers) ]
            translutions = [ T(dims)(v) for v in translvectors ]
            return STRUCT[ APPLY([t,obj]) for t,obj in zip(translations,scene) ])
            return EXPLODE0
```

```
EXPL = EXPLODE
```

Geometric Computing with a functional language Python Embedding Examples

イロト イヨト イヨト イヨト

æ

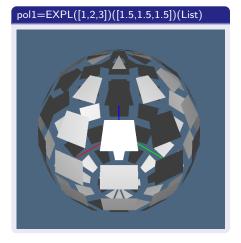
#### **EXPL** examples

List = SPLITCELLS(SPHERE(1)([8,12]))

Geometric Computing with a functional language Python Embedding Examples

#### EXPL examples

List = SPLITCELLS(SPHERE(1)([8,12]))



# pol2 = EXPL(3)(1.5)(List)

A. DiCarlo, A. Paoluzzi, G. Scorzelli

Geometry and Physics Modeling with Python

ヘロア 人間 アイボア 人間 アー

Ξ

Geometric Computing with a functional language Python Embedding Examples

#### Minkowsky sum

#### of cell complexes with a convex cell

polList1 = SPLITCELLS(SK(1)(pol1))

polList2 = SPLITCELLS(pol1)

イロト イポト イヨト イヨト

æ

Geometric Computing with a functional language Python Embedding Examples

## Minkowsky sum

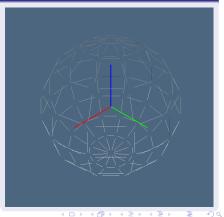
#### of cell complexes with a convex cell

polList1 = SPLITCELLS(SK(1)(pol1))

# EXPL([1.5,1.5,1.5])(polList1))

polList2 = SPLITCELLS(pol1)

#### EXPL([1.5,1.5,1.5])(polList2)



A. DiCarlo, A. Paoluzzi, G. Scorzelli

Geometry and Physics Modeling with Python

Geometric Computing with a functional language Python Embedding Examples

(日) (四) (日) (日) (日)

3

## Minkowsky sum

of cell complexes with a convex cell

def fun (poly): return COMP([ EXPL([1.5,1.5,1.5]), SPLITCELLS, (OFFSET([.1,.1,.1]) ])(poly)

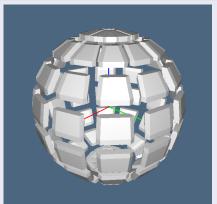
Geometric Computing with a functional language Python Embedding Examples

## Minkowsky sum

of cell complexes with a convex cell

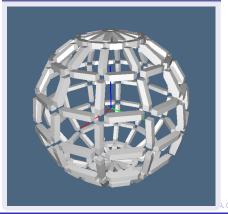
def fun (poly): return COMP([ EXPL([1.5,1.5,1.5]), SPLITCELLS, (OFFSET([.1,.1,.1]) ])(poly)

#### fun(SPHERE(1)([8,12])))



A. DiCarlo, A. Paoluzzi, G. Scorzelli

#### fun(SKELETON(1)(SPHERE(1)([8,12])))



Geometry and Physics Modeling with Python

Geometric Computing with a functional language Python Embedding Examples

(日) (四) (日) (日) (日)

3

#### Cartesian product on cell complexes

ACM/IEEE Symposium on Solid Modeling and Applications, 43–52. ACM Press, 1993.

a = 10\*[1,-5]+[1]; b = MINUS(a); P = PROD

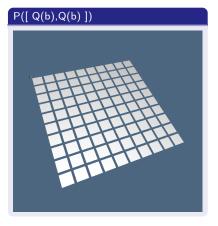
Geometric Computing with a functional language Python Embedding Examples

イロト イポト イヨト イヨト

#### Cartesian product on cell complexes

ACM/IEEE Symposium on Solid Modeling and Applications, 43–52. ACM Press, 1993.

