Does Machine Learning Amplify Pricing Errors in Housing Market?: Economics of ML Feedback Loops

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Abstract

Platforms like Zillow offer Machine Learning algorithm (ML pricing) that predict sale prices for residential homes. These ML price models learn from observed sale prices, and simultaneously also influence those same sales. We analytically model this cyclical relationship between ML price and sale price (machine – human) called the ML feedback loop. We show that the ML price has a self-fulfilling nature which results in the ML model underestimating its own pricing error and market participants over-relying on the ML price. We show that these outcomes are more adverse when adoption of ML prices among market participants is higher. We identify market conditions in which introduction of ML pricing amplifies, instead of alleviating, erratic sale prices and makes seller payoffs worse off. We also find that sellers who are impatient, risk-neutral and have high ability to price privately - have most to gain from introduction of ML pricing. Using data from Zillow we empirically validate primitive building block of our analytical model. We find that home sale prices in 2019 had a significant 15% sensitivity to Zillow's ML price (e.g., a $10,000 perturbation in the ML price shifts the sale price by $1,500). Finally, we suggest ML design choices that could correct the feedback loop. We discuss whether platforms are incentivized to follow these corrective strategies and the role of policymakers in regulating the same.

Key words: Algorithmic Price, Economics of AI, Bias-Variance, Housing Market, Zillow, Zestimate.

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1. Introduction

In the last few years, ML algorithmic pricing has been introduced to a variety of markets such as home sales (Zillow's Zestimate), vacation rentals (Airbnb's Smart Pricing), ridesharing (Uber's dynamic pricing), peer-to-peer lending (Lending Club's Model Rank), and art auctions (LiveArt's Estimate). In each example, an intermediary platform offers an ML price[1] to buyers and sellers. These ML prices can adapt dynamically to changing market conditions (Brown and McKay 2021), can crunch swaths of historical data, and can incorporate enough product features to model highly differentiated products (Bertini and Koenigsberg 2021). All these characteristics make ML pricing

[1] Algorithmic price sometimes refers to a setting where an ML algorithm output dictates (e.g., Uber) or recommends (e.g., Airbnb) the transaction price that buyer and seller should use. In case of Zillow, its ML output is not explicitly recommended as the transaction price that buyer and seller should use. Instead, it's simply a prediction of the transaction price that buyer and seller are likely to settle on. Going forward we refer to this as the ML price.

a potential boon for buyers and sellers. Notably, ML pricing can resolve pricing uncertainty, reduce friction, and democratize access to information for participants who lack pricing experience (Huang 2021, Kehoe, Larsen, and Pastorino 2018). For example, a rideshare driver and rider avoid the friction of negotiating the price for every trip because the price is set by an algorithm. An investor with optimism about the art market but no artistic expertise can purchase art pieces for an ML price, benefiting both the investor and the artist (Bailey 2020).

Given all these obvious advantages, ML pricing is being increasingly adopted in many markets (Pandey and Caliskan 2021). This paper questions if ML prices that are designed to maximize statistical accuracy automatically aid market objectives like lower pricing friction, greater and equitable surplus for heterogeneous participants. At the outset, ML prices merely crunch and summarize market information, which may otherwise be costly for participants to collect manually. But as an increasing proportion of market participants rely on ML pricing, it gains the power to shift market outcomes. In effect, ML pricing would be learning from and shifting the market outcomes simultaneously in a loop. There is a pressing need to understand ML prices embedded in such feedback loops and call attention to potential risks. This paper highlights detrimental implications to pricing and payoffs, grounded in the context of Zillow's ML pricing for residential homes. These implications will guide recommendations for intermediaries that are looking to offer ML pricing and policymakers who monitor and regulate those ML models.

We develop an analytical model of a machine-market (or machine-human) interaction. In the forward loop we incorporate how the ML price impacts distribution of market participant private valuations, sale prices and the payoff. In the backward loop we incorporate how the ML model learns from sale price. The inter-dependent market and ML model evolve simultaneously to attain an equilibrium. The distribution of private valuations, sale prices, payoffs and ML models' statistical properties are characterized at this equilibrium. We also discuss role of the exogeneous levers such as level of ML adoption[2], choice of ML capacity[3] and market characteristics, and differential implications for heterogeneous sellers. This model is grounded to one notable setting - ML prices in the housing market. This allows us to closely align with realistic institutional details of one market (housing) and the typical design choices of ML framework deployed by the dominant platform (Zillow). We are also able to provide empirical evidence to support primitives or building blocks of

[2] This paper does not model process of increasing adoption, instead just takes level of adoption as exogeneous given. We examine outcomes if this exogeneous level of adoption increases in future.

[3] Example1: A deep neural network with 100's of parameters has high model capacity, while a regression or decision tree with handful of features has low capacity. Example 2: 5-degree polynomial has higher model capacity than 2-degree polynomial

the analytical model.

We show three key results. First, the error in sale prices (relative to true equilibrium market clearing price in absence of the ML prices) after introduction of ML prices can be larger than without ML prices. These errors are not systematically positive or negative across the market, and therefore they are not observable as a (positive or negative) price bubble. Second, the error in sale prices eventually (if left un-monitored by regulators or market experts) can increase to the limit i.e., where sale prices are entirely random (devoid of any correlation with true equilibrium market clearing price in absence of the ML prices). At this feedback loop equilibrium - (i) the Platform observes perfect match its ML prices and realized sale price, (ii) it therefore convey's absolute confidence in its ML price for off market homes and (iii) market participants (homeowners and home-buyers) have absolute reliance on Platform's ML prices (even though they are entirely random and uncorrelated with true equilibrium market clearing price in absence of the ML prices). Third we show that market participants characterized by - high patience (low cost of spending time learning in the market), low pricing ability (error between private valuation and true equilibrium market clearing price) and risk averse have most to loose from introduction of ML prices.

We identify three corrective strategies that platforms could deploy: (i) choose a low-capacity ML model, which would be less responsive to self-fulfilling feedback; (ii) measure and adjust for the size of error ML over-confidence ex-post; or (iii) cut off the feedback loop altogether. Unfortunately, these strategies will either limit accuracy of ML prices or hide the ML price for at least some homes. We do not model the platform as an agent in our analytical model, but we provide some discussion on why these corrective strategies may not be in line with platforms typical revenue streams from ad sales and iBuying.

The credibility of our analytical model depends on two key primitives that – (a) buyers and sellers find it difficult to price homes, therefore they rely on ML price, and (b) the ML model learns from recent home sales. Some experts and customers are highly critical of Zillow's ML price for being inaccurate, using outdated data, missing local non digitized information and even altogether unusable (Houwzer 2021). Zillow's received customer ratings of 3.8, 2.8 and 1.6 on consumeraffairs.com, sitejabber.com and trustpilot.com. Given these concerns it's important to support these primitives to confirm that platforms like Zillow that offer an ML price in-fact have some influence on the market. If these primitives are absent or negligible than the analytical model would have feeble practical relevance.

The primitive (a) is supported by WakeField research survey (Melcher 2021) which finds that the average US homebuyer tours 15 homes and makes offers on 10 homes, spending a cumulative $845 million of work time on home search and pricing. 85% of first-time buyers say that it is challenging to make offers and stressful to be rejected or outbid. In essence, pricing homes (for buyers and sellers alike) adds friction to transactions. Participants traditionally have relied on brokers, agents, and appraisers for assistance with pricing. In last few years emergence of ML pricing, such as Zillow's Zestimate and Redfin's Estimate has offered a new option. Participants are likely to find ML pricing attractive because (i) it is free and thus highly accessible, unlike an appraiser; (ii) it seems impartial, unlike agents;[4] and (iii) the platforms (e.g., Zillow and RedFin) report low error rates (1.9 and 1.7% respectively [29, 38]) between ML price and eventual sale price. Empirically, we find an average reliance of 15% on the ML price. For example, if a home has a true value of $200,000 and an ML price of $220,000 (+10% error), then the expected sale price would be $203,000 (+ 0.15 * 10% error). We identify this effect using Zillow's own algorithm upgrades as an instrument. Zillow also displays a confidence interval with each Zestimate, thereby providing an indirect estimate of the ML error σ_z. We find that the reliance α on the ML price varies by the displayed confidence (inverse of the ML error) and across submarkets (from 10% to 50%).

Zillow discloses qualitative details but not the exact proprietary Zestimate algorithm. To support primitive (b), we reverse-engineer the algorithm using observed correlations between changes in the Zestimate for a home and new sales in the home's neighborhood. We find that if a single home with a true value of $200,000 sells for $210,000 (+5%), the ML price for approximately 25 peer homes increases by 1–1.25%. Thus, we have evidence of a feedback loop: ML pricing simultaneously learns from and shifts the sale price. Note that the empirical evidence only supports primitive assumptions or building blocks for the analytical model. It does not confirm the causal mechanism laid out in the analytical model or its conclusions.

A critical component embedded our analytical model of feedback loop is the over-confidence in ML prices. In our model this happens as a result of self fulfilling prophecy of the ML prices and a lack of ground truth in (confounded) sale price observations. The presence of such over-confidence in ML prices is consistent with anecdotal observation illustrated in Figure 1. The bottom figure shows that the error in Zillow's ML price (Zestimate) appears to be roughly decreasing over time. This is expected since the Platform continues to improve its model and gather new data-sets. In fact, in June 2021 Zillow announced on such major improvement to its Zestimate. One would

[4] In Section 2 we elaborate on prior literature on mis-aligned principal-agent incentives.

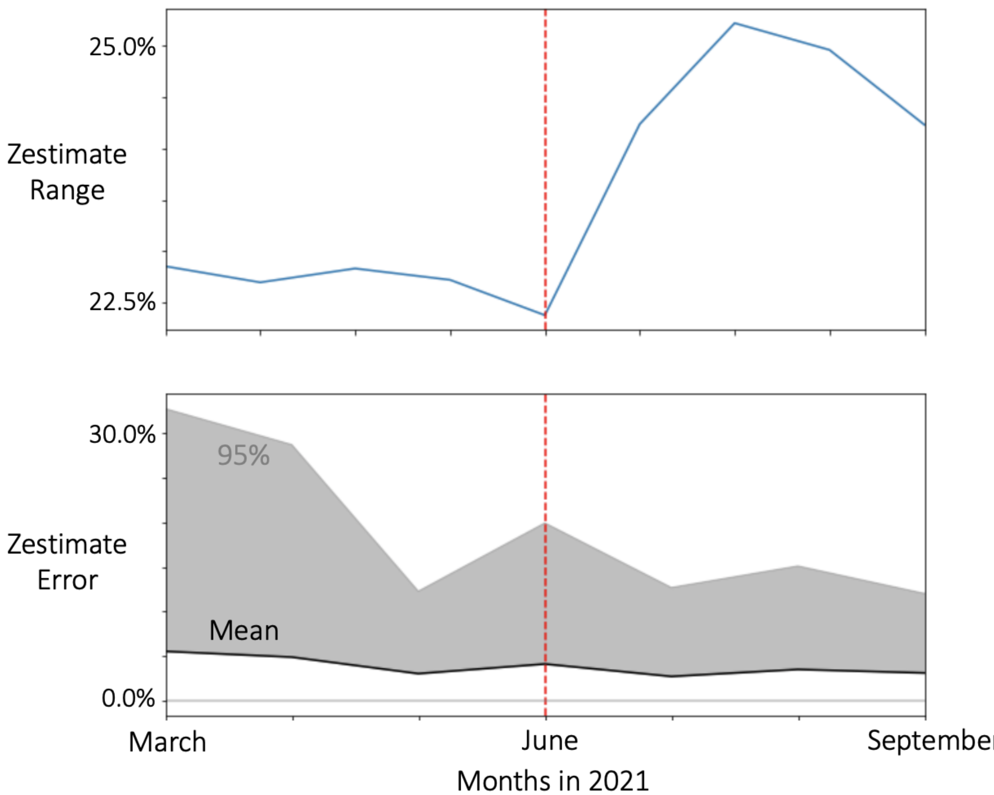

Figure 1 **(Top) The average sale price range projected around the Zestimate by Zillow increased from 22.69% (3 months prior to June 2021) to 24.66% (3 months after June 2021). (Bottom) The average actual Zestimate error (**$|Zestimate - SalePrice|/Zestimate$**) decreased from 4.36% to 3.41% during the same time. The 95 percentile Zestimate error decreased from 23.41% to 16.37%. Zillow announced a major Zestimate algorithm update on 15th June 2021.**

expect the confidence in ML prices to increase as well. The top figure shows the average confidence interval conveyed by Zillow alongside the Zestimate. Instead of narrowing, the average confidence interval jumped up. Assuming the Zestimate is in-fact improving in accuracy over time, the only plausible explanation is that the confidence interval were too *aggressive* prior to June 2021. This can be an instance of the Platform, via some internal model auditing, realizing and correcting over confidence in ML prices. While this seems to be a positive step from the Platform, there is no regulation monitoring such over confident ML prices in the industry. This motivates us to study the implications of un-regulated ML feedback loops.

ML prices that learn from and shift the market outcomes simultaneously are becoming increasingly common in housing, crowd funding, peer to peer lending and art markets. Our primary contribution is to identify the implications of these ML prices on valuations, sale prices and seller payoffs. We find that the introduction of ML pricing can add a deceptively large bias in valuations and errors in sale prices. These effects are amplified under high ML model capacity and level of adoption. We also identify that sophisticated seller (impatient, risk-neutral and high ability to price) have most to gain from introduction of ML prices. Extrapolating more broadly, this paper builds on the fact that, unlike rational human agents, algorithms are not forward looking or strategic but guided by statistical objective of good prediction accuracy. Algorithms in pursuit of this myopic objective are capable of moving the market by reinforcing selective and self-fulfilling preferences (e.g., women being recommended content on fashion and men being recommended content on sports, loan applications from historically underserved groups being tagged with higher interest rates). In the worst case, these reinforced preferences can entirely diverge from ground truth. This divergence while concealed in the short run eventually comes to bear for the market participants in the long run. Thus, the myopic statistical objective risks losing sight of long run economic outcomes for market participants.

Our findings should motivate policymakers to monitor and regulate ML pricing, especially as it becomes increasingly ubiquitous in various markets. Our research is also important for intermediary platform that may fail to account for long-term implications of ML pricing. Platform face risk to their brand and reputation when eventually bubble reinforced by ML feedback loop diverges substantially from ground truth and bursts. This paper is one of the first (i) to empirically measure the impact of ML pricing on home sale prices (in the context of Zillow and the Zestimate) and (ii) to begin to disentangle Zillow's proprietary Zestimate algorithm. These two smaller contributions should spark more academic research on and policymaker scrutiny of the dominant role of Zillow's ML pricing algorithm in the housing market. Finally, this paper is also one of the first to conceptualize the human-machine feedback loop in the housing market. We add to the computer science literature on ML risks (Sculley, D. et al. 2015, Amodei, D.et al. 2016) and feedback loops (Perdomo et al 2020, Bottou, L. et al. 2013, Wager, S. et al 2014, Sinha, A. et al. 2016). We elaborate distinction with this existing literature in the next section.

2. Related Literature

A stream of literature has looked at pricing in the housing market before introduction of ML. Linneman (1986) showed a large variance in market participants' home value estimates, which is unsurprising given that most market participants transact infrequently (say, once in 10 years), and, unlike some other assets, houses have a large diversity of features. Field surveys and experimental research tried to estimate valuation errors before the advent of pricing algorithms; error estimates include 14% (Goodman Jr, J. L., & Ittner, J. B. 1992), 5.3% (Kiel, K. A., & Zabel, J. E. 1999), and 16% (Ihlanfeldt, K. R., & Martinez-Vazquez, J. 1986). One would expect that agents, brokers, and other market experts could correct valuation errors (Han, L., & Strange, W. C. 2015), but sellers and expert agents' contract under information asymmetry. The seller's inability to observe his agent's efforts creates a moral hazard in the principal (seller) – agent contract (Anglin, P. M., & Arnott, R. 1991). When searching is costly, the agent has an incentive to undervalue the home and save on search costs; when the market is competitive, the agent has an incentive to overvalue the home to outbid competing agents. Further, the adverse selection problem prevents the seller from accurately judging whether the agent is knowledgeable about the state of the market. In this context where buyers and sellers find it challenging to accurately price homes, it is not surprising that ML pricing is influential.

