Method of False-Position

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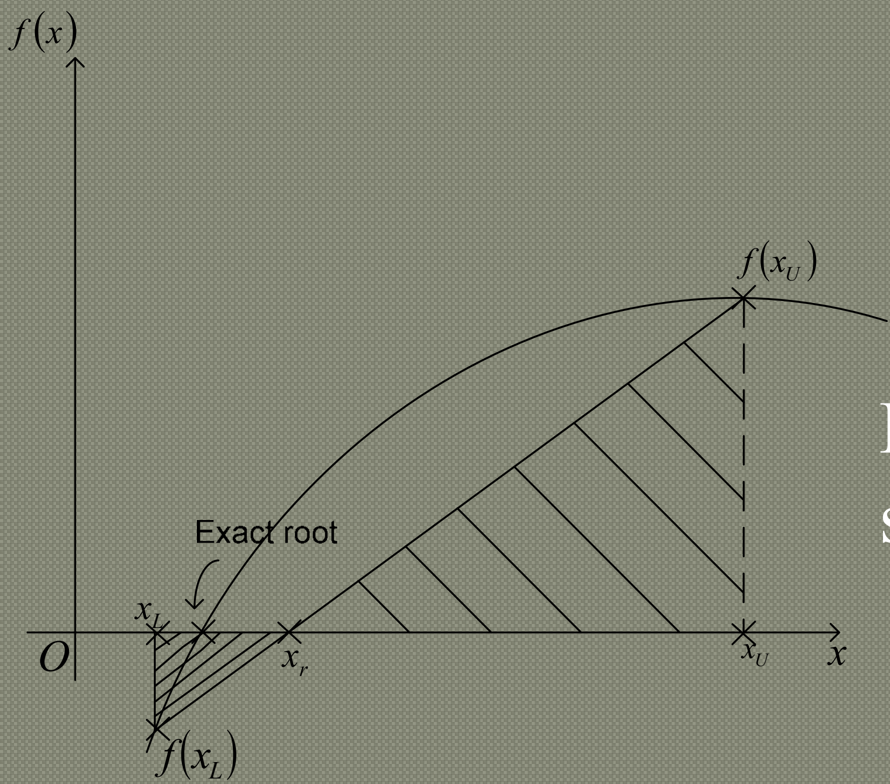


$$f(x) = 0$$

Like, Bisection method

$$f(x_L) * f(x_U) < 0$$

Based on two similar triangles, shown in Figure

$$\frac{f(x_L)}{x_r - x_L} = \frac{f(x_U)}{x_r - x_U} \quad (1)$$

Method of False -Position

From Eq. (1),

$$\left(x_r - x_L\right)f\left(x_U\right) = \left(x_r - x_U\right)f\left(x_L\right)$$

$$x_U f\left(x_L\right) - x_L f\left(x_U\right) = x_r \left\{f\left(x_L\right) - f\left(x_U\right)\right\}$$

The above equation can be solved to obtain the next predicted root

$$x_r = \frac{x_U f\left(x_L\right) - x_L f\left(x_U\right)}{f\left(x_L\right) - f\left(x_U\right)} \qquad (2)$$

Method of False -Position

The Eq. (2) can be written as

$$x_r = x_U - \frac{f(x_U)\{x_L - x_U\}}{f(x_L) - f(x_U)}$$

or

$$x_r = x_L - \frac{f(x_L)}{\left\{\dfrac{f(x_U) - f(x_L)}{x_U - x_L}\right\}}$$

Step-By-Step False-Position Algorithms

1. Choose x_L and x_U as two guesses for the root such that $f(x_L)f(x_U) < 0$

2. Estimate the root, $$x_m = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$$

3. Check the following

(a) If $f(x_L)f(x_m) < 0$, then the root lies between x_L and x_m; then $x_L = x_L$ and $x_U = x_m$

(b) If $f(x_L)f(x_m) > 0$, then the root lies between x_m and x_U; then $x_L = x_m$ and $x_U = x_U$

(c) If $f(x_L)f(x_m)=0$, then the root is x_m.
Stop the algorithm if this is true.

4. Find the new estimate of the root

$$x_m = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$$

Find the absolute relative approximate error as

$$\left|\in_a\right| = \left|\frac{x_m^{new} - x_m^{old}}{x_m^{new}}\right| \times 100$$

where

x_m^{new} = estimated root from present iteration

x_m^{old} = estimated root from previous iteration

5. $say \in_s = 10^{-3} = 0.001$. If $|\in_a| > \in_s$, then go to step 3, else stop the algorithm.

Example 1

$$f(x) = x^2 - 25$$

	Iteration No.	x_0	x_1	x_2	f_0	f_1	f_2
Initialisation		2.0	7.0		-21	24	
$x_0 \leftarrow x_2$	1	4.8	7.0	4.8	-1.96	24	-1.96
$x_1 \leftarrow x_2$	2	4.8	6.587	6.587	-1.96	18.38	18.38
$x_0 \leftarrow x_2$	3	4.9721	6.587	4.9721	-0.2782	18.38	-0.2782
$x_0 \leftarrow x_2$	4	4.9961	6.587	4.9961	-0.039	18.38	-0.039
$x_0 \leftarrow x_2$	5	4.994	6.587	4.994	-0.0053	18.38	-0.0053
$x_0 \leftarrow x_2$	6	4.9998	6.587	4.9998	-0.0019	18.38	-0.0014

Example 2

The floating ball has a specific gravity of 0.6 and has a radius of 5.5cm. Find the depth to which the ball is submerged when floating in water.

