Suppose we have a system of three linear equations in real coefficients and in two unknown:

$$a_1 x + b_1 y = c_1$$

$$a_2 x + b_2 y = c_2$$

$$a_3 x + b_3 y = c_3$$

When the system has no solution, unique solution and infinite number of solutions. (Only geometry)

Solution :- Since we have only two unknown we deal with lines only.

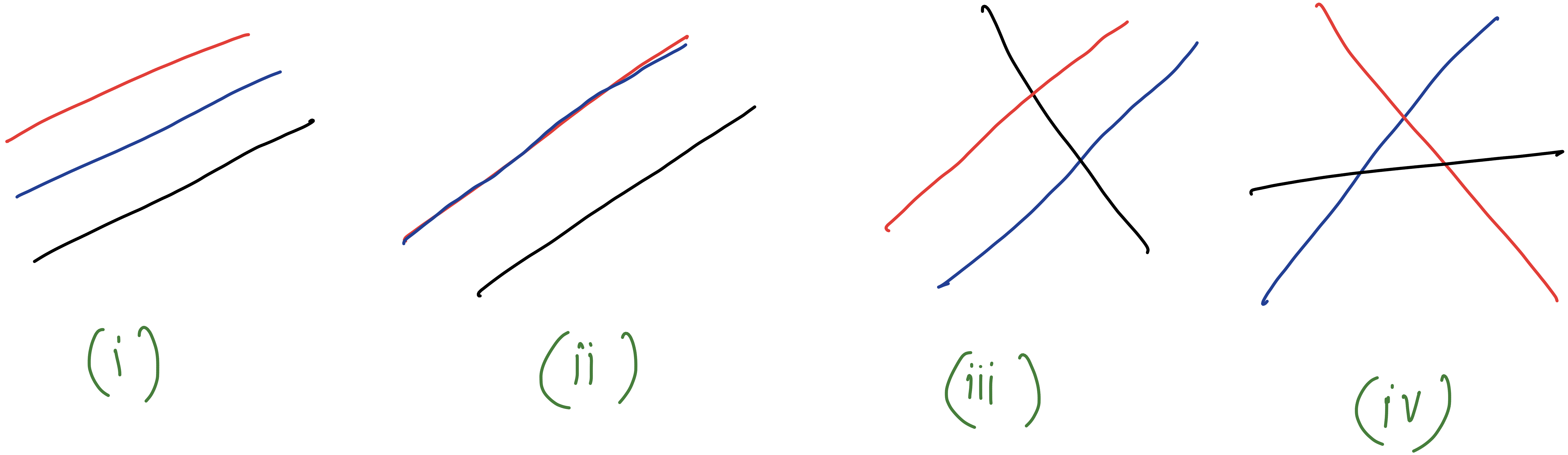
No solutions

(i) All three lines are parallel.

(ii) Two lines coincide and parallel to the third one

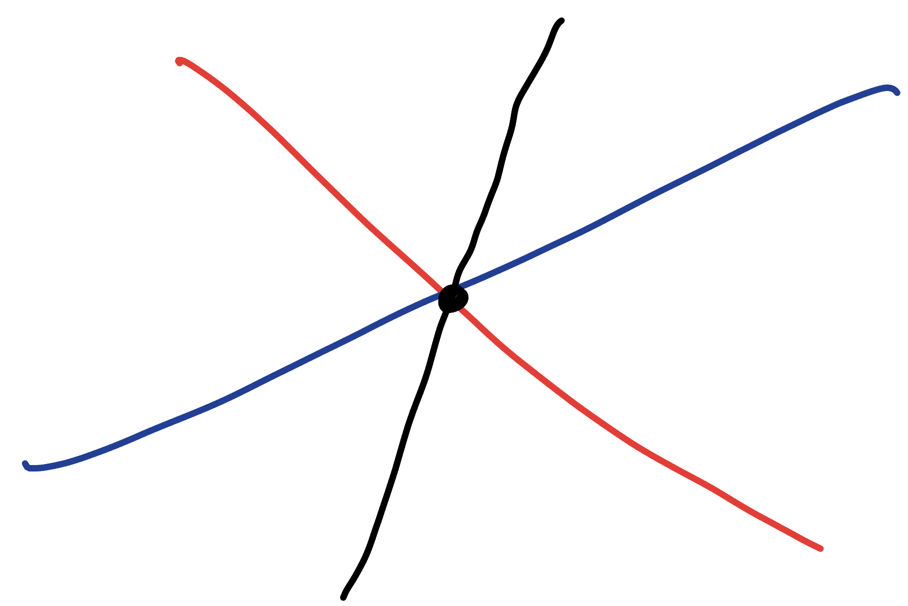
(iii) Two lines are parallel and cut the third line.

