CHAPTER 6

Image Restoration

Which of my photographs is my favorite? The one I'm going to take tomorrow.

–Imogen Cunningham

LEARNING OBJECTIVES

This chapter provides an overview of digital image restoration. It introduces the concepts related to degradation model and image restoration techniques. After studying this chapter, the reader will become familiar with the following:

- Types of degradations
- Image degradation model
- Noise models and noise removal filters
- Blurs and artefacts
- Image restoration techniques

6.1 INTRODUCTION TO DEGRADATION

No image is perfect. An image gets corrupted by different types of degradations that appear from the stage of image acquisition itself. Images are degraded by noise, blurs, and distortions or artefacts. The process of image degradation takes place in all the stages—image acquisition, image processing, image storage, and transmission. Noise is a disturbance that causes fluctuations in pixel values. Similarly, when an image-capturing system does not capture a point as a point, it results in degradation. Artefacts are extreme intensity and colour imperfections that make images meaningless. Thus, image formation itself introduces problems.

Image restoration is the process of retrieving an original image from a degraded image. The idea is to obtain an image as close to the original image as possible. This is possible by removing or minimizing degradations. This is often difficult in case of extreme noise and blurs, and is often called an inverse problem. An inverse problem aims to find the cause from effects. Often, inverse problems are ill-posed problems. Jacques Hadamard defined a well-posed problem as one that has a unique solution and its behaviour changes continuously with the initial condition. An image restoration problem is not a well-posed problem; hence there is no guarantee of a solution, and even if a solution is found, it can be unstable. Therefore, additional assumptions are made about the degraded image and incorporated as a smoothness function. This process is called regularization, which makes recovery of the original image feasible.

Let us first discuss the different types of image degradations.

6.2 TYPES OF IMAGE DEGRADATIONS

To remove image degradations, it is necessary to understand them. Degradations are of three types and are shown in Fig. 6.1.

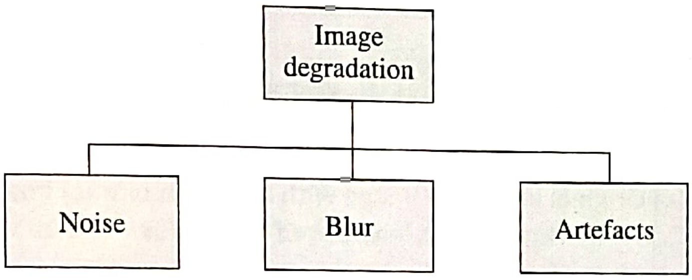


Fig. 6.1 Types of degradations

The following sections discuss the different types of image degradations.

6.2.1 Noise

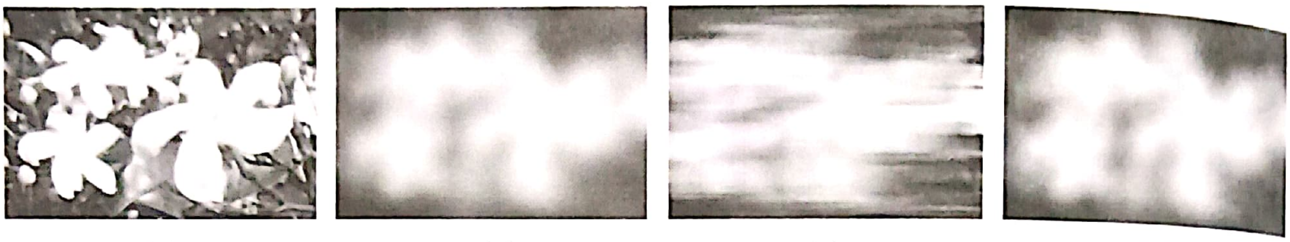
As defined in Section 6.1, noise is a disturbance that causes fluctuations in pixel values. Hence, pixel values show random variations, and this cannot be avoided. Thus, suitable strategies should be designed to model and manage noise. An original image is shown in Fig. 6.2(a). Gaussian noise is added to it, and the resultant image is shown in Fig. 6.2(b).

(a) (b)

Fig. 6.2 Effect of noise (a) Original image (b) Image with Gaussian noise

6.2.2 Blurs

A blur is a degradation that makes an image less clear, thus making the process of image analysis difficult. Some of the common blurs are Gaussian blur and motion blur. Gaussian blurs have been discussed as part of the image smoothing operation in Chapter 5. A motion blur occurs because of the movement of the object or camera during the image acquisition process. Gaussian and motion blurs are illustrated in Figs 6.3(b) and (c), respectively, for the original image shown in Fig. 6.3(a). Similarly, a lens-type blur is shown in Fig. 6.3(d).



(a) (b) (c) (d)

Fig. 6.3 Effect of blur (a) Original image (b) Image with Gaussian blur (c) Image with motion blur (d) Image with lens-out-of-focus blur

6.2.3 Artefacts

Distortions or artefacts (the term artefact is mostly used in medical imaging) are extreme intensity or colour fluctuations that make an image meaningless. Distortions involve geometrical transformations such as translation, rotation, or change in scale. A distorted image is shown in Fig. 6.4, where it can be observed that the flower in the image is slightly elongated. In medical images, an artefact is not a random noise. It is a spurious error or some systematic error that is present in the image due to the faulty image-capturing process or due to the properties of the image object. Artefacts can be mistaken for real organs, which may mislead doctors. The mishandling of X-ray films (e.g., scratches), movement of patients during the scanning or improper following of radiological procedures, and dust in the imaging system can result in systematic errors in images called artefacts.

Fig. 6.4 Image with distortion

6.3 POINT SPREAD FUNCTION AND MODULATION TRANSFER FUNCTION

The purpose of image restoration is to recover the original image from a degraded image. This requires knowledge of the point spread function (PSF) and modulation transfer function (MTF). Let us discuss these functions now. Suppose a single point (x, y) of the image, which is captured by an imaging system, is reproduced, not as a single point but as a blurred version of the point.

This can mathematically be represented as follows:

$$I_{\text{result}}(x, y) = I_{\text{original}}(x, y) * \sigma(x, y)$$

where $\sigma(x, y)$ is the PSF and the symbol '*' represents the convolution operation. *Convolution* is the superimposition of one signal over another, and involves shift and add operations. In other words, when a point is imaged, it is not captured as a point, and thus the resulting image is blurred. This effect is said to be the convolution of the signal with the blurring function. The output produced is due to the convolution of the original image with the PSF.

A blur can occur due to various reasons such as an inappropriate focus, imperfection of lens, atmospheric turbulence, motion, and Gaussian disturbance, which may affect the quality of an image. If the PSF is known, the original image can be restored using the deconvolution process. An original image is shown in Fig. 6.5(a). A PSF and its surface plot are shown in Figs 6.5(b) and (c). The result of the convolution process is shown in Fig. 6.5(d).

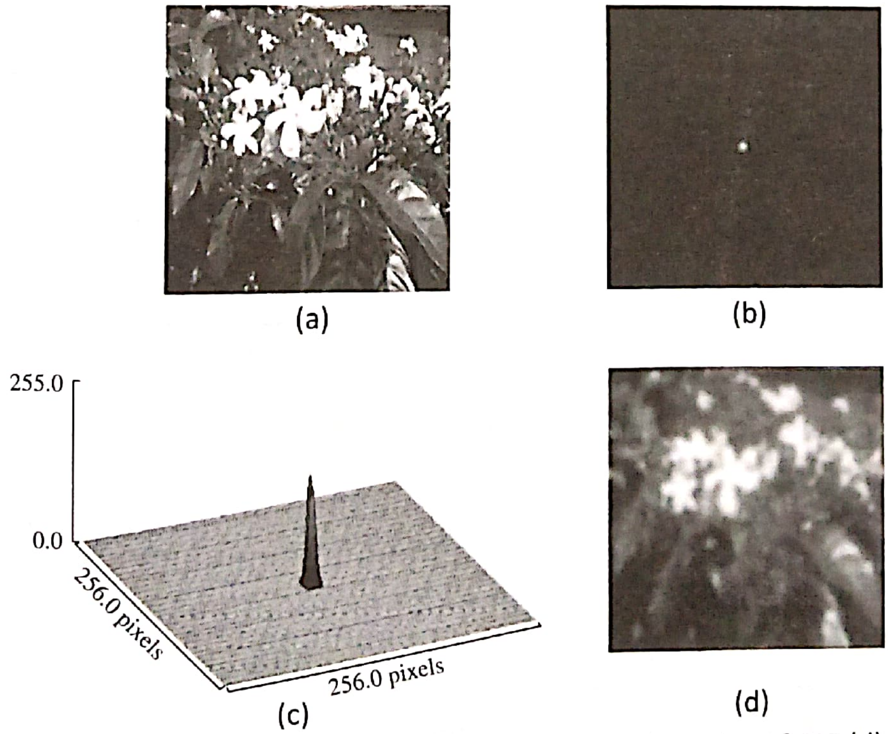


Fig. 6.5 Convolution process (a) Original image (b) PSF (c) Surface plot of PSF (d) Blurred image

Blurs can be simulated using spatial masks. Some of the popular 3 × 3 PSF masks are shown in Fig. 6.6.

$$\begin{pmatrix} & X & \\ & X & \\ & X & \end{pmatrix}, \begin{pmatrix} & & \\ X & X & X \\ & & \end{pmatrix}, \begin{pmatrix} X & & \\ & X & \\ & & X \end{pmatrix}$$

Fig. 6.6 PSF masks

The optical transfer function (OTF) is the Fourier transform of the PSF. The OTF takes a sinusoidal target and represents the plot of frequency versus relative amplitude and phase shift. The amplitude of an OTF is called the MTF, and the phase transfer function (PTF) is the phase component of the OTF. The MTF is used as an indicator of the performance of an imaging system. In film cameras, resolution can be changed by changing the film. However, in a digital camera, the sensor is not adjustable. Hence, the performance of a camera is dependent on how much information is retained and reproduced at a given spatial frequency. The MTF is an indicator of the extent to which a camera represents subtle details. As frequencies increase, the ability of the camera to reproduce the subtle details decreases. Either the subtle details are not represented or they are reproduced poorly. The cut-off point where an imaging system fails to reproduce the subtle details is an indicator of the limit of that system. This standard for determining the cut-off point depends on the nature of targets. Thus, the MTF of an imaging system can be defined as follows:

$$\text{MTF}(f) = \frac{M_{\text{out}}(f)}{M_{\text{in}}(f)}$$

where $M_{\text{in}}(f)$ is the input signal modulation and $M_{\text{out}}(f)$ the output signal modulation. The intensity $M_{\text{in}}(f)$ of the original pattern always varies between zero and the maximum value. For an 8-bit image, this varies from 0 to 255. The resolving power of a camera to distinguish the subtle details in an image is called modulation (M), which is defined as follows:

$$\text{Modulation (contrast)} = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$

where I_{max} and I_{min} are the maximum and minimum intensities, respectively, and can be determined from the intensity profile.

Example 6.1 What is the modulation of a grey-scale image?

Solution For a grey-scale image, the range is 0–255. Therefore, the modulation is $= \left(\frac{255-0}{255+0}\right)$.

Thus, the value of the input signal modulation is 1.

As the output $M_{\text{out}}(f)$ decreases with the spatial frequency, modulation for different spatial frequencies is calculated and plotted. The curve is flat up to a point, after which it starts to drop and slopes downwards; this curve indicates the quality of the imaging system (Fig. 6.7). Similarly, MTFs of other components such as sensors, lenses, and printers can be calculated. The MTF of the system is the product of the MTFs of the all components.