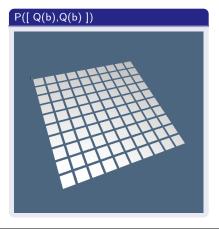
a = 10\*[1,-5]+[1]; b = MINUS(a); P = PROD



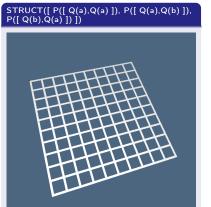
Geometric Computing with a functional language Python Embedding Examples

#### Cartesian product on cell complexes <u>ACM/IEEE Symposium on Solid Modeling and Applications</u>, 43–52. ACM Press, 1993.

a = 10\*[1,-5]+[1]; b = MINUS(a); P = PROD



1 - 1100



÷

Geometric Computing with a functional language Python Embedding Examples

3

#### Cartesian product on cell complexes

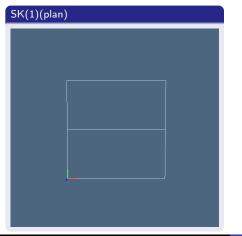
 $\begin{array}{l} \mbox{plan} = \mbox{PROD}([\ Q(10),\ Q([5,5])\ ]) & \mbox{section} = \mbox{MKPOL}([\ [[0,4],[10,4],[10,7],[5,10],[0,7],[0,0],[10,0]],\ [[1,2,3,4,5],[1,2,6,7]],\ [[1],[2]]\ ]) \\ \end{array}$ 

Geometric Computing with a functional language Python Embedding Examples

3

#### Cartesian product on cell complexes

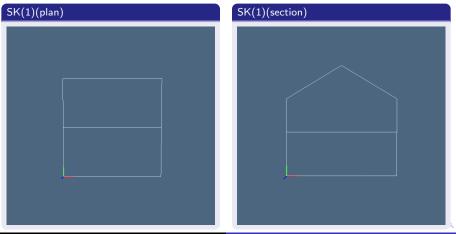
 $\begin{array}{ll} \mathsf{plan} = \mathsf{PROD}([\ \mathsf{Q}(10),\ \mathsf{Q}([5,5])\ ]) & \mathsf{section} = \mathsf{MKPOL}([\ [[0,4],[10,4],[10,7],[5,10],[0,7],[0,0],[10,0]],\ [[1,2,3,4,5],[1,2,6,7]],\ [[1],[2]]\ ]) \end{array}$ 



Geometric Computing with a functional language Python Embedding Examples

#### Cartesian product on cell complexes

 $\begin{array}{ll} \mathsf{plan} = \mathsf{PROD}([\ \mathsf{Q}(10),\ \mathsf{Q}([5,5])\ ]) & \text{section} = \mathsf{MKPOL}([\ [[0,4],[10,4],[10,7],[5,10],[0,7],[0,0],[10,0]],\ [[1,2,3,4,5],[1,2,6,7]],\ [[1],[2]]\ ]) \end{array}$ 



A. DiCarlo, A. Paoluzzi, G. Scorzelli Geometry and Physics Modeling with Python

Geometric Computing with a functional language Python Embedding Examples

イロト イポト イヨト イヨト

3

#### Cartesian product on cell complexes

Intersection of extrusions.

Special case of Cartesian product

Geometric Computing with a functional language Python Embedding Examples

#### Cartesian product on cell complexes

Intersection of extrusions.

Special case of Cartesian product

house = GPROD([[1,2,3],[1,3,2]])([Obj1,Obj2])

# EXPL([1.1,1.1,1.1])(SPLITCELLS(house))

A. DiCarlo, A. Paoluzzi, G. Scorzelli Geometry a

Geometry and Physics Modeling with Python

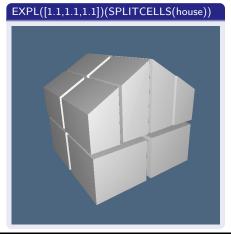
Geometric Computing with a functional language Python Embedding Examples

#### Cartesian product on cell complexes

Intersection of extrusions.