(iv) Three lines intersect three different points.



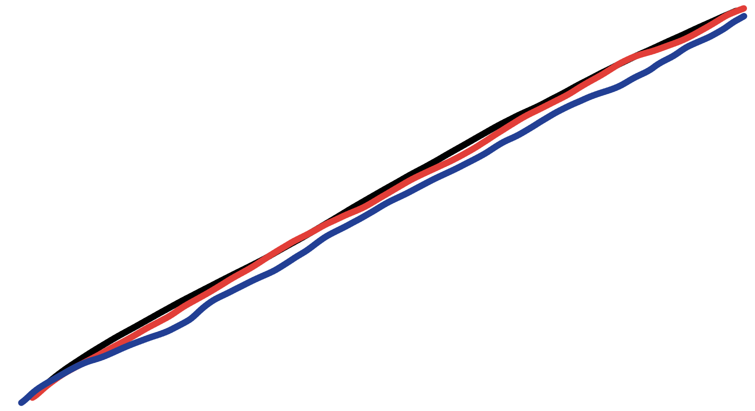
Unique Solution :-

All three lines have only one common point of intersection



Infinitely many solutions :-

(i) All three lines coincide.



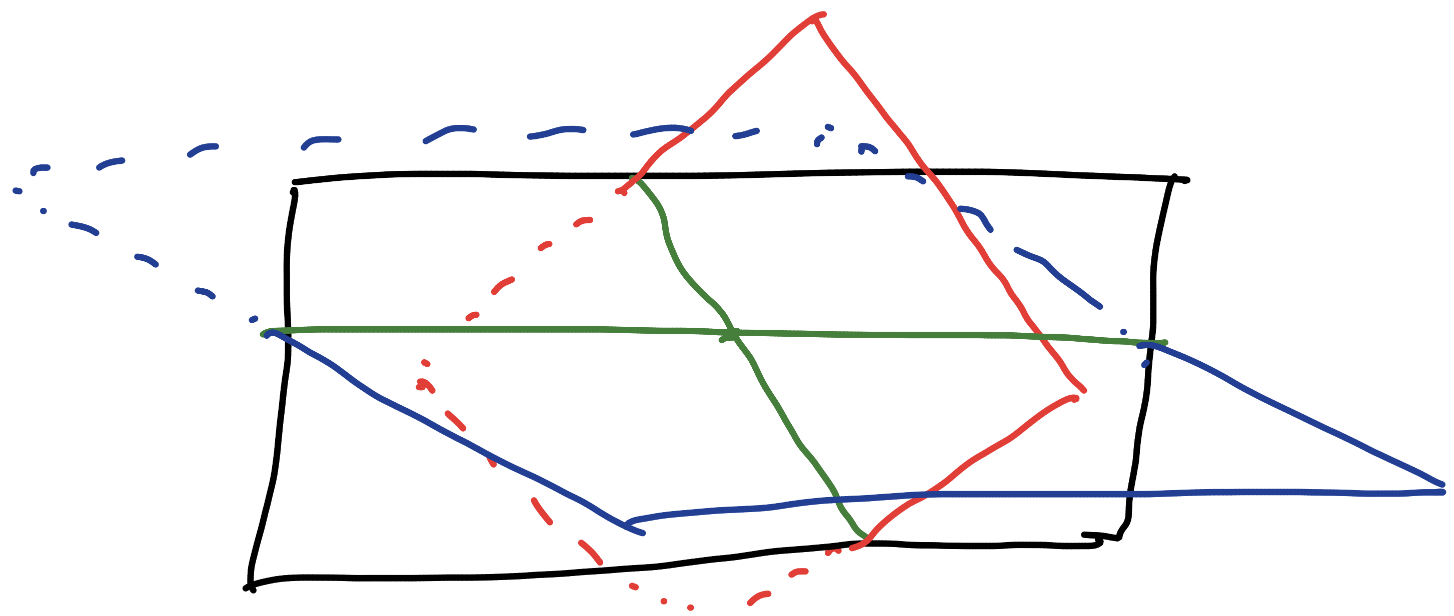
Question :- Suppose we have a system of linear equation in three unknowns in real coefficients and

$$a_1 x + b_1 y + c_1 z = d_1$$
$$a_2 x + b_2 y + c_2 z = d_2$$
$$a_3 x + b_3 y + c_3 z = d_3$$

Interpret geometrically when the system has (i) no solution (ii) unique solution (iii) infinite number of solutions.

