Group No:	Entry No:	Name:

Question-1 (4 marks):

Consider the following initial value problem (IVP)

$$\frac{dy}{dx} = x(1+y); y(0) = 1.$$

(a) Using Picard's method compute the first three successive approximations of the above IVP.

(b) Write the n-th approximation and justify it by induction (principle of mathematical induction).

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Question-2 (2 marks):

Suppose $f(x,y) = x - y^2 + e^x$ is defined on the rectangle $R = \{(x,y) : |x| \leq 1, |y| \leq 1\}$. Using the existence theorem for the initial value problem

$$\frac{dy}{dx} = f(x,y); y(0) = 0$$

find an $\alpha > 0$ such that there exists a solution of the IVP on the interval $(-\alpha, \alpha)$.

Group No:	Entry No:	Name:

Question-3 (4 marks):

Consider the vector space $\mathbb{R}^3$ over the field $\mathbb{R}$. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$T(x, y, z) = (3x + y, -2x, -x - y + 2z).$$

(a) Find all the eigenvalues of T.

(b) Justify whether T is diagonalizable.

(c) If T is diagonalizable, find an ordered basis B of $\mathbb{R}^3$ such that $[T]_B$, the matrix of T with respect to B, is diagonal.