

Indian Institute of Technology Delhi
MTL101 (Minor Test 2)
October 2015

Max Time: 1 hour

Max Marks: 20

Note: No marks will be awarded without appropriate arguments.

1. Let $A = \begin{pmatrix} 3/2 & -1/2 & 1/2 \\ 0 & 1 & 0 \\ 1/2 & -1/2 & 3/2 \end{pmatrix}$.
 - (a) Find the characteristic polynomial of A and its roots.
 - (b) Use part (a) to show that A is diagonalizable.
 - (c) Find $P \in M_3(\mathbb{R})$ such that $P^{-1}AP$ is a diagonal matrix. [2 + 2 + 2 = 6]

2. Let $\mathcal{P}_n(\mathbb{R})$ denote the space of polynomials of degree less than n with coefficients from \mathbb{R} . Consider $T : \mathcal{P}_3(\mathbb{R}) \rightarrow \mathcal{P}_4(\mathbb{R})$ defined by

$$T(f)(x) = \int_0^x f(t) dt.$$
 Find the matrix $[T]_{\mathcal{B}_1}^{\mathcal{B}_2}$ if $\mathcal{B}_1 = \{1, x, x^2\}$ and $\mathcal{B}_2 = \{1, x, x^2, x^3\}$. [2]

3. Define an inner product on \mathbb{R}^2 and use it to show that

$$\frac{[x_1x_2 + 2(x_1y_2 + x_2y_1) + 5y_1y_2]^2}{(x_1^2 + 4x_1y_1 + 5y_1^2)(x_2^2 + 4x_2y_2 + 5y_2^2)} \leq 1$$
 for all $(x_1, y_1), (x_2, y_2) \in \mathbb{R}^2 \setminus \{(0, 0)\}$. State any theorem you use. [3]

4. Consider the IVP:

$$y' = xy^2 + 1, \quad y(0) = 0.$$
 Find the first three iterates of the Picard's method of successive approximations. [3]

5. Consider the ODE

$$(xy - 1)dx + (x^2 - xy)dy = 0$$
 - (a) Find a general solution to the given ODE.
 - (b) Find a particular solution in *explicit form* to the given ODE satisfying the initial condition $y(1) = 2$.
 - (c) Using the existence-uniqueness theorem justify that the solution obtained in (b) is unique in a small enough interval $(1 - \alpha, 1 + \alpha)$. [2 + 2 + 2 = 6]

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