Total Marks: 10 ***MTL101 Quiz-1** **Time: 25 mins**

Name: **Entry No:** **Group:**

Instructions: Both questions are worth 5 marks each. Show your work.

1. Find all possible solutions of the following system of linear equations by converting the augmented matrix into its RRE form.

$$2x_1 + 3x_2 + x_3 + 2x_4 = 8$$
$$3x_1 + 2x_2 + 2x_3 + x_4 = 8$$
$$5x_2 - x_3 + 4x_4 = 8$$

$$\left(\begin{array}{cccc|c} 2 & 3 & 1 & 2 & 8 \\ 3 & 2 & 2 & 1 & 8 \\ 0 & 5 & -1 & 4 & 8 \end{array}\right) \underset{R_2 \to R_2 - R_1}{\sim} \left(\begin{array}{cccc|c} 2 & 3 & 1 & 2 & 8 \\ 1 & -1 & 1 & -1 & 0 \\ 0 & 5 & -1 & 4 & 8 \end{array}\right)$$

$$\underset{R_1 \leftrightarrow R_2}{\sim} \left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 0 \\ 2 & 3 & 1 & 2 & 8 \\ 0 & 5 & -1 & 4 & 8 \end{array}\right) \underset{R_2 \to R_2 - 2R_1}{\sim} \left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 0 \\ 0 & 5 & -1 & 4 & 8 \\ 0 & 5 & -1 & 4 & 8 \end{array}\right)$$

$$\underset{R_3 \to R_3 - R_2}{\sim} \left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 0 \\ 0 & 5 & -1 & 4 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \underset{R_2 \to \frac{1}{5}R_2}{\sim} \left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 0 \\ 0 & 1 & -1/5 & 4/5 & 8/5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\underset{R_1 \to R_1 + R_2}{\sim} \left(\begin{array}{cccc|c} 1 & 0 & 4/5 & -1/5 & 8/5 \\ 0 & 1 & -1/5 & 4/5 & 8/5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

x_3 & x_4 are independent unknowns & x_1 & x_2 are dependent.

Putting $x_3 = \lambda$, $x_4 = \mu$ gives $x_1 = \frac{8}{5} - \frac{4}{5}\lambda + \frac{1}{5}\mu$

$x_2 = \frac{8}{5} + \frac{1}{5}\lambda - \frac{4}{5}\mu$

2. Find the rank of the following matrix for every $\alpha, \beta \in \mathbb{R}$.

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ \alpha & \alpha & \beta & \beta \end{pmatrix} \underset{R_3 \to R_3 - R_2}{\overset{R_2 \to R_2 - R_1}{\sim}} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ \alpha & \alpha & \beta & \beta \end{pmatrix} \overset{\substack{R_1 \to R_1 - R_2 \\ R_3 \to R_3 - R_2}}{\sim} \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ \alpha & \alpha & \beta & \beta \end{pmatrix}$$

$$\overset{\substack{R_1 \leftrightarrow R_2 \\ R_3 \leftrightarrow R_4}}{\sim} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ \alpha & \alpha & \beta & \beta \\ 0 & 0 & 0 & 0 \end{pmatrix} \overset{R_3 \to R_3 - \alpha R_1}{\sim} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & \beta - \alpha & \beta - \alpha \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\overset{R_1 \to R_1 - R_2}{\sim} \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & \beta - \alpha & \beta - \alpha \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

If $\beta = \alpha$ then rank $= 2$

If $\beta \neq \alpha$ then rank $= 3$

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1. Find all possible solutions of the following system of linear equations by converting the augmented matrix into its RRE form.