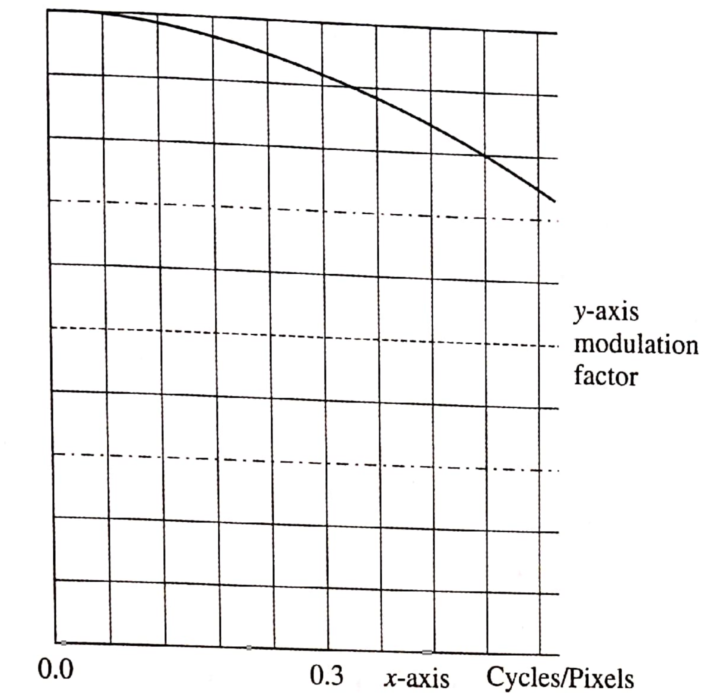


Fig. 6.7 Sample MTF plot

Instead of a sinusoidal grating, a line spread function (LSF) or an edge spread function (ESF) can be used for MTF calculation. The MTF for the LSF can be obtained by taking the Fourier spectrum of the LSF and scaling it to a maximum of 1.0. Here, only the frequency components are considered and not their logarithms. Similar to sinusoidal gratings, edges can also be used as a phantom to test the system. The intensity profile perpendicular to the edge is called an ESF. It should be converted to an LSF for MTF calculation. Plotting the MTF for a PSF is also an easy task, where a 2D Fourier transform is applied to the PSF directly and the raw Fourier spectrum component (minus the DC component) is scaled to a maximum of 1.0. Fourier transforms and their applications are discussed in detail in Chapter 4.

6.4 IMAGE DEGRADATION MODEL

Image restoration starts with an image degradation (or restoration) model, which is shown in Fig. 6.8.

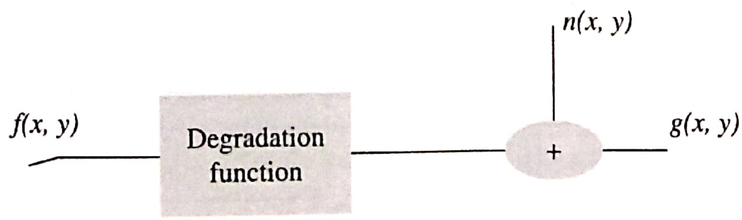


Fig. 6.8 Image degradation model

Let the original image be $f(x, y)$. When the image is acquired by the imaging system, the process of degradation starts. In a linear space-invariant (LSI) system, degradation is modelled as an operator H, which is a linear, position-invariant operator and has the following properties:

1. It is a linear operator.
2. It obeys the rules of homogeneity.
3. It is position invariant.

The degradation function is denoted as $h(x, y)$. Noise is known to affect an image and is denoted as $n(x, y)$. For an LSI system, the degraded image is expressed as follows:

$$g(x, y) = f(x, y) * h(x, y) + n(x, y)$$

Here the symbol '*' represents the convolution operation. By applying Fourier transform, the equation in the frequency domain becomes as follows:

$$G(u, v) = F(u, v) \times H(u, v) + N(u, v)$$

The original image can be retrieved by rearranging this equation to get the following expression:

$$F(u,v) = \frac{G(u,v)}{H(u,v)} - \frac{N(u,v)}{H(u,v)}$$

The original image $f(x, y)$ can be obtained by applying an inverse Fourier transform to $F(u, v)$.

6.5 NOISE MODELLING

First, let us try to categorize and model noise patterns. Some of the frequent noises that are encountered in image processing are categorized based on the criteria of distribution, correlation, nature, and source. This is shown in Fig. 6.9.

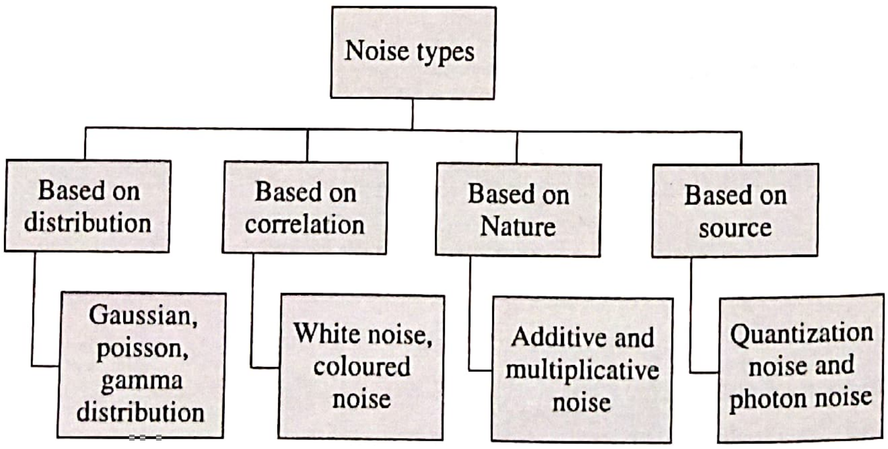


Fig. 6.9 Types of noises

6.5.1 Noise Categories Based on Distribution

Since noise is a fluctuation in pixel values, it is characterized as a random variable. A random variable probability distribution is an equation that links the values of the statistical result with its probability of occurrence. Categorization of noise based on probability distribution is very popular. On the basis of its probability distribution, noise can be classified as follows:

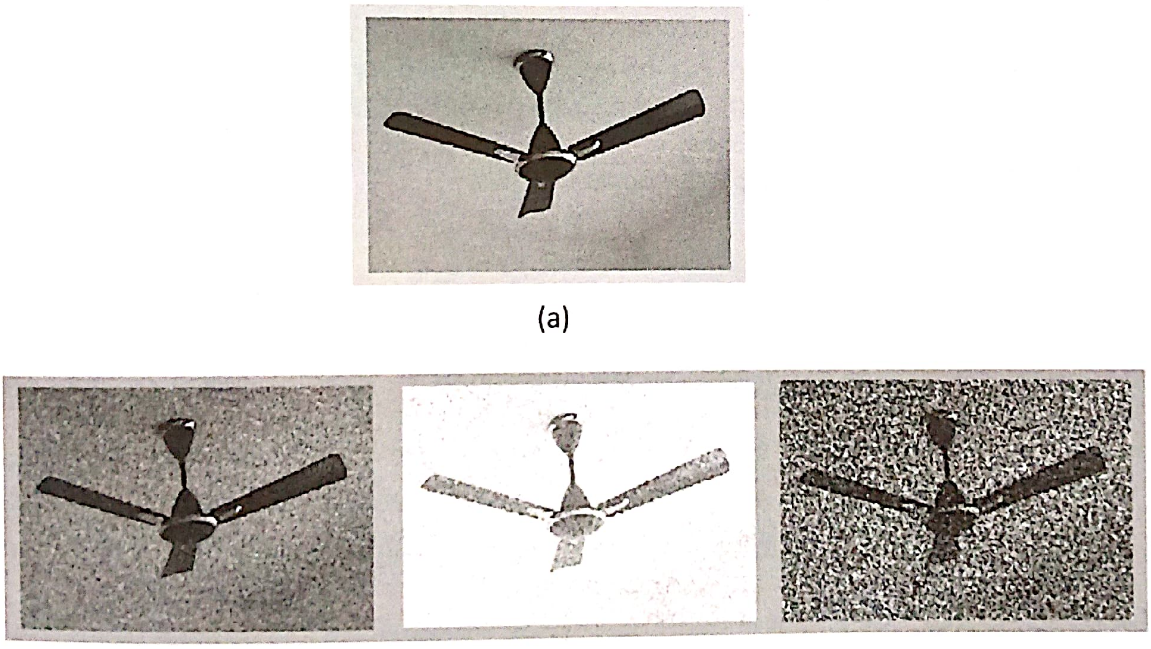
6.5.1.1 *Gaussian Noise*

A random noise that enters a system can be modelled as a Gaussian or normal distribution, which is a bell-shaped curve. Gaussian noise is mathematically denoted as follows:

$$F = f(x, y) \pm N_a$$

where N_a is the Gaussian PDF and $f(x, y)$ is the noiseless image.

Gaussian noise affects both the dark and the light areas of an image. A sample image and its corresponding Gaussian noise-affected images are shown in Figs 6.10(a) and 6.10(b)–(d), respectively.



(a)

(b) (c) (d)

Fig. 6.10 Gaussian noise (a) Original image (b) Image with Gaussian noise (default variance = 0.01) (c) Image with Gaussian noise (mean = 0.5, variance = 0.01) (d) Image with Gaussian noise (mean = 0, variance = 0.07)

The Gaussian probability density function (PDF) is given as follows:

$$P(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(z-m)^2}{2\sigma^2}}$$

Here z is the grey level of the pixel, and m and σ are the mean and standard deviation of the image, respectively. It has been observed that the error due to Gaussian noise is very small and the average of all errors is 0. Gaussian noise is also known as additive noise and can be modelled as a simple additive process.

6.5.1.2 Impulse or Salt-and-pepper Noise

This noise is known by various names such as shot noise, salt-and-pepper noise, and binary noise. It is caused by a sudden disturbance in the image signal. Mostly this noise is caused by sensor and memory problems due to which pixels are assigned incorrect maximum values. Images corrupted with salt-and-pepper noises of different densities are shown in Figs 6.11(a)–(c).

(a) (b) (c)

Fig. 6.11 Salt-and-pepper noise (a) Image with default noise density (b) Image with noise density = 0.2 (c) Image with noise density = 0.3

The PDF of impulse noise is given as follows:

$$P(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

Here z is the pixel value. Variables a and b are grey levels. If $b > a$, then level a will appear as a light spot and vice versa. If P_a or P_b is zero, then the impulse noise is called unipolar noise. If they are not zero and are approximately equal, the resulting noise is called bipolar noise or salt-and-pepper noise.

6.5.1.3 Poisson Noise

Poisson noise manifests as a random structure or texture in images, and is very common in X-ray images. The PDF of Poisson noise is given as follows:

$$P(z) = \frac{(np)^z}{z!} \times e^{-np}$$

where n is the total number of pixels and p is the ratio of noise pixels to the total number of pixels.

6.5.1.4 *Exponential Noise*

This type of noise occurs in images mostly due to illumination problems. The PDF of exponential noise is given as follows:

$$P(z) = \begin{cases} a \times e^{-az} & \text{for } z \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance are given as $\frac{1}{a}$ and $\frac{1}{a^2}$, respectively.

(a) (b)

Fig. 6.12 Illustration of exponential noise (a) Original image (b) Image with exponential noise

An original image is shown in Fig. 6.12(a) and the image with exponential noise in Fig. 6.12(b).

6.5.1.5 *Gamma Noise*

Gamma distributed noise also occurs mostly due to illumination problems. The PDF of gamma noise is given as follows:

$$P(z) = \begin{cases} \dfrac{a^b \times z^{b-1}}{(b-1)!} e^{-a^2} & \text{for } z \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance of the distribution are given as $\frac{b}{a}$ and $\frac{b}{a^2}$, respectively.

The resultant image after adding gamma noise to the original image of Fig. 6.12(a) is shown in Fig. 6.13.

Fig. 6.13 Image with gamma noise

6.5.1.6 *Rayleigh Noise*

This type of noise is mostly present in range images. Range images are used in many remote sensing applications where the pixel value indicates the distance between the object and the camera system. The PDF of Rayleigh noise is given as follows:

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-\frac{(z-a)^2}{b}} & \text{for } z \geq a \\ 0 & \text{otherwise} \end{cases}$$

The mean is given as $a + \sqrt{\frac{\pi b}{4}}$ and the variance as $\frac{b(4-\pi)}{4}$.

The image resulting after adding Rayleigh noise to the original image of Fig. 6.12(a) is shown in Fig. 6.14.

Fig. 6.14 Image with Rayleigh noise

6.5.1.7 Uniform Noise

Uniform noise is another popular noise encountered in images where different values of the noise are equally probable. It occurs due to the quantization process. The PDF of uniform noise is defined as follows:

$$P(z) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq f(x,y) \leq b \\ 0 & \text{otherwise} \end{cases}$$

The mean is $(a+b)/2$ and the variance is $(b-a)^2/12$. Here a and b are, respectively, the minimum and maximum grey-scale values of the image.

