COL 202: DISCRETE MATHEMATICAL STRUCTURES

LECTURE 8

QUIZ 1 DISCUSSION

JAN 17, 2024 | ROHIT VAISH

PROBLEM 1

Problem 1 (16 points)

Recall that the sequence of Fibonacci numbers $\{F_n\}_{n\in\mathbb{N}\cup\{0\}}$ is defined as follows:

$$F_0 = 0$$
$$F_1 = 1$$
$$\forall n \geqslant 2, F_n = F_{n-1} + F_{n-2}.$$

Prove that every positive integer can be represented as the sum of one or more *distinct* Fibonacci numbers in such a way that the sum does not include any two consecutive Fibonacci numbers. Formally, show that for any positive integer n, there exist positive integers $c_1, c_2, \ldots, c_k$ for some $k \in \mathbb{N}$, where $2 \leqslant c_1 < c_2 < \cdots < c_k$, such that $n = \sum_{i=1}^{k} F_{c_i}$.

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Correction : $\forall\, i \in \{1, 2, \ldots, k-1\}$ $c_{i+1} > c_i + 1$

non-consecutive

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Zeckendorf's theorem

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Fun fact : Zeckendorf representation is unique.

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e.g., $20 = 13 + 5 + 2$

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Why care?

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Why care? → Cool result (represent in base Fibonacci)
→ Convert kilometre to miles

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e.g., $20 = 13 + 5 + 2$

2, 3, 5, 8, 13, 22, ...

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e.g., $20 = 13 + 5 + 2$

$2, 3, 5, 8, 13, 22, \ldots$

3 miles $\approx$ 5 km
5 miles $\approx$ 8 km
$\vdots$

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e.g., $20\text{ km} = 13\text{ km} + 5\text{ km} + 2\text{ km}$

$2, 3, 5, 8, 13, 22, \ldots$

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e.g., 20 km = 13 km + 5 km + 2 km

$\downarrow$ 8 miles, $\downarrow$ 3 miles, $\downarrow$ 1 mile

$\approx$ 12 miles

2, 3, 5, 8, 13, 22, ...

3 miles $\approx$ 5 km

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Proof: (by strong induction)

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Hypothesis: $\forall n \in \mathbb{N} \quad P(n): \quad n = \sum_{i=1}^{k} F_{c_i}$

for some $2 \leq c_1 < c_2 < \dots < c_k$

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Base Case : $P(1)$ is true because

$$1 = F_2$$

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Proof: (by strong induction)

Inductive step: $\forall n \geq 1 \quad P(1) \wedge P(2) \wedge \cdots \wedge P(n) \Longrightarrow P(n+1)$.

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If $n+1$ is a Fibonacci number, we are done.

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$$F_j < n+1 < F_{j+1}.$$

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$$a = n+1 - F_j$$

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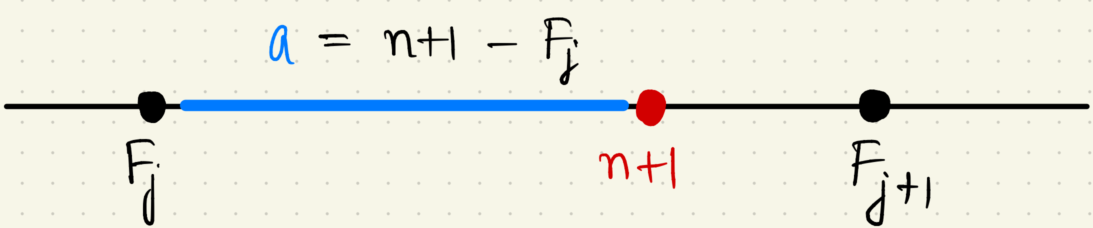

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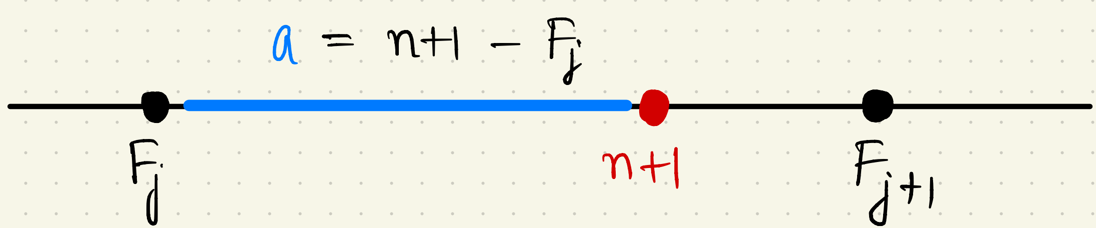
By strong induction hyp., $P(a)$ is true.



