

Algorithmic Collusion of Pricing and Advertising on E-commerce Platforms*

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Abstract

Online sellers have been adopting AI learning algorithms to automatically make product pricing and advertising decisions on e-commerce platforms. When sellers compete using such algorithms, one concern is that of tacit collusion—the algorithms learn to coordinate on higher than competitive prices which increase sellers’ profits, but hurt consumers. This concern, however, was raised primarily when sellers use algorithms to decide on prices. We empirically investigate whether these concerns are valid when sellers make pricing and advertising decisions together, i.e., two-dimensional decisions. Our empirical strategy is to analyze competition with multi-agent reinforcement learning, which we calibrate to a large-scale dataset collected from Amazon.com products.

Our first contribution is to find conditions under which learning algorithms can facilitate win-win-win outcomes that are beneficial for consumers, sellers, and even the platform, when consumers have high search costs. In these cases the algorithms learn to coordinate on prices that are lower than competitive prices. The intuition is that the algorithms learn to coordinate on lower advertising bids, which lower advertising costs, leading to lower prices for consumers and enlarging the demand on the platform.

Our second contribution is an analysis of a large-scale, high-frequency keyword-product dataset for more than 2 million products on Amazon.com. Our estimates of consumer search costs show a wide range of costs for different product keywords. We generate an algorithm usage index based on the correlation patterns in prices and find a negative interaction between the estimated consumer search costs and the algorithm usage index, providing empirical evidence of beneficial collusion. We predict that in more than 50% of the product markets, consumers benefit from tacit collusion facilitated by algorithms.

We also provide a proof that our results do not depend on the specific reinforcement learning algorithm that we analyzed. They would generalize to any learning algorithm that uses price and advertising bid exploration.

Finally, we analyze the platform’s strategic response through adjusting the ad auction reserve price or the sales commission rate. We find that reserve price adjustments will not increase profits for the platform, but commission adjustments will, while maintaining the beneficial outcomes for both sellers and consumers.

Our analyses help alleviate some worries about the potentially harmful effects of competing learning algorithms, and can help sellers, platforms and policymakers to decide on whether to adopt or regulate such algorithms.

Keywords: Artificial Intelligence, Algorithmic Collusion, Platforms, Advertising, Sponsored Product Ads, Reinforcement Learning, Q-learning, Consumer Search

1 Introduction

Reinforcement learning (RL) has been successfully applied in many fields as one of the three basic machine learning and AI paradigms¹ to maximize a reward. Classic applications include robotics

¹The other two are supervised and unsupervised learning.

(e.g., self-driving cars) (Kober et al. 2013, Pan et al. 2017, Polydoros and Nalpantidis 2017, Kiran et al. 2021), optimization of industrial processes (Li et al. 2019, Nian et al. 2020) and natural language processing (He et al. 2015, Luketina et al. 2019, Uc-Cetina et al. 2023). It is very suitable for learning in dynamic environments without much prior information, because these algorithms incorporate information about rewards as they explore the outcomes from their actions.

Recently, there have been quite successful applications of reinforcement learning in business settings within marketing including automated advertising bidding (Cai et al. 2017, Wu et al. 2018), personalized recommendations and more (Schwartz et al. 2017, Zheng et al. 2018, Bastani et al. 2022, Afsar et al. 2022, Aramayo et al. 2023, Liu 2023, Rafieian 2023, Wang et al. 2023, Cao et al. 2024). One salient example is finding the optimal price to set for a product (Kleinberg and Leighton 2003, Dubé and Misra 2017, Misra et al. 2019, Smith et al. 2023). A standard reinforcement learning approach would initially start with a given price, and explore a few other prices to not only see if they yield higher profits, but also to update what the algorithm knows about how consumers respond to prices. Over time, the algorithm will have an accurate understanding of the relationship between prices and revenues, which will allow it to find the optimal price to set. This process can be model-free and can converge without assumptions about utilities, parameters, downward-sloping demand etc.

Another salient example is finding the optimal bid to set when buying online ads—in this case the RL agent needs to learn how their bids affect their chances of winning an ad auction, but also how showing ads (perhaps to different people or for different keywords) translates to sales (Jin et al. 2018). However, the RL algorithm does not have to know how the auction operates or any other details about the economic environment in order to learn.

Most of the research on applications of reinforcement learning looked at monopoly cases of a firm optimizing its decisions. Because the theoretical properties of RL are not well-understood in competitive environments with many players such as firms, recent research used simulations to study the impact of using algorithmic decision making on competitive outcomes and their impact on consumers. One surprising outcome that emerged from this stream of research is that algorithmic pricing can lead to unintended outcomes for consumers—research by, e.g. Calvano et al. (2020), Hansen et al. (2021), Johnson et al. (2023) and Wang et al. (2023), has found that algorithms of competing firms can learn to tacitly collude on setting higher prices (even without communicating

among them), which is harmful to consumers. This issue became so widespread that the FTC even publicly stated that “Price fixing by algorithm is still price fixing.”²

In this paper, we ask whether algorithmic decision making by competing firms (of the form analyzed previously) is necessarily detrimental to consumers, or whether there are cases when consumers (as well as firms and the platform) might benefit from them. Of course, there are also many other cases where algorithmic decision making can help consumers. Well-known examples are recommendation systems and other algorithms that help match consumers and products (or consumers with other consumers) (Mullainathan and Spiess 2017, Miklós-Thal and Tucker 2019, Wang et al. 2023). However, we are interested in exactly those cases that were previously identified as harmful to consumers. One reason for asking this question is that most of the analyses of algorithmic decision making often focused on one-dimensional learning (such as learning to price, or to advertise), but in reality, firms often need to make multi-dimensional decisions. Another reason we focus on this question is that for online platforms (but also for other settings such as grocery chains and media markets), their revenue streams combine sales commissions (or another margin on sales), as well as ad revenue, and as a result the platform has some flexibility of which stream to emphasize more, which in turn affects what algorithms learn.

The analysis of competing algorithms is not straightforward, since there are many types of algorithms, and many different assumptions that one can make. To make our results comparable to previous research, we analyze a specific (yet common) reinforcement learning algorithm called Q-learning (we describe it in Section 4), the goal of which is to learn how different actions (such as prices, or bids on ads) translate to outcomes (such as sales) in different states (such as the prices of competitors, and the firm’s previous prices). Although we focus on a specific implementation of a specific algorithm, we prove that our findings are likely to generalize to other types of algorithms, if they have some level of price exploration built into them.

Our analysis combines three approaches—first, we use an analytical model to analyze a benchmark case of pure competition (without algorithmic pricing and bidding) to show the effect of ad bids and search costs. Second, we use an extensive empirical simulation to analyze algorithmic pricing and bidding to show that prices (and bids) could be lower when algorithms learn to coordi-

²Recently, the Federal Trade Commission and Department of Justice are taking different actions to fight algorithmic collusion. See detail at <https://www.ftc.gov/business-guidance/blog/2024/03/price-fixing-algorithm-still-price-fixing>.

nate in a setting with high consumer search cost. Third, we estimate search costs and algorithmic pricing adoption using a self-collected large-scale dataset from Amazon.com and provide empirical evidence that prices are indeed lower in markets with higher consumer search cost and higher algorithm usage.

In Section 3, we introduce our empirical setting—an online platform (such as Amazon.com) where consumers come to search for products to buy, and sellers come to sell their products. The platform displays ordered links to consumers, with the prominent sponsored links sold through an ad auction, while the other (organic) links are ranked based on product features (price, rating and other features). This setting is quite common in e-commerce and includes large companies such as eBay and Expedia. The sellers on the platform set prices for their products and also decide how much to bid for displaying their product in the sponsored position. When consumers search for a keyword, they see an ordered list of links, and traverse it until they find a product to buy or stop searching. We assume that the purchase decision follows a standard choice model, with consumer search costs determining the size of their consideration set. Other research has shown that this model is also consistent with optimal sequential search and other forms of search (Weitzman 1979, Ursu 2018, Lam 2021). An important assumption we make is that consumers have heterogeneous search costs (or impatience). This assumption is quite realistic in our opinion and has not been analyzed previously in the context of algorithmic pricing. We also empirically confirm that search costs are indeed high and heterogeneous in the markets we analyze, lending credibility to this assumption.

In Section 4, we compare the competition of two agents using reinforcement learning to pure competition. In order to make this comparison, in Section 3.3 we first show that in the case of competition without algorithmic decision making, search costs increase prices in equilibrium when setting prices and bids (Armstrong and Zhou 2011). The reason is that as search costs increase, the sponsored positions become more valuable and the competition for them increases. This in turn increases equilibrium bids on ads, which increases the cost that sellers incur when selling their products. Because costs are higher, prices end up also being higher.

The first interesting finding we make is in Section 4, where we analyze a multi-agent reinforcement learning setting. In the analysis, we have competing sellers who do not have prior information about the economic environment (what the demand function is, what the demand parameters or

utility functions are, or how ad auctions behave). In fact, the Q-learning agents do not even have a model—they have a mapping from their actions (prices and bids) to profits. The agents explore different prices and bids, and over time learn how their actions affect profits. When we let these algorithms run long enough, they converge to an equilibrium where they mostly use the same price/bid combination, and new information they observe does not affect their learning anymore. If we were to make predictions based on findings of previous research (Calvano et al. 2020, Johnson et al. 2023), we might expect the agents to reach an equilibrium with higher than competitive prices, because these algorithms learn that higher prices are better for both sellers. However, we find the opposite—our analysis shows that when search costs increase and consumers become more impatient and less willing to search more links, then the algorithms converge on prices lower than competitive prices. In other words, Q-learning can result in tacit collusion, but such collusion helps consumers in this case.

The intuition is interesting, and comes from the role of advertising. If the algorithms could exchange information and collude on the best outcome for them, they would have decided to evenly split the market, and bid zero on the ads. In other words, because the ads are a cost for the sellers, it is best for them to agree to lower the costs. This is the opposite case of prices, where it is beneficial for both sellers to increase prices. It turns out that when search costs are high, the benefit from colluding on lower bids outweighs the benefit from colluding on higher prices, and the equilibrium outcome is better for consumers. Unlike the case of competing over prices only, when using algorithmic pricing and bidding, lower prices result in higher seller profit, because demand increases, as well as advertising costs decrease. Tacit collusion by algorithms can also benefit the platform (Section 5) when search costs are high, because the increase in sales generates more commissions that dominate the decrease in revenue from advertising.

A concern with these results, however, is that they depend on assuming that many consumers have high search costs. To validate this assumption, in Section 6, we analyze data from more than 2,000 product search keywords on Amazon.com, yielding more than 1 million observations per day for 2 million products. We estimate the consumer search costs using the variation in observed product rankings and their sales. We use moment conditions (Sweeting 2013, Grennan 2013, Lam 2021, Yu 2024) to address the simultaneity issue between the ranking of a product on a page and

its sales.³ We find that search costs can indeed be high for many products in the data we analyze. For most keywords, most consumers will not search beyond half the search results page, and in some cases, even less.

In our second major analysis, we generate an index for algorithm usage for each product-market using the correlation patterns in prices set by sellers over time (Chen et al. 2016). We then examine the interaction between the estimated consumer search costs and the algorithm usage index, and find that the interaction effect on prices is negative. This provides empirical evidence that prices are lower in markets with higher algorithmic pricing adoption and higher search costs. When we compare the estimated consumer search costs with the critical value of consumer search costs needed for the algorithms to benefit consumers, we find that 52% of Electronics markets have consumer search costs higher than this critical value. This analysis provides additional evidence that consumers can benefit from tacit collusion facilitated by algorithms.

Finally, we also examine the possible response of the platform to prices being set by such algorithms. We analyze the platform's optimal commission rate and advertising reserve price. We find that adjusting the reserve prices might not be an effective tool for the platform, because increasing the reserve price above the Q-learning equilibrium level of bids will shift the algorithms toward coordinating on even lower bids, further reducing the advertising revenue for the platform. However, adjusting the commission rate might be effective. By increasing their commission rates, the platforms can recoup some of the lost advertising revenue due to lower bids through increased commission revenue.

To summarize, by extending the analysis to include two-dimensions (pricing and bidding) and by looking at consumers with heterogeneous search costs in a digital platform setting, we uncovered some counter-intuitive new findings and empirical evidence to support them. Unlike much of the past research, we find that consumers might benefit from tacit collusion by algorithms, as well as the platform. These findings can be useful for sellers who are concerned about adopting algorithmic pricing with multi-dimensional decision-making. In monopoly settings we would intuitively expect more advanced or flexible algorithms to improve profits. However, it is not clear ex-ante if these benefits will continue to exist in a competitive environment, and our findings can help sellers make this determination.

³If a product is ranked higher it will likely have more sales, but in turn, having more sales might rank it higher.

Our findings can also help platforms decide if they should encourage sellers to use algorithmic pricing and bidding, and even whether they should offer these algorithms themselves. The platforms have information about consumer search costs, and are best situated to benefit from our findings. When the platform needs to respond strategically, our findings provide guidance about the possible methods to consider (e.g., adjusting commission rates, adjusting reserve prices in the ad auction, or disclosing information about the winning bids). Our findings can also help the platform address regulatory concerns from policymakers, given that consumers on the platform can also benefit.

Finally, our results can provide good news for consumers because lower prices by algorithms increase consumer welfare. From a regulatory perspective, our finding that algorithmic collusion does not always harm consumers is in contrast to prevalent beliefs. This area of discussion is currently of high relevance for modern digital platforms, making our findings extremely relevant.

2 Literature Review

Our paper builds upon research in search advertising and sponsored ads (Edelman et al. 2007, Varian 2007, Katona and Sarvary 2010, Berman and Katona 2013, Sayedi et al. 2018, Choi and Mela 2019, Simonov et al. 2018, Sahni and Nair 2020, Long et al. 2022, Dai et al. 2023). These works typically assume that the value of winning the auction is exogenously given. Athey and Ellison (2011) introduced consumer search, which endogenizes the advertisers' valuation. Chen and He (2011), Kang (2021) incorporated the pricing decision into the sponsored ads auction. Our work is most closely related to Armstrong and Zhou (2011).

This study also connects with literature on algorithmic pricing and artificial intelligence. Calvano et al. (2020) demonstrate that in an oligopoly setting, Q-learning algorithms lead to prices above competitive levels. Johnson et al. (2023) investigate the ability of a platform to steer demand towards lower-priced sellers and find that algorithms result in supra-competitive prices in the absence of non-neutral platform intervention. The phenomenon of algorithmic collusion is also documented in scenarios where algorithms make sequential decisions (Klein 2021), mis-specified multi-armed bandit algorithms are used (Hansen et al. 2021), and Q-learning algorithms compete with simple heuristic algorithms (Wang et al. 2023). On the contrary, despite concerns expressed by policy makers, Miklós-Thal and Tucker (2019) find that better forecasting and algorithms can

lead to lower prices and higher consumer surplus.

To uncover the mechanism for algorithmic collusion, Asker et al. (2022) study the effect of algorithm design on collusion. Similarly, Banchio and Skrzypacz (2022) find that synchronous algorithms are less likely to converge on collusive outcomes. Banchio and Mantegazza (2022) reveal that spontaneous coupling can sustain collusion in prices and market shares.

Regarding empirical work on algorithmic pricing, Musolff (2022) documents that repricing algorithms follow Edgeworth cycles to effectively coax competitors into raising their prices, which decreases competition. Brown and MacKay (2021) find that algorithms allow for more frequent price changes, generate price dispersion, and increase price levels. Assad et al. (2024) find that algorithmic-pricing software significantly increased the margin in Germany's retail gasoline market. Calder-Wang and Kim (2023) find that algorithms set more responsive prices, leading to higher rents and lower occupancy in the U.S. multifamily rental housing market.

The seminal work on Q-learning by Watkins (1989) and Watkins and Dayan (1992) pioneered a large literature on reinforcement learning, which has been widely applied in economics research (Erev and Roth 1998, Erev et al. 1999, Waltman and Kaymak 2008). See Zhang et al. (2021) for a recent review of reinforcement learning.

This study also relates to platform rankings of products and how that influences sellers' profits and consumer surplus (Reimers and Waldfogel 2023, Lam 2021, Lee and Musolff 2021, Farronato et al. 2023). Finally, this research also relates to the seminal branch of consumer search literature (Weitzman 1979, Ursu 2018, Honka and Chintagunta 2017, Morozov et al. 2021).

3 Institutional Background and Model Setup

To motivate our empirical model, we first provide background about the settings we consider, where sellers offer their products for sale on an e-commerce platform. We then translate the institutional details into a mathematical model that we can use in analysis and also empirically fit to data.

3.1 Institutional Background

Amazon.com’s Marketplace, which the world’s largest retail digital platform, is a primary example of the setting we consider.⁴ Amazon allows third-party sellers to list and sell their products on its website, and also functions as a retailer who sells products to consumers directly. In our main analysis, if Amazon directly sells a product, we assume that consumers will treat it as just another seller. We will discuss self-preferencing (where Amazon might promote products it sells directly more) in the Conclusion.

When shopping on Amazon.com, consumers usually start a search for products by typing a product keyword into the search box. Amazon then displays a search results page relevant to the shopping query, with both organic and sponsored product listings. The default ranking of listings is “Featured” which is also the ranking we focus on in this paper. In this case, the platform’s recommendation algorithm ranks the organic products based on factors like sales performance, customer ratings, and conversion histories associated with specific keywords. Consumers can also choose other options for sorting the products.⁵ Consumers consider products sequentially following their ranking on the platform. They have heterogeneous search costs and might stop their search without considering every product on the search page. Consumers who have high search costs, or are impatient, would generally visit only the top links (which are mostly sponsored) and only consider buying the products that appear there. By contrast, consumers with low search costs (or frictions) who are patient, will consider also organic products which appear lower on the list.

Within the search results page, there are several advertising formats, which include “Sponsored Product Ads” in the “search result” section, as well as “Sponsored Display” and “Sponsored Brands,” which usually appear in a carousel in different locations on the page. We focus on Sponsored Product Ads because the sponsored products account for approximately 78% of the total ad spend among sellers on Amazon.com.⁶ and appear in the main section within the search result page.

Sponsored products are distinguished from organic results by being labeled as “Sponsored.” A

⁴In 2022, Amazon reported nearly \$514 billion in net sales revenue worldwide, and held approximately 40% of the e-commerce market share. Source: Statista, <https://www.statista.com/topics/846/amazon/#topicOverview>

⁵The options include “Price: Low to High”, “Price: High to Low”, “Avg. Customer Review”, “Newest Arrivals”, and “Best Sellers”.

⁶Statistic from *Jungle Scout*, a prominent e-commerce intelligence data provider. See <https://www.junglescout.com/blog/amazon-sponsored-product-ads/>, Accessed June 8, 2024.

product can appear in both sponsored and organic positions. Within a search results page, there are 60 products laid out in 5 columns and 12 rows for most product categories, and 22 products in the ‘Electronic’ category displayed in a vertical layout of 1 column. Figure 1 and Figure A2 present examples of a search results page with 60 products and 22 products, respectively.

Figure 1: Example of First Two Rows of an Amazon Search Result Page with 60 Products

Results
Check each product page for other buying options. Price and other details may vary based on product size and color.

