Computational Experiments for Evolutionary Pricing Games

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Abstract

Here we describe computational experiments for the project description "Evolutionary Pricing Games" (von Stengel, 2022), included in this folder. It is a documentation of the software, which at each stage represents reproducable computational experiments. It also outlines the next steps for more self-contained runs (with fewer manual interventions) of the experiments, and lists open questions.

1 What's new?

- This is the very first version of a documentation of the computational experiments, in an ad-hoc format (which will evolve). Subsequent versions may be more structured.
- Implementation of a sophisticated "guess your opponent" strategy with interesting effects on the equilibria of the tournament bimatrix game.
- Some attempt at visualizing the bimatrix game payoff pairs.

2 File inventory

directory name:	`05Aug2022/`
main Python file:	`play.py` (run with python3)
code with line numbers:	`05Aug2022-play.pdf`
output file:	`output`
computed equilibria:	`equi11x5`
plots folder:	`./PLOT`

project description:	`Evol-05Aug2022.pdf, Evol-05Aug2022.tex`
this file:	`readme.pdf, readme.tex`

3 Description of the model and investigation

The projection description `Evol-05Aug2022.pdf, Evol-05Aug2022.tex` describes a duopoly game investigated with the "strategy method" by Keser (1992).

The present text describes and documents the computational experiments to continue this research, by extending it with

- the implemention of some first pricing strategies
- equilibrium analysis
- evolutionary simulations (not yet implemented)
- machine learning (not yet implemented).

4 Purpose of this document and directory contents

The current directory `05Aug2022/`, named by date, is meant to contain a complete, self-contained snapshot of the project at each stage.

It will be copied and updated with a new date at the next iteration.

A more systematic solution might be to store it on github. However, this requires technical knowledge of branch management in git. The current code is still small and copying seems fine for now.

Its documentation in LaTeX is more important at the moment.

Text parts in red highlight points for improvement in the current context, which may become kind of "issues" (as in github) to become resolved in subsequent coding phases. Maybe they should be numbered here and later recorded as "resolved".

We can switch to Git, with the advantage that only important files are stored in the repository and very easily shared with `git push` and `git pull`, but have to clearly mark those project snapshots, perhaps as **releases**, that do work on their own. We need tutorials on branch management.

5 Current code capability

At present, the Python code `play.py` contains

- a number of duopoly pricing strategies as methods
- a method `match()` to match any two strategies against each other
- a method `tournament()` to match all low-cost strategies of player 0 to all high-cost strategies of player 1, and record the resulting payoffs to the two players in a bimatrix game (A, B). This bimatrix game is then written to standard output to find all its Nash equilibria via the program `lrsnash`, currently by using the webpage `http://banach.lse.ac.uk`. Because `lrsnash` exists as an executable C program, starting it from Python should be automated next.
- The output of `lrsnash`, via this manual route, is stored in the equilibrium file `equi11x5`.
- The output of `play.py` to standard terminal output `stdout` is recorded in the file `output`. Apart from (A, B), it prints a sample run of two strategies against each other, with the history of demand potentials, prices, and profits of the two players, and a descriptive list of the two players' strategies.
- Graphical output: In `./PLOT/`, a number of files are generated to display the payoff pairs in a suitable graphical format via the Linux program `gnuplot`. Improvement with Python packages seems appropriate.

5.1 Strategies

In `Evol-05Aug2022.pdf, Evol-05Aug2022.tex`, Sections 4 and 5 describe some pricing strategies. They are implemented as Python methods. Two minimum parameters of a strategy are player and time period (player is always the first parameter; maybe the time period should always be the second parameter for consistency), and possibly additional parameters such as a price for the first period. All strategies have access to the past demand potentials and prices of the players, but for the current time period can only access the player's own demand potential. Prices cannot be set below cost.

The following strategies are straightforward (because it is optimal, they always use the myopic monopoly price in the last period):

- the myopic monopoly price,
- a constant price,
- imitating the opponent's previous price.

A "fightback" strategy to reclaim a lost but aspired demand potential turned out to be not so easy to implement and has a first version `fight`, and a more sophisticated version `guess`.

`fight` is very aggressive because it never raises prices back, which results in oscillations and a price war when played against each other.

