

Name: _____

Entry number: _____

There are 2 questions for a total of 10 points.

1. Let S be the subset of the set of ordered pairs of integers defined recursively by

Basis step: $(0, 0) \in S$

Inductive step: If $(a, b) \in S$, then $(a + 2, b + 3) \in S$ and $(a + 3, b + 2) \in S$.

Answer the following questions:

- (a) (1 point) List the elements of S produced by the first five applications of the recursive definition.

- (b) (3 points) Use structural induction to show that 5 divides $a + b$ when $(a, b) \in S$.

2. (6 points) Prove or disprove: There exists an integer $k > 1$ such that 7^k ends with 0001 (in its decimal representation).
(Hint that you may or may not use: Consider powers of 7 modulo 10000 and apply the Pigeonhole principle.)