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Beamforming with uniform circular microphone arrays can be used for localizing sound sources over 360°. Typically, the array microphones are suspended in free space or they are mounted on a solid cylinder. However, the cylinder is often considered to be infinitely long because the scattering problem has no exact solution for a finite cylinder. Alternatively one can use a solid sphere. This investigation compares the performance of a circular array mounded on a rigid sphere with that of such an array in free space and mounted on an infinite cylinder, using computer simulations. The examined techniques are delay-and-sum and circular harmonics beamforming, and the results are validated experimentally. © *2011 Acoustical Society of America*. [DOI: 10.1121/1.3621294]

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I. INTRODUCTION

During the past decade, studies on the performance of circular arrays of microphones for localizing sound sources over 360° have been reported. For example, Meyer[1] utilized modal beamforming to generate a desired beampattern for a circular microphone array mounted around a rigid sphere. Daigle *et al.*[2] considered delay-and-sum beamforming with circular arrays mounted on the surface of sound absorbing spheres and cylinders and showed that the achieved beamwidth improved over that of arrays mounted on hard spheres and cylinders. Instead of delay-and-sum beamforming, Teutsch and Kellermann[3] analyzed various algorithms based on decomposing the sound field into a series of modes for a circular array mounted on a cylinder. Still in the field of modal beamforming Tiana-Roig *et al.*[4] adapted the theory of spherical harmonics beamforming to the two-dimensional case using circular harmonics. The resulting circular harmonics beamformer was compared to the classical delay-and-sum beamformer using both a circular array suspended in free space and one mounted on a rigid, infinite cylinder. This letter to the editor repeats the comparison for the case of a circular array mounted on a rigid sphere.

II. PLANE WAVE DECOMPOSITION

Consider a plane wave, $e^{j\mathbf{k_i}\cdot\mathbf{r}}$, generated by a source placed in the far field, at a polar angle θ_s and azimuth angle φ_s, that impinges on a rigid sphere with radius R. The pressure on the surface of the sphere, at a point with spherical coordinates $[R,\theta,\varphi]$, can be written as[5,6]

$$p(kR,\theta,\varphi) = 4\pi\sum_{q=0}^{\infty} b_q(kR)\sum_{n=-q}^{q} Y_q^n(\theta,\varphi)Y_q^n(\theta_s,\varphi_s)^*, \qquad (1)$$

where

$$b_q(kR) = (-j)^q\left(j_q(kR) - \frac{j_q'(kR)}{h_q'(kR)}h_q(kR)\right), \qquad (2)$$

$$Y_q^n(\theta,\varphi) \equiv \sqrt{\frac{(2q+1)}{4\pi}\frac{(q-n)!}{(q+n)!}}P_q^n(\cos\theta)e^{jn\varphi}. \qquad (3)$$

In the function b_n, which accounts for the effect of the rigid scatterer, j_q is a spherical Bessel function of order q, h_q is a spherical Hankel function of first kind and order q, and j_q' and h_q' are their derivatives. On the other hand, Y_q^n is a spherical harmonic, in which P_q^n is a Legendre function of degree q and order n. Note that in Eq. (1) the temporal term $e^{-j\omega t}$ is omitted; and the angles of the position of the source $[\theta_s,\varphi_s]$ are used instead of the angles of the incident wave $[\theta_i,\varphi_i]$, these being related by $\theta_s = \pi - \theta_i$ and $\varphi_s = \varphi_i + \pi$ because the unit vector of the incident wave $\hat{\mathbf{k}}_\mathbf{i}$ is opposite to the unit vector of the position of the source $\hat{\mathbf{r}}_\mathbf{s}$, $\hat{\mathbf{k}}_\mathbf{i} = -\hat{\mathbf{r}}_\mathbf{s}$.

