Minor Test 1 ::: MAL 101 ::: February 2014

Every question is compulsory.
Marks will not be awarded if appropriate arguments are not provided.
Do not write the questions before answering them.
Do not waste time describing what is not asked.

Maximum Marks: **25** Maximum Time: **one hour**

(1) Suppose W_1 and W_2 are subspaces of a vector space V over $\mathbb{R}$. Recall [2 + 2 = 4]

$$W_1 + W_2 = \{u + v \in V : u \in W_1, v \in W_2\}.$$

(a) Show that $W_1 + W_2$ is a subspace of V.

(b) Is it possible to have the following data?

$$\dim V = 4,\ \dim W_1 = \dim W_2 = 3,\ \dim(W_1 \cap W_2) = 1.$$

Justify.

$a-b-c=0$

(2) Suppose V is a vector space over $\mathbb{R}$ and $u, v, w \in V$. Let [4]

$$X = \{u, u+v, u+v+w\} \text{ and } Y = \{v, v+w, u+v-w\}.$$

Show that if X is linearly independent then Y is linearly independent.

(3) Find a basis of the range space of the linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^5$ defined by

$$T(x, y, z, w) = (x+y+z+w,\ x-y+z+2w,\ 2y+z,\ 2y-w,\ z+w).$$

Also find nullity(T). [4 + 1 = 5]

(4) (a) Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear tranformation. Is it possible to have the following data?

$$T(1,1) = (1,0);\ T(2,3) = (1,2);\ T(3,2) = (2,1).$$

Justify. [3]

(b) Find a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that the null space of T and the range space of T are the same (i.e., $\ker T = T(\mathbb{R}^2)$). [2]

(5) Consider the following system of four equations in four unknowns x, y, z, w: [7]

$$\begin{aligned} x+y+z+w &= 4 \\ x+2y+3z+4w &= 3 \\ x+y+2z &= 6 \\ y+9z+aw &= b \end{aligned}$$

(a) Apply suitable elementary row operations on the augmented matrix of this system to find an equivalent system of equations such that the coefficient matrix is row reduced echelon for every a and b in $\mathbb{R}$.

(b) Using part (a) find all possible $a, b \in \mathbb{R}$ such that the system has i) **no** solutions, ii) a **unique** solution, iii) **infinitely** many solutions.

(c) Write all the solutions of the system when the system is consistent (i.e., when it has a solution).

$ax+by=0$
$cx+dy=0$

$\frac{by}{a} = \frac{dy}{c}$

$T((a,b))=0$

::: END :::

Nullity $=$ Rk$|T|=1$ $\lambda=0$

$T(\lambda(a,b))=0$

$T(x,y)=(a,b)$

$T[\lambda(a,b)]=0$