### ACM ICPC Code Notebook

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### **Templates**

C++

18

20

21

22

23

```
#include <bits/stdc++.h>
   using namespace std;
   #define ll long long int
   #define inf 0x3f3f3f3f
   #define pb push_back
   #define mk make_pair
   #define mt make_tuple
   #define fi first
   #define se second
   #define ii pair<int, int>
   #define all(x) (x).begin(), (x).end()
   #define N 1000007 // 10e6 + 7
15
   const double PI = acos(-1.0);
16
17
```

int main(int argc, char const \*argv[]) {

return 0;

//ios::sync\_with\_stdio(false);

#### Java

```
import java.io.BufferedReader;
   import java.io.Closeable;
   import java.io.IOException;
   import java.io.InputStreamReader;
   import java.io.PrintWriter;
   import java.util.StringTokenizer;
   import java.math.*;
   import java.text.*;
   import java.util.*;
10
    public class Icpc {
11
12
            public static void main(String ... args) throws Exception {
13
14
                     FastScan sc = new FastScan (new BufferedReader (new
                         InputStreamReader(System.in)));
                     PrintWriter pw = new PrintWriter(System.out);
15
16
17
18
19
                     sc.close();
20
                     pw.close();
21
                     System.exit(0);
22
23
24
            static class FastScan implements Closeable {
25
                     private BufferedReader br;
26
                     private StringTokenizer tk;
27
28
                     public FastScan(BufferedReader br) {
29
                              \mathbf{this}. \mathbf{br} = \mathbf{br};
30
31
32
                     public int in() throws NumberFormatException, IOException
33
                              return Integer.parseInt(next());
34
35
36
                     public long ln() throws NumberFormatException, IOException
37
                              return Long.parseLong(next());
```

```
38
39
40
                    public double db() throws NumberFormatException,
                        IOException {
41
                            return Double.parseDouble(next());
42
43
                    @Override
44
                    public void close() throws IOException {
45
                            tk = null;
                            br.close();
47
48
49
                   public String next() throws IOException {
50
                            while (tk = null || !tk.hasMoreTokens()) {
51
52
                                    String line = br.readLine();
53
                                    if (line = null)
54
                                            return null;
55
                                    tk = new StringTokenizer(line);
56
57
                            return tk.nextToken();
58
59
60 }
```

## Algorithms

### Graphs

### AP & Bridges

```
int dfs(int u, int p){
        dfs_num[u] = dfs_low[u] = ++dfs_counter;
3
        for(auto v : adjList[u]){
            if(dfs_num[v]==0)
4
5
                dfs(v,u);
                if(dfs_low[v] >= dfs_num[u]){
                     articulation [u]=true;
9
                if(dfs_low[v] > dfs_num[u])
10
                    bridge = true;
                dfs_low[u] = min(dfs_low[u], dfs_low[v]);
11
12
            } else if(v!=p)
13
                dfs_low[u] = min(dfs_low[u], dfs_num[v]);
14
15
16
17
   int main() {
18
        memset(dfs_num,0,sizeof(dfs_num));
19
        memset(dfs_low,0,sizeof(dfs_low));
20
        bridge=false;
21
        dfs_counter = 0;
22
        dfs(0,-1);
23
        for (int i = 0; i < N; ++i)
24
            if(dfs_num[i]==0)
25
                bridge=true;
26
        puts (bridge ? "Yes" : "No");
27
        return 0;
28
```

### Centroid Decomposition

### Centroid Decomposition

```
1 #include <bits/stdc++.h>
2 #define MAXN 100100
3 typedef long long ll;
4
5 using namespace std;
```

```
int n, sz [MAXN];
   bool deleted [MAXN], vis [MAXN];
    char ch [MAXN];
    vector < int > g [MAXN];
11
12
    void dfs(int x, int p){
13
            if(vis[x]) return;
14
            vis[x] = true;
15
            sz[x] = 1;
16
            for(auto i : g[x]){
17
                     if(i == p || deleted[i]) continue;
18
                     dfs(i, x);
19
                     sz[x] += sz[i];
20
            //cout << x << " " << sz/x | << " \n";
21
22
23
24
    int findCentroid(int x){
25
            memset(vis, 0, sizeof(vis));
26
            dfs(x, -1);
27
            int p = -1, c = sz[x] / 2;
28
            while(true){
29
                     bool found = false;
30
                     for(auto i : g[x]){
                              if (! deleted[i] && i != p && sz[i] > c){
31
32
                                      found = true;
33
                                      p = x;
34
                                      x = i;
35
                                      break:
36
37
38
                     if(!found) return x;
39
40
41
42
    void decomp(int x, char c){
43
            int cen = findCentroid(x);
44
            ch[cen] = c;
45
            deleted [cen] = true;
46
            for(auto i : g[cen]){
47
                     if (deleted [i]) continue;
48
                     decomp(i, c + 1);
```

```
50
51
52
  int main() {
53
        #ifndef ONLINE_JUDGE
                     freopen("input.txt", "r", stdin);
54
55
            #endif
            ios_base::sync_with_stdio(false);
56
57
            cin.tie(NULL);
58
            memset(deleted, 0, sizeof(deleted));
59
        cin >> n;
60
        for (int i = 0; i < n - 1; i++)
61
            int a, b;
62
            cin \gg a \gg b;
            g[a].push_back(b);
63
64
            g[b].push_back(a);
65
        //cout \ll findCentroid(1);
66
67
        decomp(1, 'A');
68
        for (int i = 1; i \le n; i++){
            cout << ch[i] << "_";
69
70
71
   Centroid Decomposition (Eoin)
   void fill_sz(int u, int p){
2
        sz[u] = 1;
3
        for(int v : adjList[u]){
            if (v==p || mkd[v])
 5
                continue:
            fill_sz(v,u);
            sz[u]+=sz[v];
8
9
10
   int get_centroid(int u, int n, int p){
11
12
        for(int v : adjList[u]){
13
            if (v==p || mkd[v])
14
                continue:
            if(sz[v] > n/2)
15
16
                return get_centroid(v, n, u);
17
18
        return u;
19
20
21
   int decomp(int u){
22
        fill_sz(u, -1);
23
        int cent = get_centroid(u, sz[u], -1);
24
        mkd[cent] = true;
25
        for(int v : adjList[cent]){
26
            if (mkd [v])
27
                continue:
28
            int r = decomp(v);
29
            centP[r] = cent;
```

```
30 } 31 return cent; 32 }
```

