MTL101::: Minor Test 2

October, 2014

1. No marks will be awarded if appropriate justification is not provided.
2. Every question is compulsory. Maximum Time is 1 hour.

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1. Let α be a positive real number. Suppose $A = \begin{pmatrix} \alpha & -2 & 1 \\ 4 & -\alpha & 2 \\ 0 & 0 & 1 \end{pmatrix}$.

   (a) Find the characteristic polynomial of A.

   (b) Find all values of α (> 0) for which A is **not** diagonalizable. [$4 = 1 + 3$]

2. (a) Show that if λ is an eigenvalue of a matrix B, then λ^2 is an eigenvalue of B^2.

   (b) Suppose that the characteristic polynomial of B is $x^3+\mu x^2+1$. Write B^{-1} as a polynomial in B.

   (c) For B in part (b), find the trace of B^{-1}. (You may use that for any matrix, its trace is the sum of its eigenvalues.) [$4 = 1 + 1 + 2$]

3. Observe (do not prove) that $\langle \cdot | \cdot \rangle : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ defined by

$$\langle (x_1, y_1) | (x_2, y_2) \rangle := 2x_1x_2 + 5y_1y_2 + 3x_1y_2 + 3x_2y_1$$

   is linear in both the components. Show that this defines an inner product on $\mathbb{R}^2$. [2]

4. Discuss the existence, uniqueness and find the maximal interval of existence of the following initial value problem. [4]

$$y' = \frac{\sqrt{y} \ \sin(\sqrt{y})}{t^2 - 4}, \qquad y(1) = 1.$$

5. Solve the following initial value problem. [3]

$$y' = -\frac{x \tan y}{1 + x^2}, \qquad y(0) = \frac{\pi}{6}.$$

6. Suppose that a hostel houses 500 students. Following a semester break, 10 students returned to the hostel with flu and after 5 days a total of 20 students had flu. Let $y(t)$ be the number of students with flu at time t. Assume that the rate of spread of flu is proportional to

$$y(t)\left(1 - \frac{y(t)}{500}\right).$$

   Find the solution $y(t)$. [3]

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