Minor Test 1 ::: MTL 101 ::: February 2016

Every question is compulsory.
Marks will not be awarded if appropriate arguments are not provided.
Do not write the questions. Write answers only.
Do not waste time describing what is not asked.

Maximum Marks: **20**

Maximum Time: **one hour**

(1) Four persons A, B, C and D bought four articles P, Q, R and S and the following chart indicates the quantity of the individual articles they bought:

	P	Q	R	S
A	1	2	1	2
B	2	1	3	1
C	3	2	1	1
D	3	5	0	4

If A, B, C spent 34 Rs, 38 Rs and 35 Rs respectively, find how much D spent. [3]

(2) "If S is a linearly independent set of vectors in any vector space V, then V has a basis containing S." Use this statement to prove the following statement if W is a proper subspace of a finite dimensional vector space V:

There is a subspace W' of V such that $W \oplus W' = V$. [2]

(3) Suppose $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation satisfying the following

$$T(1,0,0) = (1,2,3), \qquad T(1,1,0) = (1,-1,1), \qquad T(1,1,1) = (3,0,5).$$

Find the rank and the nullity of T. [4]

(4) Find whether the following statements are true or false. Justify in each case. [4]

(a) If u, v, w are vectors then the set $\{u+v+w, u+2v+3w, u+3v+5w, u+4v+8w\}$ is linearly dependent.

(b) If A and B are subsets of a vector space satisfying $\mathrm{span}(A) = \mathrm{span}(B)$, then either $A \subseteq B$ or $B \subseteq A$.

(5) Let W_1 and W_2 be subspaces of $\mathbb{R}^4$ defined by

$$\begin{aligned} W_1 &= \{(x,y,z,w) \in \mathbb{R}^4 : x+y+z+w = 0, x-y+z-w = 0\}, \\ W_2 &= \{(x,y,z,w) \in \mathbb{R}^4 : x+z = 0, x+2w = 0\}. \end{aligned}$$

Find a basis of $W_1 + W_2$. [4]

(6) Find all possible values of λ for which the linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ given by

$$T(x,y,z,w) = (x+y+z+w,\ x-y+2z+3w,\ x+y-z+\lambda w).$$

is surjective (i.e., onto). [3]

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