DEPARTMENT OF MATHEMATICS

INDIAN INSTITUTE OF TECHNOLOGY DELHI

MTL101 (LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS)

2023-24 SECOND SEMESTER TUTORIAL SHEET-VII

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1. Which of the following functions are piecewise continuous? Which are of exponential order?
   (a) t^2 (b) e^{t^2} (c) e^{3t-2} (d) e^{t^3} (e) $e^{1/t}$ (f) $t\sin t$ (g) $\ln(t+1)$
   {Ans: EO (a),(c),(f),(g)}

2. Find the Laplace transform of the following:
   (a) $2t^2-6t+8$ (b) $\sinh 3t$ (c) $\sin^2 t$ (d) $\sqrt{t}$ (e) $\frac{\sin 2t}{2}$ (f) t^3e^{-3t} (g) $e^t\sin^2 t$ (h) $\cosh at\cos at$
   (i) $\sinh at\cos at$ (j) $\cos t\cos 2t$ (k) t^ne^{at} (l) $e^{-2t}(\cos t+6\sin t)$ (m) $te^{at}\sin at$
   {Ans: (a) $4s^3-6s^{-2}+8s^{-1}$ (b) $3/(s^2-9)$ (c) $2/s(s^2+4)$
   (d) $\sqrt{\pi}/2s^{1.5}$ (e) $1/(s^2+4)$ (f) $6/(s+3)^3$
   (g) $\frac{1}{2(s-1)}-\frac{s-1}{2(s-1)^2+8}$ (j) $\frac{s^2+5s}{(s^2+1)(s^2+9)}$ (m) $\frac{2as}{(s^2+a^2)^2}$ }

3. Find $f(t)$ whose Laplace transforms are the following:
   (a) $\frac{s}{s^2+\pi}$ (b) $\frac{s+5}{s^2+2s+5}$ (c) $\frac{1}{s(s^2+4)}$ (d) $\frac{1}{s^2(s^2-4)}$ (e) $\frac{2s-\pi}{s(s-\pi)}$ (f) $\frac{\pi^2}{s^4(s^2+\pi^2)}$ (g) $\frac{s^3}{s^4+64}$ (h) $\frac{1}{\sqrt{s}}$
   (i) $\ln\left(1+\frac{1}{s^2}\right)$ (j) $\frac{se^{-4s}}{(3s+2)(s-2)}$ (k) $\frac{6}{2s-3}-\frac{3+4s}{9s^2-16}+\frac{8-6s}{16s^2+9}$ (l) $\frac{5}{s^2}+\left(\frac{\sqrt{s}-1}{s}\right)^2-\frac{7}{3s+2}$
   { Ans: (a) $\cos(\sqrt{\pi t})$ (b) $e^{-t}(\cos 2t+2\sin 2t)$ (c) $0.5\sin^2 t$
   (d) $(1/8)(\cosh 2t-2t)$ (g) $\cosh 2t\cos 2t$
   (i) $2t^{-1}(1-\cos t)$ (j) $(1/12)e^{(-2/3)t}+(1/4)e^{2t}$
   (k) $3e^{1.5t}-0.25\sinh(4/3)t+(4/9)\cosh(4/3)t+(2/3)\sin(3/4)t-(3/8)\cos(3/4)t$}

4. Find the Laplace transform of the following functions:

   (a)
   $$f(t)=\begin{cases}\sin t & t\in(0,\pi)\\ 0 & t\geq\pi\end{cases}$$

   (b)
   $$f(t)=\begin{cases}(t-1)^2 & t>1\\ 0 & t\in(0,1)\end{cases}$$

   (c)
   $$f(t)=\begin{cases}e^t & t\in(0,5)\\ 3 & t>5\end{cases}$$

   (d)
   $$f(t)=\begin{cases}\cos(t-\frac{2}{3}) & t>\frac{2}{3}\\ 0 & t<\frac{2}{3}\end{cases}$$

{ Ans: (a) $\frac{1+e^{-\pi s}}{1+s^2}$ (c) $\frac{e^{5(1-s)}-1}{1-s}+3\frac{e^{-5s}}{s}$ }

5. Find (a) $\mathcal{L}\left(\frac{e^{-3t}\sin 2t}{t}\right)$ (b) $\mathcal{L}\left(e^{-3t}\int_0^t \frac{\sin 2t}{t}dt\right)$
{Ans: (a) $\cot^{-1}\frac{s+3}{2}$ (b) $\frac{1}{s+3}\tan^{-1}\frac{2}{s+3}$}

6. Using Laplace transform solve the following IVP's:

(a) $4y''+\pi^2 y=0,\ y(0)=2,\ y'(0)=0$
(b) $y''-2y'-3y=0,\ y(0)=2,\ y'(0)=0$
(c) $y''-ky'=0,\ y(0)=2,\ y'(0)=k$
(d) $y^{(4)}-y=1,\ y(0)=y'(0)=y''(0)=y'''(0)=0$
(e) $y'''+2y''-y'-2y=0,\ y(0)=1,\ y'(0)=y''(0)=4$

{Ans: (b) $0.5e^{3x}+1.5e^{-x}$ (d) $0.25(e^x+e^{-x})+0.5\cos x-1$
(e) $3e^x-3e^{-x}+e^{-2x}$}

7. Find Laplace transform inverse of the following:
(a) $\frac{s+12}{s^2+4s}$ (b) $\frac{3s^2-2s-1}{(s-2)(s^2+1)}$ (c) $\frac{s^3-2s^2+14s-9}{(s-1)^2(s-2)^3}$
{Ans: (a) $3-2e^{-4t}$ (b) $(7/5)e^{2t}+(8/5)\cos t+(6/5)\sin t$
(c) $-(5+4t)e^t+(20+19t)\frac{te^{2t}}{2}$}

8. Using Laplace transforms solve the following differential equations:

(a) $y''+4y'+5y=\delta(t-1);\ y(0)=0, y'(0)=3$
(b) $y''+16y=4\delta(t-\pi),\ y(0)=2,\ y'(0)=0$
(c) $y''+9y=8\sin t$, if $0<t<\pi$ and 0 if $t>\pi$, $y(0)=0,\ y'(0)=4$

{Ans: (a) $3e^{-2t}\sin t,\ 0<t<1;\ e^{-2(t-1)}\sin(t-1)+3e^{-2t}\sin t,\ t>1$
(c) $\sin t+\sin 3t,\ t<\pi;\ (4/3)\sin 3t,\ t>\pi$}

9. Using the Laplace of integrals, find the Laplace transform inverse of the following functions:
(a) $\frac{1}{s}\left(\frac{s-a}{s+a}\right)$ (b) $\frac{s}{(s^2+4)^2}$ (c) $\ln\frac{s^2+1}{(s-1)^2}$ (d) $\ln\left(\frac{s+a}{s+b}\right)$ (e) $\cot^{-1}\left(\frac{s}{w}\right)$
{Ans: (b) $\frac{t\sin 2t}{4}$ (d) $\frac{e^{-bt}-e^{-at}}{t}$ (e) $\frac{sin\omega t}{t}$ }