$$x_1 + 3x_2 + 2x_3 + 2x_4 = 5$$
$$2x_1 + 2x_2 + 3x_3 + x_4 = 5$$
$$-x_1 + 5x_2 + 4x_4 = 5$$

$$\left(\begin{array}{cccc|c} 1 & 3 & 2 & 2 & 5 \\ 2 & 2 & 3 & 1 & 5 \\ -1 & 5 & 0 & 4 & 5 \end{array}\right) \underset{R_3 \to R_3 + R_1}{\overset{R_2 \to R_2 - 2R_1}{\sim}} \left(\begin{array}{cccc|c} 1 & 3 & 2 & 2 & 5 \\ 0 & -4 & -1 & -3 & -5 \\ 0 & 8 & 2 & 6 & 10 \end{array}\right)$$

$$\overset{R_3 \to R_3 + 2R_2}{\sim} \left(\begin{array}{cccc|c} 1 & 3 & 2 & 2 & 5 \\ 0 & -4 & -1 & -3 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \overset{R_2 \to -\frac{1}{4}R_2}{\sim} \left(\begin{array}{cccc|c} 1 & 3 & 2 & 2 & 5 \\ 0 & 1 & 1/4 & 3/4 & 5/4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\overset{R_1 \to R_1 - 3R_2}{\sim} \left(\begin{array}{cccc|c} 1 & 0 & 5/4 & -1/4 & 5/4 \\ 0 & 1 & 1/4 & 3/4 & 5/4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_3 = \lambda$, $x_4 = \mu$, $x_1 = \frac{5}{4} - \frac{5}{4}\lambda + \frac{1}{4}\mu$, $x_2 = \frac{5}{4} - \frac{1}{4}\lambda - \frac{3}{4}\mu$.

2. Find the rank of the following matrix for every $a, b \in \mathbb{R}$.

$$\begin{pmatrix} a & a & b & b \\ 4 & 3 & 2 & 1 \\ 5 & 4 & 3 & 2 \\ 6 & 5 & 4 & 3 \end{pmatrix} \underset{R_4 \to R_4 - R_3}{\overset{R_3 \to R_3 - R_2}{\sim}} \begin{pmatrix} a & a & b & b \\ 4 & 3 & 2 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \overset{R_4 \to R_4 - R_3}{\sim} \begin{pmatrix} a & a & b & b \\ 4 & 3 & 2 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\overset{R_1 \leftrightarrow R_3}{\sim} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 3 & 2 & 1 \\ a & a & b & b \\ 0 & 0 & 0 & 0 \end{pmatrix} \underset{R_3 \to R_3 - aR_1}{\overset{R_2 \to R_2 - 4R_1}{\sim}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & b-a & b-a \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\underset{R_2 \to -R_2}{\overset{R_1 \to R_1 + R_2}{\sim}} \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & b-a & b-a \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

If $b = a$, rank $= 2$

If $b \neq a$, rank $= 3$

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1. Find all possible solutions of the following system of linear equations by converting the augmented matrix into its RRE form.

$$3x_1 + 2x_2 + x_3 + 2x_4 = 7$$
$$2x_1 + 3x_2 + 2x_3 + x_4 = 7$$
$$5x_1 - x_3 + 4x_4 = 7$$

$$\left(\begin{array}{cccc|c} 3 & 2 & 1 & 2 & 7 \\ 2 & 3 & 2 & 1 & 7 \\ 5 & 0 & -1 & 4 & 7 \end{array}\right) \xrightarrow{R_1 \to R_1 - R_2} \left(\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 0 \\ 2 & 3 & 2 & 1 & 7 \\ 5 & 0 & -1 & 4 & 7 \end{array}\right)$$

$$\xrightarrow[R_3 \to R_3 - 5R_1]{R_2 \to R_2 - 2R_1} \left(\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 0 \\ 0 & 5 & 4 & -1 & 7 \\ 0 & 5 & 4 & -1 & 7 \end{array}\right) \xrightarrow{R_3 \to R_3 - R_2} \left(\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 0 \\ 0 & 5 & 4 & -1 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{R_2 \to \frac{1}{5}R_2} \left(\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 4/5 & -1/5 & 7/5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_1 \to R_1 + R_2} \left(\begin{array}{cccc|c} 1 & 0 & -1/5 & 4/5 & 7/5 \\ 0 & 1 & 4/5 & -1/5 & 7/5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_3 = \lambda$, $x_4 = \mu$, $x_1 = \frac{7}{5} + \frac{1}{5}\lambda - \frac{4}{5}\mu$, $x_2 = \frac{7}{5} - \frac{4}{5}\lambda + \frac{1}{5}\mu$