The equation that gives the depth x to which the ball is submerged under water is given by

$$x^3 - 0.165x^2 + 3.993 \times 10^{-4} = 0$$

Use the false-position method of finding roots of equations to find the depth x to which the ball is submerged under water. Conduct three iterations to estimate the root of the above equation. Find the absolute relative approximate error at the end of each iteration

Solution

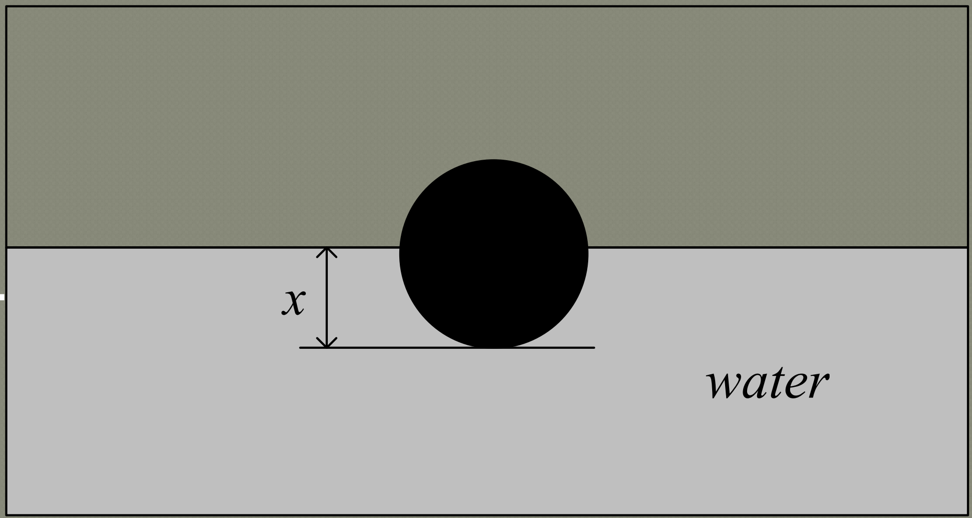
From the physics of the problem

$$0 \le x \le 2R$$

$$0 \le x \le 2(0.055)$$

$$0 \le x \le 0.11$$

Figure 2 : Floating ball problem



Let us assume

$$x_L = 0, x_U = 0.11$$

$$f(x_L) = f(0) = (0)^3 - 0.165(0)^2 + 3.993 \times 10^{-4} = 3.993 \times 10^{-4}$$
$$f(x_U) = f(0.11) = (0.11)^3 - 0.165(0.11)^2 + 3.993 \times 10^{-4} = -2.662 \times 10^{-4}$$

Hence,

$$f(x_L)f(x_U) = f(0)f(0.11) = (3.993 \times 10^{-4})(-2.662 \times 10^{-4}) < 0$$

Iteration 1

$$x_m = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$$

$$= \frac{0.11 \times 3.993 \times 10^{-4} - 0 \times \left(-2.662 \times 10^{-4}\right)}{3.993 \times 10^{-4} - \left(-2.662 \times 10^{-4}\right)}$$

$$= 0.0660$$

$$f(x_m) = f(0.0660) = (0.0660)^3 - 0.165(0.0660)^2 + \left(3.993 \times 10^{-4}\right)$$

$$= -3.1944 \times 10^{-5}$$

$$f(x_L) f(x_m) = f(0) f(0.0660) = (+)(-) < 0$$

$$x_L = 0,\ x_U = 0.0660$$

Iteration 2

$$x_m = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$$

$$= \frac{0.0660 \times 3.993 \times 10^{-4} - 0 \times \left(-3.1944 \times 10^{-5}\right)}{3.993 \times 10^{-4} - \left(-3.1944 \times 10^{-5}\right)}$$

$$= 0.0611$$

$$f(x_m) = f(0.0611) = (0.0611)^3 - 0.165(0.0611)^2 + \left(3.993 \times 10^{-4}\right)$$

$$= 1.1320 \times 10^{-5}$$

$$f(x_L)f(x_m) = f(0)f(0.0611) = (+)(+) > 0$$

Hence, $x_L = 0.0611, x_U = 0.0660$

$$\in_a = \left| \frac{0.0611 - 0.0660}{0.0611} \right| \times 100 \cong 8\%$$

Iteration 3

$$x_m = \frac{x_U f(x_L) - x_L f(x_U)}{f(x_L) - f(x_U)}$$

$$= \frac{0.0660 \times 1.132 \times 10^{-5} - 0.0611 \times \left(-3.1944 \times 10^{-5}\right)}{1.132 \times 10^{-5} - \left(-3.1944 \times 10^{-5}\right)}$$

$$= 0.0624$$

$$f\left(x_m\right) = -1.1313 \times 10^{-7}$$

$$f\left(x_L\right)f\left(x_m\right) = f\left(0.0611\right)f\left(0.0624\right) = \left(+\right)\left(-\right) < 0$$

Hence,

$$x_L = 0.0611,\ x_U = 0.0624$$

$$\in_a = \left|\frac{0.0624 - 0.0611}{0.0624}\right| \times 100 \cong 2.05\%$$

Table 1: Root of $f(x) = x^3 - 0.165x^2 + 3.993\times10^{-4} = 0$ for False-Position Method.

Iteration	x_L	x_U	x_m	$\left\|\in_a\right\|\%$	$f(x_m)$
1	0.0000	0.1100	0.0660	N/A	$-3.1944\text{x}10^{-5}$
2	0.0000	0.0660	0.0611	8.00	$1.1320\text{x}10^{-5}$
3	0.0611	0.0660	0.0624	2.05	$-1.1313\text{x}10^{-7}$
4	0.0611	0.0624	0.0632377619	0.02	$-3.3471\text{x}10^{-10}$