#### Network Flow

#### Dinic's Blocking flow

```
/**
1
2
            e-maxx's flow
3
            Dinic algorithm
4
            Complexity O(V^2*E) OR O(Vsrqt(E)) for bipartite graphs!
5
6
    */
8
    struct edge {
9
            int a, b, cap, flow;
10
    };
11
                                                               // NEED TO set n (
   int nodes, s, t, d[N], ptr[N], q[N];
        max of nodes), s source, t sink
   vector < edge > e;
13
14
   vector < int > g[N];
15
16
   void add_edge (int a, int b, int cap) {
17
            edge e1 = \{ a, b, cap, 0 \};
18
            edge e2 = \{ b, a, 0, 0 \};
19
            g[a].push_back ((int) e.size());
20
            e.push_back (e1);
21
            g[b].push_back ((int) e.size());
22
            e.push_back (e2);
23
24
   bool bfs() {
25
26
            int qh=0, qt=0;
27
            q[qt++] = s;
28
            memset (d, -1, nodes * sizeof d[0]);
29
            d[s] = 0;
30
            while (qh < qt \&\& d[t] = -1) {
31
                     int v = q[qh++];
32
                     for (size_t i = 0; i < g[v]. size(); ++i) {
33
                             int id = g[v][i],
34
                                      to = e[id].b;
35
                             if (d[to] = -1 \&\& e[id]. flow < e[id]. cap) {
36
                                      q[qt++] = to;
37
                                      d[to] = d[v] + 1;
38
39
40
41
            return d[t] != -1;
42
43
44
   int dfs (int v, int flow) {
45
            if (!flow) return 0;
46
            if (v == t) return flow;
```

```
29
47
            for (; ptr[v]<(int)g[v].size(); ++ptr[v]) {
                                                                                             vector<ii> used;
                                                                                    30
48
                     int id = g[v][ptr[v]],
                                                                                             for (int i = 0; i < N; i++)
49
                              to = e[id].b;
                                                                                    31
                                                                                                 for (int j = 0; j < N; j++)
50
                     if (d[to] != d[v] + 1) continue;
                                                                                    32
                                                                                                     if(graph[i][j] > 0 \&\& res[i][j] < graph[i][j])
51
                     int pushed = dfs (to, min (flow, e[id].cap - e[id].flow))33
                                                                                                          used.push_back(make_pair(i,j));
                     if (pushed) {
                                                                                    34
52
53
                              e[id].flow += pushed;
                                                                                        Ford Fulkerson 1
54
                              e[id^1]. flow -= pushed;
55
                              return pushed;
56
                                                                                        int ff(int u, int minE){
57
                                                                                     2
                                                                                             i f ( u==T)
58
            return 0;
                                                                                     3
                                                                                                 return minE;
59
                                                                                     4
                                                                                             vis [u]=true;
60
                                                                                     5
                                                                                             for (auto i : adjList[u]) {
61
   int dinic() {
                                                                                                 if (! vis [i] && res [u][i] > 0) {
62
            int flow = 0;
                                                                                                     if(int f = ff(i, min(minE, res[u][i])))
63
            for (;;) {
                                                                                                          res[u][i] -= f;
                     if (!bfs()) break;
64
                                                                                     9
                                                                                                          res[i][u] += f;
65
                     memset (ptr, 0, nodes * sizeof ptr[0]);
                                                                                    10
                                                                                                          return f;
66
                     while (int pushed = dfs (s, inf))
                                                                                    11
                              flow += pushed;
67
                                                                                    12
68
                                                                                    13
69
            return flow;
                                                                                    14
                                                                                             return 0;
70
                                                                                    15
                                                                                    16
   Edmond Karp
                                                                                    17
                                                                                        int main(){
                                                                                    18
                                                                                             int mf = 0;
                                                                                    19
                                                                                            while (1) {
   void aug(int u, int minE){
        if(u==S){ f=minE; return; }
                                                                                    20
                                                                                                 memset(vis,0,sizeof(vis));
                                                                                    21
                                                                                                 int f = ff(S, INF);
3
        if(p[u]!=u){
            aug(p[u], min(minE, res[p[u]][u]));
                                                                                    22
                                                                                                 if(f==0)
 4
                                                                                    23
                                                                                                     break;
            res[p[u]][u]-=f;
6
                                                                                    24
                                                                                                 mf+=f;
            res[u][p[u]]+=f;
                                                                                    25
                                                                                    26
                                                                                             printf("%d\n",mf);
8
                                                                                    27
9
   int main(){
                                                                                        Ford Fulkerson 2
11
        int mf=0;
12
        for (;;) {
13
            f=0; //Global
                                                                                        #include <bits/stdc++.h>
14
            for (int i = 0; i < N; i++)
                                                                                       #define MAXN 3000
15
                 dist[i]=INF, p[i]==i;
                                                                                        typedef long long ll;
16
            dist[S]=0;
17
            queue < int > q; q.push(S);
                                                                                     5
                                                                                        using namespace std;
18
            while (!q.empty()) {
                                                                                     6
                 int u = q. front(); q. pop();
19
                                                                                        int g[MAXN][MAXN], rg[MAXN][MAXN], parent[MAXN];
20
                 if (u=T) break;
21
                 for (int i = 0; i < N; i++)
22
                     if(res[u][i] > 0 \&\& dist[i] == INF)
                                                                                    10
                                                                                        bool bfs(int source, int sink){
23
                          \operatorname{dist}[i] = \operatorname{dist}[u] + 1, p[i] = u, q.\operatorname{push}(i);
                                                                                    11
                                                                                                 bool visited [MAXN];
24
                                                                                    12
                                                                                                 memset(visited, 0, sizeof(visited));
25
            aug(T, INF);
                                                                                    13
                                                                                                 queue < int > q;
26
            if(f==0) break;
                                                                                    14
                                                                                                 q. push (source);
27
                                                                                    15
                                                                                                 visited [source] = true;
            mf+=f;
28
                                                                                    16
                                                                                                 parent [source] = -1;
```

```
17
            while (!q.empty()) {
                                                                                     *E/x is vertex at time x
18
                    int i = q.front();
                                                                                     * L/x is depth at time x
19
                    q.pop();
                                                                                 5
20
                     for (int j = 0; j < MAXN; j++){
                                                                                 6
                                                                                    void vis(int u, int d){
21
                             if (! visited [j] && rg[i][j] > 0){
                                                                                        H[u] = vind;
22
                                                                                        E[vind] = u;
                                     q.push(j);
23
                                     parent[j] = i;
                                                                                 9
                                                                                         L[vind++] = d;
24
                                     visited [j] = true;
                                                                                 10
                                                                                         for(auto i : adjList[u]){
25
                                                                                 11
                                                                                             if(H[i]!=-1)
26
                                                                                 12
                                                                                                 continue;
27
                                                                                 13
                                                                                             vis(i,d+1);
28
            return visited [sink];
                                                                                14
                                                                                             E[vind] = u;
29
                                                                                15
                                                                                             L[vind++] = d;
30
                                                                                 16
31
   int maxFlow(int source, int sink){
                                                                                 17
32
            for (int i = 0; i < MAXN; i++){
                                                                                 18
33
                    for (int j = 0; j < MAXN; j++){
                                                                                 19
                                                                                    int LCA(int u, int v){
34
                                                                                 20
                             rg[i][j] = g[i][j];
                                                                                         if(H[u] > H[v])
                                                                                21
35
                                                                                             int t = u;
36
                                                                                 22
                                                                                             u = v;
            int \max_{-1} flow = 0;
                                                                                 23
37
                                                                                             v = t:
38
            while (bfs (source, sink)) {
                                                                                 24
                                                                                 25
39
                    int path_flow = 999999999;
                                                                                         //run some range min query on L
40
                    for(int i = sink; i != source; i = parent[i]) {
                                                                                 26
                                                                                         //between H[u] and H[v]
                                                                                 27
41
                             int j = parent[i];
                                                                                         int ind = rmq(H[u], H[v]);
42
                             path_flow = min(path_flow, rg[j][i]);
                                                                                 28
                                                                                         return E[ind];
                                                                                 29
44
                    for(int i = sink; i != source; i = parent[i]) {
                                                                                 30
                             int j = parent[i];
                                                                                31
                                                                                    int dist(int u, int v){
45
                             rg[j][i] -= path_flow;
                                                                                 32
                                                                                         int a = H[u];
46
                                                                                33
47
                             rg[i][j] += path_flow;