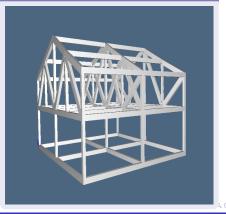
Special case of Cartesian product

house = GPROD([[1,2,3],[1,3,2]])([Obj1,Obj2])



A. DiCarlo, A. Paoluzzi, G. Scorzelli

#### sk1 = OFFSET([.2,.2,.4])(SK(1)(house))



Geometry and Physics Modeling with Python

Geometric Computing with a functional language Python Embedding Examples

(日) (四) (三) (三) (三)

## Scene graph of assemblies

Geometric Computing with a functional language Python Embedding Examples

э

## Scene graph of assemblies

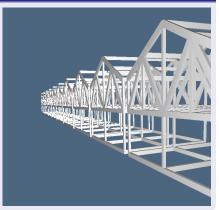
#### str1 = STRUCT(10\*[sk1,T(1)(11)])



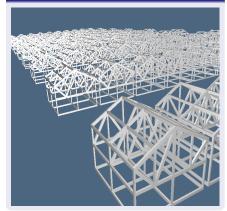
Geometric Computing with a functional language Python Embedding Examples

## Scene graph of assemblies

#### str1 = STRUCT(10\*[sk1,T(1)(11)])



#### str2 = STRUCT(10\*[str1,T(2)(31)])



・ロト ・ 日 ・ ・ ヨ ・ ・ ヨ ・

Cell complexes vs Chain complexes The Hasse Matrix Representation

## Chain complexes and Cochain complexes

- A chain complex is a sequence of linear spaces of *d*-chains, 0 ≤ *d* ≤ *n*, with a sequence of boundary operators, each mapping the space of *d*-chains to the space of (*d* − 1)-chains
- The dual of the chain complex is the cochain complex
- The duals of the boundary operators  $\partial$  are the coboundary operator  $\delta$ , that map the spaces of *d*-cochains to the spaces of (d + 1)-cochains

Cell complexes vs Chain complexes The Hasse Matrix Representation

## Chain complexes and Cochain complexes

- All meshes, say partitioning either the boundary or the interior of a model, and their associated physical fields, are properly represented by a (co)chain complex.
- Such a complex therefore gives a complete discrete representation of any type of field over any type of geometric model.
- Huge geometric structures may be properly and efficiently represented by sparse matrices, and therefore efficiently manipulated through linear computational algebra, in particular by using the last-generation of highly parallel vector GPUs.

 $\begin{array}{l} \mathsf{Pyplasm:} \ \mathsf{Plasm} \to \mathsf{Python} \\ \textbf{Modeling with Chain Complexes} \\ \mathsf{Chompy:} \ \mathsf{Python} \to \mathsf{Python} \cup \mathsf{Erlang} \\ \mathsf{Towards} \ \mathsf{Complex} \ \mathsf{Systems} \ \mathsf{Simulations} \end{array}$ 

Cell complexes vs Chain complexes The Hasse Matrix Representation

## Chain complexes and Cochain complexes

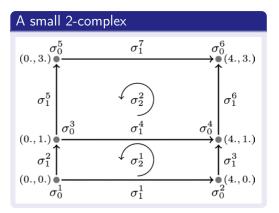
This representation apply to all cell complexes

- The (co)chain representation captures formally and unambiguously all the combinatorial relationships of abstract, geometric, and physical modelling, via the standard topological operators of boundary and coboundary.
- This representation apply to all cell complexes, without restriction of type, dimension, codimension, orientability, manifoldness, etc.
- Furthermore, this approach unifies the geometric and physical computation in a common formal computational structure.

