

COL1000: Introduction to Programming

Algorithmic Thinking (Merge & Merge Sort)

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Announcement

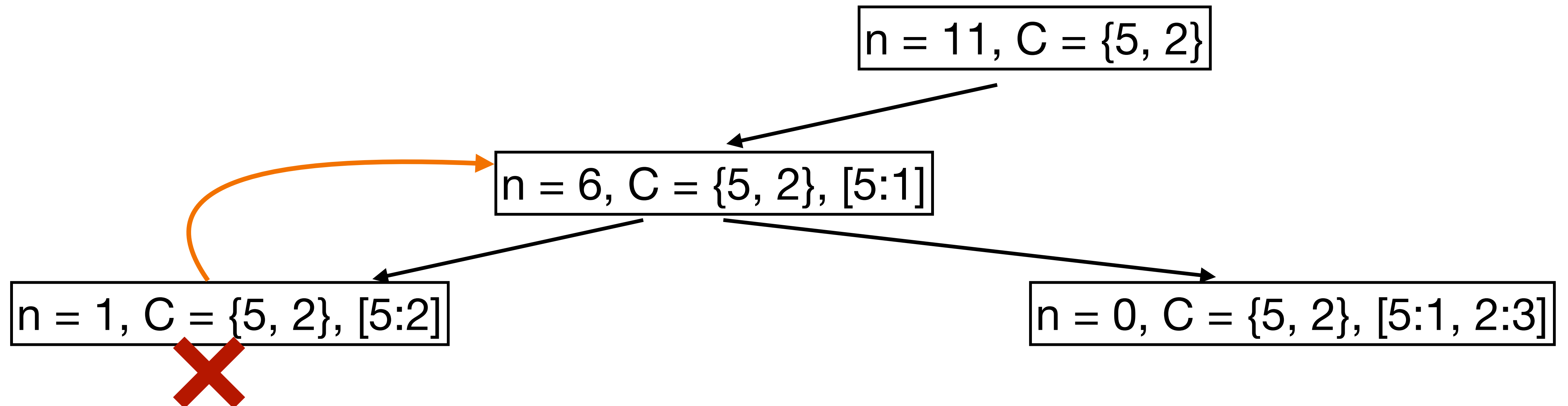
- (Usually) Monday's — 5 pm to 7 pm; Doubt learning sessions in Bharti 419
 - This time — **3 students visited me!**

Greedy is Suboptimal

- **Suboptimal** — $C = \{1, 3, 4\}$ $T = 6 \rightarrow 4, 1, 1, \rightarrow 3$ coins; $3 + 3 \rightarrow 2$ coins (optimal)
- Greedy approach is **optimal only for canonical coins**
 - Let $G(x)$ be the # of coins in greedy approach and $Opt(x)$ be the real optimal solutions to meet target x
 - Then a system is canonical *iff* $G(x) = Opt(x)$
 - **(Optional)** Look at **Kozen-Zaks theorem** to establish when Greedy can be optimal

Greedy is Suboptimal

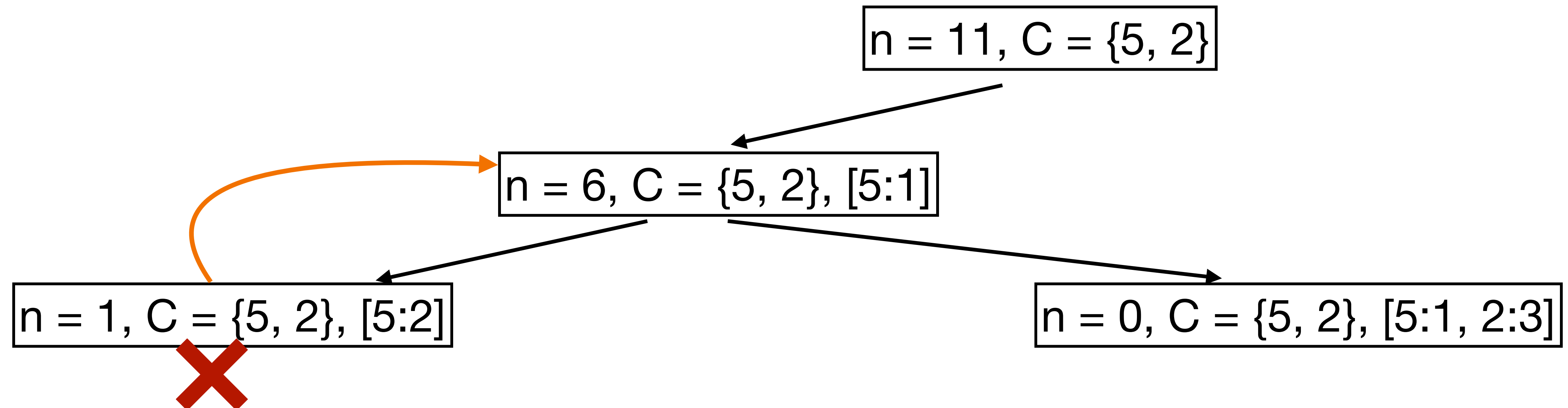
- Simple Greedy strategy is **suboptimal** — **Why?**
 - **False assumption:** that the largest coin always participate in an optimal solution
- **Alternate strategy** — **backtrack + greedy**