                                                                                         int b = H[v];
48
                                                                                 34
                                                                                         int ind = LCA(u,v);
49
                    max_flow += path_flow;
                                                                                 35
                                                                                         return abs(L[H[ind]]-L[a])
                                                                                             + abs(L[H[ind]]-L[b]);
50
                                                                                 36
                                                                                 37
51
            return max_flow;
52
                                                                                    Lowest Common Ancestor (Using Binary Lifting)
53
54
   int main(){
        #ifndef ONLINE_JUDGE
55
                                                                                    #include <bits/stdc++.h>
56
                     freopen ("input.txt", "r", stdin);
                                                                                    #define MAXN 100100
57
            #endif
                                                                                    typedef long long ll;
58
            ios_base::sync_with_stdio(false);
59
            for (int i = 0; i < MAXN; i++){
                                                                                 5
                                                                                     using namespace std;
                    for (int j = 0; j < MAXN; j++){
61
                             rg[i][j] = g[i][j] = 0;
                                                                                    int n, m, s [MAXN], depth [MAXN], anc [MAXN] [40];
62
                                                                                    vector < int > g[MAXN];
63
                                                                                 9
                                                                                    bool vis [MAXN];
64
                                                                                 10
                                                                                    int dfs(int x, int d, int p){
                                                                                11
   Lowest Common Ancestor
                                                                                12
                                                                                             vis[x] = true;
                                                                                13
                                                                                             depth[x] = d;
   Lowest Common Ancestor (Using RMQ)
                                                                                14
                                                                                             s[x] = 1;
                                                                                             anc[x][0] = p;
                                                                                15
                                                                                 16
                                                                                             for (int i = 1; pow (2, i) <= d; i++){
    *H/u is first visit of u
                                                                                 17
                                                                                                     anc[x][i] = anc[anc[x][i-1]][i-1];
```

```
18
                                                                                 73
                                                                                                       else{
19
                                                                                 74
                                                                                                               if(depth[a] = depth[b]) cout \ll s[1] - s[walk(b,
            for (int i = 0; i < g[x]. size(); i++){}
                                                                                                                   d / 2 - 1) - s[walk(a, d / 2 - 1)];
20
                    if(vis[g[x][i])) continue;
21
                    s[x] += dfs(g[x][i], d + 1, x);
                                                                                 75
                                                                                                               else cout \ll s [walk(b, d / 2)] - s [walk(b, d / 2 -
22
                                                                                                                    1)];
23
                                                                                 76
24
            return s[x];
                                                                                 77
25
                                                                                 78
                                                                                              cout \ll "\n";
26
                                                                                 79
27
                                                                                 80
   int walk(int x, int d){
28
            int i = 0;
                                                                                 81
29
            while (d) {
                                                                                     Minimum Spanning Tree (Kruskal's)
30
                    if(d \& 1) x = anc[x][i];
31
                    d /= 2;
32
                    i++;
                                                                                  1
                                                                                     struct edge {
33
                                                                                  2
                                                                                          int x, y, w;
34
            //cout \ll "\n";
                                                                                          bool operator < (edge e) const {
35
            return x;
                                                                                              return w < e.w;
36
37
                                                                                  6
                                                                                     };
   int lca(int x, int y){
39
            //cout \ll x \ll y;
                                                                                     int main(){
40
                                                                                  9
                                                                                          vector < edge > eList; //Input
41
            if(depth[x] < depth[y]) y = walk(y, depth[y] - depth[x]);
                                                                                 10
                                                                                          for (int i = 0; i < N; i++)// Set up UFDS
            if(depth[x] > depth[y]) x = walk(x, depth[x] - depth[y]);
42
                                                                                 11
                                                                                              p[i]=i;
43
            //cout \ll x \ll y;
                                                                                 12
                                                                                          vector<ii> treeList;
44
            if(x == y) return x;
                                                                                 13
                                                                                          sort(eList.begin(),eList.end());
45
            for (int i = 30; i >= 0; i --)
                                                                                 14
                                                                                          int cost = 0;
                    if(depth[x] >= pow(2, i) && anc[x][i] != anc[y][i])
46
                                                                                 15
                                                                                          int sz=N;
47
                             return lca(anc[x][i], anc[y][i]);
                                                                                 16
                                                                                          int u, v, w;
48
                                                                                 17
                                                                                          for (const auto &i : eList) {
49
                                                                                 18
                                                                                              v=i.x; u=i.v; w=i.w;
50
            return anc [x][0];
                                                                                 19
                                                                                              if(!connected(u,v)){
51
                                                                                 20
                                                                                                  join (u,v);
52
                                                                                 21
                                                                                                  treeList.push_back(\{min(u,v),max(u,v)\});
   int main() {
                                                                                 22
                                                                                                  sz --;
54
            ios_base::sync_with_stdio(false);
                                                                                 23
                                                                                                  cost+=w;
55
        cin >> n;
                                                                                 24
56
        for (int i = 0; i < n - 1; i++)
                                                                                 25
57
            int a, b;
                                                                                 26
                                                                                          if(sz!=1)
58
            cin \gg a \gg b;
                                                                                 27
                                                                                              puts ("Impossible");
            g[a].push_back(b);
59
                                                                                 28
60
            g[b].push_back(a);
61
                                                                                     Strongly Connected Components (Tarjan's)
62
        dfs(1, 0, -1);
63
        cin \gg m;
64
        for (int i = 0; i < m; i++){
                                                                                     typedef pair<int, int> ii;
65
            int a, b;
                                                                                  2
66
            cin \gg a \gg b;
                                                                                  3
                                                                                     int N.M:
67
            if(depth[a] > depth[b]) swap(a, b);
                                                                                     vector < int > adjList [MX_N];
68
            if(a == b) cout << n;
                                                                                     int dfs_num [MX_N], dfs_low [MX_N];
69
            else{
                                                                                     bool vis [MX_N];
70
                    int l = lca(a, b);
                                                                                     stack<int> scc;
71
                    int d = -2 * depth[1] + depth[a] + depth[b];
                                                                                     int dfsCounter=1;
72
                    if(d \% 2) cout << "0";
                                                                                     int sccIdx=1;
```

```
10
11
   map<int, int> sccMap;
12
13
   void tarjans (int u) {
14
         scc.push(u);
         vis [u]=true;
15
16
17
         dfs_low[u] = dfs_num[u] = dfsCounter++;
18
19
         for (int i = 0; i < adjList[u].size(); i++){
20
             int v = adjList[u][i];
21
             if(dfs_num[v]==0)
22
                  tarjans(v);
23
                  dfs_low[u] = min(dfs_low[u], dfs_low[v]);
24
             } else if (vis[v]) {
25
                  dfs_low[u] = min(dfs_low[u], dfs_num[v]);
26
27
28
         \mathbf{if}(dfs_low[u] = dfs_num[u])
29
             while (1) {
30
                  int v = scc.top(); scc.pop();
31
                  \operatorname{sccMap}[v] = \operatorname{sccId}x;
32
                  vis[v] = false;
33
                  if (v==u)
34
                      break;
35
36
             sccIdx++;
37
38
   Bipartite Check
    vector < vector < int >> adj;
```

```
vector < int > side(n, -1);
   bool is_bipartite = true;
6
   queue < int > q;
   for (int st = 0; st < n; ++st) {
        if (side[st] = -1) {
9
            q.push(st);
10
            side[st] = 0;
11
            while (!q.empty()) {
12
                int v = q. front();
13
                q.pop();
                for (int u : adj[v]) {
14
15
                     if (side [u] == -1) {
                         side[u] = side[v] ^ 1
16
                         q.push(u);
17
18
                     } else {
                         is_bipartite &= side[u] != side[v];
19
20
21
22
23
```

```
24
25
   cout << (is_bipartite ? "YES" : "NO") << endl;</pre>
   Dijkstras
   #include <bits/stdc++.h>
   #include <utility>
   #define MAXN 505
    using namespace std;
    typedef long long ll;
   typedef pair<int, int> ii;
9
   int n:
10
11
   vector < pair < int, int > g[MAXN];
    int dist[MAXN];
12
13
   void dijkstra(int x){
14
15
            for (int i = 0; i < n; i++){
16
                     dist[i] = 999999999;
17
18
            priority_queue<pair<int, int>, vector<pair<int, int> >, greater<
                pair < int, int > > pq;
19
            pq.push(\{0, x\});
20
            dist[x] = 0;
            while (!pq.empty()) {
21
22
                     pair < int, int > v = pq.top();
23
                     pq.pop();
24
                     for (int i = 0; i < g[v.second].size(); i++){
25
                             pair < int, int > u = g[v.second][i];
26
                             if(dist[v.second] + u.second < dist[u.first])</pre>
27
                                      pq.push(\{dist[u.first] = dist[v.second] +
                                          u.second, u.first });
28
29
30
31
32
   int main(){
33
        #ifndef ONLINE_JUDGE
34
                     freopen ("input.txt", "r", stdin);
35
            #endif
36
            ios_base::sync_with_stdio(false);
37
        //cin >> n:
38
   Find Cycles
1 int n;
   vector < vector < int >> adj;
   vector < char > color;
   vector < int > parent;
5 int cycle_start, cycle_end;
                                   // In O(M)
```

```
bool dfs(int v) {
8
        color[v] = 1;
        for (int u : adj[v]) {
9
10
            if (color[u] == 0) {
                parent[u] = v;
11
12
                if (dfs(u))
13
                    return true;
14
            else if (color[u] == 1) {
15
                cycle_end = v;
16
                cycle_start = u;
17
                return true;
18
19
20
        color[v] = 2;
21
        return false;
22
23
24
   void find_cycle() {
25
        color.assign(n, 0);
26
        parent.assign(n, -1);
27
        cycle_start = -1;
28
29
        for (int v = 0; v < n; v++) {
30
            if (dfs(v))
31
                break;
32
        }
33
34
        if (cycle\_start == -1) {
35
            cout << "Acyclic" << endl;
36
37
            vector <int> cycle;
38
            cycle.push_back(cycle_start);
39
            for (int v = cycle_end; v != cycle_start; v = parent[v])
                cycle.push_back(v);
40
41
            cycle.push_back(cycle_start);
42
            reverse (cycle.begin(), cycle.end());
43
            cout << "Cycle_found:_";
44
45
            for (int v : cycle)
                cout << v << "_";
46
47
            cout << endl;
48
49
```

