Problems for Recitation 4

1 Problem: The Pulverizer!

There is a pond. Inside the pond there are n pebbles, arranged in a cycle. A frog is sitting on one of the pebbles. Whenever he jumps, he lands exactly k pebbles away in the clockwise direction, where $0 < k < n$. The frog's meal, a delicious worm, lies on the pebble right next to his, in the clockwise direction.

(a) Describe a situation where the frog can't reach the worm.

(b) In a situation where the frog can actually reach the worm, explain how to use the Pulverizer to find how many jumps the frog will need.

(c) Compute the number of jumps if $n = 50$ and $k = 21$. Anything strange? Can you fix it?

2 Problem: The Fibonacci numbers.

The Fibonacci numbers are defined as follows:

$$F_0 = 0 \qquad F_1 = 1 \qquad F_n = F_{n-1} + F_{n-2} \quad (\text{for } n \geq 2).$$

Give an inductive proof that the Fibonacci numbers F_n and F_{n+1} are relatively prime for all $n \geq 0$.

3 Extra Problem: The power of 3.[1]

Let N be a number whose decimal expansion consists of 3^n identical digits. Show by induction that $3^n \mid N$. For example:

$$3^2 \mid \underbrace{777777777}_{3^2 = 9 \text{ digits}}$$

Recall that 3 divides a number iff it divides the sum of its digits.

[1] Try this if you have time!