Minor Test 2 ::: MTL 101 ::: March 24, 2017

Every question is compulsory.
Marks will not be awarded if appropriate arguments are not provided.
Do not waste time describing what is not asked.

Maximum Marks: **20** Maximum Time: **1 hour**

(1) Find the n-th Picard's iterate y_n for the following IVP. [3]

$$y' = x^2 + y, \qquad y(0) = 0.$$

(2) Discuss the existence and uniqueness of the following IVP in a neighborhood of $x_0 = 1$. [3]

$$\frac{dy}{dx} = f(x, y), \qquad y(1) = -1,$$

where

$$f(x, y) = \begin{cases} \dfrac{x + y}{\sin(x + y)} & \text{if } x + y \neq 0 \\ 1 & \text{if } x + y = 0 \end{cases}.$$

(3) Consider the following inner product on $\mathbb{R}^4$. [2+2=4]

$$\langle (x, y, z, w) | (x', y', z', w') \rangle = xx' + xy' + x'y + 2yy' + zz' + ww'.$$

Suppose $W = \{(s, t, s, t) : s, t \in \mathbb{R}\}$.

(a) Find an orthogonal basis of W.

(b) Use the basis in part (a) to find the best approximation of $(1, 1, 2, 2)$ by a vector in W.

(4) Consider the following linear operator on $M_{3\times 3}(\mathbb{R})$. [2+2+2+1=7]

$$T(A) = A - A^t \quad \text{for } A \in M_{3\times 3}(\mathbb{R}).$$

(a) Find the nullity and the null space of T.

(b) Find all the eigenvalues of T from the definition (without computing the characteristic polynomial of T), that is, find all $\lambda \in \mathbb{R}$ such that $T(A) = \lambda A$ for some $A \neq 0$.

(c) Find the dimension of each eigenspace.

(d) Is T diagonalizable?

(5) Suppose $\mathcal{P}_5 = \{a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 : a_0, a_1, a_2, a_3, a_4 \in \mathbb{R}\}$. Let [3]

$$W_1 = \{f(x) \in \mathcal{P}_5 : x^4 f(1/x) = f(x)\}$$
$$W_2 = \{f(x) \in \mathcal{P}_5 : f(-x) = f(x)\}.$$

Find $\dim(W_1)$, $\dim(W_2)$ and $\dim(W_1 + W_2)$.

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