### Arrays

### Longest Increasing Subsequence

```
1 int ls [MX.N];
2 int L[MX.N];
3 int I[MX.N];
4
5 void nlogn(){
```

```
6
        for (int i = 1; i < N+1; ++i)
7
            I [i] = INF;
8
        I[0] = -INF;
9
        int mx = 1;
10
        for (int i = 0; i < N; ++i)
            int ind = lower\_bound(I, I+N+1, ls[i]) - I;
11
12
            I[ind] = ls[i];
13
            L[i] = ind;
14
            mx = max(mx, ind);
15
16
        int prv = INF;
17
        vector<int> out;
18
        for (int i = N-1; i >= 0; —i) {
            if(ls[i] < prv && L[i]==mx){
19
20
                out.push_back(ls[i]);
21
                prv = ls[i];
22
                mx--;
23
24
25
```

### **Data Structures**

#### Fenwick Tree

```
1 int tree [MX_N];
   int N;
    int lsOne(int i) \{ return i \& (-i); \}
    void update(int k,int v){
        for (; k<MX_N; k+=lsOne(k))
 6
            tree[k]+=v;
 7
    int query(int k){
9
        int cnt = 0;
10
        for (; k; k-=lsOne(k)) {
11
            cnt+=tree[k];
12
13
        return cnt;
14 }
```

### Segment Trees

### Segment Tree Lazy

```
typedef long long ll;
   typedef pair<int, int> ii;
   const double PI = acos(-1.0);
   struct SegT {
           vector <ll> seg , lazy;
           int n;
10
           SegT () \{ \}
11
12
           SegT (int n) {
13
                   this -> n = n;
14
                   seg.resize(4*n + 1);
15
                  lazy.resize(4*n + 1);
16
17
18
           void prop (int r, int i, int j) {
19
20
                   seg[r] += lazy[r] * (j-i+1);
21
```

```
22
                     if (i!= j) {
23
                             lazy[2*r] += lazy[r];
24
                             lazy[2*r + 1] += lazy[r];
25
26
27
                    lazy[r] = 0;
28
29
30
            int a, b;
31
            ll update (int r, int i, int j, ll val) {
32
                     prop (r, i, j);
33
                     if (j < a \text{ or } i > b)
                                              return OLL;
34
35
                     if (i >= a \text{ and } j <= b) {
36
                             lazy[r] += val;
37
                             prop (r, i, j);
38
                             return seg[r];
39
                     } else {
40
                             int mid = (i + j)/2;
41
                             11 L = update (2*r, i, mid, val);
42
                             ll R = update (2*r + 1, mid + 1, j, val);
43
                             return L + R;
44
45
46
47
            ll update (int l, int r, ll val) {
48
                     a = l; b = r;
49
                     return update (1, 0, n-1, val);
50
51
52
            ll query (int l, int r) {
53
                     return update (1, r, 0);
54
55
56
   };
    Segment Tree (Eoin)
1 int tree [MX_N * 4];
2 int a [MX_N];
3 int N;
5 void construct (int p, int L, int R) {
```

```
Sparse Table
        if (L==R) {
             tree[p] = a[L];
8
             return;
                                                                                        inline int rmq(int u, int v){
9
                                                                                     2
                                                                                             if(u > v)
10
        if (R<L)
                                                                                     3
                                                                                                 return -20000000000;
11
             return;
                                                                                             int k=(int) floor (log2((double)(v-u+1)));
12
        int md = (L+R)/2;
                                                                                             if (r [mtable [u] [k]] >
13
        construct (2*p,L,md);
                                                                                                     r [ mtable [ v-(1 << k) + 1 ] [k] ] )
14
        construct(2*p+1,md+1,R);
                                                                                                 return mtable [u][k];
15
        tree[p] = min(tree[2*p], tree[2*p+1]);
                                                                                     8
                                                                                             return mtable [v-(1 << k) + 1][k];
16
                                                                                     9
17
                                                                                    10
18
    void update(int p, int L, int R, int ind, int v){
                                                                                    11
                                                                                        for (int i = 0; i < N; i++)
19
        if (L=R) {
                                                                                    12
                                                                                             mtable[i][0] = i;
20
             a[ind] = v;
                                                                                    13
                                                                                         for (int j = 1; (1 << j) <= N; j++)
21
             tree[p] = v;
                                                                                    14
                                                                                             for (int i = 0; i + (1 << j) - 1 < N; ++i)
22
             return;
                                                                                    15
                                                                                                 if (r [mtable [i] [j−1]]
23
                                                                                    16
                                                                                                          >r [ mtable [ i+(1<<(j-1)) ] [ j-1] ])
24
        int md = (L+R)/2;
                                                                                    17
                                                                                                      mtable[i][j] = mtable[i][j-1];
25
        if (ind \le md)
                                                                                    18
                                                                                                 else
26
             update (2*p, L, md, ind, v);
                                                                                    19
                                                                                                      \text{mtable}[i][j] = \text{mtable}[i+(1<<(j-1))][j-1];
27
        else
28
             update(2*p+1,md+1,R,ind,v);
                                                                                        Union Find
29
        tree[p] = min(tree[2*p], tree[2*p+1]);
30
31
                                                                                        /**
                                                                                     1
    int rmq(int p, int L, int R, int l, int r){
                                                                                     2
33
        if(r < L \mid \mid l > R)
                                                                                                 Union find algorithm
                                                                                                  Complexity O(\log n) for Join or Find.
34
             return INF;
                                                                                        */
35
        if(l>=L && r<=R)
                                                                                     4
36
             return tree[p];
37
                                                                                        int pai [N];
        int md = (1+r)/2;
38
        return \min(\text{rmq}(2*p,L,R,l,md),\text{rmq}(2*p+1,L,R,md+1,r));
39
                                                                                        void init(int n){
                                                                                     9
                                                                                                 for (int i=1; i \le n; i++){
                                                                                    10
                                                                                                          pai[i]=i;
   Iterative Segment Tree
                                                                                    11
                                                                                    12
                                                                                    13
 1 int t[2*N], n; // When debugging, the prob is most likely that you have
                                                                                    14
                                                                                        int find(int i){
        multiple n's. need this one here!
                                                                                                 if(pai[i]==i)return i;
                                                                                    15
                                                                                    16
                                                                                                 return pai[i]=find(pai[i]);
   int query(int 1, int r){ // This r is exclusive!
                                                                                    17
 4
             int ans=0;
                                                                                    18
 5
             for (l+=n, r+=n; l< r; l>>=1, r>>=1)
                                                                                    19
                                                                                        int join (int a, int b) {
                     if(1\&1)ans+=t[1++];
                                                                                    20
                                                                                                 a = find(a);
                     if(r\&1)ans+=t[--r];
                                                                                    21
                                                                                                 b = find(b);
 8
                                                                                    22
                                                                                                 pai [a]=pai [b];
9
             return ans;
                                                                                    23 }
10
11
12
   void update(int p, int v){
13
             for (t [p+=n]+=v; p>1; p>>=1){
                     t [p>>1]=t [p]+t [p^1];
14
15
16
```

