CSC311 Final Report

Fall 2020

Chunjing Zhang, Peiqing Yu

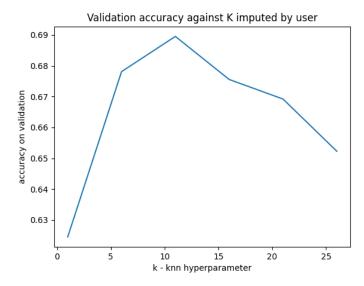
University of Toronto 2020.12.15

Part A

Question 1

(a) For $k = \{1, 6, 11, 16, 21, 26\}$, the corresponding validation accuracy imputed by user are:

```
Knn imputed by user have accuracy
[0.6244707874682472, 0.6780976573525261, 0.6895286480383855, 0.6755574372001129, 0.6692068868190799, 0
.6522720858029918]
```



Please check the relevant code at knn.py

(b) We report the test accuracy with k* having the highest performance on validation data:

For knn imputed by users, k* = 11 has the highest performance on validation data. The final test accuracy is 0.6841659610499576.

(c) For $k = \{1, 6, 11, 16, 21, 26\}$, the corresponding validation accuracy imputed by item are:

Knn imputed by question have accuracy [0.607112616426757, 0.6542478125882021, 0.6826136042901496, 0.6860005644933672, 0.6922099915325995, 0 .69037538808919]

For knn imputed by questions, k* = 21 has the highest performance on validation data. The final test accuracy is 0.6816257408975445.

Validation accuracy against K imputed by question 0.68 accuracy on validation 0.66 0.64 0.62 10 15 20 25 k - knn hyperparameter

- (d) Comparing the test performance between user and item based collaborative filtering, we can see that user-based imputation has higher test accuracy of 68.42% and item-based imputation has slightly lower test accuracy of 68.16%. Thus user-based collaborative filtering method performs better.
- (e) Limitation of KNN includes: slow computation with lots of memory occupied by training data Relatively low prediction

Question 2

(a) We can first derive the probability that question j is wrongly answered by student i:
$$p(c_{ij}=0|\theta_i,\beta_j)=1-\frac{exp(\theta_i-\beta_j)}{1+exp(\theta_i-\beta_j)}=\frac{1}{1+e^{\theta_i-\beta_j}}$$

$$p(c_{ij}=1|\theta_i,\beta_j)=\frac{exp(\theta_i-\beta_j)}{1+exp(\theta_i-\beta_j)}=\frac{e^{\theta_i}}{e^{\theta_i}+e^{\beta_j}}=\frac{1}{1+e^{\beta_j-\theta_i}}$$

Thus the likelihood of the whole training data is:

$$\prod_{k=1}^{56688} \frac{1}{1+e^{\beta_{j(k)} - \theta_{i(k)}}} \frac{c_{ij(k)}}{1} \frac{1}{1+e^{\theta_{i(k)} - \beta_{j(k)}}} \frac{1 - c_{ij(k)}}{1 - c_{ij(k)}}$$

And we can derive the log-likelihood $log p(C|\theta, \beta)$ for all students and questions:

$$\sum_{k=1}^{56688} c_{ij_{(k)}} log(1 + e^{\beta_{j_{(k)}} - \theta_{i_{(k)}}}) + (1 - c_{ij_{(k)}}) log(1 + e^{\theta_{i_{(k)}} - \beta_{j_{(k)}}})$$

The derivative of the log-likelihood with respect to theta_i and beta_j are:

$$\frac{dl}{d\theta_i} = \sum_{k=1}^{56688} \frac{e^{\theta_{i(k)}}}{e^{\beta_{j(k)}} + e^{\theta_{i(k)}}} - c_{ij(k)}$$

$$\frac{dl}{d\beta_j} = \sum_{k=1}^{56688} c_{ij(k)} - \frac{e^{\theta_{i(k)}}}{e^{\beta_{j(k)}} + e^{\theta_{i(k)}}}$$

(b) Please see the relevant code at item response.py

We chose learning rate = 0.15 and iteration = 50. The process of tuning is:

```
For learning rate = 0.1, iteration = 50, the validation accuracy is
[0.48701665255433246, 0.6905165114309907, 0.5819926615862263, 0.6655376799322608, 0.6577758961332204, 0.6364662715213096, 0.6549534292972058, 0.6566469093988145, 0.6299745977984759, 0.6747106971493085, 0.6481795088907706, 0.647897262207169, 0.6501552356759808, 0.6676545300592718, 0.6315269545582839, 0.663279706463449, 0.6543889359300028, 0.647897262207169, 0.6423934518769404, 0.6699125035280835, 0.6350550381033023, 0.6652554332486593, 0.6486028789161727, 0.6521309624611911, 0.6501552356759808, 0.6600338696020321, 0.6414055884843353, 0.65961049957663, 0.6572114027660175, 0.6453570420547559, 0.6446514253457521, 0.6718882303132938, 0.6322325712672876, 0.6622918430708439, 0.6541066892464014, 0.6517075924357889, 0.6477561388653683, 0.6689246401354784, 0.6343494213942986, 0.6622918430708439, 0.658058142816822, 0.642675698560542, 0.6512842224103866, 0.663279706463449, 0.6385831216483207, 0.6615862263618403, 0.6474738921817669, 0.6543889359300028, 0.6529777025119955, 0.6622918430708439]
```

