2202-MTL101B Quiz 1

Tatsam

TOTAL POINTS

20 / 20

QUESTION 1

1 Linear System 6 / 6

✓ **- 0 pts** *Correct [k=4 & (1+3r/2, 1-5r/2,r) is the solution]*

- 1 pts Some minor mistake

- 2 pts K=4 but the final solution is wrong or incomplete

- 4 pts The value of k is NOT correct, and the solution is NOT correct. Marks have been awarded for the process.

- 6 pts Wrong

QUESTION 2

2 Inverse of the matrix 6 / 6

✓ **+ 6 pts** *Answer is Correct.*

+ 1 pts Step1: to determine correctly that the matrix A is invertible.

+ 1 pts Step2: for explaining correctly that the inverse of the matrix A can be obtained by applying the same elementary row operations on identity matrix I which reduces A to I.

+ 4 pts Step3: applying correct elementary row operations which reduces A to I and I to inverse of A.

- 1 pts Minor mistake in Step3.

- 2 pts Major mistake in Step3.

+ 0 pts Not attempted/ incorrect answer

QUESTION 3

3 Vector Space 4 / 4

+ 0 pts Incorrect/deserves no partial marks.

+ 0 pts Not attempted.

✓ **+ 4 pts** *Verified correctly that the commutativity axiom does not hold. (Or figured out an axiom which does not hold with details)*

+ 1 pts Verified some axioms of vector space, but no correct conclusion.

QUESTION 4

4 Subspace 4 / 4

✓ **- 0 pts** *Correct*

- 4 pts Unattempted

- 4 pts Completely incorrect or no justification

- 3 pts Tried to check subspace criterion but wrong conclusion

- 1 pts Minor mistake

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Prob 1) This system of equation can be expressed as $AX = B$.

with $A = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 1 & -2 \\ 2 & 4 & 7 \end{pmatrix}$ $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $B = \begin{pmatrix} 2 \\ k \\ k+2 \end{pmatrix}$

augmented matrix $(A|B) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 3 & 1 & -2 & k \\ 2 & 4 & 7 & k+2 \end{array}\right)$

I will do elementary row operations ~~to get the row reduced echelon form of (A|B)~~.

$R_2 \longrightarrow R_2 - 3R_1$
$R_3 \longrightarrow R_3 - 2R_1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -2 & -5 & k-6 \\ 0 & 2 & 5 & k-2 \end{array}\right)$$

$R_3 \longrightarrow R_3 + R_2$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -2 & -5 & k-6 \\ 0 & 0 & 0 & 2k-8 \end{array}\right)$$

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we know, for the 3 planes to intersect in a line, we must have ∞ solutions, i.e

$$\text{rank}(A) = \text{rank}(A|B) < \underset{\substack{\downarrow \\ \text{no. of unknowns.}}}{n}$$

Also, since none of the planes are concurrent { $\because$ no row of A is a scalar multiple of the other $\Rightarrow$ if we get ∞ sol^n $\Rightarrow$ we get intersection on a line }

we see $|A| = 0$

$\therefore$ rank(A) = 2 [choose $\begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix}$ as a submatrix]

Hence rank (A|B) = 2

and $\therefore$ $2k - 8 = 0$

$\boxed{k = 4}$ otherwise rank (A|B) = 3 and no sol^n.

$$(A|B) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -2 & -5 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad \left\{\begin{array}{l} x = x_1 \\ y = x_2 \\ z = x_3 \end{array}\right\}$$

we can see x_3 is the free variable

let $x_3 = \lambda$

we have $(-2)x_2 - 5(x_3) = -2.$

$$-2x_2 = 5\lambda - 2.$$

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$$x_2 = \frac{2-5\lambda}{2}$$

and $x_1 + x_2 + x_3 = 2$

$$x_1 = 2 - x_2 - x_3 = 2 - \left(\frac{2-5\lambda}{2}\right) - \lambda$$

$$= \frac{4 - 2 + 5\lambda - 2\lambda}{2} = \frac{3\lambda + 2}{2}$$

The solution set is

$$\boxed{\begin{aligned} x_1 &= \frac{3\lambda}{2} + 1 \\ x_2 &= 1 - \frac{5\lambda}{2} \\ x_3 &= \lambda \end{aligned}} \quad \forall \lambda \in \mathbb{R}$$

$$\left\{\begin{aligned} x &= x_1 \\ y &= x_2 \\ z &= x_3 \end{aligned}\right\}$$

this gives a line

$$\boxed{\frac{x-1}{3/2} = \frac{y-1}{-5/2} = z}$$

1 Linear System **6 / 6**

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- 1 pts Some minor mistake

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Prob 2) $A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ A^{-1} exists if the row reduced echlon form of A is $\underline{I}$.