# Strings

```
KMP
                                                                                  39
                                                                                  40
                                                                                              for (int i = 0; i < (int)t.size(); i++) {
                                                                                                       while (k > 0 \text{ and } p[k] != t[i])
                                                                                  41
   KMP
                                                                                                               k = pi[k];
                                                                                  42
                                                                                  43
                                                                                  44
                                                                                                       if (p[k] = t[i])
1
                                                                                  45
                                                                                                               k++;
2
            border = proper prefix that is suffix
            p[i] = length \ of \ longest \ border \ of \ prefix \ of \ length \ i, \ s[0...i-1]_{--}^{46}
                                                                                                       if (k == m)
3
                                                                                  47
                                                                                                               report (i - m + 1);
 4
   */
                                                                                  48
 5
                                                                                  49
   typedef long long ll;
                                                                                  50
   typedef pair<int, int> ii;
                                                                                  51
   52
                                                                                     int main (void) {
   const double PI = acos(-1.0);
                                                                                  53
                                                                                              ios_base::sync_with_stdio(false);
10
                                                                                  54
   const int N = 1e6 + 6;
                                                                                  55
                                                                                              return 0;
   int pi[N];
                                                                                  56
    string p, t;
14
                                                                                     KMP (Eoin)
   void pre () {
15
            p += '#';
16
17
                                                                                     vector < int > build Failure (string s) {
            pi[0] = pi[1] = 0;
                                                                                          vector < int > T(n+1,0);
18
                                                                                          T[0] = -1;
19
            for (int i = 2; i \le (int)p.size(); i++) {
20
                    pi[i] = pi[i-1];
                                                                                  4
                                                                                          int j = 0;
21
                                                                                  5
                                                                                          for (int i = 1; i < s.size();++i){
22
                     while (pi[i] > 0 \text{ and } p[pi[i]] != p[i-1])
                                                                                              if (s[i]==s[j]) {
23
                             pi[i] = pi[pi[i]];
                                                                                                  T[i]=T[j];
24
                                                                                                  j++;
25
                     if (p[pi[i]] = p[i-1])
                                                                                  9
                                                                                              } else{
26
                             pi[i]++;
                                                                                  10
                                                                                                  T[i] = j;
27
                                                                                  11
                                                                                                  j = T[j];
28
                                                                                  12
                                                                                                  while (j >= 0 \&\& s[i]! = s[j])
29
                                                                                  13
                                                                                                      j = T[j];
                                                                                  14
   void report (int at) {
                                                                                                  j++;
31
                                                                                  15
32
                                                                                  16
33
                                                                                  17
                                                                                         T[s.size()] = j;
34
   void KMP () {
                                                                                  18
                                                                                          return T;
35
            pre ();
                                                                                  19
36
                                                                                  20
                                                                                     vector<int> search(string W, string S){
37
            int k = 0;
                                                                                  21
                                                                                          auto T=buildFailure(W);
                                                                                  22
            int m = p.size() - 1;
                                                                                          vector < int > p;
```

```
int k = 0;
24
        int j = 0;
25
        \mathbf{while}(j < S.size())
26
            if(W[k]==S[j])
27
                k++; j++;
28
                if (k==W. size ()) {
29
                    p.push_back(j-k);
30
                    k = T[k];
31
32
            }else{
33
                k = T[k];
34
                if(k < 0)
35
                    j+=1, k+=1;
36
37
38
        return p;
39
   Rabin Karp
1 // Looks for a pattern s in text t in O(n+m) time.
   // Returns where the occurences are
        const int p = 31;
        const int m = 1e9 + 9;
        int S = s.size(), T = t.size();
 8
```

```
vector < int > rabin_karp (string const& s, string const& t) {
9
        vector < long long > p_pow(max(S, T));
10
        p_{pow}[0] = 1;
11
        for (int i = 1; i < (int)p_pow.size(); i++)
12
            p_{pow}[i] = (p_{pow}[i-1] * p) \% m;
13
14
        vector < long | long> h(T + 1, 0);
15
        for (int i = 0; i < T; i++)
            h[i+1] = (h[i] + (t[i] - 'a' + 1) * p_pow[i]) \% m;
16
17
        long long h_s = 0;
18
        for (int i = 0; i < S; i++)
            h_s = (h_s + (s[i] - 'a' + 1) * p_pow[i]) \% m;
19
20
21
        vector < int > occurences;
22
        for (int i = 0; i + S - 1 < T; i++) {
23
            long long cur_h = (h[i+S] + m - h[i]) % m;
24
            if (cur_h = h_s * p_pow[i] \% m)
25
                occurences.push_back(i);
26
27
        return occurences;
28
```

### **Suffix Array**

```
\begin{array}{ccc} 1 & \textbf{void} & \texttt{countingSort} \left( \textbf{int} & k \right) \{ \\ 2 & & \textbf{int} & i \ , \texttt{sum} \ , \texttt{maxi} \!\!=\!\! \texttt{max} \! \left( 300 \ , \! N \right) ; \end{array}
```

```
memset(c, 0, sizeof(c));
4
        for (i = 0; i < N; i++)
5
             c[i+k < N ? RA[i+k] : 0]++;
6
        for (i=sum=0; i < maxi; i++){
             int t = c[i];
8
             c[i]=sum;
9
            sum+=t;
10
11
        for (i = 0; i < N; i++)
12
             tempSA[c[SA[i]+k < N]
13
                 ? RA[SA[i]+k]: 0]++] = SA[i];
14
        for (i = 0; i < N; i++)
15
            SA[i] = tempSA[i];
16
17
18
   int main(){
19
        for (int i = 0; i < N; i++)
20
            SA[i]=i, RA[i]=input[i];
21
22
        for (int k = 1; k < N; k <<= 1) {
23
             countingSort(k);
24
             countingSort(0);
25
             tempRA[SA[0]] = r = 0;
26
             for (int i = 1; i < N; i++){
27
                 tempRA [SA [ i ] ]
28
                     =(RA[SA[i]]==RA[SA[i-1]]
                     && RA[SA[i]+k]==RA[SA[i-1]+k]
29
30
                     ? r:++r);
31
32
             for (int i = 0; i < N; i++)
33
                 RA[i] = tempRA[i];
34
35
        return 0;
36
```

