Question 1: [3+2 marks]

(a) Let V be a finite dimensional vector space over a field F and, let $T : V \to V$ be a linear transformation. Let $\mathrm{Im}(T) = \{T(v) : v \in V\}$. Prove that

$$\mathrm{Im}(T) = \mathrm{Im}(T^2) \quad \text{if and only if} \quad \ker(T) + \mathrm{Im}(T) = V.$$

(b) Consider $\mathbb{R}^4$ as a vector space over $\mathbb{R}$. Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be a linear transformation such that the rank of T is 1 and $T^2 \neq 0$. Then calculate the nullity of T^2.

Solution: Part (a)

[1.5 marks] Proof of $(\Rightarrow)$: Assume $\mathrm{Im}(T) = \mathrm{Im}(T^2)$.
Since $\ker(T), \mathrm{Im}(T)$ are subspaces of V, $\ker(T) + \mathrm{Im}(T) \subset V$.
Enough to prove: $V \subset \ker(T) + \mathrm{Im}(T)$.
Let $v \in V$. Then

$$\begin{aligned} & T(v) \in \mathrm{Im}(T) = \mathrm{Im}(T^2) \\ \Rightarrow \quad & \exists w \in V \text{ such that } T(v) = T^2(w) \\ \Rightarrow \quad & T(v - T(w)) = 0 \\ \Rightarrow \quad & v - T(w) \in ker(T). \end{aligned}$$

Write $v - T(w) = u$, where $u \in \ker(T)$. Then

$$v = u + T(w) \in \ker(T) + \mathrm{Im}(T) \Rightarrow V \subset \ker(T) + \mathrm{Im}(T).$$

[1.5 marks] Proof of $(\Leftarrow)$: Assume $V = \ker(T) + \mathrm{Im}(T)$. Note that $\mathrm{Im}(T^2) \subset \mathrm{Im}(T)$ for any T.
Enough to prove: $\mathrm{Im}(T) \subset \mathrm{Im}(T^2)$.
Let $T(v) \in \mathrm{Im}(T)$ for some $v \in V$.
Since $V = \ker(T) + \mathrm{Im}(T), \exists u \in \ker(T)$ & $T(w) \in \mathrm{Im}(T)$ for some $w \in V$ such that $v = u + T(w)$.
Then $T(v) = T(u) + T^2(w) = 0 + T^2(w) \in \mathrm{Im}(T^2)$.
Therefore, $\mathrm{Im}(T) \subset \mathrm{Im}(T^2)$. Done!

Solution: Part (b):
Step-1: [1 mark] For any T, we have $T^2(V) \subset T(V)$. Then $0 \leq rank(T^2) \leq rank(T) = 1$.
Now $T^2 \neq 0 \Rightarrow rank(T^2) \neq 0$. Therefore, $rank(T^2) = 1$.
Step-2: [1 mark] Apply rank-nullity theorem for $T^2 : \mathbb{R}^4 \to \mathbb{R}^4$,

$$rank(T^2) + nullity(T^2) = 4 \Longrightarrow 1 + nullity(T^2) = 4 \Longrightarrow nullity(T^2) = 3.$$

Question-2

a) Let $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -2 & -4 \end{pmatrix}$. Find A^{-1} (the inverse of A) by using Cayley-Hamilton theorem.

Solution. The given matrix is

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -2 & -4 \end{pmatrix}.$$

The characteristic polynomial of A is

$$p(\lambda) = \det(\lambda I - A) = \begin{vmatrix} \lambda-1 & -1 & -3 \\ -1 & \lambda-3 & 3 \\ 2 & 2 & \lambda+4 \end{vmatrix}$$

$$= \lambda^3 - 14\lambda - 4 \quad \longrightarrow (1)$$

By Cayley-Hamilton theorem, A satisfies p, i.e,

$$p(A) = 0$$

i.e., $A^3 - 14A - 4I = 0$

$\Rightarrow A^2 - 14I - 4A^{-1} = 0$ (multiplying by A^{-1})

$$\Rightarrow A^{-1} = \frac{1}{4}A^2 - \frac{14}{4}I \quad \longrightarrow (1)$$

$$= \frac{1}{4}\begin{pmatrix} -4 & -2 & -12 \\ 10 & 16 & 6 \\ 4 & 0 & 16 \end{pmatrix} - \frac{7}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -1/2 & -3 \\ 5/2 & 4 & 3/2 \\ 1 & 0 & 4 \end{pmatrix} - \begin{pmatrix} 7/2 & 0 & 0 \\ 0 & 7/2 & 0 \\ 0 & 0 & 7/2 \end{pmatrix}$$

$$= \begin{pmatrix} -9/2 & -1/2 & -3 \\ 5/2 & 1/2 & 3/2 \\ 1 & 0 & 1/2 \end{pmatrix}. \longrightarrow ①$$

b) Consider $\mathbb{R}^3$, $\mathbb{R}^4$ as vector spaces over $\mathbb{R}$. Let W be the subspace of $\mathbb{R}^3$ spanned by the subset $\{(1,2,1), (0,1,1), (1,3,2)\}$. Construct a linear transformation $T: \mathbb{R}^4 \longrightarrow \mathbb{R}^3$ such that the range of T equals W.