The image resulting after adding uniform noise to the original image of Fig. 6.12(a) is shown in Fig. 6.15.

Fig. 6.15 Image with uniform noise

6.5.2 Noise Categories Based on Correlation

The statistical dependence among pixels is called a correlation. If a pixel is independent of its neighbouring pixels, it is called an uncorrelated pixel; otherwise, it is termed as a correlated pixel. Correlation is a statistical word that implies the degree of association between random variables. The degree of association or the level of dependency can be determined by a covariance matrix. If the elements of the covariance matrix of two random variables, except the diagonal elements, are 0, then the random variables are uncorrelated, which means that no two points in the time domain of the noise are related to each other. Uncorrelated noise is called white noise. Mathematically, for white noise, the noise power spectrum or power spectral density remains constant with frequency.

Coloured noise is more difficult to characterize as its origin is mostly unknown—that is, finding the correlated characteristics of coloured noise is difficult because of the involvement of many components the origins of which are uncertain. One such coloured noise is pink noise. Its power spectrum, unlike that of white noise, is not constant but is proportional to the reciprocal of frequency. This is also called $\frac{1}{f}$ or flicker noise.

6.5.3 Noise Categories Based on Nature

Based on the nature of its presence, a noise can be modelled either as an additive or as a multiplicative process.

6.5.3.1 *Additive Noise*

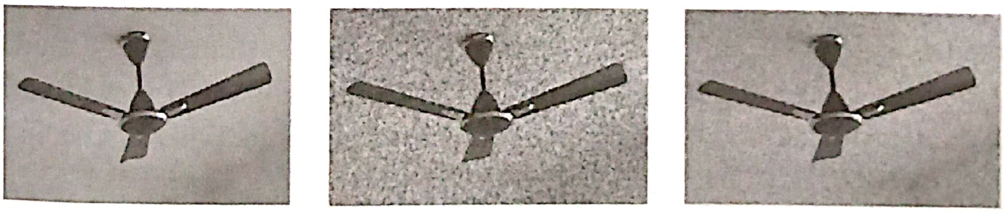
In case of additive noise, an image can be perceived as the image plus noise. This is a linear problem. Thus, the perceived image $g(x, y) = f(x, y) + n(x, y)$.

6.5.3.2 *Multiplicative Noise*

Multiplicative noise can be modelled as a multiplicative process. Speckle noise is one of the most encountered multiplicative noises in image processing. This noise is widely present in medical images, especially ultrasound images. It is a random signal in which the average amplitude increases with the overall signal intensity. It appears as bright specks in the lighter regions of an image. It can be modelled as a pixel value multiplied by the random value. An original image and its corresponding images with speckle noise are shown in Figs 6.16(a), and 6.16(b) and (c), respectively. Speckle noise can be modelled as follows:

$$I = f(x, y) + (f(x, y) \times N_g)$$

where N_g is a random noise having a zero mean Gaussian probability distributive function.



(a) (b) (c)

Fig. 6.16 Multiplicative noise (a) Original image (b) Image with speckle noise of default mean = 0 and variance = 0.04 (c) Image with speckle noise of variance = 0.01

6.5.4 Noise Categories Based on Source

Two of the noise categories, based on source, frequently encountered in image processing are quantization noise and photon noise.

Quantization noise occurs due to a difference between the actual and the allotted values, and is inherent in the quantization process.

Photon noise occurs due to the statistical nature of electromagnetic waves. Due to the statistical variation, generation of photons is not constant. This causes variations in photon count, which can be termed as photon noise. Photon noise is present in many medical images, as the pixel value is the count of photons in these images.

6.6 ESTIMATION OF DEGRADATION FUNCTIONS

Degradation functions can be estimated using the following methods:

6.6.1 Estimation by Observation

This is a method of finding information from the observed image. Identify a portion of the image that is not visually blurred. The degradation function can be estimated by applying an inverse Fourier transform to the ratio of the Fourier transform of the observed image to the Fourier transform of a sub-image.

6.6.2 Estimation by Experimentation

When the instrument used for obtaining the observed image is available, device settings that were used to acquire the degraded image can be found. This knowledge is used to obtain a simple impulse function using the same device settings. As the Fourier transform of an impulse function is a constant, the degradation function can be estimated by applying an inverse Fourier transform to the ratio of the Fourier transform of the observed image to the Fourier transform of the impulse function.

6.6.3 Estimation by Modelling

A model is a set of equations that approximate a real system. Many models have been developed for modelling degradations such as motion blurs and atmospheric turbulence. Using the constructed model of the system, degradation functions can be approximated.

Two types of techniques are available based on the estimation of the blurring function:

1. Direct estimation technique
2. Indirect estimation technique

In the direct estimation technique, the blur function is directly measured by isolating the image of an object pixel. By observing the point source object, one can get the impulse response of the blur, as it is the image of the point source object. By taking many such samples and by observation and estimation, a covariance function can be estimated for a relatively uniform region of the image.

Indirect estimation techniques, on the other hand, use strategies. One such strategy can be formulated as follows. The degradation model that has been constructed is the convolution of the image $f(x, y)$, blurring function $h(x, y)$, and additive noise. Convolution is multiplication in the frequency domain. Let us assume that there is no noise. Therefore, the logarithm of the degraded image is equal to the sum of the logarithm of the original image and that of the blur.

That is, $g(x, y) = f(x, y) \times h(x, y)$ in the spatial domain becomes $G(u, v) = F(u, v)H(u, v)$ in the frequency domain.

Taking logarithm on both sides, one gets the following relation:

$$\ln(G(u, v)) = \ln(F(u, v) + \ln(H(u, v)$$

Now, statistical approximations can be made. For example, some portions of the relatively good part of the image (u_k) and the degraded image (v_k) can be obtained. Let M be the dimension of the image. Then, the possible estimation of H is expressed as follows:

$$\log H = \frac{1}{M}\left\{\sum_{k=1}^{M}[\log|v_k| - \log|u_k|]\right\}$$

Thus, H is determined by statistical means.

6.7 IMAGE RESTORATION IN PRESENCE OF NOISE ONLY

Spatial filters, which are used in image smoothing and sharpening, are also useful for removing noise. Image restoration spatial filters are of two types—mean (or average) filters and order-statistic (rank-order) filters. The difference between these filters is that mean filters are based on the concept of convolution, whereas order-statistic filters do not use convolution but order the pixels of the neighbourhood and select a pixel value based on its order.

6.7.1 Mean Filters

These filters replace each central pixel value in the window (W) with the average of all pixel values within that window. The following five types of mean filters are available.

6.7.1.1 Arithmetic Mean Filters

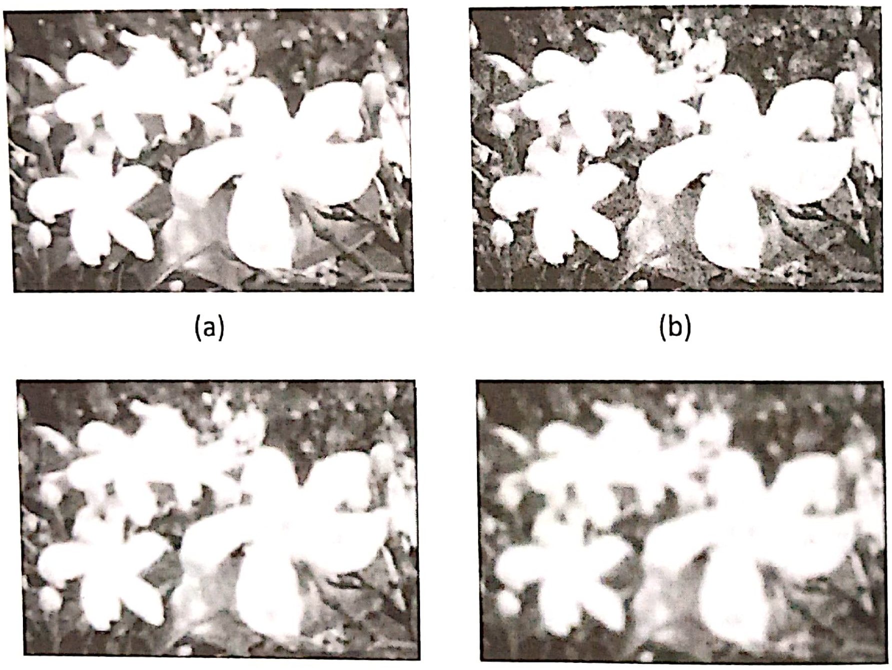
This filter removes local variations within the image. It is similar to a low-pass filter. As discussed in Chapter 5, averaging blurs an image. This filter is very useful in removing Gaussian noise and uniform noise.

An original image is shown in Fig. 6.17(a). The image with Gaussian noise is shown in Fig. 6.17(b) and the resultant image after the removal of noise is shown in Fig. 6.17(c).

(a) (b) (c)

Fig. 6.17 Application of arithmetic mean filter (a) Original image (b) Image with Gaussian noise (c) Result of arithmetic filter

The effect of masks of different sizes is illustrated in Figs 6.18(c) and (d). An original image is shown in Fig. 6.18(a). The image with Gaussian noise is shown in Fig. 6.18(b), and the resultant images after applying 3 × 3 and 5 × 5 masks are shown in Figs 6.18(c) and (d).



(a) (b)

(c) (d)

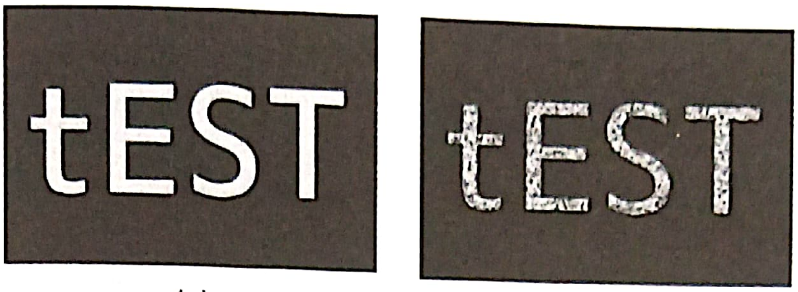
Fig. 6.18 Application of arithmetic filter (a) Original image (b) Image with Gaussian noise (c) Result of arithmetic filter with 3 × 3 mask (d) Result of arithmetic filter with 5 × 5 mask

6.7.1.2 Geometric Mean Filter

This filter eliminates Gaussian noise, but is ineffective for pepper-type noise. It is represented as follows:

$$\prod_{(x,y)\in W} f(x,y)^{\frac{1}{N^2}}$$

An original image is shown in Fig. 6.19(a), and the resultant image after the application of geometric filter is shown in Fig. 6.19(b).



(a) (b)

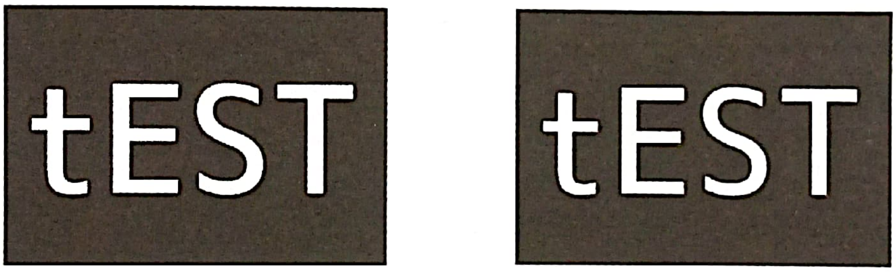
Fig. 6.19 Application of geometric mean filter (a) Original image with salt-and-pepper noise (b) Result of geometric filter

6.7.1.3 Harmonic Mean Filters

This filter is represented as follows:

$$\frac{N^2}{\sum_{(x,\, y)\in W} \frac{1}{f(x,\, y)}}$$

This works well for removing salt-type noise and Gaussian noise.



