

Department of Mathematics
 Indian Institute of Technology Delhi
MTL 101 Linear Algebra and Differential Equations: Minor I

Total marks: 20

Time: 1 hour

1. Every question is compulsory
2. No marks will be provided for answers without proper justification

1. Determine (with justification) whether the following statements are true or false. [2 × 4 = 8]

- (a) Consider \mathbb{C} as a vector space over \mathbb{Q} . Let $\omega \neq 1 \in \mathbb{C}$ be a fifth root of unity, i.e. $\omega^5 = 1$. Let

$$V = \{a_0 + a_1\omega + a_2\omega^2 + \dots + a_n\omega^{2n} \mid a_i \in \mathbb{Q} \text{ for all } 0 \leq i \leq n, n \in \mathbb{N} \cup \{0\}\}.$$

Then V is an infinite dimensional subspace of \mathbb{C} over \mathbb{Q} .

- (b) Let $P \in M_{m \times m}(\mathbb{R})$ with $P \neq 0$, $A \in M_{m \times n}(\mathbb{R})$ and $B = PA$. Then the systems of equations $AX = 0$ and $BX = 0$ have the same set of solutions.

- (c) Let B be an ordered basis of \mathbb{R}^n over \mathbb{R} . Suppose that there are vectors $v_1, \dots, v_n \in \mathbb{R}^n$ such that $[v_i]_B = e_{n-i+1}$, where $e_i \in \mathbb{R}^n$ has all entries 0 except the i -th entry which is 1, for $1 \leq i \leq n$. Then $\{v_1, \dots, v_n\}$ is linearly independent.

- (d) Let S_1, S_2 be subsets of a vector space V over \mathbb{R} . Then

$$\text{span}(S_1 \cup S_2) = \text{span}(\text{span}(S_1) \cup \text{span}(S_2)).$$

2. Let

[3+2=5]

$$A = \begin{pmatrix} 1 & 0 & 4 & 8 \\ 0 & 0 & 2 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

- (a) Using elementary row operations, find the inverse of A .

- (b) Express A as a product of elementary matrices.

3. Let W_1, W_2 be subspaces of \mathbb{R}^4 over \mathbb{R} defined as follows:

[5]

$$W_1 = \{(x, y, z, w) \in \mathbb{R}^4 : x + y + z + w = 0 \text{ and } x - y + z - w = 0\} \text{ and}$$

$$W_2 = \{(x, y, z, w) \in \mathbb{R}^4 : x + 2y + 3z + 4w = 0 \text{ and } 3y + 2z + 5w = 0\}.$$

- Find a basis for $W_1, W_2, W_1 \cap W_2$ and $W_1 + W_2$.

4. Let V be a vector space over \mathbb{R} and let B be a basis for V . Prove that B is a maximal linearly independent set.

[2]