Now a circular aperture of radius R is mounted at the equator of the rigid sphere, in the xy plane. Because the polar angle at all positions of the aperture is constant, i.e., $\theta = \pi/2$, its pressure can be represented in a Fourier series in the φ coordinate.[5] The resulting Fourier coefficients are

$$C_n = \frac{1}{2\pi}\int_0^{2\pi} p(kR,\pi/2,\varphi)e^{-jn\varphi}\mathrm{d}\varphi. \qquad (4)$$

Inserting Eq. (1), it can be shown that the coefficients become

$$C_n(kR) = \sum_{q=|n|}^{\infty}(2q+1)b_n(kR) \times \frac{(q-|n|)!}{(q+|n|)!}P_q^{|n|}(0)P_q^{|n|}(\cos\theta_s)e^{-jn\varphi_s}. \qquad (5)$$

Figure 1 shows the magnitude of the first four coefficients assuming a source located in the plane of the aperture, i.e., at $\theta_s = \pi/2$. The advantage of this configuration is that its behavior resembles the one of a circular aperture mounted

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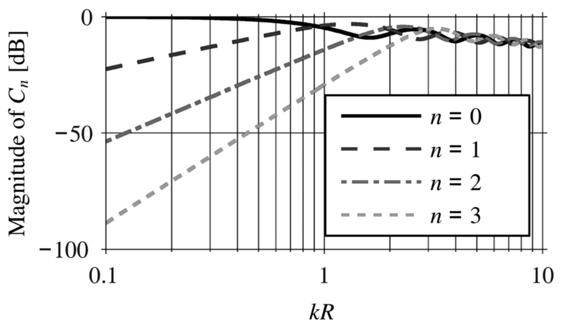

FIG. 1. Magnitude of the Fourier coefficients of the pressure on a circular aperture mounted on the equator of a rigid sphere.

on a rigid cylinder of infinite length,[4] for which all frequencies can be resolved by means of beamforming procedures.

III. BEAMFORMING ALGORITHMS

Circular harmonics beamforming (CHB) is a technique implemented specifically for circular arrays of microphones based on the decomposition of the sound field using the principles of a Fourier series. The beamformer output is[4]

$$b_{N,\mathrm{CHB}}(kR,\varphi) = \frac{A}{M}\sum_{m=1}^{M}\tilde{p}(kR,\varphi_m)\sum_{n=-N}^{N}\frac{1}{Q_n(kR)}e^{-jn(\varphi_m-\varphi)}, \tag{6}$$

where A is a scale factor, M is the number of microphones, $\tilde{p}$ and φ_m are the measured pressure and the azimuth angle of the mth microphone, and N is the maximum order taken into account. $Q_n(kR)$, which is related to the Fourier coefficients as $Q_n(kR) = C_n(kR)/e^{-jn\varphi_s}$, depends on the configuration of the array:

$$Q_n(kR) = \begin{cases} (-j)^n J_n(kR) & \text{Free space} \\ (-j)^n \left(J_n(kR) - \dfrac{J_n'(kR)H_n(kR)}{H_n'(kR)}\right) & \text{Rigid cylinder} \\ & \text{of infinite length} \\ \displaystyle\sum_{q=|n|}^{\infty}(2q+1)b_n(kR)\frac{(q-|n|)!}{(q+|n|)!} & \text{Equator of a} \\ \times P_q^{|n|}(0)P_q^{|n|}(0) & \text{rigid sphere} \end{cases} \tag{7}$$

Whereas the expressions for the array suspended in free field and mounted on an infinitely long cylinder are taken from Ref. 4, the value for the array on the sphere follows from Eq. (5). Note that in all cases the sound sources are assumed to be in the plane of the array, i.e., $\theta_s = \pi/2$.

Another technique that can be implemented for the circular geometry is delay-and-sum beamforming (DSB). This technique aligns the signals of the microphones by introducing appropriate delays and finally adds the signals together.[7] Implemented in the frequency domain using matched field processing, the beamformer output is[4]

$$b_{N,\mathrm{DSB}}(kR,\varphi) = \frac{A}{M}\sum_{m=1}^{M}\tilde{p}(kR,\varphi_m)\sum_{n=-N}^{N}Q_n^*(kR)e^{-jn(\varphi_m-\varphi)}. \tag{8}$$

The output of the unbaffled array can be also written as

$$b_{\mathrm{DSB}}(kR,\varphi) = \frac{A}{M}\sum_{m=1}^{M}\tilde{p}(kR,\varphi_m)e^{jkR\cos(\varphi_m-\varphi)}, \tag{9}$$

which is more precise than Eq. (8) because it does not imply a truncation at an order N.

Ideally, the beamformers output should be zero at all angles φ different from the angle of the sound source, φ_s. However, because a limited number of microphones is used, rather than a continuous aperture, the response exhibits a main lobe around φ_s and side lobes at other angles. Therefore, it is convenient to evaluate the performance of the beamformer in terms of resolution, defined as the −3 dB width of the main lobe, and maximum side lobe level (MSL), which is given by the difference in level between the peaks of the highest side lobe and the main lobe.

