DEPARTMENT OF MATHEMATICS, IIT DELHI
SEMESTER II 2024 − 25
MTL 101 (Linear Algebra and Differential Equations) - Minor Exam

Date: 27/02/2025 (Thursday) Time: 1:00 pm- 3:00 pm.

"As a student of IIT Delhi, I will not give or receive aid in examinations. I will do my share and take an active part in seeing to it that others as well as myself uphold the spirit and letter of the Honour Code."

Name :		BLOCK LETTER ONLY	
Entry Number:		**Group:**	
Gradescope Id:		**Lecture Hall:**	

Question 1: Prove that any maximal linearly independent set of vectors in a vector space, V (not necessarily finite dimensional), is a basis of V. [4]

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Question 2: Let $T : \mathbb{R}^4 \to \mathbb{R}^4$, be the linear transformation given by

$$T(x, y, z, w) = (x + y, z, w, w).$$

Compute the rank and nullity of T^2, where $T^2 = T \circ T$.

[3]

Question 3: Let B and B' be the following standard ordered bases of $P_2(\mathbb{R})$ and $M_{2\times 2}(\mathbb{R})$, respectively:

$$B = \{1, x, x^2\},$$

and

$$B' = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}.$$

Let $T : P_2(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$, be the linear transformation given by

$$T(f(x)) = \begin{pmatrix} f'(0) & 2f(1) \\ 0 & f''(3) \end{pmatrix}.$$

Compute the matrix of linear transformation $[T]_B^{B'}$.

[4]

Question 4: Find the eigenvalues of the following 3×3 matrix

$$M = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

Also find eigenvectors corresponding to eigenvalues that are integers.

[3]

Question 5: Let V and W be two finite dimensional vector spaces over the field $\mathbb{F}$ and $T : V \to W$ be a linear transformation.

i) Prove that, if $\text{Ker}(T) = \{0\}$, then T sends linearly independent set to linearly independent set, i.e., if $\{v_1, \ldots, v_k\}$ is linearly independent in V, then $\{T(v_1), \ldots, T(v_k)\}$ is linearly independent in W.

ii) Is the above result true if we drop the condition $\text{Ker}(T) = \{0\}$? Justify.

[3+1=4]

Question 6 Let W_1 and W_2 be subspaces of $\mathbb{R}^5$ given by

$$W_1 = \{(x_1, x_2, x_3, x_4, x_5) \mid x_1 + x_2 + 2x_3 = 0, \quad 2x_4 + x_5 = 0\}$$

and

$$W_2 = \{(x_1, x_2, x_3, x_4, x_5) \mid x_2 + 2x_3 = 0, \quad x_1 + 2x_4 + x_5 = 0\}.$$

Find $\dim(W_1 \cap W_2)$. Is $W_1 + W_2 = \mathbb{R}^5$? Justify.

[4]

Question 7 Prove (if true) or disprove (if false) the following statements.

i) If W_1, W_2 and W are subspaces of a vector space V such that

$$W_1 \oplus W = W_2 \oplus W,$$

then $W_1 = W_2$.

ii) The span of the set $S := \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 4\}$ over $\mathbb{R}$ is $\mathbb{R}^3$.

iii) If Y_1 and Y_2 are solutions of the system of linear equations

$$AX = B, \text{ where } B \neq \mathbf{0},$$

then for $a_1 \neq 0 \neq a_2$, $a_1Y_1 + a_2Y_2$ is not a solution of $AX = B$.

iv) There is a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^4$ such that

$$\begin{aligned}
T(1,1,1,1) &= (1,0,0,0), \\
T(1,0,1,0) &= (0,1,0,0), \\
T(0,-1,0,-1) &= (0,0,1,0), \\
T(0,0,0,1) &= (0,0,0,0).
\end{aligned}$$

[2+2+2+2=8]

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