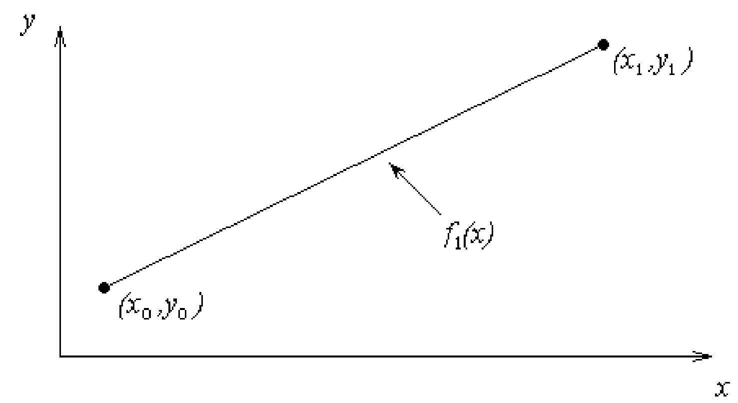
Newton’s Divided Difference Method of Interpolation

Newton's Divided Difference Method

<u>Linear interpolation</u>: Given $(x_0, y_0), (x_1, y_1),$ pass a linear interpolant through the data

$$f_1(x) = b_0 + b_1(x - x_0)$$



where

$$b_0 = f(x_0)$$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

Quadratic Interpolation

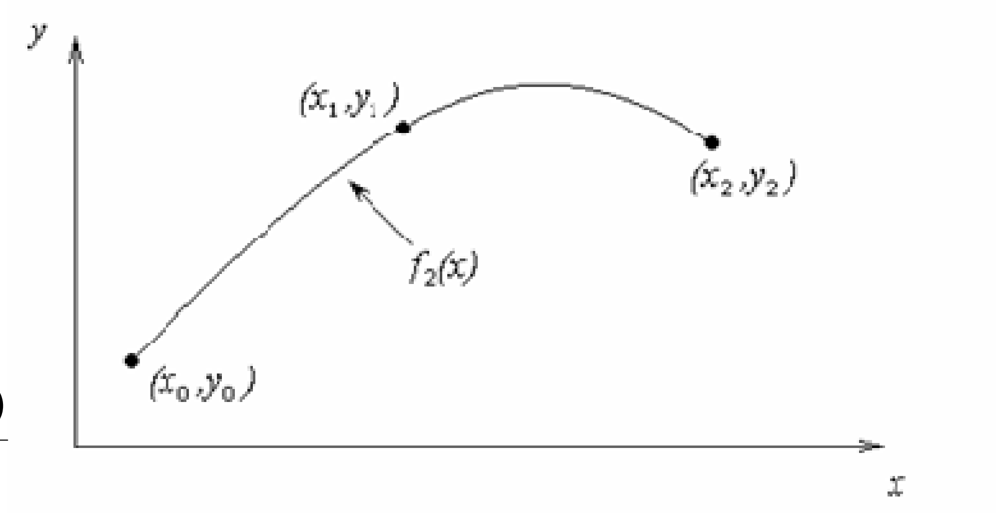
Given (x_0, y_0), (x_1, y_1), and (x_2, y_2), fit a quadratic interpolant through the data.

$$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$b_0 = f(x_0)$$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$b_2 = \frac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$



General Form

$$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

where

$$b_0 = f[x_0] = f(x_0)$$

$$b_1 = f[x_1, x_0] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$b_2 = f[x_2, x_1, x_0] = \frac{f[x_2, x_1] - f[x_1, x_0]}{x_2 - x_0} = \frac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

Rewriting

$$f_2(x) = f[x_0] + f[x_1, x_0](x - x_0) + f[x_2, x_1, x_0](x - x_0)(x - x_1)$$

General Form

Given $(n+1)$ data points, $(x_0, y_0), (x_1, y_1), \ldots\ldots, (x_{n-1}, y_{n-1}), (x_n, y_n)$ as

$$f_n(x) = b_0 + b_1(x - x_0) + \ldots. + b_n(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