Another stream of literature has looked at feedback loops in a wide range of online learning settings where an ML algorithm learns by making mistakes (Barocas et al. 2017). Such feedback designs are innocuous in settings where the ML predictions do not contaminate the ground truth label, but elsewhere, the feedback design can slowly accrue a technical debt (Sculley, D. et al. 2015) that eventually has profound effects (Amodei, D.et al. 2016). Perdomo et al. (2020) identify conditions under which a feedback loop will converge to a stable point. Our analytical framework roughly concurs, but we are less concerned with statistical properties and more concerned with the payoffs at the equilibrium. We discern how much of the covariate distribution shift comes from the evolution of intrinsic housing preferences versus from the ML feedback itself. If the latter dominates, the social surplus may be lost even as the ML algorithm reaches optimal accuracy.

ML feedback loops have been documented in settings such as - ad placement (Bottou 2013), search engine rankings (Wager et al. 2014) and recommender systems (Sinha et al. 2016, Chaney et al. 2018). Our model of market participants has similarities with the model of Schmit and Riquelme (2018), who assume that users are naïve in believing that ML recommendations are unbiased, and users are myopic and honest about their current action without regard to its impact on future states via the feedback loop. Importantly, in all these examples, individuals interact with ML predictions without an outside option. For example, a user who is seeking relevant search results does not have an alternative mechanism besides the ML algorithm; she cannot realistically achieve the same outcome by interacting with a crowd of her peers. In the housing market, however, participants can interact to determine prices in the absence of ML pricing. Thus, the introduction of ML pricing to the housing market is unique in that it replaces the wisdom of the crowd with a single correlated signal. To our knowledge, we are one of the first to evaluate the impacts of ML feedback loops on a market as a whole.

The business literature has identified consequences of ML algorithmic pricing besides the feedback loop. For example, bias propagated by the algorithmic pricing of hotels, car insurance, loans (Israeli and Ascarza 2020), and ride-hailing (Pandey and Caliskan 2021). Bertini and Koenigsberg (2021) find that customers misperceive the motives of firms that offer algorithmic pricing. Assad et al. (2020) and Brown and MacKay (2020) study whether algorithms provide competitive advantages or lead to collusive outcomes. Huang (2021) identifies settings in which algorithmic pricing may increase market friction. Yu (2020) argue that algorithmic price might mitigate racial disparities in the housing market. The present paper is unique in this literature because it does not model ML pricing or its statistical properties as static. Instead, we model ML algorithm's learning in conjunction with the evolving market as both move toward equilibrium.

3. Model

In this section, we lay out a model for - how a homeowner (prospective seller) determines pricing in the housing market and the resulting sale prices (Section 3.1). Subsequently, how a Machine Learning (ML) algorithm estimates home value to assist the homeowner (Section 3.2). The analytical model assumes two crucial primitives. First (in Section 3.1), ML prices influence the beliefs (valuations) and actions (list and offer prices) of buyers and sellers in the market. The feedback cycle would be irrelevant if ML prices had little or no influence on actions. Second (in Section 3.2), the ML price of a focal home is influenced by the recent sale prices of similar homes. The feedback loop would be absent if the pricing algorithm were driven instead by rules coded by domain experts. The feedback loop would be too slow to have practical implications if the ML price were reliant on older sales (e.g., a period of 5 years) and thus relatively static. In Appendix A, we provide empirical evidence that Zillow's Zestimate satisfies these two primitives. In Section 4, we will formulate the equilibrium of the feedback loop between the market and the ML algorithm.

3.1. Market

We model sale process of a single home incorporating seller's list price choice and buyer offers. This grounds our model to realistic institutional details of the housing market. This is necessary because ML price will influence the market by changing valuations (and therefore listing and offers) of market participants.

Buyers: We do not model how a buyer constructs valuations, learns, and determines their willingness to offer. Instead, we model all buyer that visit an on-market home in period τ using a representative buyer, the one with greatest willingness to offer from a crowd of visiting buyers. The

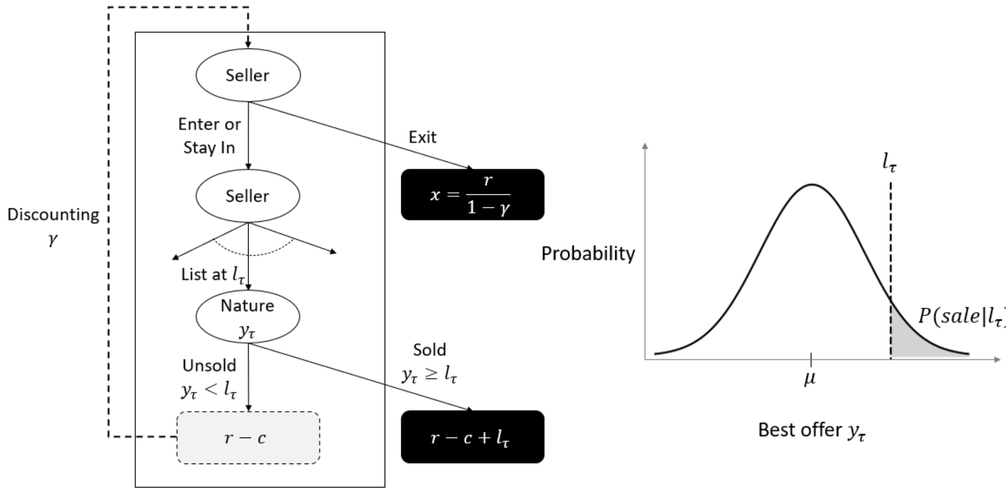

Figure 2 **(Left) The home sale process as a single-period game with the seller as the focal agent. The game repeats until the home is sold or the seller decides to exit the market. (Right) A hypothetical distribution of the list price, best offer distribution, and probability of sale.**

willingness to offer for this buyer y_τ is drawn from a distribution with mean μ and variance σ_b^2. It is sufficient to model only this one representative buyer because the homeowner will only consider their offer over all other offers in a given period. Variance σ_b^2 captures stochastic arrival of buyers with heterogeneous preferences. If the home is listed at l_τ the buyer will not reveal full willingness to offer, instead just make an offer $min(y_\tau, l_\tau)$. If $y_\tau < l_\tau$ the home stays unsold and if $y_\tau \geq l_\tau$ the home is sold at a price $min(y_\tau, l_\tau) = l_\tau$. Going forward we will refer to probability $P(y_\tau \geq l_\tau; \mu, \sigma_b^2)$ simply as $P(sale|l_\tau)$. The mean μ is unique to each home while variance σ_b^2 is common across homes. Since, homes are unique with hundreds of differentiating features, typically it would be difficult for the homeowner to exactly know μ (and therefore $P(sale|l_\tau)$) when listing the home on the market at l_τ. While individual buyers may update their valuation of homes over their search process, the aggregate offer distribution is static[5]. These assumptions about buyer crowd help with analytical simplification, but in the process, we do lose out on examining buyers more closely.

Seller: Unlike the buyer, we model individual seller in detail as shown in Figure 2. At the beginning of every period, the seller decides whether to list her home for sale and the list price l_τ. At the end of period, the home is sold at l_τ with probability $P(sale|l_\tau)$. If unsold, the seller returns to the decision of whether to keep her home listed for sale[6]. Whether or not the home receives an offer, the seller earns a flow payoff from retaining ownership of the home (e.g., rental income r) less market participation costs c (e.g., maintaining the home for visits from potential buyers). If the seller chooses

[5] Modeling aggregate offer distribution as declining over time, instead of static, do not change any conclusion

[6] In Appendix B.1, we discuss why we do not model the seller's choice to accept an offer below the list price.

to stay out of the market, she receives a lifetime payoff x from retaining ownership of the home (e.g., via rental income r at discount γ). We treat this as a terminal state (i.e., the seller's payoffs and choices remain static therefore she does not change her decision to stay out of the market). As an example, consider a seller who stays in the market and sets a sequence of list prices $l_1, ..., l_\tau, ..., l_T$ until completing the sale in period T. Her payoff π and the outside option x (had she not entered the market at all) are given by[7],

$$\pi = \frac{1-\gamma^T}{1-\gamma} \times (r-c) + \gamma^T \times l_T \quad ; \quad x = \frac{r}{1-\gamma} \tag{1}$$

The seller enters the market if her expected payoff $\tilde{\mathbb{E}}[\pi]$ is superior to the outside option x. This expectation is over all possible realizations of sale prices, time to sale and whether or not the home goes unsold. The seller sets optimal list price l^*_τ at the beginning of every period τ to maximize their expected payoff from that period onwards $\tilde{\mathbb{E}}[\pi_\tau|l_\tau]$. We use the notation $\tilde{\mathbb{E}}$ because seller's estimate can be different from the true expectation $\mathbb{E}[\pi_\tau|l_\tau]$. The seller's estimate of the expected payoff depends on their estimate of sale probability $\tilde{\mathbb{P}}(sale|l_\tau)$, which in turn depends on estimate of offer distribution parameter $\tilde{\mu}$. The seller has some uncertainty (variance) about this estimate denoted by σ_s^2. Note that this uncertainty σ_s^2 and the resulting deviation from the truth ($\tilde{\mu} \neq \mu$, $\tilde{\mathbb{P}}(sale|l_\tau) \neq \mathbb{P}(sale|l_\tau)$ and $\tilde{E}_\tau[\pi] \neq E_\tau[\pi]$) will lead to lower payoff that platform like Zillow (via its Machine Learning price) is trying to alleviate.

The seller learns and improves on her initial estimate $(\tilde{\mu}, \sigma_s^2)$. We model the updates as a Bayesian learning process in which the seller combines her current estimate $(\tilde{\mu}_\tau, \sigma^2_{s,\tau})$ with the market signal (μ, σ^2_{signal}) to develop an improved estimate $(\tilde{\mu}_{\tau+1}, \sigma^2_{s,\tau+1})$. We simplify notation for initial estimates as $\tilde{\mu} = \tilde{\mu}_{\tau=0}$ and $\sigma_s^2 = \sigma^2_{s,\tau=0}$. This signal comes from interaction with visiting buyers which may include explicit offers or implicit revelation of willingness to offer. The signal is assumed to be noisy but unbiased with respect to the buyer's offer μ. As the seller spends more time in the market, her guess improves $\tilde{\mu}_\tau \to \mu, \sigma^2_{s,\tau} \to 0$.

Table 1 provides two sets of assumptions about the distributions of offers y, seller estimate $\tilde{\mu}$ and market signal σ^2_{signal}. In the results section, we will use the "simple" model to capture the key intuition in analytical closed-form solutions. In Appendix B.3, we describe the "full" model to verify that the numerical solutions under more comprehensive assumptions are consistent with the "simple" model.

[7] Note that γ is very close to 1 if one period is 1 month (3% yearly interest rate or 0.97 yearly discounting is equivalent to 0.997 monthly discounting). Since home sales take a few months γ^T is also very close to 1. We will use this to simplify.

Table 1 **Assumptions about the distributions in our simple and full models.**

	Simple Model	Full Model
Buyer Offer Distribution $P(y)$	$U[\mu-\sigma,\mu+\sigma]$	$N(\mu,\sigma_b^2)$
Seller Estimate $P(\tilde{\mu})$	$P(\tilde{\mu}=\mu-2\sigma)=P(\tilde{\mu}=\mu)=P(\tilde{\mu}=\mu+2\sigma)=1/3$	$N(\mu,\sigma_s^2)$
Seller Learning σ^2_{signal}	0	$\kappa\sigma_b^2$

3.2. Machine Learning Algorithm

In this section, we propose a general machine learning framework for home value prediction. We then assume a specific generative model of home values over time, and use our machine learning framework to derive analytical expressions for the Machine Learning prices as a function of observed sale prices.

General machine learning framework. Our framework assumes that the machine learning model of home values is trained to (i) accurately predict the “true value” of each home in the training data, and (ii) predict similar home values for similar homes. Satisfying objective (i) increases in-sample accuracy, but could decrease out-of-sample accuracy due to overfitting. The inclusion of objective (ii) “regularizes” the model to increase out-of-sample accuracy.

Formally, let $\mathcal{H}_t$ be a set of N homes used to train the machine learning model at time t. For each home $k\in\mathcal{H}_t$, let $v_{k,t}$ be the true value of the home at time t, and $\mathbf{X}_k\in\mathbb{R}^Q$ be a Q-dimensional vector of the home's characteristics (such as its number of bedrooms and year of construction). We assume that the machine learning model parameters θ_t are given by:

$$\theta_t=\mathrm{argmin}_\theta\left[\underbrace{\sum_{k\in\mathcal{H}_t} f_k(\mathbf{X}_k|\theta)}_{\text{objective (i)}}\quad+\quad\underbrace{\sum_{m,n\in\mathcal{H}_t} g_{mn}(\mathbf{X}_m,\mathbf{X}_n|\theta)}_{\text{objective (ii)}}\right] \tag{2}$$

Here, $f_k(\mathbf{X_k}|\theta)$ is a loss function capturing the first objective of accurate in-sample predictions for each home $k\in\mathcal{H}_t$ in the training data, and $g_{mn}(\mathbf{X_m},\mathbf{X_n}|\theta)$ is a loss function capturing the second objective of similar predictions for similar homes.

Note that we make no assumptions about the functional form of either loss function or of the machine learning model. Also note that, in practice, the true home values $v_{k,t}$ are unobservable in time period t. Hence, machine learning models typically use the observed home sale prices $p_{k,t-1}$ from the previous time period as a proxy for the true home values. Using this proxy plays a role in the prediction errors that we discuss later in this section.

Example. We further clarify this framework with the following example. Let $\hat{v}_k = F(\mathbf{X}_k, \Theta_k)$ be the predicted price of each home k derived using a neural network $F(\cdot)$ with weights Θ_k. To maximize in-sample accuracy, we minimize the mean squared prediction error by setting $f_k(\mathbf{X_k}|\theta) = [\hat{v}_k - v_{k,t}]^2 = [F(\mathbf{X}_k|\Theta_k) - v_{k,t}]^2$. To improve out-of-sample accuracy, we also minimize the differences between the learned weights for pairs of similar homes. Specifically, we set $g_{mn}(\mathbf{X}_m, \mathbf{X}_n|\theta) = \lambda\|\mathbf{X}_m - \mathbf{X}_n\|_2\|\Theta_m - \Theta_n\|_2$, where $\|\cdot\|$ is the Euclidean distance or L_2 norm, and λ is a regularization hyperparameter. Note that the parameters $\theta = \{F(\cdot), \lambda\} \cup \{\Theta_k\}_{k\in\mathcal{H}_t}$ include the neural network architecture $F(\cdot)$, the regularization hyperparameter λ, and the weights Θ_k for each home $k \in \mathcal{H}_t$.

Regularization and home clustering. Solving for θ_t in Eq. 2 essentially performs joint home value prediction and home clustering. Homes in a cluster $\mathcal{C}_i$ for $i = 1, \ldots, J$ have the same learned weights Θ_k and similar characteristics $\mathbf{X}_k$ (and hence, similar predicted home values). The regularization parameter λ controls the clustering granularity: $\lambda = 0$ permits each home to form its own cluster, while $\lambda \to \infty$ leads to a single cluster containing all homes in the training data.

The clustering of homes also facilitates making predictions. Let $\bar{\mathbf{X}}_{\mathcal{C}}$ denote the centroid of cluster $\mathcal{C}$ (the average of $\mathbf{X}_k$ over all homes $k \in \mathcal{C}$), and let $\bar{v}_{\mathcal{C}}$ be the average value of all homes $k \in \mathcal{C}$. We assign each out-of-sample home with characteristics $\mathbf{X}_q$ to its "nearest" cluster with the smallest Euclidean distance $\|\mathbf{X}_q - \bar{\mathbf{X}}_{\mathcal{C}}\|_2$, and subsequently use $\bar{v}_{\mathcal{C}}$ as its predicted home value.

Modeling home values over time. We assume that the true value $v_{k,t} = G_t(\mathbf{X}_k)$ of a home k at time t is a time-evolving function of the home's characteristics $\mathbf{X}_k$. This can be interpreted as preferences for the home's characteristics evolving over time, while the home's characteristics remain unchanged. We further assume that the resulting home value evolves as a random walk:

$$v_{k,t} = v_{k,t} + e^{rw}_{k,t}, \qquad v_{k,t} = G_t(\mathbf{X}_k),\ e^{rw}_{k,t} \sim N(0, \sigma^2_{rw}) \tag{3}$$

ML Prices. The Machine Learning algorithm discussed above applied to these true values calculates ML price of home k at time $t+1$ as

$$z_{k,t+1} = \bar{p}_{\mathcal{C}_k,t} = \frac{Q}{N} \times \sum_{j\in\mathcal{C}_k} p_{j,t} \tag{4}$$

The cluster $\mathcal{C}_k$ contains exactly N/Q home sales in any period t[8]. Here N is total number of home sales in the market and Q is the number of clusters.

