COL202 Minor exam

Anish Banerjee

TOTAL POINTS

20.5 / 27

QUESTION 1

1 Problem 1 **2.5 / 3**

✓ + **3 pts** Correct

- **0.5 pts** Each Minor mistake/ Undefined variable used

+ **0 pts** Incorrect/Not attempted

- **0.5** Point adjustment

1 more condition required for there exist f(.) = 10

QUESTION 2

2 Problem 2 **2 / 2**

✓ + **0.5 pts** Mentioned proof method, and concluded the proof

✓ + **1.5 pts** Considered all cases of A and shown there is a y

+ **0 pts** Incorrect/Not attempted

QUESTION 3

3 Problem 3 **6 / 6**

✓ - **0 pts** Correct answer for both statements

- **3 pts** Wrong truth table for statement 1

- **3 pts** Wrong conclusion for statement 1

- **1 pts** Not written concluding statement for 1

- **3 pts** Wrong truth table for statement 2

- **3 pts** Wrong conclusion for statement 2

- **1 pts** Not written concluding statement for 2

QUESTION 4

4 Problem 4 **7 / 7**

✓ + **7 pts** Correct

+ **1 pts** Using the proof by contradiction.

+ **1 pts** Assuming S to be non-empty.

+ **1 pts** There exist some $$n_0$$ (smallest element in the S)

+ **3 pts** Correct by cases and using the contradiction of the minimality of S.

+ **1 pts** Concludes S is empty and proved.

+ **0 pts** Unattempted/Completely wrong.

QUESTION 5

5 Problem 5 **3 / 9**

Proof that G' is connected

+ **1.5 pts** Partially correct

+ **3 pts** Correct

Proof that G’ is acyclic

+ **1 pts** Without using maximally acyclic concept

✓ + **2 pts** Using maximally acyclic concept - considered edges of G

+ **5 pts** Using maximally acyclic concept - considered both edges and non-edges of G

✓ + **1 pts** G’ is connected and acyclic => spanning tree

+ **0 pts** Incorrect/Not Attempted

COL202: Discrete Mathematical Structures. I semester, 2022-23.
Minor exam.
28 September 2022, Maximum Marks: 29.

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Important: Please write within the box given for your answer. Answers written elsewhere on the paper will not be graded.

Problem 1 (5 marks)
We say that graph $G = (V, E)$ is *k-edge colourable* for integer $k > 0$ if there is a function $f : E \to \{1, \ldots, k\}$ such that no two edges incident on a vertex have the same "colour." i.e., the same value of $f(\cdot)$. The *edge colouring number* of G, χ_G, is the maximum k for which G is k-edge colourable. For $k > 0$, let us denote by $\mathcal{F}_k$ the set of all functions from E to $\{1, \ldots, k\}$. Use this notation to write the following statement as a predicate: $\chi_G = 10$. Note that your predicate must take only $G = (V, E)$ as an argument. Use only logic notation. You may use set inclusion, e.g. $x \in A$, if required.

$P(G(V,E)) : \exists f \in F_k : \forall u,v,w \in V ; v \neq w : (uv, uw \in E) \Rightarrow$
$((f(uv) \neq f(uw)) \wedge (f(uv) \leq 10))$

Problem 2 (2 marks)
We are given sets $A = \{a, b, c, d\}$ and $B = \{e, f, g, h\}$ and we are given predicates $p : A \times B \to \{T, F\}, r : A \to \{T, F\}, q : B \to \{T, F\}$ for which *only* the following are True: $p(a, e), p(a. f), (\forall y : p(b, y)), p(c, g), r(b). r(d), q(e), q(g), q(h)$.

