Bisection Method

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Theorem An equation $f(x)=0$, where $f(x)$ is a real continuous function, has at least one root between x_0 and x_1 if $f(x_0)\, f(x_1) < 0$.

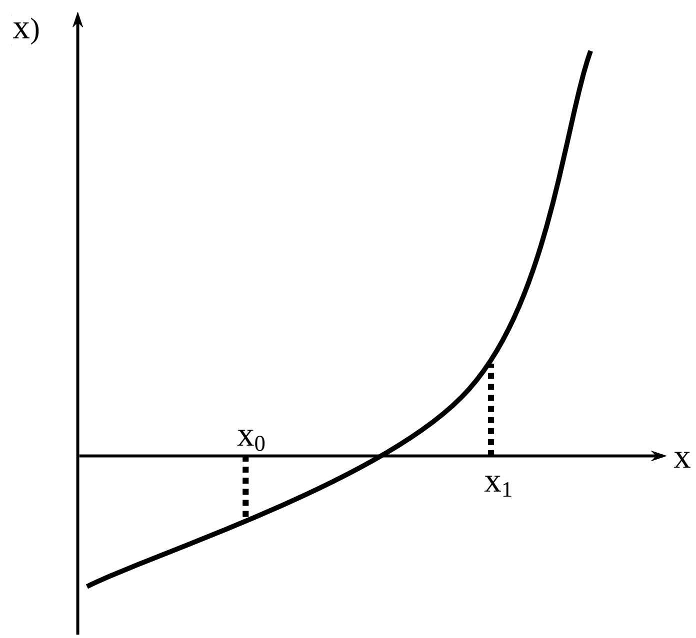


Figure 1 At least one root exists between the two points if the function is real, continuous, and changes sign.

Bisection Method

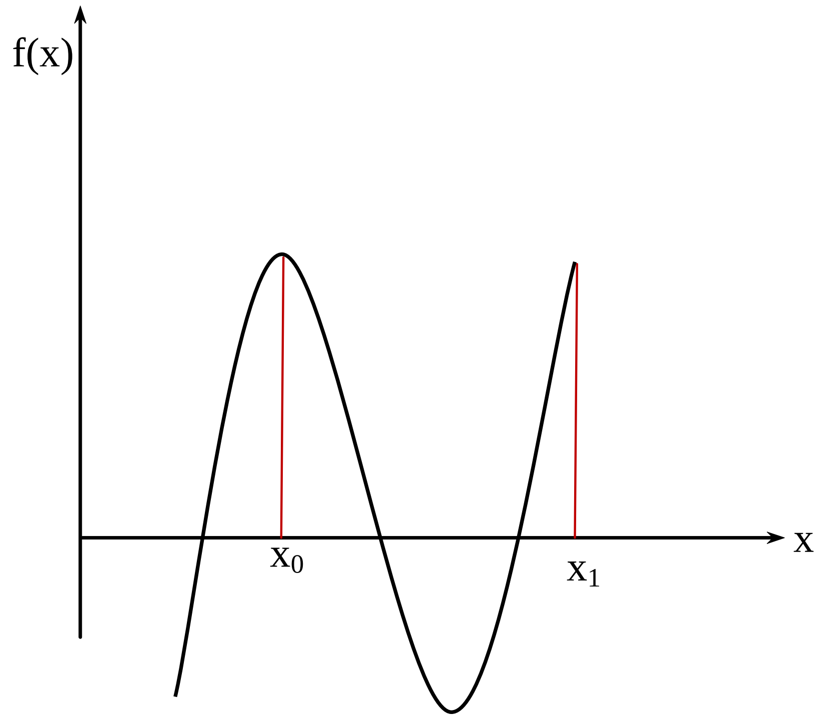


Figure 2 If function $f(x)$ does not change sign between two points, roots of the equation $f(x)=0$ may still exist between the two points.

