MTL101::: Minor Test 1

August, 2014

1. No marks will be awarded if appropriate justification is not provided.
2. Every question is compulsory. Maximum Time is 1 hour.

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1. Let V be a vector space over $\mathbb{R}$ and let $A = \{u, v, w\}$ be a subset of V. Suppose $B = \{u + v + w, u + 2v + 3w, u + 4v + 9w\}$. Show, by using the definition of linear independence, that A is linearly independent if B is linearly independent. [4]

2. Let $A \in M_{n\times n}(\mathbb{R})$ and let I be the identity matrix of size n. Suppose the augmented matrix $(A|I)$ is row equivalent to $(B|C)$ where $B \in M_{n\times n}(\mathbb{R})$ is row reduced echelon matrix. Show that if A is invertible, then $C = A^{-1}$.

   Find the inverse of the following matrix performing elementary row operations on a suitable matrix. $\begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 4 \\ 1 & 0 & 4 & 0 \\ 0 & 9 & 0 & 16 \end{pmatrix}$ [$1 + 3 = 4$]

3. Suppose the linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^4$ is defined by
   $$T(x, y, z, w) = (x - y + z - w, x + 2y + w, y + z - w, 3x + 4y + 2z).$$
   (a) Find rank(T) and nullity(T).
   (b) Use these values (rank(T) and nullity(T)) to find whether T is (i) one to one (injective), (ii) onto (surjective). [$2 + 2 = 4$]

4. (a) Let V be a vector space over $\mathbb{R}$ and let W be a subspace of V. Let $T : V \to V$ be a linear transformation. Show that U, defined by
   $$U = \{v \in V : T(v) \in W\},$$
   is a subspace of V.
   (b) Let $\mathcal{P}_3 = \{f(x) \in \mathbb{R}[x] : \deg(f(x)) \le 2\}$. Find the coordinate vector $[v]_B$ of $v = 3x^2 + 5x - 2$ with respect to the ordered basis $B = \{x - 1, x^2 + x + 1, 3\}$ (the order is as elements are written). [$2 \times 2 = 4$]

5. Find whether the following statements are True or False. Justify in each case.
   (a) Let a subset B of $\mathbb{R}^4$ be defined by $B = \{e_1 + e_2, e_2 + e_3, e_3 + e_4, e_4 + e_1\}$ where e_i denotes a 4-tuple with i-th entry equal to 1 and other entries 0. Then B is a basis of $\mathbb{R}^4$.
   (b) Let V be a finite dimensional vector space over $\mathbb{R}$. Every onto (surjective) linear transformation from V to V is one to one (injective). [$2 \times 2 = 4$]

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