Name: _____

Entry number: _____

There are 2 questions for a total of 10 points.

1. Solve the following problems. Show the working in the space provided.
   (a) (1 point) What is the last digit of 7^{150}?

   (a) _____

   (b) (1 point) What is the value of $(2^{60} \cdot 3^{100}) \ (mod \ 5)$?

   (b) _____

   (c) (1 point) What is the remainder when $\sum_{i=1}^{50}(2i)!$ is divided by 9?

   (c) _____

   (d) (2 points) <u>Prove or disprove</u>: $(2^n + 6 \cdot 9^n)$ is divisible by 7 for every $n \geq 0$.

2. (5 points) Consider the following problem:

> HALTING-INPUT: Given the description $\langle A \rangle$ of algorithm A, determine if there is a halting input for A (that is, there exists an input on which A halts).

An algorithm P is said to solve the above problem if $P(\langle A \rangle)$ halts and outputs 1 when A has a halting input, and it halts and outputs 0 otherwise.

<u>Prove</u>: There does not exist an algorithm P that solves the problem HALTING-INPUT.