(a) (b)

Fig. 6.20 Application of harmonic mean filter (a) Original image with Gaussian noise (b) Result of harmonic mean filter

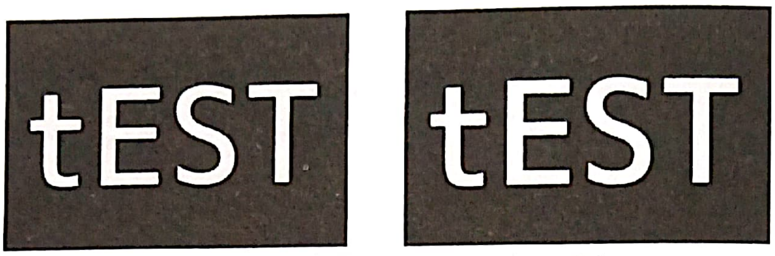
An original image is shown in Fig. 6.20 (a) and the resultant image after the application of harmonic mean filter in Fig. 6.20(b).

6.7.1.4 Contra-harmonic Mean Filters

This filter is very useful in removing either salt or pepper noise. It is represented as follows:

$$\frac{\sum\limits_{(x,\, y)\in W}^{n} f(x,\, y)^{R+1}}{\sum\limits_{(x,\, y)\in W}^{n} f(x,\, y)^{R}}$$

Here, R is the order of the filter and W the window size. A negative value of R eliminates salt-type noise and a positive value eliminates pepper-type noise.



(a) (b)

Fig. 6.21 Application of contra-harmonic mean filter (a) Original image with salt-and-pepper noise (b) Result of contra-harmonic mean filter

An original image is shown in Fig. 6.21(a) and the resultant image after the application of a contra-harmonic filter in Fig. 6.21(b).

Example 6.2 The median of a set of numbers is a value that is larger than half of the elements and smaller than the other half of the elements of that set. If there are two middle numbers, then the arithmetic mean is the median. Find the median of these sets of numbers:

$$S_1 = \{1, 5, 7, 4, 6\}$$
$$S_2 = \{1, 5, 7, 6, 7, 8\}$$

Solution The median of $S_1 = 7$. The median of $S_2 = \dfrac{7+6}{2} = 6.5$.

Example 6.3 Find the geometric and harmonic means of the set $S = \{3, 6\}$.

Solution For n numbers, the geometric mean is given as follows:

$$\sqrt[n]{a_1 \times a_2 \times a_3 \times \cdots \times a_n}$$

$\therefore$ Geometric mean $= \sqrt{3 \times 6} = \sqrt{18} = 4.243$.

Harmonic mean is given as follows: $\dfrac{N}{\dfrac{1}{a_1} + \dfrac{1}{a_2} + \cdots + \dfrac{1}{a_n}}$

Therefore, harmonic mean $= \dfrac{2}{\dfrac{1}{3} + \dfrac{1}{6}} = 4$.

6.7.1.5 Yp-mean Filter

This filter removes salt-type noise for negative values of P and pepper-type noise for positive values of P. It is represented as follows:

$$\text{Yp-mean} = \sum_{(x,y)\in W}^{n} \left(\frac{f(x,y)^P}{N^2} \right)^{\frac{1}{P}}$$

where P is the order of the filter.

(a) (b)

Fig. 6.22 Application of Yp-mean filter (a) Original image with Gaussian noise (b) Resultant image with Yp-mean filter

An original image with Gaussian noise is shown in Fig. 6.22(a) and the resultant image after the application of Yp-mean filter in Fig. 6.22(b).

6.7.2 Order-Statistic Filters

Order-statistic (also known as rank, rank-order, or order) filters are a form of filters that are not based on convolution. This is the difference between order-statistic filters and the filters that have been discussed earlier. These filters, instead of using convolution, order the pixels that come under the selected mask. Then, depending on the filter requirement, based on a predetermined value of n, the nth value of the list is chosen and this value replaces the central pixel.

Let us assume a mask of size 3 × 3. The pixel values are arranged in ascending order $I_1 \leq I_2 \leq \cdots \leq I_9$ based on the grey-scale value. The order determines the value that should replace the central pixel. For example, consider the image shown in Fig. 6.23.

12	13	15
17	14	18
19	20	21

Fig. 6.23 Sample 3 × 3 image

Using the application of order statistics, the pixel values are arranged in ascending order as follows:

$$\{\underset{\text{Minimum}}{\underset{\downarrow}{12}}, 13, 14, 15, \underset{\text{Median}}{\underset{\downarrow}{17}}, 18, 19, 20, \underset{\text{Maximum}}{\underset{\downarrow}{21}}\}$$

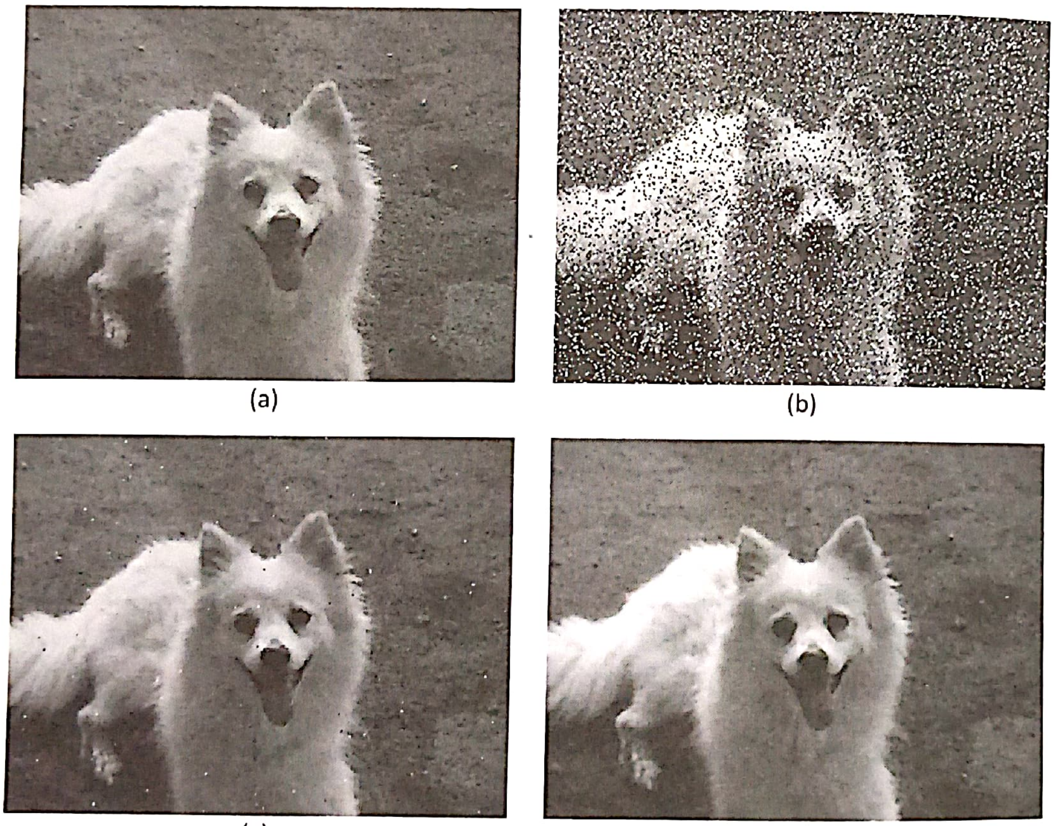
Order-statistic filters are differentiated based on how they choose the values in the sorted list. The position indicates the rank. For example, the min-filter uses the minimum value of this sorted list. Some of the popular rank-order filters are discussed in the following sections.

6.7.2.1 *Median Filter*

Median filter is an example of a non-linear filter. It does not use convolution, but simply sorts the list and finds the median. Then, the centre pixel is replaced by the median value. The procedure for implementing the median filter is as follows:

1. Read all the pixel values in the selected window.
2. Sort the values in ascending order.
3. Choose the median, that is, the central value. This value replaces the central pixel of the mask. Then, move the window by one pixel and repeat the process until the entire image is processed. Outer rows and columns are not affected by this process.

In the image shown in Fig. 6.23, median = 17. This value replaces the central pixel of the spatial mask, and then the mask is moved and the process is repeated till the entire image is processed. An original image, an image with salt-and-pepper noise, and the images obtained after applying median filters of sizes 3 × 3 and 5 × 5 are shown in Figs 6.24(a), (b), and (c) and (d), respectively. It can be observed that the 5 × 5 filter is more effective in removing the noise.



(a) (b) (c) (d)

Fig. 6.24 Median filter (a) Original image (b) Image with salt-and-pepper noise (c) Image produced by a median filter with 3 × 3 mask (d) Image produced by a median filter with 5 × 5 mask

6.7.2.2 Maximum Filters

This filter selects the largest value in the sorted list. The largest value is the last element of the list. This filter is used for removing pepper-type noise. For the given example, the largest value of the sorted list is underlined:

$$\{12, 13, 14, 15, 17, 18, 19, 20, \underline{21}\}$$

The procedure is the same as that in case of a median filter, where the mask is moved till the entire image is processed.

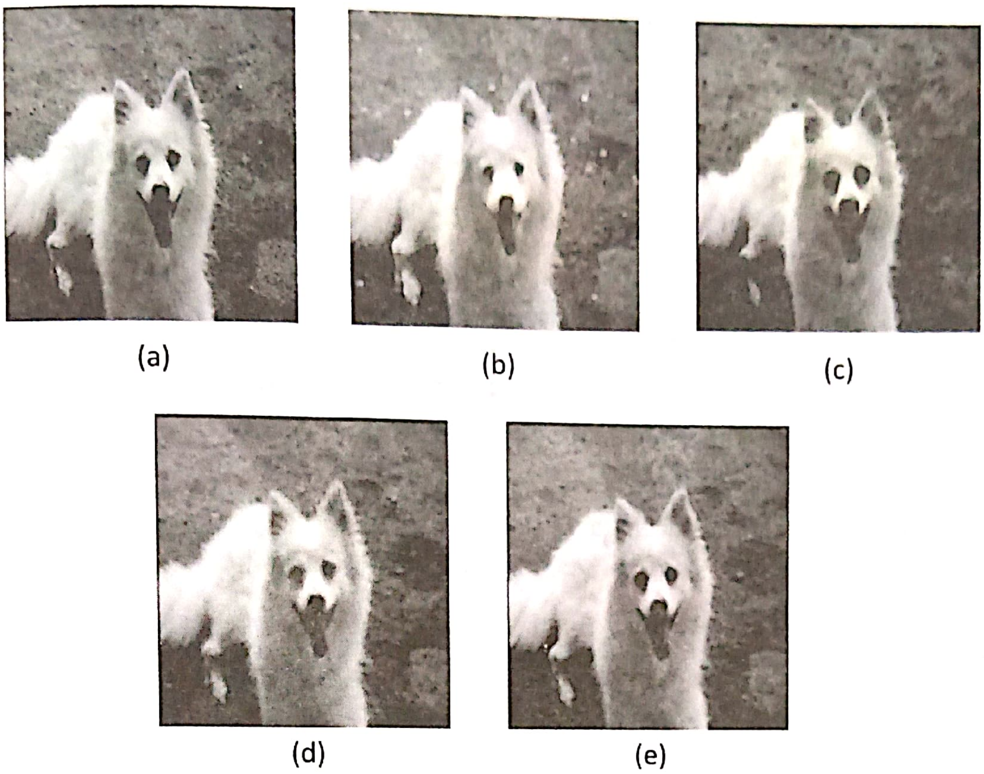


Fig. 6.25 Application of order-statistic filters (a) Original image (b) Output with max filter (c) Output with min filter (d) Output with midpoint filter (e) Output with alpha-trimmed filter

An original image is shown in Fig. 6.25(a) and the resultant of the maximum filter in Fig. 6.25(b).

6.7.2.3 Minimum Filters

This filter selects the smallest value, which is the first element of the sorted list. It is very effective in eliminating salt-type noise. An original image is shown in Fig. 6.25(a) and the resultant image of the minimum filter in Fig. 6.25(c).

6.7.2.4 Midpoint Filters

This filter selects the midpoint, which is given by $\frac{I_1 + I_{N^2}}{2}$. This is nothing but the average of the minimum and maximum values. This filter is very effective in removing Gaussian noise and uniform noise. An original image is shown in Fig. 6.25(a), and the resultant image of a midpoint filter is shown in Fig. 6.25(d).

