Pricing Game with Demand Inertia Has Strategic Substitutes

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Abstract

The two-period pricing game of a "duopoly with demand inertia" seems to have strategic substitutes, not complements as is usual in pricing games. This could explain more aggressive behavior. Multi-period play still has to be analyzed.

1 Two-stage duopoly pricing game

s-two

We consider the oligopoly game by Selten (1965) for the special duopoly case of the strategy experiments by Keser (1992), first over two periods.

As studied by Keser (1992), the game is played over fixed number T of periods, here $T = 25$. Each firm i has a *demand potential* D_i that determines its number of $D_i - p_i$ of sold units of a good when setting a price p_i (firm i's decision in each period), with a profit of $p_i - c_i$ per unit for the firm's production cost c_i. The firms have different costs, $c_1 = 57$ and $c_2 = 71$. The myopic monopoly profit maximizes $(D_i - p_i)(p_i - c_i)$ when $p_i = (D_i + c_i)/2$.

At the start of the T periods, both firms have the same demand potential $D_1 = D - 2 = 200$. After each period, the cheaper firm gains demand potential from the more expensive firm in proportion to their price difference, according to

$$\begin{aligned} D_1^{t+1} &= D_1^t + \tfrac{1}{2}(p_2^t - p_1^t)\,, \\ D_2^{t+1} &= D_2^t + \tfrac{1}{2}(p_1^t - p_2^t)\,. \end{aligned} \tag{1}$$

demand

The total profits are summed up (there is also a discount factor of 1 percent per time period that favors early profits, which we ignore).

We first consider $T = 2$ and simplify notation, and in effect decide what the players should do in the second-to-last period $T - 2$ if we number the periods

$t = 0, \ldots, T-1$. This is the last time an interaction takes place, because in the last round the players choose their myopic optimal price.

Consider one of the players (low-cost or high-cost) and let their current demand potential be d, the price they are setting p, their cost to be c, and the opponent's price be p'. The current period is $T-2$ and the last period is $T-1$.

In the current period, the player's profit is $(d-p)(p-c) = (d+c)p - p^2 - dc$. The derivative $\frac{d}{dp}$ of this is $d+c-2p$, which is zero for $p = (d+c)/2$ (the myopically optimal price), with profit $(d-c)^2/4$. In the last period, the player's demand potential has changed to $d + \frac{p'-p}{2}$ according to (1). Consequently, their last-period optimal profit will be $(d - c + \frac{p'-p}{2})^2/4$ or

$$(d-c)^2/4 + (d-c)(p'-p)/4 + (p'^2 - 2p'p + p^2)/16\,.$$

The derivative $\frac{d}{dp}$ of the sum over of these two terms over the two periods is

$$d + c - 2p - \frac{d-c}{4} - \frac{p'}{8} + \frac{p}{8} \;=\; \frac{3d}{4} + \frac{5c}{4} - \frac{p'}{8} - \frac{15p}{8} \qquad (2) \quad \boxed{\texttt{twop}}$$

which when set to zero gives

$$p = \frac{6}{15}d + \frac{10}{15}c - \frac{1}{15}p'\,, \qquad (3) \quad \boxed{\texttt{0}}$$

so p is an affine combination (like convex but with some negative coefficients) of d, c, p'. As a sanity check, if both players choose the same price ($p = p'$, which is normally not the case) in (2), then $p = \frac{3}{8}d + \frac{5}{8}c$, which means that rather than choosing the monopoly price $\frac{1}{2}d + \frac{1}{2}c$, the player should move $\frac{1}{4}$ of the distance from $\frac{d+c}{2}$ to c towards the cost c, which is quite a bit of "shading" from the monopoly price. For example, if $d = 207$ and $c = 57$ (the considered player has low cost), then $p = 113.25$, which is 18.75 below the monopoly price of 132. If $d = 193$ and $c = 71$ (the considered player has high cost), then $p = 116.75$, which is 15.25 below the monopoly price of 132 (and the two prices are not the same with these demand potentials).

The crucial observation in (3) is that a *lower* (more aggressive) opponent price p' induces a *higher* (less aggressive) best-response price p and vice versa (although the dependence is not strong), which defines the two price choices as *strategic substitutes* (rather than complements). This is therefore akin to the standard linear *Cournot* model of quantity competition. In this scenario, both players would prefer to be leader (first mover) over simultaneous action over being follower (second mover).

References

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