[8] The ML training periods t (say 1 year) is different and typically much longer than the home listing period τ (say 1 week or 1 month).

Table 2 Table of notation and definitions

Exogenous Variables

Notation	Description
v_t	True value (market clearing price) of home at time t
$\sigma_e^2 = Var_i[\tilde{v}_{i,t} - v_t]$	Variance of participant i private valuation error
$\sigma_{rw}^2 = Var_t[v_{t+1} - v_t]$	Variance of true value changes of a home over time.
$\sigma_v^2 = Var_k[v_t]$	Variance of true value across all homes in one period.
$\delta = Var[p_t]/Var_i[v_{i,t} - v_t]$	Ratio of sale price error and participant valuation errors
N	Total number of homes sold in one period.

Key Endogenous Variables

Notation	Description
$l_{\tau t}$	Sequence of list prices set by seller for home at time t
(μ, σ_b^2)	Buyer offer distribution parameters
$(\tilde{\mu}, \sigma_s^2)$	Seller guess of offer distribution parameter μ
p_t	Realized sale price (if sold)
$v_{i,t}$	Participant i valuation of home at time t
z_t	ML (Machine Learning) price of home at time t
Q	ML model hyperparameter controlling number of clusters
α	Participant reliance on ML price

Outcomes Variables

Notation	Description
$\sigma_z^2(\alpha)$	True ML price error (un-confounded $\sigma_z^2(\alpha = 0) = \sigma_z^2$)
$\hat{\sigma}_z^2(\alpha)$	Estimated ML price error (un-confounded $\hat{\sigma}_z^2(\alpha = 0) = \hat{\sigma}_z^2$)
$\Pi(\alpha)$	Payoff

4. Results

4.1. Home Value and Sale Prices

In Section 3.1 we discussed model of buyer-seller interactions grounded in the structure of the housing market. We described listing prices l_τ, offer distribution parameters (μ, σ_b^2), seller's guess of offers $(\tilde{\mu}, \sigma_s^2)$, seller's learning σ_{signal}^2, seller's characteristics (x, c) and how all of these are related. But, an external signal such as the ML price z does not convey information about the offer μ or list price l_τ, nor is it tailored to an individual seller $(\tilde{\mu}_\tau, x, c)$). Instead, the ML price z is an estimate of home value *summarized* over these market structure details and individual heterogeneity. Therefore, we translate to measure of home value $v = v_{\tau=0}$.

Theoretically the home value v is its market clearing price. This is simple for say an IBM stock with millions of identical units transacted among thousands of buyers and sellers every day. But, we only have single quantity of each unique home that is transacted only once a decade. Consider a thought experiment where the same home could be sold thousands of times, the value of the home v is the average sale price over these large number of iterations i.e., $v = \mathbb{E}[\mathbb{E}[p|\mu,\tilde{\mu}]]$. In other words, the value of a home is an expectation over all the buyer- and seller-related deviations from the "average". Consider an oracle who knows a home's true offer distribution (μ, σ_b^2) and knows that the sellers (potentially) erratic estimate. The oracle could calculate the expected sale price by integrating over seller ($\tilde{\mu}$) and stochasticity in buyer offers (embedded in $\mathbb{E}[p|\mu,\tilde{\mu}]$) as,

$$v(\mu) = \mathbb{E}[\mathbb{E}[p|\mu,\tilde{\mu}]] = \int_{\tilde{\mu}} P(\tilde{\mu}|\mu,\sigma) \times \mathbb{E}[p|\mu,\tilde{\mu}] \tag{5}$$

Following the same definition of home value, we can also express seller's estimate of her home value. Note that the seller (unlike the oracle) does not know her home's true offer distribution (μ, σ_b^2). But, she is aware about uncertainty σ_s^2 in her estimate $\tilde{\mu}$. The seller's valuation $\tilde{v} = \tilde{E}[v(\mu)]$ where the expectation is over seller's estimate of all possible true μ.[9] An external signal from Machine Learning algorithm z purports to also estimate the sale price i.e., $z = \hat{E}[p] = \hat{v}$. Notation $\hat{E}$ to differentiate Machine Learning estimation from the homeowner estimation $\tilde{E}$. The homeowner can update their valuation $\tilde{v}$ by combining their prior valuation (before observing and independent of the Machine Learning price) and the Machine Learning price z as,

$$\tilde{v} := (1-\alpha) \times \tilde{v} + \alpha \times z \tag{6}$$

Note that the ML price z does not directly convey information about the offer μ. But, the homeowner in updating their valuation indirectly also updates their estimate of offers $\tilde{\mu}$.

Lemma 1 *The true home value v and its variance is given by,*

$$v = \mathbb{E}[p] = (\mu + \sigma/3 - \sqrt{c\sigma/2}) \;; \quad Var[p] = 4\sigma^2/9 \tag{7}$$

The seller valuation and its variance is given by,

$$\tilde{v} = \tilde{E}[p] = (1-\alpha) \times \left(\tilde{\mu} + \sigma/3 - \sqrt{c\sigma/2}\right) + \alpha \times z \;; \quad Var[\tilde{v}] = 24(1-\alpha)^2\sigma^2/9 \tag{8}$$

We can express individual seller valuation as $\tilde{v}_i = E[\tilde{v}_i] + e_i$ where $e_i = (1-\alpha)(\tilde{\mu} - \mu)$ is the noise in valuation across individual sellers with $E[e_i] = 0$ and variance $\sigma_e^2 = Var[\tilde{v}]$. We can also express sale price realization as $p = E[\tilde{v}_i] + \epsilon$ where ϵ is the noise in realized sale prices (across multiple

[9] The seller can infer $P(\mu|\tilde{\mu})$ because they know that their belief is drawn as $\tilde{\mu} \sim (\mu, \sigma_s)$.

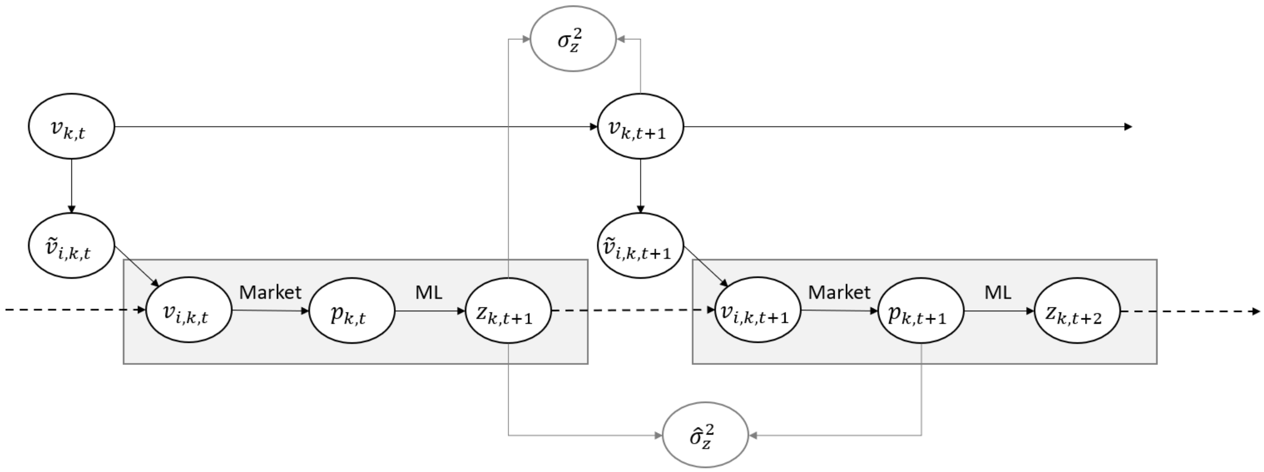

Figure 3 **All key exogeneous and endogeneous factors at a glance.**

hypothetical sale instances) with $E[\epsilon] = 0$ *and variance* $\sigma_\epsilon^2 = Var[p] = \delta \times Var[\tilde{v}_i]$ *(where* $\delta = (1/6)$*)* (Proofs in Appendix C)

There are two intermediate results worth highlighting here. First, the true expected sale price is equal to the expected seller valuation i.e., $\mathbb{E}[p] = \mathbb{E}[\tilde{v}_i]$. At $\alpha = 0$ we have $v = \mathbb{E}[p] = \mathbb{E}[\tilde{v}_i]$ i.e., prior valuations (before observing ML price) are unbiased with respect to the true value v. At $\alpha > 0$, the valuation are not unbiased anymore i.e., $\mathbb{E}[\tilde{v}_i] \neq v$ if $z \neq p$. Second, the variance in sale prices $Var[p]$ is proportional to variance (disagreement) of seller's valuation $Var[\tilde{v}]$[10]. The variance in valuations σ_e^2, and consequently variance in sale prices σ_ϵ^2 are diminished by a fraction $(1-\alpha)^2$. So overall, ML price is adding some bias but removing some variance from valuations and sale prices.

4.2. Feedback Loop Equilibrium

In this section, we will use equations 4 (z_{t+1} as function of p_t), equation 6 ($\tilde{v}_{i,t}$ as function of z_t) and Lemma 1 (p_t as function of $\tilde{v}_{i,t}$) to formulate the feedback loop equilibrium. Figure 3 visually illustrates this loop. Specifically, we will calculate at equilibrium the Machine Learning pricing errors ($\sigma_z^2 = Var[v_t - z_t]$) and its empirical estimate ($\hat{\sigma_z}^2 = Var[p_t - z_t]$). In Section 4.3, we will use the equilibrium expressions for pricing and errors to formulate payoffs for market participants.

The ML price z_t for a home is given by $z_{t+1} = \bar{p}_{K,t}$ from equation 4. Here K is the focal home's peer cluster. The error in ML price is the difference between the actual realized home value v_{t+1} and

[10] Variance (disagreement) in individual seller's valuation is a theoretical measure in the housing market setting because only one homeowner can be the seller for a unique home.

ML price z_t. This error can be decomposed into three components as,

$$v_{t+1} - z_{t+1} = \underbrace{v_{t+1} - v_t}_{\text{Random Walk}} + \underbrace{v_t - v_{K,t}}_{\text{Unpriced Features}} + \underbrace{v_{K,t} - \bar{p}_{K,t}}_{\text{Finite Sample Error}} \tag{9}$$

Home values in the current period v_t can not forecast the random walk of preferences and values into the next period v_{t+1}. We denote variance of **random walk** error as an exogenous and constant quantity $Var[v_{t+1} - v_t] = \sigma^2_{rw}$. The focal home matches homes in the peer cluster on Q features. In using the peer cluster's mean sale price, $\bar{Q} - Q$ unique features of the focal home are left unpriced i.e., an error $v_t - v_{K,t}$. Intuitively, the variance of **unpriced features** should depend on variance of all features and the number of priced features Q. The variance of all features, also the heterogeneity in housing stock, σ^2_v is treated as an exogenous constant. The choice of priced features Q explains increasingly greater proportion of the total variance. This is captured by monotonically decreasing function $h(Q)$ (possibly with positive second derivative because of diminishing returns). Thus we have $Var[v_t - v_{K,t}] = h(Q) \times \sigma^2_v$. Intuitively, as the number of "clustering features" (features used to place homes in the same cluster) increases, clusters will have fewer, very similar homes. Hence, the variance of home values in a cluster will be low. Similarly, as the number of "clustering features" decreases, clusters will have more, dissimilar homes. Hence, the variance of home values in a cluster will be high.

The ML price is effectively the sample mean of cluster sale prices. The **finite sample error** is the difference between the mean of cluster sale prices and the true cluster value i.e., $v_{K,t} - \bar{p}_{K,t}$. Since there are N/Q home sales in the cluster, we can express $\bar{p}_{K,t}$ as,

$$\begin{aligned}
\bar{p}_{K,t} &= (Q/N) \times \sum_k \left[p_{k,t} \right] \\
&= (Q/N) \times \sum_k \left[\mathbb{E}[v_{i,k,t}] + \epsilon_{k,t} \right] \\
&= (Q/N) \times \sum_k \left[\mathbb{E}[(1-\alpha) \times \tilde{v}_{i,k,t} + \alpha \times z_{k,t}] + \epsilon_t \right] \\
&= (Q/N) \times \sum_k \left[(1-\alpha) \times v_{k,t} + \alpha \times (v_{k,t} + e^z_{k,t}) + \epsilon_{k,t} \right] \\
&= (Q/N) \times \sum_k \left[v_{k,t} + \alpha e^z_{k,t} + \epsilon_{k,t} \right] \\
&= v_{K,k,t} + \alpha (Q/N) \times \sum_k e^z_{k,t} + (Q/N) \times \sum_k \epsilon_{k,t}
\end{aligned} \tag{10}$$

Note that under $\alpha = 0$, the second components disappears. Under $\alpha > 0$, this additional component captures the *confounding* of the sale price from the ML price via influencing market participants.

We can now express the variance of the finite sample error as,

$$
\begin{aligned}
Var[v_{K,t} - \bar{p}_{K,t}] &= Var\Big[(Q/N) \times \sum_k (\alpha \mathbb{E}[e^z_{k,t}] + \epsilon_{k,t})\Big] \\
&= \alpha^2 Var[(Q/N) \times \sum_k e^z_{k,t}] + Var[(Q/N) \sum_k \epsilon_{k,t}] \\
&= \alpha^2 Q \sigma_z^2/N + \delta Q(1-\alpha)^2 \sigma_e^2/N
\end{aligned} \tag{11}
$$

We can now write the full ML price error variance as,

$$
\begin{aligned}
\sigma_z^2(\alpha) &= Var\Big[(v_{t+1} - v_t) + (v_t - v_{K,t}) + (v_{K,t} - \bar{p}_{K,t})\Big] \\
&= \sigma_{rw}^2 + h(Q) \times \sigma_v^2 + \alpha^2 Q \sigma_z^2(\alpha)/N + (1-\alpha)^2 \delta Q \sigma_e^2/N \\
&= \Big(\frac{1}{1 - Q\alpha^2/N}\Big) \times (\sigma_{rw}^2 + h(Q) \times \sigma_v^2 + (1-\alpha)^2 \delta Q \sigma_e^2/N) \\
&= \sigma_{rw}^2 + h(Q) \times \sigma_v^2 + (1-\alpha)^2 \delta Q \sigma_e^2/N
\end{aligned} \tag{12}
$$

The denominator $(1 - Q\alpha^2/N)$ is very close to 1 since $Q << N << \infty$ and set to 1 going forward as a conservative assumption for analytical simplification. The ML error is increasing in random walk of home preferences σ_{rw}^2, heterogeneity in housing stock σ_v^2 and error in market's participants private valuations σ_e^2. The ML error sensitivity to Q is mixed - the unpriced feature error component $h(Q) \times \sigma_v^2$ is decreasing in Q while the finite sample error component $(1-\alpha)^2 \delta Q \sigma_e^2/N$ is increasing. We will discuss endogenization of Q below. The ML error is increasing in α. In fact, we can express the ML error in terms of the *un-confounded* (by ML price e.g., when market does not observe or use ML prices) ML error $\sigma_z^2 = \sigma_z^2(\alpha = 0)$ as,

$$
\begin{aligned}
&\sigma_z^2(\alpha) = \sigma_z^2 + \alpha(2-\alpha)\delta Q \sigma_e^2/N \\
\text{where} \quad &\sigma_z^2 = \sigma_z^2(\alpha = 0) = \sigma_{rw}^2 + h(Q) \times \sigma_v^2 + \delta Q \sigma_e^2/N
\end{aligned} \tag{13}
$$

The un-confounded ML error σ_z^2 are valid in a limited setting where the platform does not reveal the ML price or the market participants do not use the ML price at all. In the rest of the paper, we will continue to compare confounded results at $\alpha > 0$ with unconfounded results at $\alpha = 0$ to highlight impact of the confounding over the feedback loop. In comparing expressions for confounded $(\sigma_z^2(\alpha > 0))$ and unconfounded $(\sigma_z^2 = \sigma_z^2(\alpha = 0))$ ML errors, the additive term $\alpha(2-\alpha)\delta Q \sigma_e^2/N$ captures the amplification in ML error because of the confounding.