 <p>Sponsored</p> <p>LEVOIT Humidifiers for Bedroom, Quiet (3L Water Tank) Cool Mist Top Fill Essential Oil Diffuser with 25Watt for Home Large Room, 360° Nozzle, 4399</p> <p>4.5★ 76,231 6K+ bought in past month</p> <p>\$43⁹⁹</p> <p>✓prime FREE delivery Sat, Apr 20 Or fastest delivery Tomorrow, Apr 16</p> <p>Add to cart</p>	 <p>sponsored</p> <p>Humidifiers for Bedroom Smart Control - Dual Mist Cool & Warm Mist Humidifier 4L Big Capacity Top Fill Water Tank, Quie Humidifier, 12,534 300+ bought in past month</p> <p>\$78⁹⁹ List Price \$99.99</p> <p>✓prime FREE delivery Sat, Apr 20 Or fastest delivery Thu, Apr 18</p> <p>See options</p>	 <p>Sponsored</p> <p>LEVOIT Humidifiers for Bedroom Home, Smart Warm and Cool Mist Air Humidifier for Large Room, Auto Customized Humidity, Fast Sympto... 10,610 3K+ bought in past month</p> <p>\$89⁹⁹</p> <p>Save \$10.00 with coupon ✓prime FREE delivery Sat, Apr 20 Or fastest delivery Thu, Apr 18 Works with Alexa</p> <p>Add to cart</p>	 <p>Sponsored</p> <p>LEVOIT Humidifiers for Bedroom Large Room Home, (6L) Cool Mist Top Fill Essential Oil Diffuser for Baby & Plants, Smart App & Voice Contro... 26,947 3K+ bought in past month</p> <p>\$69⁹⁸ List: \$79.99</p> <p>✓prime FREE delivery Sat, Apr 20 Or fastest delivery Wed, Apr 17 Works with Alexa</p> <p>Add to cart</p>	 <p>Best Seller</p> <p>LEVOIT Top Fill Humidifiers for Bedroom, 2.5L Tank for Large Room, Easy to Fill & Clean, 28dB Quiet Cool Mist Air Humidifier for Home Baby,... 76,231 30K+ bought in past month</p> <p>\$29⁹⁸ List: \$39.99</p> <p>✓prime FREE delivery Sat, Apr 20 on \$35 of items shipped by Amazon Or fastest delivery Thu, Apr 18</p> <p>Add to cart</p> <p>More Buying Choices \$16.36 (12 used & new offers)</p>
 <p>AquaOasis® Cool Mist Humidifier (2.2L Water Tank) Quiet Ultrasonic Humidifiers for Bedroom & Large room - Adjustable -360 Rotation... 102,300 10K+ bought in past month</p> <p>\$29⁹⁷ List: \$49.99</p> <p>✓prime FREE delivery Sat, Apr 20 on \$35 of items shipped by Amazon Or fastest delivery Thu, Apr 18 Small Business</p> <p>Add to cart</p> <p>More Buying Choices \$26.97 (13 used & new offers)</p>	 <p>Humidifiers for Bedroom, Cool Mist Humidifier for Home Plant and Baby Nursery, Quiet Ultrasonic Humidifier with 360° Nozzle, Auto Shut-Off,... 5,724 10K+ bought in past month</p> <p>\$19⁹⁹ List: \$22.99</p> <p>✓prime FREE delivery Sat, Apr 20 on \$35 of items shipped by Amazon Or fastest delivery Thu, Apr 18 Small Business</p> <p>Add to cart</p> <p>More Buying Choices \$15.79 (4 used & new offers)</p>	 <p>Cool Mist Humidifiers for Bedroom - 2.2L Water Tank, Baby, Office, Quiet Ultrasonic Air Vaporizer, Adjustable Mist Level, 360 Nozzle Rotation,... 3,171 6K+ bought in past month</p> <p>\$16⁹⁹ Typical: \$19.99</p> <p>✓prime FREE delivery Sat, Apr 20 on \$35 of items shipped by Amazon Or fastest delivery Thu, Apr 18 Small Business</p> <p>Add to cart</p> <p>More Buying Choices \$15.79 (4 used & new offers)</p>	 <p>Dreo Humidifiers for Bedroom, Top Fill 4L Supersized Cool Mist Humidifier with Oil Diffuser and Nightlight, 32H Runtime, Quiet... 2,781 10K+ bought in past month</p> <p>\$39⁹⁹</p> <p>✓prime FREE delivery Sat, Apr 20 Or fastest delivery Thu, Apr 18 Small Business</p> <p>Add to cart</p> <p>More Buying Choices \$24.18 (7 used & new offers)</p>	 <p>6L Humidifiers for Bedroom Large Room Home, Cool and Warm for Baby and Plants Mist Top Fill Desk Humidifiers Essential Oil Diffuser,... 6,261 10K+ bought in past month</p> <p>\$39⁹⁹ Save 10% on 2 select item(s)</p> <p>✓prime FREE delivery Sat, Apr 20 Or fastest delivery Tomorrow, Apr 16</p> <p>Add to cart</p> <p>More Buying Choices \$36.28 (8 used & new offers)</p>

This figure presents an example of the first two rows of a search results page with 12 rows (60 products in total) on Amazon.com when an anonymous consumer enters the keyword “humidifiers for bedroom.” The products whose labels are encircled in orange rectangles are sponsored products, obtained by sellers who won these positions through bidding in ad auctions.

Amazon uses real-time auctions to decide which ads to show and their order of display for

specific product searches. According to Amazon,⁷ ads are ranked and displayed to consumers based on a combination of the advertisers' bids and the ads' relevance. Because consumers face search costs and have a higher probability of considering products in earlier positions, sellers are motivated to bid for sponsored product ad positions to capture consumers' limited attention. When buying Sponsored Product Ads to advertise their products, sellers typically choose a set of relevant keywords and specify the bid amounts for each keyword—the maximum they are willing to pay for consumer clicks on their sponsored product listing. The payment model for sponsored positions is called “Cost-per-click” (CPC), meaning that advertisers pay only when consumers click on their ads, and not for impressions (which is a display of the ad) or conversions (final sales of the product).⁸

Sellers also pay the platform a commission fee for each unit sold, which is a fixed percentage of the total sale amount.⁹ In addition to commissions, Amazon also collects other fees from sellers, such as shipping and return administration fees. We do not consider these other fees separately in our analysis, but rather include them in the manufacturing (marginal) costs, as they do not alter search result rankings or consumer search.

The sellers on the platform want to maximize their profits. The decisions they face include setting prices for their products and deciding if, and how much, they would like to bid for those sponsored positions. Sellers face two trade-offs with regard to prices. They want to avoid strong price competition with other sellers and prefer higher equilibrium prices. They also want the prices not to be too high, because otherwise consumers might choose to buy from another location (an outside option good). Regarding bids, the more sellers bid, the higher their chances of winning the sponsored link and reaching consumers who only consider the top positions. However, these bids incur advertising costs for sellers, which would lower their profits. Thus, they will want to avoid competing too aggressively on bids.

The platform's revenues come from two sources: commissions from sales and bids from ad auctions. Therefore, it has two incentive-based tools that it can use to influence revenues: adjusting the commission rate, which impacts commission revenue, and adjusting the reserve price, which

⁷Amazon Ads, How does bidding work for Sponsored Products? <https://advertising.amazon.com/library/videos/campaign-bidding-sponsored-products>

⁸Amazon uses generalized second-price auctions, where for each consumer click on an ad in the r -th sponsored position, the seller pays Amazon an amount equal to the “realized” bid of the $(r + 1)$ -th highest bidder, which is a relevance adjusted bid.

⁹Based on Amazon disclosures, commission rates differ among various product categories, typically being 15% for most categories. See <https://sell.amazon.com/pricing#referral-fees>, accessed June 8, 2024.

affects ad auction income. There are also information-based strategies to affect seller behavior, for example, the platform can disclose more or less information about the winning bids in the ad auctions which will influence the sellers' bids. In terms of objective, the platform might only care about its own profits, but it might also consider consumer surplus and sellers' profits on the platform to encourage more entry and improve long-term business growth.

When sellers use algorithms which are designed to optimize profit to guide their pricing and bidding decisions, it is unclear whether prices and bids will be high or low in equilibrium. On the one hand, sellers would like to price higher to increase profits, which increase the valuation of winning the sponsored positions leading to higher bids. On the other hand, they would like to bid lower to reduce their costs, leading to lower prices. These are two opposite forces, and a-priori it isn't obvious which one will prevail and when when they interact.

To explore the impact of competitive algorithmic decision making on prices and bids, we model an e-commerce environment in the next section and then analyze the impact of learning algorithms. We use this model to make predictions about equilibrium prices and bid, and also estimate a version of this model on large-scale data about product searches and sales from Amazon.com.

3.2 Model Setup

Our model has three key players: $n \geq 2$ **sellers**, each of which sells one differentiated product on a retail **platform**, and a unit mass of **consumers** who choose between purchasing from either one of these sellers on the platform, or an outside good.

Platform The platform's role is to display products and to determine their rankings on the search results page. For sponsored positions, rankings are based on ad auction outcomes; for organic positions, the platform determines the relative rankings of different products using a recommendation algorithm. In our model, there are multiple sponsored and organic positions. To illustrate the main effects of the model parameters, we first consider a simplified case where two sellers compete and there is one sponsored position at the top of the page followed by two organic listings. The sponsored listing is the first product consumers see and is awarded to the winning bidder in the auction. Figure A1 provides a graphical representation of the search result page layout for the simplified model.

We assume that there is a one keyword from which all sales are derived. All sellers bid to display an ad on the results page for this keyword and compete for the top (sponsored) position. Each seller has a constant variable production cost $c_i > 0$, and pays the platform a commission rate $\tau \in (0, 1)$ of the sales each time they sell a unit of the product (both sponsored and organic). Sellers interact repeatedly over time. In each period $t = 0, 1, \dots$, every seller i not only decides the bid amount to submit in the auction but also sets the price for its products. We assume sellers set a price $p_i^t \geq 0$ and a bid $b_i^t \geq 0$ simultaneously (negative prices and bids are not allowed).

For the sponsored position, the platform runs an ad auction and shows the seller with the highest *realized bid* \tilde{b}_i^t , when the bids submitted by sellers are b_i^t . To approximate the influence of the ads' relevance score as well as the uncertainty sellers face when deciding their bids, we assume that the realized bid \tilde{b}_i^t follows a log-normal distribution: $\log(\tilde{b}_i^t) \sim \mathcal{N}(\log(b_i^t), \sigma_i)$, where σ_i is the uncertainty in bid realization. We use a first-price auction in our model. While this differs from the generalized second price auction that Amazon.com uses, we do not expect this difference to be crucial for the main insights from our analysis, and we mostly make it for tractability and to avoid scenarios with multiple equilibria.¹⁰

Consumers In our model, a period t is a search by a unit mass of consumers, who wish to buy at most one product. Consumers spend one period in the market and then exit and are replaced by a new cohort. A representative consumer who buys product i in period t obtains utility

$$u_i^t = a_i - p_i^t + \epsilon_i$$

Where a_i capture vertical differentiation and p_i^t is the product's price. The outside good is indexed by 0, with utility $u_0^t = \epsilon_0$. We assume that ϵ_0 and each ϵ_i are type-I extreme value independent random variables with common scale parameter $\mu > 0$. The parameter μ is also an index of horizontal differentiation, and the case of substitutes is obtained in the limit when $\mu \rightarrow 0$. The

¹⁰Generalized second-price auctions and first-price auctions have similar theoretical strategic incentives and algorithm performance. Both are non-truthful, and algorithms behave similarly and learn to collude on lower bids Rovigatti et al. (2023), Banchio and Skrzypacz (2022), unlike in non-generalized second-price auctions Banchio and Skrzypacz (2022). Therefore, the auction format should not significantly change the results of our analyses. These model assumptions also allow for mathematical tractability. Previous research has only proved the existence of equilibrium in sponsored ads with pricing competition Kang (2021). There might be multiple or asymmetric equilibria in generalized second-price auctions. Additionally, our assumptions are similar to those imposed in Yu (2024).

resulting demand for product i in period t , assuming it is in the consumer consideration set \mathcal{N}^t (which we discuss in detail below), follows a logit demand model and is expressed as follows:

$$s_i(\mathbf{p}^t) = \frac{\exp\left(\frac{a_i - p_i^t}{\mu}\right)}{\sum_{j \in \mathcal{N}^t} \exp\left(\frac{a_j - p_j^t}{\mu}\right) + 1} \quad (1)$$

where \mathbf{p}^t is the vector of prices of products in the consumer's consideration set.

Consumers search for products to consider and purchase according to the order of products the platform displays to them. Consumers can have heterogeneous search costs, which might lead them to stop their search process in the middle of the search results page, thus considering only a subset of all products on the page. For the simplified model with one sponsored position, following Armstrong and Zhou (2011), we assume that there are two segments of consumers. A fraction θ of consumers has high search costs and considers only the product in the first position. The remaining $1 - \theta$ fraction of consumers considers products in all positions. For example, if products i and j are presented in order $\{j, i\}$ on the search results page, then the high search cost segment considers only j , and the other segment has a consideration set of $\{j, i\}$. If a product appears in both the sponsored and an organic position, we assume that consumers are sophisticated enough to recognize it as the same product and consider it as one in their consideration set, and will use the organic link. The search process represented by the two segments can be micro-founded by consumers who perform sequential search and decide to stop when the expected value from continuing is below their search cost, with a heterogeneous distribution of search costs. Appendix A.4 provides details about this micro-foundation.

Let $s_i(p_i^t)$ denote the market share of product i among consumers who only consider the top position when product i appears in the top position, and $s_i(p_i^t, p_j^t)$ denote the market share of product i among consumers considering both sponsored and organic positions. Taking the outside option into account, the market shares of product i are:

$$s_i(p_i^t) = \frac{\exp\left(\frac{a_i - p_i^t}{\mu}\right)}{\exp\left(\frac{a_i - p_i^t}{\mu}\right) + 1} \quad s_i(p_i^t, p_j^t) = \frac{\exp\left(\frac{a_i - p_i^t}{\mu}\right)}{\exp\left(\frac{a_i - p_i^t}{\mu}\right) + \exp\left(\frac{a_j - p_j^t}{\mu}\right) + 1}$$

Sellers In the simplified model, we assume that the two sellers are ex-ante symmetric in their quality and production costs ($a_i^t = a_j^t$ and $c_i^t = c_j^t$). Hence, the platform will treat the sellers identically for organic rankings and randomize the order of display of the two sellers in the two organic positions, thereby neutralizing potential ranking effects on sales. We consider the case of asymmetric sellers later in Section A.7.2. Our empirical application allows for multiple sellers who can be differentiated and asymmetric. The sellers have a common discount factor $\delta \in (0, 1)$, and maximize the cumulative discounted profit:

$$\sum_{t=0}^{\infty} \delta^t \pi_i^t(\mathbf{p}^t, \mathbf{b}^t) \quad (2)$$

The one-period profit for seller i in period t , denoted as $\pi_i^t(\mathbf{p}^t, \mathbf{b}^t)$, is given by:

$$\begin{aligned} \pi_i^t(\mathbf{p}^t, \mathbf{b}^t) = & \theta \cdot \Pr(\tilde{b}_i^t > \tilde{b}_j^t) \cdot s_i(p_i^t) \cdot \left(((1 - \tau) \cdot p_i^t - c_i) - \gamma_i \cdot \mathbb{E} [\tilde{b}_i^t | \tilde{b}_i^t > \tilde{b}_j^t] \right) \\ & + (1 - \theta) \cdot s_i(p_i^t, p_j^t) \cdot ((1 - \tau) \cdot p_i^t - c_i) \end{aligned} \quad (3)$$

where \mathbf{b}^t is the vector of bids submitted by all sellers in period t . The first term represents the profit from the sponsored position and is affected by the share of impatient consumers θ , the chance of winning the ad auction $\Pr(\tilde{b}_i^t > \tilde{b}_j^t)$, the market share of the product (vs. the outside good) $s_i(p_i^t)$ and the profit per sale, which depends on the profit margin and cost of ads. The inverse conversion rate, γ_i , indicates the number of clicks needed for a sale and is assumed common knowledge. The commission rate charged by the platform is denoted as τ . Appendix A.1 derives the expressions for the chance of winning the ad auction $\Pr(\tilde{b}_i^t > \tilde{b}_j^t)$ and the expected ad cost conditional on winning the auction the the competitor's bid $\mathbb{E} [\tilde{b}_i^t | \tilde{b}_i^t > \tilde{b}_j^t]$. The second term is the profit from consumers who consider more than just the sponsored product.

3.3 Benchmark Theoretical Results

To understand how learning algorithms interact, and the role of search costs in affecting equilibrium prices, bids, and sellers' profit, we first analyze a full competition version of the model where the sellers have complete information and do not need to learn about their environment. We focus on this setting , because it will form a good benchmark to compare to the algorithmic case in repeated

games.

We consider the following two scenarios:

1. *Full competition of pricing and bidding*: The two sellers have complete information about model parameters and consumer behavior, and compete on both prices and bids which are set simultaneously. As there is no closed-form solution because of the demand structure, we find the Nash-Bertrand equilibrium numerically. We denote the equilibrium price and bid as \mathbf{p}^N and \mathbf{b}^N , respectively.
2. *Pricing only benchmark*: To compare also to previous work, which considered pricing without advertising, we also analyze sellers who compete only on prices. In this scenario, consumer preferences remain the same, with θ representing the fraction of consumers who focus only on the first position. Each seller's probability of being displayed in the first position is $\frac{1}{2}$ without an auction. We denote the equilibrium price as \mathbf{p}^{oN} ,¹¹ and each seller solves:

$$\max_{p_i} \pi_i(\mathbf{p}) = \left(\theta \cdot \frac{1}{2} \cdot s_i(p_i) + (1 - \theta) \cdot s_i(p_i, p_j) \right) \cdot ((1 - \tau) \cdot p_i - c_i) \quad (4)$$

We present the equilibrium prices in Figure 2a and the bids in Figure 2b. We observe non-zero equilibrium bids only in the Nash-Bertrand competition.¹²

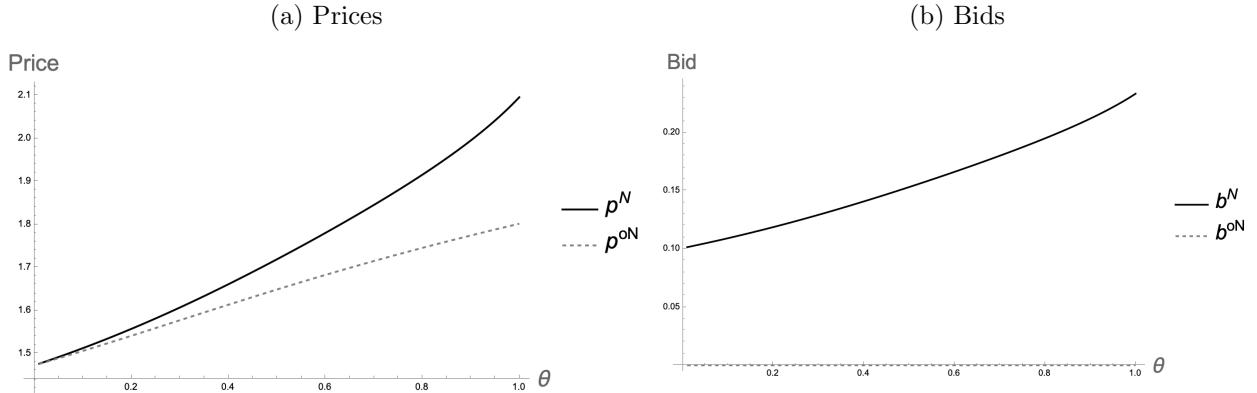
The benchmark equilibrium price (without ads) in the dashed line is lower than the Nash-Bertrand price in the solid line, because ads increase the cost and drive up the price. However, when $\theta = 0$ and there are no costly ads, the prices coincide. Importantly, both the benchmark price \mathbf{p}^{oN} and the Nash-Bertrand price \mathbf{p}^N increase with θ (the fraction of consumers who only consider the first position). This is in line with the intuition that a larger monopoly market leads to higher prices charged by the sellers. A higher θ implies a larger fraction of consumers who focus only on the first position, in which case the seller only competes with the outside option, which increases their effective monopoly power.

The result that ads increase prices over the no-ads benchmark, and that higher search costs

¹¹Here, N stands for the Nash-Bertrand equilibrium, and o indicates our baseline scenario characterized solely by price competition.

¹²When $\theta = 0$, every consumer considers both products, and the situation reduces to the case considered in Calvano et al. (2020).

Figure 2: Equilibrium Prices and Bids in One-Shot Game



The subfigure (a) and (b) show the equilibrium prices and bids as a function of θ under different scenarios, respectively. The dashed gray line, and black solid line represent the benchmark equilibrium price p^{oN} (or bid b^{oN}), and the Nash-Bertrand equilibrium price p^N (or bid b^N), respectively. For this example, we set the parameters as $a_i = a_j = 2$, $c_i = c_j = 1$, and $\mu = \frac{1}{4}$ for comparison with the results in Calvano et al. (2020). For ads-related parameters, we set $\sigma_i = \sigma_j = 0.5$ and $\gamma_i = \gamma_j = 2$.

θ increase these prices even more remains robust across different model specifications.¹³ It is consistent also with Armstrong and Zhou (2011) who in a model without an outside option and where bidders play an all-pay auction show that increased bids to win sponsored positions increase the sellers' advertising expenses and, consequently, their total costs.

With respect to equilibrium bids, b^N , in Figure 2(b) we observe that they increase with θ . A larger θ implies greater potential profits from winning the sponsored position, thereby incentivizing sellers to bid more aggressively.

4 Algorithmic Pricing and Bidding: A Multi-Agent Reinforcement Learning Approach

We now turn to explore sellers who use reinforcement learning algorithms (specifically, tabular Q-learning) to make their pricing and bidding decisions repeatedly. We use a computer-simulated environment to analyze the outcomes from a Multi-Agent setting where algorithms interact with each other. We compare the outcomes in this scenario with the cases analyzed previously in Section 3.3. First, we introduce the notation of Q-learning in a stationary single-player environment. Then,

¹³ Appendix A.3 provides more details.

we describe how we extend the single agent setting to a Multi-Agent setting, and apply it to our pricing and bidding competition with consumer search, where algorithms interact with other algorithms.

Basic Notation of Q-Learning Consider a single algorithm facing an unknown stationary Markov Decision Environment with a finite set of states $s_t \in \mathcal{S}$, a finite set of actions $a_t \in \mathcal{A}$, where $t = 1, 2, \dots$ denotes the time periods. The objective of the decision-maker is to maximize the expected present value of the stream of rewards $\pi(s_t, a_t)$. Let $a^*(s)$ denote an optimal policy. Thus, the policy $a^*(s)$ maximizes the sum of future expected discounted profits, expressed as $\mathbb{E} [\sum_{t=0}^{\infty} \delta^t \pi(s_t, a^*(s_t))]$.

Q-learning involves iteratively estimating the “action-value function” $Q^*(s, a)$ where $Q^*(s, a)$ gives the expected discounted payoffs of taking action a at state s today and then using the optimal policy function $a^*(s)$ in all future periods. Thus,

$$Q^*(s, a) = \mathbb{E} [\pi(s, a)] + \delta \mathbb{E} [V(s' | s, a)]$$

where $V(s' | s, a)$ is the future optimal value.

The optimal policy $a^*(s)$ is $a^*(s) = \arg \max_{a \in \mathcal{A}} Q^*(s, a)$.

Learning and Experimentation Since $Q^*(s, a)$ is unknown, it is estimated as follows: Starting with an initial matrix Q_0 , at time t and in state s , the algorithm determines the action to take. With a probability of $1 - \epsilon_t$, the algorithm operates in *exploitation mode*, choosing the optimal action according to the current Q-matrix. With a probability of ϵ_t , it enters *exploration mode*, uniformly randomizing over all available actions. After choosing the action a_t in s_t , the realized payoff π_t is observed, as is the new state s' . The one element of the Q-matrix corresponding to (s, a) is then updated to be

$$Q_{t+1}(s, a) = (1 - \alpha)Q_t(s, a) + \alpha \left[\pi_t(s, a) + \delta \max_{\tilde{a} \in \mathcal{A}} Q_t(s', \tilde{a}) \right]$$

where $Q_t(s, a)$ is the previous “un-updated” element of the Q-matrix, and $\alpha \in (0, 1)$ is the learning rate parameter, which captures the extent to which old information is replaced by new information.

