CHAPTER 11

Image Morphology

No one shall expel us from the paradise that Cantor has created for us.

—**David Hilbert**

LEARNING OBJECTIVES

Image morphology is an important tool in image processing. It is the study of shapes of the objects present in the image and extraction of image features. Image features are necessary for object recognition. This chapter provides an overview of morphological operators and algorithms to extract useful features for object recognition. After studying this chapter, the reader will become familiar with the following:

- *Binary image morphology operators*
- *Binary image morphology applications*
- *Grey scale image morphology operators*
- *Applications of image morphology operations*

1.1 NEED FOR MORPHOLOGICAL PROCESSING

Mathematical morphology is a very powerful tool for analysing the shapes of the objects present in images. The theory of mathematical morphology is based on set theory. We can visualize the binary image as a set. Then set theory can be applied to the sample set extracted from images.

Morphological operators take a binary image and a mask known as a structuring element as inputs. Then the set operators such as intersection, union, inclusion, and complement can be applied to the images.

A set is a group of objects or elements that share some common attributes. In a binary image, the set is a collection of coordinates where the pixel may be black or white.

Let us assume that A is a set of numbers.

$$A = \{2, 4, 6, 8, 10\}$$

If an element w, say 4, is present in a set A, it can be denoted as $w \in A$. Let us consider that the set $X = \{2, 4\}$ is a subset of A. This is represented mathematically as $X \subset A$.

The complement of the set represented by X^C, with respect to A, is a set where every member does not belong to the set X.

$$X^C = \{6, 8, 10\}$$

Two fundamental operations involving sets are set union and set intersection. Let B be a set as follows:

$$B = \{1, 3, 5\}$$

$$\text{then } A \cup B = \{1, 2, 3, 4, 5, 6, 8, 10\}$$

The union contains all the elements of the sets A and B. The intersection of A and B is null, that is, $A \cap B = \varphi$ as there are no common elements between the sets A and B. Morphological operations follow set theory laws.

Properties of sets

Some properties of sets are as follows:

1. Associative law

   The following is for the union operator:

$$\begin{aligned} A \cup B \cup C &= (A \cup B) \cup C \\ &= A \cup (B \cup C) \end{aligned}$$

   The following is for the intersection operator:

$$\begin{aligned} A \cap B \cap C &= (A \cap B) \cap C \\ &= A \cap (B \cap C) \end{aligned}$$

2. Commutative law

$$A \cup B = B \cup A$$

3. Identity law

$$A = B \text{ if } A \subset B \text{ and } B \subset A$$

4. De Morgan's laws

$$(A \cup B)^C = A^C \cap B^C$$
$$(A \cap B)^C = A^C \cup B^C$$

5. Set difference

$$A - B = A \cap B^C$$
$$= A - (A \cap B)$$

For example, if $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 3\}$, then $A - B = \{2, 4, 5\}$.

The concept of sets can be easily extended to binary images. In a binary image, the background is represented by black pixels and the foreground by white pixels. So the binary image can be represented as a set of coordinate points satisfying a property. Then, all the set theory operations can be applied to binary images, as follows:

Complement The complement of a binary image $f(x, y)$ is given as

$$g(x,y) = \begin{cases} 1 \text{ if } f(x,y)=0 \\ 0 \text{ if } f(x,y)=1 \end{cases}$$

Translation Let us assume that A is a set of pixels. Then let $w = (x, y)$ be a coordinate point. Then the translated point can be denoted as

$$A_w = \{(a, b) + (x, y): (a, b) \in A\}$$

Union The union of two binary images f and g is given as $h = f \cup g$. This is described as

$$h(x,y) = \begin{cases} 1 \text{ if } f(x,y)=1 \text{ OR if } g(x,y)=1 \\ 0 \text{ otherwise} \end{cases}$$

Intersection This operator gives the common non-zero pixels of both the images A and B and is given as $h = f \cap g$. This is described as

$$g(x,y) = \begin{cases} 1 \text{ if } f(x,y)=1 \text{ AND if } g(x,y)=1 \\ 0 \text{ otherwise} \end{cases}$$

Reflection The reflection $\hat{A}$ of a set A is denoted as

$$\hat{A} = \{(-x, -y): (x, y) \in A\}$$

11.1.1 Structuring Elements

Structuring elements are small images that are used to probe the original image. The structuring element can be a small binary image or a flat file that consists of coordinate values. Many standard structuring elements are available. Some of these are shown in Fig. 11.1.

Some standard structuring elements are as follows:

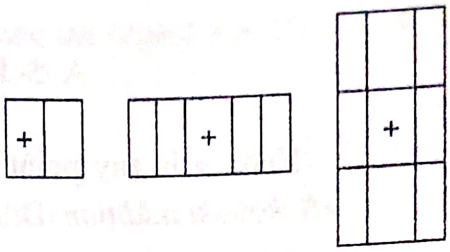

Fig. 11.1 Some general structuring elements (origin marked with +)

Line segment This is characterized by the length and orientation.

Disk This is a popular structuring element and can be described for digital grids.

Set of points A set of coordinate points can serve as a structuring element.

Composite structuring elements Composite structuring elements are two structuring elements that share a common origin.

Elementary structured mask These are square grids. The smallest square grid is of size 2×2.

Normally a structuring element is a matrix of size 3×3. It has its origin in the centre of the matrix. It is then shifted over the image and at each pixel of the image its elements are compared with the set of underlying pixels.

For the basic morphological operators, the structuring element has ones, zeros, and don't cares. These elements select or suppress the features of a certain shape, for example, removing noise from an image or selecting objects with a particular direction. Thus a morphological operator is defined by its structuring element and the applied set operator.

11.2 MORPHOLOGICAL OPERATORS

The basic morphological operators are *dilation* and *erosion*.

11.2.1 Dilation Operation

Dilation can be applied to binary as well as grey-scale images. The basic effect of this operator on a binary image is that it gradually increases the boundaries of the region, while the small holes present in the images become smaller. In short, dilation can be considered as a union operation of all the translations of the image A caused by the elements specified in the structuring element B.

$$A \oplus B = \bigcup_{b \in B}^{n} (A)_b$$

where b is any point that belongs to image B. As the dilation operator is commutative, that is, $A \oplus B = B \oplus A$, the aforementioned relation can also be written as values. Many standard structuring elements are available. Some of these are shown in Fig. 11.1.

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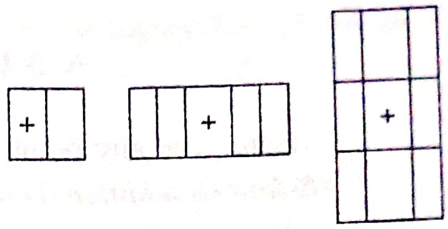

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where b is any point that belongs to image B. As the dilation operator is commutative, that is, $A \oplus B = B \oplus A$, the aforementioned relation can also be written as

$$A \oplus B = \bigcup_{a \in A}^{n} (B)_a$$

Here, a is any point corresponding to the image A. This operation is also known as *Minkowski addition*. Dilation increases the size of the object and also thickens the object.

An equivalent representation can be given as follows. Let us assume that A and B are two sets of pixels. Then the dilation of A by B can be denoted as

$$A \oplus B = \{(x, y) + (u, v): (x, y) \in A, (u, v) \in B\}$$

Let us assume that A consists of four points $\{(3,4), (3,5), (4,4), (4,5)\}$ and the structuring element is (0, 1), that is, the coordinates of the structuring element is $\{(0,0),(0,1)\}$ with the origin as underlined. Then the pixel $C = A + B$ is calculated as

$$A + B = \{A + (0, 0) \cup A + (0, 1)\}$$

The resultant of $A + (0, 0)$ operation would be

$$(3, 4) + (0, 0) = (3, 4)$$
$$(3, 5) + (0, 0) = (3, 5)$$
$$(4, 4) + (0, 0) = (4, 4)$$
$$(4, 5) + (0, 0) = (4, 5)$$

The resultant of $A + (0, 1)$ operation would be

$$(3, 4) + (0, 1) = (3, 5)$$
$$(3, 5) + (0, 1) = (3, 6)$$
$$(4, 4) + (0, 1) = (4, 5)$$
$$(4, 5) + (0, 1) = (4, 6)$$

The result would be the union of all these pixels, that is, $\{(3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\}$.

11.2.2 Erosion Operation

Erosion is another important morphological operator. This is also known as *Minkowski subtraction*. The objective of this operator is to make an object smaller by removing its outer layer of pixels. If a black pixel has a white neighbour, then all the pixels are made white. This can be described mathematically as

$$A \ominus B = \cap A_B$$

The erosion operator takes the image and the structuring element as inputs and thins the object.

Consider the structuring element $B = \{(0, 0), (1, 0)\}$ and the object $A = \{(3, 4), (4, 4), (4, 4)\}$, $A \ominus B = \{A - (0, 0) \cap A - (1, 0)\}$

Then the translations are

$$B(3, 4) = \{(3, 4), (2, 4)\}$$
$$B(4, 4) = \{(4, 4), (3, 4)\}$$
$$B(4, 6) = \{(4, 4), (3, 4)\}$$

The result is the intersection of all these coordinate points. It can be observed that only $B(3, 4)$ elements completely match with the object.

Therefore $A \ominus B = \{(3, 4)\}$

11.2.3 Approaches to Dilation and Erosion Operations

There are other approaches to the erosion and dilation operators apart from set theory. One approach is to obtain the erosion and dilation operators by modifying the median filter. Dilation is achieved by choosing a rank higher than the median. This operation expands the object and shrinks the background. Erosion is achieved by choosing a lower rank instead of the median to expand the dark regions and to shrink the object pixels. These operations are shown in Fig. 11.2, where the first element is of the lowest rank and the last element is of the highest rank.

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \Rightarrow [0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1]$$

Fig. 11.2 Erosion and dilation operators using ranks

11.2.3.1 Hit and miss approach

Another approach to implement these morphological operations is to consider them as binary correlation operation. In fact, binary correlation is not done as an arithmetic operation but instead as a logical operation. The structuring element is placed on a binary image. The value '0' in the binary image implies *don't care* condition as its values are irrelevant. Now the values of the structuring element are associated with the elements of the neighbourhood.

The three possible conditions that can result are as follows:

Fit A structuring element is said to fit when for each of its elements with value 1, the corresponding element in the image also has the value 1. Set elements whose values are 0 define points where the corresponding elements of the image are irrelevant. For example, if the structuring element is $\{1\ 1\ 1;\ 1\ 1\ 1;\ 1\ 1\ 1\}$ and the image is also $\{1\ 1\ 1;\ 1\ 1\ 1;\ 1\ 1\ 1\}$, then this represents the fit condition.

Hit This condition is satisfied when there is an exact match such that for at least one of the pixels set to 1, the corresponding pixel is also 1.

Miss This is a condition where none of the elements of the structuring element match with the elements of the image.

For example for $f = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, the structuring element $s_1 = \begin{pmatrix} & 1 & \\ 1 & 1 & 1 \\ & 1 & \end{pmatrix}$ is a hit,

whereas, the structuring element $s_2 = \begin{pmatrix} & 0 & \\ 1 & 1 & 0 \\ & 0 & \end{pmatrix}$ is a miss.

