Name: _____

ID number: _____

There are 1 questions for a total of 10 points.

---

1. Consider a simple hashing example where the universe is $U = \{0, 1, 2, 3, 4, 5\}$ and elements from this universe are supposed to be stored in a table of size 3 with indices $T = \{0, 1, 2\}$. Consider the following hash function family in this context:

$$H = \{h_{a,b} \mid a \in \{1, 2, 3, 4, 5\} \text{ and } b \in \{0, 1, 2, 3, 4, 5\}\} \quad \text{where} \quad h_{a,b}(x) = ((a \cdot x + b) \ mod \ 6) \ mod \ 3$$

As discussed in class, a random hash function from this family $h_{a,b}$ is picked by choosing a from set $\{1, 2, 3, 4, 5\}$ and b from set $\{0, 1, 2, 3, 4, 5\}$ uniformly at random. Answer the following:

(a) (2 points) State true or false: H is a 2-universal hash function family.

(a) **False**

(b) (8 points) Give reason for your answer to part (a).

Solution: Table 1 gives the value of $h_{a,b}(0)$ for various values of a, b. Table 2 gives the value of $h_{a,b}(3)$ for various values of a, b.

$a \backslash b$	0	1	2	3	4	5
1	0	1	2	0	1	2
2	0	1	2	0	1	2
3	0	1	2	0	1	2
4	0	1	2	0	1	2
5	0	1	2	0	1	2

Table 1: Table for $h_{a,b}(0)$

$a \backslash b$	0	1	2	3	4	5
1	0	1	2	0	1	2
2	0	1	2	0	1	2
3	0	1	2	0	1	2
4	0	1	2	0	1	2
5	0	1	2	0	1	2

Table 2: Table for $h_{a,b}(3)$

From these values, we can see that:

$$\mathbf{Pr}_{h_{a,b} \leftarrow H}[h_{a,b}(0) = h_{a,b}(3)] = 1 > \frac{1}{3}.$$

Hence H is not a 2-universal hash function family.