Solution. Observe that

$$\dim(W) = 2 \longrightarrow ①$$

Let $\{w_1, w_2\}$ be a basis for W & let w_3 & w_4 be any two vectors from W. Also, let $\{v_1, v_2, v_3, v_4\}$ be a basis for $\mathbb{R}^4$. Define T as

$$\left.\begin{array}{l} T(v_1) = w_1 \quad , \quad T(v_2) = w_2 \\ T(v_3) = w_3 \quad \& \quad T(v_4) = w_4 \end{array}\right\} \longrightarrow 2$$

Then extend it linearly to whole of $\mathbb{R}^4$. This T will satisfy the requirements.

Solution of Question 3 (Major-2022)

1. Here, $\frac{dy}{dx} = (x^2+1)y^{2/3}$ with $y(0) = 0$.
   Thus, $f(x,y) = (x^2+1)y^{2/3}$. To see, whether the uniqueness theorem is applicable, first we check that the given function f is Lipschitz or not with respect to y. We show that f is not a Lipschitz continuous function around $(0,0)$ with respect to y. (**1 mark**)
   Consider a rectangle $\mathcal{R}$ around $(0,0)$. Let $f(x,y)$ be Lipschitz continuous in $\mathcal{R}$. Then, there exists a positive real constant L such that, for all $y_1, y_2 \in \mathcal{R}$,

$$\begin{aligned} |f(x,y_1) - f(x,y_2)| &\leq L|y_1 - y_2|, \\ \Rightarrow |(x^2+1)y_1^{2/3} - (x^2+1)y_2^{2/3}| &\leq L|y_1 - y_2|, \\ \Rightarrow \frac{|(x^2+1)y_1^{2/3} - (x^2+1)y_2^{2/3}|}{|y_1 - y_2|} &\leq L, \end{aligned}$$

   Since, the above equation is true $\forall\ y_1$ and $y_2 \in \mathcal{R}$, thus in particular it is true for $y_2 = 0$.

$$\Rightarrow (1+x^2)\frac{|y_1^{2/3}|}{|y_1|} \leq M.$$

   So, when $y_1 \to 0$, $\frac{|y_1^{2/3}|}{|y_1|} \to \infty$. But, M is a fixed finite number. Thus, f is not Lipschitz continuous. (**1 mark**)

   Remark: It is not enough to show that $\frac{\partial f}{\partial y}$ is discontinuous or unbounded, to show the non-applicability of uniqueness theorem (as if the function is Lipschitz and $\frac{\partial f}{\partial y}$ is discontinuous, then also uniqueness theorem is applicable).

2. Given ODE $(4xy^2+3y)dx + (3x^2y+2x)dy = 0$, we compare this with general equation $Mdx + Ndy = 0$. Thus, we have $M = 4xy^2 + 3y$ and $N = 3x^2y + 2x$.

   Then, $\frac{\partial M}{\partial y} = 8xy + 3 \neq \frac{\partial N}{\partial x} = 6xy + 2$, hence ODE is not exact.
   Given an integrating factor $x^p y^q$ for some p, q, $x^p y^q((4xy^2+3y)dx + (3x^2y+2x)dy) = 0$ is exact for some p and q.
   Consider $\bar{M} = 4x^{p+1}y^{2+q} + 3x^p y^{q+1}$ and $\bar{N} = 3x^{2+p}y^{q+1} + 2x^{p+1}y^q$.