Proposition 1 *The ML error (under $\alpha > 0$) is strictly greater than un-confounded ML error (under $\alpha = 0$) i.e., $\sigma_z^2(\alpha) > \sigma_z^2$ if $N/Q > \alpha^2$.*[11]

[11] This condition is in-fact trivially true because number of homes sales in a cluster N/Q is typically much larger than the 1.

If this ML error $\sigma_z^2(\alpha)$ were directly observable, a data scientist could set Q to minimize $\sigma_z^2(\alpha)$. Unfortunately, the ML price error e_t^z was defined as $z_{t+1} - v_{t+1}$, where the true home value v_t is not observed by the platform and their data scientist. In practice, the Machine Learning platform evaluates its algorithm by comparing its ML price with the eventual sale price i.e., $\hat{e}_{t+1}^z = z_{t+1} - p_{t+1}$. The estimated ML price error can be expressed as,

$$\begin{aligned} \hat{\sigma}_z^2(\alpha) = Var[z_{t+1} - p_{t+1}] &= Var[(z_{t+1} - v_{t+1}) + (v_{t+1} - p_{t+1})] \\ &= Var[e_{t+1}^z + (v_{t+1} - (v_{t+1} + \alpha e_{t+1}^z + \epsilon_{t+1}))] \\ &= Var[(1-\alpha)e_{t+1}^z - \epsilon_{t+1}] \\ &= (1-\alpha)^2 \times \sigma_z^2(\alpha) + (1-\alpha)^2 \times \delta\sigma_e^2 \end{aligned} \tag{14}$$

Under $\alpha = 0$, $\hat{\sigma_z}^2 > \sigma_z^2$ because $\delta\sigma_e^2 > 0$. This means that the platform errs on the side of under reporting its accuracy and confidence[12]. There is no cause for alarm since the ML platform is acting in a conservative fashion. But, for $\alpha => 0$, when comparing true $(\sigma_z^2(\alpha))$ and estimated ML errors $(\hat{\sigma}_z(\alpha)^2)$, the conclusion is not trivial. The additive term $\delta\sigma_e^2$ is same is in the unconfounded setting. But, the fraction $(1-\alpha^2)$ captures the secondary effect due to the "self fulfilling prophecy" over the feedback loop. This secondary effect may dominate when α is large enough. The platform will be presenting an overly optimistic claim of accuracy to the market participants.

Proposition 2 *The un-confounded ML error estimate under $\alpha = 0$ is strictly greater than the true ML error i.e., $\hat{\sigma}_z^2 > \sigma_z^2$. But, the confounded ML error estimate can be less than the true ML error i.e., $\hat{\sigma}_z^2(\alpha) < \sigma_z^2(\alpha)$ when,*

$$\frac{\sigma_e^2}{\sigma_z^2(\alpha)} < \frac{1}{\delta} \times \Big(\frac{1}{(1-\alpha)^2} - 1\Big) \tag{15}$$

The confounded estimated ML price error $\hat{\sigma}_z^2(\alpha)$ will collapses to zero at $\alpha = 1$.

Let us now consider the choice of priced features Q. First, consider the un-confounded setting $\alpha = 0$. Q would be set to minimize $\hat{\sigma_z}^2$. The unpriced feature error $h(Q)\sigma_v^2$ is decreasing in Q while the finite sample error $\delta Q\sigma_e^2/N$ is increasing in Q. Let $Q = Q^*_{\alpha=0} = Q^*$ minimizes the error $\hat{\sigma}_z^2$, effectively trading-off these two components. At $Q = Q^* =$ we have,

$$-\frac{\partial}{\partial Q}(h(Q)\sigma_v^2)|_{Q=Q^*} = \frac{\partial}{\partial Q}(\delta Q\sigma_e^2/N)|_{Q=Q^*} \tag{16}$$

Any $Q < Q^*_{\alpha=0}$ would increase the sample size N/Q within a cluster and reduce the finite sample error, but the larger cluster size comes with more unpriced home features and a wider range of

[12] This is opposite to the final result in this paper which concludes that the platform (unknowingly) over-reports its accuracy and confidence.

heterogeneous homes within the cluster. On the other hand, any $Q > Q^*_{\alpha=0}0$ would better distinguish unique homes at the cost of an erratic estimate of the mean cluster price (due to the smaller sample). Since σ_e^2 is independent of Q, the choice of priced features Q simultaneously maximizes both the true ML price error σ_z^2 and its estimate $\hat{\sigma_z}^2$. Both these points confirm that there is no cause for alarm and the formulation captures conventional wisdom.

Now consider the confounded setting ($\alpha > 0$), the unpriced feature error remains constant while the finite sample error variance reduces. At the same $Q = Q^*$ the comparison of derivatives (similar to equation 16) now favors the unpriced feature error i.e., this component diminishes faster with increasing Q. As a result, the the unpriced feature error component will dominate more than before in determining the new $Q = Q^*_\alpha$ that minimizes the error $\hat{\sigma}_z^2(Q, \alpha)$. Thus, endogenously setting Q would result in more clusters ($Q^*_\alpha > Q^*$), smaller cluster size, fewer home sales in every cluster (N/Q) and more adverse confounding from feedback. For simplicity, we take the conservative assumption that Q is held constant under the feedback loop at $Q = Q^*$ i.e., to minimize $\hat{\sigma}^2$.

Finally, let us consider endogenizing α. Intuitively, we expect that market participants give more weight to the ML price (larger α) if the ML price is presented with lower error estimate ($\hat{\sigma}_z^2(\alpha)$). Further, if error estimate ($\hat{\sigma}_z^2(\alpha)$) is decreasing in α, we may have an alarming re-inforcing "self fulfilling prophecy".

Proposition 3 *The estimated ML price error is decreasing in α when,*

$$\alpha > 1 - \sqrt{\frac{1}{2} + \frac{N}{2Q\delta}\frac{\sigma_z^2}{\sigma_e^2}} \tag{17}$$

Let us formally endogenize α. Individuals have knowledge of the error in their own private signal σ_e^2 and the ML platform provides estimated ML price error $\hat{\sigma_z}^2$. Individual can weigh the two signals based on relative noisiness. For example, if ML price is accurate (small $\hat{\sigma_z}^2$) the individual can rely less on their private valuation. Thus, the reliance α can be endogenized as,

$$\alpha = \lambda \times \frac{\sigma_e^2}{\sigma_e^2 + \hat{\sigma_z}^2} \tag{18}$$

The quantity λ is an exogenous lever purely for exposition of results in the paper. At $\lambda = 0$, the model collapses to the results in the previous section which assumes that the platform either does not reveal the ML price or the market participants do not use the ML price at all. At $\lambda = 1$, the model represents a future state where ML price is available and used by all market participants in the rational fashion discussed above[13]. All intermediate values of λ can be roughly interpreted as

[13] The rationality is still bounded since the participant uses ML platform's estimate $\hat{\sigma_z}^2$ without adjusting for the platform's estimation limitation or presence of feedback confounding)

different degree of trust and adoption of ML prices. We will provide some numerical results with $0 < \lambda < 1$ to convey intuition. For analytical results below, we stick to $\lambda = 1$.

Proposition 4 *The estimated ML price error is always less than true ML price error under full adoption* $\lambda = 1$[14]

Now consider the feedback loop between $\hat{\sigma_z}^2$ and α. A low estimated ML price error $\hat{\sigma_z}^2$ increases weight on ML price α. This in turn shifts the $\hat{\sigma_z}^2$. Results 2(c) captures the critical range of α below which $\hat{\sigma_z}^2$ is decreasing in α and therefore the feedback loop is re-inforcing. Now we can substitute α.

Proposition 5 *The estimated ML price error is always decreasing in α under full adoption* $\lambda = 1$[15].

Under this condition estimated ML price error and α are always positively re-inforcing. We can also formulate all equilibria $(\hat{\sigma_z}^2(\alpha)^*, \alpha(\hat{\sigma_z}^2)^*)$ using equations 14 and 18.

Proposition 6 *Full reliance on ML price ($\alpha^* = 1$) and the estimated ML price error collapsed to zero is always an equilibrium of the feedback loop. Other equilibria are given by solutions to*[16]*,*

$$\frac{\sigma_z^2}{\sigma_e^2} + \delta + \frac{Q\alpha(2-\alpha)}{N} = \frac{1}{\alpha(1-\alpha)} \tag{19}$$

Full reliance on ML price ($\alpha^* = 1$) is also the only equilibrium if ,

$$\frac{\sigma_z^2}{\sigma_e^2} < 4 - \delta - \frac{Q}{N} \tag{20}$$

Let us consider a hypothetical parameter setting to understand these results better. Say a new Machine Learning algorithm is introduced in the market. Before revealing the ML prices to the participants, the platform evaluates the performance and estimates the ML error to be $\hat{\sigma}_z = \$20,000$ ($\hat{\sigma}_z^2 = 4 \times 10^8$) on \$1Mn homes. While the platform can not measure the true error, let us assume that the true error is $\sigma_z^2 = 3.5 \times 10^8$. In comparison, say individual market participants make an error $\sigma_e = \$10,000$ ($\sigma_e^2 = 1 \times 10^8$) when privately estimating the home value i.e., $\hat{\sigma_z}^2 = 4\sigma_e^2$. The platform now starts revealing these ML prices on its website. Substituting $\hat{\sigma_z}^2$ in equation 18, the reliance of market participants α on the ML price rises from 0 to 0.2. This is relatively low because the ML price error is much larger than participant's private valuation errors. Substituting α in equation 14, $\hat{\sigma_z}^2$ shrinks by a factor of 0.64 i.e., from $4\sigma_e^2$ to $2.56\sigma_e^2$. Again substituting $\hat{\sigma_z}^2$ in

[14] Detailed proof in Appendix C by substituting α from equation 18 into Proposition 2.

[15] Detailed proof in Appendix C by substituting α from equation 18 into Proposition 2

[16] Detailed proof in Appendix C.

equation 18, this underestimation of the ML error increases individuals' reliance on the ML price from 0.2 to 0.28. The feedback loop repeats until reliance and the ML error are in equilibrium at $\alpha^* = 0.5$ and $\hat{\sigma_z}^{2^*} = \sigma_e^2$. At this equilibrium the estimated ML error is at least 3.5 times smaller than the true ML error. While this is alarming, this feedback cycle did not collapse all the way to $(\hat{\sigma_z}^2, \alpha)^* = (0, 1)$ because the original ML error (unconfounded or before introduction) was large. The feedback cycle is even more acute if the original ML error is low to begin with say $\hat{\sigma_z}^2 = 2\sigma_e^2$. In this case at equilibrium individuals eventually rely entirely on the ML price $\alpha^* = 0 \rightarrow ... \rightarrow 1$ and estimated ML error collapses $\hat{\sigma_z}^2 = 2\sigma_e^2 \rightarrow ... \rightarrow 0$.

Intuition using Toy Model: To understand the key results and intuitions, consider first a baseline toy model in absence of ML pricing. A crowd of participants construct private valuations for a product (e.g., a $v = \$100$ product may have private valuations $\tilde{v}_i$ distributed in $\$100 \pm 20$). Participants then spend time to learn from the crowd to resolve this disagreement. While time consuming, correction is possible since private valuations are unbiased ($v = \mathbb{E}[\tilde{v}_i]$) and uncorrelated across participants. The learning from crowd mitigates error in valuations and subsequently in sale prices (p realized in $\$100 \pm 4$).

Introduction of an accurate ML price ($z = \$100$) provides a common signal to all. Now participants can construct their valuation by placing some reliance or weight ($\alpha = 0.5$) on the ML price ($\tilde{v}_i := (1-\alpha)\tilde{v}_i + \alpha z$). The new valuations are more narrowly distributed in $\$100 \pm 10$. This alleviates some of the costly valuation disagreement among participants. However, if the ML price has error ($z = \$110$) this is universally propagated to all participants ($\tilde{v}_i$ in $\$105 \pm 10$). Going forward we treat error in valuations ($\tilde{v}_i - v$) as split into – (i) component capturing disagreement from the crowd ($\tilde{v}_i - \mathbb{E}[\tilde{v}_i]$) called variance and (ii) component common across the crowd ($\mathbb{E}[\tilde{v}_i] - v$) called bias[17]. In the example above, introduction of ML pricing reduced variance but added bias. Participants can correct variance (disagreement or random error) in private valuation via learning from the crowd, but they cannot correct bias (common or systematic error). Thus, any valuation bias added by ML price gets propagated to sale prices ($\tilde{p}_i$ realized in $\$105 \pm 2$). Whether the sale price errors[18] ($|\tilde{p}_i - v|$) is alleviated or amplified depends on the size of ML error ($|e_z| = |z - v| = 10$)

[17] One could argue that the true product value moves up from $\mathbb{E}[v_i] \rightarrow \mathbb{E}[\tilde{v}_i]$ if an error systematically moves everyone in the market to value the product more. So, bias would always be zero by definition. But such contamination by systematic error will not sustain indefinitely. Thus, our bias measurement captures a short-run systematic error in valuations for one product. This bias (positive or negative) is not in the same direction across all products in the market. So, it does not represent inflation or deflation of (housing) market as a whole. The ML model bias-variance and product valuation bias-variance measure different quantities, but they are intricately related in our model. The ML model bias-variance will drive valuation bias-variance.

[18] The economic significance of statistical error in valuations and sale prices become clear once we elaborate payoffs.

and reliance on ML price ($\alpha = 0.5$). ML price alleviates error if reliance is optimal given the ML error (e.g., low reliance under large error or high reliance under small error) but it amplifies error if reliance is inflated (e.g., high reliance under large error). In fact, this impact is true of any signal, say real estate expert opinion, that is widely influential in the market.

The uniqueness of the ML price signal becomes apparent once we endogenize participants' reliance on the ML price α. The participants determine reliance α by observing ML error estimate $\hat{\sigma}_z$ presented alongside the ML price. The ML error $\hat{\sigma}_z$ is endogenously estimated in the ML framework by comparing ML prices with sale prices. When market participants rely on the ML price to any extent ($\alpha > 0$), the sale prices settle closer to the ML price than they would if the ML price were hidden ($\alpha = 0$). As a result, ML error $\hat{\sigma}_z$ is underestimated. The underestimation of the ML error $\hat{\sigma}_z$ inflates reliance α. In turn, an increase in reliance α further underestimates the ML error $\hat{\sigma}_z$ due to the self-fulfilling nature of the ML price, which leads to worsening over reliance, and so on as the feedback loop iterates[19]. We analytically characterized this equilibrium $(\alpha^*, \hat{\sigma}_z^*)$ of this feedback loop. This reinforcing loop depends on – (i) limited rationality of market participants who do not realize the underestimation in $\hat{\sigma}_z$ and (ii) passive behavior of platform that presents the ML price without correcting the underestimation in $\hat{\sigma}_z$. We will qualitatively discuss when and how these two assumptions stop holding true. At equilibrium with deceptively large ML error and over-reliance α^*, the ML price can overall amplify sale price errors. We refer to the feedback loop as strong when equilibrium outcomes are more adverse (large underestimation $\hat{\sigma}_z^*$ and large over-reliance α^*). This happens under high ML model capacity (unconfounded σ_z), which increases the sensitivity of ML price to individual sales. Counter to the conventional wisdom, advantage of ML pricing in alleviating sale price error can diminish or even reverse as ML model improves or the adoption increases.

4.3. Payoffs

Until now we have treated the feedback loop and resulting underestimation of the ML error as statistical issues. In this section, we discuss the implications for seller's payoffs.

Lemma 2 *The expected seller payoff is given by,*

$$\mathbb{E}[\pi(\mu)] = (\mu + \sigma - \sqrt{c\sigma/2}) - (2\sigma/3) - (2\sqrt{2c\sigma}/3) = \mu - \sigma/3 \times \left(\sqrt{17c/6\sigma} - 1\right) \tag{21}$$

In our model *price noise* in the market impacts the seller through two channels. First channel is the heterogeneity in buyer offers. The seller can benefit from this *price noise* if they can wait long enough for a high draw from the offer distribution. Second channel is noise in seller's guess.

[19] Hypothetically if a real estate expert, say Warren Buffet, becomes increasingly boisterous and influential as their prophecies are fulfilled, it would lead to the same feedback loop mechanism.

The seller has disutility from this because they may not enter the market altogether when it may have been profitable to do so or list the home at too low a price. In our simple model, the seller spends cost c in the first period and resolve this noise in price entirely $\sigma_{s,\tau} = 0 \ \forall \ \tau \geq 2$. If the cost c is small enough, the seller benefit from the first channel dominates over disutility from the second channel. We can model the cost c as linearly growing[20] with seller's price uncertainty σ as $c = \kappa\sigma$. The expected seller payoff $\mathbb{E}[\pi(\mu)]$ is decreasing in σ if,

$$\frac{\partial \mathbb{E}[\pi]}{\partial \sigma} < 0 \implies \kappa > (6/17) \tag{22}$$

We consciously choose to only examine a high cost parameter range $\kappa > (6/17)$, such that the seller has an overall disutility from *price noise*[21].