Bisection Method

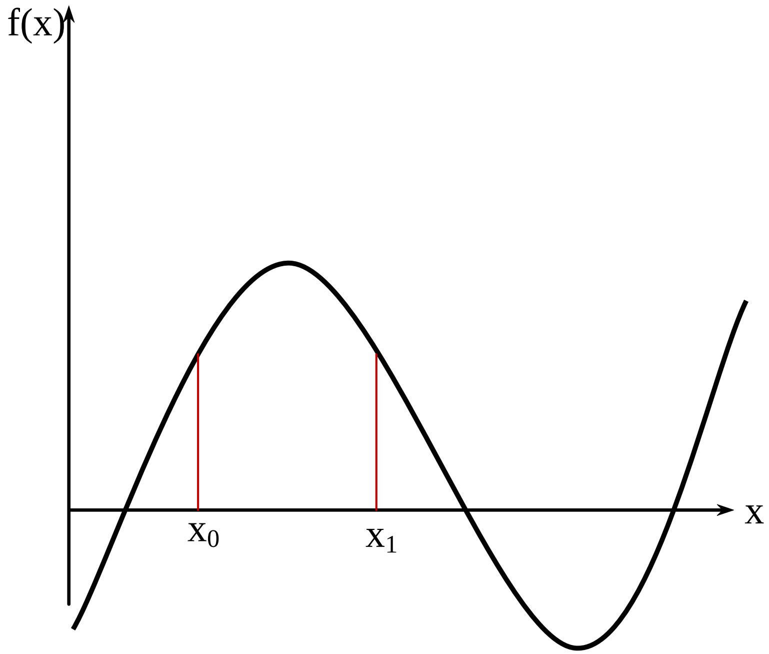


Figure 3 If the function $f(x)$ does not change sign between two points, there may not be any roots for the equation $f(x)=0$ between the two points.

Mathematical Steps for Bisection Method

Step 1

Choose x_0 and x_1 as two initial guesses for the root such that $f(x_0)\, f(x_1) < 0$, or in other words, $f(x)$ changes sign between x_0 and x_1 .

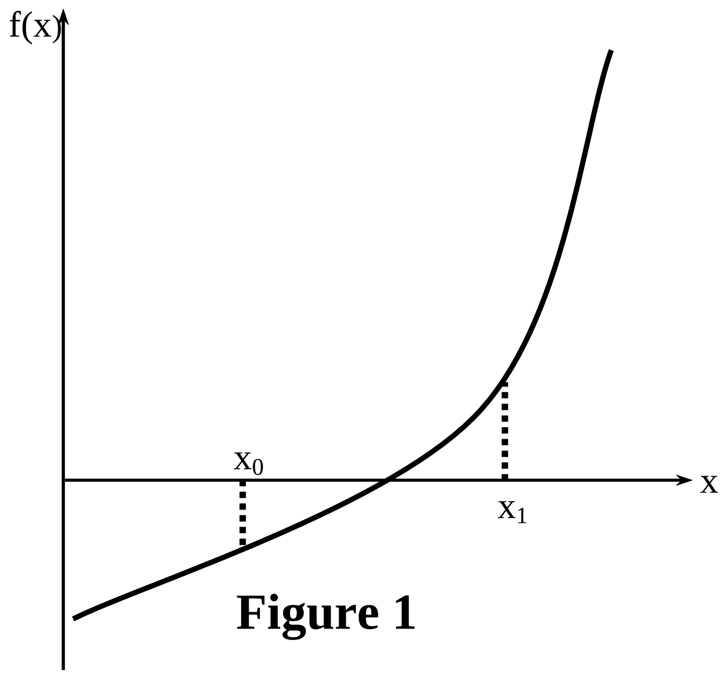
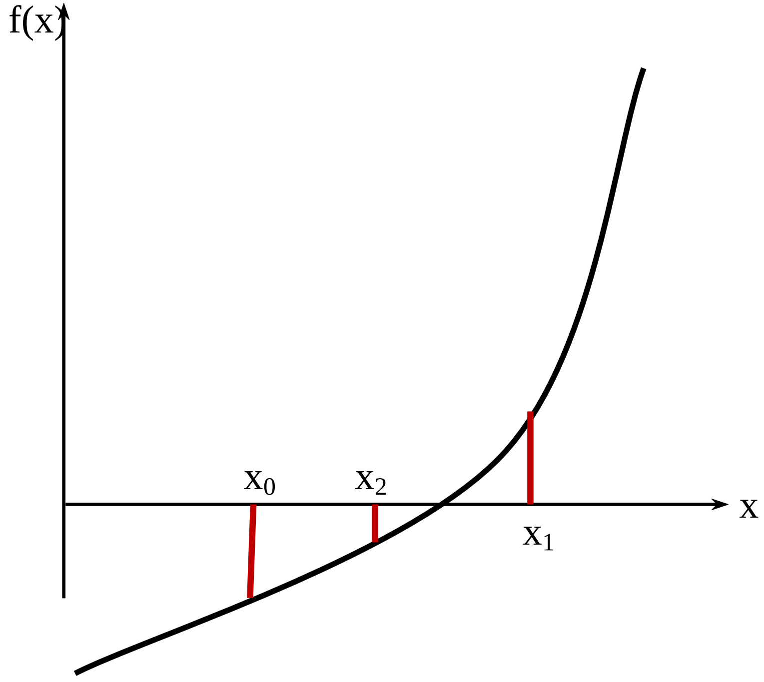