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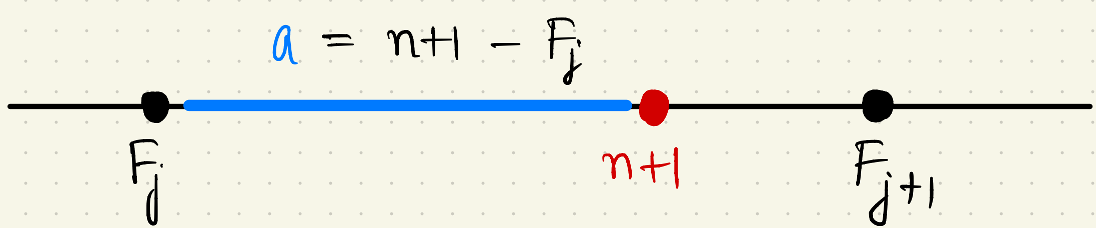
Also, $a = n+1 - F_j$



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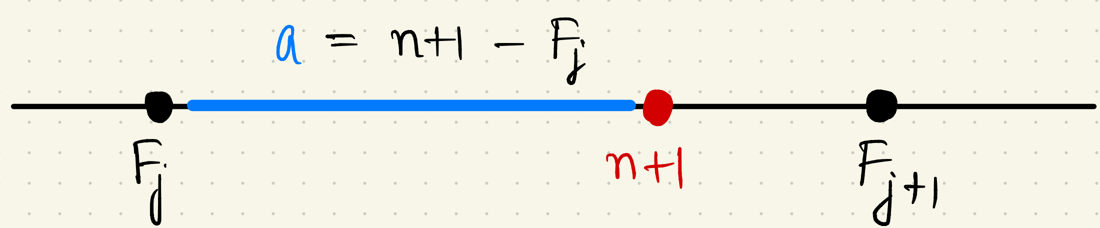
Also, $a = n+1 - F_j < F_{j+1} - F_j$



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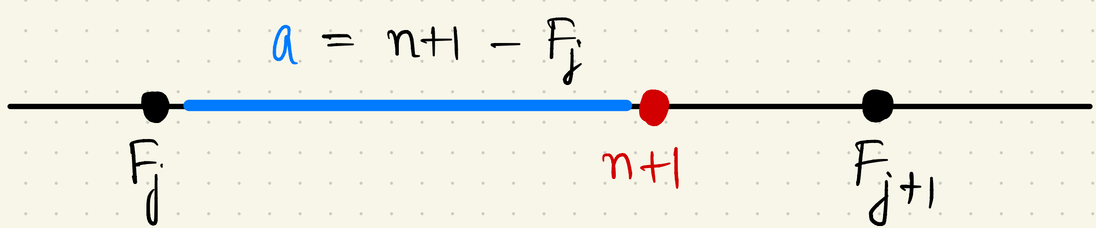

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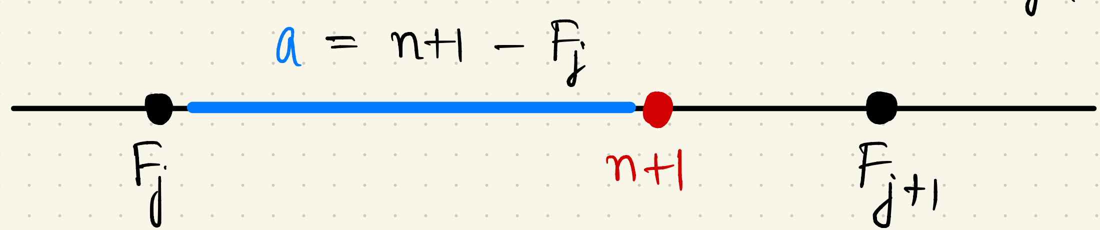
So, a's representation does not contain F_{j-1} or F_j.



