



1

Biologically-inspired approaches to robotics

Embodied artificial intelligence, neurorobotics and biorobotics

Bionics
Biomimetics
Biologically-inspired Robotics
BioRobotics
Embodied Artificial Intelligence
NeuroRobotics

Biologically-inspired robotics

using biological systems as inspiration for robotic design...



Why study animals?

SPRINTING

USAIN BOLT



27.4
MPH

OSTRICH



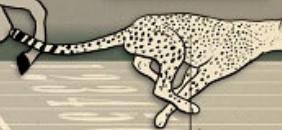
39.5
MPH

HORSE



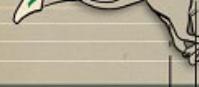
43
MPH

CHEETAH



54.7
MPH

CAT



29.8
MPH



MPH

LONG JUMP



MIKE POWELL

29.4
FEET



SNOW LEOPARD

49
FEET

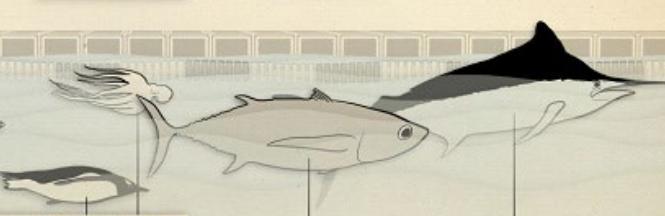
SWIMMING

CESAR CIELO



5.34
MPH

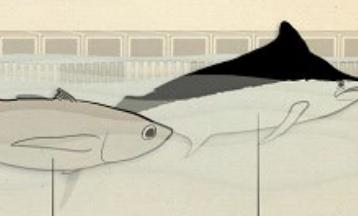
OCTOPUS



22
MPH

25
MPH

TUNA



43
MPH

BLACK MARLIN



80
MPH

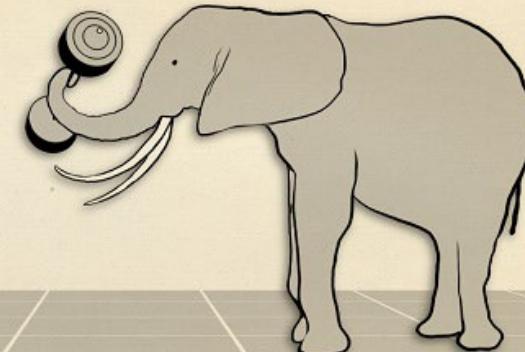
PENGUIN

LIFTING



LEONID TARANENKO

586.4
POUNDS



ELEPHANT

661
POUNDS

Why study animals?

HUMAN VISION



FLY VISION



HUMAN VISION



SHARK VISION



HUMAN VISION



SNAKE VISION



HUMAN VISION



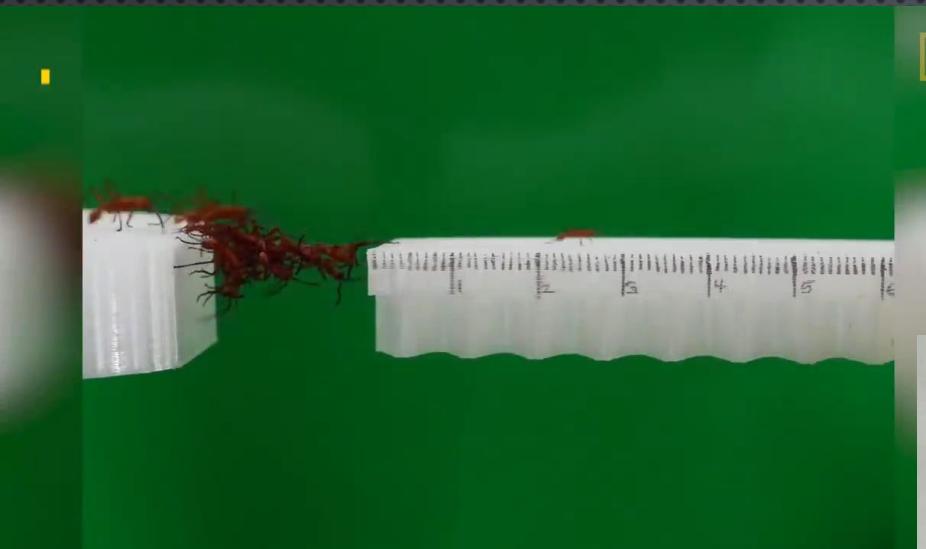
RAT VISION



EYE 1

EYE 2

Why study animals?



Still not convinced? Try doing this...



Or this...



