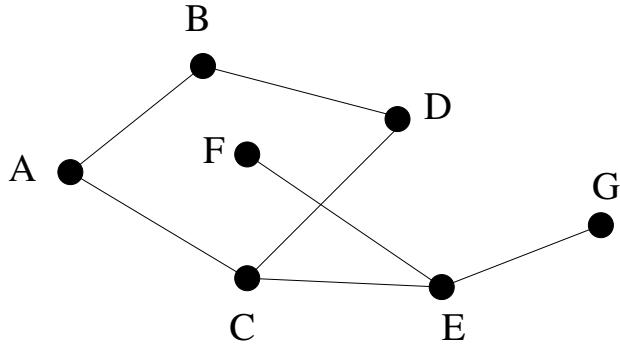


## Problems for Recitation 6

### 1 Graph Basics

Let  $G = (V, E)$  be a graph. Here is a picture of a graph.



Recall that the elements of  $V$  are called vertices, and those of  $E$  are called edges. In this example the vertices are  $\{A, B, C, D, E, F, G\}$  and the edges are

$$\{A-B, B-D, C-D, A-C, E-F, C-E, E-G\}.$$

Deleting some vertices or edges from a graph leaves a *subgraph*. Formally, a subgraph of  $G = (V, E)$  is a graph  $G' = (V', E')$  where  $V'$  is a nonempty subset of  $V$  and  $E'$  is a subset of  $E$ . Since a subgraph is itself a graph, the endpoints of every edge in  $E'$  must be vertices in  $V'$ . For example,  $V' = \{A, B, C, D\}$  and  $E' = \{A-B, B-D, C-D, A-C\}$  forms a subgraph of  $G$ .

In the special case where we only remove edges incident to removed nodes, we say that  $G'$  is the *subgraph induced on  $V'$*  if  $E' = \{(x-y | x, y \in V' \text{ and } x-y \in E)\}$ . In other words, we keep all edges unless they are incident to a node not in  $V'$ . For instance, for a new set of vertices  $V' = \{A, B, C, D\}$ , the induced subgraph  $G'$  has the set of edges  $E' = \{A-B, B-D, C-D, A-C\}$ .

### 2 Problem 1

An undirected graph  $G$  has ***width w*** if the vertices can be arranged in a sequence

$$v_1, v_2, v_3, \dots, v_n$$

such that each vertex  $v_i$  is joined by an edge to at most  $w$  preceding vertices. (Vertex  $v_j$  precedes  $v_i$  if  $j < i$ .) Use induction to prove that every graph with width at most  $w$  is  $(w + 1)$ -colorable.

(Recall that a graph is  $k$ -colorable iff every vertex can be assigned one of  $k$  colors so that adjacent vertices get different colors.)

### 3 Problem 2

A **planar graph** is a graph that can be drawn without any edges crossing.

1. First, show that any subgraph of a planar graph is planar.
2. Also, any planar graph has a node of degree at most 5. Now, prove by induction that any graph can be colored in at most 6 colors.