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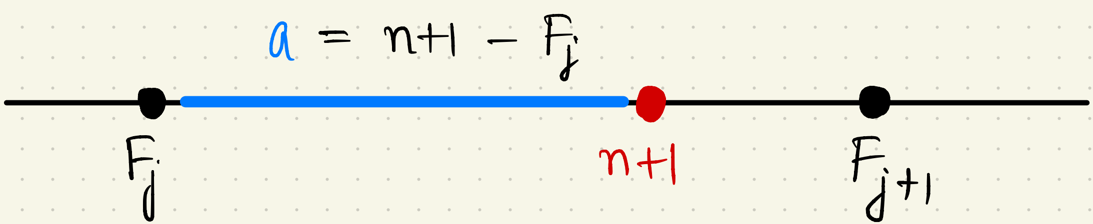
Proof: (by strong induction)

Inductive step: $\forall n \geq 1 \quad P(1) \wedge P(2) \wedge \cdots \wedge P(n) \implies P(n+1)$.

Thus, $n+1 = a + F_j$

representable without F_j or F_{j-1}.



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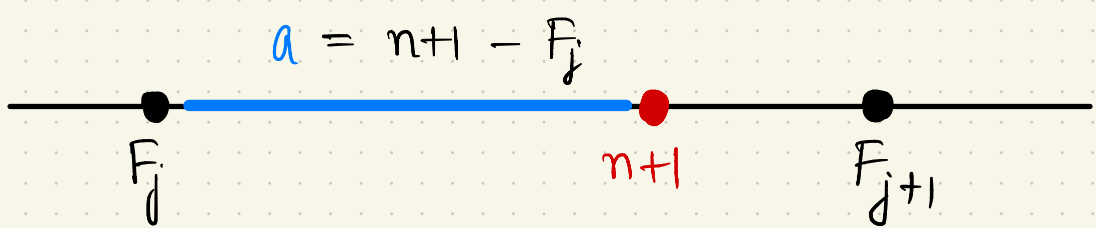
$\implies \quad P(n+1)$ is true.



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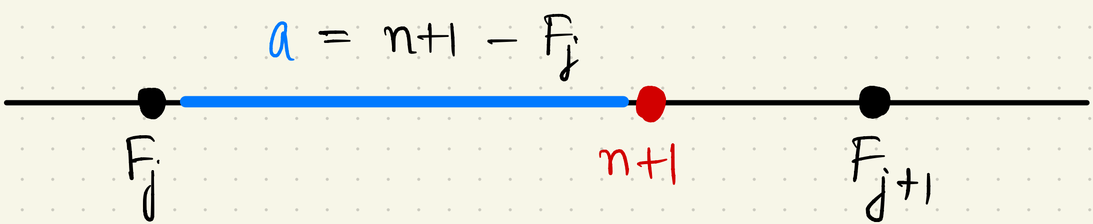

$\blacksquare$

PROBLEM 1

Total = 16 points

Identifying proof by strong induction [2 pts]

Writing $P(n)$ [2 pts]

Base case [2 pts]

Inductive step $\longrightarrow$ Applying $P(a)$ [5 pts]

$\searrow$ Observing that a's representation excludes F_j and F_{j-1}. [5 pts]

PROBLEM 2(a)

(a) **[4 points] Claim**: For any non-negative integer n, $2 \times n = 0$.

Proof: (by strong induction) Let $P(n)$ be the statement of the claim.

Base case: $P(0)$ is true since for $n = 0$, we have $2 \times 0 = 0$.

Inductive step: Want to show that for all $n \in \mathbb{N} \cup \{0\}$, $P(0) \wedge P(1) \wedge \cdots \wedge P(n) \Rightarrow P(n+1)$.

Let i and j be non-negative integers such that $n + 1 = i + j$. Then,

$$2 \times (n+1) = 2 \times (i+j) = 2 \times i + 2 \times j = 0 + 0 = 0,$$

where the second-last equality follows from the strong induction hypothesis.

This proves the inductive step and thus the claim. $\square$

Claim is incorrect for all $n \in \mathbb{N}$

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Proof is also incorrect

$P(0) \Rightarrow P(1)$ does not hold

PROBLEM 2(a)

Total = 4 points

Identify that claim is incorrect [0.5 pt]

Point out the error in the claim [0.5 pt]

Identify that erroneous step in the proof [1 pt]

Point out the error in the proof [2 pts]

PROBLEM 2(b)

(b) **[4 points]** **Claim**: All positive integers are equal.

Proof: (by induction) For any given positive integers $x_1, x_2, \ldots, x_n$, let us define the predicate $P(x_1, x_2, \ldots, x_n)$ as

$$P(x_1, x_2, \ldots, x_n) : x_1 = x_2 = \cdots = x_n.$$

Base case: $P(1, 1, \ldots, 1)$ is true because $1 = 1 = \cdots = 1$.

