Indian Institute of Technology Delhi
MTL101 (Major Test)
May 2016

Max Time: 2 hours | Max Marks: 40

Note: No marks will be awarded without appropriate arguments.

1. Let $u_1 = (1,1,-1)$, $u_2 = (1,-1,1)$, $u_3 = (2,3,4)$, $u_4 = (a,1,6)$ and $v_1 = (1,2,3)$, $v_2 = (2,3,4)$, $v_3 = (3,4,5)$, $v_4 = (10,15,b)$. Find $a, b \in \mathbb{R}$ such that there is a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ satisfying $T(u_i) = v_i$ for $i = 1,2,3,4$. [4]

2. Justify whether the following statements are true or false. [$6 = 2 \times 3$]

   (a) Let $W_1 = \{(x, 2x, 3x) \in \mathbb{R}^3 : x \in \mathbb{R}\}$ and $W_2 = \{(2x, 3x+y, 3x-y) \in \mathbb{R}^3 : x, y \in \mathbb{R}\}$. Then $\mathbb{R}^3 = W_1 \oplus W_2$.

   (b) There exists a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that range$(T) = \ker(T)$.

   (c) The matrix $\begin{pmatrix} x & x \\ y & 2y \end{pmatrix}$ is diagonalizable over $\mathbb{R}$ for all $x, y \in \mathbb{R}$.

3. Consider the IVP: $y' = x + y,\ y(0) = 0.$ [$5 = 2 + 3$]

   (a) Find the solution of the given IVP.

   (b) Using the Picard's approximation, find a formula for the n-th iterate $y_n(x)$ for the above IVP. Also, show that $y_n(x)$ converges to the solution obtained in (a).

4. Find the general solution of the following ODE using the power series method. [5]

$$y'' - 2xy' - 2y = 0.$$

5. Find the inverse Laplace transform of [4]

$$F(s) = \frac{1}{(s^2+4)(s^2+2s+2)}.$$

6. Show, using the properties of Laplace transform, that [4]

$$\mathcal{L}\left[\frac{e^t}{n!}\frac{d^n}{dt^n}(t^n e^{-t})\right] = \frac{(s-1)^n}{s^{n+1}}.$$

7. Solve the following IVP [6]

$$y'' + 4y' + 5y = [1 - u(t-10)]e^t - e^{10}\delta(t-10); \quad y(0) = 0, y'(0) = 1.$$

8. Let $Y(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}$, $A = \begin{pmatrix} -4 & 12 \\ -3 & 8 \end{pmatrix}$ and $g(t) = e^{2t}\begin{pmatrix} 6t \\ 1-t \end{pmatrix}$. [$6 = 3 + 3$]

   (a) Find two linearly independent solutions for the homogeneous system $Y' = AY$.

   (b) Use variation of parameters formula, to find a particular solution for the nonhomogeneous system $Y' = AY + g(t)$.

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