# Manacher's algorithm for longest palindromic substring

```
/* Manacher s algorithm O(N), time and memory, algorithm to find
    * longest palindromic substring
    * Transform initial string t into s,
    * puting separators between characters
6
    * Build vector p[], where p[i] is the length of the
8
    * palindrome centered at s[i]
9
10
    * Works for both, odd and even length
    * s: # a # b # a #
11
                                    # a # a #
12
    * p: 0 1 0 3 0 1 0
                                    0 1 2 1 0
13
    * p built in O(N) using the fact that elements can be simetric
14
    * given some center and p center:
```

```
* If we are in i and center c, i_{-}mirror = c - (i - c), if p[i_{-}mirror]
                                                                              70
17
    * fits in center + p[center], p[i] is p[i\_mirror], else we need to
                                                                              71
                                                                                          /* not tested */
    * check real value of p[i]
                                                                              72
                                                                                          string res;
    * If we call the border center + p[center], r. Its easy to see
                                                                              73
                                                                                          for (int i = 0; i < n; i++)
    * r is only increased, achieving the O(N) time complexity
                                                                              74
                                                                                                  if (i >= center - len and i <= center + len and s[i] != '#
21
22
    * Longest palindromic substring is the maximum element in p
                                                                              75
                                                                                                           res += s[i];
23
                                                                                          /* */
                                                                              76
24
    * */
                                                                              77
25
                                                                              78
                                                                                          cout << len << endl;
                                                                              79
                                                                                          cout << res << endl;
   typedef long long ll;
                                                                              80
   typedef pair<int, int> ii;
   81
                                                                                          return 0;
                                                                              82
   const double PI = acos(-1.0);
   const int N = 1e6 + 5;
                                                                                  Z function
   int p[2*N + 2];
33
34
   int main (void) {
                                                                               1
                                                                                              \{0, if i = 0\}
35
            ios_base::sync_with_stdio(false);
                                                                                      z[i] = \{length \ longest \ commom \ prefix \ of \ s \ and \ s[i...n-1]\}
                                                                                  */
36
37
            string s, t;
                            cin >> t;
                                                                               4
38
            s += "#";
                                                                                  typedef long long ll;
                                                                               5
39
            for (auto c : t) {
                                                                                  typedef pair<int, int> ii;
40
                    s += c;
                                                                                  41
                    s += '#';
                                                                                  const double PI = acos(-1.0);
42
                                                                               9
43
                                                                              10
                                                                                  const int N = 2e5 + 5;
            int n = s.size();
44
                                                                              11
                                                                                  string s;
           int c = 0, r = 0;
45
                                                                                 int z[N];
            for (int i = 0; i < n; i++) {
                                                                              13
47
                    int i_mirror = c - (i - c);
                                                                              14
                                                                                  void go () {
48
                                                                              15
                                                                                          int 1 = 0, r = 0;
49
                    if (i \leq r)
                                                                              16
                                                                                          int n = s.size();
50
                            p[i] = min (p[i\_mirror], r - i);
                                                                              17
                                                                                          memset (z, 0, sizeof z);
51
                    else
                                                                              18
52
                            p[i] = 0;
                                                                              19
                                                                                          for (int i = 1; i < n; i++) {
53
                                                                              20
                                                                                                  if (i \leq r)
54
                    while (i - 1 - p[i]) >= 0 and i + 1 + p[i] < n and s[i + 121]
                                                                                                           z[i] = min (z[i-l], r - i + 1);
                       + p[i] = s[i - 1 - p[i]]
                                                                              22
                                                                                                  while (z[i] + i < n \text{ and } s[z[i] + i] == s[z[i]])
                            p[i]++;
                                                                              23
                                                                                                           z[i]++;
56
                                                                              24
                                                                                                  if (r < i + z[i] - 1) {
57
                                                                              25
                                                                                                           l = i;
58
                    if (i + p[i] > r) {
                                                                              26
                                                                                                           r = i + z[i] - 1;
59
                            c = i:
                                                                              27
60
                            r = i + p[i];
                                                                              28
61
                                                                              29
62
                                                                                  Hash
64
            int len = 0, center = 0;
65
            for (int i = 0; i < n; i++)
66
                    if (p[i] > len) {
                                                                               1 typedef long long ll;
67
                            len = p[i];
                                                                               2 typedef pair <ll, ll> ii;
68
                            center = i;
69
                                                                               4 11 md (11 x, 11 mod) {
```

```
x \% = mod;
                                                                                57
                                                                                                             id = mid;
6
            if (x < 0)
                                                                                58
                                                                                                     } else {
7
                    return x + mod;
                                                                                59
                                                                                                             top = mid - 1;
8
            return x;
                                                                                60
9
                                                                                61
10
                                                                                62
11
   struct Hash {
                                                                                63
                                                                                             if (id = min (ilen, jlen) - 1) {
12
            const ll base = 31;
                                                                                64
                                                                                                     if (ilen != jlen)
13
            ll mod, *h, *pot;
                                                                                65
                                                                                                             return ilen < jlen;
14
                                                                                66
                                                                                                     return i < j;
            string s;
15
                                                                                67
            Hash () {}
                                                                                68
16
17
                                                                                69
                                                                                            return s[i + id + 1] < s[j + id + 1];
18
            void build (string s, ll mod) { // O(n)
                                                                                70
19
                                                                                71
                    this -> mod = mod;
20
                    this -> s = s;
                                                                                72 const 11 mod[2] = \{1000000007, 1000000009\};
21
                    h = new 11 [s.size() + 2];
22
                    pot = new 11 [s.size() + 2];
23
24
                    h[0] = s[0] - 'a';
25
                    for (int i = 1; i < (int)s.size(); i++)
26
                            h[i] = (h[i-1]*base + s[i] - 'a')\%mod;
27
28
                    pot[0] = 1LL;
29
                    for (int i = 1; i < (int)s.size(); i++)
30
                             pot[i] = (pot[i-1] * base)\%mod;
31
32
33
            ll query (int l, int r) { // O(1)
34
                    11 R = h[r], L = 0;
35
36
                    if (1)
                            L = (h[l-1] * pot[r - l + 1]) \% mod;
37
38
39
                    return md (R - L, mod);
40
41
42
   } h[2];
43
    // returns if s[i, i+ilen-1] is lexicographically smaller than s[j, j]
        + ilen - 1
    // not tested if ilen != jlen
   bool comp (string &s, int i, int ilen, int j, int jlen) {
47
            int bot = 0, top = min (ilen, jlen) - 1;
48
            int id = -1:
49
50
            while (bot \leq top) {
51
                    int mid = (bot + top) >> 1;
52
53
                    ii pi = ii (h[0]. query(i, i + mid), h[1]. query(i, i + mid))
                    ii pj = ii(h[0].query(j, j + mid), h[1].query(j, j + mid))
54
                    if (pi == pj) {
56
                             bot = mid + 1;
```

# Geometry

```
Convex Hull
                                                                                37
                                                                                                    while (k \ge 2 \&\& cross(H[k-2], H[k-1], P[i]) \le 0) k--; //
                                                                                                         Remove the last "=" if you want to get max points in
                                                                                                        the hull
   Convex Hull
                                                                                38
                                                                                                    H[k++] = P[i];
                                                                                39
                                                                                40
   // Implementation of Andrew's monotone chain 2D convex hull algorithm.
                                                                                            // Build upper hull
                                                                                41
   // Asymptotic complexity: O(n loq n).
                                                                                42
                                                                                            for (size_t i = n-1, t = k+1; i > 0; --i)
                                                                                43
                                                                                                    while (k \ge t \&\& cross(H[k-2], H[k-1], P[i-1]) \le 0) k--;
   typedef double coord_t;
                                // coordinate type
                                                                                                        // Remove the last "=" if you want to get max points
   typedef double coord2_t; // must be big enough to hold 2*max(|coordinate
                                                                                                        in the hull
       1)^2
                                                                                                    H[k++] = P[i-1];
                                                                                44
                                                                                45
   struct Point {
                                                                                46
            coord_t x, y;
                                                                                47
                                                                                            H. resize(k-1);
            Point(){}
                                                                                48
                                                                                            return H:
10
            Point(coord_t xx, coord_t yy){
                                                                                49
11
                    x=xx, y=yy;
                                                                                50
12
                                                                                51
            bool operator <(const Point &p) const {
13
                                                                                52
                                                                                        Note that this is using double. Its better to use long long because
14
                    return x < p.x \mid | (x = p.x \&\& y < p.y);
                                                                                         double might be TLE!
15
                                                                                53
16
   };
17
                                                                                    Convex Hull (Eoin)
   // 2D cross product of OA and OB vectors, i.e. z-component of their 3D
        cross product.
   // Returns a positive value, if OAB makes a counter-clockwise turn,
                                                                                   int main(){
                                                                                2
                                                                                        for (int i = 0; i < N; i++){
   // negative for clockwise turn, and zero if the points are collinear.
   coord2_t cross(const Point &O, const Point &A, const Point &B){
                                                                                            perm[i]=i;
22
            return (A.x - O.x) * (B.y - O.y) - (A.y - O.y) * (B.x - O.x);
                                                                                4
23
                                                                                        sort (perm, perm+N,
24
                                                                                                [](int a, int b){
   // Returns a list of points on the convex hull in counter-clockwise order.7
                                                                                                    const point &pa = V[a];
   // Note: the last point in the returned list is the same as the first one.8
                                                                                                    const point &pb = V[b];
   vector < Point > convex_hull(vector < Point > P) {
                                                                                                    if (real(pa)!=real(pb))
28
            size_t = P. size(), k = 0;
                                                                                10
                                                                                                        return real(pa) < real(pb);
29
            if (n \ll 3) return P;
                                                                               11
                                                                                                    return imag(pa) < imag(pb);
30
            vector < Point > H(2*n);
                                                                                12
                                                                                                });
31
                                                                                13
                                                                                        vector<int> L; vector<int> U;
32
            // Sort points lexicographically
                                                                                        for (int i = 0; i < N;) {
                                                                                14
33
            sort (P. begin (), P. end ());
                                                                               15
                                                                                            int t = L. size();
                                                                                            if(t \ge 2 \&\& ! ccw(V[L[t-2]],V[L[t-1]],V[perm[i]]))
34
                                                                                16
35
            // Build lower hull
                                                                                17
                                                                                                L.pop_back();
            for (size_t i = 0; i < n; ++i)
                                                                                18
                                                                                            else
```

```
19
                 L. push_back (perm [i++]);
20
21
        for (int i = N-1; i >=0;) {
22
             int t = U. size():
23
             if(t \ge 2 \&\& ! ccw(V[U[t-2]], V[U[t-1]], V[perm[i]]))
24
                 U. pop_back();
25
             else
26
                 U. push_back (perm [i--]);
27
28
        vector<int> hull;
29
        for (int i = 0; i < L. size() -1; ++i)
30
             hull.push_back(L[i]);
31
        for (int i = 0; i < U. size () -1; ++i)
32
             hull.push_back(U[i]);
33
        return 0:
34
```