Prove or disprove

$$\forall x : \exists y : r(x) \Rightarrow (p(x, y) \Rightarrow q(y)).$$

When $x = a, c$ we have $r(x)$ as False. Thus the statement is true as $F \Rightarrow T$ and $F \Rightarrow F$ are both true. — ⊛
When $x = b, d$; we need to ensure that $p(x,y) \Rightarrow q(y)$ is true
Case 1: $x = b$. We have $\forall y$ $p(b,y)$ as true. Also for $y = e, g$ or h we have $q(y)$ as true. Thus there exists a y for which $p(b,y) \Rightarrow q(y)$ is true $\{T \Rightarrow T$ is $T\}$
Case 2: $x = d$. $p(d,y)$ is false for every y. Thus $p(d,y) \Rightarrow q(y)$ is true (same reason as ⊛)
Thus the given proposition is true

Problem 3 (6 marks)
Prove or disprove the following logical statements using the truth table method.

1. $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$.
2. $((P \wedge Q) \Rightarrow R) \Leftrightarrow (P \Rightarrow (Q \Rightarrow R)$

1.

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$
F	F	F	T	T	T	T
F	T	T	F	T	F	F
T	F	T	F	F	T	F
T	T	T	F	F	F	F
			$⊛_1$			$⊛_2$

As columns $⊛_1$ and $⊛_2$ are identical, the logical statements are equivalent

$\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$

2.

P	Q	R	$P \wedge Q$	$(P \wedge Q) \Rightarrow R$	$Q \Rightarrow R$	$P \Rightarrow (Q \Rightarrow R)$
F	F	F	F	T	T	T
F	F	T	F	T	T	T
F	T	F	F	T	F	T
F	T	T	F	T	T	T
T	F	F	F	T	T	T
T	F	T	F	T	T	T
T	T	F	T	F	F	F
T	T	T	T	T	T	T
				$⊛_3$		$⊛_4$

As columns $⊛_3$ and $⊛_4$ are identical, the logical statements are equivalent

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Problem 4 (7 marks)
Use the Well Ordering Principle to show that $17^n > 0$ for every $n \in \mathbb{N} \cup \{0\}$.

Proof is by contradiction and use of well-ordering Principle.
Assume that there exists $n \in \mathbb{N}$ such that $17^n \leq 0$
Let C be the set of all such n

$$C := \{ n \in \mathbb{N} \cup \{0\} : 17^n \leq 0 \}$$

Assume that this set is non-empty. Then by well-ordering principle, it must have a least element, say n_0 such that

$$17^{n_0} \leq 0 \quad - (1)$$

As $1 \geq 0$, n_0 must be greater than 0

Divide both sides of eq^n (1) by 17

$$\frac{17^{n_0}}{17} \leq \frac{0}{17} = 0$$

$$17^{n_0 - 1} \leq 0$$

Thus $n_0 - 1$ also satisfies the property $17^n \leq 0$

As $n_0 - 1 < n_0$, this is in contradiction to our assumption that n_0 was the least element of C.

Thus C is empty.

Hence proved by contradiction $\square$

Problem 5 (9 marks)

Suppose that $G = (V, E)$ is a connected graph. Here is an algorithm we run on G:

> Create a new graph $G' = (V, E' = \emptyset)$. Now go through the edges from E in some order and try to add them into the edgeset E' one at a time. If the addition of an edge creates a cycle in G', discard the edge and move on to the next edge of E. The algorithm ends when we have either added or discarded every edge of E. Return G'.

Using the fact that every maximally acyclic graph is a tree, prove that the above algorithm returns a spanning tree of G. *If you don't use this fact you will get a 0 even if your proof is correct.*

G' spans G as $V(G') = V(G) = V$. To prove that G' is a spanning tree, we need to show that G' is maximally acyclic (Since every maximally acyclic graph is a tree).

Note that G' must be a subgraph of G as all the edges we add in E' belong to E.

Claim: G' is maximally acyclic.

Proof by contradiction. Suppose $E'+e$ remains acyclic. (where $e \in E \setminus E'$)

Then, we must have added e to E' ~~our graph~~ when we visited it in our algorithm. Thus no such e exists.

Hence G' is maximally acyclic. Hence G' is a spanning tree of G