The *price noise* among sellers is monotonically decreasing in level of reliance $\alpha > 0$ on the Machine Learning price z as (using Lemma 1),

$$\sigma^2 = Var[v_i] = (1-\alpha)^2 \sigma_e^2 = (1-\alpha)^2 \times 24\sigma^2/9 \tag{23}$$

In fact at $\alpha = 1$ we have $\sigma = 0$ and consequently the expected seller payoff is maximized at $E[\pi] = \mu$. Surprisingly, this suggest that the Machine Learning feedback loop and the resulting over-reliance on ML prices α appears to have no negative consequence on seller's payoff. In order to fully understand the impact of the ML feedback loop (and the exploding gap between confounded and unconfounded ML errors $(\hat{\sigma}_z(\alpha = 0) - \hat{\sigma}_z(\alpha))$), let us examine the resulting error in valuations and sale prices. The error in valuations can be formulated as,

$$E[(v_i - v)^2] = \underbrace{E[(v_i - E[v_i])^2]}_{\text{Price Noise}} + \underbrace{E[(E[v_i] - v)^2]}_{\text{Price Bias}} \tag{24}$$

While reliance on Machine Learning α is reducing *price noise*, it is increasing *price bias* because $E[(E[v_i] - v)^2] = \alpha^2 \sigma_z^2$. The error in valuations translates to error in sale price as,

$$E[(p - v)^2] = \delta \times \underbrace{E[(v_i - E[v_i])^2]}_{\text{Price Noise}} + \underbrace{E[(E[v_i] - v)^2]}_{\text{Price Bias}} \tag{25}$$

The corresponding variance in seller payoff is given by,

$$Var[\pi] = \underbrace{E[(\pi - E[\pi])^2 | z]}_{\text{Price Noise}} + \underbrace{E[(E[v_i] - v)^2]}_{\text{Price Bias}} \tag{26}$$

[20] If we model cost as a constant, the expected seller payoff $\mathbb{E}[\pi(\mu)]$ is decreasing in σ if $c > (24/17) \times \sigma$.

[21] We believe this better reflects individual homeowners. Institutional investors, home flippers, or i-buyers may in-fact have an overall utility.

Lemma 3 *The error in valuations, sale prices and variance in payoffs are given by,*

$$
\begin{aligned}
E[(v_i - v)^2] &= (1-\alpha)^2\sigma_e^2 + \alpha^2\sigma_z^2 \\
E[(p - v)^2] &= \delta \times (1-\alpha)^2\sigma_e^2 + \alpha^2\sigma_z^2 \\
Var[\pi] &= \Omega \times (1-\alpha)^2\sigma_e^2 + \alpha^2\sigma_z^2 \\
\textit{where} \quad \delta = (1/6) \quad \textit{and} \quad \Omega &= (2/3) + (58/27)\kappa + (2/9)\kappa^2
\end{aligned} \tag{27}
$$

Note that these sale price errors are not correlated across homes i.e., any random sample of homes will not have systematic upward or downward error. In other words, Machine Learning is not causing systematic “price bubbles” in the market. While ML is not causing price bubble, it may be increasing randomness to the sale prices and therefore the payoffs.

Lemma 4 *The total error in valuations $E[(v_i - v)^2]$ and sale prices $E[(p-v)^2]$ is always increasing with reliance α. The total variance in payoff $Var[\pi]$ is increasing in reliance α if over-reliance is large enough or ML error underestimation is severe enough at the feedback loop equilibrium as,*

$$
\alpha \geq \frac{\sigma_e^2}{\sigma_e^2 + \sigma_z^2/\Omega} \quad \textit{or} \quad \sigma_z^2 > \Omega\hat{\sigma}_z^2 \tag{28}
$$

More randomness in the seller's payoff means that the seller may get very lucky or unlucky. This is akin to the seller gambling on a coin toss instead of a deterministic payoff. Randomness is not an issue for risk-neutral sellers but is undesirable to risk-averse sellers. Consider a constant absolute risk-averse (CARA) seller with a concave utility $u(\pi) = 1 - e^{-a\pi}$ corresponding to a constant risk-aversion coefficient a. The expected utility $\Pi = E[u(\pi)]$ for this CARA seller is linearly decreasing in $Var[\pi]$ as,

$$
\Pi(a) = \mathbb{E}[\pi] - 0.5a \times \sqrt{Var[\pi]} \tag{29}
$$

Proposition 7 *The risk neutral payoff $\Pi(a=0)$ is always increasing in reliance α (and severity of feedback loop). The risk averse payoff $\Pi(a>0)$ is decreasing in reliance α on ML prices if ML error underestimation over the feedback loop is severe enough as,*

$$
\frac{\sigma_z^2}{\hat{\sigma}_z^2} > \frac{2\kappa^2}{a} \times \frac{\sqrt{Var[\pi]}}{\mathbb{E}[\pi]} + \Omega \tag{30}
$$

Note that $\sqrt{Var[\pi]}/\mathbb{E}[\pi] << 1$ thus the condition reflects a plausible setting. The RHS condition is further relaxed (i.e., payoff under ML more likely to be worse off), when - (i) seller is more risk averse (large a), (ii) seller faces low market participation cost (small c, κ, Ω) and (iii) seller has large noise in their prior valuation before observing sale price (large σ_e).

Intuition using Toy Model: We can break down payoff implications into two components – (a) ML price reduces valuation variance which minimizes need for a slow and costly learning among the crowd to resolve disagreements. This improves payoffs, particularly for impatient participants. (b) ML price, under strong feedback loop, adds a deceptively large valuation bias which makes the market resemble a coin toss or lottery where the seller may arbitrarily get a lucky or unlucky draw. This worsens payoff, particularly for highly risk averse participants. In essence ML price replace the "slow crowd learning" nature of the market with one that resembles a "quick lottery". If the crowd learning process in the market was cheap to begin with then effect (a) does not add a lot to payoffs while effect (b) dominates to make payoffs worse off. We also study sellers heterogeneous in patience[22], risk aversion and ability to price[23]. Sellers who are patient and risk-averse have most to gain from effect (a) and most to lose from effect (b). Notably, sophisticated investors (impatient, risk-neutral and high ability to price) traditionally stayed away from the housing market. The introduction of ML pricing may favor and encourage entry of such sophisticated investors. This happens as effect (b) starts to dominate under strong feedback loop. This is loosely consistent with the entry of iBuyers (1% of all US home purchases in 2019) and large real estate investors (18% of all US home purchases in Q3 2021) in residential housing market (Katz and Bokhari 2021).

5. Implications

Figure 4 illustrates implications of introducing ML price on payoff by numerically simulating on Lemma 2. First, consider introduction of ML price in (hypothetical) absence of confounding feedback (solid line). As the platform gets access to more data N, it can increase the capacity of deployed ML model e.g., from linear model to neural network. This reduces the ML price error σ_z and improves payoff. This represents the conventional wisdom. The extreme right hand side represents infeasible region because available training sample size is finite in practice $N << \infty$. Second, consider introduction of ML price in presence of confounding feedback (dashed line). As the platform gets access to more data N and increase the capacity of deployed ML model, it reduces σ_z. This strengthens the feedback and causes the adverse implications of overconfidence ML price, over-reliance and therefore inflation of true ML error.

Feedback Correction: A platform that offers ML pricing has a few options to mitigate or correct the underestimation. One option is to measure the reliance α on ML prices and use it to correct the estimated ML error σ_z^2. Unfortunately, the task of measuring α is not trivial. The

[22] An impatient individual is one who has high disutility from waiting, perhaps because learning is slow or costly.

[23] An individual with strong "ability to price" is one who can accurately guess offers a home would receive and likely sale price. Such an individual does not rely a great deal on the ML price and unaffected by errors in ML price.

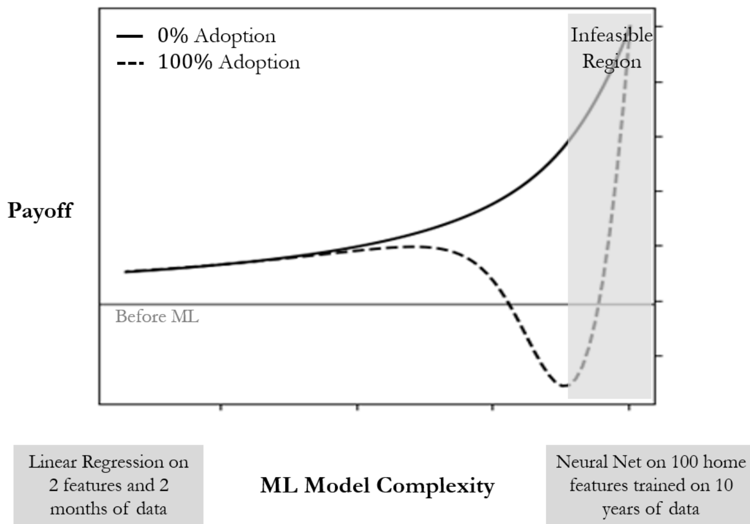

Figure 4 **The impact of introducing Machine Learning (with complexity varying on the x-axis) price on payoffs for two values of feedback strength (solid line: no adoption; dashed line: full adoption). The payoffs are compared with a scenario in which ML prices are not available**

platform could run a randomized experiment in which the ML prices for some homes are hidden while others remain available, but participants may not perceive the hidden information as a random occurrence, confounding the results. Alternatively, the platform could add random, small, positive, or negative errors to some ML prices. In fact, in Appendix A.2, we measure α with a similar natural experiment (i.e., we leverage unintentional random errors in ML prices). But it may be more challenging for the platform to intentionally and regularly add errors for the sole purpose of experimentation. Such experiments are common on websites, e-commerce platforms, and search engines, but the scrutiny in the housing market may be prohibitive.

A second and more conservative option is to calculate the ML price for a home using only a sample of historical sale prices for which the ML price was hidden from the market. This strategy would require the platform to leave a fraction of homes without an ML price so that the ML model could be trained on the unpriced homes. An increase in the fraction ρ would increase the accuracy of (and confidence in) the ML prices but would also mean that fewer homes have ML prices available. Even with this conservative approach, some feedback may seep in indirectly. For example, when an ML price is not available, market participants may look up the ML prices of similar neighboring homes. Let us assume that such limitations can be overcome using experimental manipulation or statistical solutions. Both options come with the same downside for the platform: ML prices are less visible to consumers.

Platform Incentives: We do not model the platform as an agent in the analytical model, therefore we are limited in discussing platforms incentives to correct. We provide here some plausible discussion on how correcting strategies may impact platforms revenue sources. The platforms' major revenue source is to sell ads on the website to local brokers, agents, and other real estate services. ML price presented alongside a small error (further underestimated due to the feedback loop) likely increases perceived informativeness of the site visit, site visits and ad revenue. As a result, the platform may not have incentive to limit visibility of ML prices. Further, the platform may participate as an iBuyer: an entity that purchases, upgrades, and flips (re-sells) homes at scale. The iBuyer is – (i) risk-neutral because random gains or losses over hundreds of home transactions average out, (ii) has a strong ability to price thanks to the access to data and ML model, and (iii) impatient because they prefer to flip homes quickly instead of holding large inventory. As discussed in the last section, this trio of characteristics positions the iBuyer to gain from ML pricing and an uncorrected feedback loop. In short, the platform's two sources of revenue (ads and iBuyer) do not incentivize the platform to correct the feedback and error underestimation. The platform may have other incentives to correct (e.g., the risk of long-term reputation damage, threat of regulations, or pure ethics), but we leave a more thorough examination as open questions for future research.

Price Bubbles: The analytical model enables us to identify the equilibrium but not the time required to arrive at the equilibrium. We expect market control mechanisms (e.g., expert opinions or model updates by the ML platform) to correct or reset the "self-fulfilling bubble" from time to time. Modeling such control mechanisms is outside the scope of our paper. Figure 5 depicts the simulated formation and correction of feedback bubbles defined as the divergence between true ML error σ_z and the underestimated ML error $\hat{\sigma}_z$. ML learning (and, therefore, ML feedback) is localized in submarkets (e.g., 1500–2000 sq. ft. homes in one Austin neighborhood), and the formation and correction of feedback bubbles are uncorrelated across submarkets. We would not expect these ML bubbles throughout the entire housing market simultaneously. We may see phases lasting a few months or years in local markets, as ML prices gradually become the dominant reference point for participants. If the ML price happens to be high (low), more and more sellers (buyers) will hesitate to deviate from the ML price until it becomes too difficult to make sales (purchases) at the ML price, causing a correction. Following the analytical results in Section 4, the feedback bubble is relatively mild (severe) when the level of ML price adoption is low (high).

Pricing bias in a submarket (e.g., 5% overpricing of 1000–2000 sq. ft. single-family homes in the Charlestown neighborhood of Boston) persisting for 12–36 months should be identifiable by observing summary-level demand-supply data in comparable neighborhoods or by using knowledge

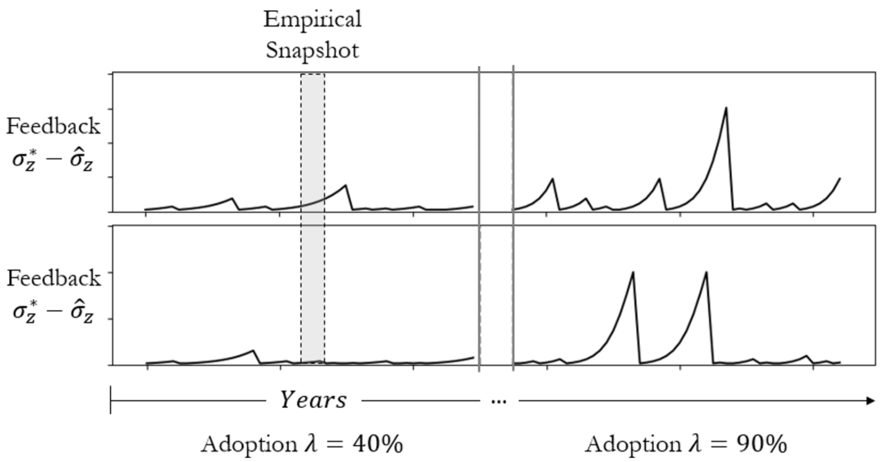

Figure 5 **The simulated, cyclical formation and correction of feedback bubbles in two submarkets with low (40%) and high (90%) rates of ML adoption. an empirical snapshot is likely too narrow to capture the cycles.**

of ML feedback (as described in this paper). However, market experts are not necessarily able to identify such bubbles. In fact, Cheng et al. (2014) show that securitized home loan managers were unaware of the growing housing bubble in 2004–06, preceding the 2007–08 collapse. The ML feedback loop is a statistically complex phenomenon, like the risk pricing of securitized home loan assets, so it would not be surprising if pricing bias from the ML feedback loop remains opaque to experts for years.

Key Drivers: We identify four drivers for the findings: (i) Preferences for the product evolve smoothly over time and across a high dimensional product feature space (such that similar products have similar value). (ii) Most buyers and sellers participate rarely in the market (e.g., a homeowner sells once in a decade; an entrepreneur raises funds infrequently). (iii) A seller needs to guess how buyers value their product. This may happen with an entrepreneur soliciting funds or loans, an art seller, or a home seller, among others. (iv) Individual buyers can determine their private valuation but need to guess how other buyers may value the product in case the current buyer needs to resell in the future.

Ingredient (i) motivates the use of an ML model to predict prices as a function of home features, and it also necessitates perpetual feedback. Ingredient (ii) ensures that participants have a limited understanding of the ML model, feedback, and potential risks (Schmit and Riquelme 2018). Many markets meet the first two requirements and have known ML feedback loops. For example, the traffic routing model in Google Maps (Lau 2020) informs changes in driver behavior, which then are observed by Google Maps and used to update its ML routing model. In fact, feedback labels are useful in a wide range of online learning settings where an ML algorithm learns by making mistakes.

In the ML routing model, a mistake is soon corrected because the resulting traffic congestion sends negative feedback to the ML model. Unfortunately, in the housing market, the feedback label is not visible. Ingredient (iii) and (iv) are relatively unique, though they apply to housing, crowdfunding, peer-to-peer lending, and art auctions. Because these markets lack ground truth prices, the ML price influences both sides of the market, thus contaminating the resulting sale prices. As more market participants rely on ML pricing, the impartial ground truth is further obscured.