The probability of experimentation ϵ_t is given by

$$\epsilon_t = e^{-\beta t},$$

where $\beta > 0$ is the experimentation parameter. This means that initially the algorithms choose in purely random fashion,¹⁴ but as time passes, they make the greedy choice more and more frequently.

Q-learning has theoretical convergence guarantees only in stationary single-player settings. In our multi-agent setting, convergence is not guaranteed because each agent continually changes its strategy by updating its Q-matrix, rendering the environment non-stationary from other agents' perspectives. Despite this, in our empirical simulations we nearly always observe convergence.

Extending to the E-Commerce Multi-Agent Setting We incorporate sponsored ads and consumer search into a framework similar to the one used in Calvano et al. (2020) and Johnson et al. (2023). We consider a symmetric duopoly ($n = 2$) with discount factor $\delta = 0.95$. Each agent has a marginal cost $c = 1$. Demand is given by Equation (1) with each firm having the same quality component $a_i = a_j = 2$ and scale parameter $\mu = \frac{1}{4}$. We normalize the quality component of the outside option to be $a_0 = 0$.

We implement Multi-Agent Reinforcement Learning (MARL) as follows: In each period t , the action space for each agent is a product of the set of possible prices and bids, that is $a_{it} = (p_{it}, b_{it})$. The set of prices is discretized into 15 equally spaced values within the range $[\mathbf{p}^{\min} - \xi(\mathbf{p}^{\max} - \mathbf{p}^{\min}), \mathbf{p}^{\max} + \xi(\mathbf{p}^{\max} - \mathbf{p}^{\min})]$.¹⁵ Here, \mathbf{p}^{\min} and \mathbf{p}^{\max} represent the minimum and maximum prices across all scenarios, respectively, for all values of θ . The parameter $\xi > 0$, set to 0.1, allows prices to extend slightly beyond the maximum and minimum values to include all possible price ranges in every scenario. The bid set contains 10 equally spaced elements in the range $[0, (1 + \xi) \cdot \mathbf{b}^{\max}]$, with the upper bound slightly above the maximum bid.¹⁶ We assume that each agent has a one-period memory. Consistent with the literature, we assume that sellers see all prices of other sellers. In our baseline scenario, we assume that an agent only knows their own bid. Thus, the state space

¹⁴When $t = 0$, $\epsilon_t = e^0 = 1$.

¹⁵In our baseline model, the minimum and maximum values are 1.47 and 2.1, respectively. Namely, $1.47 = \mathbf{p}^{\min} = \min\{\mathbf{p}^{oN}, \mathbf{p}^N, \mathbf{p}^M\}$ and $2.1 = \mathbf{p}^{\max} = \max\{\mathbf{p}^{oN}, \mathbf{p}^N, \mathbf{p}^M\}$, $\forall \theta \in [0, 1]$.

¹⁶The maximum bid \mathbf{b}^{\max} is the Nash-Bertrand bid \mathbf{b}^N , as the monopoly bid \mathbf{b}^M is zero. In our baseline model, the maximum possible bid equals 0.24. Namely, $0.24 = \mathbf{b}^{\max} = \max\{\mathbf{b}^N, \mathbf{b}^M\} = \max\{\mathbf{b}^N, 0\} = \mathbf{b}^N$

$s_{it} = (p_{it-1}, p_{jt-1}, b_{it-1})$ consists of the previous period's prices set by all agents and the bids set by the agent itself, resulting in $15^n \times 10$ elements. Conditional on the state space, each agent makes its pricing and bidding decisions at time t .

The sellers use the algorithms to set their own prices and bids. Each seller's algorithm independently maintains and updates its own Q-matrix over time. We initialize the “time zero” Q-matrix as follows: for a given agent and state, we calculate the expected period payoff for each action, assuming all other agents uniformly randomize their actions. We then divide this value by $1 - \delta$ so that the Q-matrix is set at the discounted payoff that would accrue to player i if opponents randomized uniformly:

$$Q_{i,0}(s, a_i) = \frac{\sum_{a_{-i} \in \mathcal{A}^{n-1}} \pi_i(a_i, a_{-i})}{(1 - \delta)|\mathcal{A}|^{n-1}}.$$

This aligns with the assumption that at first the choices are purely random. Unless specified otherwise, we set the default parameters to $\alpha = 0.15$ and $\beta = 10^{-5}$.

We consider convergence achieved if the induced strategy of each agent remains unchanged for 100,000 periods. For each agent i and in each period t , we examine the agent's Q-matrix. For every possible state s , we identify the action that corresponds to the highest Q-matrix payoff. This process induces a policy function for each agent in every period, $a_{i,t}(s) = \text{argmax}_a [Q_{i,t}(s, a)]$. If this policy function remains constant for each agent over a span of 100,000 periods, or after one billion periods have elapsed, we stop. Subsequently, we calculate payoffs and other relevant metrics based on these converged Q-matrices by averaging over the last 100,000-period horizon.

We repeat the procedure 1000 times for every set of parameters. In each iteration, we restart the algorithms from the initialized Q-matrices and reset experimentation levels to those at time zero. We then run the algorithms until they converge. Finally, we report the average across these 1000 iterations for all statistics we are interested in.

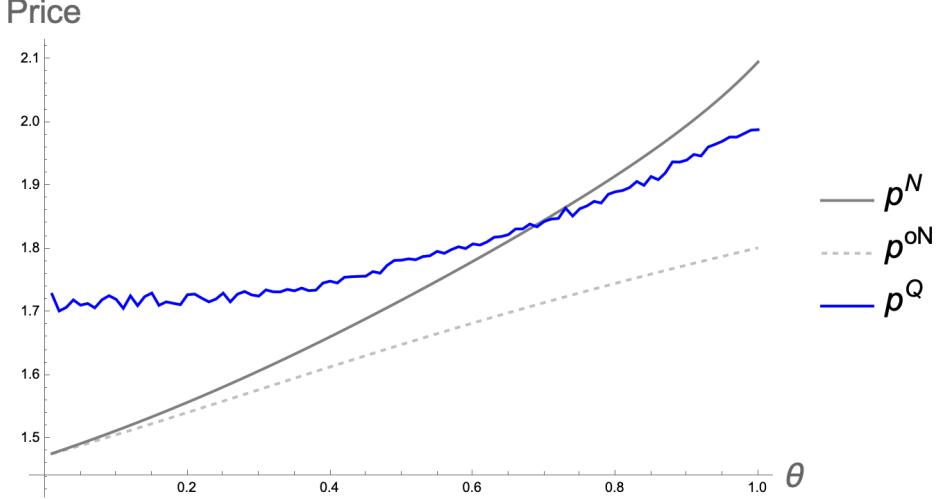
4.1 Equilibrium Prices and Bids with Reinforcement Learning

To assess the impact of sponsored ads on algorithmic prices and how algorithms respond to the increased valuation of winning the auction when search costs are higher, we use a grid search and vary θ in increments of 0.01 from zero to one.

Figure 3 plots the Q-learning equilibrium prices as a function of the fraction of consumers

focusing only on the first position, θ . To compare the Q-learning prices with the competitive price, we incorporate the prices from the one-shot game previously analyzed into the figure.

Figure 3: Q-learning Prices vs Competition Prices as a Function of Search Costs



This figure shows the Q-learning prices and theoretical prices as a function of θ . The blue solid line denote the Q-learning prices from our simulation experiments. The representation of other lines and the parameter specifications are the same as in Figure 2.

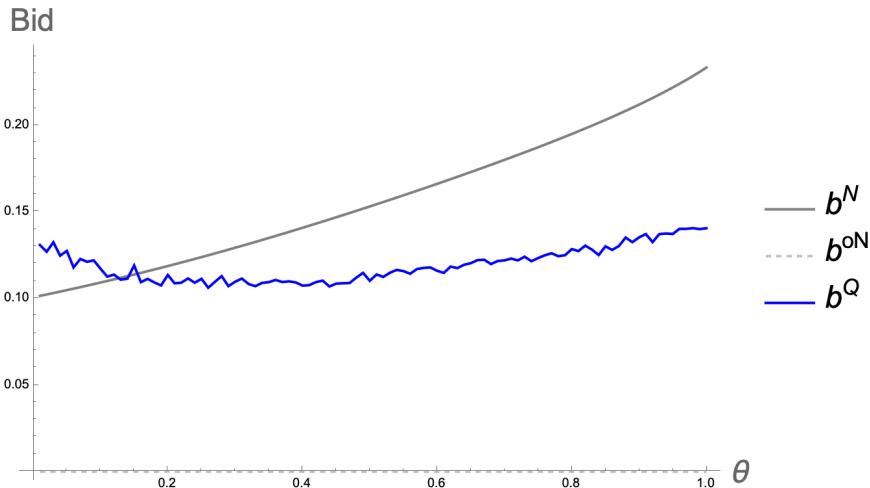
When $\theta = 0$ (the y -axis), i.e., every consumer considers the products in all positions, this corresponds to the case previously considered in the literature. The market-share weighted Q-learning equilibrium is 1.7, which is higher than the Nash-Bertrand price of approximately 1.47. This number is consistent with those reported in Calvano et al. (2020), Johnson et al. (2023), implying that the algorithms learn to set higher than competitive prices when there is no consumer search cost.

When we increase θ , for lower values of θ , Q-learning prices remain above the competitive prices. However, for higher values of θ (the case where more consumers face higher search costs in the market), Q-learning prices can fall below competitive levels. Whether algorithmic prices fall above or below fully competitive prices depends on the level of search cost heterogeneity in the market. The higher search costs are, the higher the chances that the algorithms will converge to lower than competitive prices. This finding is one of the major contributions of our paper. It is important because it goes counter to the intuition developed by past research. By considering sponsored ads and consumer search costs we are able to show scenarios where algorithms converge

to prices lower than competitive prices. Unlike other cases of algorithmic “collusion”, in our case lower prices are beneficial and not harmful to consumers. We will further illustrate the implications of this result via an analysis of consumer surplus later.

To provide an intuition to this finding, we present the average equilibrium bids in Figure 4. The bids affect the cost side of the seller’s profit. At higher values of θ , the average bids by Q-learning algorithms are significantly lower than those observed in the full competition scenario. This pattern suggests that the algorithms learn to coordinate on lower bids, thereby softening the competition in the auction. This bid coordination effectively reduces the sellers’ advertising expenses, and consequently their overall costs. Such a reduction in costs ultimately leads to lower market prices, increasing sellers’ profit.¹⁷

Figure 4: Q-learning Results: Bids



This figure shows the Q-learning bids and theoretical bids as a function of θ . The blue solid line denotes the Q-learning bids from our simulation experiments. The representation of other lines and the parameter specifications are the same as in Figure 2.

Previous research also found that Q-learning algorithms lead to bid coordination, via colluding on lower bids in either a first-price auction (Banchio and Skrzypacz 2022) or a generalized-second-price auction (Rovigatti et al. 2023), or by segmenting the market and bidding on different keywords (Banchio and Mantegazza 2022). These analyses focus on scenarios involving only bidding competition, with the valuation of winning the auction being exogenously fixed. By contrast, in our setting,

¹⁷As an aside, algorithmic bids are U-shaped because for low θ , the sponsored ad costs will be small, while it is worthwhile to price higher than competitive prices, leading to high bids in the auction. For moderate θ , advertising costs increase with θ , so bids decrease. For high θ , sponsored links are very valuable, leading to higher bids.

the valuation of winning the sponsored position is determined by price competition endogenously. Here, the pricing and bidding form an interesting “interaction”: sellers’ bids are affected by the valuation of winning the sponsored position, valuations are determined by prices, and the prices are influenced by the costs (bids). Algorithms have the tendency to increase prices, as this would increase profits. Higher prices would lead to higher bids because the valuation of sponsored position increases. At the same time, algorithms also have the tendency to lower bids, as this decreases costs. Lower costs would lead to lower prices, because the marginal cost decreases. Thus, these are two opposing forces, and the combined effect can go either way, and depends on consumer search costs. Our finding is that when the competition for the sponsored position is strong, the tendency of algorithms to lower advertising costs dominates the tendency to increase prices, ultimately resulting in lower prices than the competitive level.

4.1.1 Would Multi-Agent Q-learning Always Lead to Lower Prices?

Our previous analysis uncovered that when search costs are high, then competing learning agents can converge to charging lower than competitive prices. Because our results depended on specific simulation values, in this section we generalize this result and prove that we expect lower than competitive prices for any values of the parameters. Our analysis also allows us to shed more light about why we expect these results.

To perform the analysis, we consider a theoretical model where the two sellers actively collude on both prices and bids with complete information. In this scenario, the two sellers maximize their joint profits by jointly setting prices and bids, acting as if they are a single monopolist. The reason to perform this analysis (although the scenario is both unrealistic and illegal from an anti-trust perspective) is that it provides an upper bound on the profits of the sellers, and hence we can predict that the learning agent equilibria will be between the full competitive and the collusive one. Therefore, if we can prove that for any set of parameter values the collusive prices are lower than the competitive prices, then we would expect this outcome to also generalize to most cases of competing RL algorithms.

Following the notation in Section 3.3, the sellers’ objective function, and corresponding price

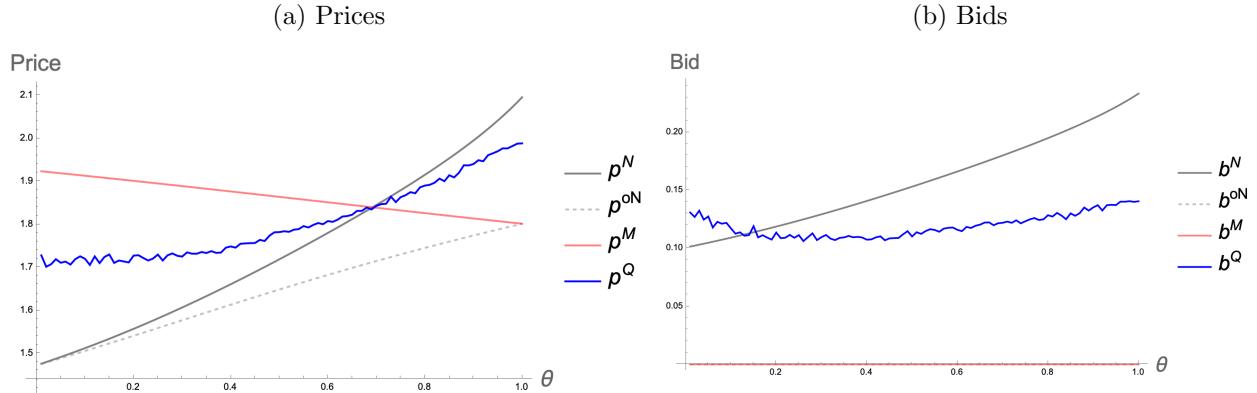
\mathbf{p}^M and bid \mathbf{b}^M are defined as¹⁸

$$\max_{p_i, b_i} \pi_M(\mathbf{p}, \mathbf{b}) = \pi_i(\mathbf{p}, \mathbf{b}) + \pi_j(\mathbf{p}, \mathbf{b}), \quad s.t. \quad p_j = p_i, b_j = b_i$$

In this case, the two sellers would agree to set their bids to the minimum, $b_i = b_j \rightarrow 0$. This strategy minimizes advertising costs while maintaining the same probability of capturing the demand from consumers who consider only the first position. Then, the two sellers would decide on the collusive prices to maximize their aggregate profits given they would both bid as low as possible. This is also equivalent to the benchmark case where the sellers only collude on prices and there are no sponsored ads, because they set their bids to zero.

Section A.5 derives the details of the analysis. Comparing the theoretical Nash-Bertrand price \mathbf{p}^N with the monopoly price \mathbf{p}^M , we find that for low values of θ , the Nash-Bertrand price is lower than the monopoly price, i.e., $\mathbf{p}^N < \mathbf{p}^M$. However, for high values of θ , the monopoly price becomes lower than the competitive price, $\mathbf{p}^M < \mathbf{p}^N$. There exists a unique value of $\tilde{\theta}$, such that $\mathbf{p}^M(\tilde{\theta}) = \mathbf{p}^N(\tilde{\theta})$. Figure 5 illustrates these results where we add the collusive outcome to the previous results. We observe a crossing of the competitive and collusive prices, and we show that there will always be a value of θ where a crossing will exist.

Figure 5: Q-learning and Theoretical Collusion



The subfigure (a) and (b) show the Q-learning vs theoretical prices and bids as a function of θ , respectively. The red solid line denotes the monopoly price \mathbf{p}^M (or bid \mathbf{b}^M). The representation of other lines and the parameter specifications are the same as in Figure 3 or Figure 4.

The analysis also reveals an additional interesting result—the monopoly prices decrease with

¹⁸where M stands for monopoly.

θ . This counterintuitive result is in the opposite direction of the competition prices analyzed in Section 3.3. The main intuition is that when $\theta = 0$, every consumer considers both products, and because of the consumer's idiosyncratic preferences, the monopolist has two chances to compete with the outside option. However, when $\theta = 1$, the monopolist only has one product to sell (because every consumer only consider one product from the first link). In this sense, the outside option becomes more competitive when $\theta = 1$, forcing the monopolist to decrease the price and obtain a lower profit margin.

This downward sloping impact of θ for the collusive model helps explain the crossing of the competitive prices and collusive prices. First, in the limiting case where $\theta = 1$, the monopoly price \mathbf{p}^M is equal to the benchmark price \mathbf{p}^{oN} , which is lower than competitive prices \mathbf{p}^N because there are no ad costs that increase prices. When $\theta = 0$ however, the collusive prices are higher than the full competition prices because the ad auctions do not affect the pricing competition as there is no ad cost, and competition leads to lower prices in this case. Effectively, we see that search costs and ad auctions create an interaction where collusive prices do not behave as we would intuitively expect.

Because the algorithmic prices are expected to be between the theoretical collusive and fully competitive prices, this analysis shows that we expect to find a value of θ for which algorithmic pricing will lead to prices that are lower than competitive, and this would benefit consumers.

4.2 Impact on Sellers and Consumers

Figure 6 presents the sellers' profit and consumer surplus¹⁹ from the Q-learning simulation. We compute the outcomes in Equations (3) and (5) using the Q-learning equilibrium prices and bids.

The sellers' profits (Figure 6(a)) from Q-learning fall below the theoretical collusion scenario, yet are higher than the levels in full competition. This indicates that the use of learning algorithms can benefit firms compared to fully competitive strategies, for all values of θ . Algorithms, whether used

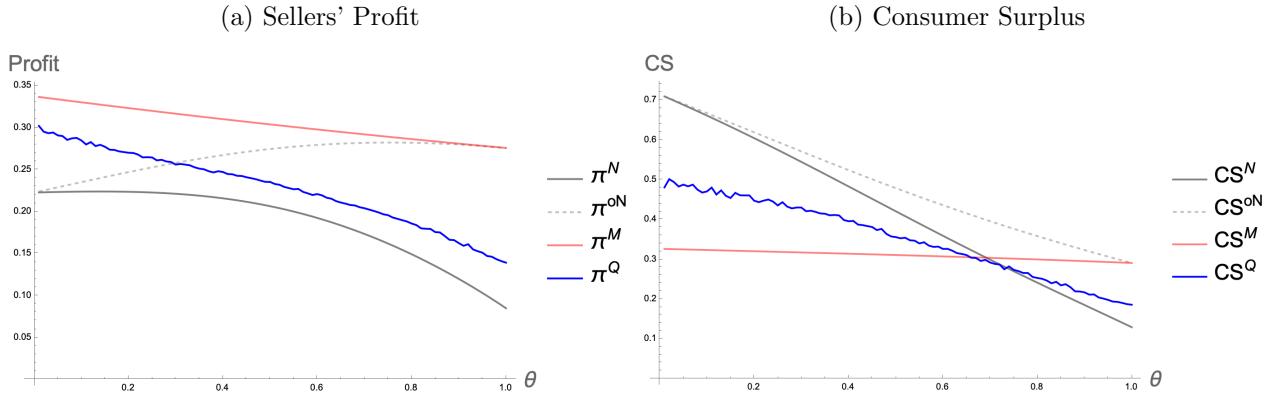
¹⁹Consumer surplus in period t is

$$\begin{aligned} CS(\mathbf{p}^t) &= \theta \cdot \sum_j \mathbf{1}\{j \in \mathcal{J}_1(\mathbf{\Gamma}^t)\} \cdot \mu \log \left[\exp \left(\frac{a - p_j^t}{\mu} \right) + 1 \right] + (1 - \theta) \cdot \mu \log \left[\sum_j \exp \left(\frac{a - p_j^t}{\mu} \right) + 1 \right] \\ &= -\theta \cdot \sum_j \mathbf{1}\{j \in \mathcal{J}_1(\mathbf{\Gamma}^t)\} \cdot \mu \log (1 - s_j(p_j^t)) - (1 - \theta) \cdot \mu \log \left(1 - \sum_j s_j(\mathbf{p}^t) \right) \end{aligned} \quad (5)$$

for single-dimensional pricing learning or multi-dimensional learning including both pricing and bidding, facilitate some degree of collusion. While this does not amount to full collusion, collusion typically advantages sellers, yielding higher profits. We also notice that algorithmic profits can be lower than the benchmark case of no ads when θ is high, because the ad costs outweigh the benefits derived from collusion on higher prices.