The bigger (and confusing) picture

Embodied artificial intelligence

Biologically-inspired robotics

Body morphology and mechanics

Biomimetic robotics

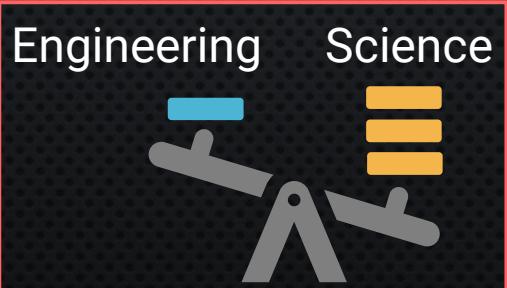
Neurorobotics

Brain computation

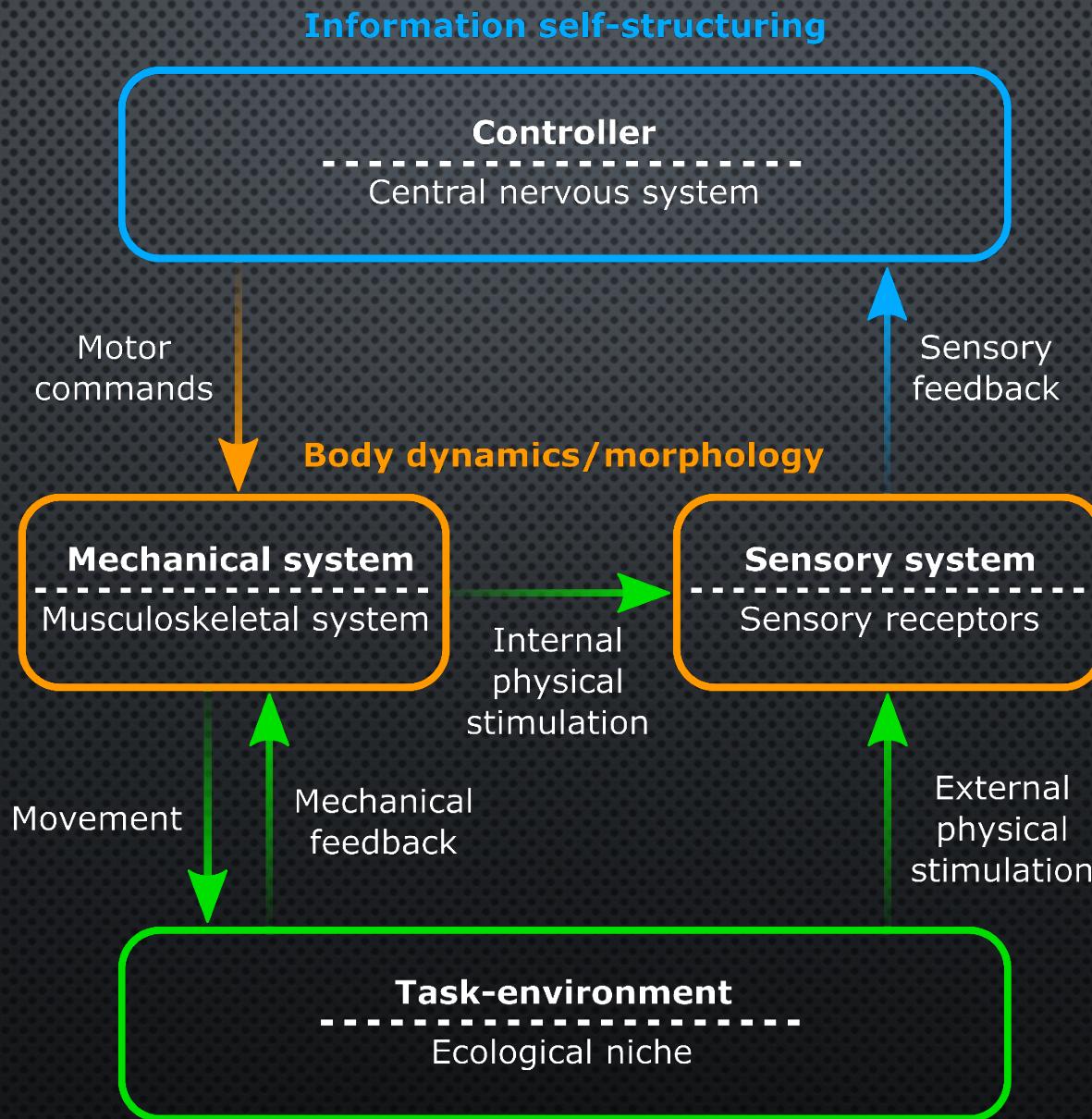
BioRobotics

Bionics

Computational neuroscience



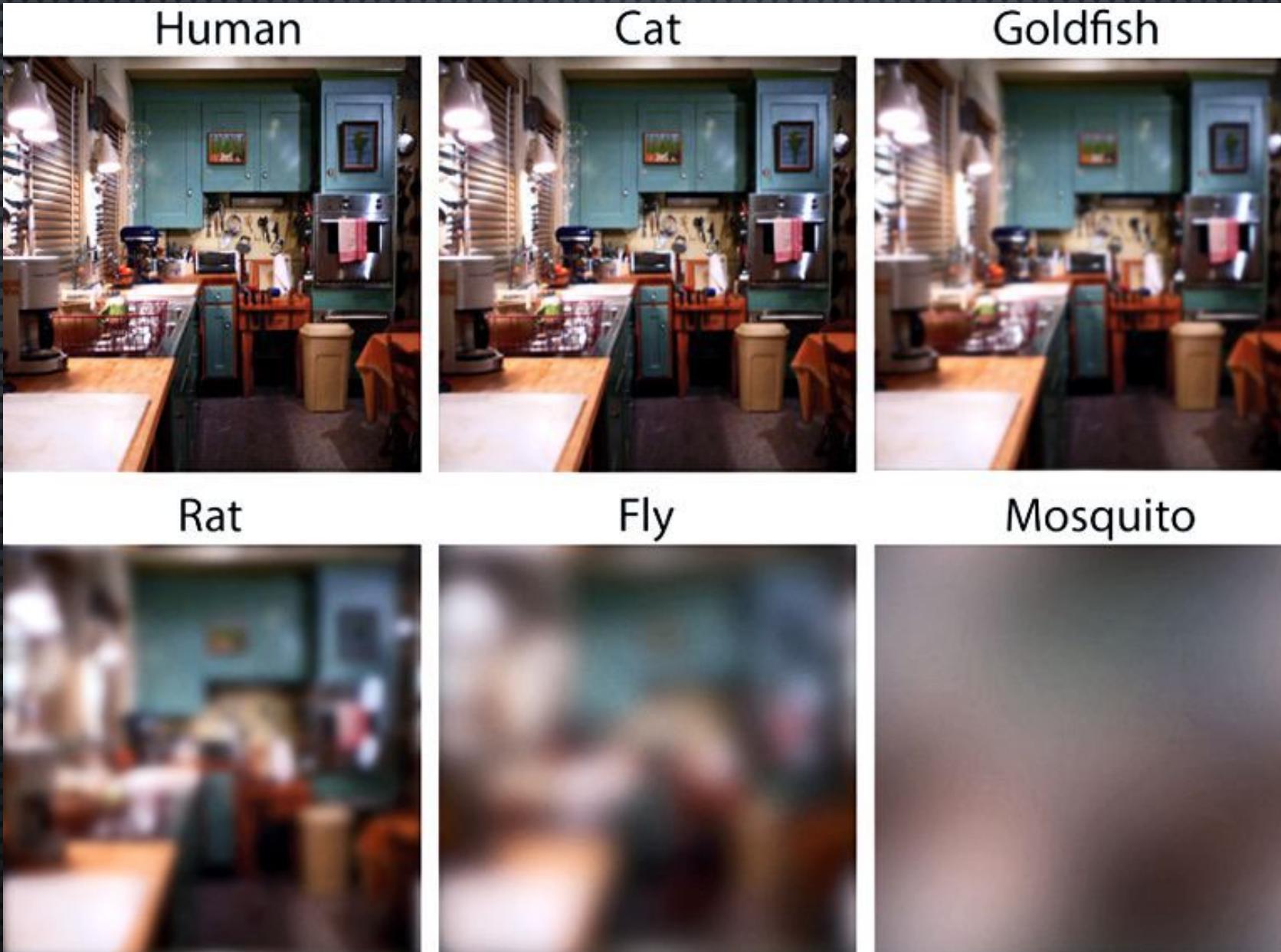
Embodied artificial intelligence



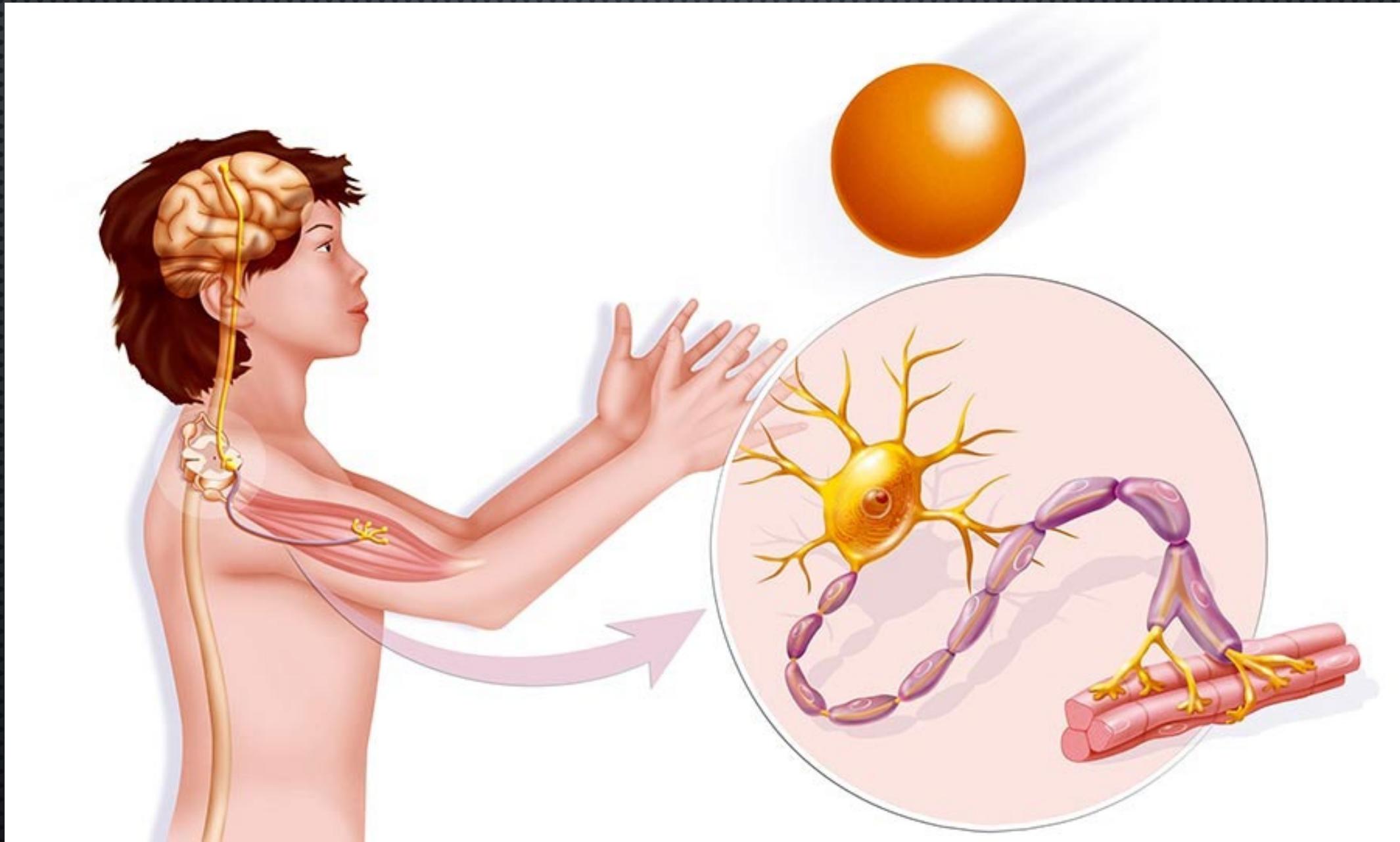
- Ecological niche is the **role a species plays in the ecosystem** and includes
 - the habitat (environment in which it is **situated**)
 - the way it **interacts** with other species as well as the environment via sensing and acting



Why is the environment / ecological niche important?



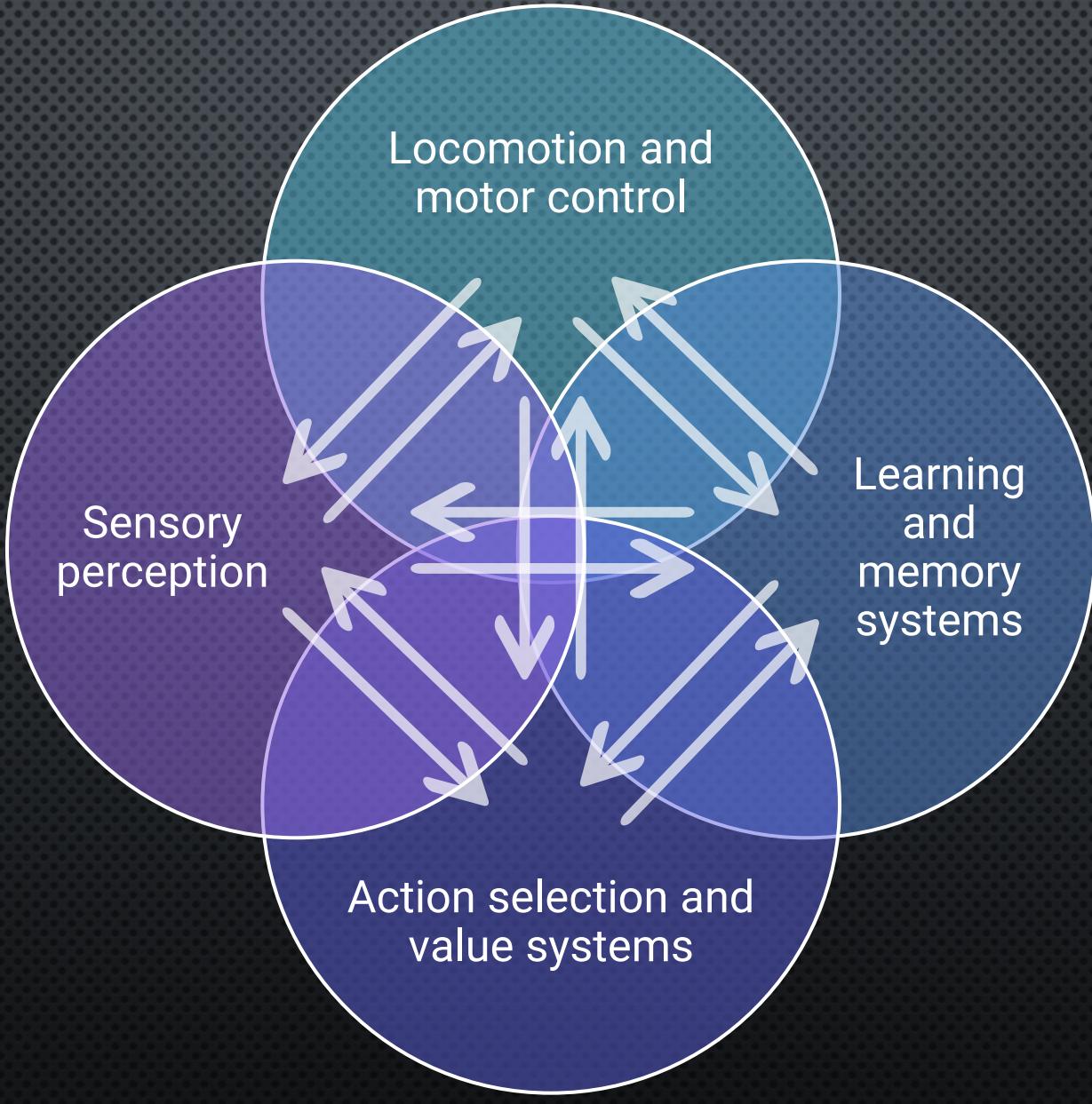
Sensorimotor control and learning: a holistic view of embodied AI