Figure 1

Step 2

Estimate the root, x_2 of the equation f (x) = 0 as the mid point between x_0 and x_1 as

$$X_2 = \frac{X_0 + X_1}{2}$$



Step 3

Now check the following

a) If , then the root lies between x_0 and x_2; then $x_0 = x_0$; $x_1 = x_2$.

b) If , then the root lies between x_2 and x_1; then $x_0 = x_2$; $x_1 = x_1$.

c) If , then the root is $x_{2.}$ Stop the algorithm if this is true.

Step 4

Find the new estimate of the root

$$X_2 = \frac{X_0 + X_1}{2}$$

Find the absolute relative approximate error

$$\left|\in_a\right| = \left|\frac{x_2^{new} - x_2^{old}}{x_2^{new}}\right| \times 100$$

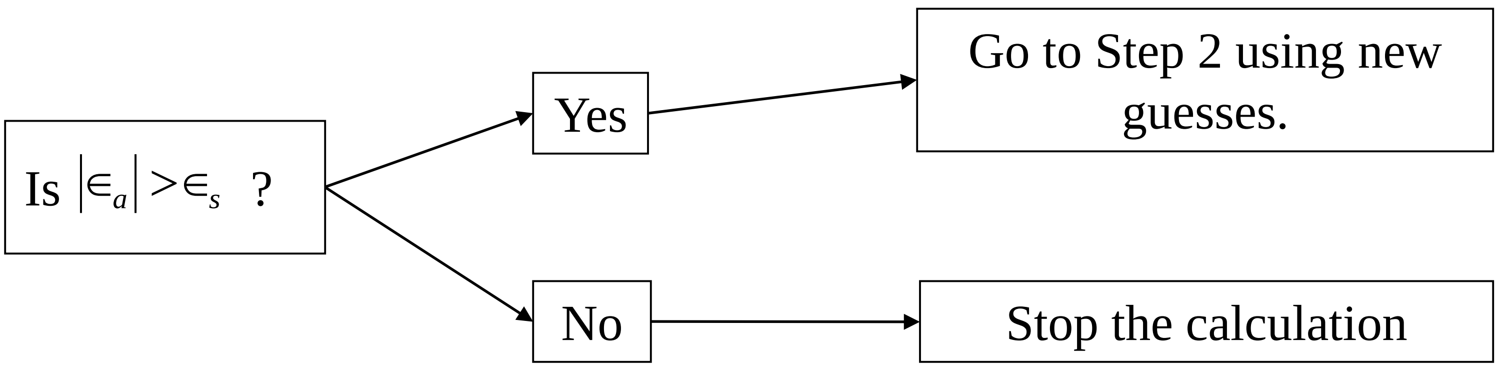
where

$$x_2^{old} = \text{previous estimate of root}$$

$$x_2^{new} = \text{current estimate of root}$$

Step 5

Compare the absolute relative approximate error $|\in_a|$ with the pre-specified error tolerance $\in_s$.



Check whether the number of iterations is more than the maximum number of iterations allowed.

