MTL 101 Linear Algebra and Differential Equations 2023-24 Second Semester

Quiz-III, April 18, 2024

Marks Obtained:

Maximum Marks: 15 Time: 45 Minutes

Name: Group No. Entry No.

Instructions: No additional sheet will be provided. All notations are standard. Use of any electronic gadget including calculator is NOT allowed. No query will be entertained.

Q1a: Construct the homogeneous differential equation having $e^x \cos x, xe^x \cos x, e^x \sin x, xe^x \sin x$ as a basis (or fundamental system) of solutions. (2)

Q1b: Let $a_1(x), a_2(x)$ be continuous functions on an open interval $I \subseteq \mathbb{R}$, and let y_1 and y_2 be two solutions of linear second order differential equation $y'' + a_1(x)y' + a_2(x)y = 0$ that satisfy the conditions $y_1(x_0) = 1$, $y_1'(x_0) = 0$, $y_2(x_0) = 0$, $y_2'(x_0) = 1$, respectively for some point x_0 in I. Prove or disprove that y_1 and y_2 are linearly independent on I. (2)

Q2: Given that $y = x(\neq 0)$ is a solution of

$$(x^2+1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0,$$

find the other linearly independent solution by reducing the order, and hence write the general solution of the differential equation. (4)

Q3: Solve the initial value problem (4)

$$y'' + \omega^2 y = a\cos bx, \ \ b \neq \omega; \ \ y(0) = y'(0) = 0.$$

Q4: Solve the initial value problem $y' = Ay$,

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, y(0) = \begin{bmatrix} 5 \\ -7 \end{bmatrix}$$

by determining the eigenvalues and eigenvectors of A. (4)