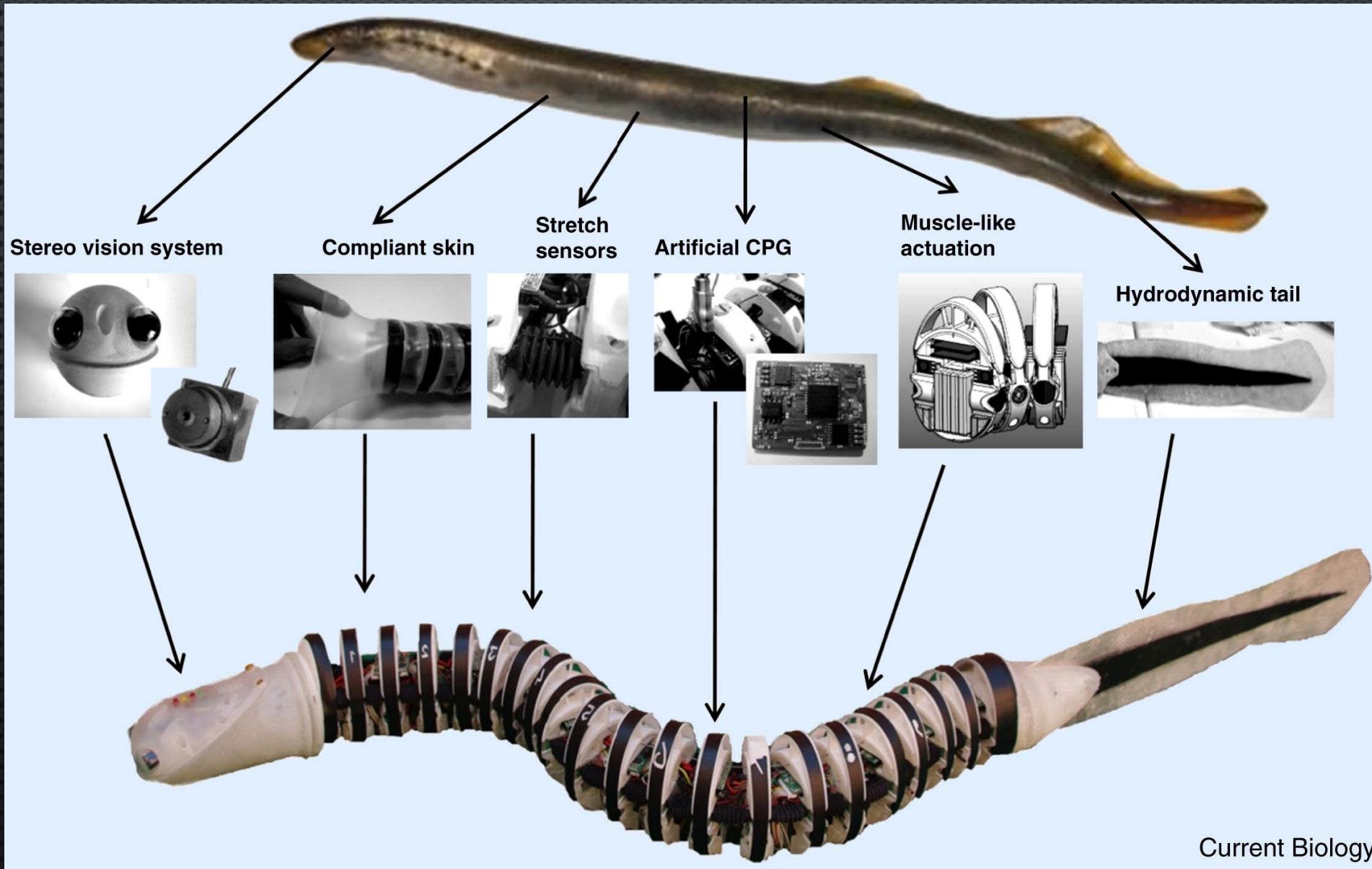
Neurorobotics

- Neurorobotics deals with the study and application of science and technology of **embodied, autonomous, brain-inspired** algorithms
- A neurorobot is a robot
 - that engages in a **behavioural task**
 - that is **situated** in a real-world environment
 - that has a means to **interact** with other agents via sensing and acting
 - whose behaviour is controlled by a simulated nervous system having a design that reflects, at some level, the brain's **architecture** and **dynamics**