Consumer surplus decreases with prices. Indeed, Figure 6(b) shows that for high θ consumer surplus can be higher when using learning algorithms compared to full competition, because the Q-learning prices are lower than the full competition prices.

Figure 6: Q-learning Results: Sellers' Profit and Consumer Surplus



The left and right figure show the sellers' profit and consumer surplus, respectively. The blue solid lines denote the Q-learning outcomes from our simulation experiments. The representation of other lines and the parameter specifications are the same as in Figure 2.

According to the Folk theorem (Fudenberg and Maskin 1986), if the sellers play the same game of setting prices and bids in an infinite repeated game, any outcome between cooperative (the theoretical collusion case in Section 4.1.1) and competitive (the full competition case in Section 3.3) in the one-shot game can be sustained as a subgame-perfect Nash equilibrium (SPE) and when the sellers are sufficiently patient when the discount factor δ is high enough.

Our theoretical results cannot predict whether the algorithmic prices and profits will be closer to the monopoly or the competition case, and to answer this question we again use simulation analysis.

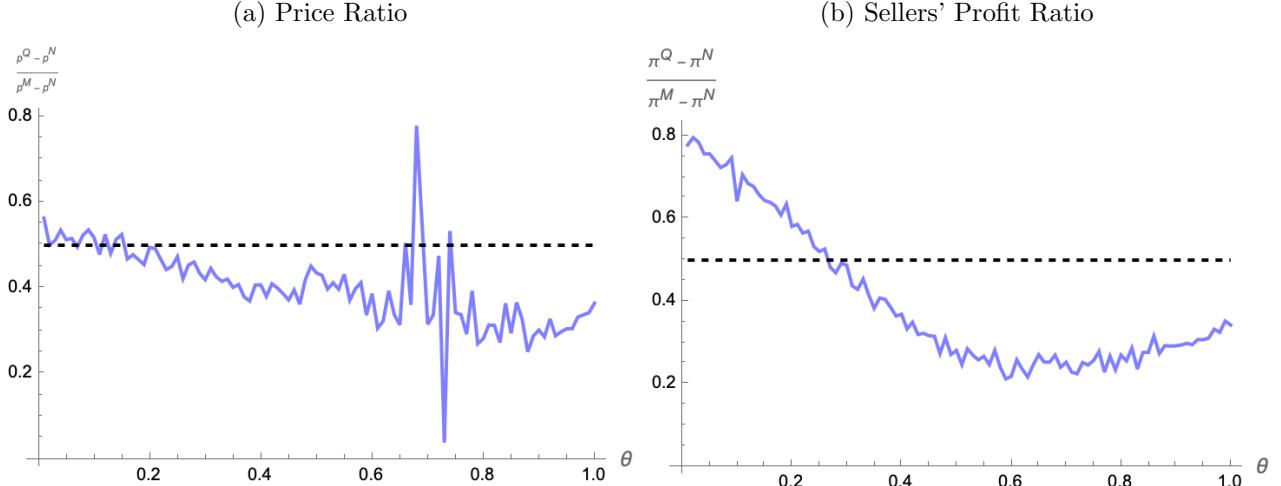
We plot the ratios of the difference between the algorithmic and competitive prices and profits, divided by the difference between the collusive and competitive prices and profits, respectively. That is, we compute $\frac{p^Q - p^N}{p^M - p^N}$ and $\frac{\pi^Q - \pi^N}{\pi^M - \pi^N}$. Given that $\frac{p^Q - p^N}{p^M - p^N}$ can be rewritten as $\frac{p^Q - p^N}{p^Q - p^N + p^M - p^Q}$, if these

ratios are smaller than 0.5, it indicates that the algorithmic outcomes are closer to competition; otherwise, they are closer to collusion.

Figure 7 presents the results. When $\theta = 0$, the algorithms can achieve roughly 60% of the price increase from the competitive price to the collusive price, and 80% of the profit increase from the competitive profit to the collusive profit. For low values of θ , we see that the ratios for both price and profit are higher than 0.5, implying that the algorithms generate outcomes closer to monopoly (collusion), which is consistent with previous findings about algorithmic collusion (Calvano et al. 2020).

However, for high values of θ , in the range where the algorithms benefit both consumers and sellers, the algorithms can only achieve profit increases that are roughly between 30% and 40% of the increase from the competitive profit to the collusive profit. This suggests that when algorithms compete in more than just one dimension of pricing, the algorithmic outcomes are closer to competition. This is the opposite of cases of $\theta = 0$, where there are no consumer search costs.

Figure 7: Q-learning Results: Ratio



The left and right figure show the ratio statistics for prices and sellers' profit, respectively. The parameter specifications are the same as in Figure 2.

4.3 Robustness to Assumptions

We also analyze a few extensions and variation of our model in Appendix A.7. We analyze alternative characterizations of consumer search costs in Subsection A.7.1, and a scenario where the sellers have differentiated products in Appendix A.7.2. Our findings, that Q-learning can benefit

both consumers and sellers, are robust to these different model setups.

Additionally, we investigate a case in Appendix A.7.3 where the state space of the Q-learning algorithm includes additional bid information. This differs from our previous assumption that algorithms make decisions based only on their own bids. The motivation for this analysis is that the platform might experiment with the auction design and disclose winning bids, or sellers could subscribe to a third-party market intelligence service to obtain bidding information and make more informed decisions. The additional information will change the sellers' bids, and we are interested in how this will affect the platform's revenue and the sellers' profits. Thus, we aim to provide guidance to platform managers on how disclosing more or less information about ad auctions can serve as an effective information-based tool to increase profits. Additionally, we explore whether sellers should subscribe to those data services.

We consider the following *full stateful* scenario, where both agents' bids are observed; that is, the state space is $s_{it} = (p_{it-1}, p_{jt-1}, b_{it-1}, b_{jt-1})$. Our results show that the additional information impacts the platform's revenue and the sellers' profits very little. The algorithms converge to almost the same equilibrium. This suggests that this information-based method might not be an effective tool for the platform, and they might want to consider more direct types of incentive-based tools to strategically respond to sellers who use algorithmic pricing, which we will analyze in detail in Section 5.

5 Platform's Strategic Response

In our previous discussion, when sellers used algorithms to determine the prices and bids, both algorithmic prices and bids fell below competitive levels. This would impact the platform's revenue from both commission fees and ad revenue, potentially adversely affecting the platform's total profit.

In the online auction setting, search engines and platforms implement various auction designs to increase revenue. Strategies include raising auction reserve prices, limiting the number of sponsored spots, or adjusting auction formats (Decarolis and Rovigatti 2021, Kobayashi and Alcobendas 2023). However, in our setting, due to interactions with pricing competition and demand, the impact of sellers using algorithms on platforms' profits and the the best direction for the platform's strategic

response are unclear. Instead, it depends on market parameters and the platform's objectives (whether the platforms only care about their own single-period payoffs or also about consumer surplus and sellers' profits to increase the long-term business growth).

In Sections 5.1 and 5.2, we first consider the scenario where the platform's objective is to maximize its own profit, and we investigate two common incentive-based methods typically used by platforms: adjusting the commission rate (which impacts the commission revenues) or the reserve price (which impacts the ad auctions revenues). The platform's profit consists of commission fees and advertising revenue, and the platform's profit in a single period is

$$\begin{aligned}\pi_p(\mathbf{p}, \mathbf{b}) &= \pi_p^{\text{Ad}}(\mathbf{p}, \mathbf{b}) + \pi_p^{\text{Com}}(\mathbf{p}, \mathbf{b}) \\ &= \theta \cdot \sum_j \Pr(\tilde{b}_j > \tilde{b}_{-j}) \cdot s_j(p_j) \cdot (\gamma_j \cdot \mathbb{E}[\tilde{b}_j | \tilde{b}_j > \tilde{b}_{-j}]) \\ &\quad + \tau \cdot \theta \cdot \sum_j \Pr(\tilde{b}_j > \tilde{b}_{-j}) \cdot p_j \cdot s_j(p_j) + \tau \cdot (1 - \theta) \cdot \sum_j p_j \cdot s_j(p_j, p_{-j})\end{aligned}\tag{6}$$

We then consider the scenario where the platform's objective is long-term growth by maximizing a weighted average of its own profit, sellers' profit, and consumer surplus in Section 5.3.

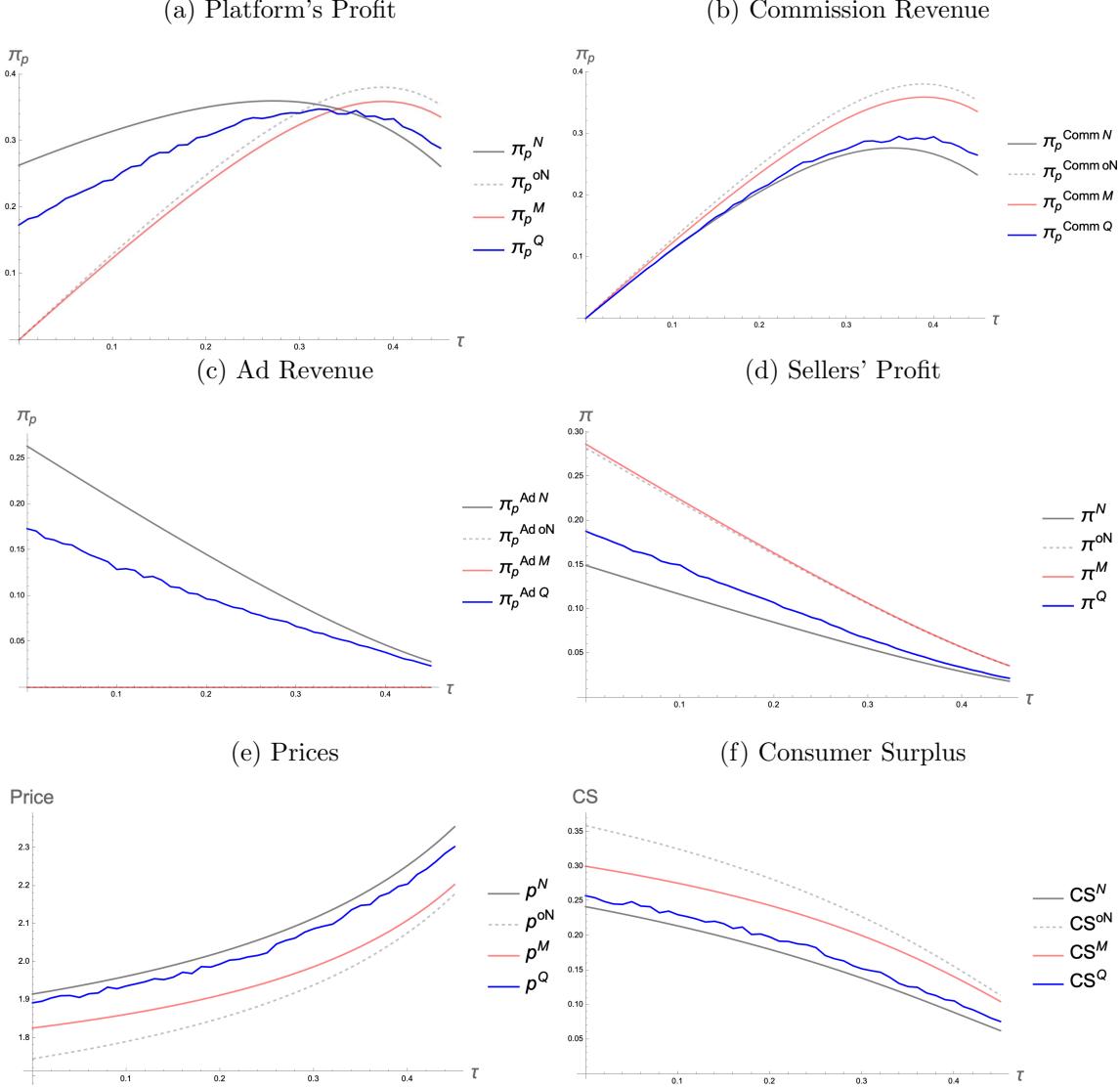
5.1 Commission Rate

In our previous discussion, when consumer search costs are high, advertising revenue might be lower because the sellers submit lower bids. At the same time, since the Q-learning prices are lower, demand is higher, and thus the platform's revenue from commission fees might be higher than that in competitive scenarios if the demand-enlarging effect dominates. Considering these two effects together, the total impact of sellers using algorithms on the platform's own profit is less clear.

Figure 8 presents the results of simulation analysis where the platform adjusts the commission rate when consumer search costs are high. As shown in Figure 8(a), the platform's profit under Q-learning can be higher than in the full competition case for high values of the commission rate τ . Decomposing the platform's profit, we see that algorithmic pricing can increase the platform's revenue from commission fees, as illustrated in Figure 8(b). This is because the Q-learning prices are lower, leading to higher demand and increased commission fees. However, advertising revenues,

presented in Figure 8(c), generally decrease as the bid coordination effect becomes more dominant.

Figure 8: Platform Strategic Response: Commission Rate



The subfigures (a) to (f) show the platform's profit, commission fee, and ads revenue, sellers' profit, market price and consumer surplus as a function of τ when $\theta = 0.8$, respectively. The blue solid line denotes the Q-learning platform profits from our simulation experiments. The representation of other lines and the parameter specifications are the same as in Figure 2.

As a result, the platform might benefit from sellers using algorithmic pricing, even if its objective is to maximize its own profit, thus not requiring any adjustment of commission rates. If its profit is lower under algorithmic pricing, the direction of the optimal adjustment depends on the market

parameters, that is, it can be optimal for the platform to either increase or decrease the commission rate.

When the platform adjusts the commission rate, a higher commission rate will result in higher prices in the market, lower sellers' profits, and lower consumer surplus, as illustrated in Figures 8(d) to 8(f). However, if the platform increases the commission rate, algorithms will continue to benefit consumers and sellers at the adjusted commission rate, compared with the full competition case. This is because the point where the competitive and collusive prices cross with each other will be at lower θ under a higher commission rate, and the same consumer search cost will still fall within the region where algorithms yield beneficial outcomes for both sellers and consumers.

5.2 Ad Auction Reserve Price

To investigate the impact of the platform using the reserve price as a tool to strategically respond to sellers employing algorithmic pricing, we conduct simulation analyses in which we allow the platform to gradually increase the reserve price from 0.

We assume that if the bids submitted by all sellers are lower than the reserve price, the platform will need to randomly display one of the products in the top position, and there are no advertising costs for the sellers. The sellers' profit function becomes

$$\begin{aligned}\pi_i^t(\mathbf{p}^t, \mathbf{b}^t) = & \theta \cdot \mathbf{1}(\tilde{b}_i^t \geq r) \cdot \Pr((\tilde{b}_i^t > \tilde{b}_j^t)) \cdot s_i(p_i^t) \cdot \left(((1 - \tau) \cdot p_i^t - c_i) - \gamma_i \cdot \mathbb{E}[\tilde{b}_i^t | \tilde{b}_i^t > \tilde{b}_j^t] \right) \\ & + \left(\frac{1}{2} \cdot \theta \cdot \mathbf{1}(\tilde{b}_i^t < r) \wedge (\tilde{b}_j^t < r) \right) \cdot s_i(p_i^t) + (1 - \theta) \cdot s_i(p_i^t, p_j^t) \cdot ((1 - \tau) \cdot p_i^t - c_i)\end{aligned}\tag{7}$$

where r denotes the reserve price set by the platform.

Figure 9 shows the results. Interestingly, the reserve price might not serve as an effective tool for the platform to strategically respond to reduced profit from algorithmic pricing and bidding. As the platform gradually increases the reserve price from 0, but still below the Q-learning bid level, the platform's commission, ads, and total revenue vary very little with the reserve price. However, as the platform raises the reserve price above the Q-learning bid level, there is a large discontinuity point in the platform's revenue, as illustrated by Figure 9(a). The reason is that the reserve price shifted the algorithms from the previous equilibrium to coordinate on a new equilibrium. When

the reserve price is above the Q-learning bid level from the scenario without a reserve price, the algorithms learn to coordinate on lower bids below the new reserve price, as shown in Figures 9(c) and 9(d). This coordination further increases the sellers' profits (as shown in Figure 9(e)), since the sellers are still displayed in the sponsored position without any ad costs if the bids submitted by all sellers are lower than the reserve price. This cost-saving effect further increases consumer surplus, as shown in Figure 9(f). Although it might seem that the algorithms can profitably deviate by bidding above the reserve price to win the sponsored positions, the competitor will respond by bidding above the reserve price as well, which causes lower profits for both sellers in the long-run. Because of the experimentation property of learning algorithms, the algorithms gradually discover that coordinating on lower bids is more profitable, eventually converging to an equilibrium of lower bidding.

To summarize, increasing the reserve price might not be an effective response for the platform, and sellers and consumers will continue to benefit from algorithmic prices. If the platform does increase the reserve price above the algorithmic equilibrium bid level, the sellers and consumers benefit even more.

5.3 Weighted Average of Total Surplus

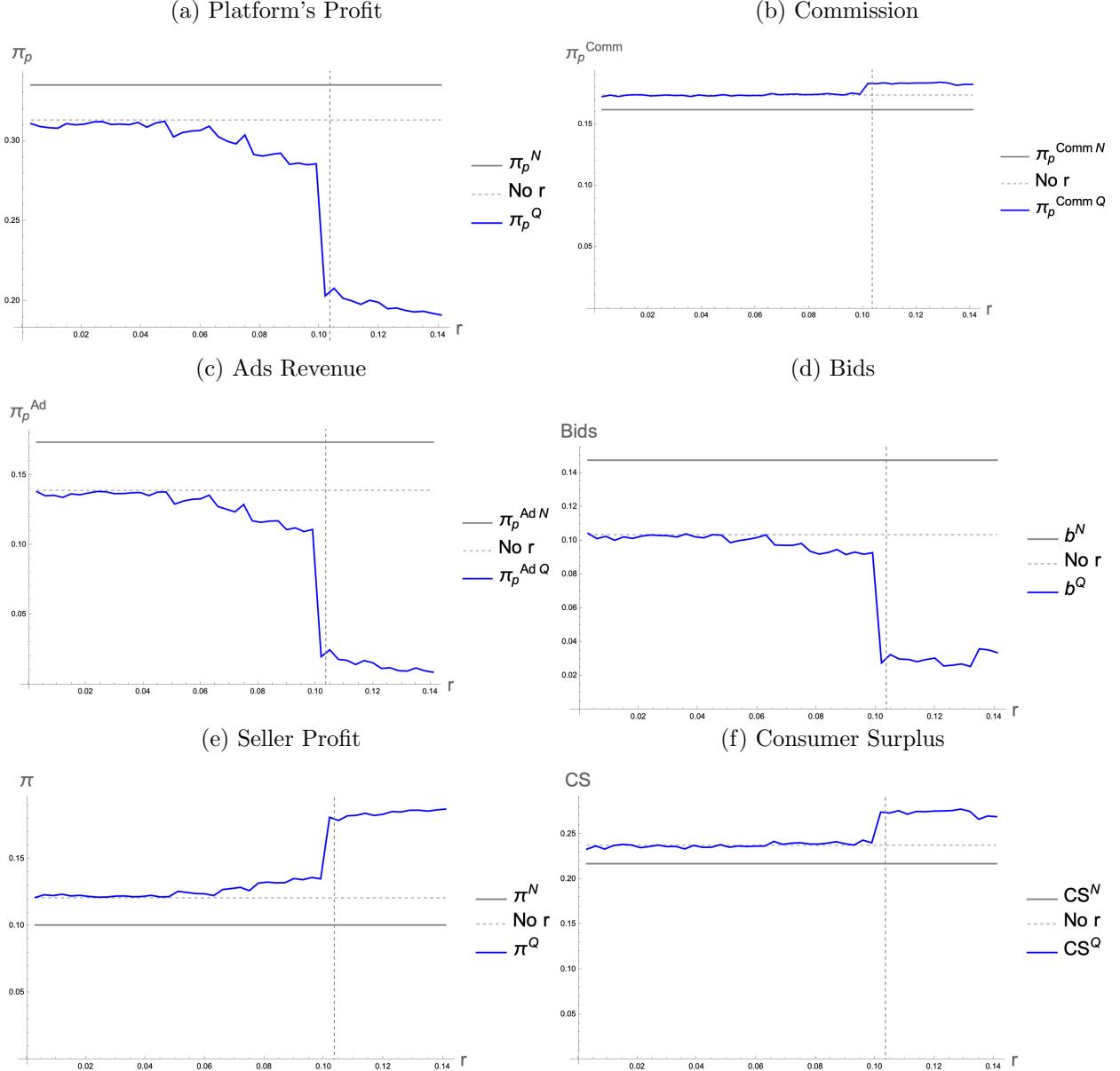
Given competition between platforms, a platform might also consider consumer surplus and seller profits to encourage entry for long-term business growth. Hence, we consider a scenario where the platform is maximizing a weighted average of its own profit, sellers' profit, and consumer surplus, with weights ω and $1 - \omega$, respectively. The platform's objective can be written as follows:

$$\begin{aligned} & \omega \cdot \pi_p(\mathbf{p}, \mathbf{b}) + (1 - \omega) \cdot \left(\sum_j \pi_j(\mathbf{p}, \mathbf{b}) + CS(\mathbf{p}) \right) \\ &= \omega \cdot \left(\pi_p^{Ad}(\mathbf{p}, \mathbf{b}) + \pi_p^{Com}(\mathbf{p}, \mathbf{b}) \right) + (1 - \omega) \cdot \left(\sum_j \pi_j(\mathbf{p}, \mathbf{b}) + CS(\mathbf{p}) \right) \end{aligned}$$

where $\pi_p(\mathbf{p}, \mathbf{b})$ is the platform's own-profit as defined in (6), $\pi_j(\mathbf{p}, \mathbf{b})$ is seller j 's profit as defined in (3), and $CS(\mathbf{p})$ is the consumer surplus as defined in (5).