let's write $(A|I_{3\times3}) = \left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 & 1 & 0 \\ -1 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$

$R_3 \to R_3 + R_2$

$R_1 \to R_1 + R_2$

$$\left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 1 & 1 \end{array}\right)$$

$R_1 \to \frac{1}{2} R_1$ $R_3 \to \frac{1}{2} R_3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 1 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} \end{array}\right)$$

$R_2 \to R_2 - R_1$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 1/2 & 0 \\ 0 & -1 & 1 & -1/2 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 1/2 & 1/2 \end{array}\right)$$

$R_2 \to -R_2$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 1/2 & 0 \\ 0 & 1 & -1 & 1/2 & -1/2 & 0 \\ 0 & 0 & 1 & 0 & 1/2 & 1/2 \end{array}\right)$$

$R_2 \to R_2 + R_3$

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$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 1/2 & 0 \\ 0 & 1 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 1 & 0 & 1/2 & 1/2 \end{array}\right)$$

$$\underbrace{\qquad\qquad}_{B}$$

$B = A^{-1}$

$\because$ we performed all the row operations on I just like on A.

$$\underbrace{f_1 f_2 f_3 f_4 f_5 f_6 f_7}_{f}(A) = I$$

$$f(A) = I$$

$$\underbrace{f(I)}_{A^{-1}} A = I \qquad \because A^{-1}A = I$$

$\therefore A^{-1} = f_1 f_2 f_3 f_4 f_5 f_6 f_7 (I)$

$$A^{-1} = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1/2 & 1/2 \end{pmatrix}$$

2 Inverse of the matrix 6 / 6

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Prob 3) for $\mathbb{R}^3$ to be a vector space over $\mathbb{R}$, it must satisfy the follow properties.

let $(x_1 x_2 x_3)$ & $(y_1 y_2 y_3) \in \mathbb{R}^3$

1. Closure over + and ·

$$(x_1, x_2 x_3) + (y_1 y_2 y_3) = (x_1 + 2y_1, x_2 + y_2, x_3 + y_3)$$
$$= (t_1, t_2, t_3)$$

where $\left.\begin{array}{l} t_1 = x_1 + 2y_1 \\ t_2 = x_2 + y_2 \\ t_3 = x_3 + y_3 \end{array}\right\} \in \mathbb{R} \Rightarrow (t_1 t_2 t_3) \in \mathbb{R}^3$

satisfied ✓

$\alpha \in \mathbb{R}$ $\quad \alpha \cdot (x_1 x_2 x_3) = (\alpha x_1, \alpha x_2, \alpha x_3)$
$$= (t_1 \; t_2 \; t_3)$$

$\left.\begin{array}{l} t_1 = \alpha x_1 \\ t_2 = \alpha x_2 \\ t_3 = \alpha x_3 \end{array}\right\} \in \mathbb{R} \Rightarrow (t_1 t_2 t_3) \in \mathbb{R}^3$

satisfied ✓

2. Commutative over +

$$(x_1 x_2 x_3) + (y_1 y_2 y_3) = (x_1 + 2y_1, x_2 + y_2, x_3 + y_3)$$

$$(y_1 y_2 y_3) + (x_1 x_2 x_3) = (y_1 + 2x_1, y_2 + x_2, y_3 + x_3)$$

However, $x_1 + 2y_1 \neq y_1 + 2x_1$

$\forall x_1, y_1 \in \mathbb{R}$ [it should have satified $\forall x, y$ and not specific situations]

Hence we need not check any further property like distributive law, additive / multiplicative identities, additive inverse,