6.7.2.5 Alpha-trimmed Mean Filters

An alpha-trimmed mean filter is based on the concept of computation of the average of the pixels that fall within the window. This is similar to a simple average filter, but the difference is that the user can clip some of the pixels by specifying an alpha value. Selection of the pixel value for replacing the central pixel value is represented by the following formula:

$$\frac{1}{N^2 - 2T} \sum_{i=T+1}^{N^2-T} f_i$$

In this filter, some pixels are excluded based on the value of T. Its value can range from 0 to $N^2 - 1$. If $T = 0$, this filter reduces to an arithmetic mean filter, and if $T = (N^2 - 1)/2$, it reduces to a median filter. This filter is very useful in removing noises such as Gaussian noise and salt-and-pepper noise. An original image is shown in Fig. 6.25(a), and the result obtained after applying this filter is shown in Fig. 6.25(e).

Similarly, many order-statistic filters can be constructed. For example, a range filter uses the difference between the maximum and minimum values.

Example 6.4 Consider the image {12, 13, 14, 15, 17, 18, 19, 20, 12}. Find the pixel value to replace the central pixel in case of a median, a max, a min, a midpoint, and an alpha-trimmed filter.

Solution For a median filter, the median value of the sorted list is underlined:

{12, 13, 14, 15, <u>17</u>, 18, 19, 20, 21}

The median filter replaces the central pixel with this median value.

For the given example, the minimum value of the sorted list is underlined:

{<u>12</u>, 13, 14, 15, 17, 18, 19, 20, 21}

The max value in the list is 21. Therefore, the min and max filters replace the central pixel with values 12 and 21, respectively.

For the given example, the mid value is $(12 + 21)/2 = 33/2 = 16.5$, which is approximately 17. Hence, the midpoint filter replaces the central pixel with 17.

For an alpha-trimmed filter, let us assume that $T = 2$, so two pixel values on either end are excluded in the sorted list to yield the following list:

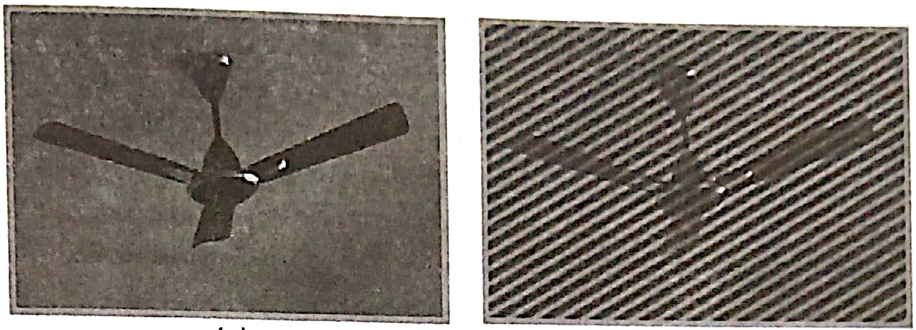
{14, 15, 17, 18, 19}

Now, the average can be calculated as 83/5. This is approximately 17. Therefore, the earlier central pixel value is not affected.

Order-statistic filters along with mean filters are based on the spatial domain. Similarly, frequency-based filters can also be used for restoration of images. One such popular filter is a band-reject filter, which is used to remove periodic noises. This is not possible using any other spatial-domain filter. Frequency-domain-based filters have been discussed in Chapter 5.

6.8 PERIODIC NOISE, AND BAND-PASS AND BAND-REJECT FILTERING

Periodic noise is sinusoidal at multiples of a specific frequency and is periodic in nature. The occurrence of uniform bars over an image is a manifestation of periodic noise. An original image and the resultant image with periodic noise are shown in Figs 6.26(a) and (b), respectively. Periodic noise mostly occurs due to electrical interferences.



(a) (b)

Fig. 6.26 Illustration of periodic noise (a) Original image (b) Image with periodic noise

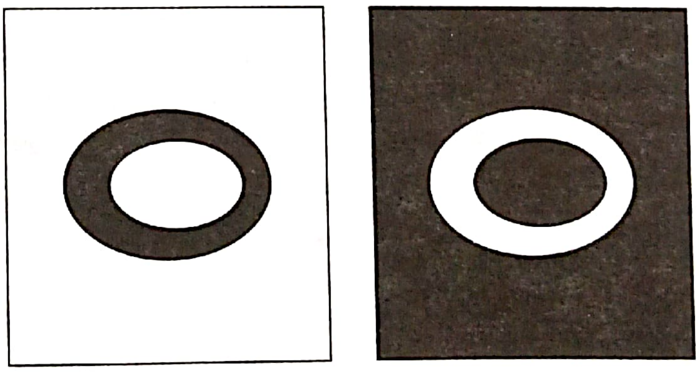
Periodic noise can be removed by band-pass, band-reject, and notch filters. Let us discuss these filters now.

6.8.1 Band-pass Filters

Band-pass filters allow frequencies within a particular range to pass through and attenuate all other frequencies. In other words, band-pass filters allow frequency components if they fall in the range $D_\ell - D_h$. This range is called a band. The transfer function for a 1D band-pass filter is given as follows:

$$H(D) = \begin{cases} 1 & \text{for } D_\ell \le D \le D_h \\ 0 & \text{for } D > D_0 \end{cases}$$

Here, D_0 is the cut-off frequency in one dimension.



(a) (b)

Fig. 6.27 Frequency-domain filters (a) Band-pass filter (b) Band-reject filter

The transfer function for a 2D band-pass filter is given as follows:

$$H(u,v) = \begin{cases} 1 & \text{if } D_0 - \frac{W}{2} \le D(u,v) \le D_0 + \frac{W}{2} \\ 0 & \text{otherwise} \end{cases}$$

Here, D_0 is the cut-off frequency for this 2D filter, $D(u, v)$ is the distance of the point (u, v) from the centre, and w is the width of the band.

Instead of applying this 1D filter along the row and column, it is better to design a single filter that can be applied radially, as discussed in Chapter 5. The shapes of the bands in 2D for band-pass and band-reject filters are shown in Figs 6.27(a) and (b), respectively.

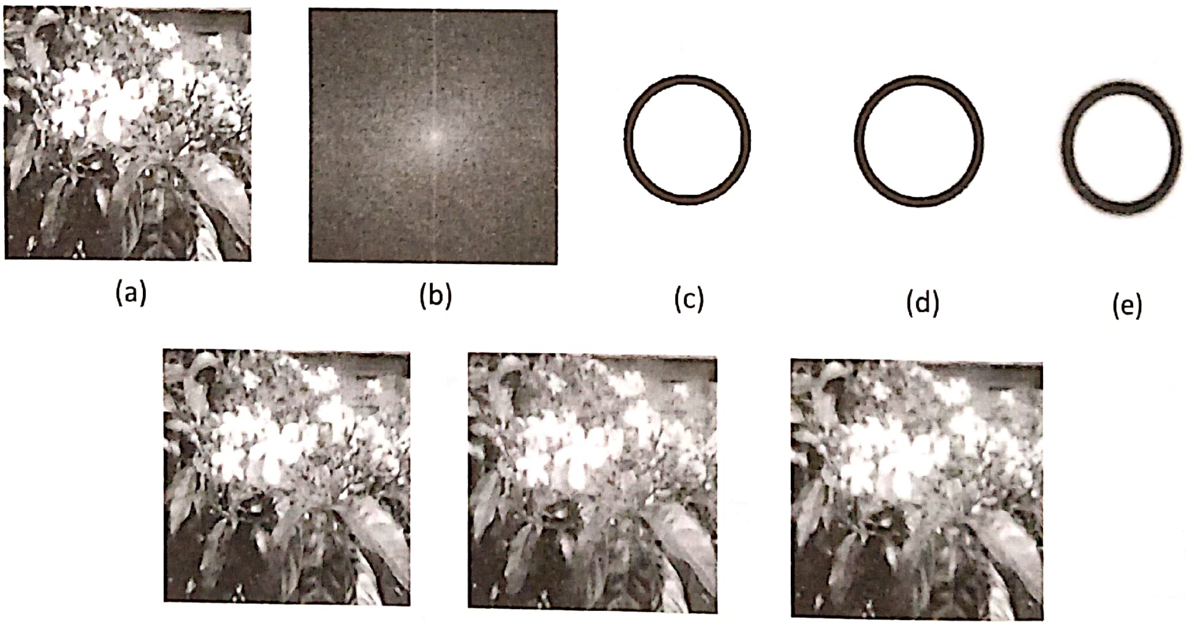
The Butterworth filter formulation is given as follows:

$$H_{\text{bp}}^{\text{Butterworth}}(u,v)=\frac{\left[\dfrac{D(u,v)w}{D^2(u,v)-D_0^2}\right]^{2n}}{1+\left[\dfrac{D(u,v)w}{D^2(u,v)-D_0^2}\right]^{2n}}$$

The Gaussian formulation of a band-pass filter is given as follows:

$$H_{\text{bp}}^{\text{Gaussian}}(u,v)=e^{-\frac{1}{2}\left[\frac{D^2(u,v)-D_0^2}{D(u,v)w}\right]^2}$$

The application of a band-pass filter is shown in Figs 6.28(a)–(h).



(a) (b) (c) (d) (e)

(f) (g) (h)

Fig. 6.28 Application of band-pass filter (a) Original image (b) FFT of original image (c) FFT of ideal band-pass filter (d) FFT of Butterworth band-pass filter (e) FFT of Gaussian band-pass filter (f) Resultant image of ideal band-pass filter (g) Resultant image of Butterworth band-pass filter (h) Resultant image of Gaussian band-pass filter

6.8.2 Band-reject Filters

A band-reject filter is the complement of a band-pass filter. The purpose of a band-reject filter is to attenuate frequencies of a limited range, while leaving the other frequencies unchanged. Band-reject filters are very effective in removing periodic noise, and the ringing effect is normally small. This filter rejects the frequencies if they fall in the range $D_\ell - D_h$. Its 1D transfer function is given as follows:

$$H(D) = \begin{cases} 0 & \text{for } D_\ell \leq D \leq D_h \\ 1 & \text{for } D > D_0 \end{cases}$$

The transfer function for a 2D band-reject filter is given as follows:

$$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \leq D(u,v) \leq D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$$

If the transfer function of a band-pass filter is given as $H_{\text{bp}}(u,v)$, then the transfer function for the band-reject filter can be given as $H_{\text{br}}(u,v) = 1 - H_{\text{bp}}(u,v)$. Thus, they are complementary in nature.

A Butterworth band-reject filter of order n is given by the following mathematical formulation:

$$H_{\text{br}}^{\text{Butterworth}}(u,v) = \frac{1}{1 + \left[\frac{D(u,v)w}{D^2(u,v) - D_0^2}\right]^{2n}}$$