`guess` remedies this, and has other features:

- When the demand potential is favorable, it lets the firm's price raise partially (weighted with a factor of 0.4 compared to the last own price with weight 0.6). This allows convergence to cooperation near the monopoly price (minus 7 for protection). It will be interesting if such a behavior, and these weights and the parameter 7, which may not be optimal, is learned in a neural net.
- Importantly, rather than reacting to the opponent's last price, which seems to induce oscillating price wars, it assumes that the determining variable for the opponent's behavior is their *sales number* (demand potential minus price). It stores this as a static variable (in Python: the global list `oppsaleguess`) which is updated with a multiplicative weight `alpha` from last time and then used to estimate the opponent price from their *current* demand potential $400 - D$ (where D is the own demand potential) to reclaim the aspired potential in the next round.

In order to test the imitation strategies and possible responses against them, they are used for the column player 1 against several constant strategies of the row player 0. In addition to the imitation strategies, each player's first strategy is the myopic strategy, and the second strategy is the sophisticated `guess` strategy. Indeed, the latter two represent a (possibly fragile) *pure Nash equilibrium*, see below. The size of (A, B) is here 11×5.

5.2 Graphical output

The generated bimatrix game (A, B) contains payoff pairs that reflect how the two players cooperate or compete for any low-cost strategy (row) against high-cost strategy (column).

We want to display the possible payoff pairs in the plane as the possible "region" of cooperation versus competition, and explore its Pareto frontier, where a gain of one player can only be achieved at the expense of the other.

I try to display these payoff pairs not just as a collection of dots but also in dependence on the player's strategies. The following can be improved but is a start.

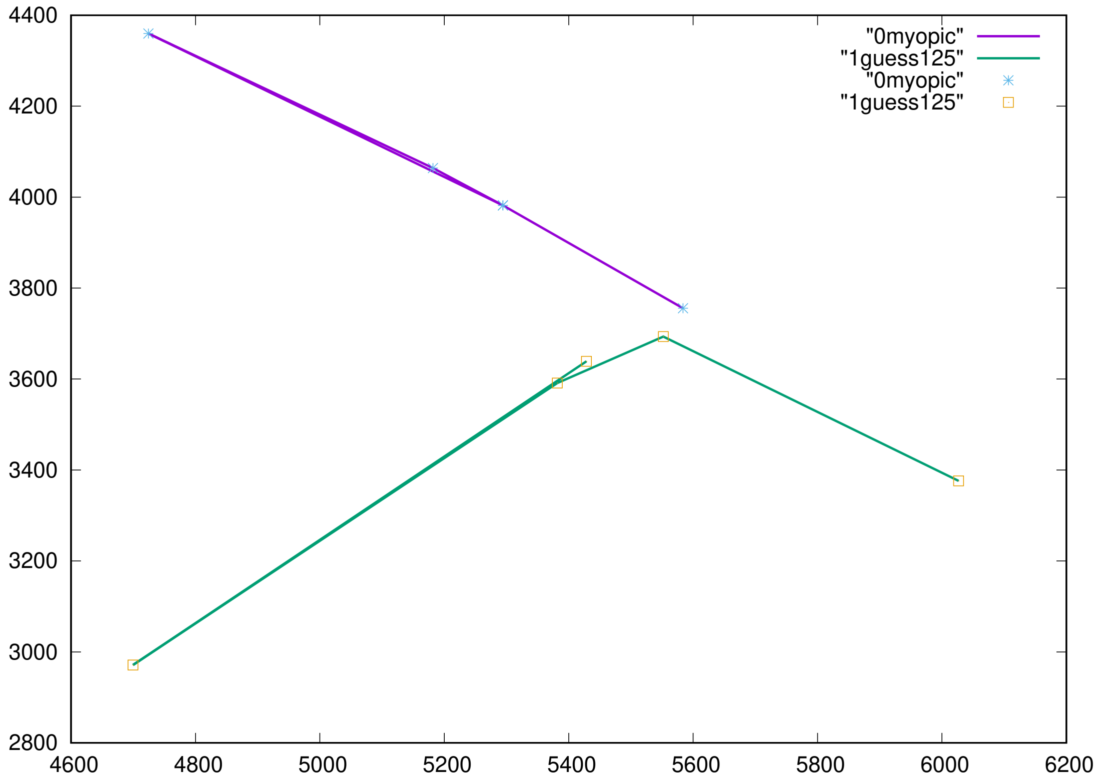
The directory `./PLOT` has to exist before `play.py` is run. It will contain m files for the m strategies of player 0, each with the pairs of payoffs in the respective row of (A, B). These are then used with the program `gnuplot` (possibly later to be replaced by Python plots) to display the payoff pairs.