The dilation of an image f using a structuring element (s) is written as

$$g(x, y) = \begin{cases} 1 & \text{if } s \text{ hits } f \\ 0 & \text{otherwise} \end{cases}$$

This operation produces a new binary image showing the locations where the structuring element hits the input image.

The erosion of the image f by the structural element s is defined as a situation where the following rule is applicable:

$$g(x, y) = \begin{cases} 1 & \text{if } s \text{ fits } f \\ 0 & \text{otherwise} \end{cases}$$

11.2.3.2 Convolution approach

Another useful approach to implement these morphological operations is to consider them similar to the convolution operation. The structuring element for dilation is shown in Fig. 11.3 where the origin is highlighted. In this approach, dilation is viewed as the logical OR operator.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} X & X & X \\ 0 & 0 & 0 \\ X & X & X \end{bmatrix}, \begin{bmatrix} X & 0 & X \\ X & 0 & X \\ X & 0 & X \end{bmatrix}, \begin{bmatrix} X & X & 0 \\ X & 0 & X \\ 0 & X & X \end{bmatrix}$$

Fig. 11.3 Masks for the dilation operator

1. If the pixel value of the input image is 1, then the centre of the mask position in the output image is 1.
2. If the pixel value of the input image is 0, then check the neighbours. If at least one of the neighbours has the value 1, then the position of the centre of the mask in the output image is set to 1.
3. If the pixel is set to 0, and none of the neighbours has the value 1, then the centre of the

mask remains at value 0.

This is illustrated in Table 11.1.

Table 11.1 Dilation as a convolution operation

Input pixel value	Neighbour's value	Output value
1	All neighbours have value 1	1 (Hit)
1	Some are 1 and some are 0	1 (Hit)
0	Some are 1 and some are 0	1 (Hit)
0	All neighbours have value 0	0 (Miss)

The masks for the erosion operation are shown in Fig. 11.4. X is the *don't care* condition, that is, its value can be either 1 or 0.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} X & X & X \\ 1 & 1 & 1 \\ X & X & X \end{bmatrix}, \begin{bmatrix} X & 1 & X \\ X & 1 & X \\ X & 1 & X \end{bmatrix}, \begin{bmatrix} X & X & 1 \\ X & 1 & X \\ 1 & X & X \end{bmatrix}$$

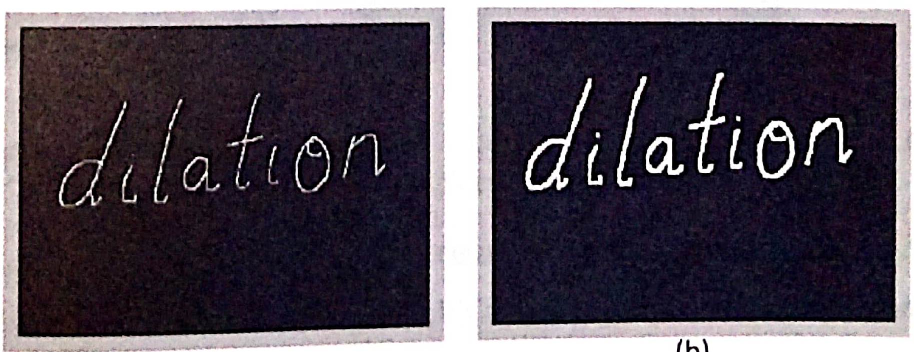
Fig. 11.4 Masks for the erosion operator

Similar to the dilation process, the masks for erosion are applied. The results of the convolution process are as shown in Table 11.2.

Table 11.2 Erosion operator

Input pixel value	Neighbour's value	Output value
1	All neighbours have value 1	1 (Hit)
1	Some are 1 and some are 0	0 (Miss)
0	Some are 1 and some are 0	0 (Miss)
0	All neighbours have value 0	0 (Miss)

An original image and the results of applying the dilation operator are shown in Figs 11.5(a)–11.5(d).



(a) (b)

Fig. 11.5 Results of the dilation operator (a) Original image (b) Result with 3 × 3 mask

(*Contd*)

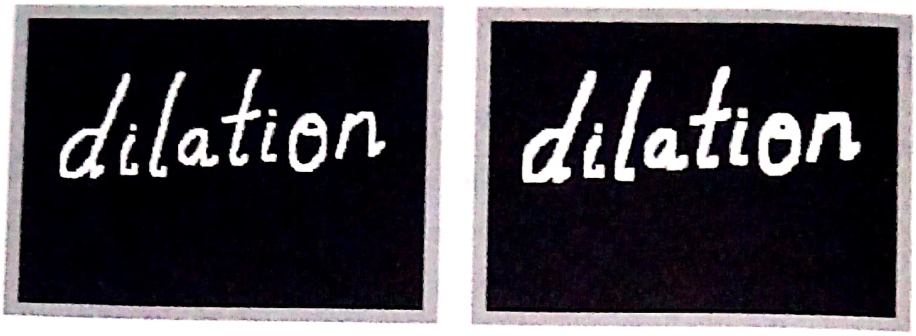
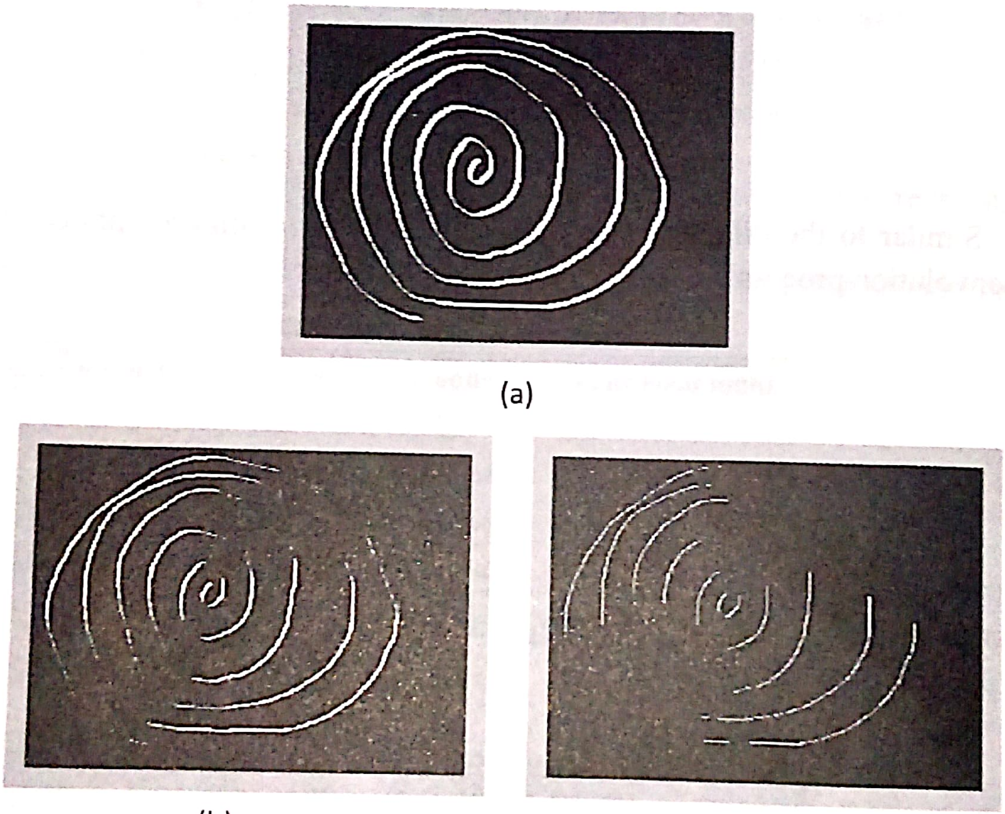


Fig. 11.5 (Contd) (c) Result with 9 × 9 mask (d) Result with 13 × 13 mask

An original image and the results of applying the erosion operator are shown in Figs 11.6(a)–11.6(c).



(a)

(b)

(c)

Fig. 11.6 Results of the erosion operator (a) Original image (b) Result with 3 × 3 mask (c) Result with 5 × 5 mask

11.2.3.3 *Properties of dilation and erosion*

Some of the essential properties of dilation and erosion are as follows:

Commutative property This is stated as

$$A \oplus B = B \oplus A$$

Associative property This is stated as

$$A \oplus (B \oplus C) = (A \oplus B) \oplus C$$
$$A \ominus (B \ominus C) = (A \ominus B) \ominus C$$

Distributive property This is stated as

$$A \oplus (B \oplus C) = (A \oplus B) \cup (A \oplus C)$$
$$A \ominus (B \ominus C) = (A \ominus B) \cup (A \ominus C)$$

Duality property Erosion and dilation are complementary.

$$(A \ominus B)^C = A^C \oplus \hat{B}$$

$$(A \ominus B)^C = A^C \ominus \hat{B}$$

Here, $\hat{B}$ is the reflection of the structuring element. If the structuring element is symmetric with respect to the origin, then $\hat{B} = B$.

Translation property This property implies that erosions and dilations are invariant to translation and preserve the order relationship between the images.

$$(A + h) \oplus B = (A \oplus B) + h$$

$$(A + h) \ominus B = (A \ominus B) + h$$

Decomposition property Suppose $B = B_1 + B_2$, then

$$(A \oplus B) = A \oplus (B_1 \oplus B_2)$$
$$= (A \oplus B_1) \oplus B_2$$

When B is decomposed into B_1 and B_2, the following additional properties hold true:

1. $(A \oplus B_1) \oplus B_2 = A \oplus (B_1 \oplus B_2)$
2. $(A \ominus B_1) \ominus B_2 = A \ominus (B_1 \ominus B_2)$

11.2.4 Opening and Closing Operations

The *opening operation* is an erosion operation followed by a dilation operation. This operation can be defined as

$$A \circ B = (A \ominus B) \oplus B$$

This is equivalent to

$$A \circ B = \cup \{B_w; B_w \subseteq A\}$$

The opening operation satisfies the following properties:

1. $(A \circ B) \subseteq A$
2. $(A \circ B) \circ B = A \circ B$

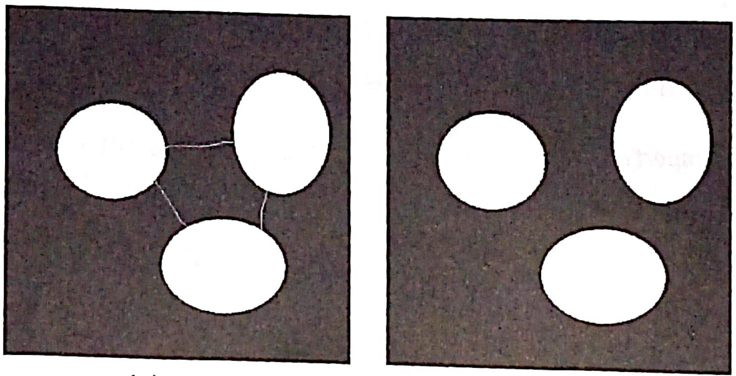
   This is called the idempotence property.
3. If $A \subseteq C$ then,

   $A \circ B \subseteq C \circ B$

Opening is useful for smoothing the edges, breaking the narrow joints, and thinning the protrusions that are present in an image. An original image and the effect of the opening operator are shown in Figs 11.7(a) and 11.7(b).

