

Tutorial Sheet 9

2. Proof By contradiction

Let v be a ~~node~~ vertex in a tournament digraph $G = (V, E)$ with maximum outdegree and suppose v is not a king.

Let $S = \{w : (v, w) \in E\}$ be the set of out-neighbours of v . Then, outdegree of $v = |S|$.

Since v is not a king, \exists a vertex $v' \notin S$, i.e. a vertex v' which v does not beat, and no out-neighbour of v beat v' . That is $(v, v') \notin E$ and $\forall w \in S, (w, v') \notin E$.

But by the definition of a tournament digraph, every vertex pair has exactly one edge between them.

$\therefore (v', v) \in E$ and $(v', w) \in E \forall w \in S$

that is, v and all out-neighbours of v are out-neighbours of v' . v' has an outgoing edge to v and to every vertex in S .

\therefore outdegree of $v' = |S| + 1 > |S| = \text{outdegree of } v$

which contradicts the assumption that v has the maximum outdegree in G .

Hence proved. ■