The first two rows of (A, B) are the myopic and the `guess` strategy of player 0, with the following payoffs:

```
A = 5584   5294   5182   4724   5294      B = 3756   3982   4064   4360   3982
    6027   5552   5382   4700   5428          3376   3693   3591   2971   3639
```

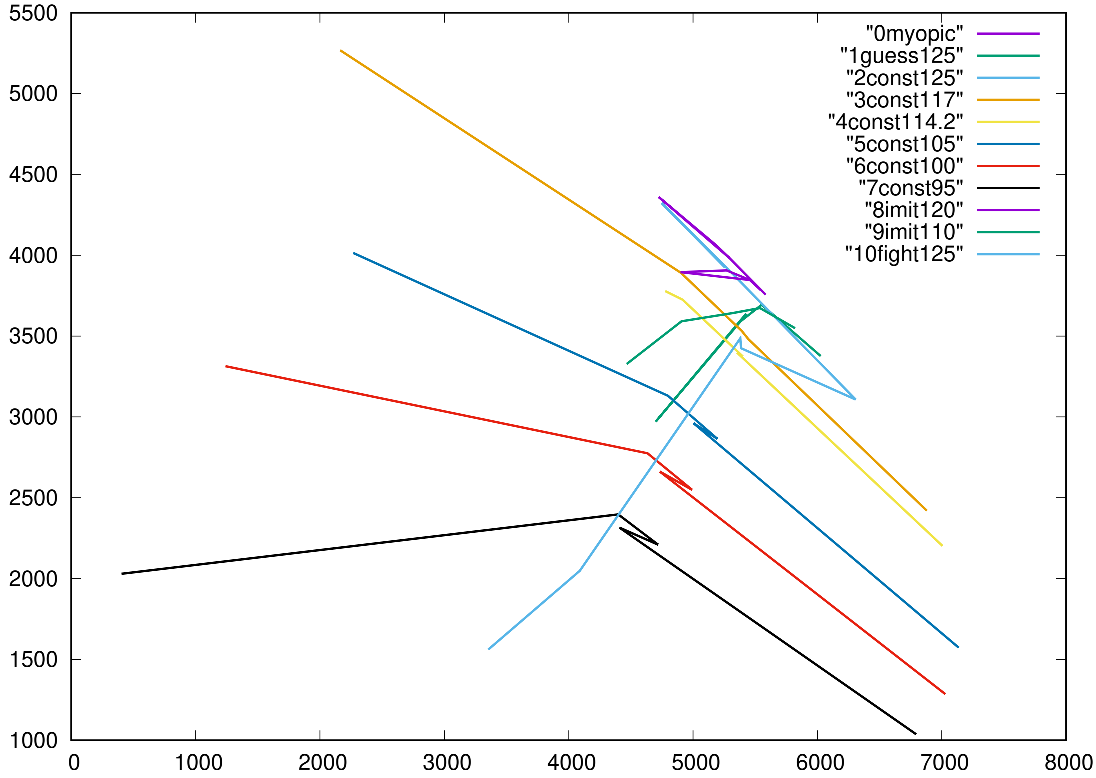
The five columns of (A, B) are the strategies named `myopic, guess130, imit131, imit114.2, fight130` of player 1, where 130, 131, 114.2, 130 are the starting prices in the first period. Against the myopic strategy of player 0, `fight130` performs the same as `guess130` (with payoff pair 5294, 3982). Usually `fight130` is much more aggressive and therefore listed last.

The graphics show each payoff pair as a point in the plane with the row player's payoff in the horizontal direction and the column player's payoff in the vertical direction. For each strategy of the row player (in the next picture only for the first two rows), we connect these points by four line segments for the five column strategies.



This plot is generated (in directory `./Plot/`) with the Linux command `gnuplot gplot-manual` to produce the output file `Pareto-manual.eps`. The choice of colors and symbols is automatic. The green line from right to left shows the performance of `guess125`, which is much better for the row player than the myopic strategy.

For the 11 strategies of the full game, the symbols (like the small yellow boxes above) are too numerous, and only the lines are displayed, as follows:



The lines, to be read from right to left, show the following:

- The right endpoint shows how each row performs against the myopic strategy (first column) of the column player. It can exploit that startegy, even with a constant price, with marginal differences when that constant price is 117, 114.2, or 105, and from then on even decreasing.
- The second point of the first line segment (from the right) shows the performance against `guess130` of the column player. It is "retaliatory" in that either both players score low or both score high.
- The left endpoint of each line shows how each row performs against the last strategy `fight130` of the column player. In fact, that strategy is able to exploit the constant-price strategies of the row player of prices 100, 105, 117 shown in red, blue, and dark yellow.

Overall, there is already a wide spectrum of possible payoff pairs and types of strategy.

6 Equilibrium analysis

This is most interesting. This game has already five mixed Nash equilibria. However, three of these will have index +1 and two have index −1. Those with index −1 will not be dynamically stable. Having the index computation alongside would therefore be a good idea.