Greedy is Suboptimal

- Alternate strategy — backtrack + greedy (two variants - any solution, optimal)

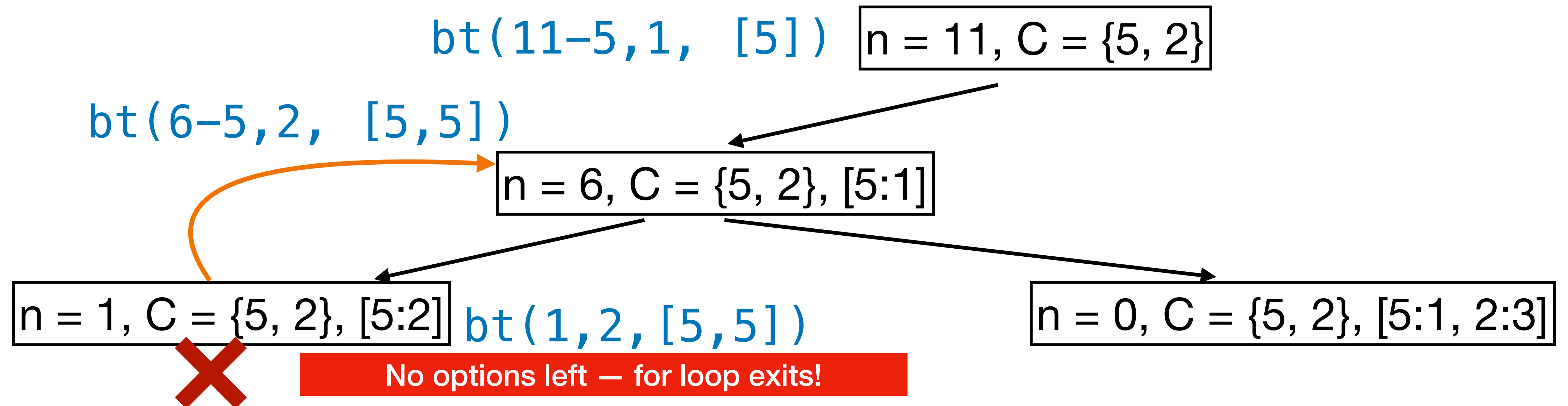
```
for coin in denom:  
    (resCnt, resCoins) = bt(remaining-coin, count+1, coins_used+[coin])
```



Greedy + Backtrack - Any Solution

- Alternate strategy — backtrack + greedy (two variants - any solution)

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Greedy + Backtrack - Any Solution

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```
for coin in denom:  
    (resCnt, resCoins) = bt(remaining-coin, count+1, coins_used+[coin])
```