Inductive step: Want to show that for all positive integers $x_1, x_2, \ldots, x_n$, $P(x_1, x_2, \ldots, x_n) \Rightarrow P(x_1 + 1, x_2 + 1, \ldots, x_n + 1)$.

By the induction hypothesis, we know that $x_1 = x_2 = \cdots = x_n$. Therefore, it follows that $x_1 + 1 = x_2 + 1 = \cdots = x_n + 1$.

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Claim is false

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Inductive step: Want to show that for all positive integers $x_1, x_2, \ldots, x_n$, $P(x_1, x_2, \ldots, x_n) \Rightarrow P(x_1 + 1, x_2 + 1, \ldots, x_n + 1)$.

By the induction hypothesis, we know that $x_1 = x_2 = \cdots = x_n$. Therefore, it follows that $x_1 + 1 = x_2 + 1 = \cdots = x_n + 1$.

This proves the inductive step and thus the claim. $\square$

Proof is also incorrect.

Nested induction $\longrightarrow$ go argument-by-argument

PROBLEM 2(b)

To prove : $\forall m \in \mathbb{N} \quad \forall n \in \mathbb{N} \quad P(m,n)$

PROBLEM 2(b)

$Q(m)$

To prove : $\forall m \in \mathbb{N} \quad \underbrace{\forall n \in \mathbb{N} \quad P(m,n)}_{Q(m)}$

PROBLEM 2(b)

To prove : $\forall m \in \mathbb{N} \; \underbrace{\forall n \in \mathbb{N} \quad P(m,n)}_{Q(m)}$

Base case : Show $Q(1)$ is true

PROBLEM 2 (b)

$Q(m)$

To prove : $\forall m \in \mathbb{N} \quad \forall n \in \mathbb{N} \quad P(m,n)$

Base case : Show $Q(1)$ is true, i.e.,

$\forall n \in \mathbb{N} \quad P(1,n)$ is true.

PROBLEM 2(b)

To prove : $\forall m \in \mathbb{N} \underbrace{\forall n \in \mathbb{N} \quad P(m,n)}_{Q(m)}$

Base case : Show $Q(1)$ is true, i.e.,

$\forall n \in \mathbb{N}$ $P(1, n)$ is true.

can prove this by induction on n

PROBLEM 2(b)

To prove : $\forall m \in \mathbb{N} \quad \underbrace{\forall n \in \mathbb{N} \quad P(m,n)}_{Q(m)}$

Base case : Show $Q(1)$ is true, i.e.,

$\forall n \in \mathbb{N} \quad P(1,n)$ is true.

can prove this by induction on n

Inductive step : Show $\forall m \in \mathbb{N} \quad Q(m) \implies Q(m+1)$

PROBLEM 2(b)

To prove: $\forall m \in \mathbb{N} \ \underbrace{\forall n \in \mathbb{N} \quad P(m,n)}_{Q(m)}$

Base case: Show $Q(1)$ is true, i.e.,

$\forall n \in \mathbb{N} \ P(1,n)$ is true.

can prove this by induction on n

Inductive step: Show $\forall m \in \mathbb{N} \quad Q(m) \Rightarrow Q(m+1)$, i.e.,

$\forall m \in \mathbb{N} \qquad \forall n \in \mathbb{N} \ P(m,n) \Rightarrow \forall n \in \mathbb{N} \ P(m+1,n).$

PROBLEM 2(b)

$Q(m)$

To prove: $\forall m \in \mathbb{N} \quad \forall n \in \mathbb{N} \quad P(m,n)$

Base case: Show $Q(1)$ is true, i.e.,

$\forall n \in \mathbb{N} \quad P(1,n)$ is true.

can prove this by induction on n

Inductive step: Show $\forall m \in \mathbb{N} \quad Q(m) \Longrightarrow Q(m+1)$, i.e.,

$\forall m \in \mathbb{N} \qquad \forall n \in \mathbb{N} \; P(m,n) \Longrightarrow \forall n \in \mathbb{N} \; P(m+1,n).$

Can use induction on n

PROBLEM 2(b)

Total = 4 points

Identify that claim is incorrect	[0.5 pt]
Point out the error in the claim	[0.5 pt]
Identify that erroneous step in the proof	[1 pt]
Point out the error in the proof	[2 pts]