where

$$b_0 = f[x_0]$$

$$b_1 = f[x_1, x_0]$$

$$b_2 = f[x_2, x_1, x_0]$$

$$\vdots$$

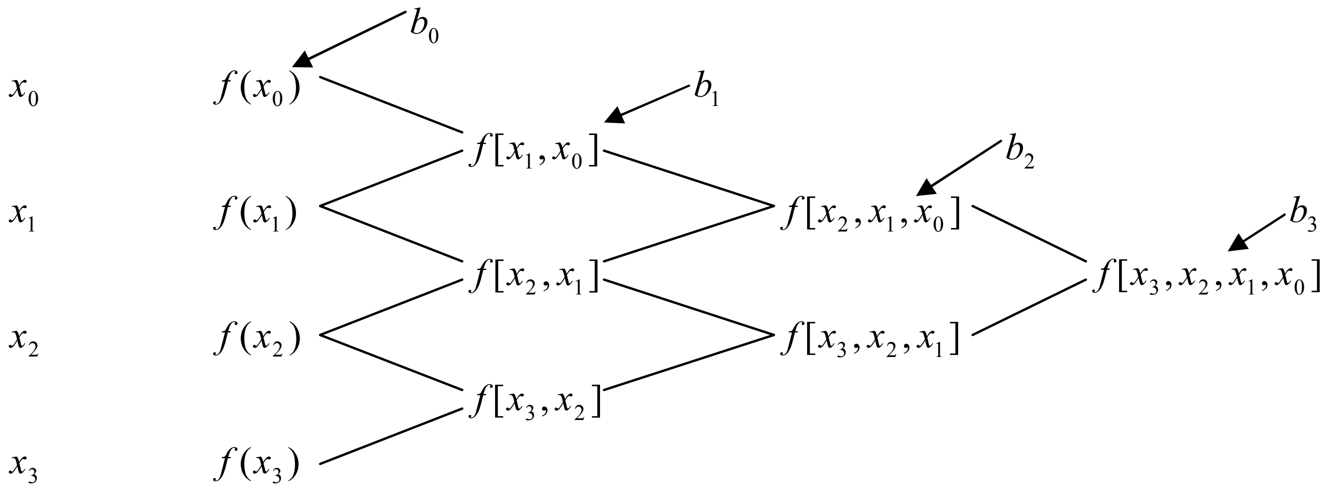
$$b_{n-1} = f[x_{n-1}, x_{n-2}, \ldots., x_0]$$

$$b_n = f[x_n, x_{n-1}, \ldots., x_0]$$

General form

The third order polynomial, given (x_0, y_0), (x_1, y_1), (x_2, y_2), and (x_3, y_3), is

$$f_3(x) = f[x_0] + f[x_1, x_0](x - x_0) + f[x_2, x_1, x_0](x - x_0)(x - x_1)$$
$$+ f[x_3, x_2, x_1, x_0](x - x_0)(x - x_1)(x - x_2)$$



Example 1

Using the following table find $f(x)$ as a polynomial in x

x	$f(x)$
-1	3
0	-6
3	39
6	822
7	1611

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–1	3
0	– 6
3	39
6	822
7	1611

The divided difference table is

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x	$f(x)$
-1	3
0	-6
3	39
6	822
7	1611

The divided difference table is

x	$f(x)$	$f[x_k, x_{k+1}]$			
-1	3				
		-9			
0	-6		6		
		15		5	
3	39		41		1
		261		13	
6	822		132		
		789			
7	1611				

Example 1

x	$f(x)$	$f[x_k, x_{k+1}]$			
-1	3				
		-9			
0	-6		6		
		15		5	
3	39		41		1
		261		13	
6	822		132		
		789			
7	1611				

$$f(x) = 3 + (x+1)(-9) + x(x+1)(6) + x(x+1)(x-3)(5) + x(x+1)(x-3)(x-6)$$

Example 1

x	$f(x)$	$f[x_k, x_{k+1}]$			
-1	3				
		-9			
0	-6		6		
		15		5	
3	39		41		1
		261		13	
6	822		132		
		789			
7	1611				

$$f(x) = 3 + (x+1)(-9) + x(x+1)(6) + x(x+1)(x-3)(5) + x(x+1)(x-3)(x-6)$$
$$= x^4 - 3x^3 + 5x^2 - 6.$$

Example 2

Newton's divided difference interpolating polynomial satisfying the following values:

x:	1	3	4	5	7	10
f(x):	3	31	69	131	351	1011

Also find f(4.5), and f(8)

Example 2

x:	1	3	4	5	7	10
f(x):	3	31	69	131	351	1011

X	First divided differences	Second divided differences
1		
	14	
3		**8**
	38	
4		12
	62	
5		16
	110	
7		22
	220	
9		

Example 2

x:	1	3	4	5	7	10
f(x):	3	31	69	131	351	1011

X	First divided differences	Second divided differences
1		
	14	
3		**8**
	38	
4		12
	62	
5		16
	110	
7		22
	220	
9		