```
# But what if a solution is found — i.e. bt returns with a valid resCnt?
```

```
return (None, None)
```

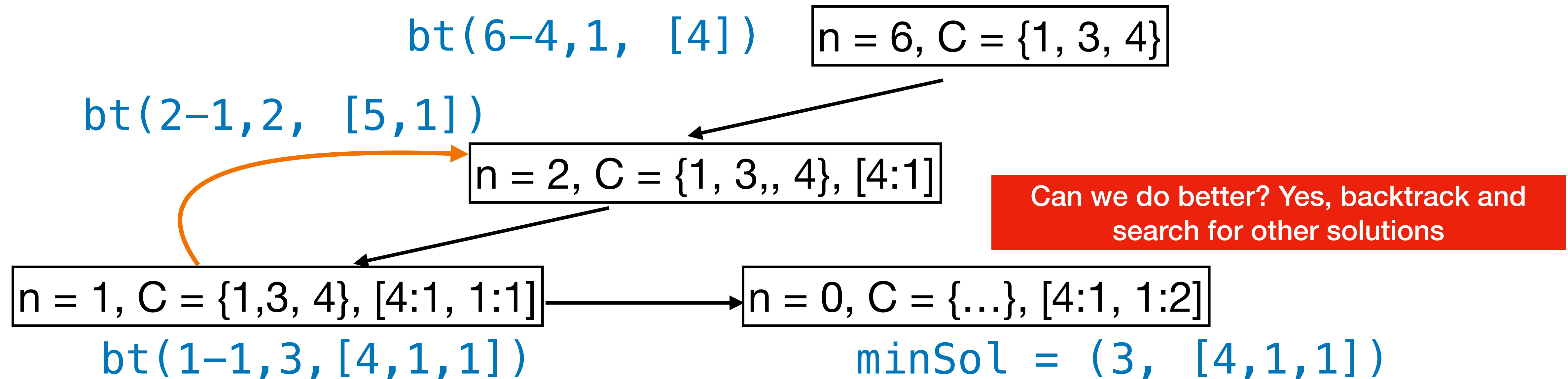

Greedy + Backtrack - Any Solution

```
def coin_change_bt_any(denom, target):  
  
    denom = sorted(denom, reverse=True)  
  
    def bt(remaining, count, coins_used):  
  
        # base case 1  
        if remaining == 0:  
            return (count, coins_used)  
  
        for coin in denom:  
  
            if remaining < coin:  
                continue  
  
            (resCnt, resCoins) = bt(remaining - coin, count+1, coins_used + [coin])  
            # If solution is found  
            if resCnt is not None:  
                return (resCnt, resCoins)  
            # Can't progress from this state -- no solution in this path  
            return (None, None)  
  
    (resCnt, resCoins) = bt(target, 0, [])  
    return (resCnt, resCoins) if resCnt is not None else (-1, [])
```


Greedy + Backtrack - Optimal Solution

- Note that for optimality (with backtrack), you must maintain the minimum (or best) solution while looking for other solutions

```
for coin in denom:  
    (resCnt, resCoins) = bt(remaining-coin, count+1, coins_used+[coin])
```



Greedy + Backtrack - Optimal Solution

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```
for coin in denom:  
    (resCnt, resCoins) = bt(remaining-coin, count+1, coins_used+[coin])  
        bt(6-4, 1, [4])  
            n = 6, C = {1, 3, 4}  
            CurrMinSol = (3, [4, 1, 1])  
            bt(2-1, 2, [5, 1])  
                n = 2, C = {1, 3, 4}, [4:1]  
                n = 1, C = {1, 3, 4}, [4:1, 1:1]  
                bt(1-1, 3, [4, 1, 1])
```

The diagram illustrates the recursive process of finding the optimal solution for a coin change problem using a greedy + backtrack approach. It shows the state of the problem at each step, including the remaining amount (n) and the current set of coins used (C). The process starts with n=6 and C={1, 3, 4}. It then branches into n=2 and C={1, 3, 4}, [4:1], and finally n=1 and C={1, 3, 4}, [4:1, 1:1]. The current minimum solution found is (3, [4, 1, 1]).

Greedy + Backtrack - Optimal Solution

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```
for coin in denom:  
    (resCnt, resCoins) = bt(remaining-coin, count+1, coins_used+[coin])  
    bt(6-3, 1, [3])          CurrMinSol = (3, [4, 1, 1])
```

$n = 6, C = \{1, 3, 4\}$

$bt(3-3, 2, [3, 3])$

$n = 3, C = \{1, 3, 4\}, [3:1]$

MinSol < CurrMinSol =>
CurrMinSol = MinSol

$n = 0, C = \{1, 3, 4\}, [3:2]$

MinSol = (2, [3, 3])