Cell complexes vs Chain complexes The Hasse Matrix Representation

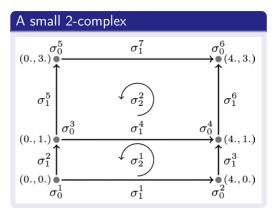
(D) (A) (A)

#### Chains/cochains over a cell complex



Cell complexes vs Chain complexes The Hasse Matrix Representation

#### Chains/cochains over a cell complex

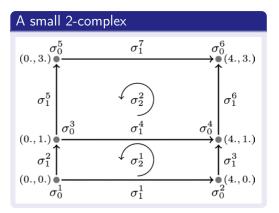


• Real-valued chains attach a signed *d*-measure to *d*-cells

(D) (A) (A)

Cell complexes vs Chain complexes The Hasse Matrix Representation

#### Chains/cochains over a cell complex

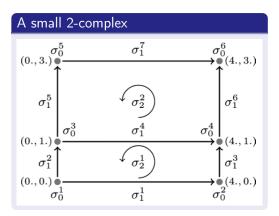


- Real-valued chains attach a signed *d*-measure to *d*-cells
- such as length to 1-cells, area to 2-cells, volume to 3-cells

(D) (A) (A)

Cell complexes vs Chain complexes The Hasse Matrix Representation

#### Chains/cochains over a cell complex



- Real-valued chains attach a signed *d*-measure to *d*-cells
- such as length to 1-cells, area to 2-cells, volume to 3-cells
- they restore part of the geometrical information left out by the purely topological construction of a cell complex

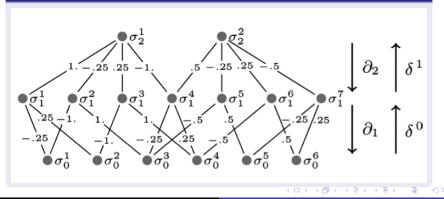
・ロト ・ 日 ・ ・ ヨ ・ ・ ヨ ・

Cell complexes vs Chain complexes The Hasse Matrix Representation

#### Hasse graph Chains/cochains over a cell complex

already used in PyPlasm as the basic data structure

#### Hasse graph of the previous 2-complex

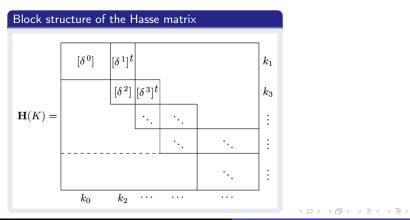


Cell complexes vs Chain complexes The Hasse Matrix Representation

#### The Hasse matrix

A complete representation of the measured incidence between all cells of all dimensions

Discrete Physics using Metrized Chains. 2009. SIAM/ACM Conf. on Geometric and Physical Modeling



A. DiCarlo, A. Paoluzzi, G. Scorzelli Geometry and Physics Modeling with Python

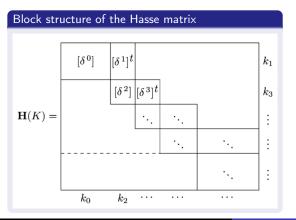
э

Cell complexes vs Chain complexes The Hasse Matrix Representation

#### The Hasse matrix

A complete representation of the measured incidence between all cells of all dimensions

Discrete Physics using Metrized Chains. 2009. SIAM/ACM Conf. on Geometric and Physical Modeling



• H(K) is the sparse bidiagonal matrix of coboundary operators

э

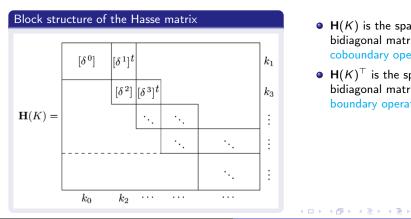
Pyplasm: Plasm  $\rightarrow$  Python Modeling with Chain Complexes Chompy: Python  $\rightarrow$  Python  $\cup$  Erlang Towards Complex Systems Simulations

Cell complexes vs Chain complexes The Hasse Matrix Representation

#### The Hasse matrix

A complete representation of the measured incidence between all cells of all dimensions

Discrete Physics using Metrized Chains. 2009. SIAM/ACM Conf. on Geometric and Physical Modeling



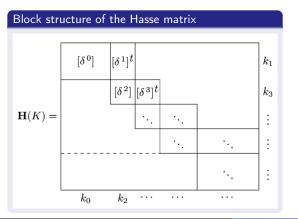
- H(K) is the sparse bidiagonal matrix of coboundary operators
- $\mathbf{H}(K)^{\top}$  is the sparse bidiagonal matrix of boundary operators

Cell complexes vs Chain complexes The Hasse Matrix Representation

#### The Hasse matrix

A complete representation of the measured incidence between all cells of all dimensions

Discrete Physics using Metrized Chains. 2009. SIAM/ACM Conf. on Geometric and Physical Modeling



- H(K) is the sparse bidiagonal matrix of coboundary operators
- H(K)<sup>⊤</sup> is the sparse bidiagonal matrix of boundary operators
- $k_0, k_2, ...$  are the sizes of *d*-skeletons of even dimensions

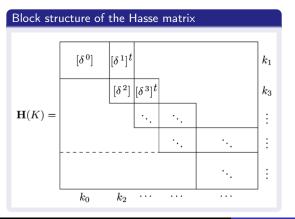
э

Cell complexes vs Chain complexes The Hasse Matrix Representation

#### The Hasse matrix

A complete representation of the measured incidence between all cells of all dimensions

Discrete Physics using Metrized Chains. 2009. SIAM/ACM Conf. on Geometric and Physical Modeling



- H(K) is the sparse bidiagonal matrix of coboundary operators
- H(K)<sup>⊤</sup> is the sparse bidiagonal matrix of boundary operators
- $k_0, k_2, ...$  are the sizes of *d*-skeletons of even dimensions
- $k_1, k_3, \dots$  are the sizes of *d*-skeletons of odd dimensions

Dataflow streaming of geometry Distributed Computing via Message Passing

イロト イポト イヨト イヨト

# $\begin{array}{l} Chompy \\ {\sf Python} \rightarrow {\sf Python} \cup {\sf Erlang} \end{array}$

Chompy: to compute with (co)chain complexes using multi-paradigm and concurrent computer languages (Python and Erlang, respectively).