Closing is a dilation operation followed by an erosion operation. This can be denoted as

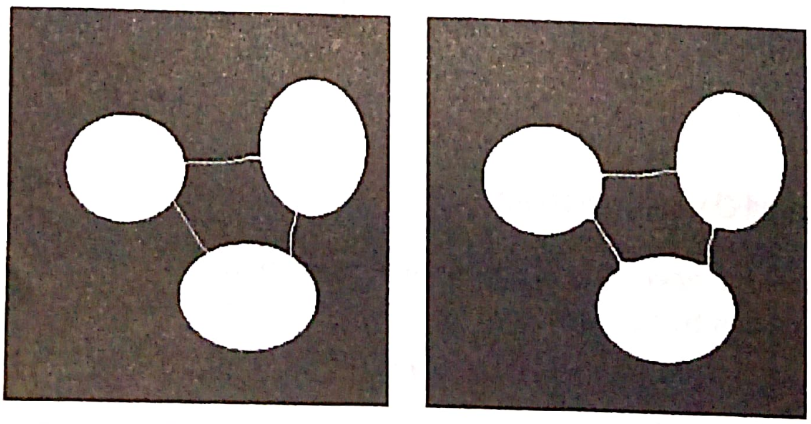
$$A \bullet B = (A \oplus B) \ominus B$$



(a) (b)

Fig. 11.7 Results of the binary opening operation (a) Original image (b) Result of binary opening—shows that the objects are separated

An original image and the effect of the opening operator are shown in Figs 11.8(a) and 11.8(b).



(a) (b)

Fig. 11.8 Results of the binary closing operation (a) Original image (b) Result of binary closing—shows the tips of the objects with the bridge more pronounced

11.2.4.1 Properties of opening and closing operations

The properties of the opening and closing operations are as follows:

1. Dual transformation:

$$(A \bullet B)^C = A^C \circ \hat{B}$$

$$(A \circ B)^C = (A^C \bullet \hat{B})$$

2. Ordering relationship:

$$A \circ B \le A \le A \bullet B$$

3. Increasing transformations:

$$A_1 \subseteq A_2 \Rightarrow A_1 \circ B \subseteq A_2 \circ B$$

$$A_1 \subseteq A_2 \Rightarrow A_1 \bullet B \subseteq A_2 \bullet B$$

4. Transform invariance:

$$(A+h) \circ B = (A \circ B) + h$$

$$(A+h) \bullet B = (A \bullet B) + h$$

5. Idempotence:

$$(A \circ B) \circ B = A \circ B$$

$$(A \bullet B) \bullet B = A \bullet B$$

11.5 HIT-OR-MISS TRANSFORM

The hit-or-miss transform is a general binary morphological operation that can be used to look for particular patterns of foreground and background pixels in an image. It is actually the basic operation of binary morphology since almost all the other binary morphological operators can be derived from it. The sample structuring masks used are shown in Fig. 11.9.

	1				1				0	0		0	0	
0	1	1		1	1	0		1	1	0		0	1	1
0	0				0	0			1				1	

Fig. 11.9 Structuring elements used for detecting corners

The algorithm for the hit-or-miss transform is similar to those of the other morphological operators as follows:

1. Translate the centre of the structuring element to all the points of the input image.
2. Compare the structuring element with the image pixels.
3. If there is a complete match, then

   The pixel underneath the structuring element (i.e., the image pixel that coincides with the centre of the structuring element) is set to the foreground colour. This operation is called a hit.

   Else

   The pixel underneath the structuring element (i.e., the image pixel that coincides with the centre of the structuring element) is set to the background colour. This operation is called a miss.

The hit-or-miss transform has many applications in more complex morphological operations such as thinning and object recognition. These structuring elements can be used as convolution templates for recognizing the objects. One such application is corner detection. The four different structuring elements shown in Fig. 11.9 can be applied to detect the corners of different orientations. Each of the structure elements should be applied, and the results are then combined (using logical OR) to get the resultant image that identifies the corner.

11.6 BASIC MORPHOLOGICAL ALGORITHMS

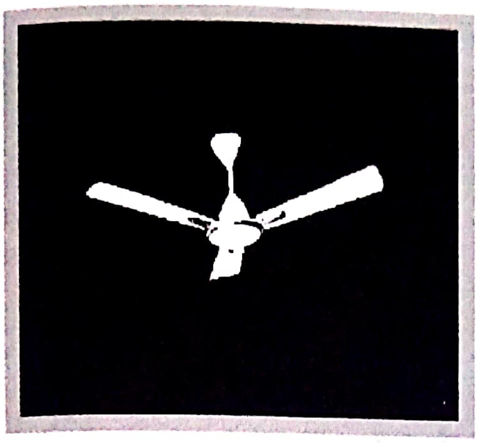
Morphological operators are immensely useful in a variety of imaging applications. They can be used to extract the boundary, remove noise, identify components, convex hull, and so on. Some useful applications are explained in Sections 11.4.1–11.4.10.

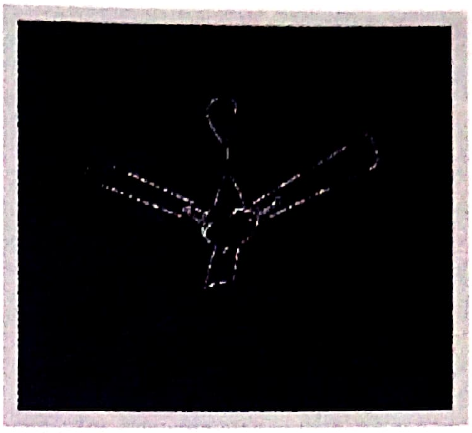
11.6.1 Boundary Extraction

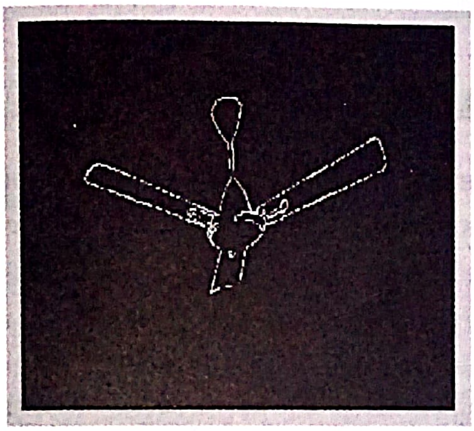
One of the major applications of morphological operators is finding the boundary. The difference between the original image and the eroded image creates a boundary. Let us assume that A is the input image and B is the structuring element. Two types of boundaries can be obtained as follows:

1. Internal boundary: $A - (A \ominus B)$
2. External boundary: $(A \oplus B) - A$

An original image and its internal boundary-extracted resultants are shown in Figs 11.10(a)–11.10(d).


(a)


(b)


(c)

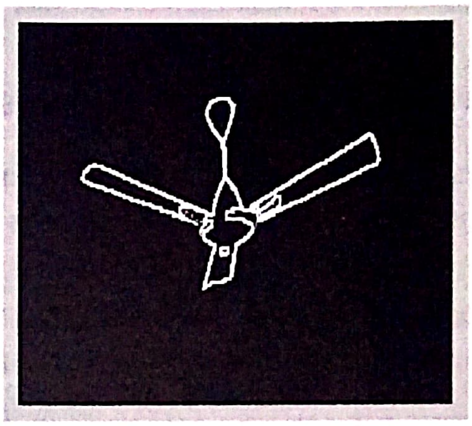

(d)

Fig. 11.10 Results of internal boundary extraction (a) Original image (b) Result with 3×3 mask (c) Result with 5×5 mask (d) Result with 13×13 mask

The external boundary for the original image in Fig. 11.10(a) is shown in Figs 11.11(a)–11.11(c).

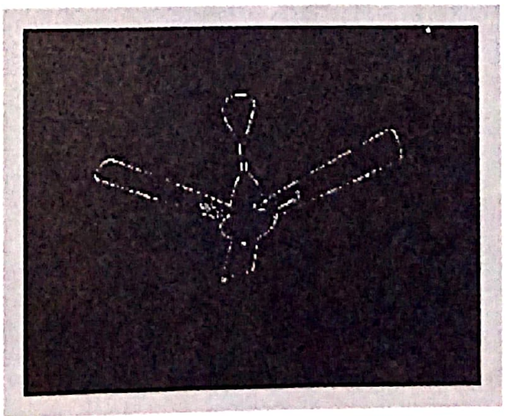
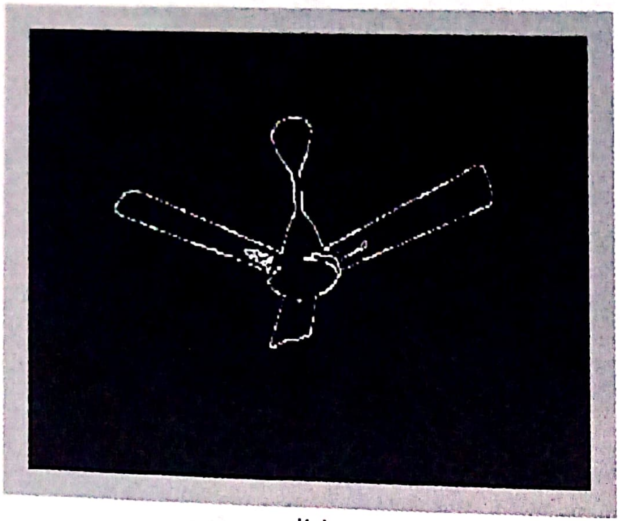
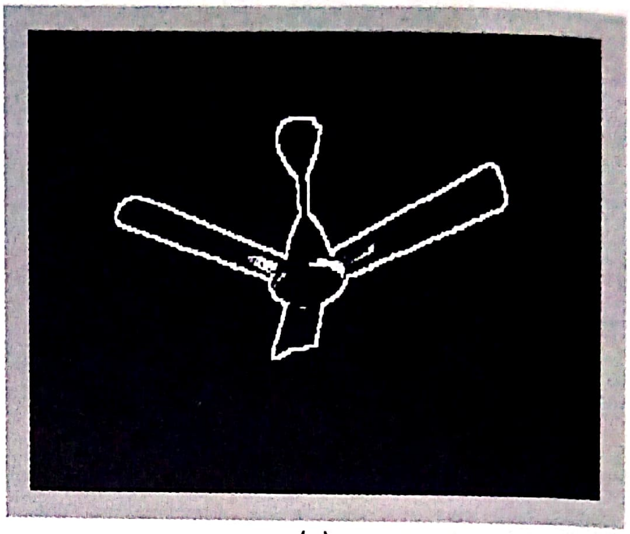

(a)

Fig. 11.11 Results of external boundary extraction (a) Result with 3×3 mask *(Contd)*



(b)



(c)

Fig. 11.11 (Contd) (b) Result with 5 × 5 mask (c) Result with 13 × 13 mask

Removal of unwanted, small scale features of the binary objects can be achieved using this operation. However, there is always a possibility that the size of the shape gets reduced.

