Name: $GUBASH KUMAR$

Entry No: _____

Indian Institute of Technology Delhi

Minor-I Exam

MTL-101 (Linear Algebra and Differential Equation)

Time: 1 Hour

Max. Marks: 25

Write your name and entry number at the place specified above. Attempt all questions. All notations are standard. All parts of a question must be answered at one place. Exhibit clearly all the steps. Any electronic gadget including calculator, mobile, etc. is NOT allowed. Attach the question paper with the answer book. No query will be entertained.

1. Let $V = C^3$, $F = C$, $W = L(S)$-linear span of $S = \{v_1, v_2, v_3, v_4\}$, where $v_1 = (1+i, 1-i, 2+3i)$; $v_2 = (i, 1, 2)$; $v_3 = (1-i, -1-i, 3-2i)$ and $v_4 = (4, -4i, 10+2i)$. Find a smallest subset T of S such that $W = L(T)$.

2. Let $V = R^5$, $F = R$, $v_1 = (1,1,1,1,1)$, $v_2 = (0,1,1,1,1)$, $v_3 = (0,0,1,1,1)$, $v_4 = (0,0,0,1,1)$, and $v_5 = (0,0,0,0,1)$. If possible express $v = (5,4,3,2,1) \in V$ as a linear combination of v_is over F.

3. Let $V = R^3$, $F = R$ and $T: V \to V$ be the mapping

$$(x, y, z) \to (x+y, -y+2z, x+2y-2z)$$

   (a) Is T a homomorphism of V to V?

   (b) Determine Kernel (T). Is T one-one?

4. (a) Determine if the matrices $\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ are linearly independent over V?

   (b) Let $V = R^4$, $F = R$, $W = \{(x, y, z, u) \in V \mid x+y+z+u = 0\}$. Is W a subspace of V?

5. Let V and W be vector spaces over a field F. If T is a homomorphism of V onto W with kernel K, then prove that $\frac{V}{K}$ is isomorphic onto W.