(**1 mark**)

Then, $\frac{\partial \bar{M}}{\partial y} = (8+4q)x^{p+1}y^{q+1} + (3q+3)x^p y^q$ and $\frac{\partial \bar{N}}{\partial x} = (6+3p)x^{p+1}y^{q+1} + (2p+2)x^p y^q$.
On equating $\frac{\partial \bar{M}}{\partial y} = \frac{\partial \bar{N}}{\partial y}$, we get, $p = 2$ and $q = 1$. Thus x^2y is an integrating factor. (**2 marks**)

$$
\begin{aligned}
\Rightarrow x^2y((4xy^2 + 3y)dx + (3x^2y + 2x)dy) = 0 \\
\Rightarrow (4x^3y^3 + 3x^2y^2)dx + (3x^4y^2 + 2x^3y)dy = 0 \\
\Rightarrow x^4y^3 + y^2x^3 = c
\end{aligned}
$$

(**1 mark**)

Q 4. (a) The general sol^n of $y^{(5)} - y^{(4)} + 2y^{(3)} - 2y^{(2)} = 0$

The char eq^n is $m^5 - m^4 + 2m^3 - 2m^2 = 0$ $\frac{1}{2}$

So, $m = 0, 0, 1, i\sqrt{2}, -i\sqrt{2}$ $\frac{1}{2}$

Thus, $y_h = \underbrace{A + Bt}_{\boxed{1/2}} + \underbrace{Ce^t}_{\boxed{1/2}} + \underbrace{D\cos(\sqrt{2}t) + E\sin(\sqrt{2}t)}_{\boxed{1/2}}$ $1\frac{1}{2}$

(b) The proposed y_p for $y^{(5)} - y^{(4)} + 2y^{(3)} - 2y^{(2)} = t + e^t$ is

$y_p = \underbrace{(P + Qt)t^2}_{\boxed{1/2}} + \underbrace{Rte^t}_{\boxed{1/2}}$ 1

Substitute y_p in the given eq^n $\frac{1}{2}$

& compare coefficients $\frac{1}{2}$

and get $P = -\frac{1}{4}$ $\frac{1}{2}$

$Q = -\frac{1}{12}$ $\frac{1}{2}$

$R = \frac{1}{3}$ $\frac{1}{2}$

Question 5

(a) $t^2y'' - 5ty' + 9y = 0$

This is a Cauchy-Euler equation.

Putting $y = t^m$, we get the characteristic eqn:

$m(m-1) - 5m + 9 = 0 \iff (m-3)^2 = 0$ [1 mark]

$\therefore$ $m = 3$ is a repeated root.

Hence, $y_1 = t^3$ [½ mark] and $y_2 = t^3 \ln|t|$ [½ mark] are two linearly independent solutions.

$\therefore$ The general soln. is $\boxed{y = c_1t^3 + c_2t^3\ln|t|}$

(b) $t^2y'' - 5ty' + 9y = t^4$

i.e. $y'' - \frac{5}{t}y' + \frac{9}{t^2}y = t^2$

Comparing with $y'' + p(t)y' + q(t)y = r(t)$,

$r(t) = t^2$ [½ mark]

By part (a), $y_1 = t^3$, $y_2 = t^3\ln|t|$ are two lin. indep. solns of the corresponding homogeneous ODE.

$y_p = u_1y_1 + u_2y_2$, where [1 mark for formula]

$u_1' = \frac{W_1}{W} = \frac{-y_2\, r(t)}{W}$

$u_2' = \frac{W_2}{W} = \frac{y_1\, r(t)}{W}$

$W = y_1y_2' - y_1'y_2 = t^3(t^2 + 3t^2\cancel{\ln|t|}) - \cancel{3t^2 \cdot t^3\ln|t|} = t^5$ [1 mark]

$\therefore$ $u_1' = \frac{-t^3\ln|t| \cdot t^2}{t^5} = -\ln|t| \Rightarrow u_1 = -t\ln|t| + t$ [1 mark]

$u_2' = \frac{t^3 \cdot t^2}{t^5} = 1 \Rightarrow u_2 = t$

$\therefore$ $y_p = (\cancel{-t\ln|t|} + t)t^3 + \cancel{t \cdot t^3\ln|t|} = t^4$ [½ mark]

Hence $\boxed{y_p = t^4}$ is a particular soln.

Remark: If $r(t)$ is taken as t^4 instead of t^2 and rest of the calculations are done correctly using that then 2.5 marks given for part (b)

Solution of Question 6 (Major)

July 4, 2022

Question 6:[6 marks]
Solve the following initial value problem (IVP) using the Laplace transform:

$$y'' + 7y' + 12y = u(t-2) + \delta(t-3); \qquad y(0) = 1, y'(0) = 3.$$

Here δ denotes the Dirac delta function and u denotes the Heaviside function.

