Minor Test 1 ::: MTL 101 ::: February 2017

Every question is compulsory.
Marks will not be awarded if appropriate arguments are not provided.
Do not waste time describing what is not asked.

Maximum Marks: **20** Maximum Time: **1 hour**

(1) (a) Find a basis for $W_1 \cap W_2$, where [3]

$$W_1 = \{(x, y, z, w) \in \mathbb{R}^4 : x - y + z - w = 0, 5x - 4y + 3z - 2w = 0\}$$
$$W_2 = \{(x, y, z, w) \in \mathbb{R}^4 : x - 2y + 3z - 4w = 0, x - 2y + 2z - w = 0\}.$$

(b) Find a basis for $\text{span}(S)$, where [2]

$$S = \{(x, y, z) \in \mathbb{R}^3 : 3x + 2y + z = 2, x + y + z = 1\}.$$

(2) Find a condition on $\alpha, \beta, \gamma, \delta$ so that

$$\{(0, 1, 0, 1), (1, 0, 1, 0), (2, 1, 1, 1), (\alpha, \beta, \gamma, \delta)\}$$

is a linearly dependent set in $\mathbb{R}^4$. [3]

(3) Let V be the vector space over the field $\mathbb{R}$ of all real-valued functions on $\mathbb{R}$.

(a) Let $S_a = \{\sin 2x, \sin ax\}$, $a \in \mathbb{R}$. Find all a such that S_a is linearly independent. [2]

(b) Let $\mathcal{B} = \{e^{3x}, \cos 2x, \sin 2x\}$ and $W = \text{span}(\mathcal{B})$. Let $f(x) = e^{3x} + \sin 2x - \cos 2x$.

(i) Show that $f'(x) \in W$, where $f'(x)$ denotes the derivative of $f(x)$. [1]

(ii) Find the coordinate vector $[f'(x)]_{\mathcal{B}}$ by treating $\mathcal{B}$ as an ordered basis for W. [1]

(4) Prove or disprove the following statements. [$6 = 3 \times 2$]

(a) Let X and Y be nonempty subsets of a vector space V. Then

$$\text{span}(X) \cup \text{span}(Y) = \text{span}(X \cup Y).$$

(b) If $\{u, v, w\}$ is linearly independent, then $\{u - 3v, 3v - w, w - u\}$ is linearly independent.

(c) For any $A \in M_{n\times n}(\mathbb{R})$, $W = \{X \in M_{n\times n}(\mathbb{R}) : AX = XA^t\}$ is a subspace of $M_{n\times n}(\mathbb{R})$, where A^t denotes the transpose of A.

(5) Write down all possible 2×3 real RRE matrices of rank 2. [2]

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