• —linear and higher order— dimension-independent simplicial complexes

Dataflow streaming of geometry Distributed Computing via Message Passing

イロト イポト イヨト イヨト

# $\begin{array}{l} Chompy \\ {\sf Python} \ \rightarrow \ {\sf Python} \ \cup \ {\sf Erlang} \end{array}$

- —linear and higher order— dimension-independent simplicial complexes
- *d*-complexes of convex cells

Dataflow streaming of geometry Distributed Computing via Message Passing

イロト イポト イヨト イヨト

# $\begin{array}{l} Chompy \\ {\sf Python} \ \rightarrow \ {\sf Python} \ \cup \ {\sf Erlang} \end{array}$

- —linear and higher order— dimension-independent simplicial complexes
- *d*-complexes of convex cells
- Cartesian product of cell complexes

Dataflow streaming of geometry Distributed Computing via Message Passing

イロト イポト イヨト イヨト

# $\begin{array}{l} Chompy \\ {\sf Python} \ \rightarrow \ {\sf Python} \ \cup \ {\sf Erlang} \end{array}$

- —linear and higher order— dimension-independent simplicial complexes
- *d*-complexes of convex cells
- Cartesian product of cell complexes
- skeleton and boundary extraction

Dataflow streaming of geometry Distributed Computing via Message Passing

イロト イポト イヨト イヨト

# $\begin{array}{l} Chompy \\ {\sf Python} \ \rightarrow \ {\sf Python} \ \cup \ {\sf Erlang} \end{array}$

- —linear and higher order— dimension-independent simplicial complexes
- *d*-complexes of convex cells
- Cartesian product of cell complexes
- skeleton and boundary extraction
- various types of local and global cell and (co)chain refinement

Dataflow streaming of geometry Distributed Computing via Message Passing

イロト イポト イヨト イヨト

# $\begin{array}{l} Chompy \\ {\sf Python} \ \rightarrow \ {\sf Python} \ \cup \ {\sf Erlang} \end{array}$

- —linear and higher order— dimension-independent simplicial complexes
- *d*-complexes of convex cells
- Cartesian product of cell complexes
- skeleton and boundary extraction
- various types of local and global cell and (co)chain refinement
- finite integration of polynomials over subcomplexes

Dataflow streaming of geometry Distributed Computing via Message Passing

イロト イポト イヨト イヨト

# $\begin{array}{l} Chompy \\ {\sf Python} \ \rightarrow \ {\sf Python} \ \cup \ {\sf Erlang} \end{array}$

- —linear and higher order— dimension-independent simplicial complexes
- *d*-complexes of convex cells
- Cartesian product of cell complexes
- skeleton and boundary extraction
- various types of local and global cell and (co)chain refinement
- finite integration of polynomials over subcomplexes
- and so on ...

