MTL 101
LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS
RE-MINOR EXAM

Total: 20 Marks   **Time: 1:00 Hrs.**

Question 1:   **(4 Marks)**

a) Let $v_1, \ldots, v_5$ be nonzero vectors in $\mathbb{R}^5$. Let $W = \{a_1 v_1 + \cdots a_5 v_5 \mid a_i \in \mathbb{R}, a_1 + a_2 + a_3 + a_4 + a_5 = 0\}$. Show that W is a subspace of $\mathbb{R}^5$.

b) Suppose $S = \{0, 1, 2, 3, 4\}$ and $V = \{f : S \to \mathbb{R}\}$ is a collection of all functions from S to $\mathbb{R}$. Find a basis for the vector space $V(\mathbb{R})$.

Question 2: (3 Marks) Let $B = \{v_1, v_2, v_3\}$ be a basis of the vector space $\mathbb{R}^3(\mathbb{R})$. Suppose $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation such that the matrix $[T]_B$ of T with respect to the basis B is given by

$$[T]_B = \begin{pmatrix} -1 & -1 & -3 \\ 11 & -2 & -6 \\ 21 & 1 & 3 \end{pmatrix}.$$

Find the kernel of T.

Question 3: (5 Marks) Consider the function $f : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ defined by

$$f((x, y, z), (x', y', z')) = 2xx' + xz' + zx' + yy' + zz'.$$

(a) Prove that f is an inner product on the vector space $\mathbb{R}^3(\mathbb{R})$.

(b) Let $B = \{e_1 = (1, 0, 0), e_2 = (0, 1, 0), e_3 = (0, 0, 1)\}$ be the standard basis of $\mathbb{R}^3(\mathbb{R})$. Find an orthonormal basis for the above inner product space by applying Gram-Schmidt orthogonalization process on the basis B.

Question 4: (5 Marks)

a) Prove or disprove whether the following function $h : \mathbb{R} \to \mathbb{R}$ is Lipschitz continuous

$$h(t) = \begin{cases} t^2 \cos \frac{1}{t^2} & \text{if } t \neq 0 \\ 0 & \text{if } t = 0 \end{cases}$$

b) Determine the maximum interval in which the solution of the following IVP exist.

$$\ln t \; x'(t) + x(t) = \cot t, \quad x\left(\frac{1}{2}\right) = \pi.$$

Question 5: (3 Marks) Find the general solution of system of ODEs:

$$\frac{d}{dt}(\vec{x}) = A\vec{x} \text{ with } A = \begin{pmatrix} 2 & 0 \\ -4 & 2 \end{pmatrix}.$$