MTL101
LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS
MINOR 1

Total Marks: 20 **Time: One Hour**

1. (4 Marks) Consider the linear system of equations in three unknowns (x, y, z):

$$\begin{aligned} 3x + 2y + az &= 2, \\ 9x + 2y + 3z &= b, \\ 6x + 8y + 5z &= 5. \end{aligned}$$

For what values of $a, b \in \mathbb{R}$, the system has:
(a) No solution.
(b) Unique solution.
(c) Infinitely many solutions.
Also, when solution(s) exists, derive the expression for solution(s).

2. (4 Marks) Find the inverse of the following 3×3 matrix using Gauss-Jordan Method:

$$\begin{pmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 5 \end{pmatrix}.$$

3. (4 Marks) Let W_1 and W_2 be two subspaces of a vector space, then PROVE or DISPROVE the following statements:
(a) $W_1 \cup W_2$ is a subspace.
(b) $W_1 \cap W_2$ is a subspace.

4. (4 Marks) Consider $\mathbb{P}_3$, the vector space of all polynomials of degree atmost 3, over $\mathbb{R}$. Expand the set

$$S = \{x + x^2, x + x^2 + x^3\},$$

to form a basis of $\mathbb{P}_3$.

5. (4 Marks) Prove that the set

$$S = \{e^x, \sin(x), \cos(x)\}$$

is Linearly Independent in vector space of continuous functions from $\mathbb{R}$ to $\mathbb{R}$.