**Name:** ________________ **Group No.**_________ **Entry No.**_______________

# DEPARTMENT OF MATHEMATICS

INDIAN INSTIUTUTE OF TECHNOLOGY DELHI

MID SEMESTER EXAMINATION 2023-24 SECOND SEMESTER

MTL101 (LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS)

**Time: 2 hours** **Max. Marks : 40**

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Write your Name and Group number and Entry number at the places specified above. Attempt all questions. All notations are standard. All parts of a question must be answered at one place. Exhibit clearly all the steps. Use of any electronic gadget including calculator is NOT allowed. Attach question paper with the answer book. Do not do any rough work on the question paper. No query will be entertained.

**Q1:** Let $V$ be a real inner product space, and let $\{w_1, w_2 \dots \dots \dots w_m\}$ be a orthonormal basis for a subspace $W$ of $V$. Let $P: V \to W$ be a linear transformation defined by

$$Pv = \sum_{j=1}^{m} \langle v, \omega_j \rangle \omega_j, \qquad v \in V$$

Show that

**a.** $v - Pv \in W^{\perp}$, for all $v \in V$. (3)

**b.** $\|v - Pv\| \le \|v - w\|$ , for all $w \in W$. (3)

**Q2a:** Let $\lambda_1$ and $\lambda_2$ be two distinct eigenvalues of $A$, and let $v_1$ and $v_2$ be the corresponding eigenvectors, respectively. Prove or disprove that $v_1 + v_2$ can never be an eigenvector of $A$. (3)

**Q2b:** Let $A$ be diagonalizable matrix such that every eigenvalue of $A$ is either $0$ or $-1$. Prove or disprove that $A^3 = A$. (2)

**Q3a:** Prove that the eigenvectors of a real symmetric matric corresponding to distinct eigenvalues are orthogonal. (2)

**Q3b:** Let $A = B + iC$, where $B$ and $C$ are Hermitian, and $i = \sqrt{-1}$. Show that $A$ is normal iff $BC = CB$. (3)

**Q4a:** Discuss the existence and uniqueness of solution of the initial value problem

$$\frac{dy}{dx} = \frac{\cos y}{1 - x^2} \text{ , } y(0) = 0$$

in the domain $R = \left\{(x, y): |x| \leq \frac{1}{2}, |y| \leq 4\right\}$. (2)

**Q4b:** Find the orthogonal trajectories of the family of curves given by $y = Ce^{x^2}$. (2)

**Q5:** Find the general solution of (4)

$$\frac{d^3y}{dx^3} + \frac{dy}{dx} = \sec^2 x, \quad 0 < x < \frac{\pi}{2}.$$

**Q6:** Find the inverse Laplace transform of (3+2)

**a.** $\ln\left(1 + \frac{1}{s}\right)$ **b.** $\frac{1}{s\sqrt{s+1}}$

**Q7a:** Solve the initial value problem (3)

$$\frac{d^2y}{dt^2} + 4y = h(t)$$

Where

$$h(t) = \begin{cases} 0, & if\ 0 < t < 5 \\ 4, & if\ t > 5 \end{cases},$$

$$y(0) = 0,\ y'(0) = 1$$

**Q7b:** Solve (3)

$$\frac{dy}{dt} + y(t) = \int_0^t \sin(t - \tau)y(\tau)\,d\tau, \quad y(0) = 1.$$

**Q8:** Using the method of undetermined coefficients solve the system of linear differential equations (5)

$$\frac{dy_1}{dt} = -3y_1 - 8y_2 + 3e^{2t}$$

$$\frac{dy_2}{dt} = 2y_1 + 5y_2 + 4e^{-t}$$