$x_i + 0 = x_i$ $\quad 0 \in \mathbb{R}$ $\quad x_i \in \mathbb{R}^3$ $\qquad 1 \cdot x_i = x_i$ $\quad 1 \in \mathbb{R}$ $\quad x_i \in \mathbb{R}$ $\qquad x_i - x_i = 0$

$\alpha \in \mathbb{R}$ $\quad x_i y_i \in \mathbb{R}^3$

$\alpha(x_i + y_i) = \alpha x_i + \alpha y_i$

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i.e. $(x_i)+((y_i)+(z_i)) = ((x_i)+(y_i))+(z_i)$ $x_i y_i z_i \in \mathbb{R}^3$

$(\alpha)(\beta x_i) = (\alpha\beta)x_i$ $x_i \in \mathbb{R}^3$ $\alpha, \beta \in \mathbb{R}$

associativity over '+' & '.' etc. since this $\mathbb{R}^3$ above these operations is not commutative and hence can not be a vector space over field $\mathbb{R}$.

prob 4) $V = C([0,1])$: over $\mathbb{R}$ of all real valued continous functions on $[0,1]$.

$\cancel{W = \{a_0 + a_1x + a_2x^2 + \cdots a x}$

on $[0,1]$

$W = \{ p : p(x)$ is a real polynomial & with odd degree $\geq 1 \} \cup \{0\}$

In order for W to be a subspace :

1. $W \neq \Phi$
2. W must contain 0 (i.e additive identity)
3. $\forall \alpha \in \mathbb{R}$ $V_1, V_2 \in W$
   $\alpha V_1 + V_2 \in W$

We see $0 \in W$ and $x^3 \in W$ $\therefore W \neq \Phi$
now we need to check if $\alpha V_1 + V_2 \in W$
$\forall \alpha \in \mathbb{R}$ $V_1 V_2 \in W$.

$\left\{ \begin{array}{l} \text{All real polynomials are real valued continous} \\ \text{fncs so, } W \subset V. \end{array} \right\}$

let us take $\alpha = 1$
$V_1 = 1 + x^3 + x^4 + x^5$ $V_2 = 1 + x^3 + x^4 - x^5$

3 Vector Space **4 / 4**

+ 0 pts Incorrect/deserves no partial marks.

+ 0 pts Not attempted.

✓ **+ 4 pts** *Verified correctly that the commutativity axiom does not hold. (Or figured out an axiom which does not hold with details)*

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i.e. $(x_i)+((y_i)+(z_i)) = ((x_i)+(y_i))+(z_i)$ $x_i y_i z_i \in \mathbb{R}^3$

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associativity over '+' & '.' etc. since this $\mathbb{R}^3$ above these operations is not commutative and hence can not be a vector space over field $\mathbb{R}$.

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~~$W = \{a_0 + a_1 x + a_2 x^2 + \dots a x$~~

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In order for W to be a subspace :

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③ $\forall \alpha \in \mathbb{R}$ $V_1, V_2 \in W$
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$\forall \alpha \in \mathbb{R}$ $V_1 V_2 \in W$.

{All real polynomials are real valued continous fncs so, $W \subset V$.}

let us take $\alpha = 1$
$V_1 = 1 + x^3 + x^4 + x^5$ $V_2 = 1 + x^3 + x^4 - x^5$

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for $x \in [0,1]$

both $v_1 v_2 \in W$ as degree $=5$, i.e odd

$\therefore \alpha v_1 + v_2 = 2 + 2x^3 + 2x^4$

this polynomial has degree 4, i.e not odd.

$\therefore \alpha v_1 + v_2 \notin W$

We found at least one case, which is enough to prove W is not a subspace of V over $\mathbb{R}$.

4 Subspace **4 / 4**

- ✓ **- 0 pts** *Correct*
- **- 4 pts** Unattempted
- **- 4 pts** Completely incorrect or no justification
- **- 3 pts** Tried to check subspace criterion but wrong conclusion
- **- 1 pts** Minor mistake

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MINOR TEST ANSWER BOOK

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प्रयोग किए गए अनुवर्ती पृष्ठों की संख्या
No. of continuation sheets used

प्रश्न सं. Q. No.	प्राप्त अंक Marks
1.	
2.	
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10.	
कुल TOTAL	

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