It turns out that for the column player, every strategy except the myopic strategy (the first column) appears in a mixed Nash equilibrium. For the row player, there are four pure strategies that appear in an equilibrium, which are `guess125`, `const114.2`, `imit120`, `imit110`.

If the strategies that do not appear in a mixed equilibrium are omitted, then the resulting 4×4 game *happens to have the same mixed Nash equilibria (including a fully mixed one). This is not normally the case: The omitted strategies may have suppressed some equilibria that could appear in the smaller game.* (Any mixed equilibrium of the larger game is also an equilibrium of the smaller game, which has fewer opportunities to deviate.) Here, the five equilibria are as follows, in a manually edited version of the output of `lrsnash`. (This could be automated: reducing the display of probabilities to 4 digits past the decimal point, and suppressing pure strategies that have probability zero.)

```
A = 5552   5382   4700   5428      B = 3693   3591   2971   3639
    5367   5383   4916   4776          3388   3387   3725   3779
    5458   5270   4900   5458          3847   3906   3895   3847
    5538   5318   4907   4467          3673   3642   3591   3328

1  P1:  (1)  0.1134  0.2613  0.3468  0.2784  EP=  5201.93
   P2:  (1)  0.2400  0.3927  0.3266  0.0407  EP=  3661.12

2  P1:  (2)  0.0501  0.1189  0.8310  0.0     EP=  5174.28
   P2:  (2)  0.0     0.5843  0.3116  0.1041  EP=  3828.49

3  P1:  (3)  0.0     0.4705  0.5294  0.0     EP=  4912.79
   P2:  (3)  0.0     0.0     0.9771  0.0229  EP=  3815.0

4  P1:  (4)  0.0     0.1957  0.0     0.8043  EP=  4938.55
   P2:  (4)  0.05    0.0     0.95    0.0     EP=  3617.22

5  P1:  (5)  1.0     0.0     0.0     0.0     EP=  5552.0
   P2:  (5)  1.0     0.0     0.0     0.0     EP=  3693.0
```

The last row shows that `(guess125, guess130)` is a pure-strategy equilibrium. Note, however, that the last row of the 4×4 game above has a payoff of 5538 which is not much lower than the equilibrium payoff of 5552, so this pure equilibrium may be fragile in the presence of other strategies.

7 Plan of work

The goal of this research is learning how to play well a complex "base game", here the pricing game, which is too complex to know in advance, against an unknown distribution of opponents who themselves also evolve.

Here are some next steps.

- **Remove the end effect.** The pricing game runs over a fixed number of 25 periods, which has an "end effect" that myopic play is optimal in the last period (which could be extended with further optimal equilibrium play in the preceding periods). It seems better to introduce a discount factor that terminates the game with a fixed probability, say 4 percent – which should be a parameter – after each round to suppress this arbitrary end effect. *In each match, this will require multiple rounds because the number of rounds of interaction with the opponent is random.* **It is probably also a good idea to start the random generator with a fixed seed to make these random runs reproducible, i.e. always giving the same payoff pairs. Dependency on this seed should be tested separately.**
- **Introduce a learning model.** In the pricing game, either Q-learning or some neural net should be trained to determine the price for the next period.
- **Evaluate the performance of a strategy.** The learning could be done, ad-hoc, in two phases:
  - play against a fixed distribution of existing strategies. The parameters of the learning model are adapted until it plays well against an existing distribution of other strategies.
  - once learning has stabilized, the new strategy is entered into the strategy pool.
  - an evolutionary dynamics, such as the replicator dynamics, is then run, which presumably changes the distribution of strategies (one would need to recognize once this has entered a certain orbit, perhaps via a stabilization of the distribution of strategies).
  - this new distribution is then used for the next learning round.

- **Alternative: no two phases of "learning" and evolution.** Alternatively, the learning could take place during the run of the evolutionary dynamics. However, this may result in interaction effects that are difficult to analyze.
- **Dependency on the starting point.** Evolutionary dynamics may result in different stationary orbits or even equilibria depending on the starting distribution. For example, the pure equilibrium above may only be an attractor in a small neighborhood of the equilibrium itself, and otherwise lead to another equilibrium.
- **Comparison of evolutionary dynamics and the tracing procedure for equilibrium computation.** The tracing procedure has a simple implementation via Lemke's algorithm and leads to Nash equilibrium (of positive index). It would be interesting to compare this, also dependent on the starting point, with where evolutionary dynamics lead to. This comparison could already be tested for the 4×4 game above.