* **Unique Solution :-**

All three planes intersect in a common point.



* The system has an infinitely many solutions :-

(i) Three planes intersect in a common line.

(ii) Two planes are coincide and intersect the other plane.

(iii) All three planes are coincide.

The system has no solutions :-

(i) All planes are parallel

(ii) Two planes are coincide and parallel to the other plane.

(iii) Two planes are parallel and the other plane intersects them.

(iv) The line of intersection of two planes is parallel to the third plane.

Now let we have a system of linear equations.

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

Then the system of linear equations can be written as $A_{m\times n} X_{n\times 1} = B_{m\times 1}$

where $A_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & \vdots & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$ $X = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ $B = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}$

The entries of matrices A & B are from a field F.

Formally, a field is a set F together with two binary operations on F called addition and multiplication.[1] A binary operation on F is a mapping F × F → F, that is, a correspondence that associates with each ordered pair of elements of F a uniquely determined element of F.[2][3] The result of the addition of a and b is called the sum of a and b, and is denoted a + b. Similarly, the result of the multiplication of a and b is called the product of a and b, and is denoted ab or a · b. These operations are required to satisfy the following properties, referred to as field axioms (in these axioms, a, b, and c are arbitrary elements of the field F):

Associativity of addition and multiplication: a + (b + c) = (a + b) + c, and a · (b · c) = (a · b) · c.

Commutativity of addition and multiplication: a + b = b + a, and a · b = b · a.

Additive and multiplicative identity: there exist two different elements 0 and 1 in F such that a + 0 = a and a · 1 = a.

Additive inverses: for every a in F, there exists an element in F, denoted −a, called the additive inverse of a, such that a + (−a) = 0.

Multiplicative inverses: for every a ≠ 0 in F, there exists an element in F, denoted by a^{-1} [redacted] or 1/a, called the multiplicative inverse of a, such that [redacted] $a \cdot a^{-1} = 1$

Distributivity of multiplication over addition: a · (b + c) = (a · b) + (a · c).

This may be summarized by saying: a field has two operations, called addition and multiplication; it is an abelian group under addition with 0 as the additive identity; t he nonzero elements are an abelian group under multiplication with 1 as the multiplicative identity; and multiplication distributes over addition.

Matrix :-

In linear algebra, a **column vector** with m elements is an $m \times 1$ matrix[1] consisting of a single column of m entries, for example,

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}.$$

Similarly, a **row vector** is a $1 \times n$ matrix for some n, consisting of a single row of n entries,

$$\boldsymbol{a} = \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix}.$$

(Throughout this article, boldface is used for both row and column vectors.)

The transpose (indicated by T) of any row vector is a column vector, and the transpose of any column vector is a row vector:

$$\begin{bmatrix} x_1 \; x_2 \; \dots \; x_m \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

and

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} x_1 \; x_2 \; \dots \; x_m \end{bmatrix}.$$

The set of all row vectors with n entries in a given field (such as the real numbers) forms an n-dimensional vector space; similarly, the set of all column vectors with m entries forms an m-dimensional vector space.

The space of row vectors with n entries can be regarded as the dual space of the space of column vectors with n entries, since any linear functional on the space of column vectors can be represented as the left-multiplication of a unique row vector.

Size :- If $A_{m\times n}$ is a matrix then $m\times n$ is called the size of the matrix A.

order :- If A is a $n\times n$ square matrix then n is called the order of the matrix A.

Trace :- If $A = (a_{ij})_{n\times n}$ is a square matrix then $\text{Trace}(A) = a_{11} + a_{22} + \cdots + a_{nn}$.

Also recall, (i) upper & lower triangular matrix.

(ii) Adjoint of a matrix.

(iii) Determinant of a matrix.

(iv) Diagonal of a matrix. etc.