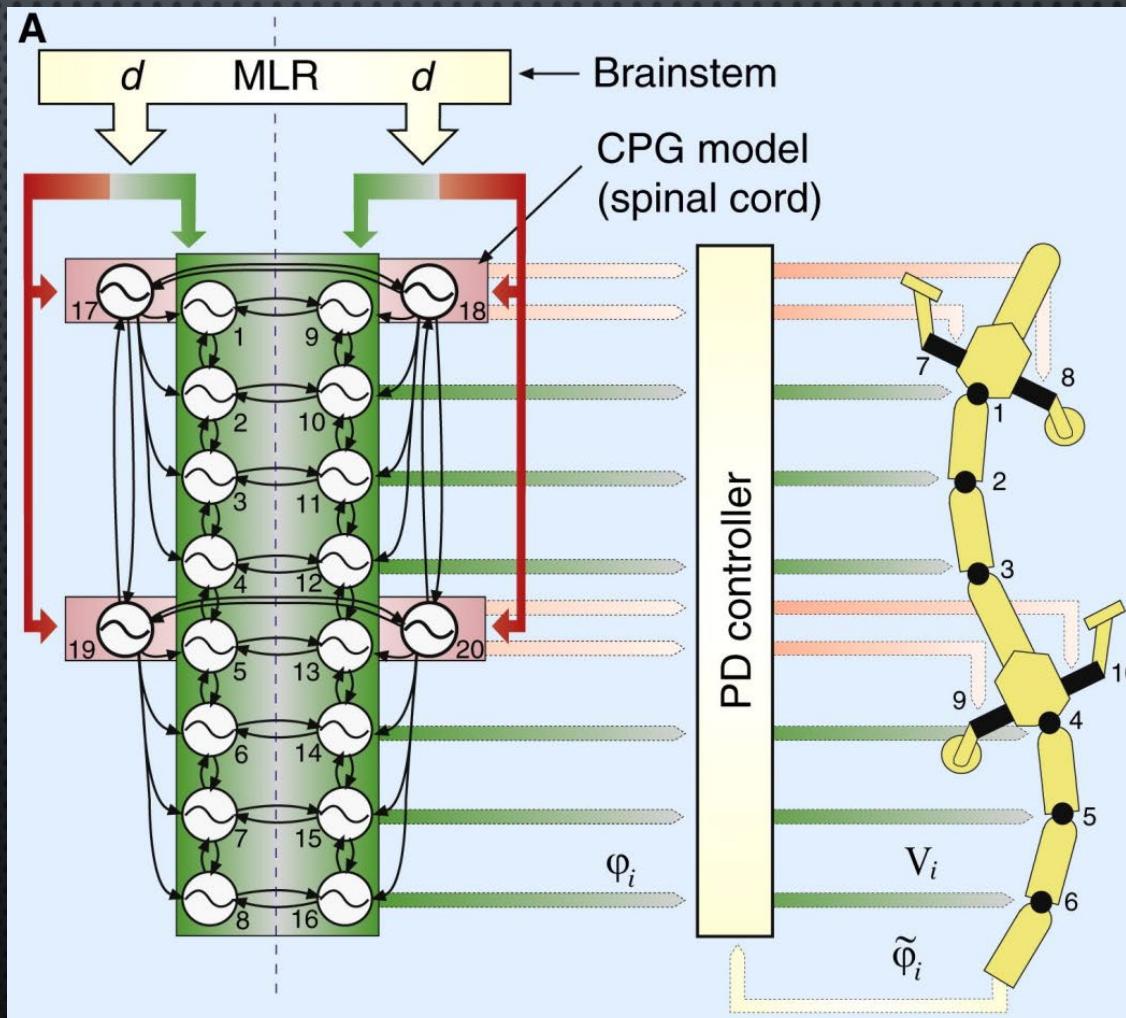
Classes of neurorobot models



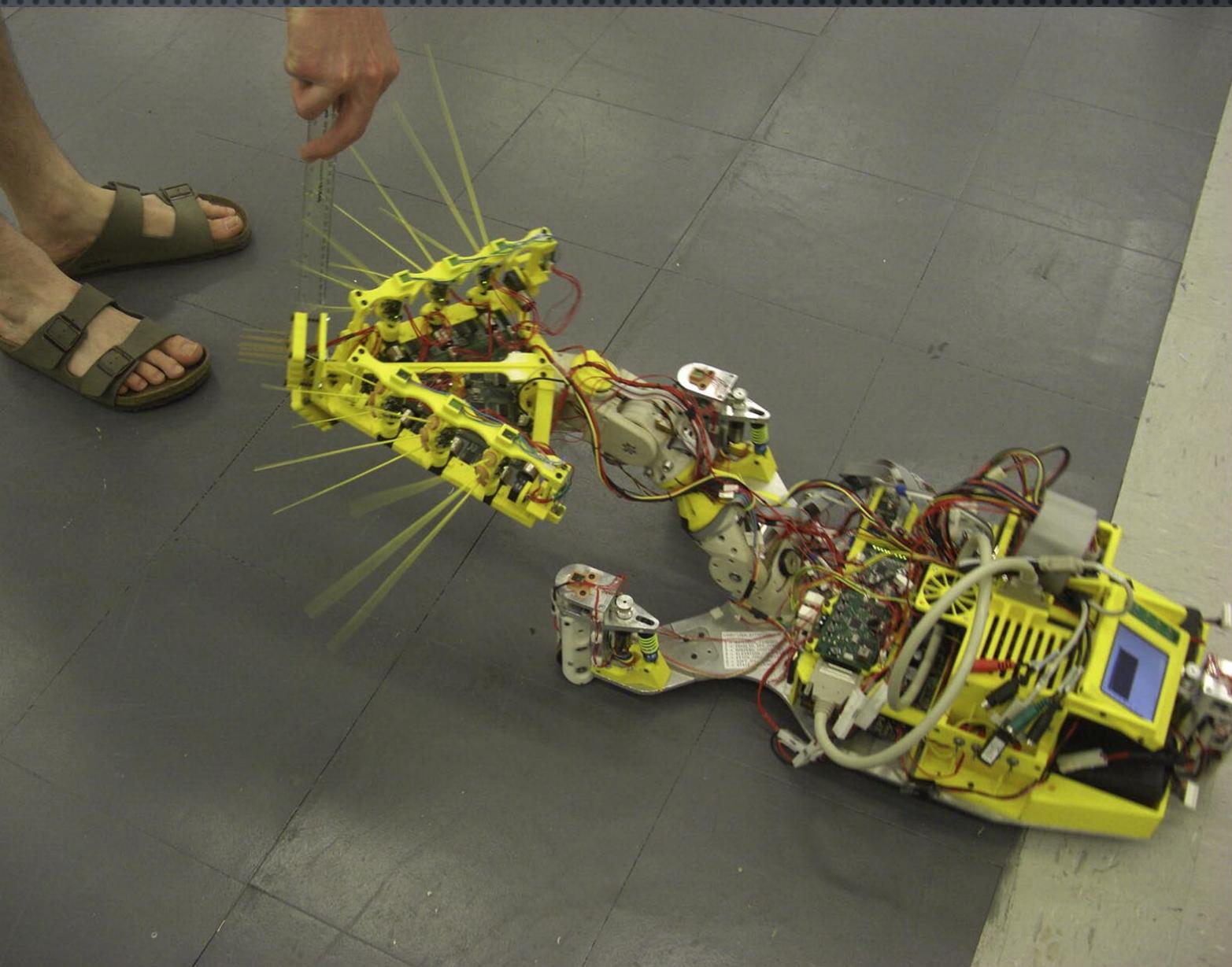
Models for locomotion



Models for locomotion



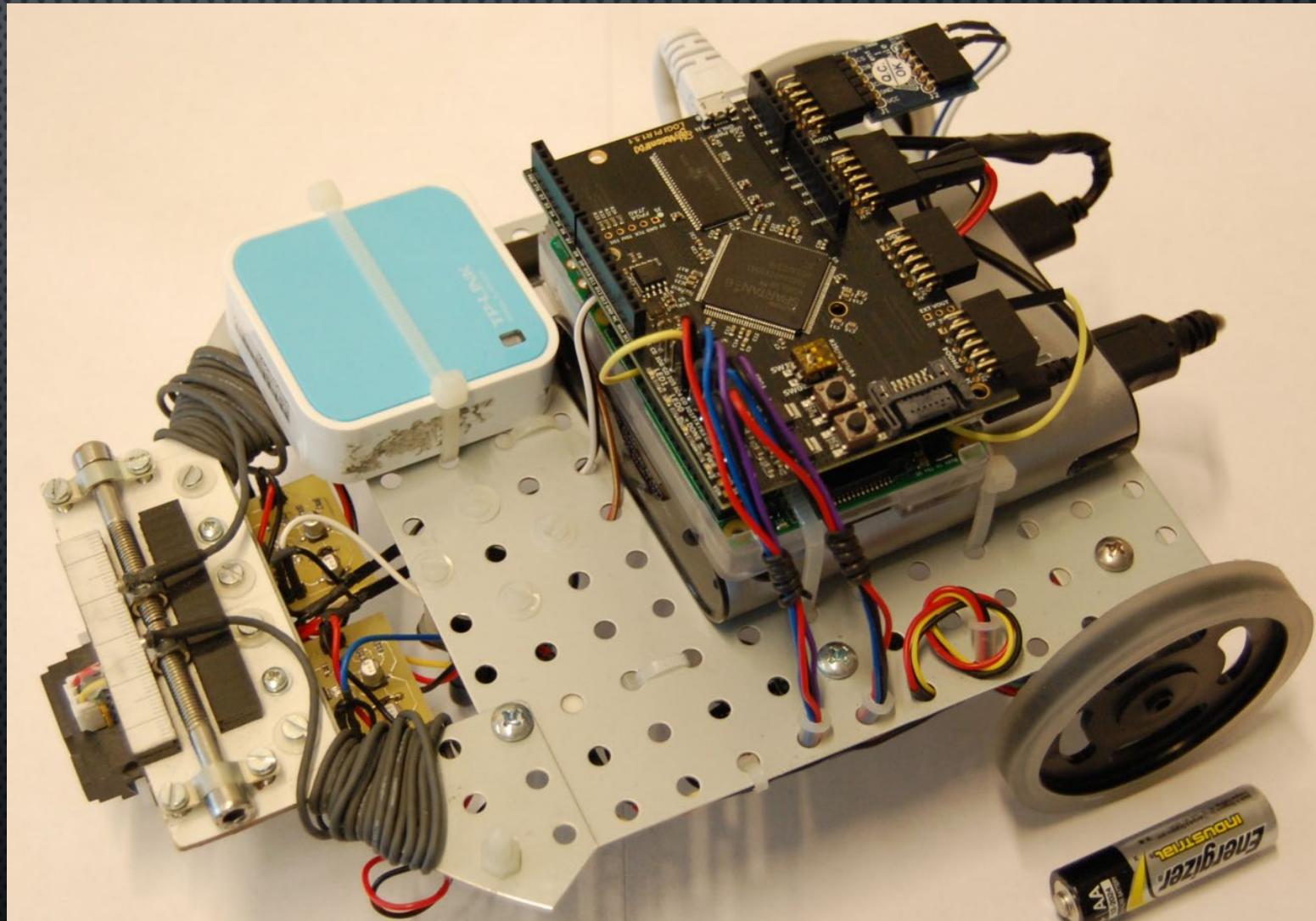
Models for sensorimotor integration



Models for perception

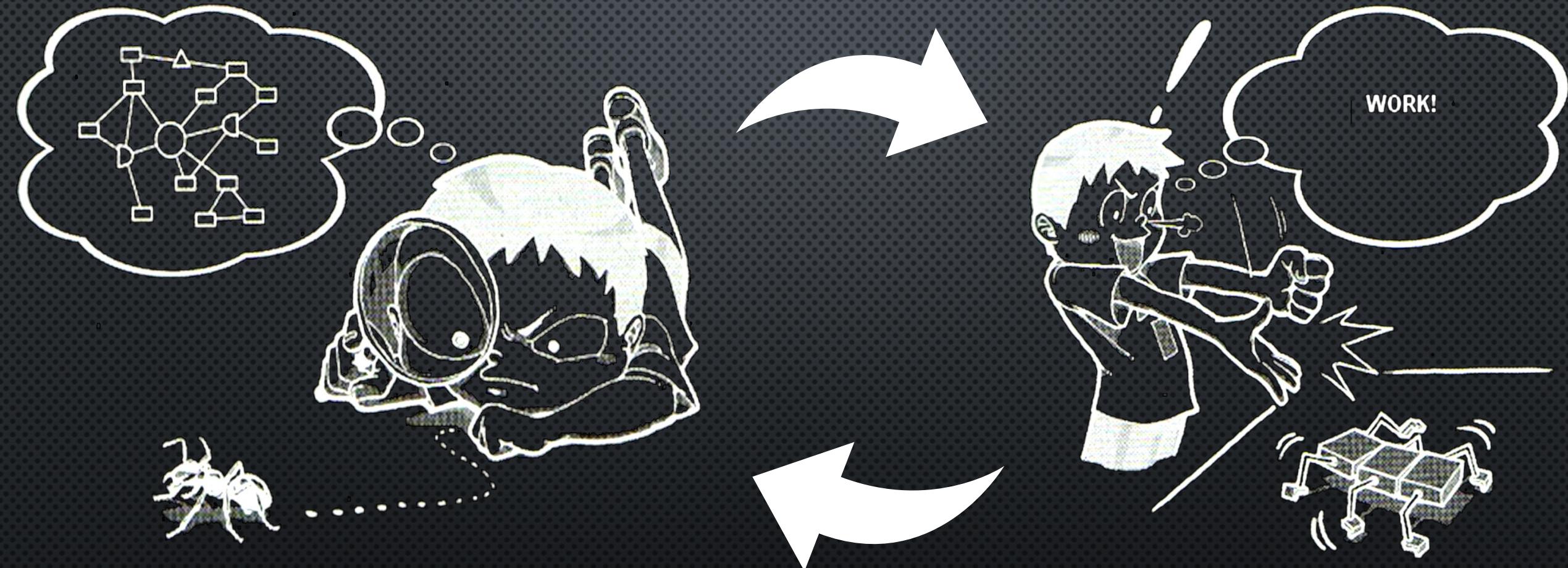


Models for perception



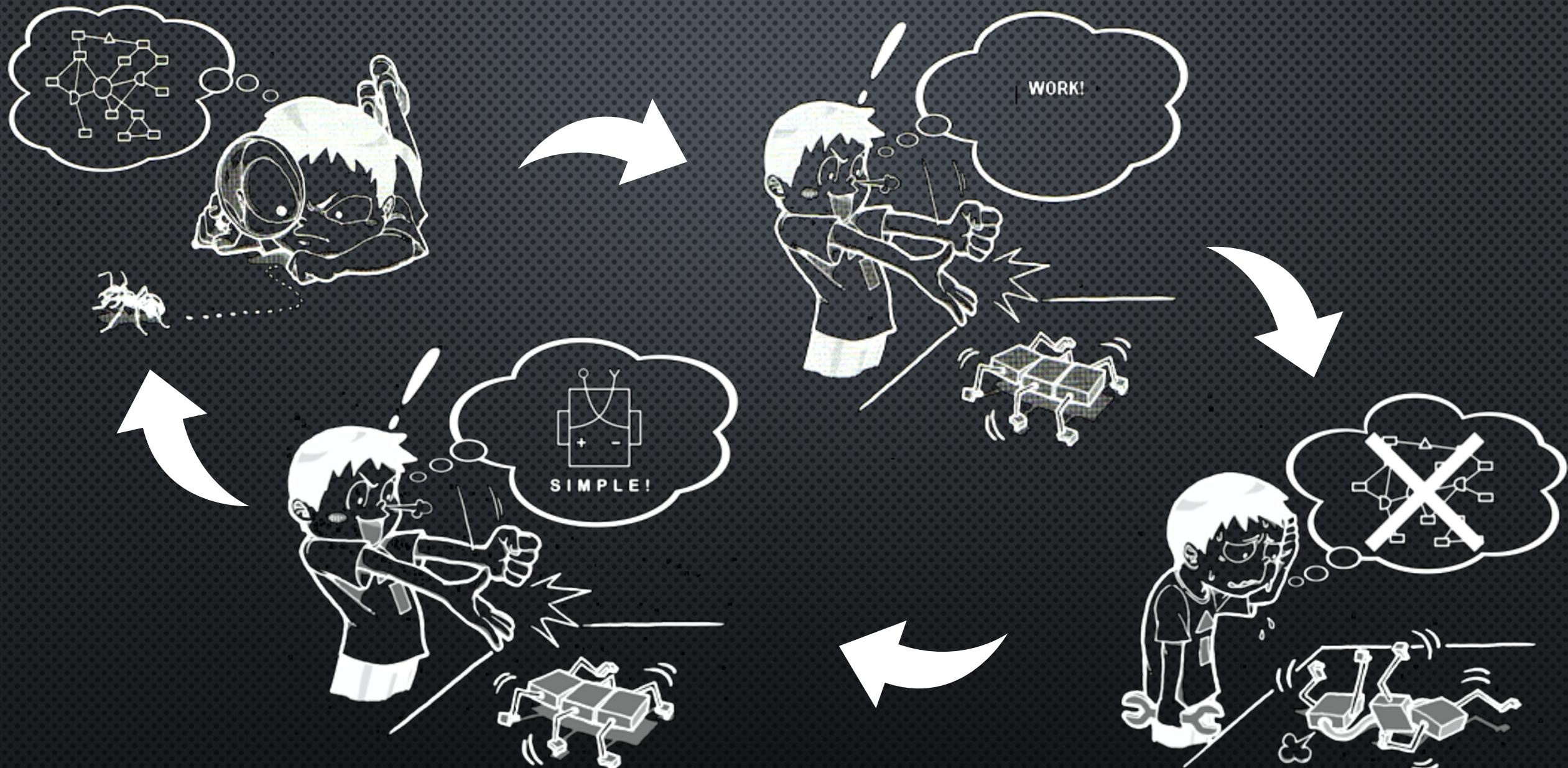
BioRobotics

using biological systems as inspiration for robotic design...



using robotic systems to understand biological design...

Reality check



A world full of model organisms!



World's first biologically-inspired robots

Elmer and Elsie (ELectroMEchanical Robot, Light-Sensitive)



William Grey Walter

Cool robot videos for motivation...



fb.com/ScienceNaturePage

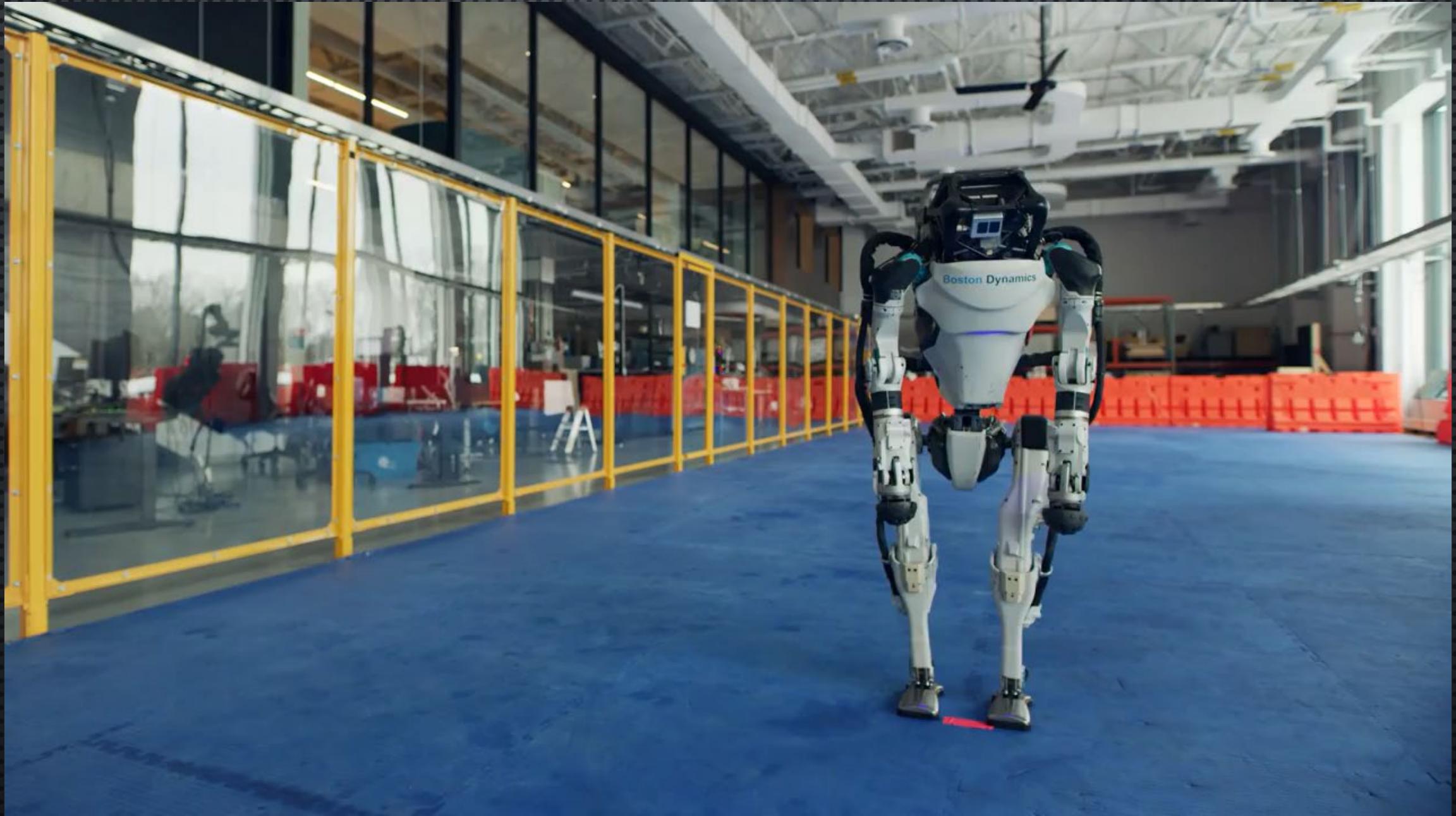


T-800: the ultimate biologically-inspired robot



MASTER_GOW

Dance break...