6. Conclusion

ML pricing purports many benefits including costless accessibility, perceived impartiality, and high reported accuracy. But the argument that ML pricing can solve pricing friction in the housing market presumes that the ML model was trained on large, independent samples. In practice, ML training samples can be confounded by its own predictions, resulting in self-fulfilling feedback. We have analytically shown three key results. First, the error in sale prices after introduction of ML prices can be larger than without ML prices. Second, the error in sale prices eventually can increase to the limit i.e., where sale prices are entirely random (devoid of any correlation with true equilibrium market clearing price in absence of the ML prices). Third, we show that market participants characterized by - high patience (low cost of spending time learning in the market), low pricing ability (error between private valuation and true equilibrium market clearing price) and risk averse have most to loose from introduction of ML prices.

The self-fulfilling ML feedback loop has similarities with the phenomena of "echo chambers" and "filter bubbles" in the personalized social media context (Eli Pariser 2010), where ML models continuously learn from user behavior while also influencing those behaviors. Such ML models tend to reinforce selective preferences, attaining good prediction accuracy (likelihood that user clicks on recommended content) but potentially losing sight of long run outcomes. Platforms that serve ML prices (Zillow or Redfin in the housing market; Google and Facebook in personalized recommendations) may not have clear incentives to address the feedback loop and reduce the associated risk. We briefly discussed options, such as statistical correction procedures and low-capacity ML models, that could decrease the impact of feedback loops. But these solutions require the platform to sacrifice revenue. We expect that regulators eventually may have to enforce ML model validation controls, as occurred with asset pricing models in financial markets. We hope future research will investigate policies to incentivize the correction of ML feedback loops. Also, our limited dataset enabled us only to verify primitives for the analytical model, but long-term empirical research is critical for uncovering how the introduction of ML pricing affects housing market characteristics.

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Table 3 Table of Content for Appendix

Section	Subsection	Title
A		Empirical Evidence
	A.1	Data Description
	A.2	Forward Loop: Zestimate Impact on List and Sale Prices
	A.3	Backward Loop: Zestimate Calculation from Sale Prices (online)
B		Model Choices
	B.1	Buyer-Seller Bargaining
	B.2	Simple Model (online)
	B.3	Full Model (online)
C		Proofs
D		Additional Tables and Figures (online)

Appendix A Empirical Evidence

A.1 Data Description

We use housing market data from Zillow.com[24], which is an online real estate database company. Zillow provides information about home features (e.g., floor size, year built), location (e.g., county, zip code, street address), historical and current listing information (e.g., list price, sale price), and a price estimate called the "Zestimate". Zillow describes the Zestimate as an "estimate of a home's market value" and provides evidence of its accuracy by presenting Zestimate alongside actual sale price outcomes. From this and other publicly available posts (Zillow.com 2020), we infer that the Zestimate is an ML-based prediction of the sale price as a function of home features, location, and the economic environment. We have access to data from over 750,000 homes in Austin (Travis County, Texas), Boston (Suffolk County, Massachusetts), and Pittsburgh (Allegheny County, Pennsylvania). Table 4 provides sample values of the features available for every home.

While home features and location are largely static, the listing information is updated every time a home is put on the market. As the listing information and economic environment evolves, the current Zestimate of a home also evolves. Further, Zillow regularly makes minor and major upgrades to the Zestimate algorithm, as a result the entire Zestimate trend shifts regularly. Consider a Boston home as an example. On January 1, 2020, Zillow presented a Zestimate trend from \$100,000 on January 1, 2010, to \$122,000 on January 1, 2020 (an annual increase of 2%). Then, following an algorithm update on February 1, 2020, the same Zestimate trend now went from \$100,000 on January 1, 2010, to \$148,000 on January 1, 2020 (an annual increase of 4%). Thus, the pre-update and post-update versions of the Zestimate assigned two different values to the same home on the same date. To

[24] Data collected by Runshan Fu, NYU Stern School of Business.

Table 4 **Sample values of the house features, location, listing information, and Zestimate**

Category	Variable	Sample Value
Location	Latitude	29.7–42.3 degrees North
	Longitude	71.0–95.3 degrees West
	Neighborhood	South Boston, Carrick, Brighton Heights, etc.
	Zip Code	15210, 15212, 15232, etc.
	County	Suffolk, Allegheny, Travis
Features	Floor Size	100–10,000 sq. ft.
	Year Built	1799–2019
	Last Remodel Year	1799–2019
	Bathrooms	0–15
	Bedrooms	0–15
	Parking	0–1000 sq. ft.
	Lot	100–10,000 sq. ft.
	Stories	0–50
	Solar Potential	0–100
	Type	Single Family, Multi-Family, Condo, etc.
	Structure Type	Colonial, Victorian, Modern, etc.
	Roof Type	Composition, Shingle, Asphalt, etc.
	Flooring	Hardwood, Carpeted, Tile, etc.
	Patio	Porch, Deck, None, etc.
	Ex-Material	Brick, Wood, Cement, etc.
Listing	List Price	$10,000$–10,000,000
	Days Listed	1–365 days
	Sale Price	$10,000$–10,000,000
ML Price	Z (Zestimate)	$10,000$–10,000,000

track algorithm updates, we take 25 snapshots of Zillow information approximately every two weeks between February 2019 and March 2020. In the next section, we use these algorithm upgrades to construct an instrument for identifying impact of Zestimate on the market.

A.2 Zestimate Impact on List and Sale Prices

The first necessary primitive for our analytical model is that Zestimate z_i has significant impact on the home sale price p_i. Both the Zestimate z_i and buyers-sellers in the market may use local information unobserved to us, thus Zestimate z_i is endogenous. To estimate the impact, we need to compare two groups of homes: those that received an erratic Zestimate over the true value, and those that received an erratic Zestimate under the true value. We cannot experimentally manipulate and add errors in Zestimate or simulate the home sale process in lab. Instead, we take advantage of the frequent upgrades to the Zestimate algorithm, from which we can infer historical instances

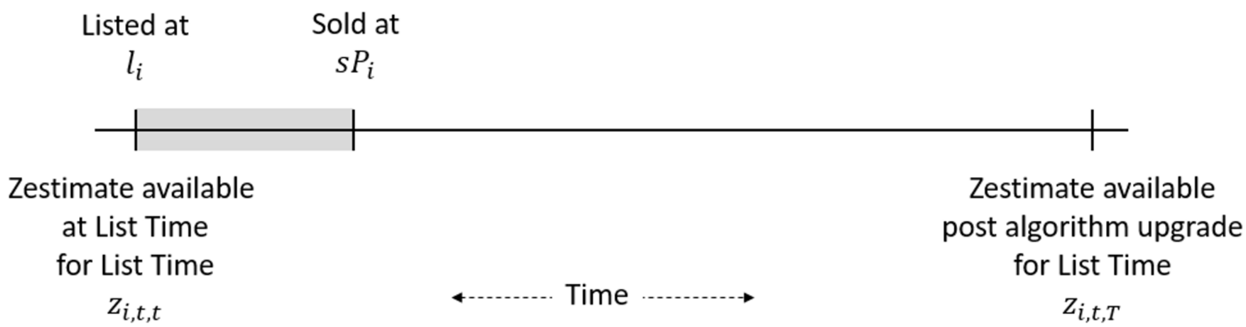

Figure 6 **A timeline of the relationship between the list price l_i, the pre-update Zestimate z_i ($z_{i,t,t}$) for home i available at list time t, and the post-update Zestimate $\bar{z}$ ($z_{i,t,T}$) at time T for home i available at list time t.**

in which the Zestimate was temporarily erratic (Figure 6). Then, we calculate the difference in the average sale price between the two groups to identify the impact of the Zestimate.

$$z_i = v_i + e_i \quad ; \quad \bar{z}_i = v_i + \bar{e}_i \quad ; \quad z_i^e = \frac{z_i - \bar{z}_i}{\bar{z}_i} \tag{31}$$

Here z_i represent the Zestimate for home i when it was listed on the market at time t. This Zestimate contains some unobserved error $e_i = z_i - v_i$ relative to true value v_i. Let $\bar{z}_i$ represent the Zestimate for the same time snapshot t after the Zestimate upgrade at T ($>> t$). This upgraded Zestimate also contains some unobserved error $\bar{e}_i = \bar{z}_i - v_i$. The change in Zestimate ($\bar{z}_i - z_i = \bar{e}_i - e_i$) is correlated with the unobserved error e_i presented on the platform when the home was on the market at time t. Using z_i^e, we construct two groups of homes: the positive-error group is defined by $z_i^e > +1\%$, while the negative-error group is defined by $z_i^e < -1\%$. These two groups can provide a useful comparison if assignment of homes into the groups is random or equivalently z_i^e is exogeneous. Since the algorithm maps home features to prices, it is plausible that algorithm upgrade corrects pricing for home features that were originally under or over-priced. So, following equation 21 we conduct propensity score matching (PSM) using the post-update Zestimate $\bar{z}_i$ (presumably less erratic than the pre-update Zestimate) and expansive set of home features $\vec{X}_i$ (such as floor area, number of bedrooms, year of construction, and many more). The validity of this pseudo randomization depends on two assumptions – (i) algorithm does not model home features that are unobserved or hidden on the platform and (ii) algorithm does not have data leakage . Zillow does not disclose the exact algorithm or its upgrade, but these assumptions are plausible given Zillow's qualitative discussion of the algorithm (Zillow.com 2020). In summary, the identification of Zestimate impact on sale prices relies on z_i^e being exogenous to local information that is observed by buyers-sellers on the ground but unobserved to us (researchers).

$$pScore_i(\boldsymbol{\phi}) = \frac{1}{1 + exp(-[\bar{z}_i, \vec{X}_i] \times \boldsymbol{\phi})} \tag{32}$$

Table 5 Positive and Negative error group comparison statistics before and after PSM. Rubin's R between 0.5 and 2.0 and Rubin's B less than 25 indicate a good match.

Statistic	**Before Matching**	**After Matching**
Mean % Std. Bias	10.21%	0.93%
Rubin's R	0.99	1.01
Rubin's B	71.82%	0.30%

Table 5 report the effectiveness of PSM. After matching, the positive-error group had an average Zestimate error of $z_i^e = 7.1\%$ while the negative-error group had an average Zestimate error of $z_i^e = -7.1\%$. Appendix D provides further details on the two groups, PSM effectiveness and descriptive statistics for dependent and independent regression variables after PSM. Figure 7, Model 1 following specification in equation 22 reports a sale price difference of 2.22% between the two groups. The difference in sale prices has a sensitivity of approximately 15% (15.5 = 2.22/14.2) to the Zestimate errors because buyers and sellers rely only partially on the Zestimate to determine their list prices and offers. In Model 3, the difference in sale prices is as high as 7.5% for small homes (500 square feet; worth about \$50,000), which tend to be more standardized and may involve buyers and sellers who rely more heavily on the Zestimate. The difference is only 1.4% for large homes (worth about \$500,000). Thus, we infer that the degree of reliance on the Zestimate ranges from 0.1 (14.2% $\rightarrow$ 1.4%) to 0.5 (14.2% $\rightarrow$ 7.5%). Going forward, we use the range 0.1 - 0.5 as a loose range of reliance on the Zestimate across housing submarkets.

$$log(p_i) = \beta_0 + \beta_1 * log(\bar{z}_i) + \beta_2 * \underbrace{(z_i^e > 1\%)}_{\text{Treatment}} + \beta_3 * pScore_i + \xi_{1,i} \tag{33}$$

Next, we use the observed list price and time to sale to uncover the impacts of the Zestimate on buyers and sellers, independently. For sellers, we examine the list prices, which are set by sellers alone (without input from buyers). As expected, Figure 8, Model 1, reports a difference of 1.4% in the initial list price between the two groups, confirming that the Zestimate impacts sellers. For buyers, we reason that if the Zestimate had no effect, then homes in the positive-error group should not be any harder to sell than homes in the negative-error group (as we conducted PSM to create groups with similar true prices). If, however, the Zestimate does affect buyers, then the sale price and time to sale should differ between the groups. Indeed, Model 2 (controlling for the initial list price) reports a difference of 1.2% in the sale price, and Model 3 reports a shorter time to sale by 4 days.

$$log(l_i) = \beta_0 + \beta_1 * log(\bar{z}_i) + \beta_2 * \underbrace{(z_i^e > 1\%)}_{\text{Treatment}} + \beta_3 * pScore_i + \xi_{2,i} \tag{34}$$

$$log(p_i) = \beta_0 + \beta_1 * log(\bar{z}_i) + \beta_2 * \underbrace{(z_i^e > 1\%)}_{\text{Treatment}} + \beta_3 * pScore_i + \beta_4 \times \left(\frac{l_i - \bar{z}_i}{\bar{z}_i}\right) + \xi_{3,i} \tag{35}$$

	Log(List Price)	Log(Sale Price)	Time to Sale	Time to Sale
Treatment ($z^e > 1\%$)	0.014*** (0.003)	0.012** (0.005)	-3.794* (2.012)	-3.791* (2.009)
List Markup ($(l_0 - z)/z$)		0.874*** (0.019)	0.304 (8.276)	
Log($\bar{z}$)	1.010*** (0.002)	1.017*** (0.003)	4.271*** (1.384)	4.273*** (1.383)
pScore	-0.013 (0.011)	-0.03 (0.02)	0.461 (8.509)	0.448 (8.5)
Constant	-0.130*** (0.028)	-0.243*** (0.041)	22.673 (17.401)	22.657 (17.393)
Observations	7670	2414	2414	2414
R2	0.967	0.977	0.006	0.006
Adj-R2	0.967	0.977	0.004	0.004
Residual Std. Error	0.122 (df=7666)	0.115 (df=2409)	49.303 (df=2409)	49.293 (df=2410)
F Statistic	75234*** (df=3;7666)	25860*** (df=4;2409)	3.445*** (df=4;2409)	4.595*** (df=3;2410)

Note: * p < 0.1, ** p < 0.05, *** p < 0.01

Figure 7 **(Model 1) Primary specification for the impact of the Zestimate error on the sale price (equation 22). (Model 2) Additional interaction of the Zestimate error with the home size. (Model 3) Additional interaction of the Zestimate error with the Zestimate. (Model 4) Additional regression controls for home features (more than 50 estimates skipped here).**

	Log(Sale Price)	Log(Sale Price)	Log(Sale Price)	Log(Sale Price)
Treatment ($z^e > 1\%$)	0.022*** (0.006)	0.379*** (0.11)	0.368*** (0.11)	0.024*** (0.06)
Treatment * Log(Floor Size)	. .	-0.049** (0.015)		
Treatment * Log($\bar{z}$)			-0.027*** (0.009)	
Log($\bar{z}$)	1.022*** (0.004)	1.025*** (0.005)	1.037*** (0.007)	1.030*** (0.009)
pScore	-0.067** (0.027)	-0.076** (0.027)	-0.066** (0.027)	-0.063** (0.032)
Log(Floor Size)		0.008 (0.011)		-0.035** (0.015)
Constant	-0.287*** (0.055)	-0.384*** (0.092)	-0.478*** (0.082)	-0.026 (0.283)
Observations	2414	2414	2414	2414
R2	0.958	0.958	0.958	0.960
Adj-R2	0.958	0.958	0.958	0.959
Residual Std. Error	0.157 (df=2410)	0.156 (df=2410)	0.156 (df=2410)	0.154 (df=2346)
F Statistic	18297*** (df=3;2410)	11039*** (df=5;2408)	13776*** (df=4;2409)	844*** (df=67;2346)

Note: * p < 0.1, ** p < 0.05, *** p < 0.01

Figure 8 **(Model 1) Impact of the Zestimate error on the list price (equation 23). (Model 2) Impact of the Zestimate error on the sale price, controlling for the list price (equation 24). (Models 3 and 4) Impact of the Zestimate error on the time to sale (in days)**