When $\omega = \frac{1}{2}$, i.e., the platform puts equal weight on its own profit versus the sum of sellers'

Figure 9: Platform Strategic Response: Reserve Price



The subfigures (a) - (f) show the platform's profit, commission fee, ads revenue, bids, sellers' profit, and consumer surplus as functions of the reserve price when $\theta = 0.8$ and $\tau = 0.15$, respectively. The blue solid line denotes the Q-learning platform profits from our simulation experiments. The representation of other lines and the parameter specifications are the same as in Figure 2. The dashed vertical lines represent the Q-learning bids in the algorithmic pricing scenario.

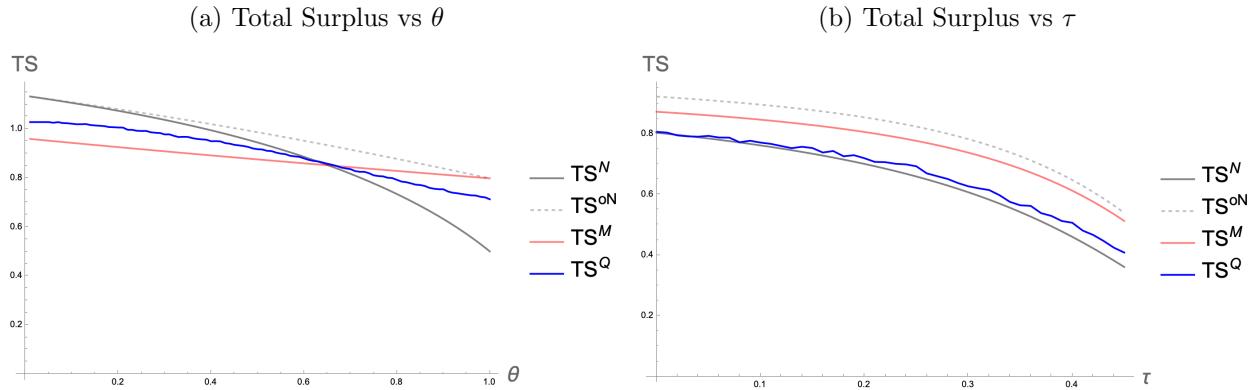
profits and consumer surplus, then its objective is equivalent to maximizing the total surplus.²⁰

²⁰Note that since the commission fee and advertising costs are transfers between sellers and consumers, they cancel out when $\omega = \frac{1}{2}$. Therefore, the total surplus in this scenario is also equivalent to the duopoly case where there is no platform, no commission fee, and no advertising costs.

$$\begin{aligned}
& \sum_j \left(\theta \cdot \Pr(\tilde{b}_j > \tilde{b}_{-j}) \cdot s_j(p_j) + (1 - \theta) \cdot s_j(p_j, p_{-j}) \right) \cdot (p_j - c_j) \\
& - \theta \cdot \sum_j \mathbf{1}\{j \in \mathcal{J}_1(\boldsymbol{\Gamma})\} \cdot \mu \cdot \log(1 - s_j(p_j)) - (1 - \theta) \cdot \mu \cdot \log \left(1 - \sum_j s_j(\boldsymbol{p}) \right)
\end{aligned} \tag{8}$$

Figure 10a shows the total surplus as function of θ when commission rate is 15%, i.e., $\tau = 0.15$.²¹ When consumer search costs are high, compared with the full competition case, using learning algorithms always increase the total surplus. This increase is due to the fact that, compared to the outside option, products on the platform become cheaper and hence more attractive. Figure 10b plots the total surplus when the platform adjusts its commission rate, given a high value of θ .²² Therefore, algorithmic pricing consistently yields a higher total surplus than the full competition case, even when the platform is adjusting the commission rate.

Figure 10: Platform Strategic Response: Total Surplus



The subfigure (a) shows the total surplus as a function of consumer search friction when $\tau = 0.15$, and (b) shows the total surplus as a function of the reserve price when $\theta = 0.8$. The blue solid line denotes the Q-learning platform profits from our simulation experiments. The representation of other lines and the parameter specifications are the same as in Figure 2.

Thus, even if the platform's own single-period profit is lower, considering the sellers' profit and consumer surplus, the platform will not make any adjustments, and the beneficial outcomes for sellers and consumers will remain.

²¹This is the rate for most product categories on Amazon.com

²²The minimum value of θ where the total surplus is higher than in the competitive case is $\tilde{\theta}$, such that when $\tau = 0$, $p_N(\tilde{\theta}) = p_M(\tilde{\theta})$. The total surpluses TS^M and TS^N intersect at the same value of θ as do the prices p^M and p^N .

6 Estimation of Search Costs on Amazon.com

Our results about the benefits of algorithmic pricing and bidding for consumers and sellers depend on search costs being high enough on the e-commerce platform. In this Section, we collect and analyze a large-scale dataset from Amazon.com to demonstrate robust evidence of high consumer search costs.

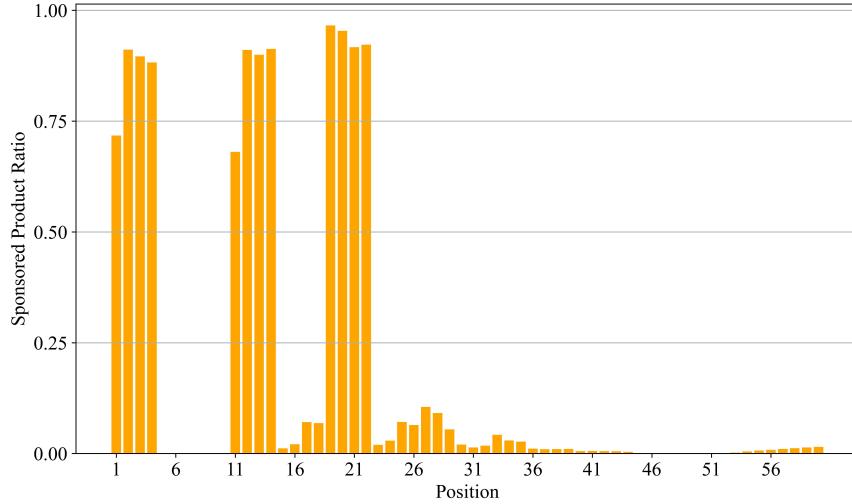
6.1 Data

We collected data from Amazon.com from April to June 2024. The data includes 1,918 highly searched keywords across all product categories. Our scraper navigated to Amazon.com, representing a typical consumer shopping journey without requiring a login. The scraper submitted a query request by entering a keyword and then navigated to the first search result page. On the search results page, the scraper recorded the ranking of each product (both sponsored and organic) within the first page. The page usually contains other product information, such as price, whether it is on sale or has a coupon available, product ratings, the number of reviews, whether it is an Amazon Prime product, and delivery information. To obtain a representative distribution of the sponsored product, we repeated the search for each keyword once every three hours, generating more than 15,000 requests per day.

We obtained additional product information from Keepa.com for each product that appears in the search results. The key information is the daily best sales rank within each product category reported by Amazon. The best sales rank is generally considered a reliable proxy for quantity of sales in literature (Chevalier and Goolsbee 2003). We then converted the best seller rank to daily sales. We also obtained keyword information from one of the biggest E-commerce data intelligence companies *Jungle Scout*. For each keyword, Jungle Scout provides information about the monthly search volume, suggested pay-per-click, and related keywords, etc.

Figure 11 and Figure A6 show the ratio of sponsored products for each position on product search result pages containing 60 and 22 products, respectively. The positions that are most likely to be sponsored positions in our data are 1-4, 11-14, and 19-22 for a product page with 60 products, and 1, 2, 7, 12, 17, and 22 for a product page with 22 products. Figure A7 and A8 present the corresponding heatmaps of the density of sponsored products within a product result page.

Figure 11: Distribution of Sponsored Product Ads: Page Layout 60



This figure shows the ratios of sponsored products by position for product result pages with 60 products.

6.2 Descriptive Analysis

6.2.1 Comparing Sponsored and Organic Products

To compare the differences between sponsored and organic products, we plot the average prices, number of ratings, ratings, and number bought last month for a given position in Figure 12. Compared with organic products in the same positions, the sponsored products have higher prices, fewer reviews, and lower ratings, and fewer number bought last month.

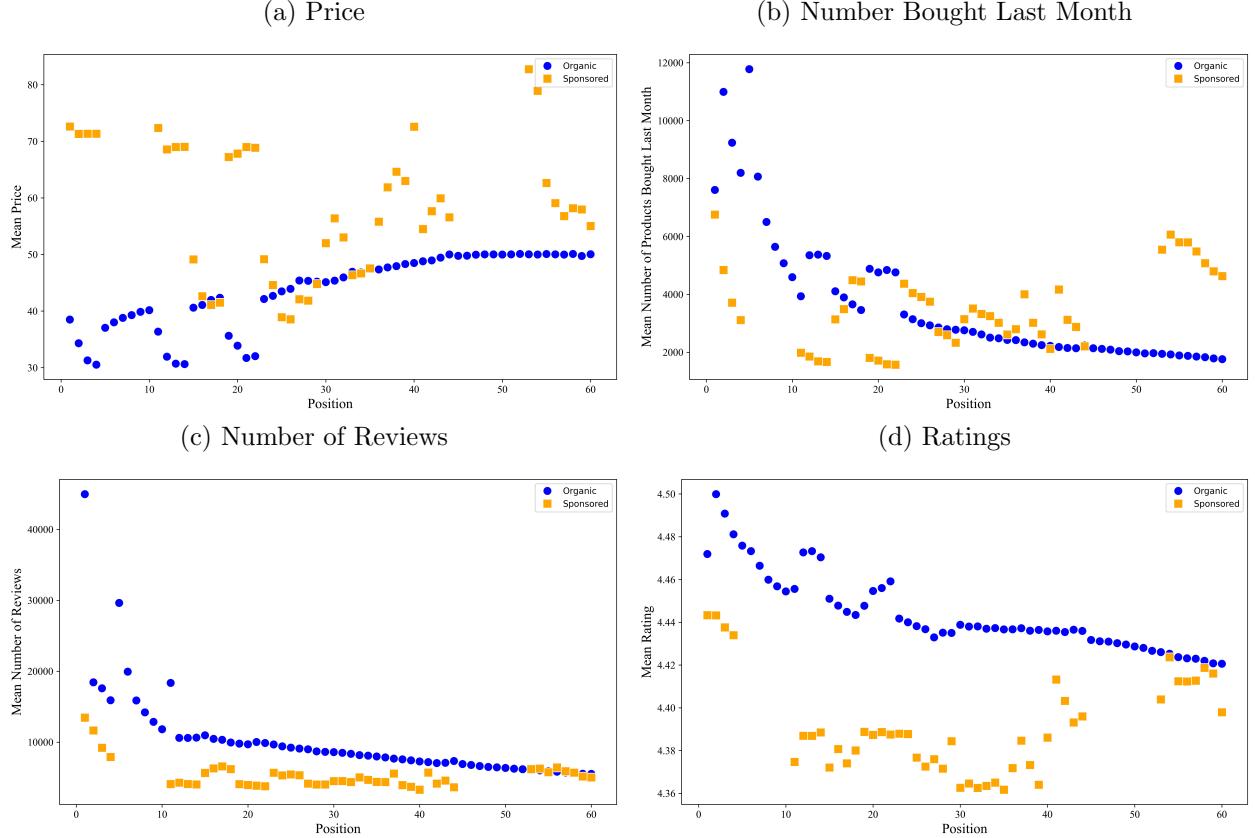
This intuitively makes sense, as sponsored product ads are additional costs for sellers, which would increase their marginal costs and, consequently, the prices in the market. This is also consistent with our theoretical prediction that sponsored product ads will increase equilibrium prices compared with the scenario without ads.

Compared with their organic counterparts in the same positions, sponsored products have lower number of reviews and number of products bought last month, indicating that they are more likely to be less established products and might be buying ads to gain awareness from consumers.

Finally, the average ratings decrease with position, with sponsored products having lower average ratings than organic products in the same position. However, ratings might be a noisy indicator of quality, and sellers might be buying fake reviews (He et al. 2022). Without demand

estimation based on consumer revealed preferences, we are cautious about drawing the conclusion that sponsored products have lower mean quality.

Figure 12: Position vs Observables



The subfigures (a) - (d) show the mean product prices, number of products bought last month, number of reviews, and ratings by each position for sponsored and organic products. The blue circle represents the organic products, while yellow squares denote the sponsored products.

6.2.2 Correlation of Product Position and Sales

We investigate the relationship between prices, search result positions, and sales by regressing the logarithm of sales of a product on its position, logarithm of price, and their interaction, after controlling for each product's fixed effect. This regression is at daily level.

$$\log(sales_{jt}) = position_{jt} + \log(price_{jt}) + position_{jt} \cdot \log(price_{jt}) + FE_j + \epsilon_{jt} \quad (9)$$

As one would expect, the coefficient of the position is significantly negative, indicating that the later position a product appears in the results page, the fewer sales it can get. However, the

coefficient is likely to be biased for the following reason: if there is a positive demand shock, the sales of a product will increase. Moreover, if a product has more sales in a period, the recommendation algorithms are likely to rank the product in a higher position, which in turn brings it more sales. This creates simultaneity issues, motivating us to build a structural model in the following section to correctly estimate consumer search costs, and the causal effect of rank on sales.

6.3 Estimation of Search Costs

In this part, we first introduce the notations and assumptions in our structural model, then describe the identification strategy, and finally present our estimation results and implications.

Model Setup Suppose there are k_1, k_2, \dots, k_n keywords. Let $\bar{\mathcal{K}}$ be the set of keywords we scrape. We assume that each keyword represents a market. The monthly search volumes for each keyword are $V_{k_1}, V_{k_2}, \dots, V_{k_n}$, respectively. Using the daily bestseller rank information reported by Amazon and the Rank to Sales estimation tool from *Jungle Scout*, we obtained the imputed daily sales, q_{jt} , for a product j . Next, we assume that the market size of a keyword is its search volume. Hence, the market share of a product j in market k at time t (day) can be written as:

$$s_{jkt} = \frac{q_{jt}}{V_k/30} \quad (10)$$

In each market, products are indexed by $j \in \mathcal{J}_k$. When our scraper searches a keyword k , they usually see N products in the first page of the search results, with $N = 60$ for most of the product categories and $N = 22$ for the category ‘Electronics.’ For each keyword, we conduct a search once every three hours, resulting in a total of $S = 8$ searches each day, with each search indexed by s . Let \mathcal{R}_k^{st} be the ordered ranking of products in search s of keyword k in day t , where $r_{nk}^{st} \in \mathcal{J}_k$ is the product in the n -th position. Then, $\mathcal{J}_n(\mathcal{R}_k^{st}) = \{r_{1k}^{st}, r_{2k}^{st}, \dots, r_{nk}^{st}\} \subset \mathcal{J}_k$ is the set of products in the first n positions. Among the N different positions, there are both sponsored and organic positions. The number of sponsored positions is 12 when $N = 60$, and 6 when $N = 22$.

Consumers face search costs and might stop at a certain position without continuing to the next position on the search results page. We assume that there is a unit mass of consumers and that the mass of consumers who stops exactly at the n -th position follows an exponential distribution,

i.e., $\lambda \cdot e^{-\lambda \cdot n}$, where λ is the rate parameter and n is the position. Hence, the mass of consumers who stop at position n or before is given by the cumulative distribution function of the exponential distribution, $F(\lambda, n) = 1 - e^{-\lambda \cdot n}$. And the mass of consumers with consideration set of size n is $F(\lambda, n) - F(\lambda, n - 1) = e^{-\lambda \cdot (n-1)} - e^{-\lambda \cdot n}$.

Consumer i 's utility of purchasing product j at time t is:

$$u_{ijt} = \underbrace{X'_{jt} \cdot \beta - \alpha \cdot p_{jt} + \xi_{jt}}_{\delta_{jt}} + \epsilon_{ijt} \quad (11)$$

where X_{jt} are product characteristics, p_{jt} is the price of product j at time t , and ξ_{jt} is an unobserved demand shock. ϵ_{ijt} is an idiosyncratic preference shock following a Type I extreme value distribution. The mean utility of the outside option, or not buying any product on the first page of the search results, is normalized to zero.

To approximate the average position at which a product appears in different searches, we take the average of the search results and express product j 's market share for keyword k at time t as

$$\text{share}_{jkt} = \frac{1}{|S|} \sum_{s=1}^S \sum_{n=1}^N \mathbf{1}\{j \in J_n(\mathcal{R}_{kt}^s)\} \cdot (F(\lambda, n) - F(\lambda, n - 1)) \cdot \frac{e^{\delta_{jt}}}{1 + \sum_{j' \in J_n(\mathcal{R}_{kt}^s)} e^{\delta_{j't}}} \quad (12)$$

Identification To overcome the endogeneity issue and follow the common assumption in the literature (Sweeting 2013, Grennan 2013, Lam 2021, Yu 2024), we assume that the unobserved quality of a product follows an AR(1) process, i.e., $\xi_{jt} = \eta_{jt} + \rho \cdot \xi_{jt-1}$. By definition of the AR(1) process, the contemporaneous shock η_{jt} is uncorrelated with the lagged unobserved quality ξ_{jt-1} . Additionally, the contemporaneous shock η_{jt} should also be uncorrelated with the previous period's organic rank r_{jt-1} . The rationale is that the platform cannot predict the next period's shock in the unobserved quality, and even if it could, the platform does not have any incentive to incorporate that into the current period's organic ranking. Putting these two points together, we have the following moment condition.

$$\mathbb{E} \begin{pmatrix} \eta_{jt} \cdot \xi_{jt-1} \\ \eta_{jt} \cdot r_{jt-1} \end{pmatrix} = 0 \quad (13)$$

And the estimation steps are

1. Start with a guess of λ , according to (12), solve for all the $\tilde{\delta}_{jt}$.
2. Given $\tilde{\delta}_{jt}$, regress $\tilde{\delta}_{jt}$ on X_{jt} , and obtain the residual $\tilde{\xi}_{jt}$.
3. Given a guess of ρ , construct $\eta_{jt} = \tilde{\xi}_{jt} - \rho \cdot \tilde{\xi}_{jt-1}$.
4. Construct moment condition according to (13), where r_{jt-1} is the rank of product j at time $t-1$. Search for λ and ρ .

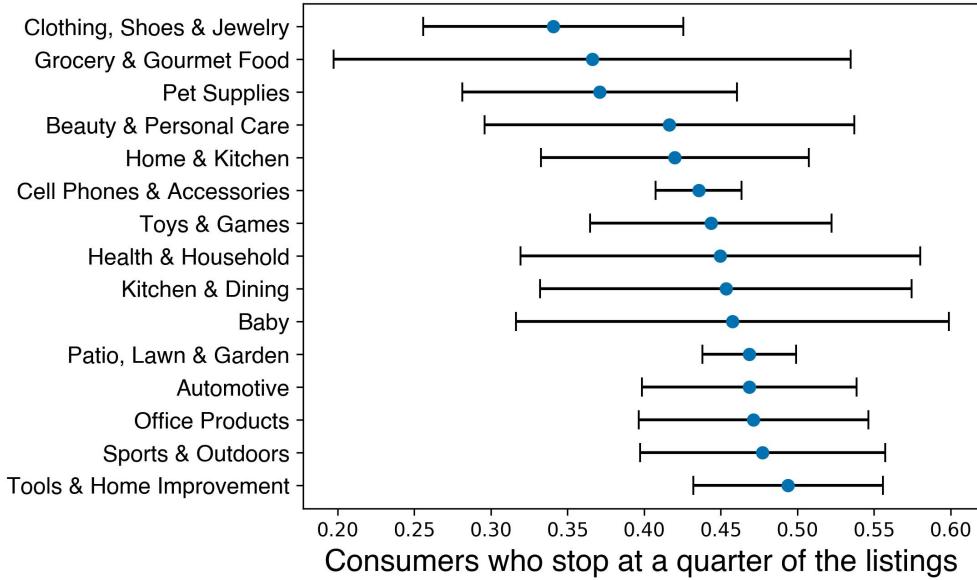
Estimation Results We estimate the consumer search cost parameter by market and then calculate the empirical percentiles across different markets. Figure 13 shows the estimation results for the mass of consumers who stop at or before a quarter of the listings in the results page. We find substantial consumer search costs across different product categories. When comparing different categories, we find that ‘Clothing, Shoes & Jewelry,’ ‘Pet Supplies,’ and ‘Beauty & Personal Care’ are the categories with higher consumer search costs, and consumers tend to stop earlier on the product search results page. In contrast, ‘Office Products,’ ‘Sports & Outdoors,’ and ‘Tools & Home Improvement’ are the categories with lower search costs, and consumers search more products before making a purchase decision. These results are consistent with the idea that consumers search fewer products when there is little new differentiation to be found, or when the benefit of continuing their search is low.

In Appendix A.6, we provide estimation results for an alternative characterization of consumer search costs, after applying a similar moment condition to address the endogeneity issue. We find similar and robust evidence of a position effect. Combining these with our main estimation results, this implies that the beneficial outcomes of algorithms who learn to collude on prices and bids could hold for many, if not most, of the product markets we analyzed.

6.4 Empirical Evidence: Negative Interaction of Consumer Search Costs and Algorithm Usage on Prices

In this section, we provide empirical evidence that algorithm usage tends to impact markets with different levels of consumer search costs differently, and that the algorithmic collusion of both pricing and advertising can exist in real data.