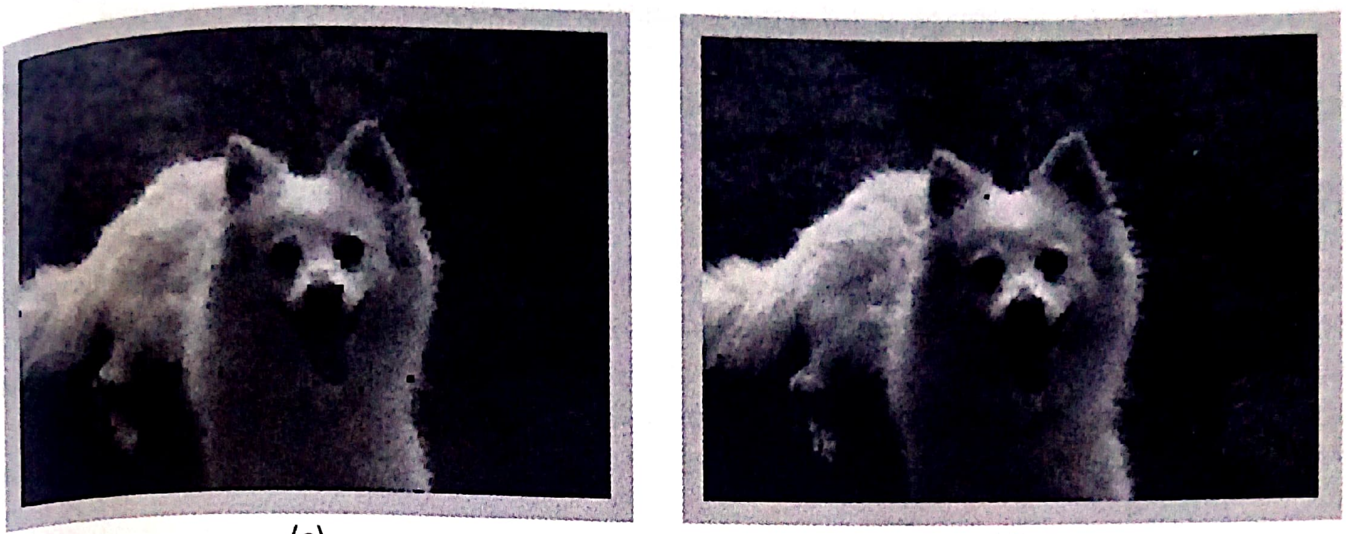
11.6.2 Noise Removal

If the image is corrupted by impulse noise, morphological operations are helpful in removing such noise. Impulse noise is discussed in Chapter 5. The morphological opening followed by a closing operation can remove the noise. An original image and the effect of the morphology filter are shown in Figs 11.12(a)–11.12(d).

(a)

(b)

Fig. 11.12 Results of morphology filter operation (a) Original image (b) Image with salt and pepper noise (*Contd*)



(c) (d)

Fig. 11.12 (Contd) (c) Result with 5 × 5 mask (d) Result with 3 × 3 mask

11.6.3 Thinning

This operation is applied only to binary images. Thinning is a morphological operator that is used to remove irrelevant foreground pixels present in binary images. The aim is to tidy up all the lines to a single pixel thickness. The performance of the thinning algorithm depends on the nature of the structuring element.

The thinning operation can be expressed in terms of the hit-or-miss transform as

$$\text{Thinning}(A, B) = A - \text{Hit-or-miss transform}(A, B)$$

Here A is the image and B is the structuring element. The operation subtraction here is logical subtraction (i.e., $A - B = A \cap NOT\ (B)$), that is, a set difference operation.

Theoretically, this is a single-pass algorithm. In reality, this operator is applied repeatedly till a condition of convergence is achieved. This is different from pruning, where only a limited number of iterations are used.

11.6.4 Thickening

Thickening is a dual morphological operation of the thinning operation. This operation is normally applied to binary images only. This operation is also related to the hit-or-miss transform and is used to grow some selected foreground pixels in binary images.

The relationship between the thickening operation and the hit-or-miss transform can be expressed as

$$\text{Thickening}(A, B) = A \cup \text{Hit-or-miss transform}(A, B)$$

Here A is the image and B is the structuring element.

This operation is repeatedly applied till convergence is achieved. Alternatively, iteration can be defined for some predefined value.

11.6.5 Convex Hull

A region is called convex if a line drawn between any two parts of the region also lies within the region. *Convex hull* is the smallest polygon that encompasses the region completely like an elastic band or an envelope. Identifying the convex hull is viewed as an extension of the morphological thinning operation and is implemented as a thickening operation.

Let B_i be the structuring element where $i = 1, 2, 3$, and 4. The convex hull involves a process

$$X_k^i = (X_{k-1}^i \otimes B^i) \cup A$$

for $k = 1, 2, 3, \ldots, n$.

This process is repeated till there is a convergence. Let D^i be X^i.

In other words, use the structuring element repeatedly and apply the hit-or-miss transform till there is no change. Then repeat this with the next structuring element which is a rotation of the previous mask by 90°. Perform this for the entire structuring element, producing the intermediate D. Perform union of all four parts of D to form the convex hull, that is, the connected component $C(A) = \bigcup_{i=1}^{4} D^i$.

11.6.6 Skeletonization

A skeleton (or medial axis) is a geometrical description that describes a shape using lesser number of pixels than the original. Skeletonization is useful in applications where image recognition and compression are used. However, the skeleton is sensitive to noise and hence changes in the image affect the result. Since it is possible to reconstruct the original from the skeleton, it is ideally suited for the lossless compression process.

Skeletonization and medial axis transform (MAT) are often mentioned in together literature. However, the difference between them is in the nature of the output image and the processes involved. Medial axis transform always produces a grey scale image where the pixel value represents the distance between the object point and the nearest boundary point but skeletonization produces a binary image. Hence medial axis transform is closer to distance transform and skeletonization is based more on repeated thinning operations. Medial axis transform is discussed along with distance transform in Section 11.4.7.

Skeletonization using morphological operations can be illustrated using a popular example known as *prairie-fire*. Suppose a slow-burning material is stacked up in all directions in the background of a non-flammable material and is lit up, then the fire starts burning the material from all the directions till they meet together at a point. This point is called the *quench line* and it represents the skeleton.

A skeleton can be obtained using a technique called *maximal disk*. A disk is maximal if it fills the object inside which is placed fully and there is no other disk that can be placed inside the object that can cover it at that point. The centre of such a maximal disk forms a skeleton point. The idea is to place all possible maximal disks inside the object and finding their centres. The skeleton of the object is a set of all such centre points of possible maximal disks. If B is the structuring element that represents the maximal disk, then $0B$ represents the origin of the maximal disk. Then B, $2B$, $3B$,..., nB are all possible maximal disks. Let skel(S) be a set of all centres of the maximal disks. So the skeleton (S) of the object can be denoted as the union of all skeleton subsets, as follows:

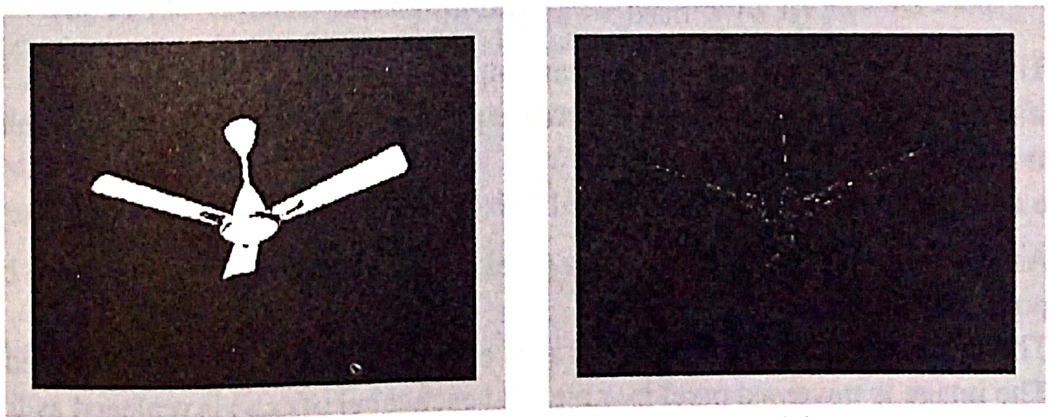
$$\text{Skeleton}(S) = \bigcup_{n=0}^{\infty} \text{skel}(S)$$

The skeleton as discussed earlier can be viewed as a repeated thinning operation. This can be described as $\text{skel}(S) = (A \ominus nB) - [(A \ominus nB) \circ B]$. Here B is the structuring element representing the maximal disk and $A \ominus nB$ indicates the successive erosions of image A by B. Now the final skeleton is

$$Skeleton(S) = \bigcup_{n=0}^{\infty} (S \ominus nB) - [(S \ominus nB) \circ B].$$

An original image and some results are shown in Figs 11.13(a)–11.13(d). If all the skeleton subsets are available, then the original set S can be constructed as

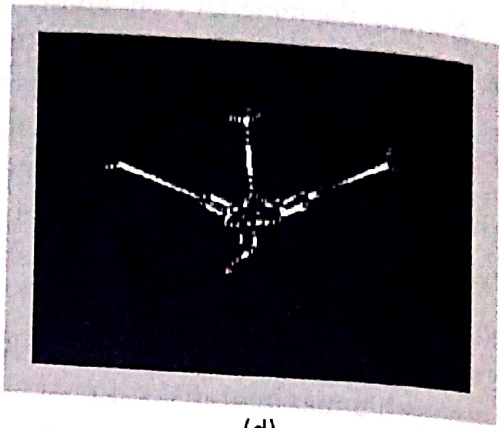
$$S = \bigcup_{n=0}^{\infty} \text{skel}(S) \oplus nB$$



(a) (b)

Fig. 11.13 Skeletonization (a) Original image (b) Result with 3 × 3 mask *(Contd)*

(c)



(d)

Fig. 11.13 (Contd) (c) Result with 5 × 5 mask (d) Result with 13 × 13 mask

11.6.7 Distance Transform

Another way of skeletonizing an image is by using the distance transform.

The distance transform takes a binary image as input and generates a greyscale image as output, where each pixel represents the distance between that pixel and the nearest background pixel. One can recollect from Chapter 3 that the distance measures between two points p and q with coordinate position (x, y) and (s, t) are given as follows:

The Euclid distance between the pixels p and q can be defined as

$$D_e = \sqrt{(x-s)^2 + (y-t)^2}$$

The D_4 distance or city block distance can be simply calculated as

$$|x-s| + |y-t|$$

The D_8 distance or chessboard distance can be calculated as

$$D_8(p, q) = \max(|x-s|, |y-t|)$$

The ridge of the distance transform output is the skeleton of the object. The idea is similar to the erosion operation, but here the pixel is replaced by the iteration number rather than being removed from the image.

The distance transform is implemented through these steps:

1. Assign a label $d(x, y)$ to each of the pixel (x, y) that represents the distance between the point and region R.
2. The labels of region R are assigned zero and all the other labels are assigned ∞.
3. The image is traversed pixel by pixel and each label of (x,y) is replaced as follows:

   $\min\{d(x,y), d(x+1,y)+1, d(x-1,y)+1, d(x,y+1)+1\}$ with $\infty + 1 = \infty$.
4. Step 3 is repeated till all the labels are assigned integer values.

   Thus, the distance can be computed from every pixel to the pixel of the region R.