Example 2

x:	1	3	4	5	7	10
f(x):	3	31	69	131	351	1011

X	First divided differences	Second divided differences	Third divided differences	Fourth divided differences
1				
	14			
3		**8**		
	38		**1**	
4		12		0
	62		1	
5		16		0
	110		1	
7		22		
	220			
9 10				

Example 2

Since the fourth divided differences are zeroes, f(x) is of degree 3 and it is obtained as,

$$f(x)=f(x_0)+(x-x_0)f(x_0,x_1)+(x-x_0)(x-x_1)f(x_0,x_1,x_2)$$
$$+(x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_3)$$

$$f(x_0)=f(1)=3;\ f(x_0,x_1)=14;\ f(x_0,x_1,x_2)=8 \quad and \quad f(x_0,x_1,x_2,x_3)=1$$

$$f(x)=3+(x-1)\times 14+(x-1)(x-3)\times 8+(x-1)(x-3)(x-4)\times 1$$

$$f(x)=x^3+x+1$$

Hence, $f(4.5)=(4.5)^3+4.5+1=96.625$ and $f(8)=(8)^3+8+1=521$

Newton Forward difference Polynomial

$$(x_0, f_0), (x_1, f_1), (x_2, f_2) \dots (x_n, f_n)$$

where x values are equally spaced as $\boldsymbol{x_i = x_0 + ih}$

$i = 0,1,2, \dots n$

n^{th} degree polynomial may be written as

$$\begin{aligned} P_n(x) = a_0 + a_1(x - x_0) \\ + a_2(x - x_0)(x - x_1) + \cdots \\ + a_n\{(x - x_0)(x - x_1)(x - x_2) \dots (x \\ - x_{n-1})\} \end{aligned}$$

Newton Forward difference Polynomial

$$P_n(x_0) = f_0 = a_0 \Longrightarrow a_0 = f_0$$

Newton Forward difference Polynomial

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$$P_n(x_1) = f_1 = a_0 + a_1(x - x_0)$$

Newton Forward difference Polynomial

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$$P_n(x_1) = f_1 = a_0 + a_1(x - x_0)$$

$$a_1 = \frac{f_1 - f_0}{x_1 - x_0} = \frac{\nabla f_0}{h}$$

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$$P_n(x_1) = f_1 = a_0 + a_1(x - x_0)$$

$$a_1 = \frac{f_1 - f_0}{x_1 - x_0} = \frac{\nabla f_0}{h}$$

$$a_2 = \frac{\nabla^2 f_0}{h^2 2!}, \dots \dots \; , a_n = \frac{\nabla^n f_0}{h^n n!}$$

Newton Forward difference Polynomial

‘s’ be an interpolating variable, where

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$$x = x_0 + sh \quad \Longrightarrow \; s = \frac{x - x_0}{h} \quad \Longrightarrow (x - x_0) = sh$$

Newton Forward difference Polynomial

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$$x = x_0 + sh \implies s = \frac{x - x_0}{h} \implies (x - x_0) = sh$$

$$x_1 = x_0 + h = x - sh + h$$

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$$(x - x_1) = h(s - 1) \quad a_1(x - x_0) = \frac{\nabla f_0}{h} \cdot sh = s\nabla f_0$$

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$$(x - x_1) = h(s - 1) \quad a_1(x - x_0) = \frac{\nabla f_0}{h} \cdot sh = s\nabla f_0$$

$$a_2(x - x_0)(x - x_1) = \frac{\nabla^2 f_0}{h^2 2!} \cdot sh \cdot h(s - 1) = s(s - 1)\frac{\nabla^2 f_0}{2!}$$

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$$a_n(x - x_0)(x - x_1) \ldots (x - x_{n-1})$$

$$= \frac{s(s - 1)(s - 2) \ldots (s - n + 1)}{n!} \nabla^n f_0$$

Newton Forward difference Polynomial

$$a_n(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$
$$= \frac{s(s-1)(s-2)\ldots(s-n+1)}{n!}\nabla^n f_0$$

n^{th} degree polynomial

$$P_n(x)$$
$$= f_0 + s\nabla f_0 + \frac{s(s-1)}{2!}\nabla^2 f_0 + \cdots$$
$$+ \frac{s(s-1)(s-2)\ldots(s-n+1)}{n!}\nabla^n f_0$$