Figure 13: Estimation of Consumer Search Costs



This figure shows the fraction of consumers who stop at or before a quarter of the listings in the search results page. The estimate is by market, and we then obtain the mean and standard deviation within each product category in our sample.

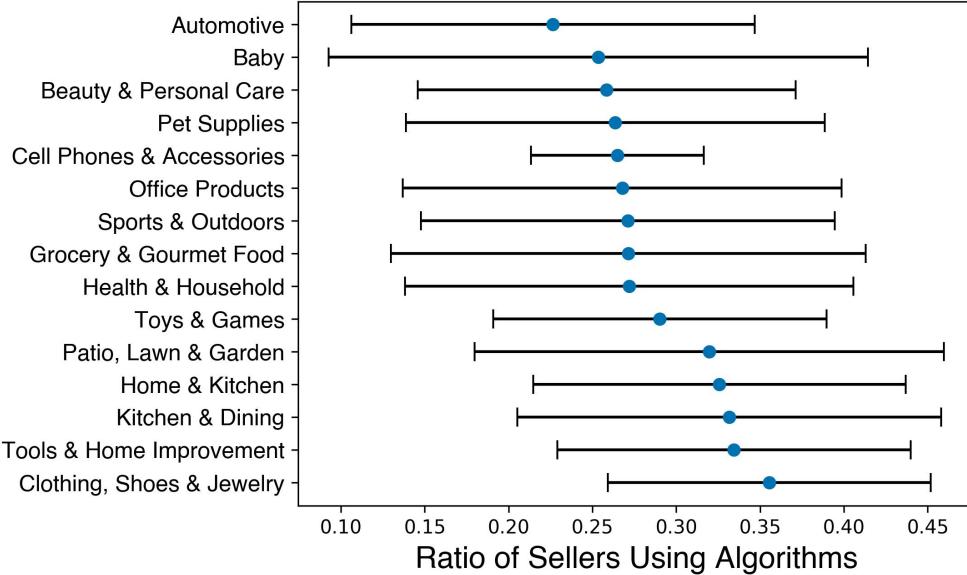
We first generate an algorithm usage index derived from the correlation in the pricing pattern, following Chen et al. (2016). The intuition behind this is that sellers using algorithmic pricing are likely to base their prices, at least partially, on the prices set by other sellers. Since we scraped the data at a very high frequency, if we observe that the prices of products are highly correlated at this frequency, it would suggest that the products are likely using pricing algorithms. We compute the correlation of the price vector of a product, \vec{p}_j , with the market average price, $\vec{p}_{j'}$. If the correlation is greater than a threshold ρ , we infer that the seller of product j is using algorithms to adjust the price. Otherwise, we infer that the seller is not using algorithms to adjust the price. We then take the average of all products in the market as the market average algorithm usage index.

$$algok = \frac{1}{|J_k|} \sum_{j \in J_k} \mathbf{1} \{ \text{corr} (\vec{p}_j, \vec{p}_{j'}) \geq \bar{\rho} \}$$

The fraction of sellers using algorithms depends on the threshold ρ that we choose. When $\rho = 0.3$, the mean ratio of sellers using algorithms across all keywords is 31.1%, with a standard deviation of 12%. Figure 14 shows the ratios of sellers using algorithms in each product keyword

category. The empirical evidence is robust to different thresholds.

Figure 14: Algorithm Usage Index

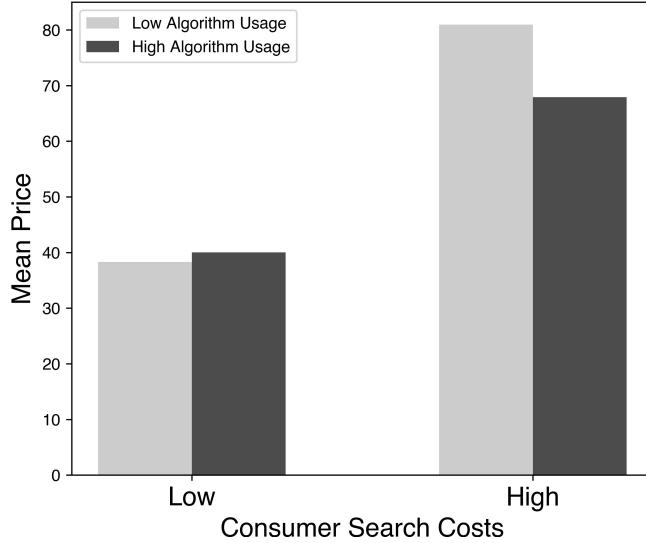


This figure shows the fraction of sellers using algorithms in each product keyword market. The estimate is by market, and we then obtain the mean and standard deviation within each product category in our sample.

After we obtain the imputed algorithm usage index, we interact the algorithm usage with estimated consumer search costs, and we find a negative interaction effect on prices. To see this, we split the markets with low vs high consumer search costs and algorithm usage, and plot the mean prices in the markets in Figure 15. The results show that the algorithms do tend to impact markets with different levels of consumer search costs differently. In the markets with low consumer search costs, high algorithm usage increases the prices, while in markets with high consumer search costs, high algorithm usage decreases the prices. This indicates a negative interaction effect of consumer search costs and algorithm usage on prices, consistent with our theory prediction. It provides empirical evidence for the existence of both pricing and advertising collusion forces in our data.

One concern is that products with higher consumer search costs may also tend to be more expensive or have higher production costs. To address this, we run the following equivalent regressions,

Figure 15: Interaction of Consumer Search Costs on Prices



This figure shows the mean prices in markets with low versus high consumer search costs and algorithm usage.

controlling for category fixed effects.

$$price_k = \beta_0 + \beta_1 \cdot \text{search cost high}_k + \beta_2 \cdot \text{algo high}_k + \beta_3 \cdot \text{search cost high}_k \cdot \text{algo high}_k + \epsilon_k \quad (14)$$

Table 1 presents the results. The interaction of algorithm usage and consumer search costs on prices is significantly negative when controlling for category fixed effects.

Table 1: Search Cost and Algorithm Usage Interaction

	Price	
search cost high	63.36*** (5.942)	42.55*** (5.809)
algo high	2.502 (5.942)	1.797 (5.727)
algo high · search cost high	-22.41*** (8.404)	-26.83*** (7.819)
constant	30.45*** (4.077)	42.31*** (3.900)
Category FE	No	Yes
Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1		

6.5 Predictions of Markets with Beneficial Algorithmic Outcomes

In this section, to predict whether algorithmic pricing can be lower, we link the estimated consumer search costs with the crossing condition needed for the algorithms to generate beneficial outcomes for consumers.

The algorithmic pricing can be lower than the competitive price for multiple sellers. Note that the algorithms always facilitate some collusive outcomes between the collusive and competitive prices. The crossing value needed for algorithms to benefit consumers is the same value where the competitive and collusive prices intersect. This intersection depends only on the market parameters we can estimate, not on the specific details about the algorithms. Thus, we can compare the estimated consumer search costs with the crossing value where the competitive and collusive prices intersect to predict in which product markets algorithmic pricing can be lower.

We compute the crossing value of $\hat{\lambda}$ where the competitive prices and collusive prices intersect.²³ If the consumer search costs are greater than the crossing condition needed, we would predict that the algorithms can generate prices lower than the competitive prices in that market.

Figure 16 presents the results. The figure shows the distribution of the fraction of consumers who stop after viewing a quarter of the listings. The vertical dashed line represents the condition needed for algorithms to generate beneficial outcomes for consumers. The results show that for 52% of the Electronics product markets, algorithmic pricing can be lower. For the rest of the market, where consumer search costs are low, the takeaway is that regulation can be helpful in those markets, as the tendency for algorithms to coordinate on high prices still dominates.

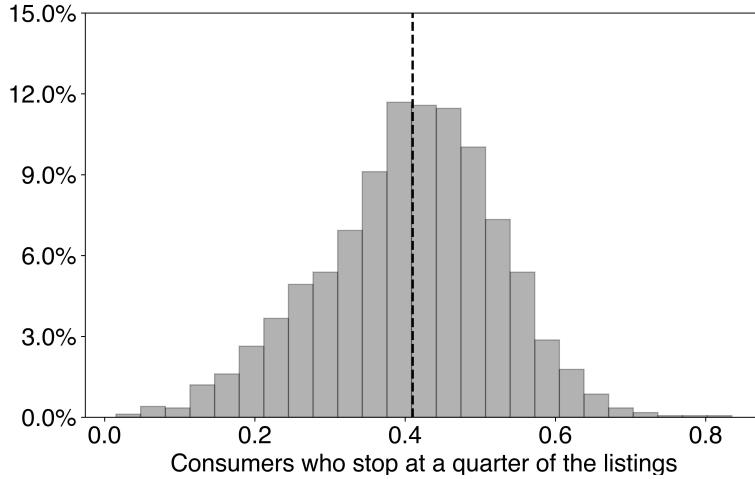
7 Discussion and Conclusion

In this paper we analyzed competing sellers who need to decide on product prices and ad-auction bids in an e-commerce setting. Many of these sellers opt to using algorithms to make these decisions, and past research raised concerns that the outcome might be harmful for consumers.

Our findings show that when consumers have heterogeneous search costs, and enough consumers have substantial costs, then employing reinforcement learning algorithms can surprisingly result in prices that are lower than competitive prices, because the algorithms learn to collude on lower

²³We calibrate the quality of the products to the empirical estimates.

Figure 16: Comparison of Consumer Search Costs and the Crossing Conditions



This figure presents the distribution of the fraction of consumers who stop after viewing a quarter of the listings across different markets. The vertical dashed line illustrates the average condition needed for algorithms to benefit consumers.

advertising bids, which lower the marginal cost for pricing. Lower prices are beneficial to consumers, but can also be beneficial to the e-commerce platform because they increase the demand from consumers which compensates for lower advertising revenues. The sellers also benefit, but mostly from saving on advertising costs.

By analyzing a large-scale dataset of over 2 million products on Amazon.com using a structural model, we also found evidence that for many categories and products, consumer search costs are significant. Our results suggest consumers can benefit when sellers use algorithmic pricing and bidding in 52% of the Electronics product markets we analyzed. And we find a negative interaction between consumer search costs and algorithm usage on pricing, providing empirical validation of our theory.

Platforms can try to strategically respond to sellers using such algorithms in multiple ways. One way is to limit the amount of information available to the sellers (for example, by not disclosing competing bids) to make it harder for the algorithms to learn. We show that this strategy is not likely to make much difference, which is why we focused on incentive based strategies: changing sales commissions or auction reserve prices. On reserve prices we made an interesting observation—counter to classic results, increasing the auction reserve price hurts the platform, because it doesn't

make too much difference if it's low, but lowers revenues dramatically when it is too high, because it causes the algorithm to coordinate on even lower bids. Among the strategies we analyze, only adjusting commission rates can allow the platform to recoup some of the lost revenue due to lower advertising bids. When we analyzed the impact of changes in commissions on sellers and consumers, we found that higher commissions erode the benefits from achieving lower prices through algorithmic collusion, but they still remain lower than without these algorithms.

There are a few limitations to our analysis. First, we focused on two specific decisions of sellers: pricing and advertising, while in reality, sellers might need to learn to make others decisions. However, the principles that we uncovered regarding advertising should operate for most decisions that are realized as costs to the sellers, because sellers are better off agreeing to lower their costs if they do not affect demand. Second, we analyzed a specific reinforcement learning algorithm, and the results might not generalize to any learning algorithm. Although this is a promising direction for more research, we can identify one feature of reinforcement learning that we expect to generalize the results to other algorithms. Reinforcement learning includes an exploration property, where the algorithm always explores other decisions with some probability. This exploration causes the algorithm to converge to a specific non-competitive equilibrium. Even when starting the algorithm at the competitive equilibrium prices and providing it with the true state-action value function (Q matrix), the algorithm will gradually shift to non-competitive equilibrium because of exploration. We believe that in other cases where exploration is an inherent property of the algorithm, it will learn to converge on non-competitive outcomes.

Our paper also opens the door to a few additional research questions. For example, platforms might also act as a seller, selling their own private label products. This self-preferencing (Farronato et al. 2023, Lee and Musolff 2021, Dubé 2022, Lam 2021) incentive will add another layer of complexity which is promising for analysis in future work.

One implication of our findings for research is to emphasize the impact of consumer search and heterogeneous search costs on learning algorithms. These costs create a unique interaction between pricing and advertising, that cause competition and collusion to yield unexpected outcomes. We believe that this finding makes a unique contribution to the growing literature on algorithmic collusion.

Another implication of our findings affects mostly platforms and consumers. There is currently

an ongoing debate about the role of algorithmic decision making on platforms, and whether they are harmful or beneficial to consumers (Halaburda et al. 2018, Berman and Katona 2020, Zhang et al. 2021, Fu et al. 2022, Calder-Wang and Kim 2023, Zhong 2023, Assad et al. 2024, Yang et al. 2024). Our findings provide a nuanced view on this question. Algorithms indeed create tacit collusion when they compete, but some level of collusion is not necessarily bad for consumers (or platforms).

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Appendix

A.1 Model Detail

For simplicity, we drop the time superscript in the one-shot game. To model this uncertainty in auction outcomes, we assume that when a seller submits a bid b_i , the realized bid \tilde{b}_i is stochastic and follows $\tilde{b}_i = \omega_i b_i$, where $\omega_i > 0$ reflects the deviation of the realized bid from the submitted bid. The vector $\boldsymbol{\omega} = (\omega_1, \omega_2)$ follows an exogenous distribution denoted as $F(\cdot)$. Furthermore, we assume when seller i submits a bid b_i , the realized bid in search is subject to randomness and follows a log-normal distribution: $\log(\tilde{b}_i) \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(\log(b_i), \sigma^2)$

Hence, when sellers i and j submit bids b_i and b_j , respectively, the probability that the realized bid of seller i (denoted as \tilde{b}_i) is higher than that of seller j is given by $\Pr\{\tilde{b}_i > \tilde{b}_j\} = \Pr\{\log(\tilde{b}_i) > \log(\tilde{b}_j)\} = \Phi\left(\frac{\log\left(\frac{b_i}{b_j}\right)}{\sqrt{2}\sigma}\right)$

Next, consider the expected Cost Per Click (CPC) that seller i needs to pay, given that he wins the auction, $E[\tilde{b}_i | \tilde{b}_i > \tilde{b}_j]$. Let $u = \frac{\log(\omega_i)}{\sigma} \sim \mathcal{N}(0, 1)$ and $v = \frac{\log(\omega_j)}{\sigma} \sim \mathcal{N}(0, 1)$.

$$\begin{aligned} E\left[\tilde{b}_i | \tilde{b}_i > \tilde{b}_j\right] &= \frac{1}{\Pr\{\tilde{b}_i > \tilde{b}_j\}} \int_{\frac{\log(b_i)}{\sigma} + u > \frac{\log(b_j)}{\sigma} + v} b_i \omega_i \phi(u) \phi(v) du dv \\ &= \frac{1}{\Pr\{\tilde{b}_i > \tilde{b}_j\}} \int_{\frac{\log(b_i)}{\sigma} + u > \frac{\log(b_j)}{\sigma} + v} b_i \exp(\sigma u) \phi(u) \phi(v) du dv \\ &= \frac{b_i}{\Pr\{\tilde{b}_i > \tilde{b}_j\}} \int_{-\infty}^{+\infty} \exp(\sigma u) \phi(u) du \int_{-\infty}^{\frac{\log(b_i)}{\sigma} + u - \frac{\log(b_j)}{\sigma}} \phi(v) dv \\ &= \frac{b_i}{\Pr\{\tilde{b}_i > \tilde{b}_j\}} \int_{-\infty}^{+\infty} \Phi\left(\frac{\log\left(\frac{b_i}{b_j}\right)}{\sigma} + u\right) \exp(\sigma u) \phi(u) du \end{aligned} \quad (15)$$

Thus, the demand is

$$D_i(\mathbf{p}, \mathbf{b}) = \theta \cdot \mathbf{1}\{\tilde{b}_i > \tilde{b}_j\} \cdot s_i(p_i) + (1 - \theta) \cdot s_i(p_i, p_j) \quad (16)$$

In expectation, the demand becomes,

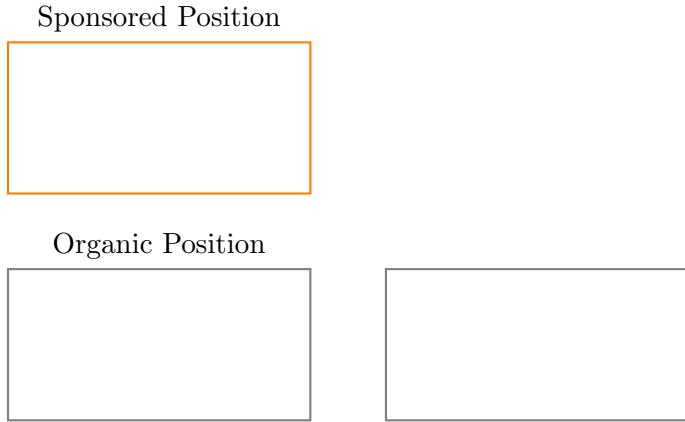
$$D_i(\mathbf{p}, \mathbf{b}) = \theta \cdot \Pr\{\tilde{b}_i > \tilde{b}_j\} \cdot s_i(p_i) + (1 - \theta) \cdot s_i(p_i, p_j) = \theta \cdot \Phi\left(\frac{\log\left(\frac{b_i}{b_j}\right)}{\sqrt{2}\sigma}\right) \cdot s_i(p_i) + (1 - \theta) \cdot s_i(p_i, p_j)$$

Hence, the expected profit for the seller i is

$$\begin{aligned}
& \pi_i(\mathbf{p}, \mathbf{b}) \\
&= \theta \cdot \Phi\left(\frac{\log\left(\frac{b_i}{b_j}\right)}{\sqrt{2}\sigma}\right) \left((1 - \tau) \cdot p_i - c_i - \gamma_i \cdot \mathbb{E}[\tilde{b}_i | \tilde{b}_j < \tilde{b}_i] \right) \cdot s_i(p_i) + (1 - \theta) \cdot ((1 - \tau) \cdot p_i - c_i) \cdot s_i(p_i, p_j) \\
&= \theta \cdot \Phi\left(\frac{\log\left(\frac{b_i}{b_j}\right)}{\sqrt{2}\sigma}\right) \cdot s_i(p_i) \left((1 - \tau) \cdot p_i - c_i - \gamma_i \cdot \underbrace{\frac{b_i}{\Phi\left(\frac{\log\left(\frac{b_i}{b_j}\right)}{\sqrt{2}\sigma}\right)}}_{\tilde{c}_i} \int_{-\infty}^{+\infty} \Phi\left(\frac{\log\left(\frac{b_i}{b_j}\right)}{\sigma} + u\right) \exp(\sigma u) \phi(u) du \right) \\
&\quad + (1 - \theta) \cdot ((1 - \tau) \cdot p_i - c_i) \cdot s_i(p_i, p_j)
\end{aligned} \tag{17}$$

A.2 Sponsored Ads Examples

Figure A1: Graphical Representation of the Theoretical Model



This figure provides a graphical representation of the search result page layout. There is a sponsored position on the top, followed by organic positions.

Figure 1 illustrates an example of an Amazon search results page as viewed on a desktop or laptop browser, which contains a mix of both sponsored and organic products. Key information presented on the search results page includes the product's image, price, sales, customer ratings, and delivery options. The mobile app experience is similar, displaying a mix of sponsored and organic products vertically that consumers can scroll down or click on for more details.

Figure A2: Example of Amazon Search Result Page With 22 Products

Results
Check each product page for other buying options.



COSORI Pro III Air Fryer Dual Blaze, 6.8-Quart, Precise Temps Prevent Overcooking, Heating Adjusts for a True Air Fry, Bake, Roast, and Broil, Even and Fast Cooking, In-App Recipes, 1750W, Dark gray

★★★★★ ✓ 3,465
1K+ bought in past month

\$179⁹⁹
Save \$25.00 with coupon
✓prime One-Day
FREE delivery Tomorrow
Works with Alexa

Add to cart



Nuwave Brio 10-in-1 Air Fryer 7.25Qt with Patented Linear T Thermal Technology for Crisping, Roasting, Dehydrating, and Reheating Non-Stick, Dishwasher Safe Basket, and App with 100+ Recipes - Black

★★★★★ ✓ 10,416
1K+ bought in past month

\$84⁹⁹ List: \$99.99
Or \$28.33/month for 3 months (no fees or interest)
✓prime Overnight
FREE delivery Overnight 4 AM - 8 AM

Add to cart

Best Seller



Ninja AF101 Air Fryer that Crisps, Roasts, Reheats, & Dehydrates, for Quick, Easy Meals, 4 Quart Capacity, & High Gloss Finish, Grey

★★★★★ ✓ 53,183
30K+ bought in past month

\$89⁹⁵ List: \$129.99
Or \$29.99/month for 3 months (no fees or interest)
✓prime Overnight
FREE delivery Overnight 4 AM - 8 AM

Add to cart
More Buying Choices
\$68.00 (12 used & new offers)



Air Fryers 4 Qt, Fabulette 9 Customizable Smart Cooking Programs Compact Air Fryer, Shake Reminder, Powerful 1550W Electric Oilless Cooker, Tempered Glass Display, Dishwasher-Safe & Nonstick

★★★★★ ✓ 3,093
800+ bought in past month

\$59⁹⁹ List: \$99.99
✓prime One-Day
FREE delivery Tomorrow

Add to cart



Instant Vortex 6QT XL Air Fryer, 4-in-1 Functions that Crisps, Roasts, Reheats, Bakes for Quick Easy Meals, 100+ In-App Recipes, is Dishwasher-Safe, from the Makers of Instant Pot, Black

★★★★★ ✓ 11,883
1K+ bought in past month

\$79⁹⁵ List: \$119.99
✓prime Overnight
FREE delivery Overnight 4 AM - 8 AM

Add to cart
More Buying Choices
\$57.43 (8 used & new offers)

This figure presents an example of the first 5 products of a search results page with 22 product on Amazon.com when an anonymous consumer enters the keyword “humidifiers for bedroom.” The products whose labels are encircled in orange rectangles are sponsored products, obtained by sellers who won these positions through bidding in auctions.

