

Marks Obtained:

Maximum Marks: 15

Time: 45 Minutes

Name:

Group No.

Entry No.

Instructions: No additional sheet will be provided. All notations are standard. Use of any electronic gadget including calculator is NOT allowed. No query will be entertained.

Q1: Let B be an $m \times n$ real matrix. Show that the eigenvalues of $B^T B$ are nonnegative. (2)

Q2: Let

$$A = \begin{bmatrix} 3 & -2 & 0 \\ -1 & 3 & -1 \\ 5 & 7 & -1 \end{bmatrix}$$

- (i) Determine eigenvalues of and corresponding eigenvectors for A . (3)
- (ii) Is A diagonalizable? Give reason(s). (1)

Q3: Solve the initial value problem (3)

$$\frac{dy}{dx} = 1 - \sin(y - x), \quad y(0) = 1$$

Q4a: Assume M and N , and their first order partial derivatives are continuous. If $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = Mg(y) - Nf(x)$, where g is a function of y alone and f is a function of x alone, then prove that $Mdx + Ndy = 0$ has an integrating factor, $I.F = e^{-(\int y(y) dy + f(x) dx)}$. (2)

Q4b: Using 4a, find the general solution of the differential equation (4)

$$y(x^3 - y) - x(x^3 + y) dy = 0.$$