We see that for learning rate = 0.1, the validation accuracy increases quickly and then oscillates a lot. Thus we try smaller learning rate.

```
For learning rate = 0.01, iteration = 50, the validation accuracy is

[0.5033869602032176, 0.635196161445103, 0.6610217329946373, 0.6742873271239063, 0.6817668642393452, 0,

c.6875529212531752, 0.6902342647473892, 0.694044594976009, 0.6979960485464296, 0.7003951453570421, 0,

c.7050522156364663, 0.7056167090036692, 0.7061812023708721, 0.7071690657634773, 0.7084391758396839, 0,

c.7074513124470787, 0.7080158058142817, 0.707310189105278, 0.7064634490544736, 0.70575783234547, 0,

c.7053344623200677, 0.7050522156364663, 0.7044877222692634, 0.7040643522438611, 0.7050522156364663, 0,

c.7044877222692634, 0.7043465989274627, 0.7042054755856618, 0.7040643522438611, 0.7037821055602597, 0,

c.7039232289020604, 0.7042054755856618, 0.7037821055602597, 0.7047699689528648, 0.7058989556872707, 0,

c.7060400790290714, 0.7064634490544736, 0.7061812023708721, 0.7060400790290714, 0.7058989556618685, 0,

c.7049110922946655, 0.7046288456110641, 0.7047699689528648, 0.7049110922946655, 0.7050522156364663]
```

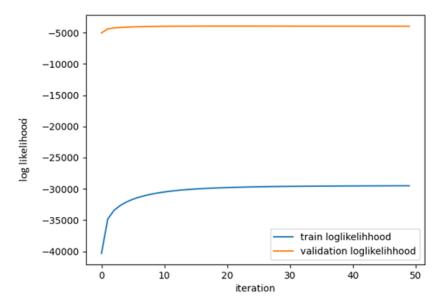
For learning rate = 0.1, we increase the iteration to check whether we miss the final convergence.

```
For learning rate = 0.01, iteration = 100, the validation accuracy is
[0.49562517640417725, 0.6450747953711544, 0.6707592435788879, 0.6819079875811459, 0.6902342647473892, 0
.6960203217612193, 0.7001128986734406, 0.7020886254586508, 0.7036409822184589, 0.7047699689528648, 0
.707874682472481, 0.7075924357888794, 0.7071690657634773, 0.7077335591306803, 0.7090836692068868, 0
.70985804092576913, 0.7101326559412927, 0.7104149026248942, 0.7106747493084956, 0.70985840992576913, 0
.7085802991814846, 0.707874682472481, 0.7074513124470787, 0.707310189195278, 0.7068868190798758, 0
.70745131224470787, 0.70874682472481, 0.7081569291560824, 0.707874682472481, 0.70753951306803, 0
.7070279424216765, 0.7067456957380751, 0.7070279424216765, 0.7064634490544736, 0.7058989556872707, 0
.7061812023708721, 0.7060400790290714, 0.7061812023708721, 0.7064634490544736, 0.7066045723962744, 0
.7067456957380751, 0.7063223257126728, 0.7061812023708721, 0.7063223257126728, 0.7067456957380751, 0.7061812023708721, 0.7063223257126728, 0.7063223257126728, 0.7066045723962744, 0
.70575783234547, 0.7066045723962744, 0.7064634490544736, 0.7063223257126728, 0.7063223257126728, 0.7066045723962744, 0.7064634490544736, 0.7063223257126728, 0.7065232357126728, 0.7066045723962744, 0.7064634490544736, 0.7063223257126728, 0.70653233257126728, 0.7064634490544736, 0.7063223257126728, 0.70653233257126728, 0.7064634490544736, 0.7063223257126728, 0.70653233257126728, 0.70653233257126728, 0.7064634490544736, 0.7063223257126728, 0.7065045723962744, 0.7053233257126728, 0.7063223257126728, 0.7063223257126728, 0.70640549036692, 0.7054755856618685, 0.7054755856618685, 0.7055167090036692, 0.7055167090036692, 0.7055167090036692, 0.7055167090036692, 0.70553344623200677, 0.7055167090036692, 0.7055167090036692, 0.7055364623200677, 0.705536167090036692, 0.7055167090036692, 0.7055167090036692, 0.7055167090036692, 0.7055167090036692, 0.7055167090036692, 0.7055167090036692, 0.7055167090036692, 0.7055167090036692, 0.7055167090036692, 0.7055167090036692, 0.7055167090036692, 0.70551
```

The results show that validation accuracy stables before the first 50 iterations. So, we set the iteration to be 50 and increase the learning rate at a small amount to 0.15.

```
For learning rate = 0.015, iteration = 50, the validation accuracy is
[0.49689528648038384, 0.6587637595258256, 0.6782387806943269, 0.6909931414055885, 0.698278295230031, 0
.703076488851256, 0.7046288456110641, 0.7042054755856618, 0.7044877222692634, 0.7068868190798758, 0
.7053344623200677, 0.7063223257126728, 0.7061812023708721, 0.7060400790290714, 0.7064634490544736, 0
.705757832334547, 0.7056167090036692, 0.7058989556872707, 0.7061812023708721, 0.7061812023708721, 0
.7058989556872707, 0.7061812023708721, 0.7058989556872707, 0.70575783234547, 0.7061812023708721, 0
.7061812023708721, 0.7063223257126728, 0.7060400790290714, 0.70575783234547, 0.70575783234547, 0.7056167090036692,
0.7066045723962744, 0.7066045723962744, 0.7067456957380751, 0.7068868190798758, 0.7067456957380751, 0
.7064634490544736, 0.7064634490544736, 0.7063223257126728, 0.7064634490544736, 0.7058989556872707, 0
.7060400790290714, 0.7058989556872707, 0.70644334490544736, 0.7060400790290714, 0.7060400790290714, 0.7059575783234547, 0.705575783234547, 0.705575783234547, 0.705575783234547, 0.705575783234547, 0.705575783234547, 0.705575783234547, 0.705575783234547, 0.705757583234547, 0.705757583234547, 0.705757583234547, 0.7054755856618685, 0.7053344623200677]
```