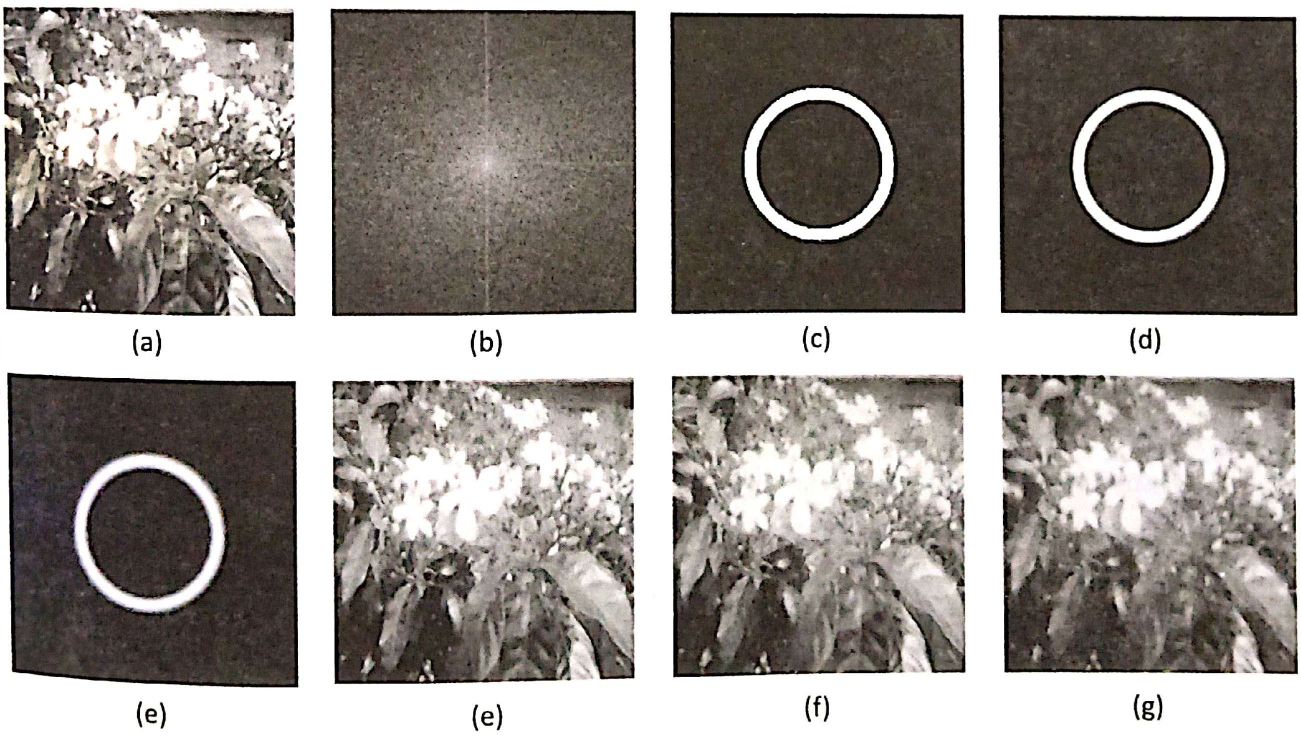


Fig. 6.29 Application of band-reject filter (a) Original image (b) FFT of original image (c) FFT of ideal band-reject filter (d) FFT of Butterworth band-reject filter (e) FFT of Gaussian band-reject filter (f) Resultant image of ideal band-reject filter (g) Resultant image of Butterworth band-reject filter (h) Resultant image of Gaussian band-reject filter

The mathematical formulation of a Gaussian band-reject filter is given as follows:

$$H_{\text{br}}^{\text{Gaussian}}(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D^2(u,v)-D_0^2}{D(u,v)w}\right]^2}$$

The application of a band-reject filter is shown in Figs 6.29(a)–(h).

6.8.3 Notch Filters

A notch filter is a special form of a band-reject filter. Instead of removing the entire range of frequencies, it removes only selective frequency components. In other words, it selectively attenuates or allows frequencies within a specific range around the central frequency. It is useful in removing a periodic signal of a clearly defined frequency, like the interference patterns caused by electrical disturbances.

Assuming an ideal filter of radius D_0 with centre (u_0, v_0), the filter that rejects the frequencies is given as follows:

$$H_{\text{nr}}^{\text{ideal}} = \begin{cases} 0 & \text{if } D_1(u,v) < D_0 \text{ or } D_2(u,v) > D_0 \\ 1 & \text{otherwise} \end{cases}$$

Here,

$$D_1(u,v) = \left[\left(\left(u - \frac{M}{2}\right) - u_0\right)^2 + \left(\left(v - \frac{N}{2}\right) - v_0\right)^2\right]^{\frac{1}{2}}$$

$$D_2(u,v) = \left[\left(\left(u - \frac{M}{2}\right) - u_0\right)^2 + \left(\left(v - \frac{N}{2}\right) - v_0\right)^2\right]^{\frac{1}{2}}$$

A Butterworth notch filter of order n is given as follows:

$$H_{\text{nr}}^{\text{Butterworth}}(u,v) = \frac{1}{1 + \left[\frac{D_0^2}{D_1(u,v)D_2(u,v)}\right]^n}$$

A Gaussian notch filter is given as follows:

$$H_{\text{nr}}^{\text{Gaussian}}(u,v) = 1 - e^{-\frac{1}{2}\left(\frac{D_1(u,v)D_2(u,v)}{D_0^2}\right)}$$

Notch pass filters are complementary to notch reject filters. A notch pass filter can be computed as follows:

$$H_{\text{notch-pass}}(u,v) = 1 - H_{\text{notch-reject}}(u,v)$$

Here, $H_{\text{notch-reject}}(u,v)$ is the transfer function of the notch reject filter.

Let us now review the fundamentals of image restoration and some of the deconvolution algorithms.

6.9 IMAGE RESTORATION TECHNIQUES

The different types of image restoration techniques are shown in Fig. 6.30.

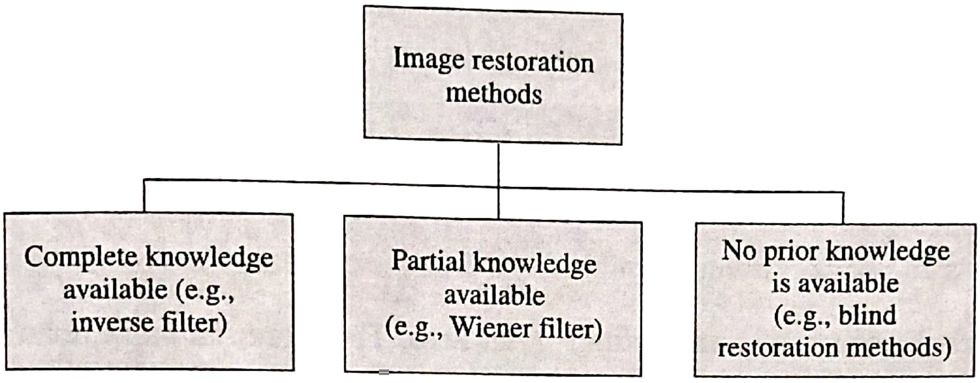


Fig. 6.30 Types of image restoration methods

Based on the knowledge available for the blurring function, the following three scenarios are possible:

1. Complete knowledge of the blurring function is available. This is a simple case in which an original image can be retrieved by applying an inverse filter. This approach is called deconvolution (or inverse filtering).
2. Only partial knowledge of the blurring function H is available. A Wiener filter is helpful in this scenario.
3. There is no knowledge of the blurring function. Approaches like blind restoration techniques are used in this scenario. Blind restoration (or blind deconvolution) is a procedure where an attempt is made to deconvolve the degraded image without any prior knowledge of the cause of degradation.

Let us first review some of the fundamentals of image restoration algorithms. Algebraic method is very popular for image restoration, as it uses the concepts of matrices and linear algebra instead of those of integrals. The two types of algebraic methods are as follows:

1. Unconstrained method
2. Constrained method

6.9.1 Unconstrained Method

As the name suggests, the unconstrained approach has no constraints. The basic approach in the unconstrained method is that the approximate image ($\hat{f}$) of the original image f is recovered by minimizing the noise. As discussed in Section 6.4, the general image degradation model is given as follows:

$$g = Hf + n$$

Therefore, $n = g - Hf$

In the absence of any knowledge regarding n, it is desirable to seek $\hat{f}$ such that $H\hat{f}$ approximates g in the least square sense. The norm of $\|n\|^2$ is given by $n^T n$. Therefore,

$$\|n\|^2 = \|g - H\hat{f}\|^2$$

where
$$\|g - H\hat{f}\|^2 = (g - H\hat{f})^T (g - H\hat{f})$$

This can be equivalently stated as a minimization problem and can be written with respect to $\hat{f}$, as an image restoration minimization problem, as $J(\hat{f}) = \|g - H\hat{f}\|^2$. Minimization involves differentiating and setting $\hat{f}$ to zero. Therefore, minimization of $\hat{f}$ gives the following relation:

$$\frac{\partial J(\hat{f})}{\partial \hat{f}} = 0$$

$$-2H^T (g - H\hat{f}) = 0$$

$$-2H^T g + 2H^T H\hat{f} = 0$$

This simplifies to the following relation:

$$HH^T\hat{f} = H^T g$$

Further simplification leads to the following equation:

$$\hat{f} = (H^T H)^{-1} H^T g$$
$$= H^{-1} g$$

The expression $f = H^{-1} g$ is used by an inverse filter for implementing a simple degradation.

).2 Constrained Method

Constrained restoration method is another image restoration process that models image restoration as a minimization function of the form $|Q\hat{f}|^2$, where Q is a linear operator of f that measures the smoothness of an image in the form of first- or second-order derivatives of the image.

Q is subjected to the optimality constraint $\|Q\hat{f}\|^2 = \|n\|^2$. Unlike the unconstrained restoration method, the focus of the constrained method is to minimize the error between the approximate and the original images. This is equivalent to the following minimization problem:

$$J(\hat{f}) = \left(\|Q\hat{f}\|^2 + \alpha\left(\|g - H\hat{f}\| - \|n\|^2\right)\right)$$

Here, α is called the Lagrange multiplier, $\|Q\hat{f}\|^2$ is called the smoothness term or regularization term, and the rest is called the restoration term. This method is called regularization, which is performed to solve ill-posed image restoration problems. The regularization term aims to force g to the closest original image f. The rest of the term enforces the prior knowledge of the image. Using calculus, differentiation of this equation yields the following result:

$$\frac{\partial(J(\hat{f}))}{\partial\hat{f}} = 0$$

$$= 2Q^T\hat{f} - 2\alpha H^T\left(g - H\hat{f}\right)$$

Solving this equation for $\hat{f}$, the following relation is obtained:

$$\hat{f} = \left(H^T H + \gamma Q^T Q\right)^{-1} H^T g$$

where $\gamma = \frac{1}{\alpha}$. This parameter is important and should be tuned to satisfy the optimality constraint. In other words, an increase in γ emphasizes the restoration term and a decrease emphasizes the smoothness term. It is possible to convert the algebraic solution to the frequency domain by applying Fourier transform.

Let us now review some of the filters based on the algebraic method.

6.9.3 Inverse Filters

The process of removing blurs and noise is called deconvolution (or inverse filtering). A simple deconvolution starts with an assumption that a blur is characterized by the PSF or the impulse response of the system. It assumes that most blurs are linear, and the output of the imaging system is the convolution of the impulse response and the input image. In the presence of noise, the degraded image is given as follows:

$$g(x, y) = f(x, y) * h(x, y) + n(x, y)$$

By applying Fourier transform, this equation yields the following relation:

$$G(u, v) = F(u, v)\, H(u, v) + N(u, v)$$

Therefore, $F(u, v) = G(u, v)/H(u, v) - N(u, v)/H(u, v)$. If noise is assumed to be zero, then the deconvolution is equivalent to the following expression:

$$G(u, v) = F(u, v)H(u, v)$$

Therefore, $$F(u,v) = \frac{1}{H(u,v)} G(u,v)$$

In other words, an inverse filter is the inverse of the degradation function. The inverse filter acts as a high-pass filter, causing blurring and increasing noise. One major problem is that $H(u, v)$ may be zero or closer to zero, and hence, the inverse may result in infinity. Therefore, one possible solution is to design an inverse filter not with the transfer function $\frac{1}{H(u,v)}$, but with the following transfer function:

$$\hat{H}(u,v) = \begin{cases} \frac{1}{H(u,v)} & \text{if } |H(u,v)| \geq \varepsilon \\ 1 & \text{if } |H(u,v)| < \varepsilon \end{cases}$$

Here, ε is the threshold value. It is used to mitigate the effect of zeros in the degraded function. An original image is shown in Fig. 6.31(a) and the distorted image is shown in Fig. 6.31(b). The restored image using an inverse filter is shown in Fig. 6.31(c).

(a) (b) (c)

Fig. 6.31 Application of inverse filter (a) Original image (b) Image with distortion (c) Restored image using inverse filter

6.9.4 Wiener Filters

When partial knowledge of the degradation function is available, Weiner filters can be used for image restoration. Let us assume that the image has additive white Gaussian noise, and the aim is to minimize the mean square error between the original and the restored image. Therefore, to design a Wiener filter, an estimation of the original image and the additive noise is required. In other words, this method requires prior knowledge of the power spectra of the noise and original image. It can be observed that the spectral power density is a function of power distribution over different frequencies.

Similar to the unconstrained method, a Wiener filter finds an estimate $\hat{f}(x, y)$ of the original image $f(x, y)$ such that the mean square error is minimized. Thus, a Wiener filter is an optimal filter in terms of the least mean square of the estimated and the original image.

