COL202 Quiz 2

TOTAL POINTS

4.5 / 5

QUESTION 1

Bandwidth 5 pts

1.1 Definition predicate 2 / 2

✓ **- 0 pts** **Correct**

- 0.5 pts Did not mention $$\exists f \in F$$ for which the predicate is true.

- 0.5 pts Did not mention $$\forall (u, v) \in E$$

- 2 pts Incorrect Predicate, one correct predicate is: $$ \exists f \in \mathcal{F}: \forall (u,v) \in E: |f(u) - f(v)| \leq k$$

- 2 pts Did not attempt

1.2 The bandwidth of a cycle 2.5 / 3

✓ **- 0 pts** **Correct**

- 0.5 pts Incorrect/No argument that bandwidth cannot be 1

- 0.5 pts Did not follow proof guidlines

- 2 pts Did not show construction/Incorrect Construction for bandwidth = 2

- 1 pts DId not show proof of construction/Incorrect proof of construction

- 3 pts Did not attempt

- 0.5 **Point adjustment**

Show for a general vertex that difference with neighbors is <= 2

COL202: Discrete Mathematical Structures. I semester, 2022-23.
Quiz 2, 12 September 2022, Maximum Marks: 5

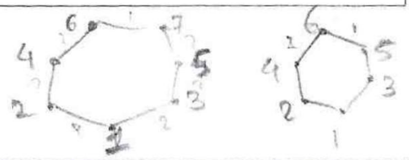
Name	Ent. No.

Important: Answer within the boxes. Anything written outside the box will be treated as rough work.

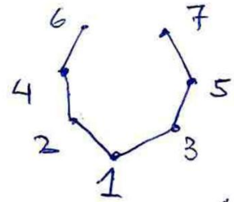
Problem 1.1 (2 marks)
The *bandwidth* of a graph is defined as follows: Find a numbering of the vertices of a graph such that the maximum difference between the numbers assigned to two vertices connected by an edge in the graph is minimized. This minimum value is called the bandwidth of the graph. Write the following as a predicate: The bandwidth of $G = (V, E)$ is at most k. You *must* use the following notation: $\mathcal{F}$ is the set of functions from V to $\{1, \ldots, |V|\}$; $\texttt{bandwidth}(G, k)$ is the name of the predicate you define.

bandwidth(G,k) : $\min_{f \in F} \left(\max_{xy \in E} (|f(x) - f(y)|) \right) \leq k \quad (x, y \in V)$

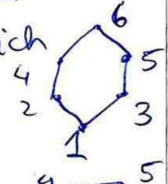
Problem 1.2 (3 marks)
Prove that a cycle on n vertices has bandwidth 2.



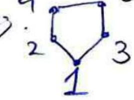
To show that a cycle on n vertices has bandwidth, we will show that such a numbering exists and that a numbering does not exist such that bandwidth is 1.
To show bandwidth = 2 :
Choose a starting vertex v and number it 1.
Number both of its neighbours 2 and 3.
Now keep numbering vertices such that the difference between its number and its neighbour is 2.



If n is even, there will be a last vertex remaining which can be numbered as $|V|$ eg.
Max. difference is still 2.



If n is odd, there will be two vertices left in the end which can be numbered such that difference is still 2. eg:



We have proved that such a numbering exists.
To show bandwith cannot be 1:
It can only be 1 if all differences between vertices is 1. So if a vertex is numbered as n, its neighbours would have to be numbered n+1 and n−1. Since n+1 already has a neighbour n, its other neighbour would have to be n+2. Thus, from one vertex two paths emerge such that one has increasing numbers and other decreasing. Since no number can be on both paths, those cannot meet. But in a cycle, those two paths meet. It is a contradiction. Hence, proved.