Solution:

STEP 1: Taking Laplace transform of the given ODE.

$$\mathcal{L}\{y''\}(s) + 7\mathcal{L}\{y'\}(s) + 12\mathcal{L}(\{y\}(s) = \mathcal{L}\{u(t-2)\}(s) + \mathcal{L}\{\delta(-3)\}(s).$$

STEP 2: Simplifying the above equation and computing $\mathcal{L}\{y\}(s)$ using the following facts:

1) $\mathcal{L}\{y''\}(s) = s^2\mathcal{L}y(s) - sy(0) - y^{'}(0)$

2) $\mathcal{L}\{y'\}(s) = s\mathcal{L}\{y\}(s) - y(0)$

3) $\mathcal{L}\left\{u(t-a)\right\}(s) = \dfrac{e^{-as}}{s}$

4) $\mathcal{L}\left\{\delta(t-a)\right\}(s) = e^{-as}$

This gives

$$\begin{aligned} s^2\mathcal{L}y(s) - sy(0) - y^{'}(0) + 7\left(s\mathcal{L}\{y\}(s) - y(0)\right) + 12\mathcal{L}\{y\}(s) &= \frac{e^{-2s}}{s} + e^{-3s} \\ (s^2+7s+12)\mathcal{L}\{y\}(s) - s - 10 &= \frac{e^{-2s}}{s} + e^{-3s} \end{aligned}$$

whence

$$\mathcal{L}\{y\}(s) = \frac{e^{-2s}}{s(s+4)(s+3)} + \frac{e^{-3s}}{(s+4)(s+3)} + \frac{s+10}{(s+4)(s+3)}.$$

STEP 3: Taking the inverse Laplace transform.

$$\begin{aligned} y(t) = \mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s(s+4)(s+3)}\right\} + \mathcal{L}^{-1}\left\{\frac{e^{-3s}}{(s+4)(s+3)}\right\} \\ + \mathcal{L}^{-1}\left\{\frac{s+10}{(s+4)(s+3)}\right\}. \end{aligned}$$

Using

$$\mathcal{L}^{-1}\left\{e^{-as}F(s)\right\}(t) = u(t-a)\mathcal{L}^{-1}\left\{F(s)\right\}(t-a) = u(t-a)f(t-a)$$

we get

$$\begin{aligned} y(t) &= u(t-2)\mathcal{L}^{-1}\left\{\frac{1}{12s} + \frac{1}{4(s+4)} - \frac{1}{3(s+3)}\right\}(t-2) \\ &+ u(t-3)\mathcal{L}^{-1}\left\{\frac{1}{s+3} - \frac{1}{s+4}\right\}(t-3) + \mathcal{L}^{-1}\left\{\frac{7}{s+3} - \frac{6}{s+4}\right\} \\ &= u(t-2)\left(\frac{1}{12} + \frac{1}{4}e^{-4(t-2)} - \frac{1}{3}e^{-3(t-2)}\right) + u(t-3)\left(e^{-3(t-3)} - e^{-4(t-3)}\right) \\ &+ 7e^{-3t} - 6e^{-4t} \end{aligned}$$

MTL 101, MAJOR, PROBLEM 7

Let $\mathcal{L}$ and $\mathcal{L}^{-1}$ denote the Laplace and the Laplace inverse.

(a) Using the convolution property of the Laplace transform find $\mathcal{L}^{-1}\left(\frac{4}{(s^2+4s+8)^2}\right)$

Solution. Let $\mathcal{L}(f(t) = F(s) = \frac{4}{(s^2+4s+8)^2} = \frac{4}{((s+2)^2+2^2)^2}$. Then

$$\mathcal{L}(e^{2t}f(t)) = F(s-2) = \left(\frac{2}{s^2+2^2}\right)^2$$

Thus

$$e^{2t}f(t) = \mathcal{L}^{-1}\left(\frac{2}{s^2+2^2}\right)^2 = g(t) * g(t)$$

where $g(t) = \sin 2t$ and $\mathcal{L}(\sin 2t) = \frac{2}{s^2+2^2}$. Thus

$$\begin{aligned} e^{2t}f(t) = g(t) * g(t) &= \int_0^t \sin 2\tau sin2(t-\tau)d\tau \\ &= \int_0^t 1/2\left(\cos(2\tau - 2(t-\tau)) - \cos(2\tau + 2(t-\tau))\right)d\tau \\ &= 1/2\int_0^t (\cos(4\tau - 2t) - \cos 2t)d\tau \\ &= \frac{1}{4}\sin 2t - \frac{1}{2}t\cos 2t \end{aligned}$$

Thus $f(t) = \frac{1}{4}e^{-2t}\sin 2t - \frac{1}{2}e^{-2t}t\cos 2t$ □

(b) Show that $\mathcal{L}\left(\int_0^t f(\tau)d\tau\right)(s) = \frac{1}{s}\mathcal{L}(f)(s)$

Solution. Write $g(t) = \int_0^t f(\tau)d\tau$. Then $g'(t) = f(t)$ and $g(0) = 0$.

$$\begin{aligned} \mathcal{L}(g') &= \mathcal{L}(f) \\ s\mathcal{L}(g) - g(0) &= F(s) \\ \mathcal{L}(g) &= \frac{\mathcal{L}(f)(s)}{s} \end{aligned}$$

Remark 0.1. You can also solve the problem using integration by parts or by convolution.