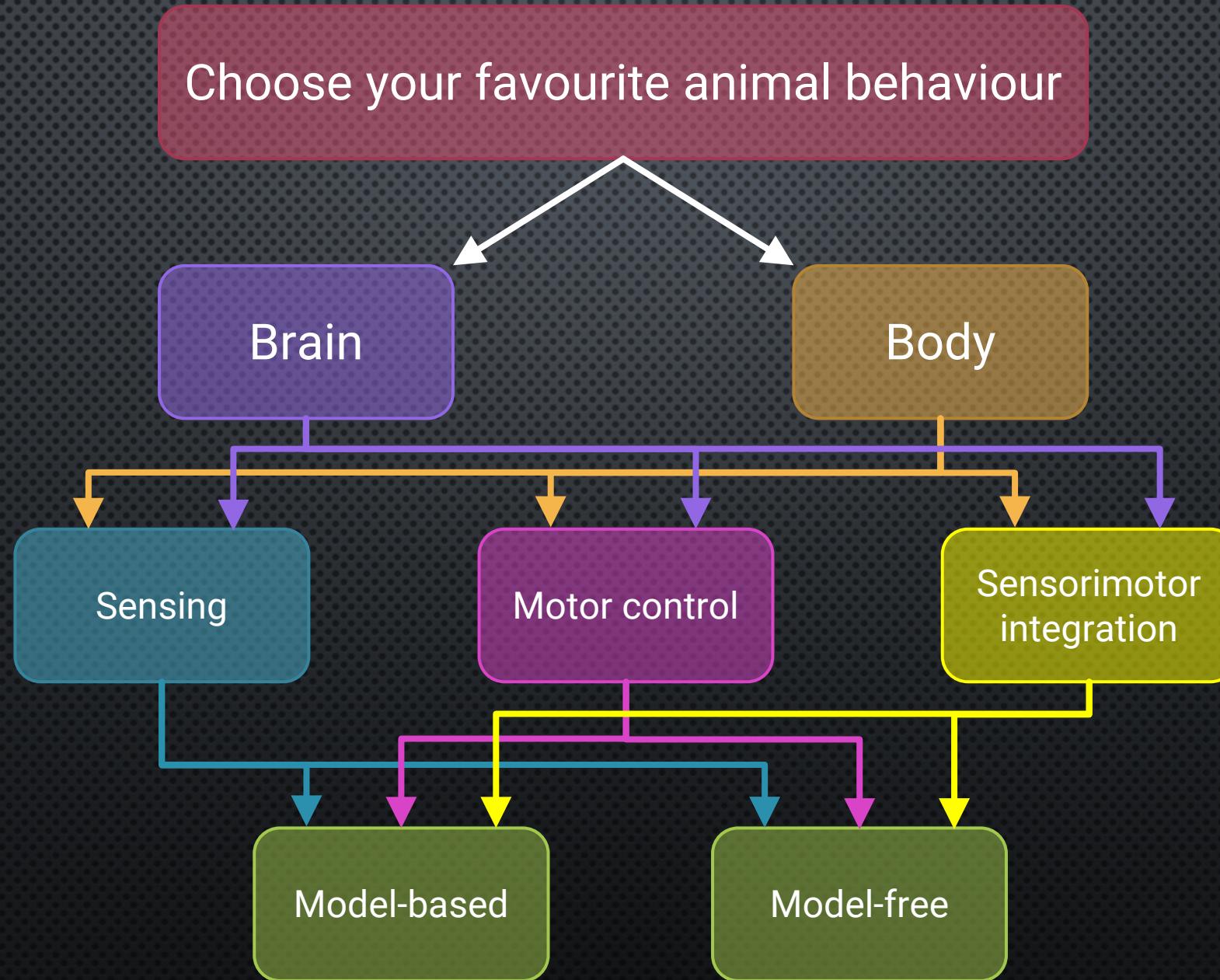


Reality check...

Desert Locusts

(*Schistocerca gregaria*)

So how and where do I start?



Meet the robot...



8-microphone array

- Alexa voice recognition
- sound direction estimation

18-LED array

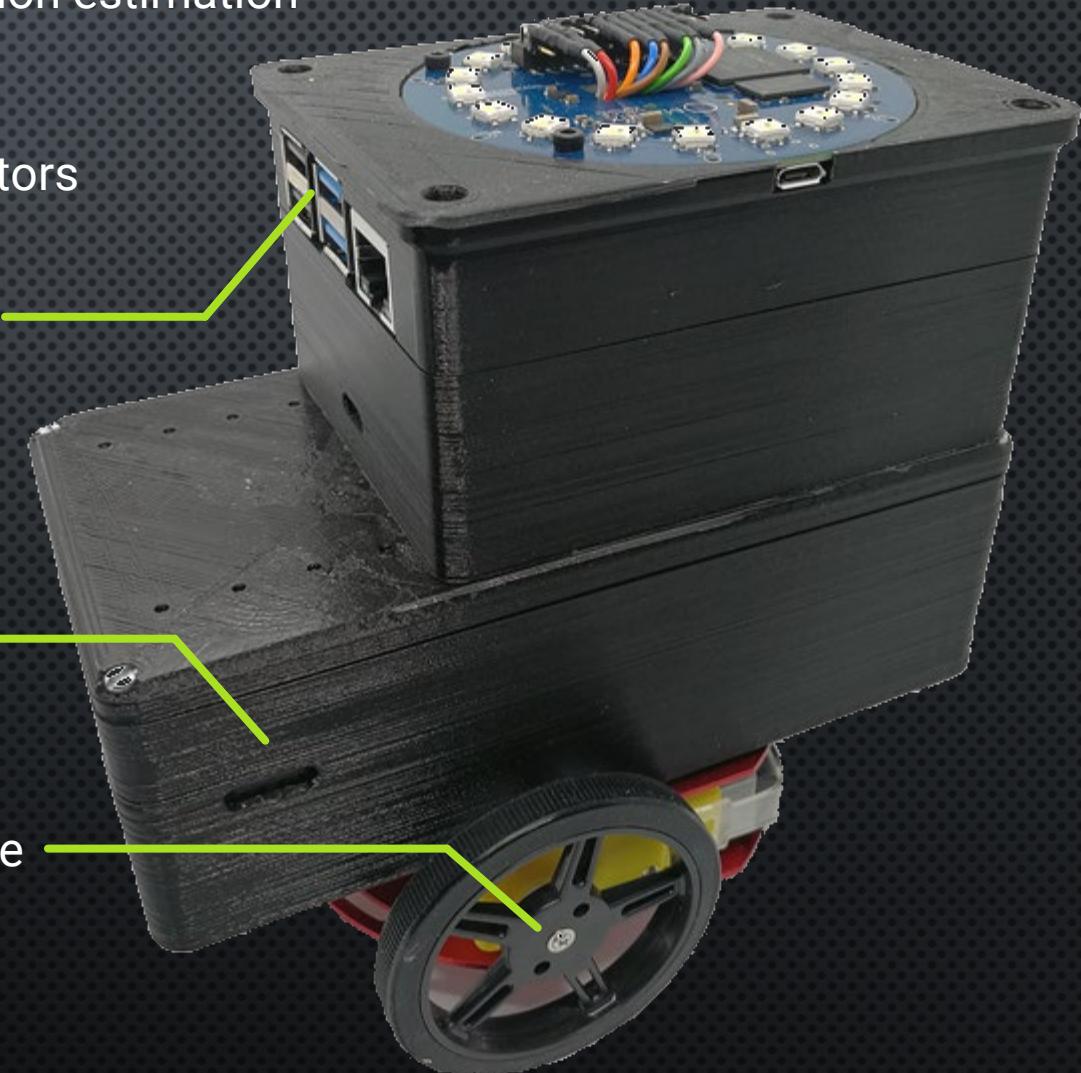
- visual indicators

Raspberry Pi 4

RGB camera

30000 mAh
rechargeable
battery

Differential drive
kinematics



Questions?



2 Biological neurons and their functional engineering models

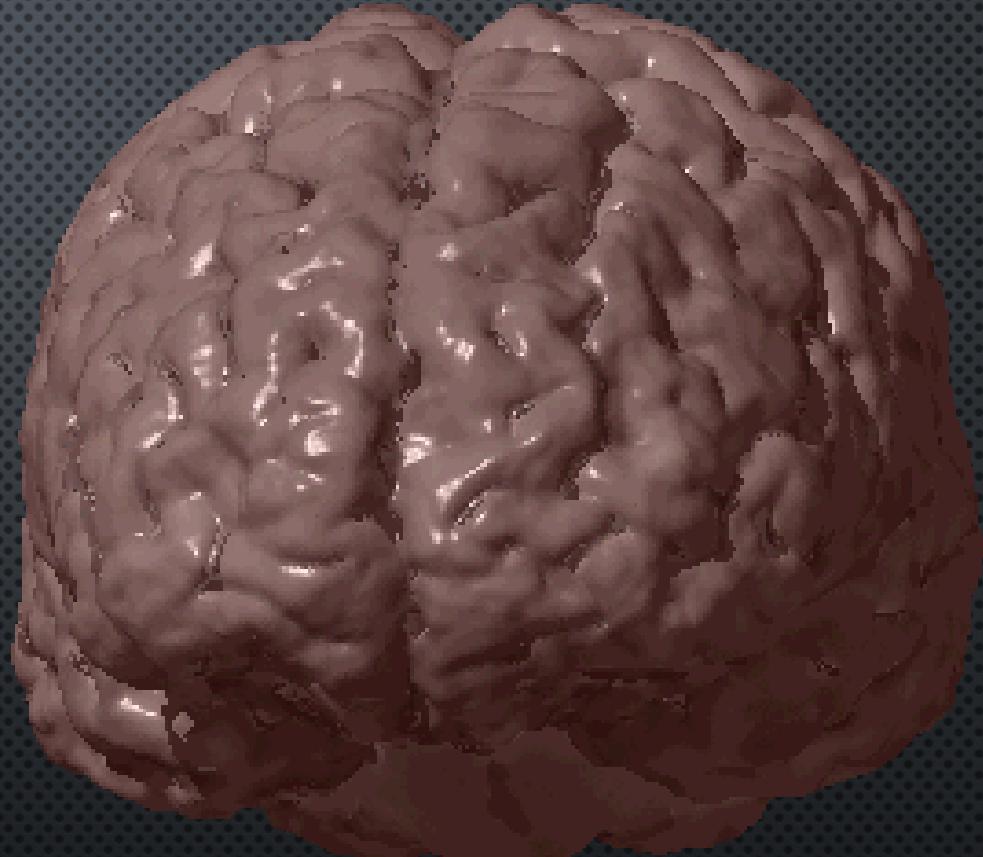


Neurobiology ahead
(get used to it)

