MTL101: Minor II, Semester I, 2016-17
October 08, 2016

Max Time: 1 hr

Max Marks: 20

Attempt all question.

1. Consider the vector space $\mathbb{R}^4$ over the field $\mathbb{R}$. Let

$$W = \text{span}\{\, v_1 = (1,0,1,1),\ v_2 = (-3,3,7,1),\ v_3 = (-1,3,9,3),\ v_4 = (-5,3,5,-1)\,\}.$$ [3]

Find a subset of $\{v_1, v_2, v_3, v_4\}$ that forms a basis of W.

2. Let $V = \{p \mid p(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4,\ a_i \in \mathbb{R},\ i = 0,1,\ldots,4\}$ be a vector space of all polynomial functions of degree less than or equal to 4 over the field $\mathbb{R}$. Let

$$W_1 = \{p \in V \mid p(1) + p(-1) = 0\} \quad \text{and} \quad W_2 = \{p \in V \mid p'(0) = 0,\ p(1) - p(-1) = 0\},$$

where p' denotes the derivative of p. Determine a basis of $W_1 \bigcap W_2$ containing a polynomial $(1 - x^4)$. Also find the dimension of the subspace $W_1 + W_2$ of V. [5]

3. Let A be a real 4×4 matrix whose row reduced Echelon form is given by

$$\begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

If $c_1 = \begin{pmatrix} -1 \\ 1 \\ -1 \\ 0 \end{pmatrix}$ and $c_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \end{pmatrix}$ are respectively the first and the second columns of A then find the third and the fourth columns of A. [3]

4. Let $A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 2 & 4 & 1 & 0 \\ 3 & 6 & 2 & 0 \\ -4 & -8 & 1 & -6 \end{pmatrix}$, and $S = \{X \in \mathbb{R}^{4\times 4} \mid AX = A\}$.

Determine the row reduced Echelon form of A. Use this form to find rank(s) of every $X \in S$. Completely describe row space(s) of every $X \in S$. [5]

5. Check whether the following vector spaces V with defined maps $\langle\ ,\ \rangle$ forms an inner product space over the specified fields F ? Justify your answer.

   (a) $V = C[-1,1]$, the space of all continuous functions $f : [-1,1] \to \mathbb{R}$, $F = \mathbb{R}$, and $\langle f, g\rangle = \int_0^1 f(t)g(t)\,dt$.

   (b) $V = M_{2\times 2}(\mathbb{R})$, the space of all 2×2 real matrices, $F = \mathbb{R}$, and $\langle A, B\rangle = \text{trace}(AB^t)$. [4]