Quiz-1 : Question 1

$$A^{-1}B^{-1}AB = c\,I_{n\times n}$$

$$\Longrightarrow AB = c\,BA \quad \text{—— (1) mark}$$

Taking Determinant

$$\text{Det}(AB) = \text{Det}(c\,BA)$$

$$\text{Det}(c\,BA) = c^n\,\text{Det}(BA) \quad \text{—— (2) marks}$$

$$\text{Det}(AB) = \text{Det}(BA) = \text{Det}(A)\,\text{Det}(B) \neq 0$$

$$\therefore \quad c^n = 1 \quad \text{—— (1) mark}$$

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Ques 2 (3 marks) What is the value of the determinant

$$\det\left(\begin{bmatrix} 2a+4b & 2a+5b & 2a+6b \\ 2a+5b & 2a+6b & 2a+7b \\ 2a+6b & 2a+7b & 2a+8b \end{bmatrix}\right).$$

Sol^n

Let
$$A = \begin{bmatrix} 2a+4b & 2a+5b & 2a+6b \\ 2a+5b & 2a+6b & 2a+7b \\ 2a+6b & 2a+7b & 2a+8b \end{bmatrix}$$

② marks

Applying row operations $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$B = \begin{bmatrix} 2a+4b & 2a+5b & 2a+6b \\ b & b & b \\ 2b & 2b & 2b \end{bmatrix}$$

$$\cancel{= \begin{bmatrix} 2a+4b & 2a+5b & 2a+6b \end{bmatrix}}$$

① mark

We have $\det(A) = \det(B) = 2\det\left(\overbrace{\begin{bmatrix} 2a+4b & 2a+5b & 2a+6b \\ b & b & b \\ b & b & b \end{bmatrix}}^{C}\right)$

Since two rows of matrix C is equal,

$\therefore \det(C) = 0 \Rightarrow \det(A) = 0$

Qn 3

Prove that $W = \{(x, y, z) \in \mathbb{R}^3 : ax+by+cz=d$
is a subspace of $\mathbb{R}^3$ if and only if $d=0$.

Suppose W is a subspace of $\mathbb{R}^3$.

Then $(0, 0, 0) \in W$

$\Rightarrow d = 0.$

$\boxed{1 \text{ Mark}}$

Suppose $d=0$, and let

$$W = \{(x, y, z) \in \mathbb{R}^3 : ax+by+cz=0\}.$$

To prove that W is a subspace,
it is enough to prove that

- $W \neq \phi$
- $u, v \in W, \ \alpha \in \mathbb{R} \Rightarrow \alpha u + v \in W.$

Write the details, correctly.

$\boxed{2 \text{ Marks}}$

If the above condition is clearly written as "if and only if condition" and $d=0$ is derived, then full credit 3 marks will be given. If above criteria is used but only one side implication, namely $d=0$, is derived, then 2 marks.