### **Geometry Functions**

```
1 typedef complex<double> pt;
   typedef complex<double> vec;
   typedef vector <pt> pgon;
   typedef struct { pt p,q; } lseg;
   double cross (const vec& a, const vec &b) {
        return x(a)*y(b)-y(a)*x(b);
6
7
   }
    //cross\ product\ of\ (b-a)\ and\ (c-b),\ 0\ is\ collinear
   int orientation (const pt& a,
10
            const pt& b, const pt& c){
11
        double v = cross(b-a, c-b):
12
        if (abs (v-0.0) < EPS)
13
            return 0;
        return v > 0 ? 1 : 2;
14
15
    //Line segment intersection
   bool intersects (const lseg& a, const lseg& b) {
17
18
        if(a.q = b.p \mid\mid b.q = a.p)
19
            return false;
20
        if (orientation (a.p.a.q.b.p)
21
                != orientation (a.p,a.q,b.q)
22
                && orientation (b.p,b.q,a.p)
23
                != orientation(b.p,b.q,a.q))
24
            return true;
25
        return false;
26
   //Area of polygon
   double area (const pgon& p) {
29
        double area = 0.0:
30
        for (int i = 1; i < p. size(); ++i)
31
            area = cross(p[i-1],p[i]);
32
        return abs(area)/2.0;
33
    //If a \rightarrow b \rightarrow c is a counterclockwise turn
   double ccw(const point& a, const point& b,
```

```
36
                               const point& c){
37
                     if (a==b | b==c | a==c)
38
                               return false;
39
                     point relA = b-a:
40
                     point relC = b-c;
41
                     return cross(relA, relC) >= 0.0;
42
43
         //Returns if point p is in the polygon poly
          bool in Poly (const pgon& poly, const pt& p) {
45
                     for (int i = 0; i < poly.size()-1; i++){}
46
                               if (! ccw ( poly [ i ] , p , poly [ i +1]) )
47
                                         return false:
48
49
                     return true;
50
         //Distance from p to line (a,b)
51
          double distToLine(const pt& p, const pt& a,
53
                               const pt &b){
                     vec ap = p-a;
54
55
                     vec ap = b-a;
                     double u = dot(ap, ab)/dot(ab, ab);
56
57
                     //Ignore for non-line segment
58
                     if (u < 0.0) //Closer to a
59
                               return abs(a-p);
60
                     if (u > 1.0) //Closer to b
61
                               return abs(b-p);
62
                     pt c = a+ab*u; // This is the point
63
                     return abs(c-p);
64 }
        double area (vector < Point > v) {
                                                                                               // Return the area of the convex hull in O
                     (n).
                               double ret = 0.0;
 3
                               int n=v.size();
                               for (int i=0; i < v . size(); i++){
                                                    ret+=v[i].x*(v[(i+1+n)\%n].y-v[(i-1+n)\%n].y);
 7
                               return abs(ret/2);
 8
 9
10
         double perimeter (vector < Point > v) {
                                                                                                                     // Return the perimeter of the
                     convex hull in <math>O(n).
11
                               double ans = 0.0;
12
                               v.pb(v[0]);
13
                               for (int i=0; i < v \cdot size() -1; i++){
14
                                                    ans = sqrt((v[i].x-v[i+1].x)*(v[i].x-v[i+1].x)+(v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v[i].y-v
                                                               +1].y)*(v[i].y-v[i+1].y));
15
16
                               return ans:
17
18
         Point rotate (Point c, Point po, double ang) {
                                                                                                                                         // Rotate po, around c by
                    anq.
20
21
                                  * C is the center of the rotation
```

