MTL 101
LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS
MAJOR EXAM

Total: 25 Marks **Time: 1:00 Hrs.**

Question 1: (5 Marks) Assume f is such the system of equations:

$$x_1' = x_2, \quad x_2' = x_3, \quad x_3' = x_4, \quad x_4' = f(x_1),$$

with initial conditions

$$x_1(t_0) = a_1, \quad , x_2(t_0) = a_2, \quad x_3(t_0) = a_3, \quad x_4(t_0) = a_4.$$

has unique solution for any t_0, a_1, a_2, a_3 and a_4. Show that any solution $(x_1(t), x_2(t), x_3(t), x_4(t))$ of the problem:

$$x_1' = x_2, \quad x_2' = x_3, \quad x_3' = x_4, \quad x_4' = f(x_1),$$

which satisfy,

$$x_2(0) = 0, \quad x_4(0) = 0.$$

implies that $x_1(t)$ is even.

Question 2 (**5 Marks**) Show that the solutions of following ODE do not change sign.

$$x' = (1 - e^{-x^2})(\sin(x^2) + \cos(x^2)).$$

Question 3: (5 Marks) Using Laplace transform, solve the system of ODE's

$$x' = x + 2y,$$
$$y' = -y + 2\delta(t),$$

with $x(0) = 1$ and $y(0) = 0$. Here $\delta(t)$ is the Dirac Delta function at $t = 0$.

Question 4: (5 Marks) Consider a homogeneous system of two Linear ODEs with continuous coefficients. State and prove the Abel's Lemma for Wronskian of the solutions.

Question 5: (5 Marks) Solve the following ODE:

$$t\sin(x)dt + (t + 1)\cos(x)dx = 0$$