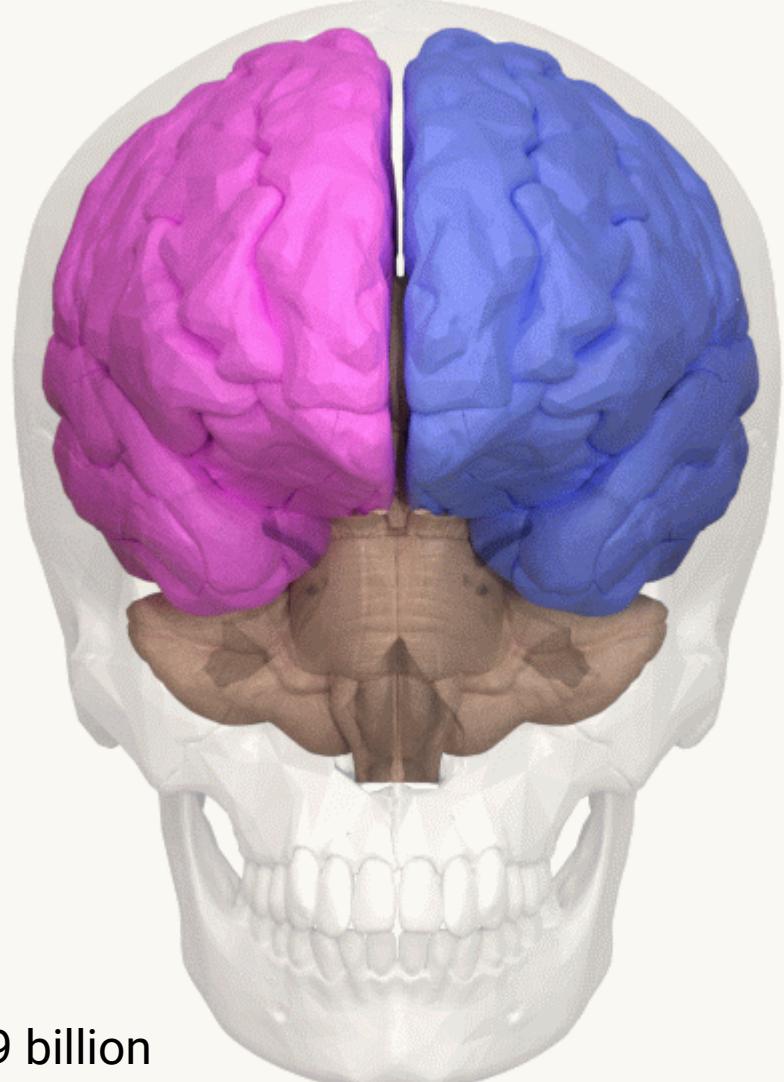


The human brain

- Represents only **2% of the body weight** (1.2 - 1.4 kg) but receives 15% of the cardiac output, 20% of total body oxygen consumption, and 25% of total body glucose utilization
- Composed of neurons, glial cells, neural stem cells, and blood vessels
- Contains about **86 ± 8 billion neurons** and **85 ± 10 billion non-neuronal cells**

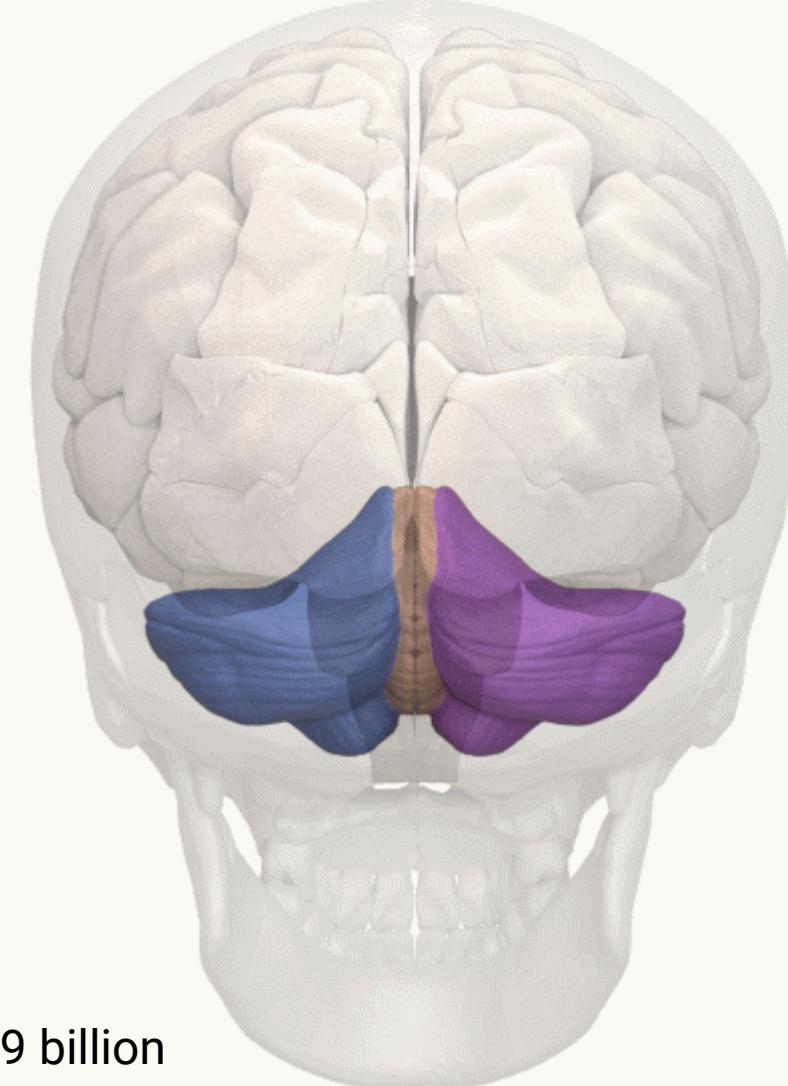


What makes the brain so special?