Newton Backward difference Polynomial

$(x_0, f_0), (x_1, f_1), (x_2, f_2) \dots (x_n, f_n)$

where x values are equally spaced as $\boldsymbol{x_i = x_0 + ih}$

$i = 0,1,2, \dots n$

n^{th} degree polynomial may be written as

$$
\begin{aligned}
P_n(x) = a_0 + a_1(x - x_0) \\
+ a_2(x - x_0)(x - x_{-1}) + \cdots \\
+ a_n\{(x - x_0)(x - x_{-1})(x \\
- x_{-2}) \dots (x - x_{-(n-1)})\}
\end{aligned}
$$

Newton Backward difference Polynomial

$x = x_0 \qquad P_b(x_0) = f_0 = a_0$

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$$x = x_{-1} \qquad P_b(x_{-1}) = f_{-1} = a_0 + a_1\overbrace{(x_{-1} - x_0)}^{-h}$$

Newton Backward difference Polynomial

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$$ha_1 = a_0 - f_{-1} = f_0 - f_{-1}$$

Newton Backward difference Polynomial

$$x = x_0 \qquad P_b(x_0) = f_0 = a_0$$

$-h$

$$x = x_{-1} \qquad P_b(x_{-1}) = f_{-1} = a_0 + a_1(x_{-1} - x_0)$$

$$h a_1 = a_0 - f_{-1} = f_0 - f_{-1}$$

$$a_1 = \frac{f_0 - f_{-1}}{h} = \frac{\nabla f_0}{h} = a_1$$

Newton Backward difference Polynomial

$x = x_0$ $\quad\quad$ $P_b(x_0) = f_0 = a_0$

$x = x_{-1}$ $\quad\quad$ $P_b(x_{-1}) = f_{-1} = a_0 + a_1\overbrace{(x_{-1} - x_0)}^{-h}$

$$ha_1 = a_0 - f_{-1} = f_0 - f_{-1}$$

$$a_1 = \frac{f_0 - f_{-1}}{h} = \frac{\nabla f_0}{h} = a_1$$

$x = x_{-2}$

$$\begin{aligned} P_b(x_{-2}) &= f_{-2} \\ &= a_0 + a_1(x_{-2} - x_0) \\ &\quad + a_2(x_{-2} - x_0)(x_{-2} - x_{-1}) \end{aligned}$$

Newton Backward difference Polynomial

$$
\begin{aligned}
P_b(x_{-2}) &= f_{-2} \\
&= a_0 + a_1(x_{-2} - x_0) \\
&+ a_2(x_{-2} - x_0)(x_{-2} - x_{-1})
\end{aligned}
$$

Newton Backward difference Polynomial

$$P_b(x_{-2}) = f_{-2}$$
$$= a_0 + a_1(x_{-2} - x_0)$$
$$+ a_2(x_{-2} - x_0)(x_{-2} - x_{-1})$$

$$= f_0 + \frac{f_0 - f_{-1}}{h}(-2h) + a_2(-2h)(-h)$$

Newton Backward difference Polynomial

$$P_b(x_{-2}) = f_{-2}$$
$$= a_0 + a_1(x_{-2} - x_0)$$
$$+ a_2(x_{-2} - x_0)(x_{-2} - x_{-1})$$

$$= f_0 + \frac{f_0 - f_{-1}}{h}(-2h) + a_2(-2h)(-h)$$

$$f_{-2} = f_0 - 2f_0 + 2f_{-1} + a_2 2h^2$$

Newton Backward difference Polynomial

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\begin{aligned}
P_b(x_{-2}) = f_{-2} \\
&= a_0 + a_1(x_{-2} - x_0) \\
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\end{aligned}
$$

$$= f_0 + \frac{f_0 - f_{-1}}{h}(-2h) + a_2(-2h)(-h)$$

$$f_{-2} = f_0 - 2f_0 + 2f_{-1} + a_2 2h^2$$

$$
\begin{aligned}
a_2 2h^2 &= f_{-2} + f_0 - 2f_{-1} \\
&= (f_0 - f_{-1}) - (f_{-1} - f_{-2})
\end{aligned}
$$