### Math

```
Ternary Search
                                                                                24
                                                                                                 p = mod_exp(p, 2, n);
                                                                                25
                                                                                                 \mathbf{i} \mathbf{f} (p = n-1LL) 
                                                                                                     all_fail = false;
                                                                                26
   double ternary_search (double 1, double r) {
                                                                                27
                                                                                                     break:
2
        double eps = 1e-9;
                                         //set the error limit here
                                                                                28
3
        while (r - l > eps) {
                                                                                29
            double m1 = 1 + (r - 1) / 3;
                                                                                30
                                                                                             if (all_fail)
            double m2 = r - (r - 1) / 3;
                                                                                31
                                                                                                 return false;
                                     //evaluates the function at m1
 6
            double f1 = f(m1);
                                                                                32
            double f2 = f(m2);
                                     //evaluates the function at m2
                                                                                33
                                                                                         return true;
            if (f1 < f2)
                                                                                34
9
                l = m1;
10
            else
                                                                                    Fast fourier transform
11
                r = m2;
12
                                         //return the maximum of f(x) in [l, r]_1
13
        return f(l);
                                                                                    /* emaxx implementation */
14
                                                                                     /* Multiplication with arbitrary modulos
                                                                                           use ntt if mod is prime and can be written as 2**k*c+1
   Miller Rabin primality test
                                                                                           if not, use Chinese Reminder Theorem
                                                                                           or transform A(x) = A1(x) + A2(x)*c decompose into A(x)/c and A(x)\%c
                                                                                                        B(x) = B1(x) + B2(x)*c
   void factor(ll x, ll& e, ll& k){
                                                                                               where c = sqrt \pmod{1}
                                                                                              A * B = A1*B1 + c*(A1*B2 + A2*B1) * c**2(A2*B2)
        while (x%2LL==0LL) {
                                                                                 8
3
            x/=2LL;
                                                                                               with all values < sqrt (mod) subpolynomials have coefficientes <
4
            ++e;
                                                                                         mod * N after fft multiply decreasing changes of rounding error
                                                                                10
6
        k = x;
                                                                                11
                                                                                12
                                                                                    const double PI=acos(-1);
                                                                                13
    //increase x for higher certainty, 5 works well
                                                                                14
                                                                                    typedef complex<double> base;
   bool is_prime(ll n, int x){
                                                                                15
11
        if (n&2LL==0 | n==1LL)
                                                                                16
                                                                                    void fft (vector < base > & a, bool invert) {
12
            return false;
                                                                                17
                                                                                             int n=(int) a.size();
13
        if (n==2 | | n==3 | | n==5 | | n==7)
                                                                                18
                                                                                             for (int i=1, j=0; i< n; ++i) {
                                                                                19
                                                                                                     int bit=n>>1;
14
            return true;
15
        ll e, k;
                                                                                20
                                                                                                     for (;j>=bit;bit>>=1)
16
        factor (n-1,e,k);
                                                                                21
                                                                                                             j-=bit;
17
        while (x-->0)
                                                                                22
                                                                                                     i+=bit:
                                                                                23
18
            11 a = (rand())\%(n-5LL) + 2LL;
                                                                                                     if (i < j)
                                                                                24
                                                                                                              swap(a[i],a[j]);
19
            ll p = mod_exp(a, k, n);
20
            if (p==1LL | p==n-1LL)
                                                                                25
                                                                                             for (int len=2; len<=n; len<<=1) {
21
                                                                                26
                continue:
22
            bool all_fail = true;
                                                                                27
                                                                                                     double ang = 2*PI/len * (invert ? -1 : 1);
23
            for (int i = 0; i < e-1; ++i){
                                                                                28
                                                                                                     base wlen(cos(ang), sin(ang));
```

```
for (int i=0; i< n; i+=len) {
                                                                                 18
                                                                                              11 tmp[NMAT][NMAT];
30
                             base w(1);
                                                                                 19
                                                                                              for (int i = 0; i < n; i++)
31
                             for (int j=0; j < len/2; ++j) {
                                                                                 20
                                                                                                      for (int j = 0; j < n; j + +)
32
                                      base u=a[i+j], v=a[i+j+len/2]*w;
                                                                                 21
                                                                                                               ans [i][j] = i = j;
                                     a[i+j]=u+v;
                                                                                 22
                                                                                              while(b) {
                                                                                 23
                                                                                                      if (b&1) {
34
                                     a[i+j+len/2]=u-v;
35
                                     w*=wlen;
                                                                                 24
                                                                                                               mu(ans, mat, tmp, _n);
36
                                                                                 25
                                                                                                               for (int i=0; i<_n; i++)
37
                                                                                 26
                                                                                                                       for (int j=0; j<_n; j++)
                                                                                 27
38
                                                                                                                                ans [i] [j] = tmp [i] [j];
39
            if (invert)
                                                                                 28
                    for (int i=0; i< n; ++i)
40
                                                                                 29
                                                                                                      mu(mat, mat, tmp, _n);
41
                             a[i]/=n;
                                                                                 30
                                                                                                      for (int i=0; i<_n; i++)
42
                                                                                 31
                                                                                                               for (int j=0; j<_n; j++)
                                                                                 32
43
                                                                                                                       mat[i][j]=tmp[i][j];
   // a, b \Rightarrow coefs to multiply, res \Rightarrow resulting coefs
                                                                                 33
                                                                                                      b>>=1;
   // a[0], b[0], res[0] = coef x^0
                                                                                 34
   // Doesnt work with negative coefs
   void multiply (const vector < int > & a, const vector < int > & b, vector < int > &
         res) {
                                                                                     Math Tricks
48
            vector < base > fa (a.begin(), a.end()), fb (b.begin(), b.end());
49
            size_t n=1;
50
            while (n < max(a.size(),b.size())) n < <=1;
                                                                                  1 ll fexp(ll a, int x, ll mod){
                                                                                                                                // Fast exponenciation returns a^x
51
            n < < =1;
                                                                                          \% mod
52
            fa.resize(n),fb.resize(n);
                                                                                              if(x==0)return 111;
53
            fft (fa, false); fft (fb, false);
                                                                                              if(x\%2==0){
            for (size_t i=0; i< n; ++i)
54
                                                                                                      11 y=fexp(a, x/2, mod);
55
                    fa[i]*=fb[i];
                                                                                                      return (y*y)%mod;
            fft (fa, true);
56
57
            res.resize (n);
                                                                                              return (a*fexp(a, x-1, mod))\%mod;
            // avoid precision errors, mess up with negative values of coefs
59
            for (size_t i = 0; i < n; ++i)
60
                    res[i] = int(fa[i].real() + 0.5);
                                                                                     ll divv(ll a, ll b, ll mod){ // Division with mod returns a/b % mod
61 }
                                                                                 11
                                                                                              return (a*fexp(b, mod-2, mod))\%mod;
                                                                                 12
   Matrix Exponentiation
                                                                                 13
                                                                                 14 ll f[N];
                                                                                 15
   typedef long long ll;
                                                                                     11 fat(11 a, 11 mod){
                                                                                                                       // Calculates factorial and stores in f %
   const int NMAT=2;
                                                                                         mod
   const int mod=1;
                                                                                 17
                                                                                              if(a \le 1)return 1;
                                                                                 18
                                                                                              return f[a]? f[a]: (f[a] = (a*fat(a-1, mod))%mod);
   /* c = a * b * /
                                                                                 19
   void mu(11 a[][NMAT], 11 b[][NMAT], 11 c[][NMAT], int _n) {
                                                                                 20
            for (int i = 0; i < n; i++)
                                                                                     ll choose(ll n, ll k, ll mod){ // Returns n choose k % mod
8
                                                                                 22
                                                                                              return divv(fat(n, mod), (fat(k, mod)*fat(n-k, mod))%mod, mod)%mod
                     for (int j=0; j<_n; j++) {
9
                             c[i][j]=0;
                                                                                 23
10
                             for (int h=0;h<_n;h++) {
11
                                     c[i][j] += (a[i][h]*b[h][j])%mod;
                                                                                 24
12
                                                                                     ll gcd(ll a, ll b){ // Greatest common divisor
                                     c[i][j]\%=mod;
13
                                                                                 26
                                                                                              return b?gcd(b, a%b):a;
14
                                                                                 27
15
                                                                                 29
                                                                                    ll lcm(ll a, ll b){ // Least common multiple
   /*returns ans=mat^b*/
17 void power(ll ans[][NMAT], ll mat[][NMAT], ll b, int _n) {
                                                                                 30
                                                                                             return (a*b)/gcd(a, b);
```

```
9
31
32
                                                                                    10
33
                                                                                    11
   /* Fast factorization */
35
36
   int p[N];
37
                                                                                    14 G must be closed
   void start_fast(int MAX){
                                      // Runs O(nlog(n)) Needs to be called to
        use\ fast\_fact\ or\ ammount\_of\_divisors.
39
            for (int i=2; i \le MAX; i++){
40
                     if(p[i]==0){
41
                              for(int j=i; j<=MAX; j+=i){
42
                                      p[j]=i;
43
44
45
46
47
    vector<int>fast_fact(int x){
48
                                      // Fast factorization in O(\log 2(x))
49
             vector<int>ret;
            \mathbf{while}(x>1){
50
51
                     ret.pb(p[x]);
52
                     x/=p[x];
53
54
            return ret;
55
56
57
   int amount_of_divisors (int x) { // Calculate the amount of divisors of a
        number in O(\log 2(x)) assume already ran start-fast.
58
             if(x==1)return 1;
59
             vector < int > v = fast_fact(x);
60
            int ret=1, curr=2;
61
            for (int i=1; i < v . size(); i++){
62
                     if(v[i]==v[i-1])curr++;
63
                     else{
64
                              ret *= curr;
65
                              curr = 2;
66
67
68
            return ret*curr;
69
```

### Theorems

```
1 Picks theorem
2 Given a certain lattice polygon with non-zero area.
  We denote its area by S, the number of points with integer coordinates
      lying strictly inside the polygon by I and the number of points lying
      on polygon sides by B.
  Then, the Pick's formula states:
  S=I+B/2 1
```

Burnsides Lemma

each g in G

Let G be the finite group of operations we can perform on X

The number of orbits of X is the average of the number of fixed points for