Minor Test 1 ::: MTL 101 ::: February 2017

Every question is compulsory.
Marks will not be awarded if appropriate arguments are not provided.
Do not waste time describing what is not asked.

Maximum Marks: **20** Maximum Time: **1 hour**

(1) Write down all possible 3×3 real RRE matrices of rank 2. [2]

(2) Prove or disprove the following statements. [$6 = 3 \times 2$]

(a) For any $A, B \in M_{n\times n}(\mathbb{R})$, $W = \{X \in M_{n\times n}(\mathbb{R}) : AXB = BXA\}$ is a subspace of $M_{n\times n}(\mathbb{R})$.

(b) Let A and B be nonempty subsets of a vector space V. Then

$$\text{span}(A) \cap \text{span}(B) = \text{span}(A \cap B).$$

(c) If $\{u, v, w\}$ is linearly independent, then $\{u-2v, 2v-w, w-u\}$ is linearly independent.

(3) (a) Find a basis for $W_1 \cap W_2$, where [3]

$$W_1 = \{(x, y, z, w) \in \mathbb{R}^4 : x + y + z + w = 0, 2x + 3y + 4z + 5w = 0\}$$
$$W_2 = \{(x, y, z, w) \in \mathbb{R}^4 : 4x + 3y + 2z + w = 0, x + 2y + 2z + w = 0\}.$$

(b) Find a basis for span(S), where [2]

$$S = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 1, x + 2y + 3z = 2\}.$$

(4) Find a condition on a, b, c, d so that

$$\{(1, 1, 1, 1), (1, 0, 1, 0), (1, 1, 0, 1), (a, b, c, d)\}$$

is a linearly dependent set in $\mathbb{R}^4$. [3]

(5) Let V be the vector space over the field $\mathbb{R}$ of all real-valued functions on $\mathbb{R}$.

(a) Let $S_a = \{\sin 3x, \sin ax\}$, $a \in \mathbb{R}$. Find all a such that S_a is linearly independent. [2]

(b) Let $\mathcal{B} = \{e^{2x}, \sin 3x, \cos 3x\}$ and $W = \text{span}(\mathcal{B})$. Let $f(x) = e^{2x} + \sin 3x - \cos 3x$.

(i) Show that $f'(x) \in W$, where $f'(x)$ denotes the derivative of $f(x)$. [1]

(ii) Find the coordinate vector $[f'(x)]_{\mathcal{B}}$ by treating $\mathcal{B}$ as an ordered basis for W. [1]

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