Newton Backward difference Polynomial

$$\begin{aligned} P_b(x_{-2}) = f_{-2} \\ &= a_0 + a_1(x_{-2} - x_0) \\ &+ a_2(x_{-2} - x_0)(x_{-2} - x_{-1}) \end{aligned}$$

$$= f_0 + \frac{f_0 - f_{-1}}{h}(-2h) + a_2(-2h)(-h)$$

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$$\begin{aligned} a_2 2h^2 &= f_{-2} + f_0 - 2f_{-1} \\ &= (f_0 - f_{-1}) - (f_{-1} - f_{-2}) \end{aligned}$$

$$a_2 = \frac{\nabla f_0 - \nabla f_{-1}}{2h^2} = \frac{\nabla^2 f_0}{h^2 2!}$$

Newton Backward difference Polynomial

$$P_b(x_{-2}) = f_{-2}$$
$$= a_0 + a_1(x_{-2} - x_0)$$
$$+ a_2(x_{-2} - x_0)(x_{-2} - x_{-1})$$

$$= f_0 + \frac{f_0 - f_{-1}}{h}(-2h) + a_2(-2h)(-h)$$

$$f_{-2} = f_0 - 2f_0 + 2f_{-1} + a_2 2h^2$$

$$a_2 2h^2 = f_{-2} + f_0 - 2f_{-1}$$
$$= (f_0 - f_{-1}) - (f_{-1} - f_{-2})$$

$$a_2 = \frac{\nabla f_0 - \nabla f_{-1}}{2h^2} = \frac{\nabla^2 f_0}{h^2 2!} \qquad a_n = \frac{\nabla^2 f_{-n}}{h^n n!}$$

Newton Backward difference Polynomial

‘s’ be an interpolating variable, where

$$x = x_0 + sh \quad \Longrightarrow \; s = \frac{x - x_0}{h}$$

$$\Longrightarrow (x - x_0) = sh$$

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$$a_2(x - x_0)(x - x_1) = \frac{\nabla^2 f_0}{h^2 2!} \cdot sh \cdot h(s+1) \; = s(s+1)\frac{\nabla^2 f_0}{2!}$$

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$$a_2(x - x_0)(x - x_1) = \frac{\nabla^2 f_0}{h^2 2!} \cdot sh \cdot h(s + 1) \; = s(s + 1)\frac{\nabla^2 f_0}{2!}$$

$$a_n\{(x - x_0)(x - x_1) \ldots (x - x_{n-1})\}$$

$$= s(s + 1) \ldots (s + n - 1)\frac{\nabla^n f_0}{n!}$$

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's' be an interpolating variable, where

$$x = x_0 + sh \quad \Longrightarrow \; s = \frac{x - x_0}{h}$$

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$$a_n\{(x - x_0)(x - x_1) \ldots (x - x_{n-1})\}$$

$$= s(s+1) \ldots (s+n-1)\frac{\nabla^n f_0}{n!}$$

,

Newton Backward difference Polynomial

$$P_b(x) = f_0 + s\nabla f_0 + \frac{s(s+1)}{2!}\nabla^2 f_0 + \cdots + \frac{s(s+1)\ldots(s+n-1)}{n!}\frac{\nabla^n f_0}{n!}$$

The population of a town in the decimal census was as given below. Estimate the population for the year 1895.

Year x:	*1891*	*1901*	*1911*	*1921*	*1931*
Population y: (in thousands)	*46*	*66*	*81*	*93*	*101*

$$P_n(x) = f_0 + s\nabla f_0 + \frac{s(s-1)}{2!}\nabla^2 f_0 + \cdots + \frac{s(s-1)(s-2)\ldots(s-n+1)}{n!}\nabla^n f_0$$

$$s = \frac{1895 - 1891}{10} = 0.4$$

x	y	
1891	$\boxed{46}$	
1901	66	
1911	81	
1921	93	
1931	101	

$$s = \frac{1895 - 1891}{10} = 0.4$$

x	y	Δy
1891	$\boxed{46}$	$\boxed{20}$
1901	66	15
1911	81	12
1921	93	8
1931	101	

$$s = \frac{1895 - 1891}{10} = 0.4$$

x	y	Δy	$\Delta^2 y$
1891	$\boxed{46}$		
		$\boxed{20}$	
1901	66		$\boxed{-5}$
		15	
1911	81		-3
		12	
1921	93		-4
		8	
1931	101		

$$s = \frac{1895 - 1891}{10} = 0.4$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1891	$\boxed{46}$				
		$\boxed{20}$			
1901	66		$\boxed{-5}$		
		15		$\boxed{2}$	
1911	81		-3		$\boxed{-3}$
		12		-1	
1921	93		-4		
		8			
1931	101				

$$P_n(x) = f_0 + s\nabla f_0 + \frac{s(s-1)}{2!}\nabla^2 f_0 + \cdots + \frac{s(s-1)(s-2)\dots(s-n+1)}{n!}\nabla^n f_0$$

$$= 46 + (.4)(20) + \frac{(.4)(.4-1)}{2}(-5)$$

$$+ \frac{(.4)(.4-1)(.4-2)}{6}(2) + \frac{(.4)(.4-1)(.4-2)(.4-3)}{24}(-3)$$

$$= 54.8528 \text{ thousands}$$

The population of a town was as given. Estimate the population for the year 1925.