It not only restores the image, but also removes noise by image smoothing. The minimized error is given as follows:

$$e^2 = E\{f(x,y) - \hat{f}(x,y)^2\}$$

where $E(\cdot)$ is the expected value. To know the error, the correlation matrices of f and n are required. Let us assume that R_f and R_n are the correlation matrices of f and n, respectively. These correlation matrices are denoted, in terms of expectation operation, as follows:

$$R_f = E\{f \cdot f^T\}$$

$$R_n = E\{n \cdot n^T\}$$

The use of the unconstrained form and substitution of $Q^+Q = R_f^{-n}R_n$ give the following estimation:

$$\hat{f} = (H^T H + \gamma R_f^{-1} R_n)^{-1} H^{-1} g$$

This filter in the frequency domain corresponds to the following expression:

$$F(u,v) = \left[\frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + \gamma \left| \dfrac{S_n(u,v)}{S_f(u,v)} \right|} \right] \cdot G(u,v)$$

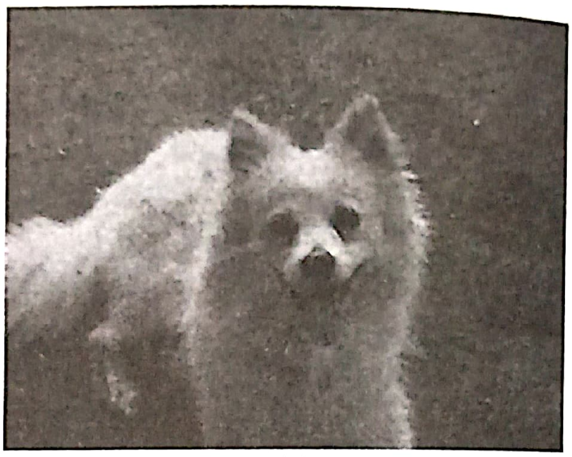
where $S_n(u, v)$ and $S_f(u, v)$ are the spectral power densities of the noise and image, respectively. A cruder version of the Wiener filter often works well in situations where the ratio of $\dfrac{S_n(u,v)}{S_f(u,v)}$ is assumed to be a constant K. This equation is given as follows:

$$F(u,v) \cong \left[\frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + \gamma K} \right] \cdot G(u,v)$$

In the absence of any knowledge of noise, K can be assumed to be the inverse of the SNR of the image, which is averaged over all frequencies. Thus, K is related to the low-frequency aspect of the filter, and if $K = 0$, the filter becomes an inverse filter. The value of γ is very crucial, and it should be tuned so that $\|n\|^2 = \|g - H\hat{f}\|^2$. If $\gamma = 0$, the expression simply reduces to a simple inverse filter. The expression $\gamma = 1$ represents a Wiener filter, and when γ assumes different values, this leads to a parametric Wiener filter.

(a)

(b)



(c)

Fig. 6.32 Application of Wiener filter (a) Image produced by Wiener filter with 3 × 3 mask (b) Image produced by Wiener filter with 5 × 5 mask (c) Image produced by Wiener filter with 9 × 9 mask

Images obtained using Wiener filters having different mask sizes are shown in Figs 6.32(a)–(c).

6.9.5 Constrained Least Square Filters

The problem with a Wiener filter is that it is necessary to know the power spectrum of the original image and noise. The constrained approach is used for implementing this technique. This technique also tries to enforce a constraint to represent some degree of smoothness so that the resultant image is smooth and noise free. Let Q be represented as follows:

$$Q = \min\left\{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[\nabla^2 f(x, y)\right]^2\right\}$$

It can be observed that Q is selected to approximate a Laplacian or second-order derivative of the image, so the problem becomes minimization of Q.

The approximation of the second-order derivative, already discussed in Chapter 5, is possible by convolving the image with a mask $p(x, y) = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{pmatrix}$. Let $P(u, v)$ be the Fourier transform of the matrix.

Minimization of the second-order derivative of the image subject to the constraint $\|g - H\hat{f}\|^2 = \|n\|^2$ yields a solution $\|Q\hat{f}\| = \hat{f}^T Q^T Q\hat{f}$. This leads to a transfer function in the frequency domain, which is given as follows:

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)^2| + \gamma|p(u,v)|^2}\right] G(u,v)$$

Here, γ is the parameter used to tune the degree of smoothness. Let the residual difference be $r_s = H - g\hat{f}$. Then, $\|r_s\|^2 = \|n\|^2 \pm c$, where $\|n\|^2 = MN\left(\sigma_n^2 + m_n^2\right)$ for an optimal filter. Here, M and N are the dimensions of the image, and s_n and m_n are the mean and variance of the image, respectively.

For obtaining the optimal filter, the parameter γ should be tuned. The procedure for tuning is as follows:

1. Specify an initial value of γ.
2. Compute $\hat{f}$ and $\|r\|^2$.
3. Check whether $R(u, v) = G(u, v) - H(u, v)\,\hat{F}(u, v)$; if yes, then stop.
   If $\|r\|^2 < \|n\|^2$, then increase the value of γ.
   Else, if $\|r\|^2 > \|n\|^2 + a$, then decrease the value of γ.

6.9.6 Iterative Image Restoration

Unlike the linear image restoration model, the iterative approach produces successive improvements to the restoration process until a termination condition is reached. Van Cittert introduced this filter in 1930 using the following equation:

$$f_{k+1} = f_k + (g - h * f_k)$$

Here, f is the blur kernel, and when $n = 0, f^0$ is the initial guess and the initial observed degraded image; k is a variable that represents the number of iterations. We will discuss a modified procedure here.

The process starts with a guess, and it repeatedly updates the guess until it arrives at a solution by solving a least square solution iteratively. The process starts with an initial guess based on the observed image g. Let $\hat{f}_0$ be the initial guess and let $\hat{f}_0(x,y) = \gamma g(x,y)$. The important issue here is the selection of γ. For faster convergence, the value of γ should be selected based on the following condition:

$$|1 - \gamma H(u,v)| < 1$$

Here, γ is a positive constant and its value ranges between 0 and 1. Then, the guess is updated as follows:

$$\hat{f}_{k+1}(x,y)=\hat{f}_k(x,y)+\gamma(g(x,y)-\hat{f}_k(x,y)*h(x,y))$$

Here, the symbol '*' denotes a convolution operation. The idea is that if the approximation $\hat{f}_k$ is correct, then the entire term multiplied by γ becomes zero and the term itself vanishes, leading to convergence. The equation in the frequency domain is given as follows:

$$\hat{F}_{k+1}(u,v)=\hat{F}_k(u,v)+\gamma(G(u,v)-\hat{F}_k(u,v)\times H(u,v))$$

The main advantage of this method is the rapidity of recovering the original image from its degraded image with few assumptions, and it also uses fewer operations. However, this method has some disadvantages, for example, it is sensitive to noise and tends to be unstable as the number of iterations increases.

The Lucy–Richardson deconvolution technique is one of the simplest algorithms for iterative image restoration, which is used for restoring astronomical images. The process is illustrated as follows:

$$f_{k+1}=f_k \bullet \left(h \bullet \frac{g}{h \bullet f_k}\right)$$

Here, k is the number of iterations. It can be observed that in every iteration, the method uses the previous degraded image. The Lucy–Richardson method is useful in the presence of noise also. However, the noise increases with an increase in the number of iterations.

An original image is shown in Fig. 6.33(a) and the degraded image in Fig. 6.33(b). The image restored using an iterative filter is shown in Fig. 6.33(c).

(a) (b) (c)

Fig. 6.33 Application of iterative filter (a) Original image (b) Degraded image (c) Image restored using Lucy-Richardson filter

6.9.7 Blind Image Restoration

All the image restoration techniques that have been discussed so far are based on the premise that either the complete knowledge of the blurring function is available (inverse filter) or partial knowledge is available (Wiener filter). However, there may be some situations where we may not have any knowledge of the blurring function. The technique of restoring images in the absence of any knowledge of the degradation function is known as the blind restoration (or blind deconvolution) technique. Often, restoration of images in the absence of any knowledge of the degradation function is a difficult task. To apply blind restoration, an initial guess is made about the degradation function. Often this is done using homomorphic ideas. Blind convolution techniques aim to estimate H. One such popular estimation technique breaks the input image f and degraded image g into M smaller blocks, and computes the Fourier transforms. Then, $\log(H)$ is computed by taking the average of the sum of the differences between degraded image blocks and input image blocks. After this estimation of $\log(H)$ is completed, a Wiener filter can be used to restore the images.

(a) (b) (c)

Fig. 6.34 Blind image restoration (a) Original image (b) Blurred image (c) Result of blind convolution

An original image is shown in Fig. 6.34(a) and its degraded image in Fig. 6.34(b). The image restored using blind convolution is shown in Fig. 6.34(c).

6.10 GEOMETRICAL TRANSFORMATIONS FOR IMAGE RESTORATION

Geometrical transformations include spatial transforms and image interpolation algorithms. Spatial transforms are useful when pixels can be rearranged. Let us assume that (x, y) represent the original coordinates and (x', y') the deformed image. The spatial transformation can then be represented as follows:

$$x' = T_1(x, y)$$

and

$$y' = T_2(x, y)$$

The transformations are represented by the following equations:

$$T_1(x, y) = c_1x + c_2y + c_3xy + c_4$$

and

$$T_2(x, y) = c_5x + c_6y + c_7xy + c_8$$

These equations can be solved for the unknown coefficients, to get the transformation function for the given requirement.

SUMMARY

- Images are degraded by noise, blurs, and artefacts. Image restoration is the process of recovering the original image from a degraded image.
- Degradations can be noise, blurs, and artefacts.
- The impulse response of a system is called the point spread function (PSF). The Fourier transform of the PSF is called the optical transfer function (OTF). The amplitude of an OTF is called the modulation transfer function, which is an indicator of the performance of an imaging system.
- Distortions and artefacts are extreme intensity and colour imperfections that make images meaningless.
- Distortions can be modelled as image degradation models. Degradations can be estimated by observation, experimentation, and modelling.
- Noise is a disturbance. Noises can be classified based on their distribution, correlation, nature, and source.
- Mean filters and order-statistic filters are used to restore an image in the presence of noise only.
- Mean filters compute the average of the pixels that fall within the selected window and replace the central pixel. The different types of mean filters are arithmetic mean filters, geometric mean filters, harmonic filters, contra-harmonic filters, and Yp-mean filters.
- An order-statistic filter is a non-convolution-based filter that orders the pixels within the selected window and selects a value that replaces the central pixel.
- Order-statistic filters include median, maximum, minimum, midpoint, and alpha-trimmed filters.
- Band-pass filtering is a combination of both low- and high-pass filters. A band-pass filter allows all frequencies except those present in the narrow band defined by the cut-off values. On the other hand, a band-reject filter attenuates the frequencies that falls within the band.
- An inverse filter removes blurs and noise using the deconvolution process.
- The two types of algebraic methods used for image restoration are unconstrained and constrained methods.
- A Wiener filter is an optimal filter. It finds an estimate $\hat{f}(x, y)$ of the original image $f(x, y)$ using the least mean square.
- The iterative approach recovers the original image from a degraded image in successive iterations.
- In situations where there is little or no prior knowledge of blurs, blind image restoration methods are useful. Here, restoration is carried out by estimating the blurring functions either directly or indirectly.
- Geometrical transforms include spatial and image interpolation algorithms.

KEY TERMS

Artefacts The extreme intensity or colour fluctuations that make images meaningless

Band-pass filter A filter that selectively allows components within a selected range (called a band)

Band-reject filter A filter that removes only selective frequencies that fall within the specified band (a range of frequency)

Blind image restoration A method that recovers an image without any knowledge of the blurring function

Correlation The statistical dependence among pixels

Inverse filter A filter that uses the deconvolution process to recover the original image from a degraded image

Iterative filter A filter that improves an image in successive iterations.

