MTL 101 Linear Algebra and Differential Equations 2023-24 Second Semester

Quiz-I, January 30, 2024

Marks Obtained:

Maximum Marks: 15 Time: 45 Minutes

Name: Group No. Entry No.

Instructions: No additional sheet will be provided. All notations are standard. Use of any electronic gadget including calculator is NOT allowed. No query will be entertained.

Q1: Define a row reduced Echelon matrix. (2)

Q2: Reduce the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 1 & 3 & 1 \\ 0 & 2 & -2 & 1 \end{bmatrix}$$

to it row reduced echelon form, and hence solve the homogenous system $Ax = 0$. (3)

Q3: Let $V = \mathbb{R}^2$ be a vector space, and $W = \{(x, x) : x \in \mathbb{R}\}$ be a subspace of V. Find two different subspaces U_1 and U_2 such that $V = U_1 \oplus W$ and $V = U_2 \oplus W$. Justify your answer. (3)

Q4: Let

$$W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a + b + c + d = 0 \right\}$$

be a subset of the vector space V of all 2×2 matrices over the field of real numbers $\mathbb{R}$. Show that W is a subspace of V, and find a basis for W. (4)

Q5: Let u, v and w be linearly independent in $\mathbb{R}^3$. Find all the values of $k \in \mathbb{R}$ for which the vectors $v - u$, $kw - v$ and $u - w$ are linearly independent. (3)