A.3 Competitive Price and Benchmark Price

Nash-Bertrand equilibrium with price competition only

The profit function for seller i is

$$\pi_i(\mathbf{p}) = \left(\theta \cdot \frac{1}{2} \cdot s_i(p_i) + (1 - \theta) \cdot s_i(p_i, p_j) \right) \cdot ((1 - \tau) \cdot p_i - c_i) \quad (18)$$

Take the derivative of seller i 's profit with respect to p_i

$$\begin{aligned} \frac{\partial \pi_i(\mathbf{p})}{\partial p_i} &= \underbrace{\left(\frac{\theta}{2} \cdot s_i(p_i) \left[(1 - \tau) - (1 - s_i(p_i)) \cdot \frac{(1 - \tau) \cdot p_i - c_i}{\mu} \right] \right)}_{D\pi_i^{top}} \\ &\quad + \underbrace{(1 - \theta) \cdot s_i(p_i, p_j) \left[(1 - \tau) - (1 - s_i(p_i, p_j)) \cdot \frac{(1 - \tau) \cdot p_i - c_i}{\mu} \right]}_{D\pi_i^{all}} \end{aligned} \quad (19)$$

Let $p_i^{oN}(\theta)$ denote the Nash-Bertrand equilibrium price given θ , and specifically, let $p_i^{\tilde{oN}}$ denote the price when $\theta = 1$. By definition, we have

$$\frac{\partial \pi_i(\mathbf{p})}{\partial p_i}|_{\theta=1} = \frac{1}{2} \cdot s_i(p_i^{\tilde{oN}}) \left[(1 - \tau) - (1 - s_i(p_i^{\tilde{oN}})) \cdot \frac{(1 - \tau) \cdot p_i^{\tilde{oN}} - c_i}{\mu} \right] = 0 \quad (20)$$

Since $s_i(p_i^{\tilde{oN}}) > 0$, thus we have

$$(1 - \tau) - (1 - s_i(p_i^{\tilde{oN}})) \cdot \frac{(1 - \tau) \cdot p_i^{\tilde{oN}} - c_i}{\mu} = 0 \quad (21)$$

The left-hand side of (21) is monotonically decreasing in $p_i^{\tilde{oN}}$, thus there exists a unique $p_i^{\tilde{oN}}$ such that (21) holds. Similarly, $D\pi_i^{all}$ is also monotonically decreasing in p_i . Thus, $\frac{\partial \pi_i(\mathbf{p})}{\partial p_i}$ is monotonically decreasing in p_i . This implies that, for all values of θ , there exists a unique $p_i^{oN}(\theta)$.

Next, for $\theta < 1$, if we plug in $p_i^{\tilde{oN}}$,

$$\begin{aligned} \frac{\partial \pi_i(\mathbf{p})}{\partial p_i}|_{\theta<1, p_i=p_i^{\tilde{oN}}, p_j=p_j^{\tilde{oN}}} &= (1 - \theta) \cdot s_i(p_i^{\tilde{oN}}, p_j^{\tilde{oN}}) \left[(1 - \tau) - (1 - s_i(p_i^{\tilde{oN}}, p_j^{\tilde{oN}})) \cdot \frac{(1 - \tau) \cdot p_i^{\tilde{oN}} - c_i}{\mu} \right] \\ &<_{s_i(p_i^{\tilde{oN}}) > s_i(p_i^{\tilde{oN}}, p_j^{\tilde{oN}})} (1 - \theta) \cdot s_i(p_i^{\tilde{oN}}, p_j^{\tilde{oN}}) \left[(1 - \tau) - (1 - s_i(p_i^{\tilde{oN}})) \cdot \frac{(1 - \tau) \cdot p_i^{\tilde{oN}} - c_i}{\mu} \right] \\ &= 0 \end{aligned} \quad (22)$$

The FOC $\frac{\partial \pi_i(\mathbf{p})}{\partial p_i}|_{\theta<1, p_i=p_i^{\tilde{oN}}, p_j=p_j^{\tilde{oN}}}$ is negative. Combined with the fact that $\frac{\partial \pi_i(\mathbf{p})}{\partial p_i}$ is monotonically decreasing in p_i , implies that at $\theta < 1$, it must be $p_i^{oN}(\theta) < p_i^{\tilde{oN}}$ such that $\frac{\partial \pi_i(\mathbf{p})}{\partial p_i}|_{\theta, p_i=p_i^{oN}(\theta), p_j=p_j^{oN}(\theta)} =$

0.

Next, for $\theta_1 < \theta_2$, we are going to show that $p_i^{oN}(\theta_1) < p_i^{oN}(\theta_2)$. Letting θ_1 approach θ_2 from the left and taking the limit, we have monotonicity.

By definition,

$$\begin{aligned} & \frac{\partial \pi_i(\mathbf{p})}{\partial p_i} \Big|_{\theta_1, p_i = p_i^{oN}(\theta_1), p_j = p_j^{oN}(\theta_1)} \\ &= \underbrace{\frac{\theta_1}{2} \cdot s_i(p_i^{oN}(\theta_1)) \left[(1 - \tau) - (1 - s_i(p_i^{oN}(\theta_1))) \cdot \frac{(1 - \tau) \cdot p_i^{oN}(\theta_1) - c_i}{\mu} \right]}_{D\pi_i^{top}} \\ &+ \underbrace{(1 - \theta_1) \cdot s_i(p_i^{oN}(\theta_1), p_j^{oN}(\theta_1)) \left[(1 - \tau) - (1 - s_i(p_i^{oN}(\theta_1), p_j^{oN}(\theta_1))) \cdot \frac{(1 - \tau) \cdot p_i^{oN}(\theta_1) - c_i}{\mu} \right]}_{D\pi_i^{all}} = 0 \end{aligned}$$

In must be the case that the first term $D\pi_i^{top} > 0$ and the second term $D\pi_i^{all} < 0$. Otherwise, suppose $D\pi_i^{top} < 0$, it must be $D\pi_i^{all} < 0$ as $s_i(p_i^{oN}(\theta)) > s_i(p_i^{oN}(\theta), p_j^{oN}(\theta))$. So, $\frac{\partial \pi_i(\mathbf{p})}{\partial p_i} = 0$ cannot hold.

Next, at θ_2 , if we plug in $p_i^{oN}(\theta_1)$,

$$\begin{aligned} & \frac{\partial \pi_i(\mathbf{p})}{\partial p_i} \Big|_{\theta_2, p_i = p_i^{oN}(\theta_1), p_j = p_j^{oN}(\theta_2)} \\ &= \frac{\theta_2}{2} \cdot s_i(p_i^{oN}(\theta_1)) \left[(1 - \tau) - (1 - s_i(p_i^{oN}(\theta_1))) \cdot \frac{(1 - \tau) \cdot p_i^{oN}(\theta_1) - c_i}{\mu} \right] \\ &+ (1 - \theta_2) \cdot s_i(p_i^{oN}(\theta_1), p_j^{oN}(\theta_2)) \left[(1 - \tau) - (1 - s_i(p_i^{oN}(\theta_1), p_j^{oN}(\theta_2))) \cdot \frac{(1 - \tau) \cdot p_i^{oN}(\theta_1) - c_i}{\mu} \right] \\ &> 0 \end{aligned} \tag{23}$$

Recall that $\frac{\partial \pi_i(\mathbf{p})}{\partial p_i}$ is monotonically decreasing in p_i . It must be the case that $p_i^{oN}(\theta_2) > p_i^{oN}(\theta_1)$ such that $\frac{\partial \pi_i(\mathbf{p})}{\partial p_i} \Big|_{\theta_2, p_i = p_i^{oN}(\theta_2), p_j = p_j^{oN}(\theta_2)} = 0$ holds. Thus, for all values of θ , i.e., $\forall \theta \in [0, 1]$, we have $\frac{\partial p_i^{oN}}{\partial \theta} \geq 0$, i.e., the benchmark price \mathbf{p}^{oN} is monotonically increasing in θ .

Nash-Bertrand equilibrium with price and bid competition

We will show that for $\theta \in [0, 1]$, $p_i^N(\theta) \geq p_i^{oN}(\theta)$

Since $\tilde{c}_i = \gamma_i \cdot \frac{b_i}{\Phi\left(\frac{\log\left(\frac{b_i}{b_j}\right)}{\sqrt{2}\sigma}\right)} \int_{-\infty}^{+\infty} \Phi\left(\frac{\log\left(\frac{b_i}{b_j}\right)}{\sigma} + u\right) \exp(\sigma u) \phi(u) du \geq 0$. Thus, given a specific

value of θ , if we plug in $p_i^{oN}(\theta)$,

$$\begin{aligned} \frac{\partial \pi_i(\mathbf{p}, \mathbf{b})}{\partial p_i^N} \Big|_{p_i^{oN}(\theta)} &= \frac{\theta}{2} \cdot s_i(p_i^{oN}) \left[(1 - \tau) - (1 - s_i(p_i^{oN})) \cdot \frac{(1 - \tau) \cdot p_i^{oN} - (c_i + \tilde{c}_i)}{\mu} \right] \\ &\quad + (1 - \theta) \cdot s_i(p_i^{oN}, p_j^{oN}) \left[(1 - \tau) - (1 - s_i(p_i^{oN}, p_j^{oN})) \cdot \frac{(1 - \tau) \cdot p_i^{oN} - c_i}{\mu} \right] \\ &> 0 \end{aligned} \tag{24}$$

Given that $\frac{\partial \pi_i(\mathbf{p}, \mathbf{b})}{\partial p_i}$ is also monotonically decreasing in p_i . It must be the case that $p_i^N(\theta) > p_i^{oN}(\theta)$ such that $\frac{\partial \pi_i(\mathbf{p}, \mathbf{b})}{\partial p_i} \Big|_{\theta, p_i = p_i^N(\theta), p_j = p_j^N(\theta)} = 0$ holds.

A.4 Micro-Foundation of θ

At each position n , consumers will form the expectation of the incremental utility of continuing their search to the next position, which is $\log \left(1 + e^{\delta_1} + e^{\delta_2} + \dots + E \left[e^{\hat{\delta}_{n+1}} | \delta_1, \delta_2, \dots, \delta_n \right] \right) - \log \left(1 + e^{\delta_1} + e^{\delta_2} + \dots + e^{\delta_n} \right) = \log \left(\frac{E \left[e^{\hat{\delta}_{n+1}} | \delta_1, \delta_2, \dots, \delta_n \right]}{1 + e^{\delta_1} + e^{\delta_2} + \dots + e^{\delta_n}} + 1 \right)$, and compare with the cost s of search one product. If the cost is lower than the expected incremental utility, they will continue to search for the product in position $n + 1$; otherwise, they will stop at position n .

For the two-product example, suppose that the cumulative distribution function of the cost is F_s .

- Suppose that every consumer search at least one product, then

$$\log \left(1 + E \left[\hat{\delta}_1 \right] \right) - 0 > s$$

would always hold.

- Consumers search the first position but not the second

$$\log(1 + \delta_1) - 0 > s > \log \left(1 + \delta_1 + E \left[\hat{\delta}_2 | \delta_1 \right] \right) - \log(1 + \delta_1)$$

- Consumers search the second

$$\log \left(1 + \delta_1 + E \left[\hat{\delta}_2 | \delta_1 \right] \right) - \log(1 + \delta_1) > s$$

Then θ and $1 - \theta$ can be expressed as

$$\theta = \frac{F_s \left(\log \left(1 + E \left[\hat{\delta}_1 \right] \right) \right) - F_s \left(\log \left(1 + \delta_1 + E \left[\hat{\delta}_2 | \delta_1 \right] \right) - \log(1 + \delta_1) \right)}{F_s \left(\log \left(1 + E \left[\hat{\delta}_1 \right] \right) \right)}$$

$$1 - \theta = \frac{F_s \left(\log \left(1 + \delta_1 + E \left[\hat{\delta}_2 | \delta_1 \right] \right) - \log (1 + \delta_1) \right)}{F_s \left(\log \left(1 + E \left[\hat{\delta}_1 \right] \right) \right)}$$

A.5 Theoretical Collusion

Monopoly/Collusion:

The sellers' objective function are

$$\max_{p_i, p_j, b_i, b_j} \pi_M(\mathbf{p}, \mathbf{b}) = \pi_i(\mathbf{p}, \mathbf{b}) + \pi_j(\mathbf{p}, \mathbf{b}) \quad (25)$$

First notice the two sellers would agree to set their bids to the minimum, $b_i = b_j \rightarrow 0$. This strategy minimizes advertising costs while maintaining the same probability of capturing the demand from consumers who consider only the first position. The monopolist profit becomes:

$$\begin{aligned} & \max_{p_i, p_j = p_i} \pi_M(\mathbf{p}, \mathbf{0}) \\ &= \left(\frac{\theta}{2} \cdot (s_i(p_i) + s_j(p_j)) + (1 - \theta) \cdot (s_i(p_i, p_j) + s_j(p_i, p_j)) \right) \cdot ((1 - \tau) \cdot p_i - c_i) \\ &\stackrel{p_j = p_i}{=} (\theta \cdot s_i(p_i) + (1 - \theta) \cdot 2 \cdot s_i(p_i, p_i)) \cdot ((1 - \tau) \cdot p_i - c_i) \end{aligned} \quad (26)$$

Take the derivative of the monopolist's profit with respect to p_i

$$\begin{aligned} \frac{\partial \pi_M(\mathbf{p})}{\partial p_i} &= \underbrace{\theta \cdot s_i(p_i) \left[(1 - \tau) - (1 - s_i(p_i)) \cdot \frac{(1 - \tau) \cdot p_i - c_i}{\mu} \right]}_{D\pi_i^{sponsored}} \\ &+ \underbrace{(1 - \theta) \cdot s_i(p_i, p_i) \left[(1 - \tau) - (1 - 2 \cdot s_i(p_i, p_i)) \cdot \frac{(1 - \tau) \cdot p_i - c_i}{\mu} \right]}_{D\pi_i^{all}} \end{aligned} \quad (27)$$

Let $p_i^M(\theta)$ denote the monopoly price given θ , and specifically, let \tilde{p}_i^M denote the price when $\theta = 1$. Then, we have

$$\frac{\partial \pi_M(\mathbf{p})}{\partial p_i}|_{\theta=1} = s_i(\tilde{p}_i^M) \left[(1 - \tau) - (1 - s_i(\tilde{p}_i^M)) \cdot \frac{(1 - \tau) \cdot \tilde{p}_i^M - c_i}{\mu} \right] = 0 \quad (28)$$

Since $s_i(\tilde{p}_i^M) > 0$, we have

$$(1 - \tau) - (1 - s_i(\tilde{p}_i^M)) \cdot \frac{(1 - \tau) \cdot \tilde{p}_i^M - c_i}{\mu} = 0 \quad (29)$$

Rearranging

$$\underbrace{\left(\exp\left(\frac{a_i - p_i^M}{\mu} \right) + 1 \right)}_{\text{inverse of outside option market share}} - \underbrace{\frac{(1 - \tau) \cdot p_i^M - c_i}{\mu \cdot (1 - \tau)}}_{\text{effective margin}} = 0 \quad (30)$$

The above equation can be interpreted as the effective margin equals the inverse of the outside option market share.

The left-hand side is strictly decreasing in p_i^M , which implies that there exists a unique \tilde{p}_i^M such that (29) holds. Similarly, $D\pi_i^{all}$ is also monotonically decreasing in p_i . Thus, $\frac{\partial \pi_M(\mathbf{p})}{\partial p_i}$ is monotonically decreasing in p_i . This implies that, for all values of θ , there exists a unique $p_i^M(\theta)$.

Next, for $\theta < 1$, if we plug in p_i^M ,

$$\begin{aligned} \frac{\partial \pi_M(\mathbf{p})}{\partial p_i} \Big|_{\theta < 1, p_i = \tilde{p}_i^M} &= (1 - \theta) \cdot s_i(p_i^M, p_i^M) \left[(1 - \tau) - \left(1 - 2 \cdot s_i(p_i^M, p_i^M) \right) \cdot \frac{(1 - \tau) \cdot p_i^M - c_i}{\mu} \right] \\ &> \underbrace{s_i(p_i^M) < 2 \cdot s_i(p_i^M, p_i^M)}_{(1 - \theta) \cdot s_i(p_i^M, p_i^M)} \left[(1 - \tau) - \left(1 - s_i(p_i^M) \right) \cdot \frac{(1 - \tau) \cdot p_i^M - c_i}{\mu} \right] \\ &= 0 \end{aligned} \quad (31)$$

The FOC $\frac{\partial \pi_i(\mathbf{p})}{\partial p_i} \Big|_{\theta < 1, p_i = \tilde{p}_i^M, p_j = \tilde{p}_j^M}$ is positive. Combined with the fact that $\frac{\partial \pi_M(\mathbf{p})}{\partial p_i}$ is monotonically decreasing in p_i , implies that at $\theta < 1$, it must be $p_i^M(\theta) > \tilde{p}_i^M$ such that $\frac{\partial \pi_M(\mathbf{p})}{\partial p_i} \Big|_{\theta < 1, p_i = p_i^M(\theta)} = 0$.

Next, for $\theta_1 < \theta_2$, we are going to show that $p_i^M(\theta_1) > p_i^M(\theta_2)$. Letting θ_1 approach θ_2 from the left and taking the limit, we have monotonicity.

By definition,

$$\begin{aligned} &\frac{\partial \pi_M(\mathbf{p})}{\partial p_i} \Big|_{\theta_2, p_i = p_i^M(\theta_2), p_j = p_j^M(\theta_2)} \\ &= \underbrace{\theta_2 \cdot s_i(p_i^M(\theta_2)) \left[(1 - \tau) - (1 - s_i(p_i^M(\theta_2))) \cdot \frac{(1 - \tau) \cdot p_i^M(\theta_2) - c_i}{\mu} \right]}_{D\pi_i^{sponsored}} \\ &\quad + \underbrace{(1 - \theta_2) \cdot s_i(p_i^M(\theta_2), p_i^M(\theta_2)) \left[(1 - \tau) - (1 - 2 \cdot s_i(p_i^M(\theta_2), p_i^M(\theta_2))) \cdot \frac{(1 - \tau) \cdot p_i^M(\theta_2) - c_i}{\mu} \right]}_{D\pi_i^{all}} = 0 \end{aligned} \quad (32)$$

In must be the case that $D\pi_i^{sponsored} < 0$ and $D\pi_i^{all} > 0$. Otherwise, suppose $D\pi_i^{all} < 0$, it must be $D\pi_i^{sponsored} < 0$ as $s_i(p_i^M(\theta)) < 2 \cdot s_i(p_i^M(\theta), p_i^M(\theta))$. So, $\frac{\partial \pi_M(\mathbf{p})}{\partial p_i} = 0$ cannot hold.

Next, at θ_1 , if we plug in $p_i^M(\theta_2)$,

$$\begin{aligned} & \frac{\partial \pi_M(\mathbf{p})}{\partial p_i} \Big|_{\theta_1, p_i=p_i^M(\theta_2), p_j=p_j^M(\theta_2)} \\ &= \frac{\theta_1}{2} \cdot s_i(p_i^M(\theta_2)) \left[(1 - \tau) - (1 - s_i(p_i^M(\theta_2))) \cdot \frac{(1 - \tau) \cdot p_i^M(\theta_2) - c_i}{\mu} \right] \\ &+ (1 - \theta_1) \cdot s_i(p_i^M(\theta_2), p_j^M(\theta_2)) \left[(1 - \tau) - (1 - s_i(p_i^M(\theta_2), p_j^M(\theta_2))) \cdot \frac{(1 - \tau) \cdot p_i^M(\theta_2) - c_i}{\mu} \right] \\ &> 0 \end{aligned} \quad (33)$$

Recall that $\frac{\partial \pi_M(\mathbf{p})}{\partial p_i}$ is also monotonically decreasing in p_i . It must be the case that $p_i^M(\theta_1) > p_i^M(\theta_2)$ such that $\frac{\partial \pi_M(\mathbf{p})}{\partial p_i} \Big|_{\theta_1, p_i=p_i^M(\theta_1), p_j=p_j^M(\theta_1)} = 0$ holds. Thus, for all values of θ , i.e., $\forall \theta \in [0, 1]$, we have $\frac{\partial p_i^M}{\partial \theta} \leq 0$. The monopoly price \mathbf{p}^M is monotonically decreasing in θ .

The first-order condition of the monopolist's profit with respect to price can be interpreted as the inverse of the outside market share equaling a weighted profit margin. Let \tilde{p}_i^M denote the monopoly price when $\theta = 1$, that is, $\tilde{p}_i^M = p_i^M(\theta = 1)$. When $\theta = 1$, and all consumers only consider the top product, the outside option market share is $1 - s_i(\tilde{p}_i^M)$. However, when $\theta = 0$, assuming the monopolist still charges the price as if $\theta = 1$, the outside option market share is $1 - 2 \cdot s_i(\tilde{p}_i^M, \tilde{p}_i^M)$, which is smaller than $1 - s_i(\tilde{p}_i^M)$, i.e., $1 - 2 \cdot s_i(\tilde{p}_i^M, \tilde{p}_i^M) < 1 - s_i(\tilde{p}_i^M)$. In this sense, when θ is smaller, the outside option is less competitive, allowing the monopolist to further increase the price and obtain a larger profit margin. Regarding general values of θ , the partial derivative of the monopolist with respect to price is a convex combination of the sponsored position part and the both positions part. At the optimal monopoly price $p_i^M(\theta)$, the partial derivative of the sponsored position part is always negative, while that of both positions is positive; otherwise, the first-order condition cannot be zero. Hence, the larger the fraction of consumers considering both positions (small values of θ), the higher the price the monopolist can charge.