A simple distance transform can be illustrated using a 1D image. Let us consider the 1D image shown in Fig. 11.14(a).

In the first pass, the scanning is done from left to right and the foreground pixel is replaced with the distance between the foreground pixel and the background. This changes the values as shown in Fig. 11.14(b). In the second pass, the scanning is done from right to left and the current distance between the foreground and background pixels is calculated and updated. This gives the values shown in Fig. 11.14(c).

0	0	0	1	1	1	0	0	0	0

(a)

0	0	0	1	2	3	0	0	0	0

(b)

0	0	0	1	2	1	0	0	0	0

(c)

Fig. 11.14 Distance transform (a) Original 1D image (b) Scanning from left to right—stores the minimum distance (c) Scanning from right to left and final result

Similar to 1D, a 2D distance transform can also be implemented. A 2D transform is shown in Figs 11.15(a)–11.15(c). This is based on a D_8 measure. The local maxima or ridge of the final result represents the skeleton. This is highlighted in Fig. 11.15 (c).

1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1

(a)

1	1	1	1	1	1
1	2	2	2	2	1
1	2	2	2	2	1
1	2	2	2	2	1
1	1	1	1	1	1

(b)

1	1	1	1	1	**1**
1	**2**	2	2	**2**	1
1	2	**3**	**3**	2	1
1	**2**	2	2	**2**	1
1	1	1	1	1	**1**

(c)

Fig. 11.15 Distance transform (a) Original 2D image (b) First iteration (c) Distance transform result—ridge highlighted

Some of this distance measures computation can be implemented faster by applying masks. The results of this mask are more or less similar to actual distance measure computation and are shown in the Figs 11.16(a)–11.16(b).

	1	
1	0	1
	1	

(a)

1	1	1
1	0	1
1	1	1

(b)

Fig. 11.16 Distance measures (a) D_4 distance mask (b) D_8 Distance mask

11.6.7.1 Faster implementation using Chamfer approach

A faster version of the distance transform with either 3 × 3 or 5 × 5 Chamfer masks can be formulated as described as follows.

$$\begin{pmatrix} 4 & 3 & 4 \\ 3 & 0 & \\ & & \end{pmatrix}\begin{pmatrix} & & \\ & 0 & 3 \\ 4 & 3 & 4 \end{pmatrix}$$

(a)

$$\begin{pmatrix} & 11 & & 11 & \\ 11 & 7 & 5 & 7 & 11 \\ & 5 & 0 & & \\ & & & & \\ & & & & \end{pmatrix}\begin{pmatrix} & & & & \\ & & & & \\ & & 0 & 5 & \\ 11 & 7 & 5 & 7 & 11 \\ & 11 & & 11 & \end{pmatrix}$$

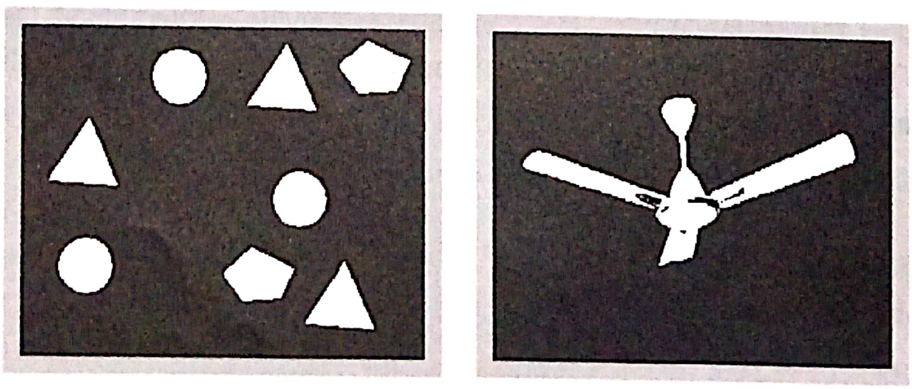
(b)

Fig. 11.17 Chamfer distance masks (a) Chamfer distance using 3 × 3 mask shown as two masks (forward and backward) (b) Chamfer distance using 5 × 5 mask shown as two masks (forward and backward)

The masks for finding distance measure for 3 × 3 and 5 × 5 are shown in Figs 11.17(a) and 11.17(b), respectively, as a combination of two masks.

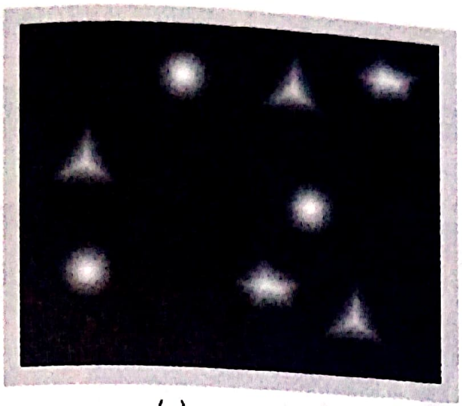
The forward mask in the first pass moves from left to right and works on the image from top to bottom and in the second pass the backward mask moves in the right to left direction, but works from bottom to top. When the mask value is 0, no operation is done. In this case the existing values of the image remain intact. Otherwise, the sum of the mask value and the pixel value is calculated. Then the centre of the mask is replaced with the minimum of the sums. This process is repeated till the entire image is processed.

Two original images and the results of applying the distance transform on them are shown in Figs 11.18(a)–11.18(d).



(a) (b)

Fig. 11.18 Results of the distance transform (a) Original image of shapes (b) Original image of a fan



(c)

(d)

Fig. 11.18 (Contd) (c) Result of applying distance transform on the shapes (d) Distance transform on the fan

11.6.8 Region Filling

Region filling is the process of filling up an image. The process that achieves this is as follows:

1. Let p be the point of the region to be filled. Initially $p = X_0$. Let A be subset of elements that represents the region. Let k denote the iteration.
2. Let B be the structuring element.
3. Repeat steps 4–6.
4. Set $k = k + 1$.
5. $X_k = (X_{k-1} \oplus B) \cap A^C$ and store the result.
6. If $X_k = X_{k-1}$, then stop.
7. The union of the final X_k with the original image gives the filled region.
8. Exit.

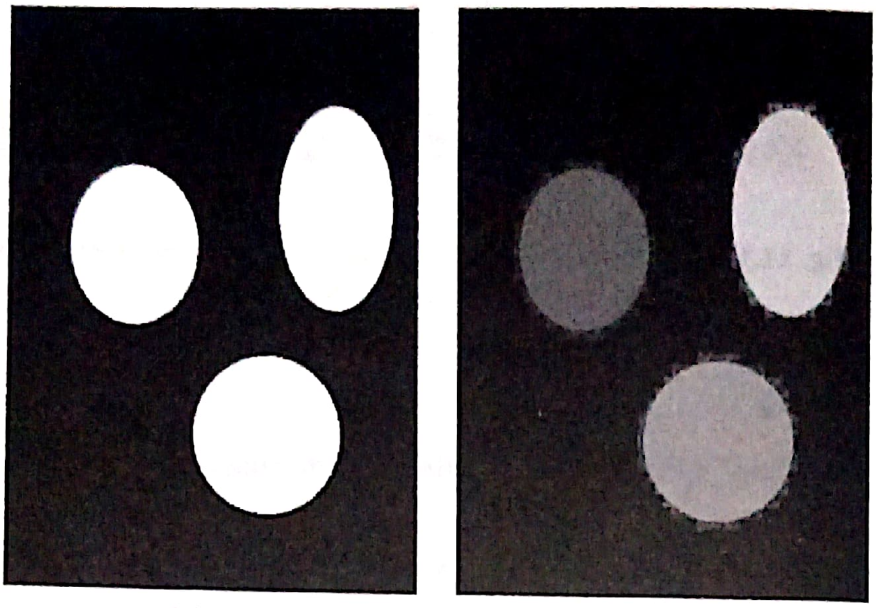
11.6.9 Extraction of Connected Component

This is similar to the region filling operation. Let us assume that p is a point of the connected component C. The process is the same as region filling except that the original image rather than the complement is used.

1. Let p be a point in the connected component C that needs to be extracted. Let $p = X_0$.
2. Let B be the structuring element.
3. Repeat steps 4–6.
4. $k = k + 1$.
5. $X_k = (X_{k-1} \oplus B) \cap A$
   The difference with the previous algorithm is that A is used instead of its complement.
6. If $X_k = X_{k-1}$, then stop.

7. Assign $c = X_k$.
8. Exit.

An original image and the result of connected component extraction are shown in Figs 11.19(a) and 11.19(b), respectively.



(a) (b)

Fig. 11.19 Connected component extraction (a) Original image (b) Extracted component with colours artificially applied—the spurs can be observed at the edges (Refer to the OUPI website for colour images)

11.6.10 Pruning

Morphological operations create some tail pixels that affect the topology of the object. These pixels are also called spurs or parasitic components. The process of removing these extra tail pixels is called *pruning*. This process is an extension of the thinning process. After the thinning process is applied to the image, the resultant can be thinned with the following structuring masks whose primary aim is to detect the end points and remove them. The pruning structuring element masks are shown in Fig. 11.20.

$$B_1 = \begin{pmatrix} 0 & 0 & 0 \\ X & 1 & X \\ 1 & 1 & 1 \end{pmatrix}, B_2 = \begin{pmatrix} X & 0 & 0 \\ 1 & 1 & 0 \\ X & 1 & X \end{pmatrix}, B_3 = \begin{pmatrix} 1 & X & 0 \\ 1 & 1 & 0 \\ 1 & X & 0 \end{pmatrix}, B_4 = \begin{pmatrix} 1 & 1 & X \\ 1 & 1 & 0 \\ X & 0 & 0 \end{pmatrix}$$

$$B_5 = \begin{pmatrix} 1 & 1 & 1 \\ X & 1 & X \\ 0 & 0 & 0 \end{pmatrix}, B_6 = \begin{pmatrix} X & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & X \end{pmatrix}, B_7 = \begin{pmatrix} 0 & X & 1 \\ 0 & 1 & 1 \\ 0 & X & 1 \end{pmatrix}, B_8 = \begin{pmatrix} 0 & 0 & X \\ 0 & 1 & 1 \\ X & 1 & 1 \end{pmatrix}$$

Fig. 11.20 Structuring element masks

11.7 GREY SCALE MORPHOLOGY

$$B = \begin{pmatrix} X_3 & X_2 & X_1 \\ X_4 & X & X_0 \\ X_5 & X_6 & X_7 \end{pmatrix}$$

Fig. 11.21 Erosion mask for grey scale morphology

The idea of a binary image can be extended to grey scale images. A grey scale image is the input for the grey scale morphology operations. Similar to the binary morphology operation, the mask moves across the image. The pixel-by-pixel process is done and the resultant is produced in the output image.

The structuring element can be a square matrix of size 3×3, 5×5, or larger depending upon the application. The erosion mask is shown in Fig. 11.21.

Let A be the given image and B be the structuring element with mask weights as shown in Fig. 11.21; then the output value of erosion process is the minimum value of all the nine factors for erosion. The nine factors are

$$\begin{aligned} O(x, y) = A \ominus B = \text{minimum}(&X + A(x, y), X_0 + A(x+1, y), X_1 + A(x+1, y-1), \\ &X_2 + A(x, y-1), X_3 + A(x-1, y-1), X_4 + A(x-1, y), \\ &X_5 + A(x-1, y+1), X_6 + A(x, y+1), X_7 + A(x+1, y+1)) \end{aligned}$$

The numerical values of the point and its eight neighbours are evaluated and the minimal value among these nine replaces the value of the central pixel in the output image. Grey scale morphological operations are described in the following discussion.