2. Find the rank of the following matrix for every $p, q \in \mathbb{R}$.

$$\begin{pmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ p & p & q & q \end{pmatrix} \underset{R_3 \to R_3 - R_2}{\overset{R_2 \to R_2 - R_1}{\sim}} \begin{pmatrix} 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ p & p & q & q \end{pmatrix} \underset{R_3 \to R_3 - R_2}{\overset{R_1 \to R_1 - 2R_2}{\sim}} \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ p & p & q & q \end{pmatrix}$$

$$\underset{R_3 \leftrightarrow R_4}{\overset{R_1 \leftrightarrow R_2}{\sim}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ p & p & q & q \\ 0 & 0 & 0 & 0 \end{pmatrix} \overset{R_3 \to R_3 - pR_1}{\sim} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & q-p & q-p \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\overset{R_1 \to R_1 - R_2}{\sim} \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & q-p & q-p \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

If $p = q$ then rank $= 2$

If $p \neq q$ then rank $= 3$

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1. Find all possible solutions of the following system of linear equations by converting the augmented matrix into its RRE form.

$$2x_1 + x_2 + 3x_3 + 2x_4 = 6$$
$$x_1 + 2x_2 + 2x_3 + 3x_4 = 6$$
$$4x_1 - x_2 + 5x_3 = 6$$

$$\left(\begin{array}{cccc|c} 2 & 1 & 3 & 2 & 6 \\ 1 & 2 & 2 & 3 & 6 \\ 4 & -1 & 5 & 0 & 6 \end{array}\right) \underset{R_3 \to R_3 - 4R_2}{\overset{R_1 \to R_1 - 2R_2}{\sim}} \left(\begin{array}{cccc|c} 0 & -3 & -1 & -4 & -6 \\ 1 & 2 & 2 & 3 & 6 \\ 0 & -9 & -3 & -12 & -18 \end{array}\right)$$

$$\overset{R_1 \leftrightarrow R_2}{\sim} \left(\begin{array}{cccc|c} 1 & 2 & 2 & 3 & 6 \\ 0 & -3 & -1 & -4 & -6 \\ 0 & -9 & -3 & -12 & -18 \end{array}\right) \underset{R_2 \to \frac{1}{3}R_2}{\overset{R_3 \to R_3 - 3R_2}{\sim}} \left(\begin{array}{cccc|c} 1 & 2 & 2 & 3 & 6 \\ 0 & 1 & 1/3 & 4/3 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\overset{R_1 \to R_1 - 2R_2}{\sim} \left(\begin{array}{cccc|c} 1 & 0 & 4/3 & 1/3 & 2 \\ 0 & 1 & 1/3 & 4/3 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_3 = \lambda$, $x_4 = \mu$, $x_1 = 2 - \frac{4}{3}\lambda - \frac{1}{3}\mu$

$x_2 = 2 - \frac{1}{3}\lambda - \frac{4}{3}\mu$

2. Find the rank of the following matrix for every $x, y \in \mathbb{R}$.

$$\begin{pmatrix} x & x & y & y \\ 5 & 4 & 3 & 2 \\ 6 & 5 & 4 & 3 \\ 7 & 6 & 5 & 4 \end{pmatrix} \underset{R_4 \to R_4 - R_3}{\overset{R_3 \to R_3 - R_2}{\sim}} \begin{pmatrix} x & x & y & y \\ 5 & 4 & 3 & 2 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\underset{R_4 \to R_4 - R_3}{\overset{\substack{R_1 \to R_1 - xR_3 \\ R_2 \to R_2 - 5R_3}}{\sim}} \begin{pmatrix} 0 & 0 & y-x & y-x \\ 0 & -1 & -2 & -3 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \underset{R_2 \to -R_2}{\overset{R_1 \leftrightarrow R_3}{\sim}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & y-x & y-x \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\overset{R_1 \to R_1 - R_2}{\sim} \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & y-x & y-x \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

If $y = x$ then rank $= 2$

If $y \neq x$ then rank $= 3$.