Total Marks: 10 ***MTL101 Quiz-2** Time: 20 mins

Name: Entry No: Group:

1. Consider the following nonhomogeneous ODE. [2 + 3 = 5]

$$y''' - 3y'' + 3y' - y = e^t.$$

(a) Find the general solution of the corresponding homogeneous ODE.

(b) Use the method of undetermined coefficients to find a particular solution to the given equation.

(a) $y''' - 3y'' + 3y' - y = 0$

Putting $y = e^{mt}$ gives characteristic eqn:

$$m^3 - 3m^2 + 3m - 1 = 0 \iff (m-1)^3 = 0$$

which has $m = 1$ as a triple root.

Thus the general solution is

$$\boxed{y = c_1 e^t + c_2 t e^t + c_3 t^2 e^t}$$

(b) Let $y_p = At^3 e^t$

Then $y_p' = Ae^t(t^3 + 3t^2)$

$y_p'' = Ae^t(t^3 + 6t^2 + 6t)$

& $y_p''' = Ae^t(t^3 + 9t^2 + 18t + 6)$

Putting in the nonhomog. eqn gives

$$Ae^t\left[(\cancel{t^3} + \cancel{9t^2} + \cancel{18t} + 6) - 3(\cancel{t^3} + \cancel{6t^2} + \cancel{6t}) + 3(\cancel{t^3} + \cancel{3t^2}) - \cancel{t^3}\right] = e^t$$

$$\Rightarrow 6Ae^t = e^t \Rightarrow A = \frac{1}{6}$$

Hence $\boxed{y_p = \frac{1}{6} t^3 e^t}$ is a particular solution.

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2. Find the general solution of: [5]

$$y'' + 4y = \operatorname{cosec}(2x).$$

Characteristic eqn. for the homog. ODE $y''+4y=0$ is $m^2+4=0$ which has $m=\pm 2i$ as roots.

Hence, $y_1 = \cos(2x)$ & $y_2 = \sin(2x)$ are two L.I. solns.

$$W(y_1, y_2)(x) = \begin{vmatrix} \cos(2x) & \sin(2x) \\ -2\sin(2x) & 2\cos(2x) \end{vmatrix} = 2$$

By variation of parameters method,

$$y_p = u_1 y_1 + u_2 y_2$$

where $u_1 = -\int \frac{y_2 r}{W} dx = -\int \frac{\sin(2x)\operatorname{cosec}(2x)}{2} dx = -\frac{1}{2}x$

& $u_2 = \int \frac{y_1 r}{W} dx = \int \frac{\cos(2x)\operatorname{cosec}(2x)}{2} dx$

$$= \frac{1}{4}\int \frac{2\cos(2x)}{\sin(2x)} dx = \frac{1}{4}\ln|\sin(2x)|$$

$\therefore \quad y_p = -\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x)\ln|\sin(2x)|$

and the general soln. is $y = c_1 y_1 + c_2 y + y_p$

i.e. $\boxed{y = c_1\cos(2x) + c_2\sin(2x) - \frac{1}{2}x\cos 2x + \frac{1}{4}\sin(2x)\ln|\sin(2x)|}$

Total Marks: 10 | MTL101 Quiz-2 | Time: 20 mins

Name: Entry No: Group:

1. Consider the following nonhomogeneous ODE. [2 + 3 = 5]

$$y''' + 3y'' + 3y' + y = e^{-t}.$$

(a) Find the general solution of the corresponding homogeneous ODE.

(b) Use the method of undetermined coefficients to find a particular solution to the given equation.

(a) Homog. ODE : $y''' + 3y'' + 3y' + y = 0$

Putting $y = e^{mt}$ gives characteristic eqn

$m^3 + 3m^2 + 3m + 1 = 0$ ie. $(m+1)^3 = 0$

which has $m = -1$ as a triple root.

Thus the general solution is

$$\boxed{y = c_1 e^{-t} + c_2 t e^{-t} + c_3 t^2 e^{-t}}$$

(b) Let $y_p = A t^3 e^{-t}$

Then $y_p' = A e^{-t}(-t^3 + 3t^2)$

$y_p'' = A e^{-t}(t^3 - 6t^2 + 6t)$

& $y_p''' = A e^{-t}(-t^3 + 9t^2 - 18t + 6)$

Putting in the nonhomog. eqn gives

$$A e^{-t}\left[(\cancel{-t^3} + \cancel{9t^2} - \cancel{18t} + 6) + 3(\cancel{t^3} - \cancel{6t^2} + \cancel{6t}) + 3(\cancel{-t^3} + \cancel{3t^2}) + \cancel{t^3}\right] = e^{-t}$$

$\Rightarrow 6Ae^{-t} = e^{-t} \Rightarrow A = \frac{1}{6}$

Hence, $\boxed{y_p = \frac{1}{6} t^3 e^{-t}}$ is a particular solution

:::PTO:::

2. Find the general solution of: [5]

$$y'' + 9y = \text{cosec}(3x).$$

Characteristic eqn for the homog. ODE $y'' + 9y = 0$ is $m^2 + 9 = 0$ which has $m = \pm 3i$ as roots.

Hence, $y_1 = \cos(3x)$, $y = \sin(3x)$ are two linearly independent solutions.

$$W(y_1, y_2)(x) = \begin{vmatrix} \cos(3x) & \sin(3x) \\ -3\sin(3x) & 3\cos(3x) \end{vmatrix} = 3$$

By variation of parameters method,

$$y_p = u_1 y_1 + u_2 y_2$$

where $u_1 = -\int \frac{y_2 r}{W} dx = -\int \frac{\sin(3x)\,\text{cosec}(3x)}{3} dx = -\frac{1}{3}x$

and $u_2 = \int \frac{y_1 r}{W} dx = \int \frac{\cos(3x)\,\text{cosec}(3x)}{3} dx$

$$= \frac{1}{9}\int \frac{3\cos(3x)}{\sin(3x)} dx = \frac{1}{9}\ln|\sin(3x)|$$

$$\therefore y_p = -\frac{1}{3}x\cos(3x) + \frac{1}{9}\sin(3x)\ln|\sin(3x)|$$

The general soln. is $y = c_1 y_1 + c_2 y_2 + y_p$

i.e. $$\boxed{y = c_1\cos(3x) + c_2\sin(3x) - \frac{1}{3}x\cos(3x) + \frac{1}{9}\sin(3x)\ln|\sin(3x)|}$$