Dataflow streaming of geometry Distributed Computing via Message Passing

(日) (同) (三) (三)

#### Erlang language Concurrent processing done rigth

• Powerful (open source and multi-platform) concurrent functional programming language and runtime system

Dataflow streaming of geometry Distributed Computing via Message Passing

(ロ) (同) (三) (三)

- Powerful (open source and multi-platform) concurrent functional programming language and runtime system
- Purely functional (single assignment, dynamic typing), easy to understand and to debug

Dataflow streaming of geometry Distributed Computing via Message Passing

・ロト ・ 同ト ・ ヨト ・ ヨト

- Powerful (open source and multi-platform) concurrent functional programming language and runtime system
- Purely functional (single assignment, dynamic typing), easy to understand and to debug
- Fits well with multicore CPUs, clusters and SMP architectures

Dataflow streaming of geometry Distributed Computing via Message Passing

- Powerful (open source and multi-platform) concurrent functional programming language and runtime system
- Purely functional (single assignment, dynamic typing), easy to understand and to debug
- Fits well with multicore CPUs, clusters and SMP architectures
- Even hot swapping of programs is supported code can be changed without stopping a system

 $\begin{array}{l} \mathsf{Pyplasm:} \ \mathsf{Plasm} \to \mathsf{Python} \\ \mathsf{Modeling} \ \mathsf{with} \ \mathsf{Chain} \ \mathsf{Complexes} \\ \mathbf{Chompy:} \ \mathsf{Python} \to \mathsf{Python} \cup \mathsf{Erlang} \\ \mathsf{Towards} \ \mathsf{Complex} \ \mathsf{Systems} \ \mathsf{Simulations} \end{array}$ 

Dataflow streaming of geometry Distributed Computing via Message Passing

- Powerful (open source and multi-platform) concurrent functional programming language and runtime system
- Purely functional (single assignment, dynamic typing), easy to understand and to debug
- Fits well with multicore CPUs, clusters and SMP architectures
- Even hot swapping of programs is supported code can be changed without stopping a system
- Developed by Ericsson to support distributed, fault-tolerant, soft real-time, non-stop applications

Dataflow streaming of geometry Distributed Computing via Message Passing

イロト イポト イヨト イヨト

### Disco: Erlang $\cup$ Python

Distributed computing framework developed by Nokia Research Center to solve real problems in handling massive amounts of data

• Disco users start Disco jobs in Python scripts.

Dataflow streaming of geometry Distributed Computing via Message Passing

### Disco: Erlang $\cup$ Python

- Disco users start Disco jobs in Python scripts.
- Jobs requests are sent over HTTP to the master.

Dataflow streaming of geometry Distributed Computing via Message Passing

### Disco: Erlang $\cup$ Python

- Disco users start Disco jobs in Python scripts.
- Jobs requests are sent over HTTP to the master.
- Master is an Erlang process that receives requests over HTTP.

Dataflow streaming of geometry Distributed Computing via Message Passing

(D) (A) (A) (A)

### Disco: Erlang $\cup$ Python

- Disco users start Disco jobs in Python scripts.
- Jobs requests are sent over HTTP to the master.
- Master is an Erlang process that receives requests over HTTP.
- Master launches another Erlang process, worker supervisor, on each node over SSH.

Dataflow streaming of geometry Distributed Computing via Message Passing

(D) (A) (A) (A)

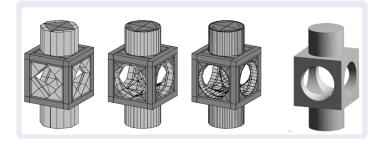
### Disco: Erlang $\cup$ Python

- Disco users start Disco jobs in Python scripts.
- Jobs requests are sent over HTTP to the master.
- Master is an Erlang process that receives requests over HTTP.
- Master launches another Erlang process, worker supervisor, on each node over SSH.
- Worker supervisors run Disco jobs as Python processes.

