

Max Marks: 25

Max Time: 1 hr

1. Let the solution of the equations $x_1v_1 + x_2v_2 + x_3v_3 + x_4v_4 = b$ be given by

$$(x_1, x_2, x_3, x_4)^t = (12, 0, 8, 0)^t + (6, 1, 0, 0)^t x_2 + (-2, 0, 5, 1)^t x_4.$$

Is $\{v_1, v_2, v_3, v_4\}$ LI? Is $v_1 \in \text{span}\{v_2, v_3, v_4, b\}$? Justify both answers. [4]

2. State TRUE or FALSE with appropriate reason(s) for each of the following statements.

- (a) If S is the set of p vectors in \mathbb{R}^m and $p \geq m$, then S is a LD set.
- (b) If $1 \leq k < n$, and S is the set of k vectors in \mathbb{R}^n , then S can always be enlarged to become a basis of \mathbb{R}^n .
- (c) If A is $m \times n$ matrix, then the column space $C(A)$ is a subspace of \mathbb{R}^n and the row space $R(A)$ is a subspace of \mathbb{R}^m .
- (d) The set $W = \{(a, b, c) | a \geq b\}$ is a subspace of \mathbb{R}^3 over the field \mathbb{R} .
- (e) The set of skew Hermitian matrices forms a subspace of the vector space of square matrices over the field of complex numbers.

$\rightarrow R_2 + R_1 + R_3$

$$0 \quad 0 \quad 6 \quad 12 \quad -12$$

[5]

3. Let $A = \begin{pmatrix} 1 & -2 & 2 & 3 & -1 \\ -3 & 6 & -1 & 1 & -7 \\ 2 & -4 & 5 & 8 & -4 \end{pmatrix}$. Find a basis of row space of A . Also find a basis of $N(A)$. Show the complete working. [5]

4. For what value(s) of λ the set of vectors $S = \{(1, 5, -2), (0, 6, \lambda), (3, 13, -3)\}$ is LI. For all such value(s) of λ , express $(3, 5, 11)$ as a linear combination of the vectors of S . [5]

5. Let $U = \{(x, y, x+y, x-y) | x, y \in \mathbb{R}\}$, a subspace of \mathbb{R}^4 . Find a subspace W of \mathbb{R}^4 such that $\mathbb{R}^4 = U \oplus W$. [6]

$$2 \times 1 \quad -2 \quad 2 \quad 3 \quad -1$$