Problems for Recitation 2

1 Problem: A Geometric Sum

Perhaps you encountered this classic formula in school:

$$1 + r + r^2 + r^3 + \ldots + r^n = \frac{1 - r^{n+1}}{1 - r}$$

First use the well ordering principle, and then induction, to prove that this formula is correct for all real values $r \neq 1$.

Prepare a complete, careful solution. You'll be passing your proof to another group for "constructive criticism"!

2 Problem: Surveyevor

In a new reality TV series called *Surveyevor*, a group of contestants is placed on a small island. Before the series begins, each contestant agrees to have a small purple or red tattoo, in the shape of an eye, applied to the middle of his or her forehead. In all, there are $p \geq 1$ purple eyes and $r \geq 0$ red eyes. However, none of the contestants knows the color of his or her third eye, nor how many total purple and red eyes there are. Furthermore, there are no mirrors and no one is allowed to discuss the tattoos ever. Therefore, everyone knows the colors of everyone else's third eye, but not their own. Good thing, because a contestant who learns that he or she has a purple eye must leave the island at the end of the show that day, and is therefore no longer eligible to win the $1 million cash prize at the end of the show!

The contestants live in uneasy ignorance for several weeks. As time goes on, however, most of them lose their fear of being exiled, adapt to island living, and even make friends with one another. Things are going quite well for the islanders, but as you might suppose, the television audience grows bored, and the show's ratings plummet. When the network threatens to cancel the series, the producer decides she needs to do something, fast: on the next show, to the surprise of the happy islanders, the producer herself appears and convenes a meeting. Very loudly, she proclaims, "I see that at least one person here has a purple eye." Assuming that all the contestants are master logicians, what happens?

Use induction to prove that your conclusion is correct. We suggest a hypothesis $P(n)$ that asserts all of the following are true on day n:

1. If $p > n$, then ____.
2. If $p = n$, then ____.
3. If $p < n$, then ____.

(We leave the task of filling in the blanks to you.)