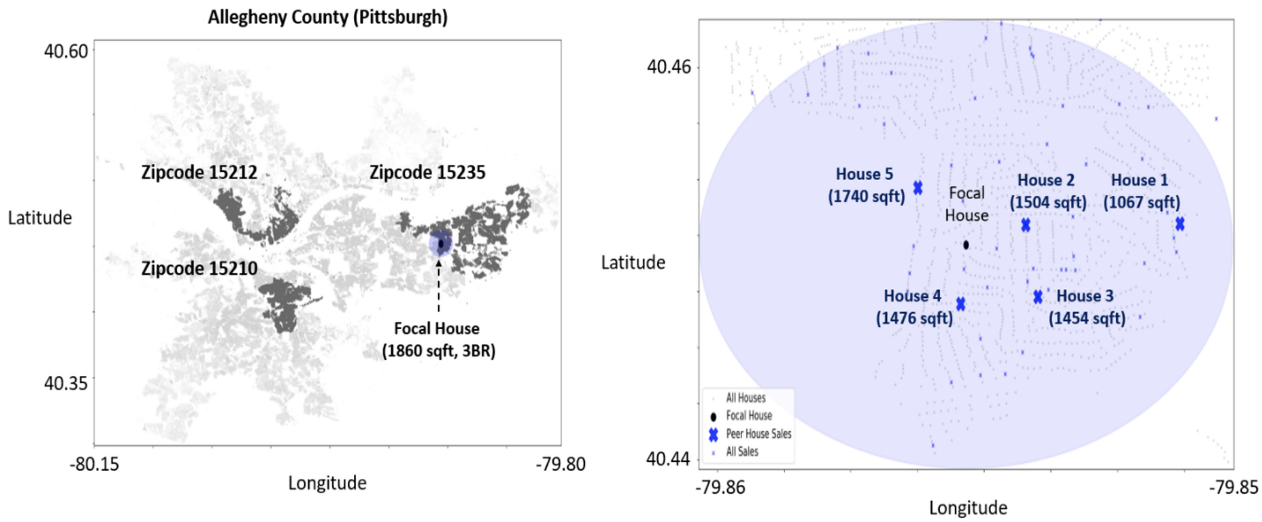

Figure 9 **(Left) All homes in Allegheny County (small grey dots); our dataset includes homes from three zip codes (small dark grey dots). Consider an example of a focal home: a 3-bedroom, 1660 square-foot home in zip code 15235 (large black dot). (Right) The focal home's five peers are located within 2 km of the focal home (the shaded circular region), sold in the past 12 months (small blue x), and have a similar floor size (large blue X).**

A.3 Zestimate Calculation from Sale Prices

The Zestimate algorithm is proprietary to Zillow and thus opaque to us. Zillow describes the Zestimate as composed of both expert-driven economic modeling and data-driven predictive ML. The feedback loop phenomenon is relevant only if the data-driven predictive ML component is a significant driver. We want to understand if data-driven predictive ML methodology is a significant driver of Zestimate. We know that for every Zestimate, Zillow reports the sales of 4 or 5 "peer homes" that are in the geographic vicinity and have similar features as the focal home. We first attempt to reverse engineer the choice of these peer sales. Then we test a hypothesis that Zestimate is driven primarily by a simple variable: the average sale price of peer homes. If so, it would greatly simplify the data-driven methodology behind the Zestimate calculation.

Model of Peer Sales: We analyze 1346 houses ($i \in [1, 1346]$) in Allegheny County, each observed over seven time snapshots t. In Allegheny County, we observed approximately 1400 home sales ($j \in [1, 1400]$) at the same time going up to 12 months back. All these sales j are candidate for peer set for every house i. Among the 1400 candidates only 4-5 houses are selected as peer sales for each house. We use $isPeer_{i,j,t} = 0, 1$ to represent if house sale j is tagged as a peer sale of house i at time snapshot t. Given the characteristics of house i and j, we want to predict if the pair would be tagged as peers. Figure 9 illustrate an example home and its five peers.

$$P\Big(isPeer_{i,j,t} = 1\Big) = (t - saleTime_j < 12) \times (Dist_{i,j} < 2)$$

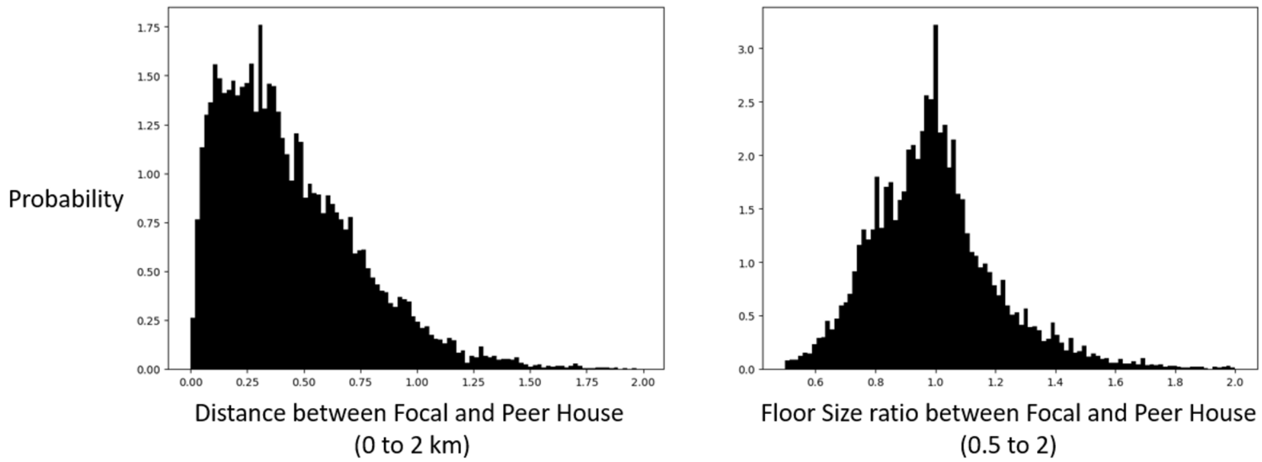

Figure 10 **Probability distribution of geographical distance (Left) and floor size ratio (Right) between focal and peer sales. The distributions are empirically calculated using Zillow's reported peer sales for houses in Zip Codes (15210, 15212, 15235) between March and August 2019.**

$$\times(0.5 < \frac{floorSize_j}{floorSize_i} < 2) \times \frac{1}{1 + exp(-y_{i,j,t})}$$

$$y_{i,j,t} = \beta_0 + \beta_1 * Dist_{i,j} + \beta_2 * (ZipCode_j = ZipCode_i) + \beta_3 * abs\Big(log\Big(\frac{floorSize_j}{floorSize_i}\Big)\Big)$$
$$+\beta_4 * (bedrooms_j - bedrooms_i) + \beta_5 * abs(bathrooms_j - bathrooms_i) \qquad (36)$$

We infer from simple descriptive analysis (Figure 10) that a house has a very high likelihood of being tagged as a peer ($isPeer_{i,j,t} = 1$) if – (i) it is sold within past 12 months, (ii) it is within 2 km of the focal house and (iii) it has a floor size that is no less than half and no more than double of the focal house. To further predict $isPeer_{i,j,t}$ among houses that satisfy the three criteria above, we use a simple logistic regression model using – distance between house i and j, binary indicator whether the two houses are in the same zipcode, ratio of floor sizes, absolute difference of number of bedrooms and bathrooms.

Figure 11 reports that homes in close geographical vicinity (distance and zip code) and similar house features (size, bedrooms, bathrooms) are more likely to be peers. An accuracy of 98.7% and an F1 score of 0.32 (compared with F1 score of 0.02 for a random model) suggest that our predictive model can successfully pick out peers of a house (Table 6 and 7).

Model of Zestimate from Peer Sales: The results identify peer sales for a home with high confidence. Next, we hypothesize that Zestimate z_i is driven primarily by a simple variable: the average sale price of peer homes $\bar{p}_j$. To establish the role of $\bar{p}_j$, we could consider a pair of adjacent similar homes A and B. At time t_1, both homes have the same Zestimate and the same peer sets

	Peer Match	
	OLS	logistic
	(1)	(2)
Peer Distance	-0.090***	-3.716***
	(0.0003)	(0.016)
Peer Floor Ratio	-0.069***	-3.049***
	(0.001)	(0.038)
Peer Bedrooms Diff	-0.010***	-0.425***
	(0.0002)	(0.010)
Peer Bathrooms Diff	-0.017***	-0.822***
	(0.0002)	(0.010)
Peer ZipCode Match	0.008***	0.013
	(0.0004)	(0.019)
Constant	0.170***	0.756***
	(0.001)	(0.022)
Observations	1,383,795	1,383,795
R2	0.086	
Adjusted R2	0.086	
Log Likelihood		-136,567.200
Akaike Inf. Crit.		273,146.500
Residual Std. Error	0.172 (df = 1383789)	
F Statistic	26,137.130*** (df = 5; 1383789)	
Note:		*p<0.1; **p<0.05; ***p<0.01

Figure 11 **Model of whether a house, which satisfies sale time within 12 months, distance within 2 km and floor area between 0.5 to 2 times of focal house, is a peer to the focal house.**

Table 6 **Confusion Matrix and classification evaluation metrics (TPR, FPR, Accuracy, F1 Score) for the peer prediction model.**

Confusion Matrix		Actual	
		Peer	Not a Peer
Predicted	Peer	4	16
	Not a Peer	1	1379

Table 7 **Performance of logistic regression components $y_{i,j,t}$ within the overall predictive model**

True Positive Rate	80%
False Positive Rate	1.14%
Precision	20%
Accuracy	98.7%
F1 Score	0.32

$(J=1,2,3,4)$, so they have the same $\bar{p}_j$. At time t_2, the peer set for one home is the same while the peer set for the other home has changed. Given the homes geographical proximity and similarity in features, any difference in the Zestimate $(z_{B,t_2} - z_{A,t_2})$ at time t_2 must arise from the change in the peer set $(J_B = 1,2,3,4 \to 1,3,4,5)$. Thus, we can isolate the role of the average peer sale price $\bar{p}_j$ as a dominant driver of the Zestimate z_i.

$$pScore_i(\boldsymbol{\phi}) = \frac{1}{1+exp\Big(\Big[(\bar{p}_j)_{i,t}, z_{i,t_1}, \theta_{i,t_1}, \Delta\theta_i, \vec{X}_i\Big] * \boldsymbol{\phi}\Big)} \quad ; \quad (\bar{p}_j)_{i,t} = \Big(\sum_{j \in \text{peers of } i \text{ at } t} p_j\Big) \tag{37}$$

We create a control group (no change in the peer set) and a treatment group (a change in the peer set that causes $\bar{p}_j$ to increase by 0–5%; the average increase is 2.3%). We use Propensity Score

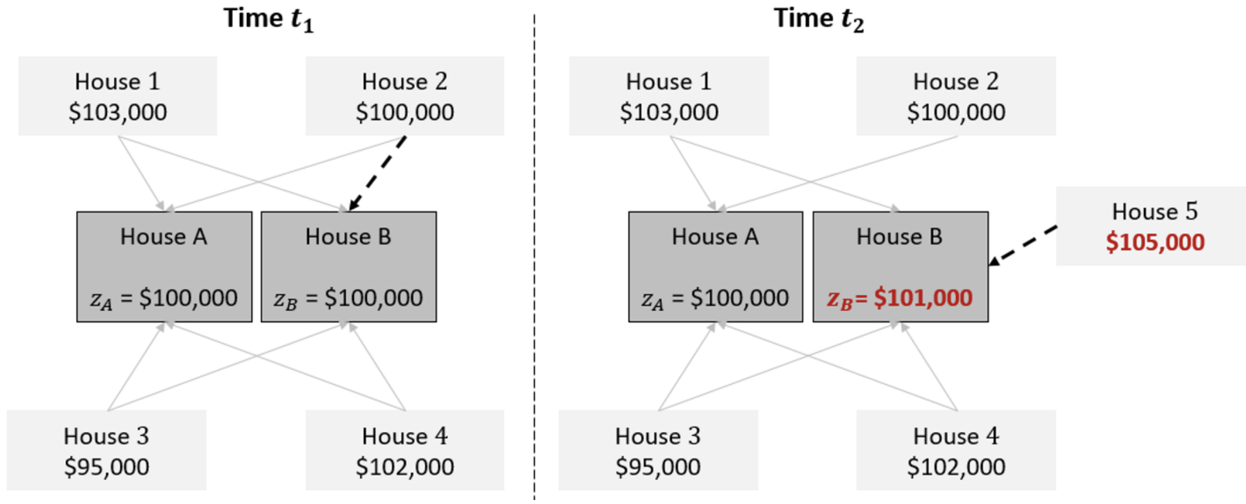

Figure 12 **A hypothetical example. At time t_1, Homes A and B have the same Zestimate, the same set of peer homes $1, 2, 3, 4$, and the same average peer sale price ($\bar{p}_j$). At time t_2, House 5 replaces House 2 in the peer set of Home B, causing the $\bar{p}_j$ for Home B to increase by $1,250 and its Zestimate to increase by $1,000.**

	Zestimate			
	(1)	(2)	(3)	(4)
Avg. Peer Sale Price	1.050***	1.049***	1.042***	0.770***
	(0.002)	(0.002)	(0.002)	(0.007)
Time Since Sale Weight		0.934***	1.015***	
		(0.142)	(0.136)	
Peer Distance Weight		-1.395***	-1.186***	
		(0.137)	(0.132)	
Peer Floor Ratio Weight		-0.900***	-1.065***	
		(0.095)	(0.091)	
Avg. Peer Time Since Sale			-323.955***	
			(64.591)	
Avg. Peer Floor Size			12.474***	
			(0.543)	
Avg. Peer List Price				-0.003
				(0.004)
Tax Estimate				0.268***
				(0.005)
Constant	-5,916.929***	-5,751.926***	-3,907.004***	-3,517.140***
	(372.910)	(368.627)	(422.500)	(307.487)
Observations	9,383	9,383	9,366	9,203
R2	0.958	0.959	0.963	0.973
Adjusted R2	0.958	0.959	0.963	0.973
Residual Std. Error	12,730.440 (df = 9381)	12,578.420 (df = 9378)	12,016.530 (df = 9359)	10,240.530 (df = 9199)
F Statistic	214,093.000*** (df = 1; 9381)	54,882.640*** (df = 4; 9378)	40,180.470*** (df = 6; 9359)	109,863.700*** (df = 3; 9199)
Note:				*p<0.1; **p<0.05; ***p<0.01

Figure 13 **Model of Zestimate using average peer sale price, weighted peer sale prices, average peer floor size, average peer list price and county tax estimate.**

Matching (PSM) to create a sample that is matched on the peer set at t_1, Zestimate at t_1, home features X_i, and other potential drivers (e.g., estimated taxes) of a change (θ) in the Zestimate. Now we can measure the treatment effect of $\Delta\bar{p}_j$ as β_2 in equation below.

$$\Delta z_i = (z_{i,t_2} - z_{i,t_1}) = \beta_0 + \beta_1 * \Delta\theta_i + \beta_2 * \underbrace{(\Delta\bar{p}_j)}_{\text{Treatment}} + \beta_3 * pScore_i \tag{38}$$

The average increase of 2.3% in $\bar{p}_j$ corresponds to a 1.9% increase in the Zestimate. We repeat the same steps with two alternative treatment groups: 5–10% increase in $\bar{p}_j$ and 10–15% increase in

Table 8 Treatment effects using three different treatment groups with different ranges of the change in the average peer sale price $\bar{p}_j$.

Treatment Group	Range of Change in $\bar{p}_j$	Mean Change in $\bar{p}_j$	Treatment Effect (β_2)	$\beta_2/\bar{p}_j$
I	0–5 %	2.3%	1.9%	0.83
II	5–10 %	6.7%	4.4%	0.66
III	10–15%	11.5%	8.6%	0.75

	Zestimate (1)	(2)	(3)
Avg. Peer Sale Price	1.051***	1.050***	1.030***
	(0.001)	(0.001)	(0.002)
Peer Sale Price Deviation	-0.000	-0.004	-0.022
	(0.004)	(0.008)	(0.172)
Peer Price Dev*Peer Distance		0.033***	
		(0.004)	
Peer Price Dev*Peer Floor Ratio		0.119***	
		(0.027)	
Peer Price Dev*Time Since Sale		-0.009***	
		(0.001)	
Peer Price Dev:Ind w1			0.018*
			(0.011)
Peer Price Dev:Ind w2			-0.008
			(0.172)
Peer Price Dev:Ind w3			0.067
			(0.173)
Peer Price Dev:Ind w4			0.133
			(0.124)
Constant	-6,152.341***	-6,692.196***	-4,586.921***
	(160.929)	(185.685)	(1,705.038)
Observations	46,735	46,735	10,609
R2	0.961	0.961	0.961
Adjusted R2	0.961	0.961	0.961
Residual Std. Error	12,233.420 (df = 46732)	12,184.070 (df = 46726)	11,954.880 (df = 10598)
F Statistic	575,691.000*** (df = 2; 46732)	145,139.200*** (df = 8; 46726)	26,283.050*** (df = 10; 10598)

Figure 14 Model of Zestimate using different weighted peer sale prices.