Limiting Case In the limiting case where all consumers focus only on the first position, i.e., $\theta = 1$, the monopoly price \mathbf{p}^M is equal to the benchmark price \mathbf{p}^{oN} , i.e., $\mathbf{p}^M = \mathbf{p}^{oN}$.

This is because in both cases, the advertising costs are zero. In the monopoly case, the colluding sellers bid as low as possible, while in the benchmark scenario, there are no sponsored ads, thus the advertising cost is zero by definition. When $\theta = 1$, consumers in both the benchmark and monopoly cases consider only one product. Consequently, this gives both monopolists and duopolists in price competition identical objective functions when setting prices, results in the same equilibrium prices in the market.

Single-crossing The competition price \mathbf{p}^N with ads is always greater than or equal to the benchmark price, i.e., $\mathbf{p}^N \geq \mathbf{p}^{oN}$. Combined with the limiting case, it follows that when $\theta = 1$, $\mathbf{p}^N > \mathbf{p}^{oN} = \mathbf{p}^M$; and combined the monotonicity property, we get when $\theta = 0$, $\mathbf{p}^M > \mathbf{p}^{oN} = \mathbf{p}^N$.

The monopoly price \mathbf{p}^M monotonically decreases in θ , and the Nash-Bertrand \mathbf{p}^N monotonically

increases in θ , which implies that there is a threshold $\tilde{\theta} < 1$ such that $\mathbf{p}^N(\tilde{\theta}) = \mathbf{p}^M(\tilde{\theta})$. When θ is larger than this threshold, the monopoly prices become lower than the competition prices, i.e., when $\theta > \tilde{\theta}$, $\mathbf{p}^N(\theta) > \mathbf{p}^M(\theta)$.

To see why the benchmark price \mathbf{p}^{oN} is always the lowest: $\mathbf{p}^N \geq \mathbf{p}^{oN}$. Additionally, the monopoly price \mathbf{p}^M monotonically decreases in θ , and the benchmark \mathbf{p}^N monotonically increases in θ , when $\theta = 1$, $\mathbf{p}^{oN} = \mathbf{p}^M$, which implies that $\mathbf{p}^M \geq \mathbf{p}^{oN}, \forall \theta$.

A.6 Casual Effect of Rank on Sales Model

Following Reimers and Waldfogel (2023), we consider an alternative, more of a reduced-form characterization of consumer search costs. Specifically,

The consumer's utility for product j when ranked at r_j is given by:

$$u_{ij} = X'_j \beta - \alpha p_j + \zeta r_j + \zeta' r_j^0 + \xi_j + \epsilon_{ij}$$

where the outside good has utility 0, ξ_j represents unobserved product quality, r_j^0 is the initial rank of product j , and ϵ_{ij} is an extreme value error. Here, ζ is the causal effect of rank on sales, while ζ' reflects the systematic product quality variation across ranks that is not accounted for by x_j .

The mean expected utility of product j when ranked at r is $\delta_j(r) = X'_j \beta - \alpha p_j + \zeta' r_j^0 + \zeta r$. Even if product j were moved to a different rank r_j , the part of utility reflected by $\zeta' r_j^0$ would remain, while the part reflected by ζr would change. The “rank-independent expected mean utility” is

$$\delta_j^0 \equiv X'_j \beta - \alpha p_j + \zeta' r_j^0 = \delta_j(r) - \zeta r.$$

Product j 's market share when ranked r^{th} is given by $s_j(r) = \frac{e^{\delta_j(r)}}{1 + \sum e^{\delta_j(r)}}$. The way in which the products are ranked affects both consumer well-being and the propensity for consumers to purchase.

Then assume that the ad auction part is the same as the one we consider in our benchmark model, while the consumer choice is from this model. In the full competition scenario, where sellers compete on both prices and bids, the profit function becomes:

$$\begin{aligned} \pi_i^t(\mathbf{p}^t, \mathbf{b}^t) &= \Pr(\tilde{b}_i^t > \tilde{b}_j^t) \cdot \frac{e^{\frac{a_i - p_i^t}{\mu}}}{e^{\frac{a_i - p_i^t}{\mu}} + e^{\frac{a_j - p_j^t - \zeta}{\mu}} + 1} \cdot \left(((1 - \tau) \cdot p_i^t - c_i) - \gamma_i \cdot \mathbb{E} \left[\tilde{b}_i^t \mid \tilde{b}_i^t > \tilde{b}_j^t \right] \right) \\ &\quad + \Pr(\tilde{b}_i^t < \tilde{b}_j^t) \cdot \frac{e^{\frac{a_i - p_i^t - \zeta}{\mu}}}{e^{\frac{a_i - p_i^t - \zeta}{\mu}} + e^{\frac{a_j - p_j^t}{\mu}} + 1} \cdot \left(((1 - \tau) \cdot p_i^t - c_i) \right) \end{aligned} \tag{34}$$

In the scenario where there are no sponsored ads and the platform randomly displays one of

the products in the top position, the sellers' profit function is:

$$\pi_i^t(\mathbf{p}^t, \mathbf{b}^t) = \frac{1}{2} \cdot \left(\frac{e^{\frac{a_i - p_i^t}{\mu}}}{e^{\frac{a_i - p_i^t}{\mu}} + e^{\frac{a_j - p_j^t - \zeta}{\mu}}} + \frac{e^{\frac{a_i - p_i^t - \zeta}{\mu}}}{e^{\frac{a_i - p_i^t - \zeta}{\mu}} + e^{\frac{a_j - p_j^t}{\mu}}} \right) \cdot (((1 - \tau) \cdot p_i^t - c_i))$$

In this model, the consumer surplus can be expressed as

$$U(\mathbf{p}^t) = \Pr(\tilde{b}_i^t > \tilde{b}_j^t) \cdot \mu \cdot \log \left[e^{\frac{a_i - p_i^t}{\mu}} + e^{\frac{a_j - p_j^t - \zeta}{\mu}} + 1 \right] + \Pr(\tilde{b}_i^t < \tilde{b}_j^t) \cdot \mu \cdot \log \left[e^{\frac{a_i - p_i^t - \zeta}{\mu}} + e^{\frac{a_j - p_j^t}{\mu}} + 1 \right]$$

Figure A3 presents the results. As we can see from the figure, the prices, sellers' profit, and consumer surplus follow the same pattern as those in Figure 2. In the limiting case, when the causal effect of rank on sales term $\zeta \rightarrow \infty$ ($\zeta \rightarrow 0$), it is equivalent to the case where the fraction of consumers who only care about the first position $\theta \rightarrow 1$ ($\theta \rightarrow 0$).

Despite the modeling details regarding consumer search costs, we show that the robustness of our findings, that competitive prices can be higher than collusive prices when consumer search costs are high, holds under various model assumptions.

A.7 Robustness

A.7.1 Alternative Characterization of Consumer Search Costs

In Reimers and Waldfogel (2023), the authors consider an alternative, more reduced-form characterization of consumer search costs. In Appendix A.6, we present the details of that model and demonstrate the correspondence between their model and ours. We show that the details of modeling consumer search costs do not change our main findings in this section: when consumer search costs or the effects of rank on sales are strong, collusive prices can fall below competitive prices.

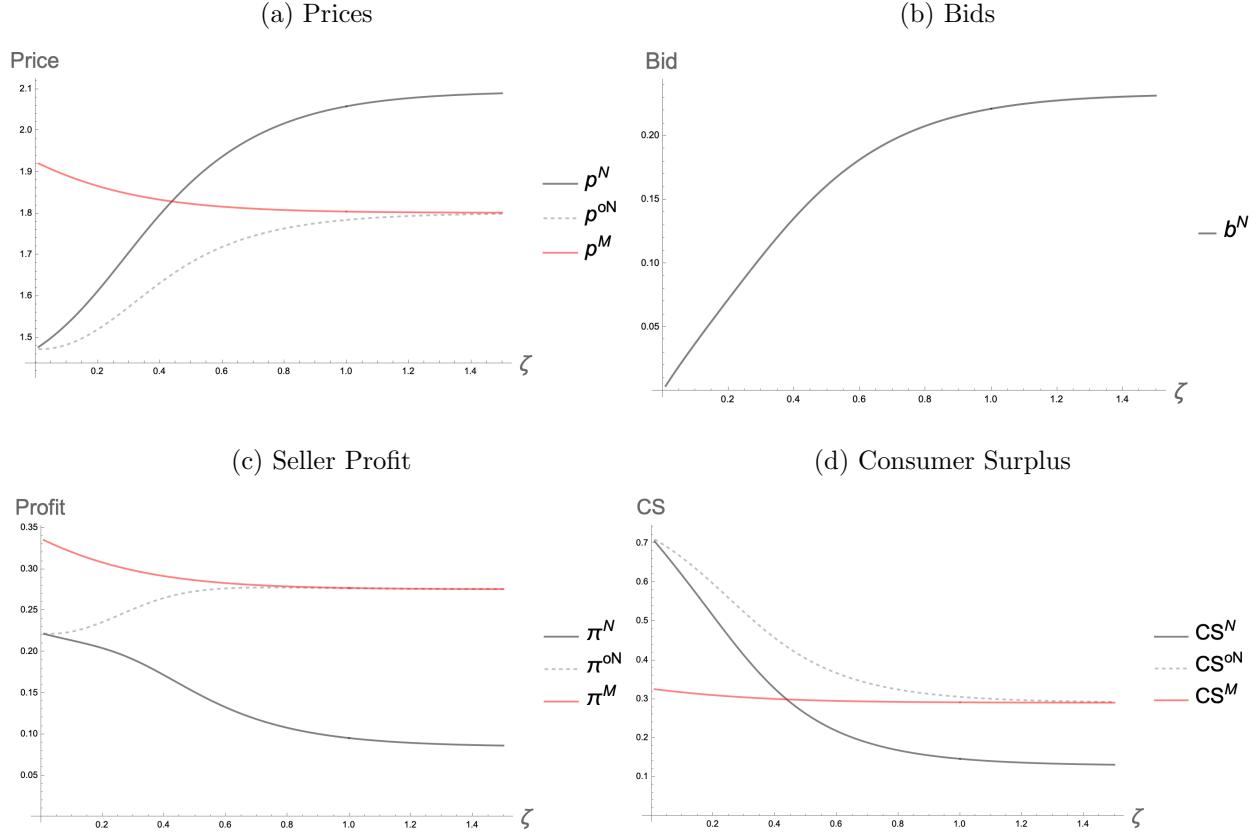
A.7.2 Asymmetric Sellers

Now, we consider a case of sellers having products of differentiated quality. In the benchmark case without ads, the rankings of products appearing in the organic positions depend on the platform's recommendation system. For example, the platform always displays the product with the highest quality or does so probabilistically. Similarly, in the collusive case, how sellers collude may vary.²⁴ Thus, we only present the results of the full competition case and the algorithmic pricing case.

Figure A4 shows the sellers' profit and consumer surplus, previously. Similar to our previous results, algorithms benefit both sellers. And when consumer search costs are high, algorithmic pricing can benefit consumers, generating a higher consumer surplus.

²⁴To maximize the joint profit, the seller with high quality should be displayed first. However, that would hurt the other product, and they might not be able to reach a collusion agreement. Hence, the sellers might agree to take turns in the first position, with probability proportional to their quality.

Figure A3: Casual Effect of Rank Model



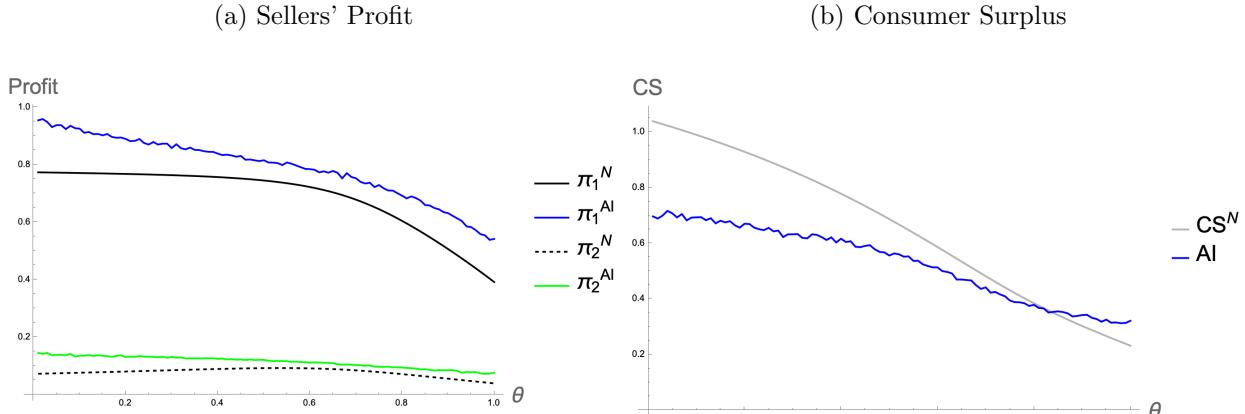
The subfigures (a) - (d) show the equilibrium prices, bids, sellers' profit, and consumer surplus as functions of the casual effect on sales parameter ζ , respectively. The representation of other lines and the parameter specifications are the same as in Figure 2.

A.7.3 Alternative Bid Information in State Space

We previously assume that the algorithms make decisions based only on their own bids. The platform might experiment with the auction design to disclose the winning bids, or the sellers could subscribe to a third-party market intelligence data service to obtain more bidding information in the market and make more informed decisions. The additional information will change the sellers' bids, and we are interested in how it will affect the equilibrium outcomes. Hence, we also consider the following *full stateful* scenario, where both agents' bids are known; that is, the state space is $s_{it} = (p_{it-1}, p_{jt-1}, b_{it-1}, b_{jt-1})$.

Figure A5 presents the results of the *full stateful* scenario in comparison to the benchmark scenario. We find that the outcomes facilitated by the algorithms, which are beneficial to both sellers and consumers, are robust to assumptions regarding the bid information available in the algorithm's state space. In our setting, the *full stateful* condition yields outcomes closer to the theoretical full collusion case. That is, it generates higher profits for the sellers, bids closer to zero,

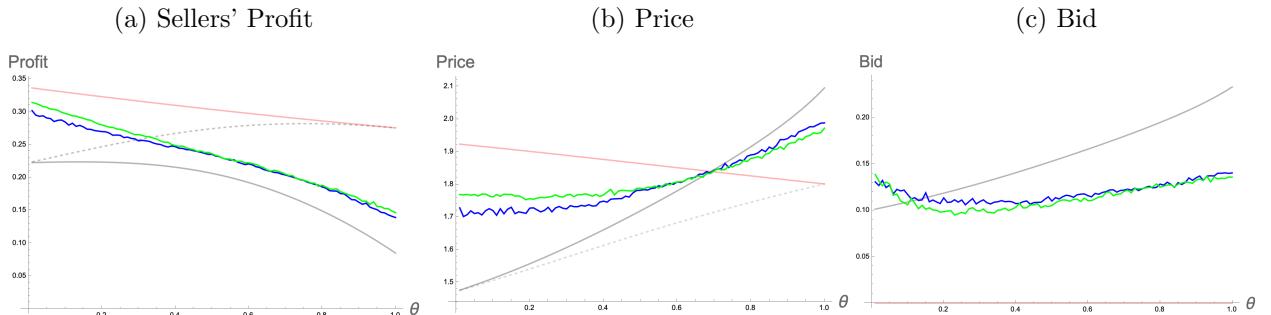
Figure A4: Heterogeneous Sellers



The subfigures (a) and (b) show the Q-learning and theoretical sellers' profits and consumer surplus as functions of θ , respectively. For this experiment, we set $a_i = 3$ and $a_j = 2$, and the rest of the parameter specifications are the same as in Figure 2. In subfigure (a), the black line and the blue line represent seller 1's profits in the full competition and Q-learning cases, respectively; the dashed black line and the green line represent seller 2's profits in the full competition and Q-learning cases, respectively. In subfigure (b), the solid black line represents the consumer surplus in the full competition case, while the solid blue line denotes the Q-learning consumer surplus from our simulation experiments.

and higher prices for small values of θ , and lower prices otherwise, compared with the benchmark state space.

Figure A5: Full Stateful vs Benchmark Bid Information in State Space



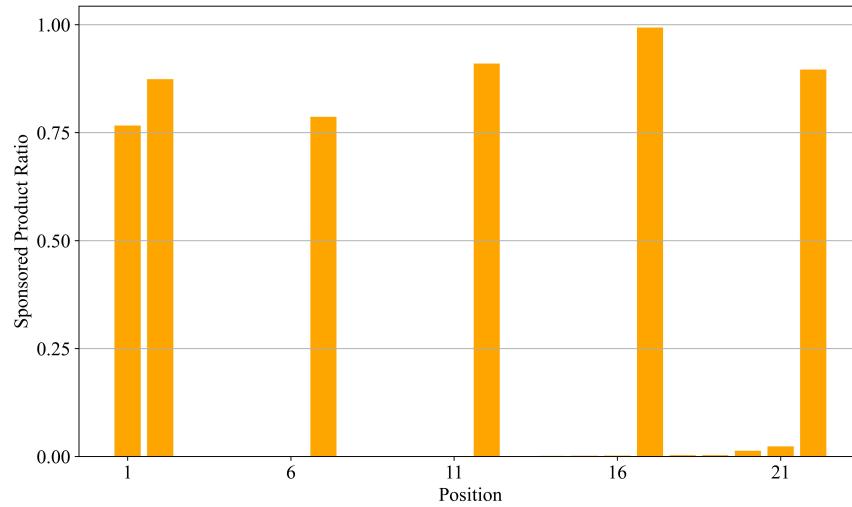
The subfigures (a), (b), and (c) show the sellers' profit, price, and bid, respectively. The blue solid lines represent the benchmark scenario, while the green lines represent the *full stateful* scenario. The representation of other lines and the parameter specifications are the same as in Figure 2.

A.8 Additional Figures and Tables

A.9 Casual Effect of Rank on Sales

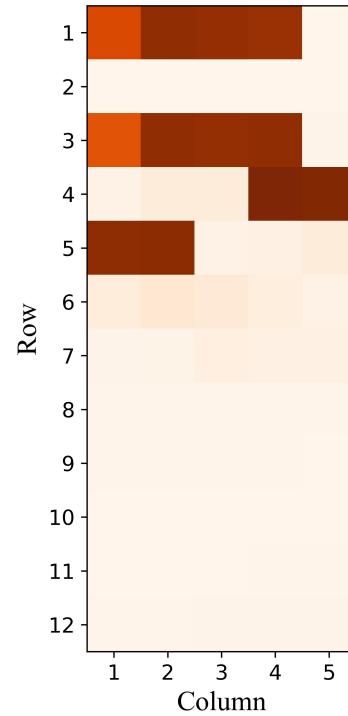
$$u_{ijt} = X'_{jt} \cdot \beta - \alpha \cdot p_{jt} + \zeta \cdot r_j + \xi_{jt} + \epsilon_{ijt} \quad (35)$$

Figure A6: Distribution of Sponsored Product Ads: Page Layout 22



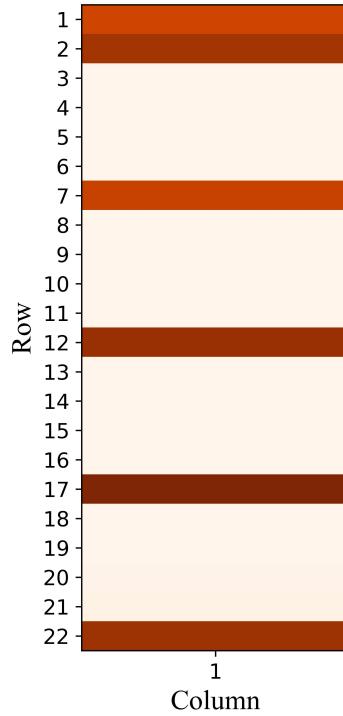
This figure shows the ratios of sponsored products by position for product result pages with 22 products.

Figure A7: Distribution of Sponsored Product Ads: Page Layout 60



This figure shows the heatmap plot of the ratios of sponsored products by position for product result pages containing 60 products.

Figure A8: Distribution of Sponsored Product Ads: Page Layout 22



This figure shows the heatmap plot of the ratios of sponsored products by position for product result pages containing 22 products.

ζ is significantly negative

$$\log(s_{jt}) - \log(s_{0t}) = X'_{jt} \cdot \beta - \alpha \cdot p_{jt} + \zeta \cdot r_j + \xi_{jt} \quad (36)$$

With instrument

1. Solve for all the $\tilde{\delta}_{jt}$, given $\tilde{\delta}_{jt}$, and a guess of ζ , regress $\tilde{\delta}_{jt} - \zeta \cdot r_j$ on X_{jt} , and obtain the residual $\tilde{\xi}_{jt}$
2. Given a guess of ρ , construct $\eta_{jt} = \tilde{\xi}_{jt} - \rho \cdot \tilde{\xi}_{jt-1}$
3. Construct moment condition $\mathbb{E} \begin{pmatrix} \eta_{jt} \cdot \tilde{\xi}_{jt-1} \\ \eta_{jt} \cdot r_{jt-1} \end{pmatrix} = 0$, where r_{jt-1} is the rank of product j at time $t-1$. Search for ζ and ρ .