COL202: Discrete Mathematical Structures. II semester, 2024-25.
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Tutorial Sheet 2: Proofs.
16 January 2025

Important: The question marked with a ♠ is this week's quiz. The start time for the quiz is 1PM and the end time for the quiz is 1:12PM.

Problem 1 ([LLM18], Prob 1.2(b))
Using the first method of proving implications, prove: $1 = -1 \Rightarrow 2 = 1$. We know, of course, that we can trivially claim that this is a valid implication since the antecedent is false, i.e., since $1 \neq -1$, but nonetheless, please write out the proof as a sequence of assertions using deduction rules.

Problem 2 ([BSD05], Prob 7, pp 115)
Prove using the contrapositive method that for all real numbers x, if $x^2 - 2x \neq -1$ then $x \neq 1$.

Problem 3 ([LLM18], Prob 1.7)
Prove by cases that

$$\max\{r, s\} + \min\{r, s\} = r + s$$

for all real r, s.

Problem 4 ([BSD05], Prob 8, pp 115)
Prove using the proof by contradiction method that for all real numbers x, if $x^2 - 2x \neq -1$ then $x \neq 1$.

Problem 5 ([LLM18], Prob 1.18)
Prove by contradiction that $\sqrt{3} + \sqrt{2}$ is irrational.

Problem 6 ([LLM18], Prob 1.19)
Given integers $a_0, \ldots, a_{m-1}$ prove that any real root of $a_0 + a_1 x + \cdots + a_{m-1} x^{m-1} + x^m$ is either an integer or irrational. Show that this implies that $\sqrt[m]{k}$ is irrational whenever k is not the mth power of some integer.

Problem 7 ([BSD05], Prob 15, pp 115)
Given function $f, g, h : \mathbb{Z}_+ \to \mathbb{R}_+$ Recall that $f(n) = O(g(n))$ by definition if

$$\exists c \in \mathbb{R}_+ : \exists n_0 \in \mathbb{Z}_+ : \forall n \geq n_0 f(n) \leq c g(n).$$

Prove that if $f(n) = O(g(n))$ and $g(n) = O(h(n))$ then $f(n) = O(h(n))$.

Problem 8 ♠ ([BSD05], Prob 1, pp 126)
Use the Well Ordering Principle to prove that

$$\frac{2}{3} + \frac{2}{9} + \cdots + \frac{2}{3^n} = 1 - \left(\frac{1}{3}\right)^n,$$

for all $n \geq 1$.

Problem 9 ([LLM18], Problem 2.5)
Use the Well Ordering Principle to prove that there is no positive integer solution for

$$4a^3 + 2b^3 = c^3.$$

Problem 10
Let us assume that the postal department has only two denominations of stamps: Rs 3 and Rs 5. We call a number n *postal* if we can create postage worth Rs n using the two denominations that are available. Prove using the Well Ordering Principle that every $n \geq 8$ is postal.

References

[BSD05] K. Bogart, S. Drysdale, C. Stein. Discrete Math for Computer Science Students. 2005.

[LLM18] E. Lehman, F. T. Leighton, and A. R. Meyer. Mathematics for Computer Science, June 2018, MIT Open Courseware.