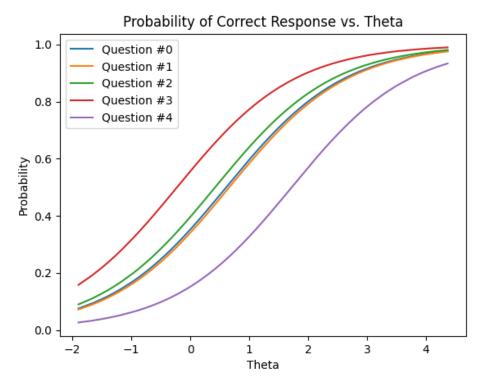
Here is the training curve that shows the training and validation log-likelihoods as a function of iteration with learning rate = 0.15, iteration = 50



(c) The final test and validation accuracy are:

With chosen learning rate = 0.015 and iteration = 50, The final validation accuracy is 0.7053344623200677 The final test accuracy is 0.7064634490544736

(d) The probability of the correct response of question # 0, 1, 2, 3, 4 are:



We can see that the shape of the curves is sigmoid. This shape indicates that as the student ability increases, the probability of correctly answered the specific questions increases. This shape is consistent with all the randomly chosen question.

3

Option 2: Neural Networks.

While running nn, we got the UserWarning: nn.functional.sigmoid is deprecated. Use torch.sigmoid instead. However these two functions are identical, so we keep nn.functional.sigmoid, which is provided in the starter code.

(a)

ALS (Alternating Lesat Squares) is used in unsupervised learning, while neural networks is used for supervised learning.

ALS is a method used in a matrix multiplication to get to the final answer. While neural networks can be used more widely in other situations (universality).

ALS is updating one parameter at a time while fixing the other ones and alternating between the parameters, while neural networks update all the parameters at the same time using a matrix form.

(b)

Codes are in py file.

(c)

Test learning rate $\{0.01, 0.05, 0.1, 0.5\}$ and num_epoch under 30 (mainly around 10) for each k. The best validation accuracy with corresponding hyperparameters are below:

| k | lr | num_epoch | accuracy |
|-----|------|-----------|-----------|
| 10 | 0.1 | 10 | 0.6862828 |
| 50 | 0.05 | 10 | 0.6868473 |
| 100 | 0.05 | 7 | 0.6803556 |
| 200 | 0.05 | 9 | 0.6765453 |
| 500 | 0.1 | 9 | 0.6732995 |

According to results above, $k^* = 50$.

```
k = 10:
Epoch: 0
            Training Cost: 13509.171195
                                         Valid Acc: 0.6193903471634208
            Training Cost: 12262.740651 Valid Acc: 0.6395709850409258
Epoch: 2
            Training Cost: 11545.053267 Valid Acc: 0.658058142816822
            Training Cost: 10942.724810 Valid Acc: 0.6670900366920689
Epoch: 4
            Training Cost: 10450.880671 Valid Acc: 0.6724527236804968
            Training Cost: 10044.467480 Valid Acc: 0.68077900084674
Epoch: 5
Epoch: 6
            Training Cost: 9703.641267
                                         Valid Acc: 0.6817668642393452
            Training Cost: 9410.230672
            Training Cost: 9142.778405
                                         Valid Acc: 0.6838837143663562
                                         Valid Acc: 0.6862828111769687
Epoch: 9
```

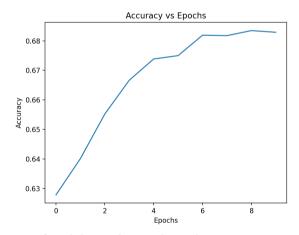
| Training Cost: | 13340.221335 | Valid Acc: | 0.6271521309624611 |
|----------------|--|--|--|
| Training Cost: | 12292.556435 | Valid Acc: | 0.642111205193339 |
| Training Cost: | 11588.957260 | Valid Acc: | 0.6510019757267852 |
| Training Cost: | 10923.860900 | Valid Acc: | 0.665961049957663 |
| Training Cost: | 10311.696580 | Valid Acc: | 0.6752751905165114 |
| Training Cost: | 9753.755806 | Valid Acc: | 0.6804967541631386 |
| Training Cost: | 9235.304152 | Valid Acc: | 0.6867061812023709 |
| Training Cost: | 8743.937502 | Valid Acc: | 0.6874117979113745 |
| Training Cost: | 8274.097998 | Valid Acc: | 0.6869884278859724 |
| Training Cost: | 7824.574489 | Valid Acc: | 0.6868473045441716 |
| | Training Cost: | Training Cost: 13340.221335 Training Cost: 12292.556435 Training Cost: 11588.957260 Training Cost: 10923.860900 Training Cost: 10311.696580 Training Cost: 9753.755806 Training Cost: 9235.304152 Training Cost: 8743.937502 Training Cost: 8274.097998 Training Cost: 7824.574489 | Training Cost: 12292.556435 Valid Acc: Training Cost: 11588.957260 Valid Acc: Training Cost: 10923.860900 Valid Acc: Training Cost: 9753.755806 Valid Acc: Training Cost: 9235.304152 Valid Acc: Training Cost: 8243.937502 Valid Acc: Training Cost: 8274.997998 Valid Acc: |

k = 100:

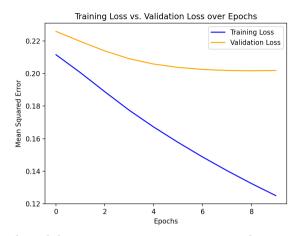
```
Training Cost: 12410.300181 Valid Acc: 0.6416878351679368
Epoch: 1
           Training Cost: 11614.630996 Valid Acc: 0.6589048828676263
Epoch: 2
           Training Cost: 10856.562655 Valid Acc: 0.6661021732994638
           Training Cost: 10151.541637
                                       Valid Acc: 0.6756985605419137
           Training Cost: 9480.503721
                                      Valid Acc: 0.6776742873271239
Epoch: 5
           Training Cost: 8818.394983
k = 200:
Epoch: 0
           Training Cost: 13807.696970
                                       Valid Acc: 0.6169912503528083
Epoch: 1
           Training Cost: 12684.508000 Valid Acc: 0.6354784081287045
           Training Cost: 11820.206228 Valid Acc: 0.6487440022579735
           Training Cost: 11065.518775 Valid Acc: 0.6598927462602314
Epoch: 3
Epoch: 4
           Training Cost: 10352.613084 Valid Acc: 0.6642675698560542
           Training Cost: 9632.534934
Epoch: 5
                                       Valid Acc: 0.6732994637313011
           Training Cost: 8881.009156
                                       Valid Acc: 0.6714648602878917
           Training Cost: 8100.275627
                                       Valid Acc: 0.6735817104149027
Epoch: 7
Epoch: 8
           Training Cost: 7312.334899
                                       Valid Acc: 0.676545300592718
k = 500:
            Training Cost: 15355.295413
           Training Cost: 13455.437402 Valid Acc: 0.6240474174428451
           Training Cost: 12329.551403
                                      Valid Acc: 0.6346316680779001
Epoch: 2
           Training Cost: 11443.095316 Valid Acc: 0.6480383855489698
Epoch: 3
           Training Cost: 10657.886461 Valid Acc: 0.6573525261078182
           Training Cost: 9837.048679
Epoch: 6
           Training Cost: 8894.015684
                                       Valid Acc: 0.672311600338696
           Training Cost: 7817.088929
                                      Valid Acc: 0.6732994637313011
Epoch: 7
           Training Cost: 6677.067158
```

(d)

Need to plot training loss, validation loss, and validation accuracy. Validation accuracy and epoch:



Train & validation loss and epoch:



The validation accuracy increases as epoch increases. While the loss for both train and validation decrease as epoch increases. However, the MSE for validation decrease more and more slowly, nearly becomes flat at the end.

(e)

For $k^* = 50$, lr = 0.05, $num_epoch = 10$, got this for each λ in list $\{0.001, 0.01, 0.1, 1\}$:

| λ | validation acc | test acc |
|-----------|----------------|-----------|
| 0.001 | 0.6879763 | 0.6838837 |
| 0.01 | 0.6860006 | 0.6771098 |
| 0.1 | 0.6907988 | 0.6827547 |
| 1 | 0.6241885 | 0.6226362 |

Here are results for each λ :

```
\lambda = 0.001:
             Training Cost: 13329.817492
                                          Valid Acc: 0.6308213378492803
            Training Cost: 12269.545421 Valid Acc: 0.6406999717753317
Epoch: 1
Epoch: 2
            Training Cost: 11546.206075
                                         Valid Acc: 0.6584815128422241
            Training Cost: 10883.661262
                                         Valid Acc: 0.6679367767428732
            Training Cost: 10292.029622
                                         Valid Acc: 0.6752751905165114
            Training Cost: 9758.018521
Epoch: 5
                                         Valid Acc: 0.679226644086932
            Training Cost: 9258.252411
Epoch: 6
                                         Valid Acc: 0.682895850973751
                                          Valid Acc: 0.6855771944679651
            Training Cost: 8319.096074
                                          Valid Acc: 0.687694044594976
Epoch: 9
            Training Cost: 7875.928446
                                          Valid Acc: 0.6879762912785775
0.6838837143663562
```

```
\lambda = 0.01:
            Training Cost: 13432.491016
                                          Valid Acc: 0.627575500987863
            Training Cost: 12384.960312 Valid Acc: 0.6392887383573242
Epoch: 1
Epoch: 2
            Training Cost: 11696.357103 Valid Acc: 0.6556590460062094
Epoch: 3
            Training Cost: 11050.797025
                                          Valid Acc: 0.6662432966412645
            Training Cost: 10461.818962
                                         Valid Acc: 0.6725938470222975
            Training Cost: 9934.681867
                                         Valid Acc: 0.6776742873271239
Epoch: 5
Epoch: 6
                                          Valid Acc: 0.6806378775049393
            Training Cost: 8986.494424
                                          Valid Acc: 0.6830369743155518
            Training Cost: 8542.530875
                                          Valid Acc: 0.6861416878351679
            Training Cost: 8116.903748
                                          Valid Acc: 0.6860005644933672
Enoch: 9
```