Mean filter A filter that selects the mean value of the pixels that fall within the selected window and replaces the central pixel with it

Median filter A filter that selects the middle value of the sorted list of the pixels that fall within the selected window and replaces the central pixel with it

Midpoint filter A filter that uses the midpoint, which is the average of the first and the last pixels, of the sorted list to replace the central pixel

Modulation transfer function The amplitude of OTF

Modulation The contrast of an image, which is given by $(I_{max} - I_{min})\backslash(I_{max} + I_{min})$, where I_{max} and I_{min} are the maximum and minimum intensities, respectively

Noise A disturbance that causes fluctuations in pixel values

Notch filter A filter that removes the select frequency

Optical transfer function The Fourier transform of the point spread function

Order-statistic filter A filter that selects a value within a selected window based on a predetermined order

Periodic noise A noise that is a sinusoid at multiples of a specific frequency and is periodic in nature

Photon noise A noise that occurs due to the statistical nature of EM waves

Pink noise A noise whose power spectrum is not constant and is proportional to the frequency ($1/f$); it is also called $1/f$ or flicker noise

Point spread function A factor that causes blurring of a point captured by an imaging system

Spatial transforms Transforms that are used to recover the original coordinates from a deformed image

Speckle noise A random noise that appears as bright specks in the lighter regions of an image

White noise An uncorrelated noise

Wiener filter A filter that recovers the original image from a degraded image using the limited prior knowledge of the image

REVIEW QUESTIONS

1. What are the factors that can cause image degradations?
2. Classify degradations and explain the classification briefly.
3. What is the role of the point spread function?
4. What is a blur? What are the types of blurs?
5. What is the significance of the MTF and an MTF plot?
6. List out some of the kernels or masks of the PSF.
7. What is an artefact?
8. Explain in detail an image degradation model.
9. How are degradation functions estimated? List out the techniques.
10. What is noise? How can noises be classified?
11. Elucidate the types of noise based on its probability distribution.
12. What are white noise and pink noise?
13. What are the sources of image noise? Can noise be controlled? Justify.
14. What is speckle noise? What kind of noise is this?
15. Describe in detail the types of mean filters.
16. Write about the types of mean filters and noises they remove from images.
17. What is an order-statistic filter? How is it different from convolution-based filters?
18. Explain briefly the types and implementation of order-statistic filters.
19. List out order-statistic filters and the types of noise that they remove.
20. What is periodic noise? How is it removed using band-pass and band-reject filters?

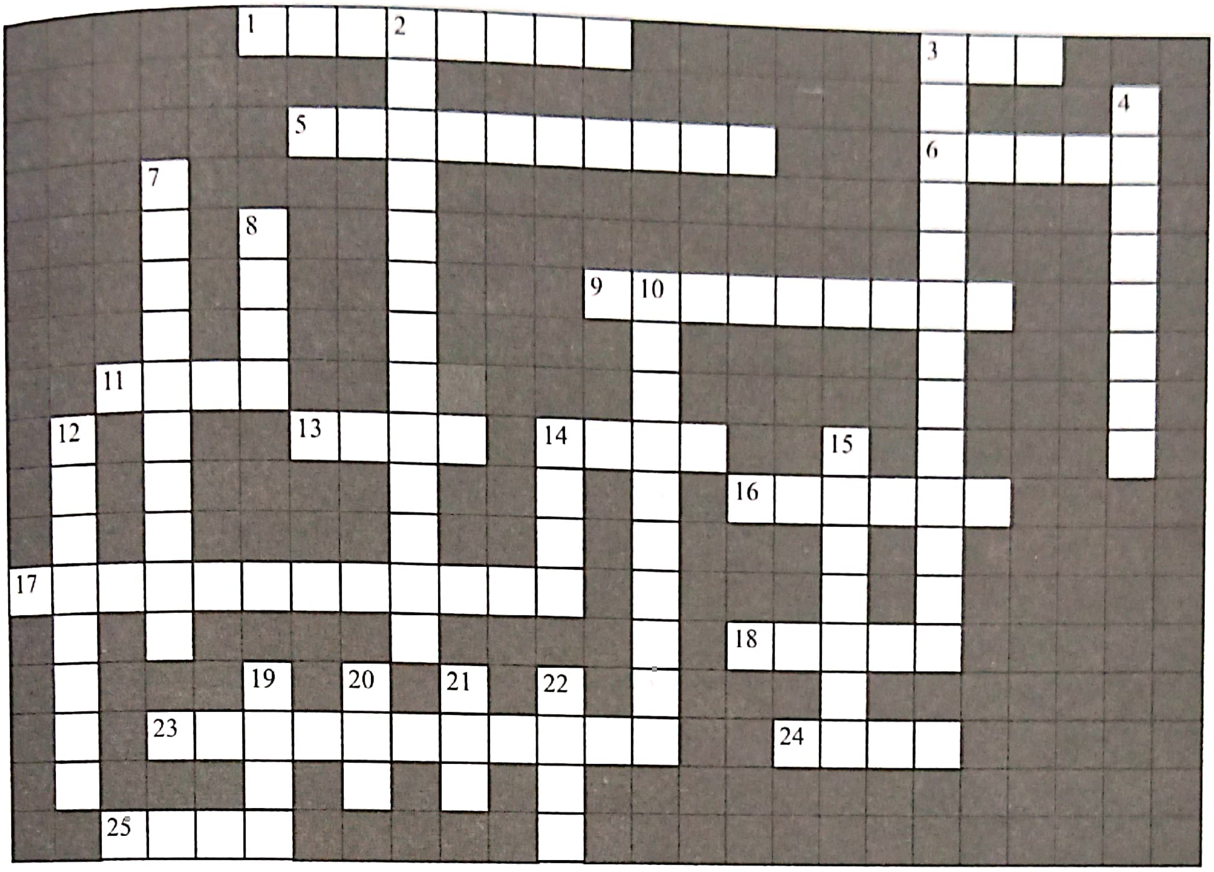
21. What is a notch filter? What is the significance of notch filters?
22. What is an inverse filter?
23. What is a Wiener filter? Write in detail the construction and implementation of a Wiener filter. List out the advantages and disadvantages of a Wiener filter.
24. Explain the concept of algebraic image restoration methods.
25. What is meant by the blind restoration method?
26. Explain the role of geometrical transforms in image restoration.

NUMERICAL PROBLEMS

1. Consider the following list of elements:
   $A = \{12, 23, 45, 45, 67, 89, 89, 90, 100\}$
   Compute the arithmetic mean, geometric mean, and harmonic mean of list A.
2. Consider the following list of elements:
   $A = \{22, 43, 65, 75, 87, 80, 81, 92, 100\}$
   Compute the median, maximum, minimum, and midpoint of list A.
3. Consider the following list of elements:
   $A = \{15, 15, 25, 25, 55\ 79, 80, 80, 110\}$
   Apply alpha-trimming filter wiht a clipping factor $T = 2$ find the replacing pixel value.
4. What is the result of applying an order-statistic filter on the following image?

$$\begin{pmatrix} 1 & 3 & 5 \\ 4 & 4 & 3 \\ 5 & 2 & 2 \end{pmatrix}$$

CROSSWORD



Across

1. A filter that allows frequency components within the band is called __________ filter.
3. A factor that causes blurring
5. Iterative methods restore images in __________ iterations.
6. An order filter that selects the difference of the maximum and minimum values of the sorted list of all pixel values of the window is called __________ filter.
9. The __________ of OTF is called MTF.
11. Inverse filter assumes that complete knowledge of the blurring function is available. (True/False)
13. Geometric transformations can restore images. (True/False)
14. Artefacts make huge fluctuations in pixel values. (True/False)
16. A noise due to statistical nature of EM waves is called __________ noise.
17. A noise that appears as bright spots in the lighter image is called __________.
18. A disturbance that cause fluctuations in pixel values
23. The statistical dependance of pixels is called __________.
24. Modulation is also called contrast. (True/False)
25. Max filter is an example of order filter. (True/False)

Down

2. Inverse filter uses __________ process.
3. A noise that appears in multiples of a specific frequency is called __________.
4. A band is a set of __________ frequencies.
7. A filter that removes the frequency components of the band is called __________ filter.
8. Wiener assumes that partial knowledge of the blurring function is available. (True/False)
10. Contrast is also known as __________.
12. An order filter that selects the average of the first and last element of the sorted list of all pixel values of the window is called __________ filter.
14. White noise is uncorrelated. (True/False)
15. OTF is the __________ transform of PSF.
19. Order filter selects values within the specified window. (True/False)
20. Pink noise spectrum is not constant. (Yes/No)
21. An order filter that selects the max of the sorted list of all pixel values of the window is called __________ filter.
22. A noise whose spectrum is proportional to $1/f$ is called __________ noise.

WORD SEARCH PUZZLE

Some of the important terms in this chapter are present in the following word jumble. Identify the words. Diagonal words are possible.

Z	J	H	C	M	T	Z	P	E	R	I	O	D	I	C	N	O	I	S	E	R	M	H	M	O
Z	Y	Y	O	T	I	N	C	W	U	N	T	M	X	B	S	J	P	N	G	U	O	Q	A	J
A	H	A	R	T	H	D	S	M	G	S	N	N	V	M	F	Y	L	Y	K	C	D	N	X	G
M	A	L	R	R	A	I	P	I	X	M	W	L	W	T	R	U	E	I	Z	V	U	P	S	F
P	Q	F	E	U	Y	W	A	O	S	P	E	C	K	L	E	N	O	I	S	E	L	R	K	T
L	Y	Q	L	E	T	N	I	O	I	Y	P	H	O	T	O	N	O	P	P	D	A	A	W	R
I	A	X	A	U	U	U	S	J	B	N	X	J	H	S	M	S	U	D	D	P	T	N	H	U
T	O	A	T	B	J	L	F	U	Q	A	T	Y	I	N	T	C	I	N	S	H	I	G	V	E
U	X	G	I	Z	A	Z	D	E	C	O	N	V	O	L	U	T	I	O	N	U	O	E	F	M
D	W	U	O	M	Z	N	W	M	D	C	Z	D	R	C	D	H	S	A	G	V	N	N	O	L
E	O	X	N	P	Z	J	D	T	R	U	E	L	P	S	C	N	F	E	A	X	S	S	U	Y
N	H	B	E	S	N	B	R	R	U	C	B	S	G	A	X	J	L	H	L	T	W	G	R	E
N	U	M	G	Z	M	U	Z	T	E	B	T	Y	S	S	S	E	P	P	K	E	L	G	I	S
E	T	T	Q	M	U	V	W	W	S	J	W	R	O	I	S	S	V	I	V	S	C	Q	E	X
F	O	R	F	F	K	L	V	U	I	X	E	J	U	R	V	W	S	N	P	R	S	T	R	T
X	J	U	T	R	U	E	P	A	Z	K	V	C	G	E	V	E	T	K	N	O	I	S	E	F
O	K	E	A	T	U	U	B	B	T	R	U	E	T	B	R	V	L	N	S	N	C	M	X	D

Hints

1. A disturbance that causes fluctuations in pixel values
2. A factor that causes blurring
3. OTF is the __________ transform of PSF.
4. The __________ of OTF is called MTF.
5. Contrast is also known as __________.
6. Modulation is also called contrast. (True/False)
7. Artefacts make hugh fluctuations in pixel values. (True/False)
8. The statistical dependance of pixels is called __________.
9. White noise is uncorrelated. (True/False)
10. Pink noise spectrum is not constant. (Yes/No)
11. A noise whose spectrum is proportional to $1/f$ is called __________ noise.
12. A noise that appears as bright spots in the lighter image is called __________.
13. A noise that appears in multiples of a specific frequency is called __________.
14. A noise due to statistical nature of EM waves is called __________ noise.
15. Order filter selects values within the specified window. (True/False)
16. Max filter is an example of order filter. (True/False)
17. An order filter that selects the average of the first and last element of the sorted list of all pixel values of the window is called __________ filter.
18. An order filter that selects the difference of the maximum and minimum values of the sorted list of all pixel values of the window is called __________ filter.
19. An order filter that selects the max of the sorted list of all pixel values of the window is called __________ filter.
20. A band is a set of __________ frequencies.
21. A filter that allows frequency components within the band is called __________ filter.
22. A filter that removes frequency components of the band is called __________ filter.
23. Inverse filter uses __________ process.
24. Inverse filter assumes that complete knowledge of the blurring function is available. (True/False)
25. Wiener filter assumes that partial knowledge of the blurring function is available. (True/False)
26. Iterative methods restore images in __________ iterations.
27. Geometric transformations can restore images. (True/False)