IV. SIMULATION STUDY

The performance of CHB and DSB using a circular array mounted on the equator of a rigid sphere has been compared to other configurations such as a circular array suspended in free space or a circular array mounted on a rigid cylinder of infinite length by means of computer simulations. The impinging plane waves were perpendicular to the plane of the array and were created by a source placed at $\theta_s = 90°$ and $\varphi_s = 180°$. (Note, however, that the azimuth angle has a very limited influence on the results.) The maximum order of the algorithms followed $N \approx kR$ for CHB and $N \approx kR + 1$ for DSB,[4] up to a maximum $N = M/2 - 1$ to satisfy the Nyquist criterion: $\lambda/2 > d$, where d is the distance between the microphones, or equivalently, $M > 2kR$.[8,9] An array with 10 cm of radius and 10 microphones was considered.

The left panels of Fig. 2 show the performances of the unbaffled array and the array mounted on a sphere using CHB and considering ideal conditions, i.e., without background noise. The behavior of the array mounted on a rigid cylinder is not depicted because the curves coincide with the ones of the array on the sphere in the frequency range of interest. As can be seen, by mounting the array on the sphere, the performance is very similar to the one of the unbaffled array but improves particularly at those frequencies where the unbaffled array presents peaks. Note that from 2.7 kHz on the MSL worsens dramatically because in this range the Nyquist criterion is no longer fulfilled and consequently aliasing occurs.

The resolution and the MSL with DSB can be seen in the right column of Fig. 2. In this case, the response of the array mounted on a cylinder of infinite length is also shown. With this technique, the performance of the array mounted on the sphere is better than the one with an unbaffled array, especially toward low frequencies. However, it is not as good as in the case of the array mounted on an infinitely long cylinder. Actually, by mounting the array on a cylinder or on a sphere, the apparent distance between microphones increases, and this improves the performance of DSB at low frequencies. This is in agreement with the observations of Daigle *et al.*, although they claimed that the array has an effectively larger aperture when mounted on a physical