Grey scale dilation For dilation, the mask value can range from 0 to 255. The output value is the maximum value of all the nine addends. This is given as

$$\begin{aligned} O(x, y) = A \oplus B = \text{maximum}(&X + A(x, y), X_0 + A(x+1, y), X_1 + A(x+1, y-1), \\ &X_2 + A(x, y-1), X_3 + A(x-1, y-1), X_4 + A(x-1, y), \\ &X_5 + A(x-1, y+1), X_6 + A(x, y+1), X_7 + A(x+1, y+1)) \end{aligned}$$

The numerical values of the point and its eight neighbours are evaluated and the maximum value among these nine replaces the value of the central pixel in the output image. A comparison of the grey scale erosion and dilation operations is given in Table 11.3.

Table 11.3 Comparison of grey scale erosion and dilation operations

Erosion	Dilation
Reduces the size of the objects with respect to the background	Increases the size of the objects
Eliminates noise spikes and ragged edges	Eliminates noise spikes and ragged edges
Darkens the bright objects	Brightens the objects
Increases the size of holes and sharpens corners	Connects objects, bridges gaps, smoothens edges, fills holes, and creates outlines in an image

Dilation can also be realized using masks. The generalized grey scale dilation operation is described as follows. The mask value of the structuring element can range from 0 to 255. However, the value is typically zero. This is used to extract the maximum of the input pixel group. The purpose is to brighten the object so that small objects become bigger and exaggerated.

Generalized erosion The generalized grey scale erosion operation is described as follows. The mask value of the structuring element can range from –255 to 0. However, the value is typically zero. The value of the structuring element mask is a 3×3 array of ones. This is used to extract the minimum of the input pixel group. The purpose of this is to darken the input image so that the small brighter regions would disappear.

Generalized opening and closing The combined operation of grey scale erosion followed by grey scale dilation is called the *opening* operation. It is denoted as $A \circ B$. Similarly grey scale dilation followed by grey scale erosion is called the *closing* operation, denoted as $A \bullet B$.

The purpose of this operation is to darken the object and to remove single-pixel objects. This operation can be applied multiple times and can be used to remove the larger object anomalies.

An original image and the results of the grey scale morphological operations are shown in Figs 11.22(a)–11.22(e).

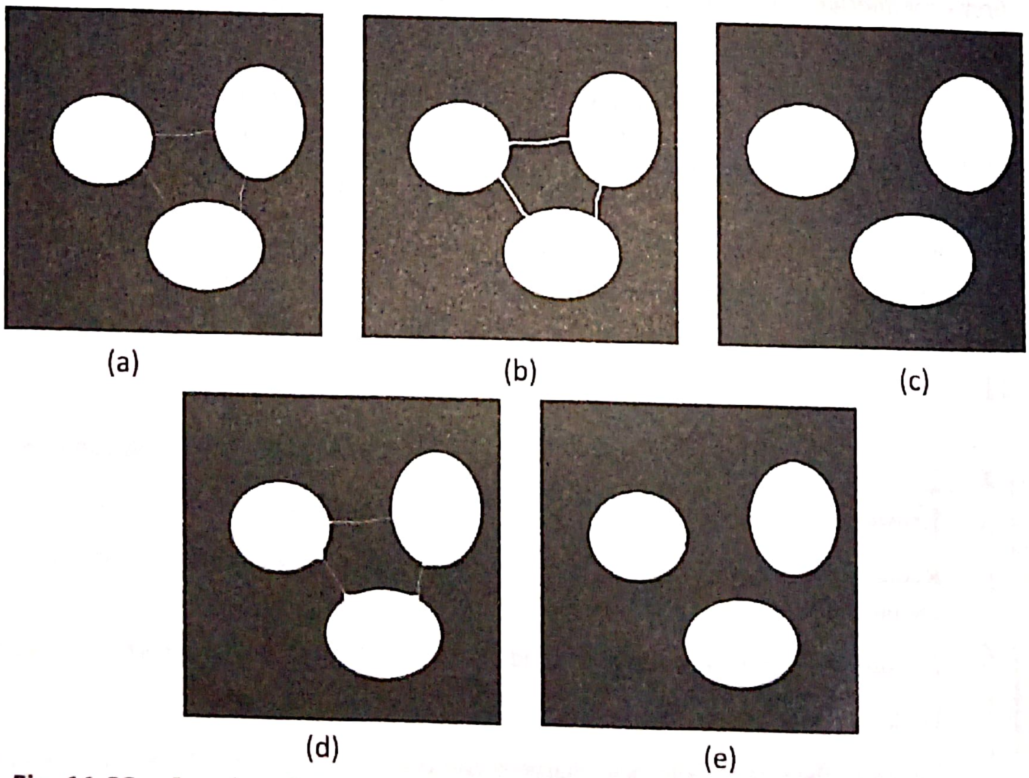


Fig. 11.22 Results of the grey scale morphology operations (a) Original image (b) Dilation (c) Erosion (d) Opening (e) Closing

Example 11.1 Consider the following image A and structuring element B. Show the result of grey scale dilation and erosion.

$$A = \begin{pmatrix} 7 & 8 & 2 & 4 \\ 6 & 4 & 3 & 3 \\ 7 & 3 & 6 & 6 \\ 4 & 4 & 2 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Solution The grey scale dilation, represented as $A \oplus B$, can be obtained by adding the image and the structuring element and the centre of the mask can be replaced by the maximum value. If the mask range goes beyond the image range, it can be ignored. The first overlapping of the structuring element over the image gives the following result:

$$\begin{pmatrix} 7+1 & 8+1 & 2+1 \\ 6+1 & 4+2 & 3+1 \\ 7+1 & 3+1 & 6+1 \end{pmatrix} = \begin{pmatrix} 8 & 9 & 3 \\ 7 & 6 & 4 \\ 8 & 4 & 7 \end{pmatrix} \Rightarrow \begin{pmatrix} & & \\ & 9 & \\ & & \end{pmatrix}$$

It can be observed that 9 is the maximum. Then the structuring element can be shifted once and the operations can be repeated. This gives the second result as

$$\begin{pmatrix} 8+1 & 2+1 & 4+1 \\ 4+1 & 3+2 & 3+1 \\ 3+1 & 6+1 & 6+1 \end{pmatrix} = \begin{pmatrix} 9 & 3 & 5 \\ 5 & 5 & 4 \\ 4 & 7 & 7 \end{pmatrix} \Rightarrow \begin{pmatrix} & & \\ & 9 & \\ & & \end{pmatrix}$$

Further shifts of the mask cause the structuring mask to cross the image range and are hence ignored. Then the structuring element is moved down and the operations are repeated to give the further two results as

$$\begin{pmatrix} 6+1 & 4+1 & 3+1 \\ 7+1 & 3+2 & 6+1 \\ 4+1 & 4+1 & 2+1 \end{pmatrix} = \begin{pmatrix} 7 & 5 & 4 \\ 8 & 5 & 7 \\ 5 & 5 & 3 \end{pmatrix} \Rightarrow \begin{pmatrix} & & \\ & 8 & \\ & & \end{pmatrix}$$

and

$$\begin{pmatrix} 4+1 & 3+1 & 3+1 \\ 3+1 & 6+2 & 6+1 \\ 4+1 & 2+1 & 3+1 \end{pmatrix} = \begin{pmatrix} 5 & 4 & 4 \\ 4 & 8 & 7 \\ 5 & 3 & 4 \end{pmatrix} \Rightarrow \begin{pmatrix} & & \\ & 8 & \\ & & \end{pmatrix}$$

Further shifts make the structuring element cross the domain of the image. So those operations are ignored. The final result is $\begin{pmatrix} 9 & 9 \\ 8 & 8 \end{pmatrix}$.

Then the grey scale erosion, represented as $A \ominus B$, can be obtained by subtracting the values of the structuring element from the image and the centre of the mask is replaced by the minimum value. Similar to the grey scale dilation process, if the mask range goes beyond the image range during the shift process then the result is ignored.

$$\begin{pmatrix} 7-1 & 8-1 & 2-1 \\ 6-1 & 4-2 & 3-1 \\ 7-1 & 3-1 & 6-1 \end{pmatrix} = \begin{pmatrix} 6 & 7 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 5 \end{pmatrix} \Rightarrow \begin{pmatrix} & & \\ & 1 & \\ & & \end{pmatrix}$$

$$\begin{pmatrix} 8-1 & 2-1 & 4-1 \\ 4-1 & 3-2 & 3-1 \\ 3-1 & 6-1 & 6-1 \end{pmatrix} = \begin{pmatrix} 7 & 1 & 3 \\ 3 & 1 & 2 \\ 2 & 5 & 5 \end{pmatrix} \Rightarrow \begin{pmatrix} & & \\ & 1 & \\ & & \end{pmatrix}$$

$$\begin{pmatrix} 6-1 & 4-1 & 3-1 \\ 7-1 & 3-2 & 6-1 \\ 4-1 & 4-1 & 2-1 \end{pmatrix} = \begin{pmatrix} 5 & 3 & 2 \\ 6 & 1 & 5 \\ 3 & 3 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} & & \\ & 1 & \\ & & \end{pmatrix}$$

$$\begin{pmatrix} 4-1 & 3-1 & 3-1 \\ 3-1 & 6-2 & 6-1 \\ 4-1 & 2-1 & 3-1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 4 & 5 \\ 3 & 1 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} & & \\ & 1 & \\ & & \end{pmatrix}$$

Further shifts make the structuring element cross the domain of the image. So those operations are ignored. The final result is $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$.

11.7.1 Morphological Gradient

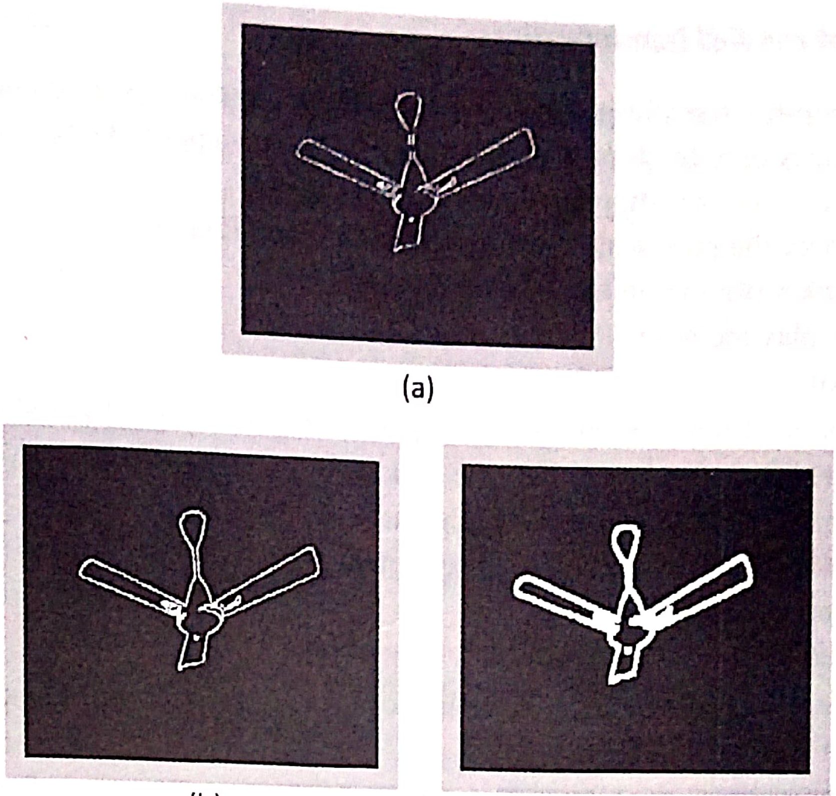
This operation is used to create edge-enhanced images. Edges represent the transitions from dark to bright regions or vice versa in an image. The procedure for obtaining the morphological gradient is as follows:

1. Create a duplicate of the original image. Let it be B.
2. Apply the erosion operation to the original image.
3. Apply the dilation operation to the duplicate image.
4. Edge = dilated image – eroded image, that is,

$$(A \oplus B) - (A \ominus B)$$

5. Display the gradient image.
6. Exit.

The results of this gradient operation are shown in Figs 11.23(a)–11.23(c).