Dataflow streaming of geometry Distributed Computing via Message Passing

### Progressive BSP (Binary Space Partition) tree Back to Chompy dataflow streaming

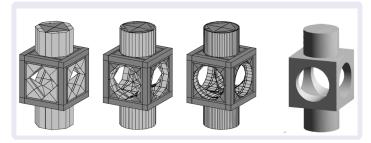
#### 3 types of geometry nodes: FULL, EMPTY and FUZZY cells



Dataflow streaming of geometry Distributed Computing via Message Passing

### Progressive BSP (Binary Space Partition) tree Back to Chompy dataflow streaming

3 types of geometry nodes: FULL, EMPTY and FUZZY cells

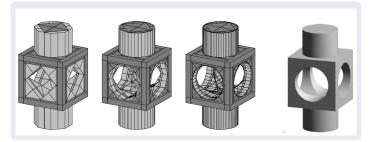


• The FUZZY cells, to be split at next step, are in light gray

Dataflow streaming of geometry Distributed Computing via Message Passing

# Progressive BSP (Binary Space Partition) tree Back to Chompy dataflow streaming

3 types of geometry nodes: FULL, EMPTY and FUZZY cells

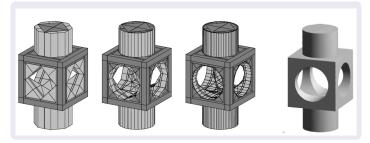


- The FUZZY cells, to be split at next step, are in light gray
- The FULL cells are in dark gray

Dataflow streaming of geometry Distributed Computing via Message Passing

# Progressive BSP (Binary Space Partition) tree Back to Chompy dataflow streaming

3 types of geometry nodes: FULL, EMPTY and FUZZY cells

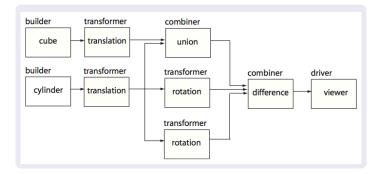


- The FUZZY cells, to be split at next step, are in light gray
- The FULL cells are in dark gray
- The EMPTY cells are not shown (of course :o)

Dataflow streaming of geometry Distributed Computing via Message Passing

э

### Dataflow graph of the generating expression

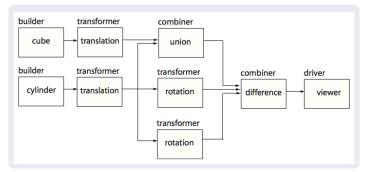


Dataflow streaming of geometry Distributed Computing via Message Passing

イロト 不得下 イヨト イヨト

э

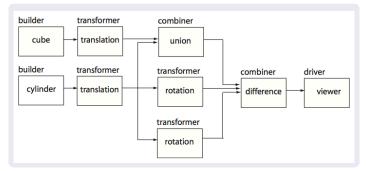
### Dataflow graph of the generating expression



• Dataflow graph of the pyplasm expression that produces the mechanical piece

Dataflow streaming of geometry Distributed Computing via Message Passing

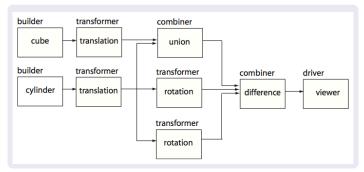
# Dataflow graph of the generating expression



- Dataflow graph of the pyplasm expression that produces the mechanical piece
- The dataflow generation is from source preprocessing

Dataflow streaming of geometry Distributed Computing via Message Passing

# Dataflow graph of the generating expression

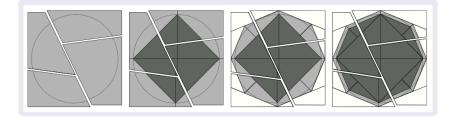


- Dataflow graph of the pyplasm expression that produces the mechanical piece
- The dataflow generation is from source preprocessing
- The various processes will run concurrently in an Erlang environment

Dataflow streaming of geometry Distributed Computing via Message Passing

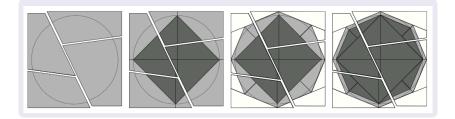
・ロン ・四と ・ヨン ・ヨン

# Progressive BSP: generation of the 2-circle Splitting of both (a) the model and (b) the computation



Dataflow streaming of geometry Distributed Computing via Message Passing

# Progressive BSP: generation of the 2-circle Splitting of both (a) the model and (b) the computation

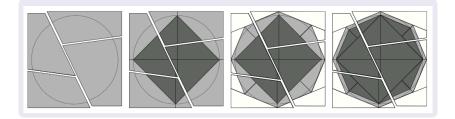


• Dataflow refinement based on progressive splits of convex cells with BSP tree nodes (hyperplanes)