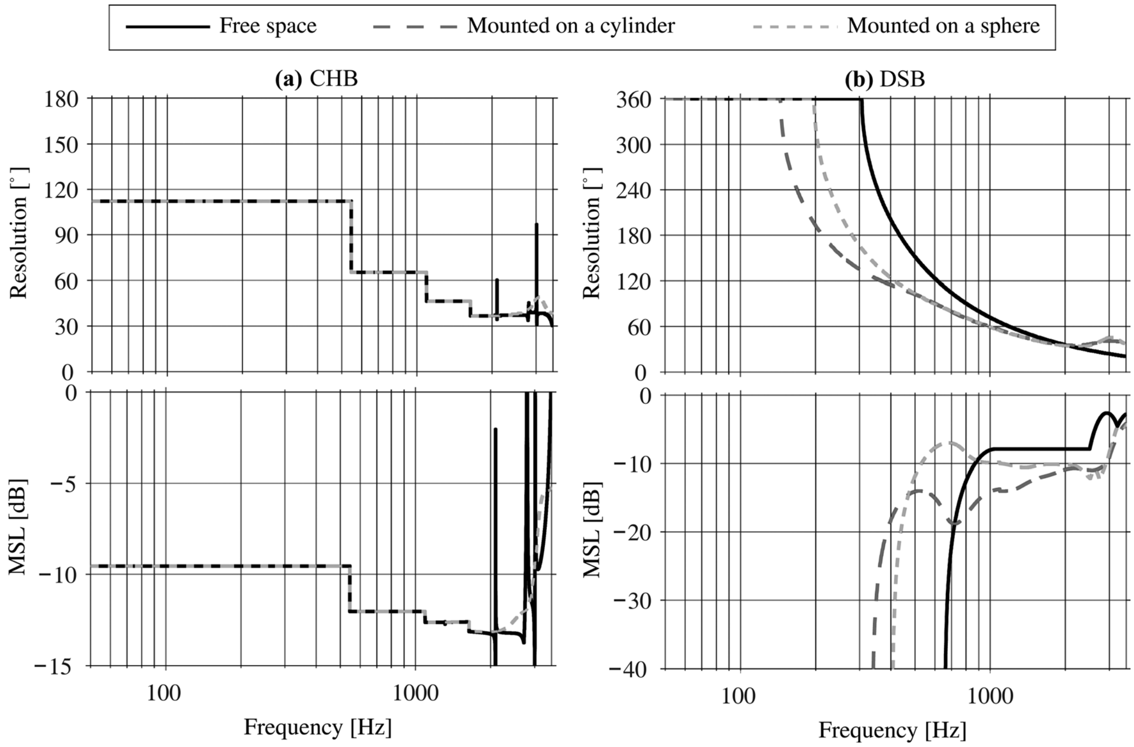

FIG. 2. Resolution and MSL using CHB (left) and DSB (right) and a circular array of radius 10 cm and 10 microphones when the microphones are suspended in free space, mounted on the equator of a rigid sphere, and mounted on a rigid cylinder of infinite length. For ease of comparison, the case of the cylinder with CHB is not plotted because it coincides with the case of the sphere. In all cases, the source is in the far field at $[\theta_s, \varphi_s] = [90°, 180°]$.

structure independently of the beamforming technique,[2] whereas the present study has revealed that this is not the case with CHB.

V. EXPERIMENTAL RESULTS

A circular array mounted on a rigid sphere has been tested in an anechoic room. The prototype array consisted of 16 1/4 in. microphones, Brüel & Kjær (B&K) type 4958, mounted on the equator of a rigid sphere with a radius of 9.75 cm, corresponding to a microphone for every 22.5°. With this configuration, the array can operate free of aliasing up to about 4.5 kHz.

The array and the source, a loudspeaker, were controlled by a B&K PULSE analyzer. The loudspeaker was driven by a signal from the generator, pseudorandom noise of 1 s of period, 6.4 kHz of bandwidth, and 1 Hz of resolution. The microphone signals were recorded with the analyzer and post-processed with the beamforming algorithms CHB and DSB.

Figure 3 shows the output of the array using CHB and DSB when a source is located 4 m away but at the very same height and at an azimuth angle $\varphi_s = 180°$. It can be seen that with the two techniques, the array is capable of localizing the sound source in the frequency range of interest with exception of DSB at the frequencies below about 300 Hz due to the fact that this technique behaves omnidirectionally at such values.

The performance of the array is also illustrated in Fig. 4, where the resolution and the MSL for both CHB and DSB are shown. The predictions made with computer simulations are also depicted. To account for the background noise introduced in the measurements, the simulations were carried out with a signal-to-noise ratio (SNR) of 30 dB at the input of each microphone due to uniformly distributed noise.

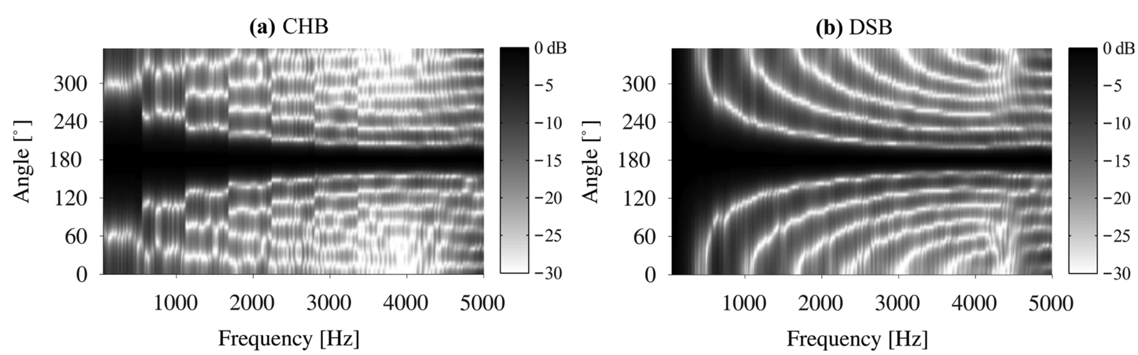

FIG. 3. Normalized output with CHB (left) and DSB (right) using a circular array with 16 microphones mounted on the equator of a rigid sphere with radius of 9.75 cm. The source is in the far field at $[\theta_s, \varphi_s] = [90°, 180°]$.

