MTL101::: Major Test

November, 2014

1. No marks will be awarded if appropriate justification is not provided.
2. Total marks is 45. Maximum Time is 2 hours.

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1. Consider the following IVP [4 = 2 + 2]

$$y' = \sqrt{t} + y^2, \quad y(0) = y_0 = 0.$$

   (a) Discuss the existence and uniqueness of this IVP.

   (b) Find the first two Picard's iterations y_1 and y_2.

2. Find the general solution of [4]

$$t^2 y'' - 4ty' + 6y = \sin(\ln t).$$

3. Solve the following IVP [4]

$$y'' - 4y' + 4y = \delta(t-2) + H_1(t), \quad y(0) = y'(0) = 0.$$

4. Find the general solution of $\vec{x}\,' = A\vec{x}$, where $A = \begin{pmatrix} 8 & 12 & -2 \\ -3 & -4 & 1 \\ -1 & -2 & 2 \end{pmatrix}$. [6]

5. Find, by using the method of variation of parameters, the general solution of [4]

$$y'' + y = \tan(t).$$

6. Let

$$f(t) = \mathcal{L}^{-1}\left[\frac{e^{-\pi s/2} - e^{-3\pi s/2}}{s(s^2+2s+5)}\right].$$

   Find the value of $f(2\pi)$. [4]

7. Find the general solution of [4]

$$\begin{aligned} x' &= 2x + 6y + e^t \\ y' &= x + 3y - e^t \end{aligned}$$

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8. Show that the following system of linear equations have infinitely many solutions and find all solutions. [4]

$$\begin{aligned} x+2y+3z+w &= 4 \\ 2x+3y+z-w &= 4 \\ 3x+4y-z+2w &= 5 \\ 2x+3y+z+4w &= 5 \end{aligned}$$

9. Let [4]

$$W_1 = \{(x,y,z,w) \in \mathbb{R}^4 : x = 0, y+z+w = 0\}$$

and

$$W_2 = \{(x,y,z,w) \in \mathbb{R}^4 : x-y = 0, x+y+z+w = 0\}.$$

Find a basis B_1 of $W_1 \cap W_2$ and find a basis B_2 of $W_1 + W_2$ containing B_1.

10. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation. Suppose [4]

$$\begin{aligned} T(1,1,1) &= (1,-1,0), \\ T(1,-1,1) &= (0,1,-1), \\ T(0,1,-1) &= (-1,0,1). \end{aligned}$$

Find the nullity of T and $T(0,0,1)$.

11. Find the span of the set $\{(t,t^2,t^3) \in \mathbb{R}^3 : t \in \mathbb{R}\}$ in $\mathbb{R}^3$. [3]

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