 $\lambda = 0.1$:

```
Epoch: 0 Training Cost: 14133.327981 Valid Acc: 0.6257408975444538 Epoch: 1 Training Cost: 13232.630282 Valid Acc: 0.6394298616991251 Epoch: 2 Training Cost: 12706.057663 Valid Acc: 0.6521309624611911 Epoch: 3 Training Cost: 12237.017430 Valid Acc: 0.661727349703641 Epoch: 4 Training Cost: 11830.097083 Valid Acc: 0.661727349703641 Epoch: 5 Training Cost: 11480.480174 Valid Acc: 0.6696302568444821 Epoch: 6 Training Cost: 11171.502251 Valid Acc: 0.681742875271239063 Epoch: 7 Training Cost: 10886.335584 Valid Acc: 0.681768642393452 Epoch: 8 Training Cost: 10614.760466 Valid Acc: 0.684984278859724 Epoch: 9 Training Cost: 10352.705603 Valid Acc: 0.68997987581145921 0.6827547276319503 Valid Acc: 0.6997987581145921 0.6827547276319503 Valid Acc: 0.6233418007338414 Epoch: 0 Training Cost: 18829.117218 Valid Acc: 0.6233418007338414 Epoch: 1 Training Cost: 15533.271581 Valid Acc: 0.6182613604290149
```


It seems like the model performs slightly better for $\lambda = 0.1$, which makes validation accuracy reach 0.69.

For this question, we used the neural networks as base model. First, we used np.random.randint() to randomly select n students with replacement for three times, where n is the total number of students. Then trained three base models corresponding to the three groups of selected data (selected rows from the data matrix where each row represents a selected students with all questionss). Then get prediction by the three models' average for evaluation.

```
Epoch: 0
            Training Cost: 13571.452437
                                          Valid Acc: 0.6079593564775614
Epoch: 1
            Training Cost: 12145.219665
                                          Valid Acc: 0.6038667795653401
Epoch: 2
            Training Cost: 11077.611763
                                          Valid Acc: 0.5970928591589049
Epoch: 3
            Training Cost: 10169.012415
                                          Valid Acc: 0.5906011854360711
Epoch: 4
            Training Cost: 9452.807407
                                          Valid Acc: 0.581145921535422
            Training Cost: 8879.751910
                                          Valid Acc: 0.581145921535422
Epoch: 5
            Training Cost: 8407.603848
                                          Valid Acc: 0.5817104149026249
Epoch: 6
                                          Valid Acc: 0.5798758114592154
Epoch: 7
            Training Cost: 8010.175925
Epoch: 8
            Training Cost: 7670.466305
                                          Valid Acc: 0.5786057013830087
Epoch: 9
            Training Cost: 7376.891085
                                          Valid Acc: 0.578464578041208
                                          Valid Acc: 0.6011854360711262
Epoch: 0
            Training Cost: 13467.668539
Epoch: 1
            Training Cost: 12387.411854
                                          Valid Acc: 0.6007620660457239
Epoch: 2
            Training Cost: 11526.810686
                                          Valid Acc: 0.596528365791702
Epoch: 3
            Training Cost: 10690.442652
                                          Valid Acc: 0.5930002822466836
Epoch: 4
            Training Cost: 9994.157496
                                          Valid Acc: 0.586931978549252
            Training Cost: 9415.572773
                                          Valid Acc: 0.5824160316116286
Epoch: 5
Epoch: 6
            Training Cost: 8923.523271
                                          Valid Acc: 0.5776178379904036
Epoch: 7
            Training Cost: 8501.869490
                                          Valid Acc: 0.5771944679650014
Epoch: 8
            Training Cost: 8138.308293
                                          Valid Acc: 0.5767710979395992
Epoch: 9
            Training Cost: 7822.550102
                                          Valid Acc: 0.574795371154389
Epoch: 0
            Training Cost: 14249.315764
                                          Valid Acc: 0.6079593564775614
Epoch: 1
            Training Cost: 12882.573984
                                          Valid Acc: 0.6104995766299746
                                          Valid Acc: 0.6064069997177534
            Training Cost: 11897.499172
Epoch: 2
            Training Cost: 11055.499102
                                          Valid Acc: 0.6034434095399379
Epoch: 3
            Training Cost: 10356.602646
                                          Valid Acc: 0.6013265594129269
Epoch: 4
Epoch: 5
            Training Cost: 9763.409961
                                          Valid Acc: 0.5992097092859159
            Training Cost: 9253.768347
                                          Valid Acc: 0.599633079311318
Epoch: 6
Epoch: 7
            Training Cost: 8810.833066
                                          Valid Acc: 0.5997742026531189
Epoch: 8
            Training Cost: 8423.033213
                                          Valid Acc: 0.5977984758679086
            Training Cost: 8082.173427
                                          Valid Acc: 0.5953993790572961
Epoch: 9
0.5935647756138865
0.5901778154106689
```

The final test accuracy is 0.5901778154106689 which is much lower than the single neural networks model. The final validation accuracy is 0.5935647756138865, also much lower than before.

Therefore, actually we didn't obtain better performance using the ensemble, and the running time for running ensemble once is quite long.

We think one main reason why accuracy decline could because when sampling with replacement, some data is not sampled into all of the three samples. Also, some model with wrong prediction may have higher weight for its wrong prediction which lead to a wrong results finally.