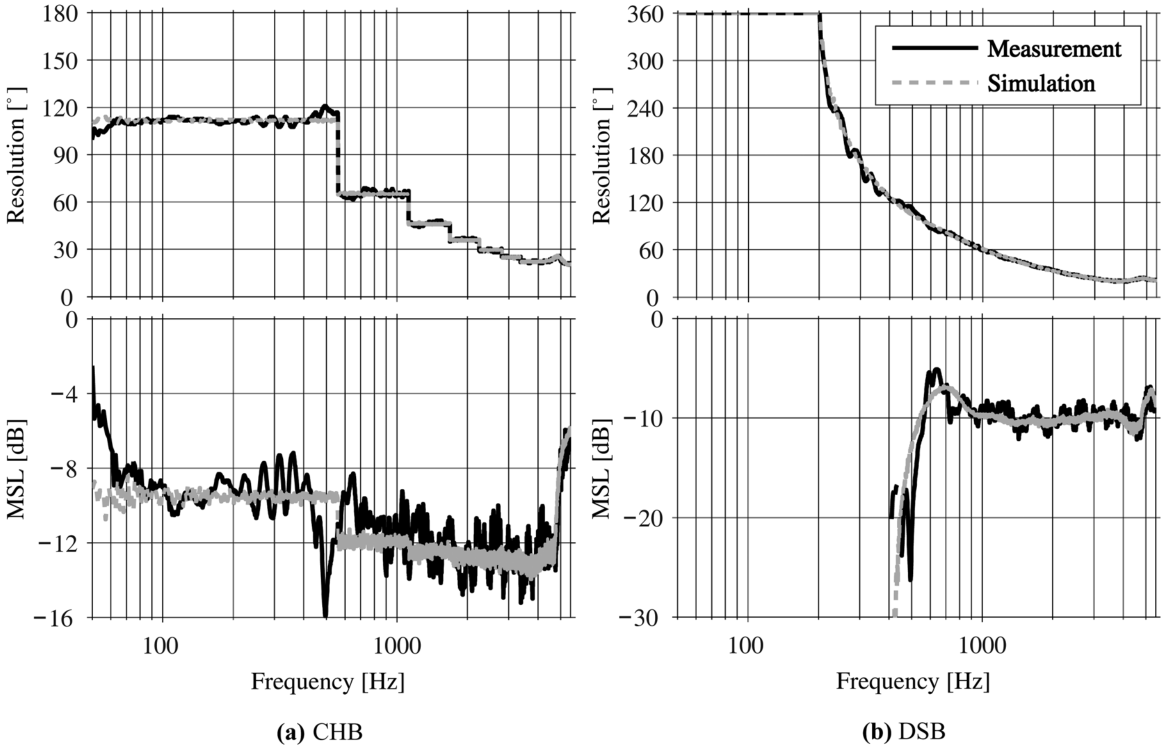

FIG. 4. Resolution and MSL with CHB (left) and DSB (right) using a circular array of radius 9.75 cm and 16 microphones mounted on the equator of a rigid sphere. The source is in the far field at $[\theta_s, \varphi_s] = [90°, 180°]$. For the simulation, the SNR in each microphone is 30 dB.

For CHB, the simulations and the measurements agree in terms of resolution in most of the frequency range. Small deviations from the expected values are observed at the lowest frequencies and around 500 Hz. In terms of MSL, the measurements oscillate around the expected values as occurs with circular arrays suspended in free space.[4] The differences detected in the resolution at low frequencies and about 500 Hz also appear in the MSL. At low frequencies, the difference is attributed to a presence of background noise higher than expected. At about 500 Hz, the performance is better than expected because the MSL is much lower than the predictions. The good agreement with the simulations is also found in the case of DSB. Just in the range from 400 to 600 Hz, the MSL differs from the expected value.

Other measurements with the source placed at different positions revealed that the behavior of the array is practically independent of its azimuth angle. Because the beamforming algorithms given in Sec. III expect a source placed at a polar $\theta_s = 90°$, i.e., at the plane of the array, the performance is optimal when this happens. However, it can be shown that sources placed in the range $\theta_s = 90° \pm 45°$ can still be localized.

VI. CONCLUSIONS

A beamformer consisting of a uniform circular array of microphones mounted on the equator of a rigid sphere has been examined using CHB and DSB. A simulation study has revealed that this configuration is an improved version of a circular array suspended in free space. Particularly, with CHB, the array mounted on the sphere behaves identically to the unrealistic case of an array with the same dimensions mounted on a rigid cylinder of infinite length. Therefore, the array on the sphere is a simple solution of special interest as alternative to beamformers based on cylinders of finite length because these are often approximated by infinitely long cylinders to overcome the problem that an exact analytical expression for such cylinders does not exist.

Various experiments using a prototype array have proved the validity of the model.

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