

COL1000: Introduction to Programming Functions

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Co-Develop: Binary Search

- Given a **list of sorted** elements and an element **x**
 - Find whether x exists in the list and if so where?
- Algorithm:
 1. Search x at the mid of the list: If found then return the index, else go to step (2)
 2. If the element at mid $>$ x, then `lst = lst[mid+1:]` and go to step 1
 3. Else, `lst = lst[:mid]` and go to step 1

Non-linear Recursion: Tree Recursion

- Fibonacci
 - $\text{Fib}(n) = \text{Fib}(n-1) + \text{Fib}(n-2)$
 - **Recursive call stack grows in the form of a binary tree**
 - **Exponential Time Θ**
- Count the number of paths in a grid $m \times n$, moving right and down
 - $P(m,n) = 1$ if $m = 1$ or $n = 1$ else $P(m-1, n) + P(m, n-1)$

Non-linear Recursion: Tree Recursion

- **Fibonacci**

- $\text{Fib}(n) = \text{Fib}(n-1) + \text{Fib}(n-2)$

```
def recFib_slow(n: int) -> int:
    if n < 2:                # base
        return n
    return recFib_slow(n-1) + recFib_slow(n-2)

print(recFib_slow(100))
```


Non-linear Recursion: Tree Recursion

- Fibonacci — cached version (**more efficient**)
- $\text{Fib}(n) = \text{Fib}(n-1) + \text{Fib}(n-2)$

```
from functools import lru_cache
```

```
@lru_cache(maxsize=None)
```

```
def fib_cached(n: int) -> int:
```

```
    if n < 2:
```

```
        return n
```

```
    return fib_cached(n-1) + fib_cached(n-2)
```

```
print(fib_cached(100))
```



Decorator for memoization

Non-linear Recursion: Tree Recursion

- Fibonacci — cached version (**more efficient**) — **manual memoization**
- $\text{Fib}(n) = \text{Fib}(n-1) + \text{Fib}(n-2)$

```
_cache = {0: 0, 1: 1}
```

Dictionary serving as a cache
Also records the base case

```
def fib(n: int) -> int:
    if n < 0:
        raise ValueError("n must be non-negative")
    if n in _cache:
        return _cache[n]
    _cache[n] = fib(n - 1) + fib(n - 2)
    return _cache[n]
```

Checking cache first

Non-linear Recursion: Tree Recursion

- Paths in an $m \times n$ grid
 - Convince yourself of the base case and the induction step!

```
def paths(m: int, n: int) -> int:
    if m < 1 or n < 1:
        raise ValueError("m, n must be >= 1")
    if m == 1 or n == 1:
        return 1
    return paths(m - 1, n) + paths(m, n - 1)
```

Tail Recursion

- Special form of recursion that is the **very last computation** performed in the function

- `def factorial(n:int) -> int:`

- `if n == 0: return 1`

- `elif n < 0:`

- `raise Error("Not defined on neg values")`

- `else: return n * factorial(n-1)`



Not Tail Recursive

Tail Recursion: Why it matters?

- Memory efficient (uses $O(1)$ space)
 - Reuses the same stack frame
 - Therefore no stack overflow
- Unfortunately, Python **does NOT** perform tail call optimisation

Functions: Exceptions

- Exceptions in functions **denote errors or unexpected conditions**
- There are two ways of handling exceptions in the code:
 - Explicitly **raising the errors with an appropriated exception message**
 - **Catching** them to handle them gracefully
 - Let us see examples of each!