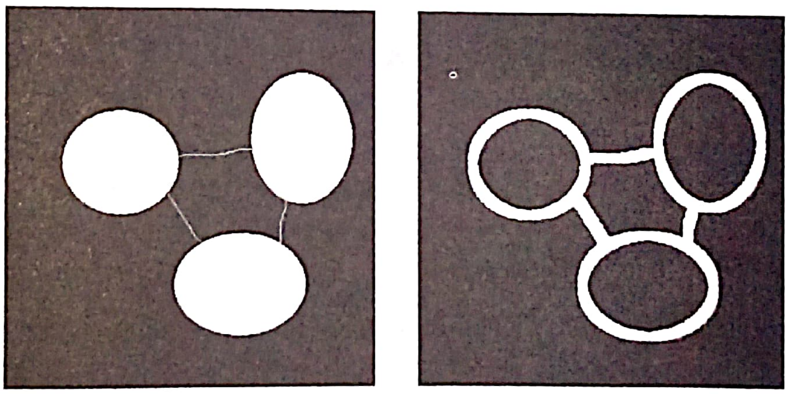


(a)

(b) (c)

Fig. 11.23 Results of the grey scale morphology gradient operation (a) Gradient with 3 × 3 mask (b) Gradient with 5 × 5 mask (c) Gradient with 13 × 13 mask

Figures 11.24(a) and 11.24(b) show another original image and the results of applying this gradient.



(a) (b)

Fig. 11.24 Results of grey scale morphology gradient operation (a) Original image (b) Morphological gradient with 11 × 11 mask

Morphological gradient is also used to sharpen an image whose edges are replaced by peaks. It is expressed as a sharpening operation as follows:

$$\text{Morphological gradient} = \frac{1}{2}(\text{Max(Image)} - \text{Min(Image)})$$

11.7.2 Top-hat and Well Transformations

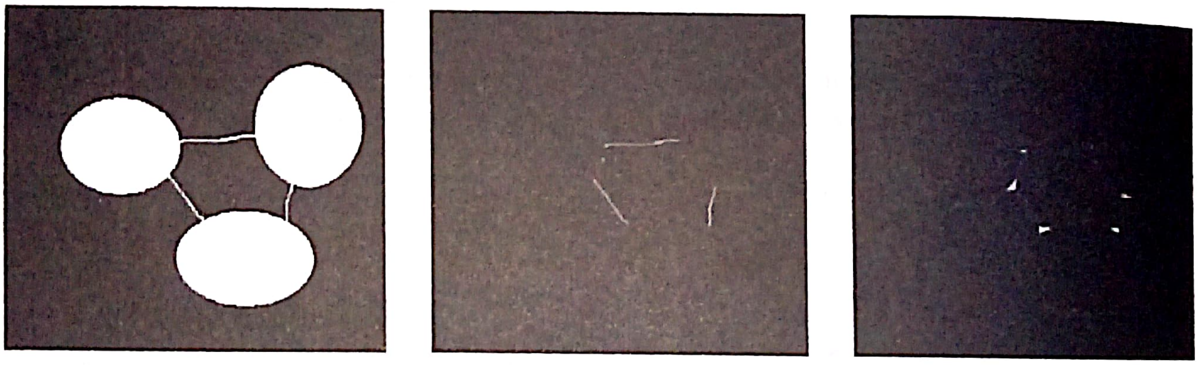
The top-hat transformation is a variant of grey scale opening and closing operations that produces only the bright peaks of an image. It is called the *peak detector*. The process of top-hat transformation is given as follows:

1. Apply the grey scale opening operation to an image.
2. Peak = original image – opened image.
3. Display the peak.
4. Exit.

The well transformation is the opposite of the top-hat transformation and produces the dark valleys of an image (i.e., the dark features in the image). The procedure is as follows:

1. Apply the grey scale closing operation to an image.
2. Valley = original image – closed image.
3. Display the valley.
4. Exit.

An original image and the results of the top-hat and well transformations are shown in Figs 11.25(a)–11.25(c).



(a) (b) (c)

Fig. 11.25 Top-hat and well transformation (a) Original image (b) Top-hat transformation (c) Well transformation

11.7.3 Morphological Reconstruction

This procedure involves two images. One image is the given original image and the other is called the marker. The marker is the starting point of the transformation. Only elementary operations such as erosion and dilation are used. The transformations force the original image to remain within the limits of the second image. The size of the second image is immaterial. The entire operation depends on the selection of the marker image. The selection of the marker image depends on various factors such as the knowledge of the expected result, source image, user interaction, and transformations.

Morphological reconstruction is very useful in various operations such as opening and closing using reconstruction, top-hat transformation, detection of holes, and clearing of objects commented by borders.

11.7.4 Watershed Algorithm

A grey scale image can be viewed as a topological surface, where the value of a pixel is its height. Pixels having high values denote peaks, whereas pixels having low values denote valleys. In a complement image, the peak becomes the valley.

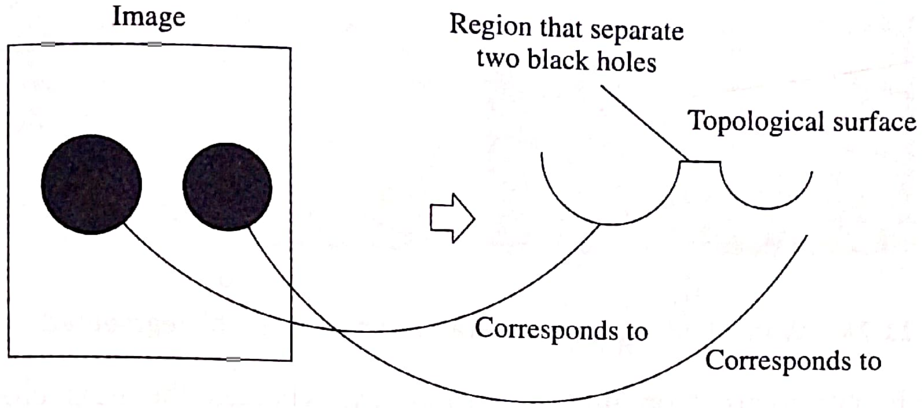


Fig. 11.26 Illustration of topological surface

The terminologies of this algorithm are largely derived from geography. This algorithm visualizes that the topological surface is filled with water. In terms of image processing, water is colour. The valley is called catchment basin and the structure that separates two basins is called watershed. This is shown in Fig. 11.26.

It can be observed that the black holes correspond to two catchment basins and the region between the holes is the watershed or dam and is represented as a barrier line between the valleys. Now, when water starts filling the distinct catchment basins, dams need to be built wherever it is necessary to prevent the merging of two adjacent basins. In other words, colour is filling adjacent regions, and if two regions have the same colour then the regions merge. This process is repeated for the entire image till water fills up the entire topographical surface. Once the surface is immersed in water, the dams outline the watershed lines. Thus, watershed lines mark the boundaries of the catchment basins. These segment the image into the desired regions by showing the waterlines.

The stages of the watershed segmentation are shown in Fig. 11.27.

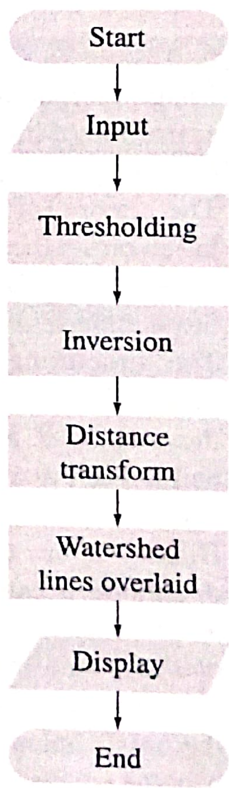
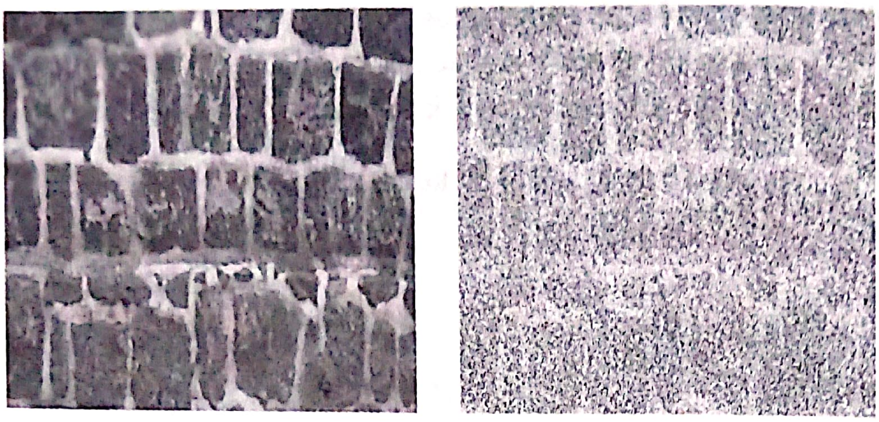


Fig. 11.27 Watershed segmentation process

One disadvantage of this method is that it leads to over-segmentation. This is also because of noise. Hence it is better if the images are smoothed before the segmentation process, and post-segmentation algorithms are implemented to connect the smaller regions to prevent over-segmentation.

An example of an image and its segmentation are shown in Figs 11.28(a) and (b), respectively.



(a) (b)

Fig. 11.28 Watershed segmentation (a) Original image (b) Segmented image

This chapter focused on image morphology, whereas the next chapter focuses on extraction of features from the segmented regions.