□

Qn-8

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} 7 & -3 & 1 \\ 8 & -3 & 2 \\ -1 & 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad A = \begin{pmatrix} 7 & -3 & 1 \\ 8 & -3 & 2 \\ -1 & 0 & 3 \end{pmatrix}$$

$$|\lambda I - A| = 0 \Rightarrow \begin{vmatrix} \lambda - 7 & 3 & -1 \\ -8 & \lambda + 3 & -2 \\ 1 & 0 & \lambda - 3 \end{vmatrix} = 0 \quad \text{------ } (1)$$

$$\Rightarrow -6 + (\lambda+3) + (\lambda-3)\left[(\lambda-7)(\lambda+3) + 24\right] = 0$$

$$\Rightarrow (\lambda-3)(\lambda-7)(\lambda+3) + 25\lambda - 3 \times 25 = 0$$

$$\Rightarrow (\lambda-3)\left[(\lambda-7)(\lambda+3) + 25\right] = 0$$

$$\Rightarrow (\lambda-3)(\lambda-2)^2 = 0$$

$$\lambda = 2, 2, 3 \quad \text{------ } (1)$$

For $\lambda = 3$

Let $A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 3 \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$$4x_1 - 3x_2 + x_3 = 0$$
$$8x_1 - 6x_2 + 2x_3 = 0$$
$$-x_1 = 0$$

$$\therefore \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ x_2 \\ 3x_2 \end{pmatrix} = x_2 \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \quad \text{------ } (1)$$

For $\lambda = 2$

Let $A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 2 \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$$\left.\begin{aligned} 5x_1 - 3x_2 + x_3 &= 0 \\ 8x_1 - 5x_2 + 2x_3 &= 0 \\ -x_1 + x_3 &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} x_3 &= x_1 \\ 6x_1 &= 3x_2 \end{aligned}$$

$$\therefore \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ 2x_1 \\ x_1 \end{pmatrix} = x_1 \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \quad \text{------ } (1)$$

Now solve $(A - 2I)\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ ---(*), then $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ or any vector $\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$ satisfying (*) ------ (1)

The general soln becomes.

$$x(t) = c_1 e^{3t} \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + c_3 e^{2t} \left[t \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \right] \quad \text{------ } (1)$$

Question 9: [3 marks]
Suppose the function given by the power series $\sum_{n=0}^{\infty} a_n x^n$ is the solution of the following initial value problem (IVP):

$$y'' + xy' + x^2 y = 1 + x; \quad y(0) = 7, y'(0) = 11.$$

Find the values of a_i for $i \leq 5$.

Solution: [Step-1: 1 mark] Write $y(x) = \sum_{n=0}^{\infty} a_n x^n$ is the solution of given IVP, we have

$$y'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1} \text{ and } y''(x) = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}.$$

and

$$\begin{array}{rcl} (\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}) + x(\sum_{n=1}^{\infty} n a_n x^{n-1}) + x^2(\sum_{n=0}^{\infty} a_n x^n) & = & 1 + x \\ \Rightarrow \quad (\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}) + (\sum_{n=1}^{\infty} n a_n x^n) + (\sum_{n=0}^{\infty} a_n x^{n+2}) & = & 1 + x \quad --- (*) \end{array}$$

[Step-2: 1 mark] Using initial conditions we get the following:
(1) $y(0) = 7 \Rightarrow a_0 = 7$.
(2) $y'(0) = 11 \Rightarrow a_1 = 11$.
[Step-3: 1 mark] Comparing the constant term, coefficients of x, x^2, x^3 on both sides of $(*)$, we get
(1) $2a_2 = 1 \Rightarrow a - 2 = 1/2$.
(2) $6a_3 + a_1 = 1 \Rightarrow 6a_3 + 11 = 1 \Rightarrow a_3 = -10/6 = -5/3$.
(3) $12a_4 + 2a_2 + a_0 = 0 \Rightarrow 12a_4 + 1 + 7 = 0 \Rightarrow a_4 = -2/3$.
(4) $20a_5 + 3a_3 + a_1 = 0 \Rightarrow 20a_5 - 5 + 11 = 0 \Rightarrow a_5 = -6/20 = -3/10$.