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**COL202: Discrete Mathematical Structures**  
**Tutorial/Homework: 14**

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1. Discuss Quiz-11 questions.
2. Complete discussion of Tutorial-13 problems in case needed.
3. We will use the following notion of independence of random variables:

**Definition 14.0.1 (Independent random variables)** *Random variables  $X, Y$  on a sample space  $S$  are independent iff*

$$\mathbf{Pr}[X = r_1 \text{ and } Y = r_2] = \mathbf{Pr}[X = r_1] \cdot \mathbf{Pr}[Y = r_2].$$

Use the above definition in the problem below.

$k$  objects are picked independently at random with replacement from a set of  $n$  distinct objects. For  $1 \leq i < j \leq k$ , let  $X_{ij}$  denote the indicator random variable that is 1 if the  $i^{th}$  and  $j^{th}$  objects are the same otherwise 0. Show that for any  $i < j$  and  $p < q$  such that  $(i, j) \neq (p, q)$ , the random variables  $X_{ij}$  and  $X_{pq}$  are independent.

4. **(Coupon-collector problem)** Every time you go to the superstore, you get a random coupon out of  $n$  distinct coupons. What is the expected number of times you have to visit the store to be able to collect all distinct coupons?
5. **(Balls and bins)**  $n$  balls are thrown randomly into  $n$  bins. Let  $E$  be the event that no bin has more than  $\frac{3 \ln n}{\ln \ln n}$  balls. Show that  $\mathbf{Pr}[E] \geq (1 - 1/n)$ .
6. **(Universal Hashing)** Hashing is a technique used to store elements from a large universe  $U = \{0, \dots, m-1\}$  using a small table  $T = \{0, \dots, n-1\}$  using a hash function  $h : U \rightarrow T$  such that the number of collisions are minimized <sup>1</sup>.

Using a fixed hash function might does not work. So, we use a *family* of hash functions  $H$  and then pick a hash function randomly from this family. A hash function family  $H$  is called 2-universal if

$$\forall x, y \in U, x \neq y, \mathbf{Pr}_{h \leftarrow H}[h(x) = h(y)] \leq 1/n.$$

Show how a 2-universal hash function family is useful in hashing and give an example of such a family.

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<sup>1</sup>Assume that collisions are resolved using auxiliary data structure