SUMMARY

- Image morphology is an important tool in the extraction of image features. Image features are necessary for the recognition of objects.
- The theory of mathematical morphology is based on set theory. We can visualize the binary object as a set. Then set theory can be applied to the sample set.
- The structuring element is usually a matrix of size 3×3. It has its origin at the centre of the matrix. It is then shifted over the image and at each pixel of the image its elements are compared with the set of the underlying pixels.
- Dilation is one of the two basic morphological operators. It can be applied to binary as well as grey scale images. The basic effect of the operator on a binary image is that it gradually increases the boundaries of the region while the small holes that are present in the image become smaller.
- One can view the morphological operations as a binary correlation operation involving logical elements.
- The difference between the original image and the eroded image creates a boundary.
- Opening is an erosion operation followed by a dilation operation.
- Closing is a dilation operation followed by an erosion operation.
- The hit-or-miss transform is a general binary morphological operation that can be used to look for particular patterns of foreground and background pixels in an image.
- Thinning is a morphological operator that is used to remove irrelevant foreground pixels that are present in binary images.
- The idea of binary image can be extended to grey scale images. A grey scale image is the input for grey scale morphology operations. Similar to binary morphology operations, the mask moves across the image. The pixel-by-pixel process is performed and the resultant is produced in the output image. The minimum value of the factors is used for the erosion operation and the maximum value is used for the dilation operation.
- The combined operation of grey scale erosion followed by grey scale dilation is called opening operation. It is denoted as $A \circ B$. Similarly,

the combined operation of grey scale dilation followed by the grey scale erosion is called grey scale closing and is denoted as $A \bullet B$.

- Top-hat transformation is a variant of grey scale opening and closing operations that produces only the bright peaks of an image.
- Well transformation is the opposite of the top-hat transformation and produces the dark valleys of an image (i.e., the dark features in the image).

KEY TERMS

Boundary The difference between the original and the eroded images gives the boundary. Boundaries separate a component from another component.

Dual process This means that the operations involved have opposing effects.

Fit This condition is satisfied when there is an exact match for at least one of its pixels set to 1 and the corresponding pixels that is also 1

Hit A structuring element is said to be intersect or hit the image if its pixels correspond to the image pixel that is also '1'.

Miss This is a condition where none of the elements of the structuring element matches with the elements of the image.

Morphological gradient These are the edges that represent the transitions from dark to bright and bright to dark regions in an image.

Over-segmentation This is a process where unnecessary regions are identified in the intended region of interest.

Pruning This is a process of removing the extra tail pixels in an image.

Reconstruction This is a process that restrains the image processing operations with the help of a reference image.

Region filling This is a process of filling up a portion of the image with constant intensity.

Skeleton This is a geometrical description that describes a shape using lesser number of pixels than the original.

Structuring element This is a mask that probes the image.

Thinning This is a process of removing irrelevant pixels.

Top-hat This is a peak detector that locates the bright spots of an image.

Watershed This is a process of locating the object using watersheds.

Well This is a valley detector that locates the dark spots of an image.

REVIEW QUESTIONS

1. What is the role of morphology in image processing?
2. What are the multiple ways of finding dilation and erosion for binary images?
3. What is a structuring element?
4. What is a hit-or-miss transform?
5. The combined operation of erosion followed by dilation is called an opening operation. Are not these operations the reverse of each other and hence do they restore the original image? Justify.
6. What is boundary extraction?
7. What is the difference between outlining and boundary?
8. What is a morphological gradient?
9. What is the difference between thinning and thickening?
10. What is a top-hat transform?
11. What is meant by distance transform?

NUMERICAL PROBLEMS

1. Consider the image $A = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ and let the structuring element be [1 1]. Perform the morphological operations erosion and dilation.
2. Consider the image

$$A = \begin{pmatrix} 11 & 18 & 13 & 12 \\ 12 & 2 & 22 & 22 \\ 22 & 22 & 22 & 2 \\ 1 & 68 & 70 & 6 \end{pmatrix}$$

and let the structuring element be

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Perform the morphological operations erosion and dilation.

3. What is the distance transform of a 7×7 square matrix and a 7×5 rectangular matrix of 1s surrounded by a background of 0s?

dilatn

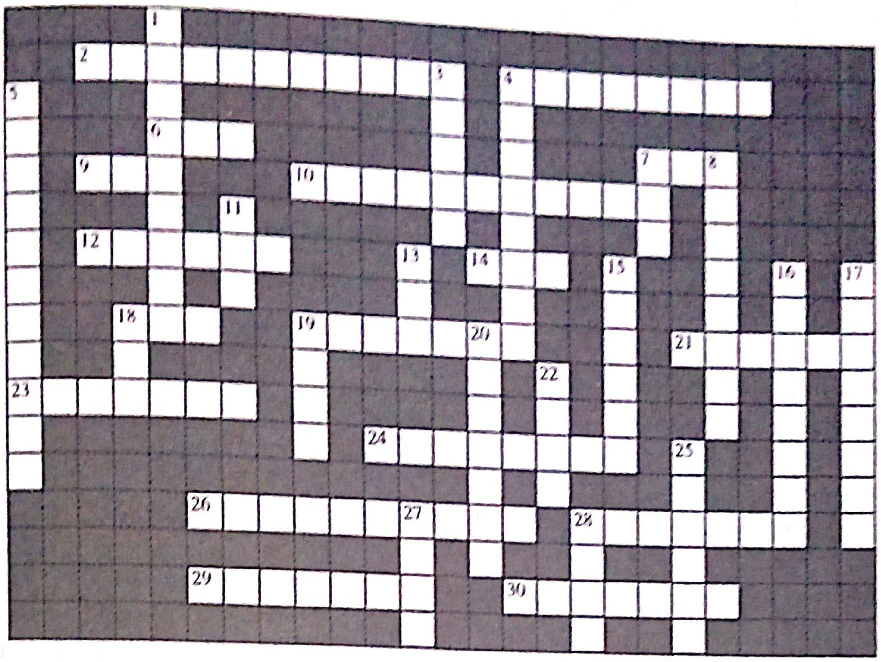
12	19	14	13
13	3	23	23
23	23	23	3
2	69	71	7

$$\begin{bmatrix} 23 & 23 \\ 71 & 71 \end{bmatrix}$$

10	17	12	11
11	1	21	21
21	21	21	1

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

CROSSWORD



Across

2. The mask in image morphology is called ________ elements.
4. If structuring element s hits image f, it results in ________ operation.
6. Is connected component algorithm related to region fill algorithm? (Yes/No)
7. The dilation and erosion operations are complementary. (Yes/No)
9. Region fill can be done using morphology operations. (Yes/No)
10. ________ law states that $A \cup B = B \cup A$.
12. Image morphology is a power tool for analysing object ________.
14. The condition of exact match of an image for at least one of the pixels set to 1 and the corresponding element that is also 1 is called ________.
18. The set complement, say B, of set A is a set of all objects that do not belong to set A. (Yes/No)
19. If structuring element s fits image f, it results in one in ________ operation.
21. Skeleton is also called ________ axis.
23. The dilation operation followed by erosion operation is called ________ operation.
24. The difference between the dilated and the original image is called external ________.
26. Thinning is used to remove ________ (foreground/background) pixels present in the image.
28. The process of removing extra pixels that form spur or parasiting components is called ________.
29. Morphological operations can remove ______ noise.
30. Morphological gradient ________ images.

Down

1. The output of distance transform is a ________ (binary/greyscale/colour) image.
3. A set is a ________ of objects.
4. The operation that makes small holes in an image to become smaller is called ________.
5. Image is viewed as ________ surface in watershed segmentation.
7. A set of coordinate points can serve as a structuring element. (Yes/No)
8. Convex hull is the ________ polygon that encompasses the region completely like an elastic band.
11. The operations of image morphology are based on ________ theory.
13. Morphological operations can be implemented using convolution operation implemented in a logical manner. (Yes/No)
15. Well transformation results in _____ of a image.
16. Repeated thinning results in _____operation.
17. Greyscale ______ operation creates outlines in an image.
18. Watershed lines separate catchment basins. (Yes/No)
19. The dilated image minus eroded image results in ________.
20. The erosion operation followed by dilation operation is called ________ operation.
22. Thickening is a ______ of thinning operation.
25. A region is called ______, if a line that connects two parts of the region lies within the region.
27. Watershed segmentation may result in ______ (over/under) segmentation.
28. Top-hat transformation is a ________ detector.

WORD SEARCH PUZZLE

Some of the important terms in this chapter are present in the following word jumble. Identify the words. Diagonal words are possible.

D	O	D	U	A	L	P	W	Q	T	H	O	W	J	Z	S	H	A	R	P	E	N	R	R	Y
C	W	R	N	W	M	G	L	R	W	J	X	C	O	M	M	U	T	A	T	I	V	E	U	E
Y	E	S	U	U	N	A	D	N	M	W	D	Y	O	P	E	N	I	N	G	R	J	X	J	S
A	S	P	A	I	I	T	O	F	O	R	E	G	R	O	U	N	D	B	V	Q	G	B	N	I
C	L	S	N	D	U	I	O	U	W	L	A	U	F	J	N	U	T	O	V	H	G	S	L	Q
Y	V	U	E	Y	T	N	B	P	C	I	K	D	W	K	S	L	D	U	A	P	Y	E	S	A
M	R	M	F	A	E	I	G	O	O	S	M	R	N	G	K	I	S	N	L	N	H	V	Y	E
P	N	I	L	E	Y	S	D	F	N	L	P	P	X	G	R	V	T	D	L	M	X	S	E	B
J	B	I	R	F	O	M	H	I	G	F	O	N	U	G	U	E	R	A	E	H	U	M	S	W
Q	D	H	V	J	E	O	K	A	L	R	F	G	A	L	G	S	U	R	Y	Y	T	A	D	P
D	B	R	I	G	H	T	E	N	P	A	O	R	I	D	S	V	C	Y	A	I	P	L	I	C
A	Z	Y	X	C	O	N	V	E	S	E	T	U	E	C	V	E	T	F	H	Y	E	L	L	D
U	N	T	H	I	N	N	I	N	G	Q	S	I	P	O	A	L	U	T	B	W	A	E	A	Q
G	A	G	R	E	Y	S	C	A	L	E	D	Q	O	A	J	L	R	P	N	B	K	S	T	T
Q	E	O	T	H	I	J	M	R	N	H	X	I	G	N	N	E	I	Y	E	S	N	T	I	U
O	E	T	O	V	D	O	E	R	O	S	I	O	N	O	V	R	N	Y	E	S	X	N	O	T
R	Z	R	C	P	S	G	S	E	T	V	M	N	I	O	R	X	G	C	L	O	S	I	N	G

Hints

1. Image morphology is useful in analysing __________ of the objects.
2. __________ is the primary data structure used for morphological operators.
3. __________ elements are small images that are used to probe the original image.
4. Structuring elements are __________ of pixels that can be used to probe larger images.
5. Dilation operator is __________.
6. __________ operator is a union of operation.
7. The condition of exact matching of the image for at least one of its elements that is 1 and the corresponding pixels that is also 1 is called __________.
8. The basic morphological operators are __________ and __________.
9. The __________ operation is an erosion operation followed by a dilation operation.
10. The __________ operation is a dilation operation followed by erosion operation.
11. The difference between the original image and its eroded image creates a __________.
12. __________ axis transform produces a skeleton of an object.
13. Image __________ is necessary for many character recognition projects in image processing.
14. Morphological operations can be extended to __________ images also.
15. Morphological gradient operations can be used to detect an __________ and to __________ an image.
16. Top-hat transformation is called __________ detector.
17. The difference between original image and closed image is called a __________.