MTL101: Quiz II, Semester I, 2016-17
November 08, 2016

Max Marks: 10 Max Time: 30 minutes

- Answer the question in only the separate answer sheet provided to you.
- Do mention SET number on TOP of your answer script; failing which (-2) mark will be awarded to you.
- The final answer to each question must be clearly mentioned in the first line of the answer.
- Each answer must be supported by its working in brief. You must clearly mention the formula and all important steps involved, failing which no marks will be awarded even if your reported final answer is correct.
- If you commit some mistake in calculations/working and arrive at the final answer which is wrong, then also no marks will be awarded for partially correct working of the answer.
- Each question is of 2 marks. For each answer, either you score 0 or full marks 2; there is no step marking.
- You must answer questions in order starting from 1, 2, 3, ... only; (-2) marks will be awarded if you do not answer questions in the order specified.
- If any of you is found in talking even for a second or cheating of any form then on spot (-20) marks will be awarded without any warning.
- At the end of exam, keep sitting and the invigilator will come to you to collect the paper. Do not rush towards the invigilator.

SET: 1

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1. Let $V = C[-1,1]$, $F = \mathbb{R}$, and the inner product be defined as $\langle \phi, \psi \rangle = \int_{-1}^{1} \phi(x)\psi(x)\,dx$. If $f(x) = 7x^3 + \lambda x$ and $g(x) = x^3$ are orthogonal in V then $||f||^2$ equals __________.

2. Let $V = M_{2\times 2}(\mathbb{R})$, $F = \mathbb{R}$, and the inner product be defined by $\langle A, B \rangle = \text{trace}(AB^t)$. Let $S = \left\{ \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \right\}$. Then, by the Gram-Schmidt orthogonalization process, the orthogonal set corresponding to S is __________.

3. Let $V = \mathbb{R}^3$, $F = \mathbb{R}$, $W = \{(x, y, z) \in V : x + y - z = 0, x - y + z = 0\}$. The vector in W closest to the vector $v = (1, 0, -1)$ is __________.

4. Consider the standard inner product on $V = \mathbb{C}^3$ over the field $F = \mathbb{C}$. The orthogonal complement of the subset $W = \{(1, i, 2i), (i, -1, 2)\}$ of V is __________.

5. Let u, v, w be vectors in an inner product space V satisfying $||u|| = ||v|| = ||w|| = 2$ and $\langle u, v \rangle = 0$, $\langle u, w \rangle = 1$, $\langle v, w \rangle = -1$. Then the set $S = \{(\alpha, \beta, \gamma) \in \mathbb{R}^3 : \alpha u + \beta v + \gamma w = 0\}$ is equal to __________.

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SET: 2

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1. Let $V = C[-1,1]$, $F = \mathbb{R}$, and the inner product be defined as $\langle \phi, \psi \rangle = \int_{-1}^{1} x^2 \phi(x)\psi(x)\,dx$. If $f(x) = x^3 + 5\lambda x$ and $g(x) = x$ are orthogonal in V then $||f||^2$ equals __________.

2. Let $V = M_{2\times 2}(\mathbb{R})$, $F = \mathbb{R}$, and the inner product be defined by $\langle A, B \rangle = \text{trace}(A^t B)$. Let $S = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} \right\}$. Then, by the Gram-Schmidt orthogonalization process, the orthogonal set corresponding to S is __________.

3. Let $V = \mathbb{R}^3$, $F = \mathbb{R}$, $W = \{(x,y,z) \in V : 2x + y - z = 0,\ x - 2y + z = 0\}$. The vector in W closest to the vector $v = (0,0,1)$ is __________.

4. Consider the standard inner product on $V = \mathbb{C}^3$ over the field $F = \mathbb{C}$. The orthogonal complement of the subset $W = \{(1,\ i,\ 0), (0,\ 1,\ -1)\}$ of V is __________.

5. If u and v are vectors such that $||u|| = \sqrt{2}$, $||v|| = \sqrt{3}$ and the angle between them is $\frac{\pi}{6}$, then the angle between the vectors $u + v$ and $u - v$ is __________.

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SET: 3

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1. Let $V = \mathbb{C}^2$, $F = \mathbb{C}$, and the inner product be defined by $\langle v_1, v_2 \rangle = v_1 A \overline{v_2}^t$, where $A = \begin{pmatrix} 2 & i \\ -i & 2 \end{pmatrix}$ is the given matrix. If $v_1 = (i + \lambda, \mu - i)$, $\lambda, \mu \in \mathbb{C}$, and $v_2 = (1 + i, 1 - i)$ are orthogonal with respect to the given inner product then the set of ordered pairs (λ, μ) equals (1, 1)

2. Let $V = M_{2\times 2}(\mathbb{R})$, $F = \mathbb{R}$, and the inner product be defined by $\langle A, B \rangle = \text{trace}(AB^t)$. Let $S = \left\{ \begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix} \right\}$. Then, by the Gram-Schmidt orthogonalization process, the orthogonal set corresponding to S is $\frac{-1}{\sqrt{2}}$. $\frac{1}{\sqrt{2}}$