Part B

1 Formal Description

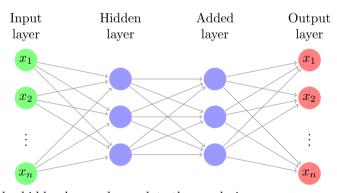
We have tried multiple ways to modify the implemented algorithm in part A to increase the test accuracy on students' answers to the diagnostic questions. Above all, we tried to consider the improvement on the neural networks algorithm. Since tuning hyperparameter is already done in Part A, we tried something else. For example, first we tried to add one hidden layer. Then we tried to use other activation functions like ReLU, Softmax, and LogSigmoid. Also, we tried different regularizer (ie. L_1 regularization). Finally we also thought about normalize the input data; however, since the input data is not in different scale we didn't modify the normalization.

After examining all the possible modifications on the neural network algorithm, we took a deeper look at the the student metadata, student_meta.csv. This dataset is composed of the user id, gender, date of birth and premium pupil. As the training accuracy of the baseline algorithms in part A is not too good, we hypothesized that there might exists a mixture of data where each group of data has its own density for the provided training, validation and test dataset. If we train the model with the combined data, the model may be weak to capture the overall existed pattern, and this may cause underfitting. The gender and age of users may both be the factor that affect the probability of answering questions correctly. This makes sense as younger students normally have lower accuracy, and as children get older they tend to have higher accuracy. Therefore, we separate the training, validation and test data into individual dictionaries based on gender and age, then train the item-response probability algorithm on each of them.

The separation of dataset based on age is at age.py and the separation of dataset based on gender is at gender.py. We expect the separate training may increase the train accuracy and reduce underfit, and optimally increase the test accuracy.

2 Figure or Diagram

Neural Network Modification



Add a hidden layer, also update the regularizer:

```
# Define linear functions.
self.g = nn.Linear(num_question, k)
self.s = nn.Linear(k, k)
self.h = nn.Linear(k, num_question)
```

```
g_w_norm = torch.norm(self.g.weight, 2)
s_w_norm = torch.norm(self.s.weight, 2)
h_w_norm = torch.norm(self.h.weight, 2)
```

Tried different activation functions with the added new layer:

```
activate_g = self.g(inputs)
output1 = F.sigmoid(activate_g)
# output = torch.sigmoid(activate_g)
# m = nn.ReLU()
# m = nn.LogSigmoid()
# m = nn.LogSoftmax()
# output = m(activate_g)
activate_s = self.s(output1)
output2 = F.sigmoid(activate_s)
activate_f = self.h(output2)
out = F.sigmoid(activate_f)
# out = torch.sigmoid(activate_f)
# out = m(activate_f)
```

Group separation using Student Metadata

This is the code snippets implemented the separerate training of each group data based on gender information provided in the student metadata.

```
# separate the train, validation, and test data by gender
boy, girl = separate_gender()
b_test, g_test = get_dic(boy, girl, test_data)
b_train, g_train = get_dic(boy, girl, train_data)
b_val, g_val = get_dic(boy, girl, val_data)

# train each male and female datasets separately
boy = irt(b_train, b_val, 0.015, 50)
girl = irt(g_train, g_val, 0.015, 50)
print(f"For learning rate = 0.015, iteration = 50, the train accuracy for boys is\n{boy[2]}")
print(f"For learning rate = 0.015, iteration = 50, the train accuracy for girls is\n{girl[2]}")
```

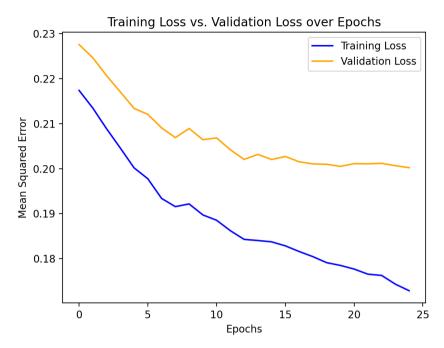
3 Comparison or Demonstration

Neural Network Modification

Here are results for using Sigmoid with an additional hidden layer:

```
Epoch: 0
           Training Cost: 14173.529993 Valid Acc: 0.6277166243296641
           Training Cost: 13453.212581 Valid Acc: 0.6305390911656789
Epoch: 1
Epoch: 2
           Training Cost: 13106.100574 Valid Acc: 0.6349139147615016
           Training Cost: 12803.304464 Valid Acc: 0.6412644651425345
Epoch: 3
Epoch: 4
           Training Cost: 12560.058697 Valid Acc: 0.648461755574372
Epoch: 5
           Training Cost: 12356.876511 Valid Acc: 0.6549534292972058
           Training Cost: 12266.673736 Valid Acc: 0.6668077900084673
Epoch: 6
           Training Cost: 12193.919032 Valid Acc: 0.668077900084674
Epoch: 7
Epoch: 8
           Training Cost: 12119.076354 Valid Acc: 0.668077900084674
           Training Cost: 12077.401896 Valid Acc: 0.6737228337567034
Epoch: 9
           Training Cost: 12133.146861 Valid Acc: 0.6721704769968952
Epoch: 10
Epoch: 11
           Training Cost: 12035.328566 Valid Acc: 0.6747106971493085
Epoch: 12
           Training Cost: 11957.835894 Valid Acc: 0.6776742873271239
Epoch: 13
           Training Cost: 11905.195812 Valid Acc: 0.677109793959921
Epoch: 14
           Training Cost: 11822.732113 Valid Acc: 0.6806378775049393
Epoch: 15
           Training Cost: 11807.365512 Valid Acc: 0.6779565340107254
Epoch: 16
           Training Cost: 11770.325193 Valid Acc: 0.6827547276319503
Epoch: 17
           Training Cost: 11752.178491 Valid Acc: 0.6816257408975445
Epoch: 18
           Training Cost: 11741.590530 Valid Acc: 0.682895850973751
Epoch: 19
           Training Cost: 11678.233868 Valid Acc: 0.6844482077335591
Epoch: 20
           Training Cost: 11618.268810 Valid Acc: 0.6833192209991532
Epoch: 21
           Training Cost: 11622.833282 Valid Acc: 0.6827547276319503
Epoch: 22
           Training Cost: 11655.201063 Valid Acc: 0.6864239345187694
Epoch: 23
           Training Cost: 11599.434348  Valid Acc: 0.6857183178097658
Epoch: 24
           Training Cost: 11532.738863 Valid Acc: 0.6850127011007621
0.6782387806943269
0.6850127011007621
```

