DEPARTMENT OF MATHEMATICS, IIT DELHI
SEMESTER II 2024 − 25
MTL 101 (Linear Algebra and Differential Equations) - Quiz 2

Date: 27/03/2025 (Thursday) Time: 6:45 PM - 7:30 PM.

**"As a student of IIT Delhi, I will not give or receive aid in examinations. I will do my share and take an active part in seeing to it that others as well as myself uphold the spirit and letter of the Honour Code."**

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| **Name :** | | | BLOCK LETTER ONLY |
| **Entry Number:** | | **Group:** | |
| **Gradescope Id:** | | **Lecture Hall:** | |

**Question 1:** Let $T$ be the linear operator on $\mathbb{R}^4$ given by the matrix

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ s & 0 & 0 & 0 \\ 0 & t & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

with respect to the standard basis.

(i) Find all values of $s$ and $t$ for which $T$ is diagonalizable.

(ii) Find an ordered basis $\mathcal{B}$ of $\mathbb{R}^4$ such that $[T]_{\mathcal{B}}$ is a diagonal matrix, whenever $T$ is diagonalizable.

[4+2=6]

**Question 2:** Consider the initial value problem (IVP):

$$y' = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ \frac{1}{2}, & (x,y) = (0,0) \end{cases}$$

with initial condition:

$$y(x_0) = 0.$$

(i) For what values of $x_0$ does this IVP have a solution according to the existence theorem?

(ii) Additionally, for such, $x_0$ find the largest positive $\alpha$ such that the solution exists in the interval $(x_0 - \alpha, x_0 + \alpha)$.

[2+2=4]

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**Question 3:** Let $A$ be the $3 \times 3$ matrix given by

$$A = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & 3 \end{bmatrix}.$$

Using Cayley-Hamilton theorem show that $A$ is invertible and find the inverse. [3]