Instruction: Justify all your statements. Remember that you will be graded on what you write on the answer sheet, NOT on what you intend to write.

Group No:	Entry No:	Name:

Question-1 (3+1+1 marks):

Consider the homogeneous system of equations $AX = \mathbb{0}$ with coefficient from $\mathbb{R}$, where

$$A = \begin{pmatrix} 1 & 5 & 1 & 5 & 1 \\ -1 & -5 & 1 & -1 & 0 \\ 2 & 10 & 1 & 8 & 1 \\ 1 & 5 & 2 & 7 & 0 \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \text{ and } \mathbb{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

(a) By converting A into its row reduced echelon (RRE) matrix find all the free (or independent) variables in the given system.
(b) Find the rank of A.
(c) Write all the solutions of the given system $AX = \mathbb{0}$.

Instruction: Justify all your statements. Remember that you will be graded on what you write on the answer sheet, NOT on what you intend to write.

Group No:	Entry No:	Name:

Question-2 (2+1 marks):

For any positive integer n, let $M_n(\mathbb{R})$ be the vector space of all $n \times n$ matrices over the field $\mathbb{R}$. Let $A^t, Tr(A)$ and $\det(A)$ denote the transpose, the trace and the determinant of A, respectively, for $A \in M_n(\mathbb{R})$.

(a) Is the subset $W_1 = \{A \in M_3(\mathbb{R}) : Tr(A + 2A^t) = 0\}$ a subspace of $M_3(\mathbb{R})$?

(b) Is the subset $W_2 = \{A \in M_2(\mathbb{R}) : \det(A) = 0\}$ a subspace of $M_2(\mathbb{R})$?

Instruction: Justify all your statements. Remember that you will be graded on what you write on the answer sheet, NOT on what you intend to write.

Group No:	Entry No:	Name:

Question-3 (2 marks):

Let $\mathbb{R}[X]$ be the vector space of all polynomials over the field $\mathbb{R}$. Determine whether the subset

$$X = \{1, 1 + X, 1 + X + X^3, 1 + X^3\}$$

of $\mathbb{R}[X]$ is linearly dependent. Justify your answer.