 $\begin{array}{l} \mathsf{Pyplasm:} \ \mathsf{Plasm} \to \mathsf{Python} \\ \mathsf{Modeling} \ \mathsf{with} \ \mathsf{Chain} \ \mathsf{Complexes} \\ \mathbf{Chompy:} \ \mathsf{Python} \to \mathsf{Python} \cup \mathsf{Erlang} \\ \mathsf{Towards} \ \mathsf{Complex} \ \mathsf{Systems} \ \mathsf{Simulations} \end{array}$ 

Dataflow streaming of geometry Distributed Computing via Message Passing

# Progressive BSP: generation of the 2-circle Splitting of both (a) the model and (b) the computation

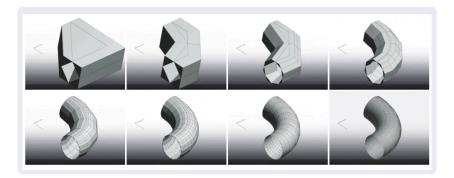


- Dataflow refinement based on progressive splits of convex cells with BSP tree nodes (hyperplanes)
- model partition induced by the BSP subtree closest to the root, to be detailed independently on different computational nodes

Dataflow streaming of geometry Distributed Computing via Message Passing

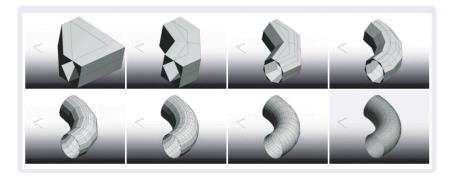
・ロト ・ 日 ・ ・ ヨ ・ ・ ヨ ・

#### Progressive BSP: biquadratic rational B-spline



Dataflow streaming of geometry Distributed Computing via Message Passing

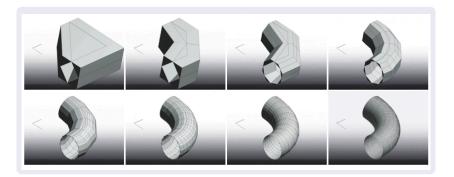
### Progressive BSP: biquadratic rational B-spline



 each refinement is generated by splitting and is contained within the previous cell

Dataflow streaming of geometry Distributed Computing via Message Passing

### Progressive BSP: biquadratic rational B-spline



- each refinement is generated by splitting and is contained within the previous cell
- In this case (approximation of a surface with a solid mesh) all of the cells are either EMPTY or FUZZY, i.e. there are no solid cells

Dataflow streaming of geometry Distributed Computing via Message Passing

Progressive BSP: the Leaning tower of Pisa Parallel solid modeling using BSP dataflow. 2008. *Journal of Comp. Geometry and Appl.* 



Dataflow streaming of geometry Distributed Computing via Message Passing

Progressive BSP: the Leaning tower of Pisa Parallel solid modeling using BSP dataflow. 2008. *Journal of Comp. Geometry and Appl.* 



 Dataflow refinement of convex cells with BSP tree nodes (hyperplanes)

Dataflow streaming of geometry Distributed Computing via Message Passing

Progressive BSP: the Leaning tower of Pisa Parallel solid modeling using BSP dataflow. 2008. *Journal of Comp. Geometry and Appl.* 



- Dataflow refinement of convex cells with BSP tree nodes (hyperplanes)
- model partition induced by the BSP subtree closest to the root, detailed on different computational nodes

A. DiCarlo, A. Paoluzzi, G. Scorzelli Geometry and Physics Modeling with Python

The ProtoPlasm framework

#### The next step:

ProtoPlasm: A Parallel Language for Scalable Modeling of Biosystems. 2009. *Philosophical Transactions of the Royal Society A*, Vol. 366. Issue "The virtual physiological human: building a framework for computational biomedicine I"

(D) (A) (A) (A) (A)

 $\begin{array}{l} \mathsf{Pyplasm:} \ \mathsf{Plasm} \to \mathsf{Python} \\ \mathsf{Modeling} \ with \ \mathsf{Chain} \ \mathsf{Complexes} \\ \mathsf{Chompy:} \ \mathsf{Python} \to \mathsf{Python} \cup \mathsf{Erlang} \\ \mathsf{Towards} \ \mathsf{Complex} \ \mathsf{Systems} \ \mathsf{Simulations} \end{array}$ 

The ProtoPlasm framework

Thanks for your attention !!

イロン イヨン イヨン イヨン

æ