

Problems for Recitation 22

1 Properties of Variance

In this problem we will study some properties of the variance and the standard deviation of random variables.

- a. Show that for any random variable R , $\text{Var}[R] = \mathbb{E}[R^2] - \mathbb{E}^2[R]$.
- b. Show that for any random variable R and constants a and b , $\text{Var}[aR + b] = a^2 \text{Var}[R]$. Conclude that the standard deviation of $aR + b$ is a times the standard deviation of R .
- c. Show that if R_1 and R_2 are independent random variables, then

$$\text{Var}[R_1 + R_2] = \text{Var}[R_1] + \text{Var}[R_2].$$

- d. Give an example of random variables R_1 and R_2 for which

$$\text{Var}[R_1 + R_2] \neq \text{Var}[R_1] + \text{Var}[R_2].$$

- e. Compute the variance and standard deviation of the Binomial distribution $H_{n,p}$ with parameters n and p .
- f. Let's say we have a random variable T such that $T = \sum_{j=1}^n T_j$, where all of the T_j 's are mutually independent and take values in the range $[0, 1]$. Prove that $\text{Var}(T) \leq \mathbb{E}(T)$. We'll use this result in lecture tomorrow. *Hint: Upper bound $\text{Var}[T_j]$ with $\mathbb{E}[T_j]$ using the definition of variance in part (a) and the rule for computing the expectation of a function of a random variable.*