DEPARTMENT OF MATHEMATICS

MTL 101: QUIZ 1

20 Marks total

Instructions: Write down all the steps of the solution clearly.

Problem 1 [6 marks] For what value of $k \in \mathbb{R}$, the planes

$$x + y + z = 2, \ 3x + y - 2z = k, \ 2x + 4y + 7z = k + 2.$$

intersect in a line? For that value of k, find the solution set.

Problem 2 [6 marks] Using elementary row operations find the inverse of the matrix (if exists)

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{pmatrix}.$$

Problem 3 [4 marks] On $\mathbb{R}^3 = \{(x_1, x_2, x_3) : x_1, x_2, x_3 \in \mathbb{R}\}$ define

$$(x_1, x_2, x_3) + (y_1, y_2, y_3) = (x_1 + 2y_1, x_2 + y_2, x_3 + y_3) \quad \text{and} \quad \alpha(x_1, x_2, x_3) = (\alpha x_1, \alpha x_2, \alpha x_3).$$

Is $\mathbb{R}^3$ a vector space over $\mathbb{R}$ under the above operations?

Problem 4 [4 marks] Let $V = C([0, 1])$ be the vector space (over $\mathbb{R}$) of all real-valued continuous functions defined on $[0, 1]$. Let W be the set of all real polynomials of odd degrees more than one. Verify whether $W \cup \{0\}$ is a subspace of V.