COL202: Discrete Mathematical Structures. II semester, 2024-25.
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Problem sheet for pre-midterm practice
13 February 2025

Problem 1
Prove or disprove the following logical statements using the truth table method.

1. $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$.
2. $((P \wedge Q) \Rightarrow R) \Leftrightarrow (P \Rightarrow (Q \Rightarrow R)$

Problem 2
We are given sets $A = \{a, b, c, d\}$ and $B = \{e, f, g, h\}$ and we are given predicates $p : A \times B \to \{T, F\}, r : A \to \{T, F\}, q : B \to \{T, F\}$ for which *only* the following are True: $p(a, e), p(a, f), (\forall y : p(b, y)), p(c, g), r(b), r(d), q(e), q(g), q(h)$.

Prove or disprove

$$\forall x : \exists y : r(x) \Rightarrow (p(x, y) \Rightarrow q(y)).$$

Problem 3
Let S be the set of all IITD students. Write logical sentences for each of the following english sentences. Define any predicates you use, e.g., you may say "Define $\mathsf{Btech} : E \to \{T, F\}$, where $\mathsf{BTech}(x)$ is True is x is a BTech student and False otherwise."

Problem 3.1 (2 marks)
BTech students enter IITD through JEE.

Problem 3.2 (2 marks)
A student is either a BTech student or a Dual student an MTech student or an MSR student or a PhD student.

Problem 3.3 (2 marks)
MTech students do not enter IITD through JEE.

Problem 3.4 (2 marks)
IITD has some MTech students.

Problem 4
Assuming all the four statements of Problem 3 are true, prove that the following statement is valid: "There are some students in IITD who have not entered through JEE." First write this as a logical sentence and then argue that it is true.

Problem 5
Prove that $3n^2+5n+1$ is odd for all $n \in \mathbb{N}$. Your proof *must* explicitly use the Well-Ordering Principle.

Problem 6
What does the algorithm in the box return? Prove that your answer is correct by defining an appropriate loop invariant and proving its correctness by induction.

Require: : Given an integer linked list ℓ.

initialise an empty list ℓ'

while ℓ is not empty **do**

  Remove the last node of ℓ and insert it at the head of ℓ'.

end while

Return ℓ'

Problem 7

Prove that if an acyclic graph has $n - k$ edges, it has k components.

Problem 8

A graph $G = (V, E)$ is called *bipartite* if there is a partition (U_1, U_2) of V such that every edge has one endpoint in U_1 and the other endpoint in U_2. Prove that every tree is bipartite. Give an example that shows that not every bipartite graph is a tree.

Problem 9

Given two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ we define their product graph $G_1 \times G_2 = (V, E)$ as follows: $V = V_1 \times V_2$ and $((x_1, y_1), (x_2, y_2)) \in E$ if $(x_1, x_2) \in E_1$ or $(y_1, y_2) \in E_2$. Prove or disprove the following statements:

1. The product of two regular graphs is regular. Recall a graph is called regular if all vertices have the same degree.
2. The product of two trees is a tree.
3. The product of two bipartite graphs is a bipartite graph.