Where final validation accuracy is 0.6850127011007621 and test accuracy is 0.6782387806943269.

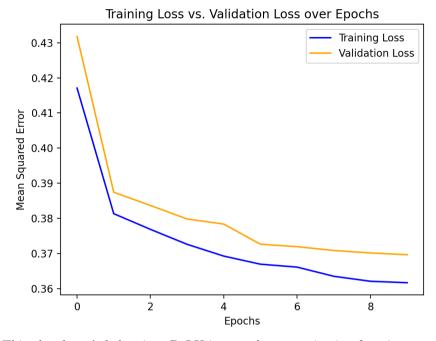


However, there's no obvious changing for using this.

Then by using ReLU as activation function, we got this:

| Epoch: 0 | Training | Cost: | 43074.088623 | Valid Acc: | 0.5193338978267006 |
|-------------------|----------|-------|--------------|------------|--------------------|
| Epoch: 1 | Training | Cost: | 35417.084000 | Valid Acc: | 0.5397967823878069 |
| Epoch: 2 | Training | Cost: | 33547.767464 | Valid Acc: | 0.5438893593000282 |
| Epoch: 3 | Training | Cost: | 33337.407709 | Valid Acc: | 0.5467118261360429 |
| Epoch: 4 | Training | Cost: | 32972.840151 | Valid Acc: | 0.5496754163138583 |
| Epoch: 5 | Training | Cost: | 32852.308329 | Valid Acc: | 0.5491109229466554 |
| Epoch: 6 | Training | Cost: | 32645.523170 | Valid Acc: | 0.5517922664408693 |
| Epoch: 7 | Training | Cost: | 32485.007030 | Valid Acc: | 0.5534857465424782 |
| Epoch: 8 | Training | Cost: | 32290.693451 | Valid Acc: | 0.5532034998588766 |
| Epoch: 9 | Training | Cost: | 32172.030867 | Valid Acc: | 0.554614733276884 |
| 0.563646627152131 | | | | | |

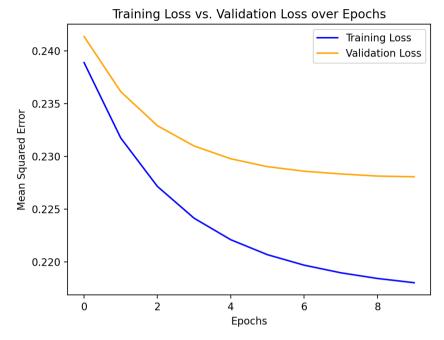
Validation accuracy is 0.554614733276884 and test accuracy is 0.563646627152131.



This also doesn't help, since ReLU is not a better activation function compared to Sigmoid. We then tried L_1 regularization:

```
Epoch: 0
            Training Cost: 27082.602104
                                          Valid Acc: 0.6123341800733841
Epoch: 1
            Training Cost: 20425.982297
                                          Valid Acc: 0.6233418007338414
Epoch: 2
            Training Cost: 20050.000346
                                          Valid Acc: 0.6253175275190517
Epoch: 3
            Training Cost: 19795.116239
                                         Valid Acc: 0.6270110076206604
Epoch: 4
                                          Valid Acc: 0.6270110076206604
            Training Cost: 19618.048839
Epoch: 5
            Training Cost: 19489.839846
                                          Valid Acc: 0.6275755009878634
                                          Valid Acc: 0.626728760937059
Epoch: 6
            Training Cost: 19395.148801
Epoch: 7
            Training Cost: 19324.223396
                                          Valid Acc: 0.6264465142534575
                                          Valid Acc: 0.6258820208862546
Epoch: 8
            Training Cost: 19267.709228
Epoch: 9
            Training Cost: 19223.142982
                                          Valid Acc: 0.6254586508608524
0.6223539373412362
```

Validation accuracy is 0.6254586508608524 and test accuracy is 0.6223539373412362.



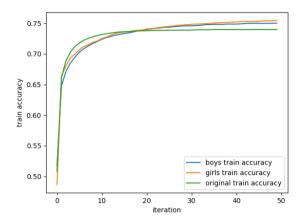
Similarly, this doesn't help to improve our results.