$\bar{p}_j$. Table 8 reports the results from all three treatment specifications. Overall, a 1% increase in $\bar{p}_j$ corresponds to an increase of 0.66–0.83% in the Zestimate.

To further substantiate the role of $\bar{p}_j$ in driving Zestimate z_i, we test the out-of-sample explanatory power, and we find that $\bar{p}_j$ alone explains almost 96% of all variation in the Zestimate. Although the actual Zestimate model may be significantly more sophisticated than our simple approximation, the extremely high out-of-sample explanatory power ($R^2 \approx 0.96$) suggests that significant fraction of full model's output is driven by information contained in peer sales. We iterate over various alternative predictors – (i) weighted (instead of unweighted) average of peer sale prices, (ii) House Features X_i, (iii) Location (Neighborhood, ZipCode, County) fixed effects, (iv) Time fixed effects, (v) Tax Estimate. We find that additional features (House Features, Location fixed effects, Time fixed effects, and Tax Estimate do not contain significant Zestimate explanatory power on their own. Table 9 shows that all these features and weighted peer sale price add very little to the explanatory power when used alongside the simple average peer sale price $\bar{p}_j$ metric.

Table 9 Alternative Models and features to predict Zestimate

Features	Out of Sample	
	Linear Model	Support Vector Model
Average Peer Sale Price only	**96.15**	95.97
+ Kernel Weights	96.59	96.61
+ All other covariates	96.79	97.34

Appendix B Model Choices

B.1 Buyer-Seller Bargaining

We model seller's choice of list price such that - if an offer meets the list price the home sells otherwise the home does not sell. We make a three assumptions here – (a) The seller does not receive offers above the list price, (b) The seller cannot reject an offer at (or above their list price) and (c) The seller cannot accept an offer below the list price.

Let's consider the assumption (a). Since, the seller's list price is visible to the buyer, even if the buyer has a higher willingness to pay, they have no reason to make an offer above the list price. A buyer may make an offer above the list price if they face competition from other buyers bidding for the same home. Empirically we observe that 81% of homes sell below their list price. On an average homes seller 0.5% below their list price (Figure 15). Given these observations we choose to not model buyer competition and resulting offers above list price. Regarding (b), if the buyer makes an offer at the list price, we assume that seller cannot reject the offer i.e., "enforced full price offer contract" ([Guerra 2018]). In practice, contracts and regulations do not allow enforcing a seller to accept offers at list price. However, brokers may demand commission if the seller chooses to reject an offer at list price. This indirectly discourages the seller from listing lower than their reservation price and subsequently studying offers and making choices to accept or reject. We do not model broker commission, instead we directly assume that seller does not have a choice to reject a full price offer and therefore does not list below their reservation price.

Let's consider the assumption (c). The seller could choose to list at a very high price, well above their reservation price. Once they receive the offers, they could choose to accept if the offer is above the reservation price. In doing so, the seller doesn't reveal their reservation price to the buyer. In such a model, the buyer is forced to reveal their full willingness to pay or engage in bargaining as both the buyer and seller choose to not reveal their reservation price. In practice, listing very high has some negative implications – (i) Buyer's may not want to enter lengthy negotiations, (ii) Broker may not be willing to spend effort in advertising a home where an offer meeting the list price is unlikely, (iii) The home may be left out of buyer's consideration, since similar homes priced more competitively

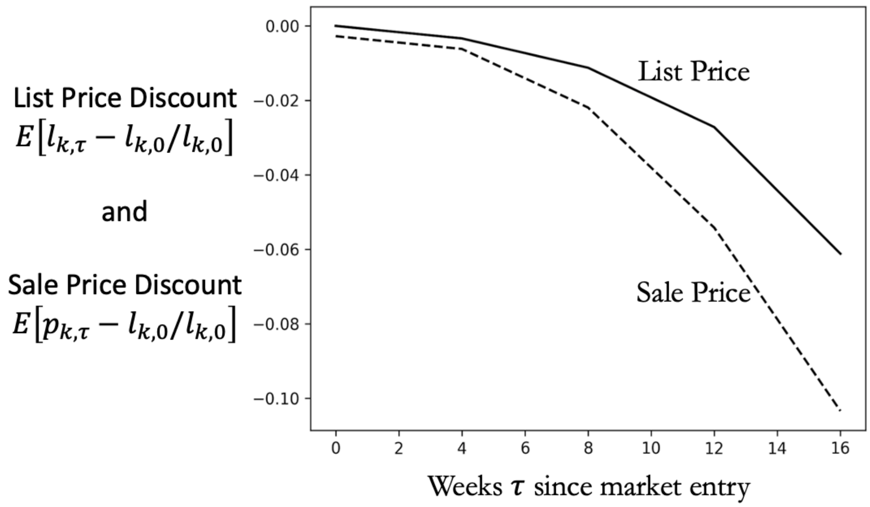

Figure 15 **Average list price discount** $\mathbb{E}[(l_{k,\tau} - l_{k,\tau=0})/l_{k,\tau=0}]$ **and average sale price discount** $\mathbb{E}[(p_{k,\tau} - l_{k,\tau=0})/l_{k,\tau=0}]$ **relative to the first list price.**

substitute out the home with the high list price. We do not model seller's competition against similar homes, instead we directly assume that seller does not list significantly above their reservation price. The Figure 15 below shows the empirically observed evolution of list price and corresponding sale prices. Empirically we observe that 55% of homes sell at or withing 1% of their list price. The evidence reinforces the assumption that seller does not receive and accept offer significantly below their list price. However, this assumption is not always true. We do observe that 10.7% of homes sell more than 5% below their list price. Our analytical model is limited in representing these outcomes.

Appendix C Proofs

Proposition 3(a)

We can express Proposition 2 as,

$$\begin{aligned}
\hat{\sigma}_z^2(\alpha) &= (1-\alpha)^2 \times \left(\sigma_z^2(\alpha) + \delta\sigma_e^2\right) \\
\frac{\hat{\sigma}_z^2(\alpha)}{(1-\alpha)^2} - \delta\sigma_e^2 &= \sigma_z^2(\alpha) > \hat{\sigma}_z^2(\alpha) \\
\hat{\sigma}_z^2(\alpha)\Big(\frac{1}{(1-\alpha)^2} - 1\Big) &> \delta\sigma_e^2 \\
\frac{1-(1-\alpha)^2}{(1-\alpha)^2} &> \delta\frac{\sigma_e^2}{\hat{\sigma}_z^2(\alpha)}
\end{aligned} \tag{39}$$

We can express $\sigma_e^2/\hat{\sigma}_z^2(\alpha)$ in terms of α using equation 18 as,

$$\begin{aligned}
\alpha &= \frac{\sigma_e^2}{\sigma_e^2 + \hat{\sigma_z}^2} \\
\frac{\sigma_e^2}{\hat{\sigma_z}^2} &= \frac{\alpha}{1-\alpha}
\end{aligned} \tag{40}$$

We can now substitute $\sigma_e^2/\hat{\sigma}_z^2(\alpha)$ in the condition that satisfies 2(b) at equilibrium,

$$
\begin{aligned}
\frac{1-(1-\alpha)^2}{(1-\alpha)^2} &> \delta \frac{\sigma_e^2}{\hat{\sigma}_z^2(\alpha)} \\
\frac{1-(1-\alpha)^2}{(1-\alpha)^2} &> \delta \frac{\alpha}{1-\alpha} \\
1-(1-\alpha)^2 &> \delta\alpha(1-\alpha) \\
\alpha(2-\alpha) &> \delta\alpha(1-\alpha) \\
\alpha &< \frac{2}{1+\delta}
\end{aligned} \tag{41}
$$

Since $\delta < 1$, the RHS is greater than 1. Therefore this condition is always satisfied.

Proposition 3(b)

Substituting α from equation 18 into Proposition 2.

$$
\begin{aligned}
\sigma_z^2(\alpha) &> \sigma_e^2 \delta \frac{(1-\alpha)^2}{1-(1-\alpha)^2} \\
\sigma_z^2 + \alpha(2-\alpha)\delta\sigma_e^2/n &> \sigma_e^2 \delta \frac{(1-\alpha)^2}{1-(1-\alpha)^2} \\
\sigma_z^2 &> \Big[\frac{(1-\alpha)^2}{1-(1-\alpha)^2} - \frac{1-(1-\alpha)^2}{n}\Big]\sigma_e^2\delta/n \\
\frac{n\sigma_z^2}{\delta\sigma_e^2} &> \Big[\frac{(1-\alpha)^2}{1-(1-\alpha)^2} - \frac{1-(1-\alpha)^2}{n}\Big]
\end{aligned} \tag{42}
$$

Rewriting Proposition 2.

$$
\begin{aligned}
\sqrt{\frac{1}{2} + \frac{n\sigma_z^2}{\delta\sigma_e^2}} &> 1-\alpha \\
\frac{n\sigma_z^2}{\delta\sigma_e^2} &> \Big[(1-\alpha)^2 - 1/2\Big]
\end{aligned} \tag{43}
$$

Compare RHS conditions on $n\sigma_z^2/\delta\sigma_e^2$ for results 2(b) and 2(c). The condition for Proposition 2 is always larger (stricter) than result 2 because for any $\alpha \in [0,1]$ and $n \geq 2$ we have,

$$
\frac{(1-\alpha)^2}{1-(1-\alpha)^2} > 1-\alpha \text{ and } \frac{1-(1-\alpha)^2}{n} < 1/2 \tag{44}
$$

Thus any α that satisfies Proposition 2 will also satisfy Proposition 2.

Proposition 4

We can reformulate α in equation 18 as,

$$
\begin{aligned}
\alpha &= \frac{\sigma_e^2}{\sigma_e^2 + \hat{\sigma_z}^2} \\
\frac{\hat{\sigma_z}^2}{\sigma_e^2} &= (1/\alpha) - 1
\end{aligned} \tag{45}
$$

In the expression of $\hat{\sigma}_z^2(\alpha)$ we can reuse above as,

$$
\begin{aligned}
\hat{\sigma}_z^2(\alpha) &= (1-\alpha)^2 \times (\sigma_z^2(\alpha) + \delta\sigma_e^2) \\
\hat{\sigma}_z^2(\alpha) &= (1-\alpha)^2 \times \Big(\sigma_z^2 + \delta\sigma_e^2(1+\alpha(2-\alpha)/n)\Big) \\
(1/\alpha) - 1 &= (1-\alpha)^2 \times \Big(\frac{\sigma_z^2}{\sigma_e^2} + \delta + \frac{\alpha(2-\alpha)}{n}\Big) \\
(1-\alpha) \times \Big[(1-\alpha)\Big(\frac{\sigma_z^2}{\sigma_e^2} + \delta + \frac{\alpha(2-\alpha)}{n}\Big) - \frac{1}{\alpha}\Big] &= 0
\end{aligned} \tag{46}
$$

$\alpha^* = 1$ is one solution for this equation. The remaining solutions must satisfy,

$$
\begin{aligned}
\Big[(1-\alpha)\Big(\frac{\sigma_z^2}{\sigma_e^2} + \delta + \frac{\alpha(2-\alpha)}{n}\Big) - \frac{1}{\alpha}\Big] &= 0 \\
\alpha(1-\alpha)\Big(\frac{\sigma_z^2}{\sigma_e^2} + \delta + \frac{\alpha(2-\alpha)}{n}\Big) &= 1 \\
\frac{\sigma_z^2}{\sigma_e^2} + \delta + \frac{\alpha(2-\alpha)}{n} &= \frac{1}{\alpha(1-\alpha)}
\end{aligned} \tag{47}
$$

The minimum values of RHS is 4 and the maximum values of LHS is $(\sigma_z^2/\sigma_e^2) + \delta + (1/n)$. There are no other solutions α^* (except $\alpha^* = 1$) if,

$$
\begin{aligned}
\frac{\sigma_z^2}{\sigma_e^2} + \delta + \frac{1}{n} &< 4 \\
\frac{\sigma_z^2}{\sigma_e^2} &< 4 - \delta - \frac{1}{n}
\end{aligned} \tag{48}
$$

Proof for Lemma 4

We can evaluate derivative of $E[(p-v)^2]$ (from Lemma 3) with α and set greater than zero as,

$$
\begin{aligned}
\frac{\partial E[(p-v)^2]}{\partial \alpha} = \frac{\partial}{\partial \alpha}\Big(\delta \times (1-\alpha)^2\sigma_e^2 + \alpha^2\sigma_z^2\Big) &> 0 \\
-2\delta(1-\alpha)\sigma_e^2 + 2\alpha\sigma_z^2 &> 0 \\
\alpha\sigma_z^2 > \delta(1-\alpha)\sigma_e^2 \implies \alpha &> \frac{\sigma_e^2}{\sigma_e^2 + \sigma_z^2/\delta}
\end{aligned} \tag{49}
$$

We can substitute for α as,

$$
\alpha = \frac{\sigma_e^2}{\sigma_e^2 + \hat{\sigma}_z^2} > \frac{\sigma_e^2}{\sigma_e^2 + \sigma_z^2/\delta} \implies \sigma_z^2 > \delta\hat{\sigma}_z^2 \tag{50}
$$

Since $\delta < 1$ and $\sigma_z^2 > \hat{\sigma}_z^2$ from Proposition 4, we have $E[(p-v)^2]$ always increasing in α. Following a similar procedure, $E[(v_i - v)^2]$ and $Var[\pi]$ are increasing in α if $\sigma_z^2 > \hat{\sigma}_z^2$ and $\sigma_z^2 > \Omega\hat{\sigma}_z^2$. While the former is always true from Proposition 4, the latter is not guaranteed.

Proof for Proposition 7

The payoff for risk averse participant is given by,

$$
\Pi(a) = \mathbb{E}[\pi] - 0.5a \times \sqrt{Var[\pi]} \tag{51}
$$

The risk neutral payoff $\Pi(a=0)$ is always increasing in reliance α because $\mathbb{E}[\pi]$ is decreasing in σ (from Lemma 2), which in turn is decreasing in α (from Lemma 1).

The risk averse payoff $\Pi(a>0)$ is decreasing in reliance α if,

$$\begin{aligned}\frac{\partial \Pi}{\partial \alpha} = \frac{\partial}{\partial \alpha}\Big(\mathbb{E}[\pi] - 0.5a \times \sqrt{Var[\pi]}\Big) < 0 \\ \frac{\partial \mathbb{E}[\pi]}{\partial \alpha} < 0.5a\frac{\partial \sqrt{Var[\pi]}}{\partial \alpha} \\ \frac{\partial \mathbb{E}[\pi]^2}{\partial \alpha} < \Big(0.5a\frac{\mathbb{E}[\pi]}{\sqrt{Var[\pi]}}\Big)\frac{\partial Var[\pi]}{\partial \alpha}\end{aligned} \tag{52}$$

We can formulate the components as,

$$\frac{\partial \mathbb{E}[\pi]^2}{\partial \alpha} = 2\kappa^2(1-\alpha)\sigma_e^2 \quad \text{and} \quad \frac{\partial Var[\pi]}{\partial \alpha} = -2\Omega(1-\alpha)\sigma_e^2 + 2\alpha\sigma_z^2 \tag{53}$$

Here κ is a constant when cost c is linearly growing with σ. For example, $\kappa = (1/3)$ when $c = 24c/17$. We can substitute as,

$$\begin{aligned}2\kappa^2(1-\alpha)\sigma_e^2 < \Big(0.5a\frac{\mathbb{E}[\pi]}{\sqrt{Var[\pi]}}\Big) \times \Big(-2\Omega(1-\alpha)\sigma_e^2 + 2\alpha\sigma_z^2\Big) \\ \frac{2\kappa^2 \times \sqrt{Var[\pi]}}{a \times \mathbb{E}[\pi]} + \Omega < \frac{\alpha}{1-\alpha} \times \frac{\sigma_z^2}{\sigma_e^2}\end{aligned} \tag{54}$$

This is satisfied if ML error underestimation over the feedback loop is severe enough as,

$$\frac{\sigma_z^2}{\hat{\sigma}_z^2} > \frac{1}{a}\Big(\frac{2\kappa^2 \times \sqrt{Var[\pi]}}{\mathbb{E}[\pi]}\Big) + \Omega \tag{55}$$