About 19 billion
(~19%) neurons

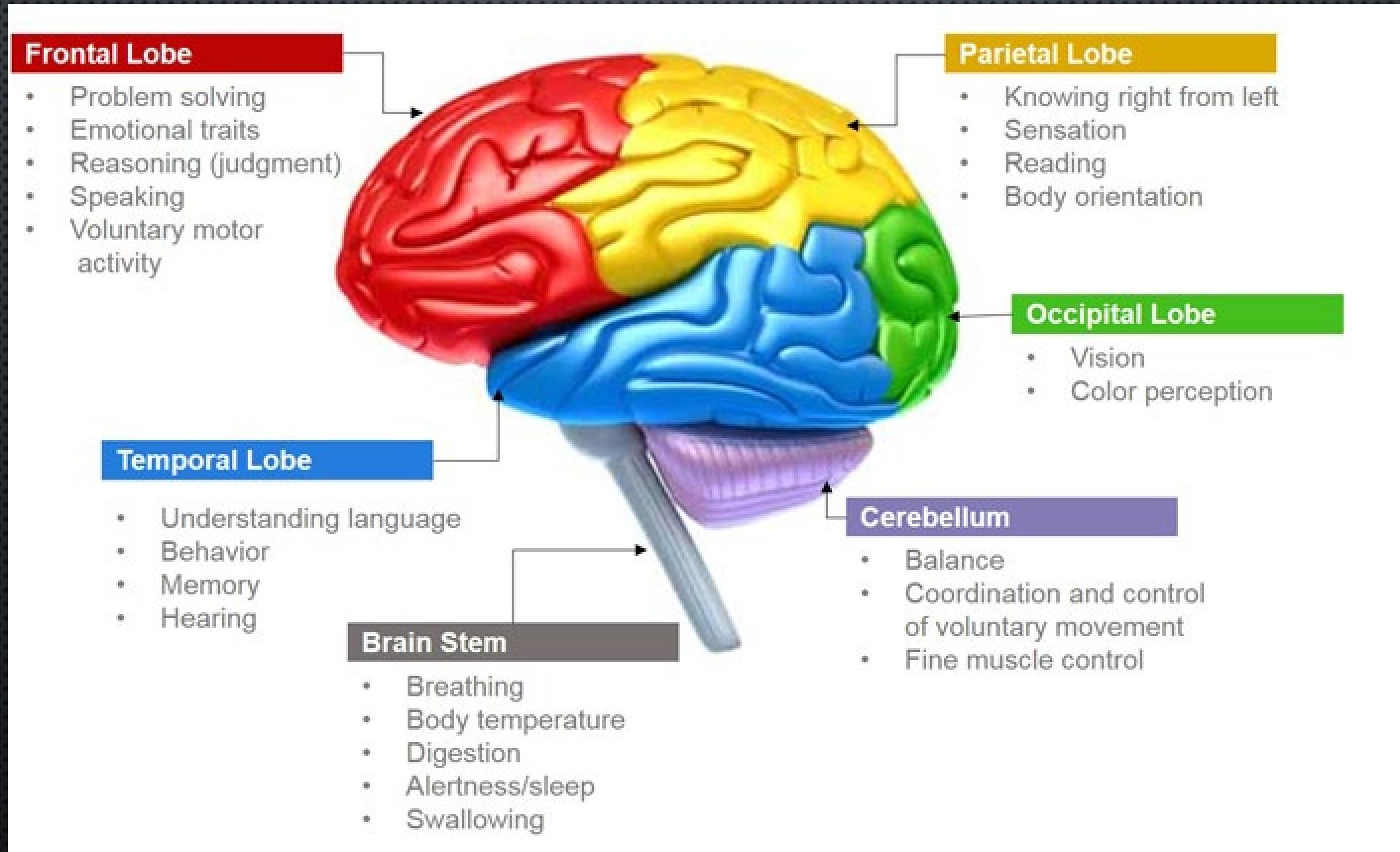
Cerebrum



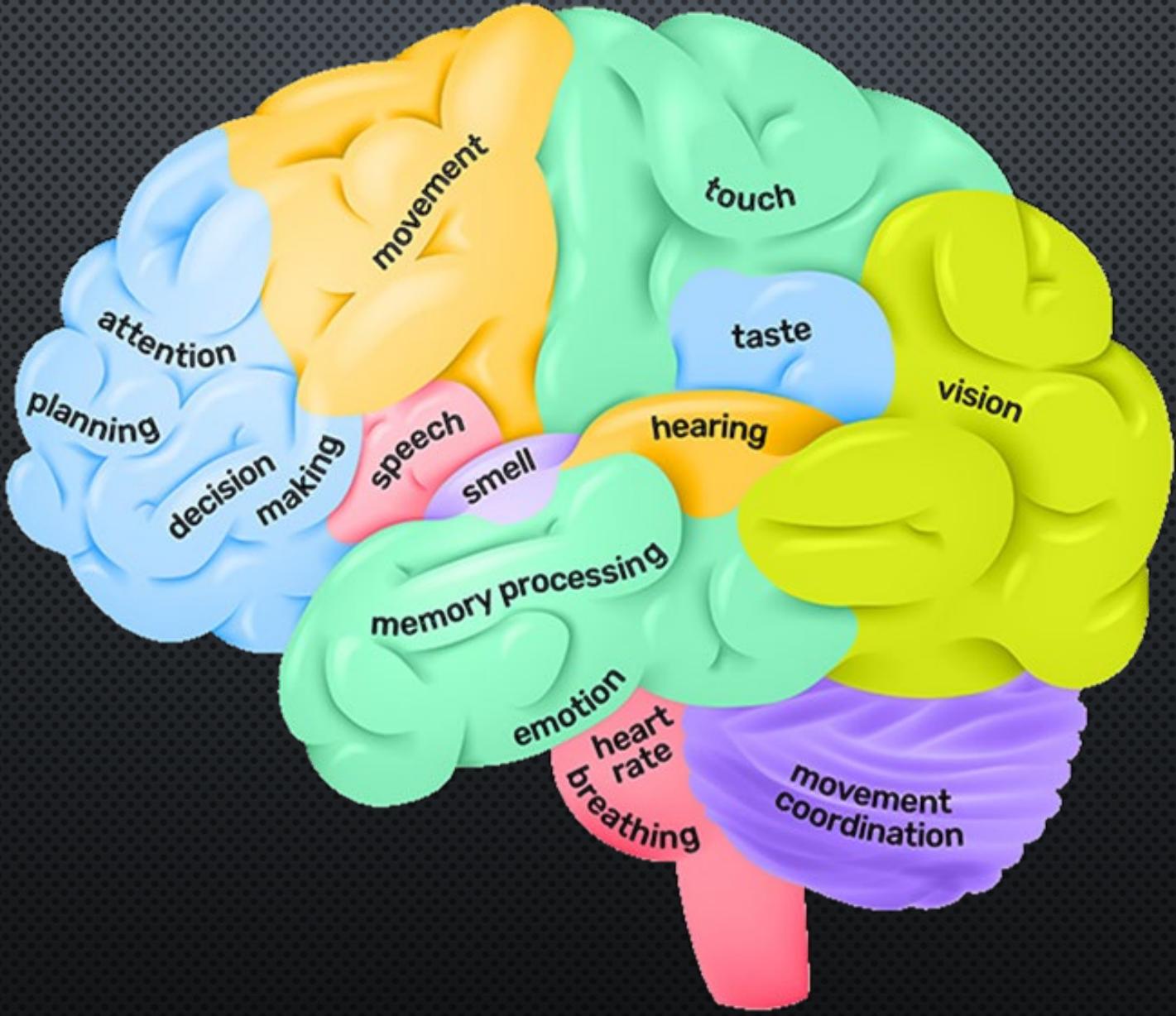
About 69 billion
(~80%) neurons

Cerebellum

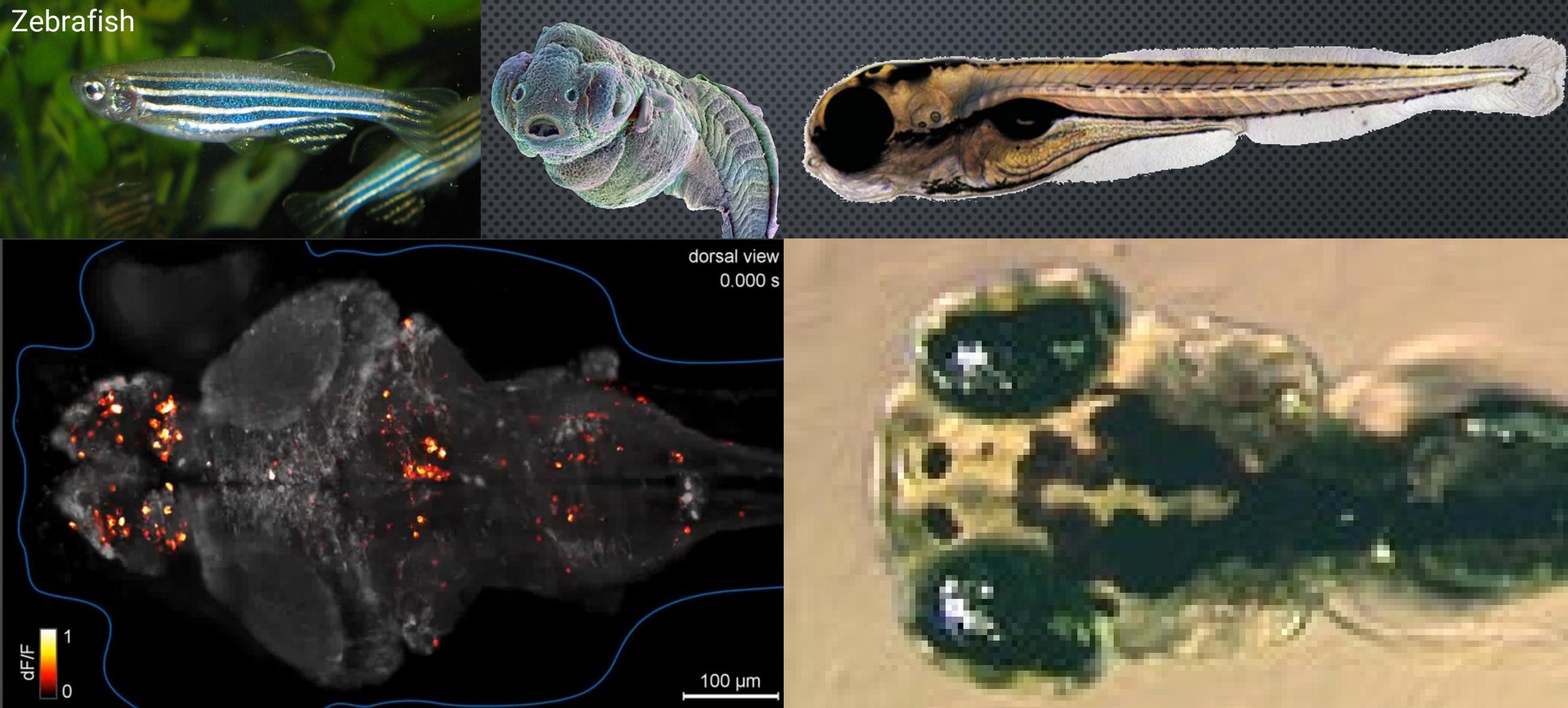
Brain anatomy and function



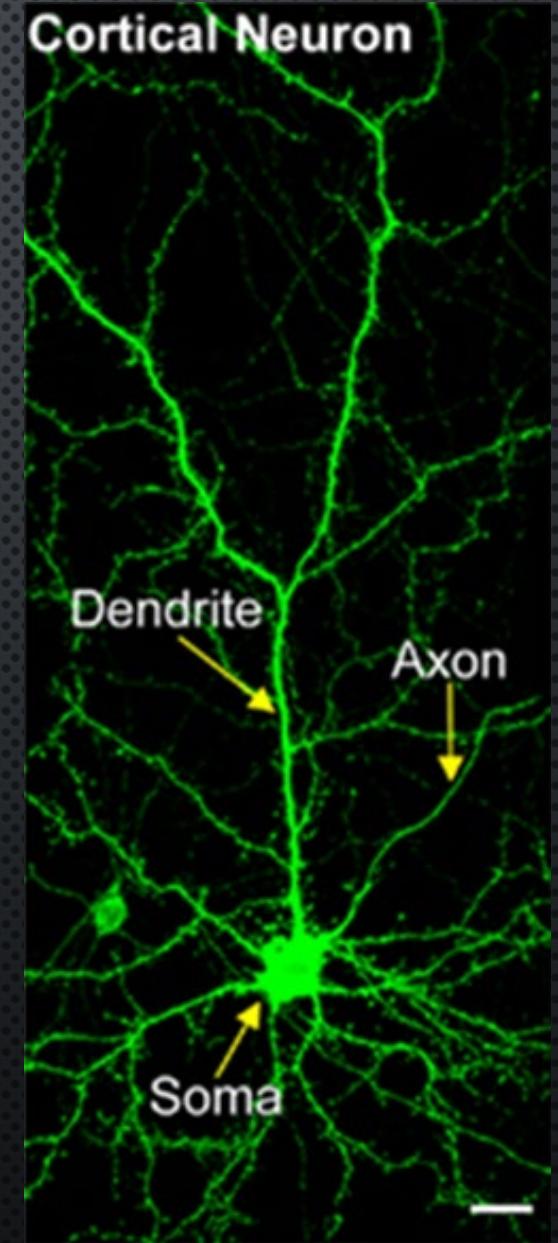
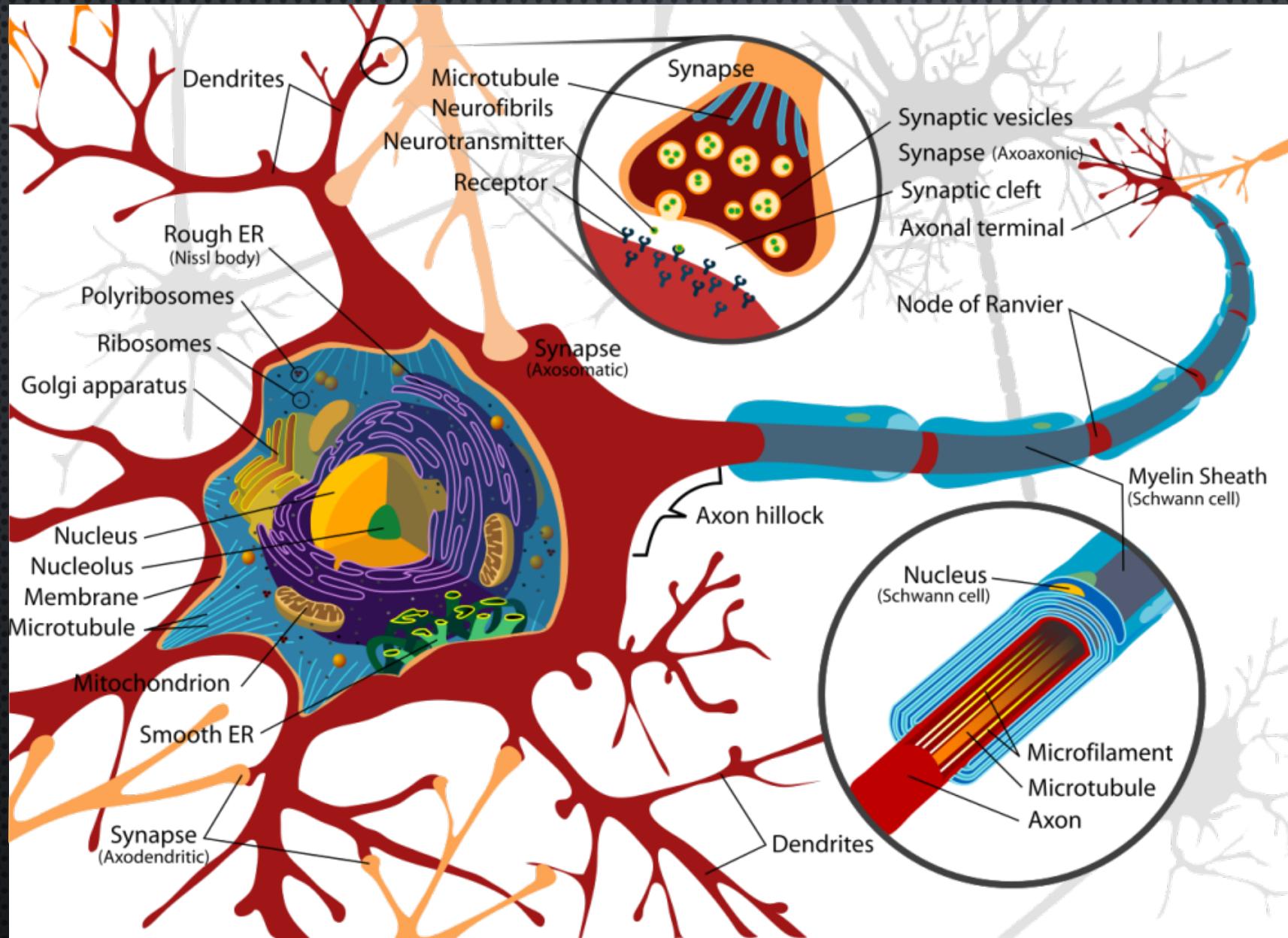
Implications for robotics



Visualising the brain



What is a neuron?