Year (x):	*1891*	*1901*	*1911*	*1921*	*1931*
Population (y):	*46*	*66*	*81*	*93*	*101*

(in thousands)

$$P_b(x) = f_0 + s\nabla f_0 + \frac{s(s+1)}{2!}\nabla^2 f_0 + \cdots + \frac{s(s+1)\ldots(s+n-1)}{n!}\frac{\nabla^n f_0}{n!}$$

$$s = \frac{1925 - 1931}{10} = -0.6$$

x	y
1891	46
1901	66
1911	81
1921	93
1931	$\boxed{101}$

$$s = \frac{1925 - 1931}{10} = -0.6$$

x	y	∇y
1891	46	
		20
1901	66	
		15
1911	81	
		12
1921	93	
		$\boxed{8}$
1931	$\boxed{101}$	

$$s = \frac{1925 - 1931}{10} = -0.6$$

x	y	∇y	$\nabla^2 y$
1891	46		
		20	
1901	66		-5
		15	
1911	81		-3
		12	
1921	93		$\boxed{-4}$
		$\boxed{8}$	
1931	$\boxed{101}$		

$$s = \frac{1925 - 1931}{10} = -0.6$$

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		$\boxed{-3}$
		12		$\boxed{-1}$	
1921	93		$\boxed{-4}$		
		$\boxed{8}$			
1931	$\boxed{101}$				

$$s = \frac{1925 - 1931}{10} = -0.6$$

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		$\boxed{-3}$
		12		$\boxed{-1}$	
1921	93		$\boxed{-4}$		
		$\boxed{8}$			
1931	$\boxed{101}$				

$$P_b(x) = f_0 + s\nabla f_0 + \frac{s(s+1)}{2!}\nabla^2 f_0 + \cdots + \frac{s(s+1)\ldots(s+n-1)}{n!}\,\frac{\nabla^n f_0}{n!}$$

$$= 101 + (-.6)(8) + \frac{(-.6)(.4)}{2!}(-4) + \frac{(-.6)(.4)(1.4)}{3!}(-1)$$

$$+ \frac{(-.6)(.4)(1.4)(2.4)}{4!}(-3)$$

$= 96.8368$ thousands.

Find the unique polynomial P(x) of degree 2 such that:

P(1) = 1, P(3) = 27, P(4) = 64

Use the Lagrange method of interpolation.

Here, $x_0 = 1, \qquad x_1 = 3, \qquad x_2 = 4$

$f(x_0) = 1, \quad f(x_1) = 27, \quad f(x_2) = 64$

$$P(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} f(x_0) + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} f(x_1)$$

$$+ \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} f(x_2)$$

$$= \frac{(x-3)(x-4)}{(1-3)(1-4)}(1) + \frac{(x-1)(x-4)}{(3-1)(3-4)}(27) + \frac{(x-1)(x-3)}{(4-1)(4-3)}(64)$$

$$= \frac{1}{6}(x^2 - 7x + 12) - \frac{27}{2}(x^2 - 5x + 4) + \frac{64}{3}(x^2 - 4x + 3)$$

$$= 8x^2 - 19x + 12$$

Using Newton's divided difference formula, find a polynomial function satisfying the following data:

x:	-4	-1	0	2	5
$f(x)$:	1245	33	5	9	1335

Hence find f(1).

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
-4	$\boxed{1245}$				
		$\boxed{-404}$			
-1	33		$\boxed{94}$		
		-28		$\boxed{-14}$	
0	5		10		$\boxed{3}$
		2		13	
2	9		88		
		442			
5	1335				

$$f_n(x) = b_0 + b_1(x - x_0) + \ldots. + b_n(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

$$f(x) = 1245 + (x + 4)(-404) + (x + 4)(x + 1)\,94$$
$$+ (x + 4)(x + 1)(x - 0)(-14) + (x + 4)(x + 1)x(x - 2)(3)$$
$$= 3x^4 - 5x^3 + 6x^2 - 14x + 5$$

Hence, $f(1) = 3 - 5 + 6 - 14 + 5 = -5$.

THE END