Bi-section Method – Example

Solve, $f(x) = 2^x - 3x$ in the interval [0,2] and assume ε=0.01

0	2	0	1	-1
2	0.5	0	0.25	0.43920712
4	0.5	0.375	0.4375	0.04175555
6	0.46875	0.4375	0.453125	0.00962742
8	0.4609375	0.453125	0.45703125	0.00162042

Problems: Bi-section Method

1. $x^3 - 4x - 10 = 0 \qquad :[0,3] \qquad (Root: 2.762696)$

Advantages

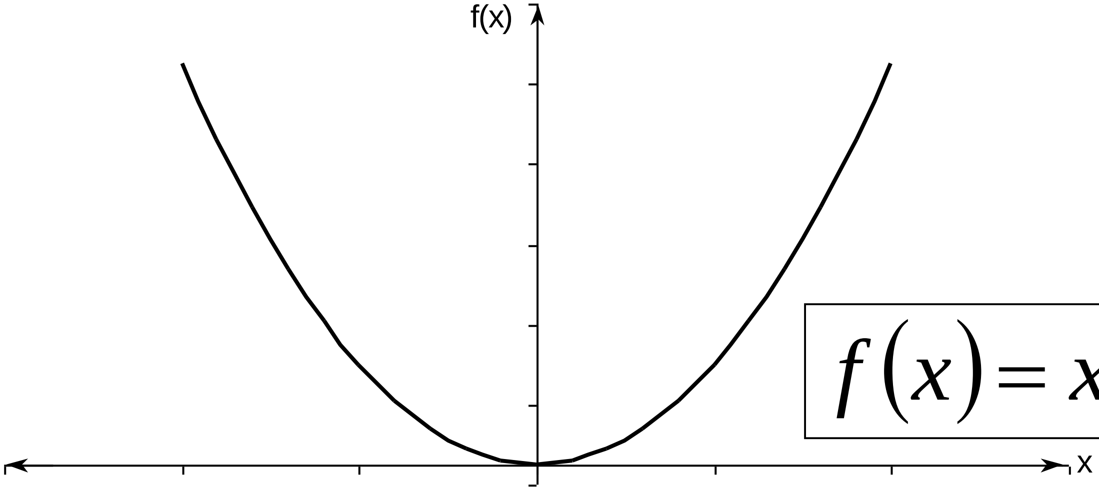
- Always convergent
- The root bracket gets halved with each iteration .

Drawbacks

- Slow convergence
- If one of the initial guesses is close to the root, the convergence is slower

Drawbacks (continued)

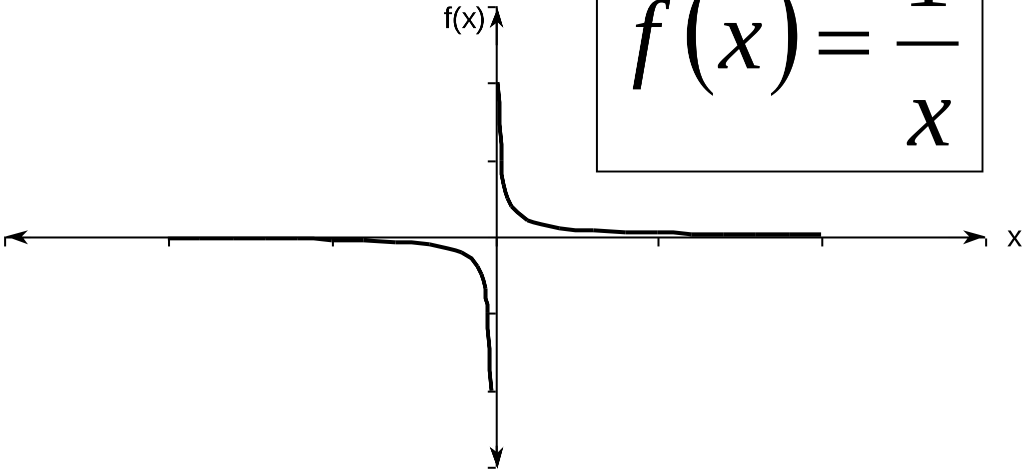
- If a function f(x) is such that it just touches the x-axis it will be unable to find the lower and upper guesses.



$$f(x) = x^2$$

Drawbacks (continued)

- Function changes sign but root does not exist



$$f(x) = \frac{1}{x}$$

THE END