3. Let $V = \mathbb{R}^3$, $F = \mathbb{R}$, $W = \{(x, y, z) \in V : x - y + z = 0\}$. The orthogonal complement of W is the subspace {x − i n}

4. Let $V = M_{2\times 3}(\mathbb{C})$, $F = \mathbb{C}$, and the norm be defined as $||A||_1 = \max_{i=1,2} \sum_{j=1}^{3} |a_{ij}|$. For $\lambda > 0$, if $X = \begin{pmatrix} 1 & i & -2\lambda i \\ 3 + 4i & \lambda & -i \end{pmatrix}$, then the set $S = \{\lambda : ||X||_1 = 10\}$ is equal to ________.

5. Let u, v, w be vectors in an inner product space V satisfying $||u|| = ||v|| = ||w|| = \sqrt{2}$ and $\langle u, v \rangle = 1$, $\langle u, w \rangle = -1$, $\langle v, w \rangle = -2$. Then the set $S = \{(\alpha, \beta, \gamma) \in \mathbb{R}^3 : \alpha u + \beta v + \gamma w = 0\}$ is equal to ________.

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SET: 4

1. Let $V = \mathbb{C}^2$, $F = \mathbb{C}$, and the inner product be defined by $\langle v_1, v_2\rangle = v_1 A \overline{v_2}^t$, where $A = \begin{pmatrix} 2 & -i \\ i & 2 \end{pmatrix}$ is the given matrix. If $v_1 = (2+i, 2-i)$ and $v_2 = (\lambda + i, \mu - i)$, $\lambda, \mu \in \mathbb{C}$, are orthogonal with respect to the given inner product then the set of ordered pairs (λ, μ) equals ________.

2. Let $V = P_2(\mathbb{R})$ (real polynomials of degree less than or equal to 2), $F = \mathbb{R}$, and the inner product be defined by $\langle p, g\rangle = \int_0^1 p(x)g(x)dx$. Let $S = \{1+x, x-x^2\}$. Then, by the Gram-Schmidt orthogonalization process, the orthogonal set corresponding to S is ________.

3. Let $V = \mathbb{R}^3$, $F = \mathbb{R}$, $W = \{(x,y,z) \in V : x + 2y - z = 0\}$. The orthogonal complement of W is the subspace ________.

4. Let $V = M_{3\times 2}(\mathbb{C})$, $F = \mathbb{C}$, and the norm be defined as $||A||_1 = \max_{j=1,2} \sum_{i=1}^{3} |a_{ij}|$. For $\lambda > 0$, if $X = \begin{pmatrix} 1 & i \\ 3+4i & \lambda i \\ -i & 2\lambda \end{pmatrix}$, then the set $S = \{\lambda : ||X||_1 = 25\}$ is equal to ________.

5. If u and v are vectors such that $||u|| = 1$, $||v|| = \sqrt{3}$ and the angle between them is $\frac{\pi}{6}$, then the angle between the vectors $u + 2v$ and $2u + v$ is ________.

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SET: 5

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1. Let $V = M_{2\times2}(\mathbb{R})$, $F = \mathbb{R}$, and the inner product be defined by $\langle A, B\rangle = \text{trace}(AB^t)$. If the matrix $\underbrace{\begin{pmatrix} \lambda & 0 \\ 1 & \lambda \end{pmatrix}}_{A}$ and $\begin{pmatrix} 1 & \lambda \\ 1 & 1 \end{pmatrix}$ are orthogonal then $||A||^2$ is equal to ________.

2. Let $V = P_2(\mathbb{R})$ (real polynomials of degree less than or equal to 2), $F = \mathbb{R}$, and the inner product be defined by $\langle p, g\rangle = a_0b_0 + a_1b_1 + a_2b_2$, $p(x) = a_0 + a_1x + a_2x^2$, $g(x) = b_0 + b_1x + b_2x^2$. Then, by the Gram-Schmidt orthogonalization process, the orthogonal set corresponding to $S = \{1 + x, x - x^2\}$ is ________.

3. Let $V = \mathbb{R}^3$, $F = \mathbb{R}$, $W = \{(x, y, z) \in V : 2x + y - z = 0, x - y + 2z = 0\}$. The vector in W closest to the vector $v = (1, 0, 2)$ is ________.

4. Consider the standard inner product on $V = \mathbb{C}^3$ over the field $F = \mathbb{C}$. The orthogonal complement of the subset $W = \{(i, -i, 1), (1, i, -i)\}$ of V is ________.

5. If u and v are vectors such that $||u|| = 3$, $||v|| = \sqrt{2}$ and the angle between them is $\frac{\pi}{4}$, then the angle between the vectors $2u + v$ and $u - 2v$ is ________.