

Name: \_\_\_\_\_

Entry number: \_\_\_\_\_

There are 2 questions for a total of 10 points.

---

1. Solve the following problems. Show the working in the space provided.

(a) (1 point) What is the last digit of  $7^{100}$ ?

(a) \_\_\_\_\_

(b) (1 point) What is the value of  $(2^{100} \cdot 3^{60}) \pmod{5}$ ?

(b) \_\_\_\_\_

(c) (1 point) What is the remainder when  $\sum_{i=1}^{100} (i)!$  is divided by 9?

(c) \_\_\_\_\_

(d) (2 points) Prove or disprove:  $(2^n + 6 \cdot 9^n)$  is divisible by 7 for every  $n \geq 0$ .

2. (5 points) Consider the following problem:

**HALTING-INPUT:** Given the description  $\langle A \rangle$  of algorithm  $A$ , determine if there is a halting input for  $A$  (that is, there exists an input on which  $A$  halts).

An algorithm  $P$  is said to solve the above problem if  $P(\langle A \rangle)$  halts and outputs 1 when  $A$  has a halting input, and it halts and outputs 0 otherwise.

Prove: There does not exist an algorithm  $P$  that solves the problem HALTING-INPUT.