COL202 : Discrete Mathematical Structures

Name: T Pratham
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Group: 2

Tutorial Sheet -10

2.

$$T(n) = 3T(\lfloor n/3 \rfloor) + n$$

$$\Rightarrow T(n) = 3T\left(\frac{n}{3} + \left(\left\lfloor \frac{n}{3} \right\rfloor - \frac{n}{3}\right)\right) + n$$

Here, $a_1 = 3$, $b_1 = \frac{1}{3} < 1$, $h_1(n) = \left\lfloor \frac{n}{3} \right\rfloor - \frac{n}{3}$

and $g(n) = n$
↳ polynomial

$h_1(n) = \left\lfloor \frac{n}{3} \right\rfloor - \frac{n}{3} = -\left\{ \frac{n}{3} \right\}$ (where $\{.\}$ denotes fractional part)

$\therefore 0 \leq |h_1(n)| < 1 \quad \forall\ n \in \mathbb{R}$

i.e. $h_1(n) = O\left(\frac{n}{\log^2 n}\right)$. $\left[\because \lim_{n\to\infty} \frac{\{\frac{n}{3}\}}{\frac{n}{\log^2 n}} = 0 \Rightarrow \{\frac{n}{3}\} = O\left(\frac{n}{\log^2 n}\right)\right]$

$\therefore$ Akra-Bazzi formula applies to $T(n)$.

Now, $a_1 b_1^p = 1 \Rightarrow 3 \cdot \left(\frac{1}{3}\right)^p = 1 \Rightarrow p = 1$

$$\therefore \int_1^n \frac{g(u)}{u^{p+1}}\, du = \int_1^n \frac{u}{u^2}\, du = \int_1^n \frac{1}{u}\, du = \log n$$

Hence, by Akra-Bazzi formula, $T(n) = \Theta\left(n^p + n^p \int_1^n \frac{g(u)}{u^{p+1}}\, du\right)$

$$= \Theta(n + n\log n)$$
$$= \Theta(n\log n)$$

$\therefore T(n) = \Theta(n\log n)$