Major Test::: MAL 101::: November 2013

There are ten questions. Every question is compulsory.
Marks will not be awarded if appropriate arguments are not provided.

Maximum Marks: **50** Maximum Time: **Two hours**

(1) Suppose $\rho_1, \rho_2, \ldots, \rho_n$ are elementary row operations such that $\rho_n \circ \rho_{n-1} \circ \cdots \circ \rho_1(A) = I$ for a square matrix A (here, I is the identity matrix). Describe the inverse of A using these row operations. Justify. Use this to find the inverse of the following matrix (indicate the row operations applied in each step). [5 = 2 + 3]

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

(2) Suppose $\mathcal{P}_4$ is the real vector space $\{a + bx + cx^2 + dx^3 : a, b, c, d \in \mathbb{R}\}$. Suppose $D : \mathcal{P}_4 \to \mathcal{P}_4$ is the derivative operator [5 = 3 + 1 + 1]

$$D(a + bx + cx^2 + dx^3) = b + 2cx + 3dx^2.$$

Let $T = D^2 - D$. Find the null space of T and $\operatorname{rank}(T)$. What are all the eigenvalues of T? Write eigen-space corresponding to each eigenvalue.

(3) Prove the following statements: [5 = 2 + 3]

(a) If W_1 and W_2 are subspaces of a vector space V over $\mathbb{R}$ then $W_1 + W_2$ is the span of $W_1 \cup W_2$.

(b) If $\{u + v, v + w, w + u\}$ is a linearly independent subset of a vector space V then $\{u, v, w\}$ is linearly independent.

(4) Describe Picard iteration method to find approximate solutions of an initial value problem (of a first order ODE). Find the first three iterations y_1, y_2, y_3 of the following IVP using Picard method: [5 = 1 + 2 + 2]

$$y' - xy = x, \quad y(0) = 0.$$

What is y_n for $n \in \mathbb{N}$? Justify it by the method of mathematical induction (i.e., assuming the formula for n, prove it for $n + 1$).

(5) Solve the following second order ODE using the method of undetermined coefficients. [5]

$$y'' + y = 2\cos t, \quad y(0) = 1, \quad y'(0) = 1.$$

PTO

$y(0) = 1$

$y_1 = 1 + \int$

$y_2 = 1 +$

$y = e^x + c$ $y = Ae^x$

$y' = e^x$

$y = y' + c$

$y' - y + c$