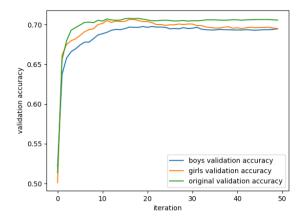
Group separation using Student Metadata

We can see that there is significant increase of training accuracy for all the gender groups. We can observe that the highest train accuracy for boy group has reached 75%. However, the validation accuracy have shown no improvement. The test accuracy of one group increases 2% and the other group stays the same.

```
For learning rate = 0.015, iteration = 50, the train accuracy for boys is
[0.5176057984568623, 0.6478372691138649, 0.6714051905541267, 0.6852466682253916, 0.6946457797521627, 0
.7029693710544774, 0.7085807809212065, 0.7130699088145896, 0.7169043722235212, 0.7203647416413373, 0
.723871872808043, 0.7267243394902969, 0.7286883329436521, 0.7305120411503391, 0.7319616553659107, 0
.7330371755903671, 0.7343465045592705, 0.7359831657703998, 0.7375263034837503, 0.7389291559504325, 0
.7398643909282208, 0.7409866729015665, 0.7417816226326864, 0.7424830488660276, 0.7434182838438158, 0
.743558569090484, 0.7443535188216039, 0.7449146598082769, 0.7457096095393968, 0.7457563712882862, 0
.7460369417816226, 0.7462239887771802, 0.7467851297638531, 0.7477203647416414, 0.747907411737199, 0
.7482815057283142, 0.7481879822305354, 0.7481879822305354, 0.7487491232172083, 0.7487491232172083, 0.7487491232172083, 0.749764554594341, 0.7496843581949965, 0.7498714051905542, 0.7500584521861118, 0
.749964928688333, 0.7498246434416648, 0.7500116904372224, 0.7501519756838906, 0.7502922609305588]
```

With chosen learning rate = 0.015 and iteration = 50,
The final test accuracy for age 7-13 is 0.6238532110091743
The final test accuracy for age 13-16 is 0.7026178010471205
The final test accuracy for age 16-18 is 0.7263479145473042





The train accuracy of all the age group separation increased to a higher extent, and we can observe that the highest train accuracy for age 7-13 has reached 76.8%. However, the validation accuracy and test accuracy have shown an overall no improvement.

```
For learning rate = 0.015, iteration = 50, the train accuracy for age 7-13 is

[0.4785867237687366, 0.637503823799327, 0.6494340776996024, 0.6563169164882227, 0.6648822269807281, 0,

c.671612113796268, 0.6751300091771184, 0.6789538085041297, 0.6838482716427042, 0.6882838788620373, 0,

c.6916488222698073, 0.6951667176506577, 0.6991434689507494, 0.7018966044661976, 0.7055674518201285, 0,

c.7106148669317834, 0.7129091465279902, 0.7184154175588865, 0.7213215050474151, 0.7245334964821046, 0,

c.7274395839706332, 0.7294279596206791, 0.7315692872438054, 0.7330988069746099, 0.7358519424900581, 0,

c.737840318140104, 0.7396757418170694, 0.7416641174671154, 0.7438054450902417, 0.7454879167941266, 0,

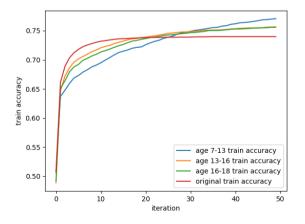
c.7470174365249311, 0.7486999082288162, 0.7503823799327012, 0.752370755582747, 0.7534414193943102, 0,

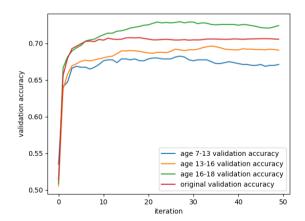
c.7543591312327929, 0.755429795044356, 0.7561945549097583, 0.7565004588559192, 0.7578770266136433, 0,

c.7601713062098501, 0.760477210156011, 0.7615478739675742, 0.7617008259406546, 0.7641480575099419, 0,

c.7649128173753441, 0.7661364331599878, 0.767207096971551, 0.7670541449984705, 0.7678189048638727]
```

With chosen learning rate = 0.015 and iteration = 50,
The final test accuracy for age 7-13 is 0.6238532110091743
The final test accuracy for age 13-16 is 0.7026178010471205
The final test accuracy for age 16-18 is 0.7263479145473042





4 Limitations

- Adding hidden layer requires tuning for hyperparameters again. Since the original k may lead to overfitting, while the original learning rate and number of iterations may lead to relatively longer running time. However, adding 1 layer seems to help less, doesn't contribute to both accuracy and loss in our experiment.
- One possible extension could be, trying to add more hidden layers, then tune hyperparameters to avoid overfitting. We think this may help to improve accuracy. Also, maybe mixing all methods (ie. k, number of hidden layers, activation function) we tried to use could also bring improvements. Since there could be some better combinations if try more.
- The data processing for converting given train dataset into spare matrix is time consuming and non-efficient.
- Even though we reduce the underfitting and increase train accuracy, the Group separation using student metadata is not modifying the algorithm, but focuses on data processing. The improved train accuracy would not apply to a new dataset.

Contribution

Group Member: Peiqing Yu

- Part A: knn.py
- Part A: item_response.py
- Part A: report question 1, question 2
- Part B: irt_partB.py
- Part B: modified_nn.py
- Part B: half of the report

Group Member: Chunjing Zhang

- Part A: neural_network.py
- Part A: esemble.py
- Part A: report question 3, question 4
- Part B: age.py
- Part B: gender.py
- Part B: half of the report