

1. (**Universal Hashing**) Hashing is a technique used to store elements from a large universe  $U = \{0, \dots, m-1\}$  using a small table  $T = \{0, \dots, n-1\}$  using a hash function  $h : U \rightarrow T$  such that the number of collisions are minimized <sup>1</sup>.

Using a fixed hash function might does not work. So, we use a *family* of hash functions  $H$  and then pick a hash function randomly from this family. A hash function family  $H$  is called 2-universal if

$$\forall x, y \in U, x \neq y, \Pr_{h \leftarrow H}[h(x) = h(y)] \leq 1/n.$$

Show how a 2-universal hash function family is useful in hashing and give an example of such a family.

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<sup>1</sup>Assume that collisions are resolved using auxiliary data structure