Basic types of neurons



Unipolar

Transmit signals from outside of body to brain (found in insect brains)



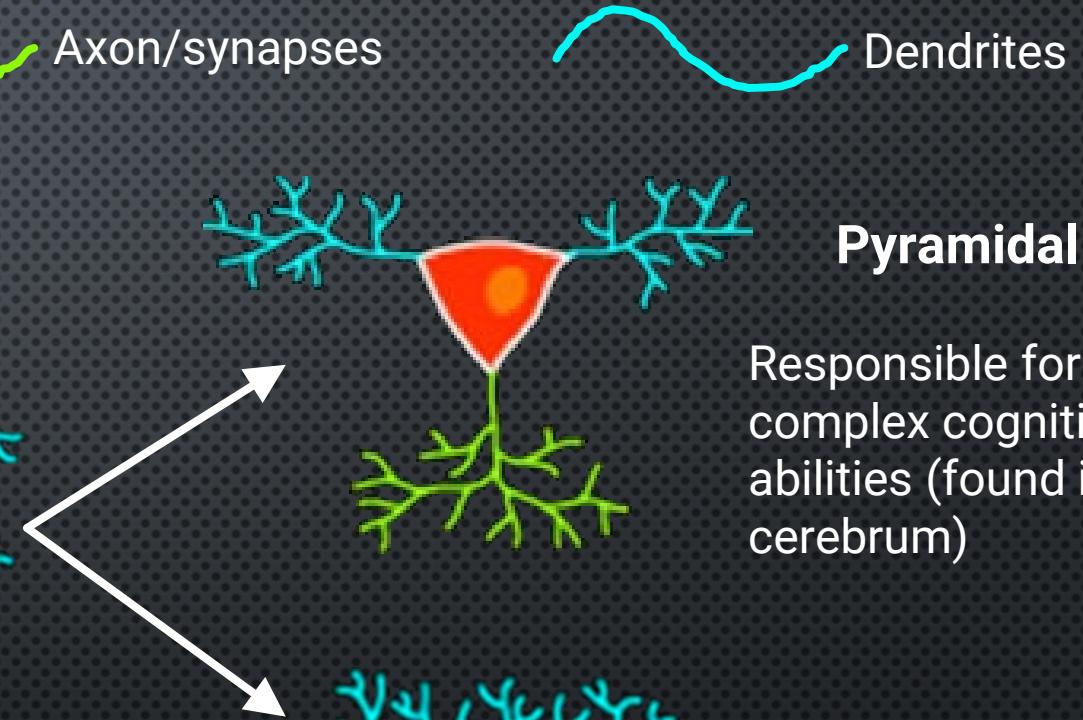
Bipolar

Transmit signals from sensory appendages (eyes, ears etc.) to brain



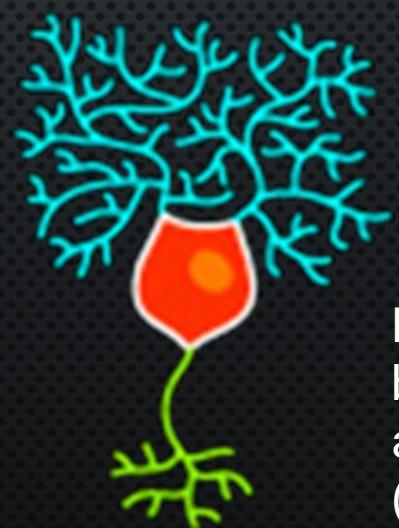
Multipolar

Transmit signals within the brain as well as from brain to muscles



Pyramidal

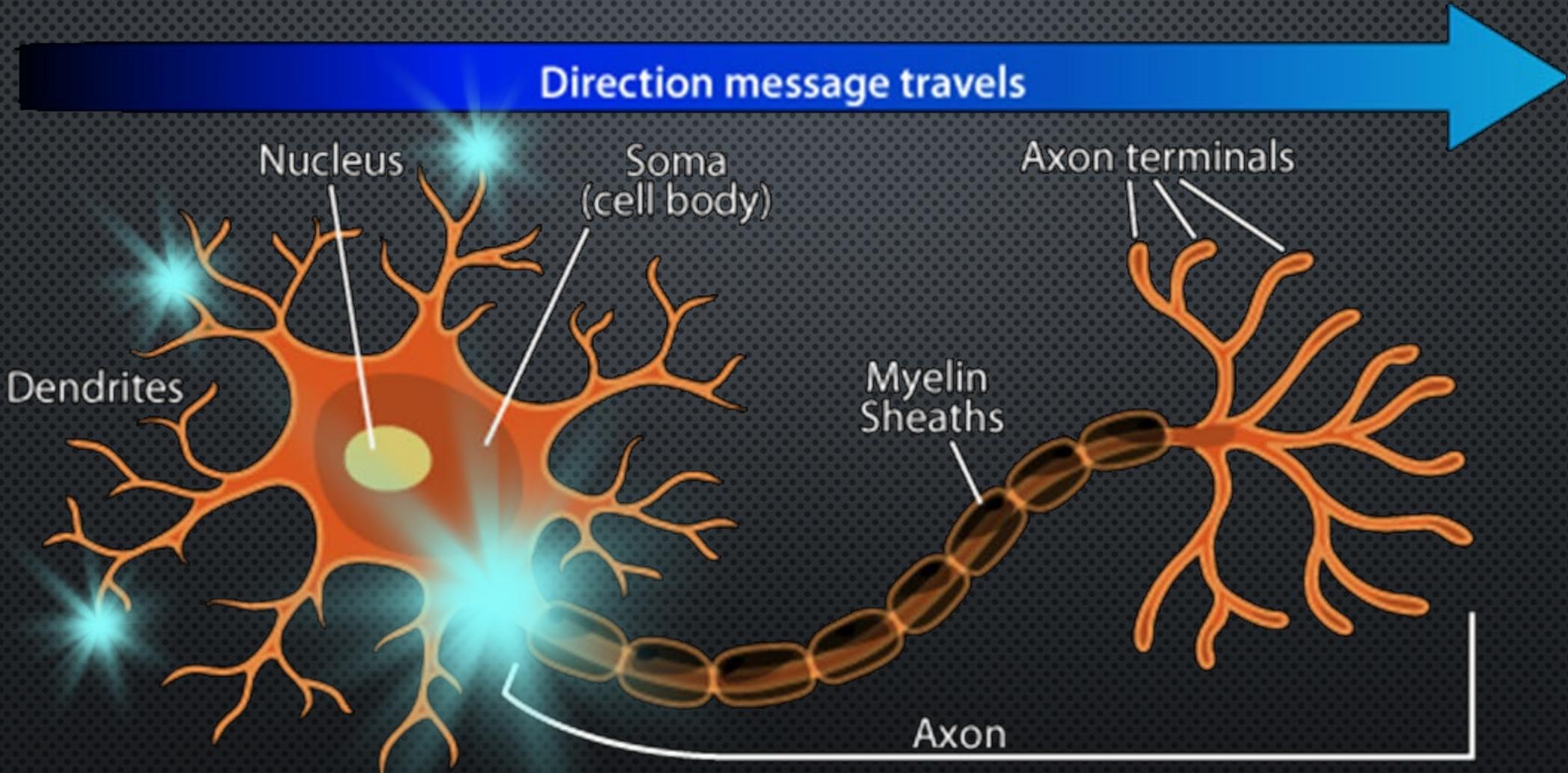
Responsible for complex cognitive abilities (found in cerebrum)



Purkinje

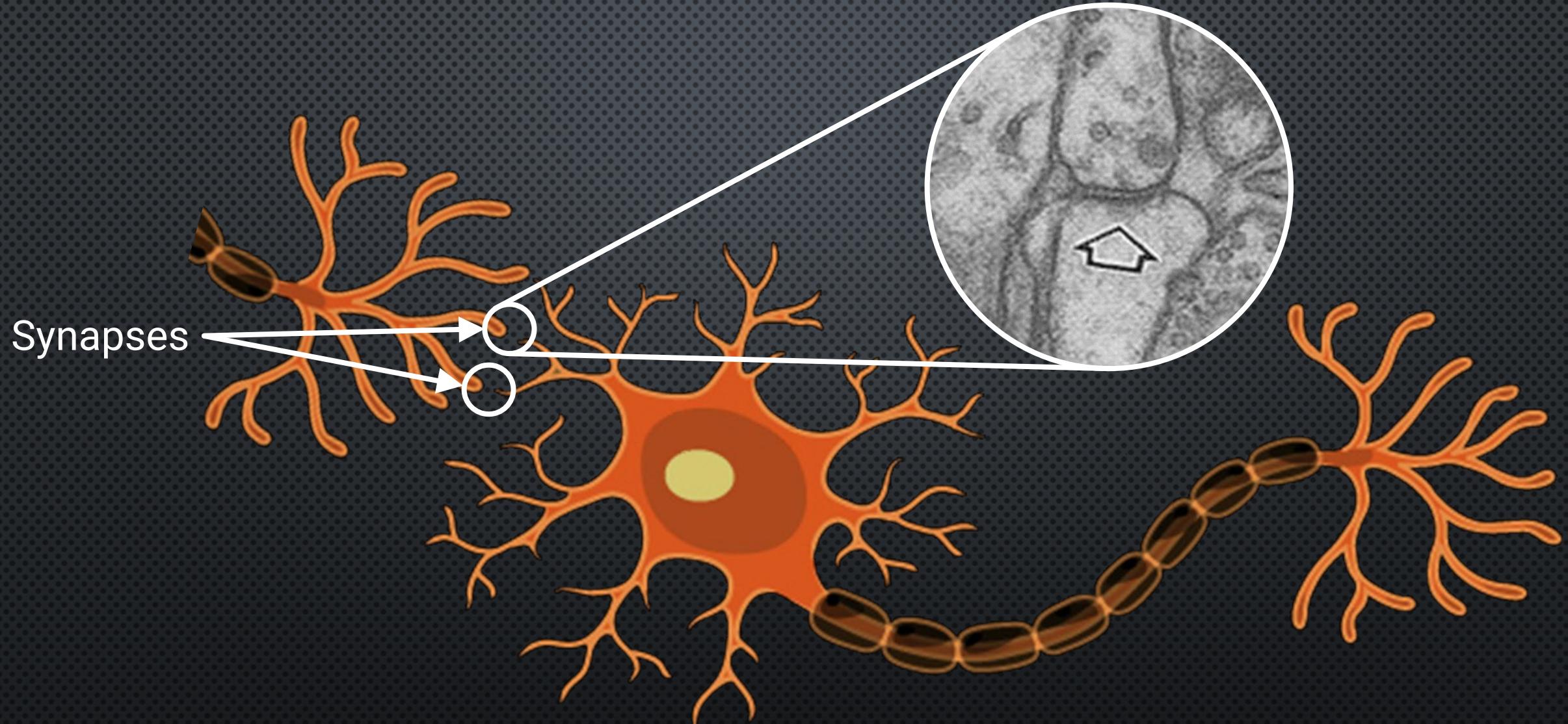
Responsible for balance, coordination and timing of actions (found in cerebellum)

Neurons integrate and transmit information



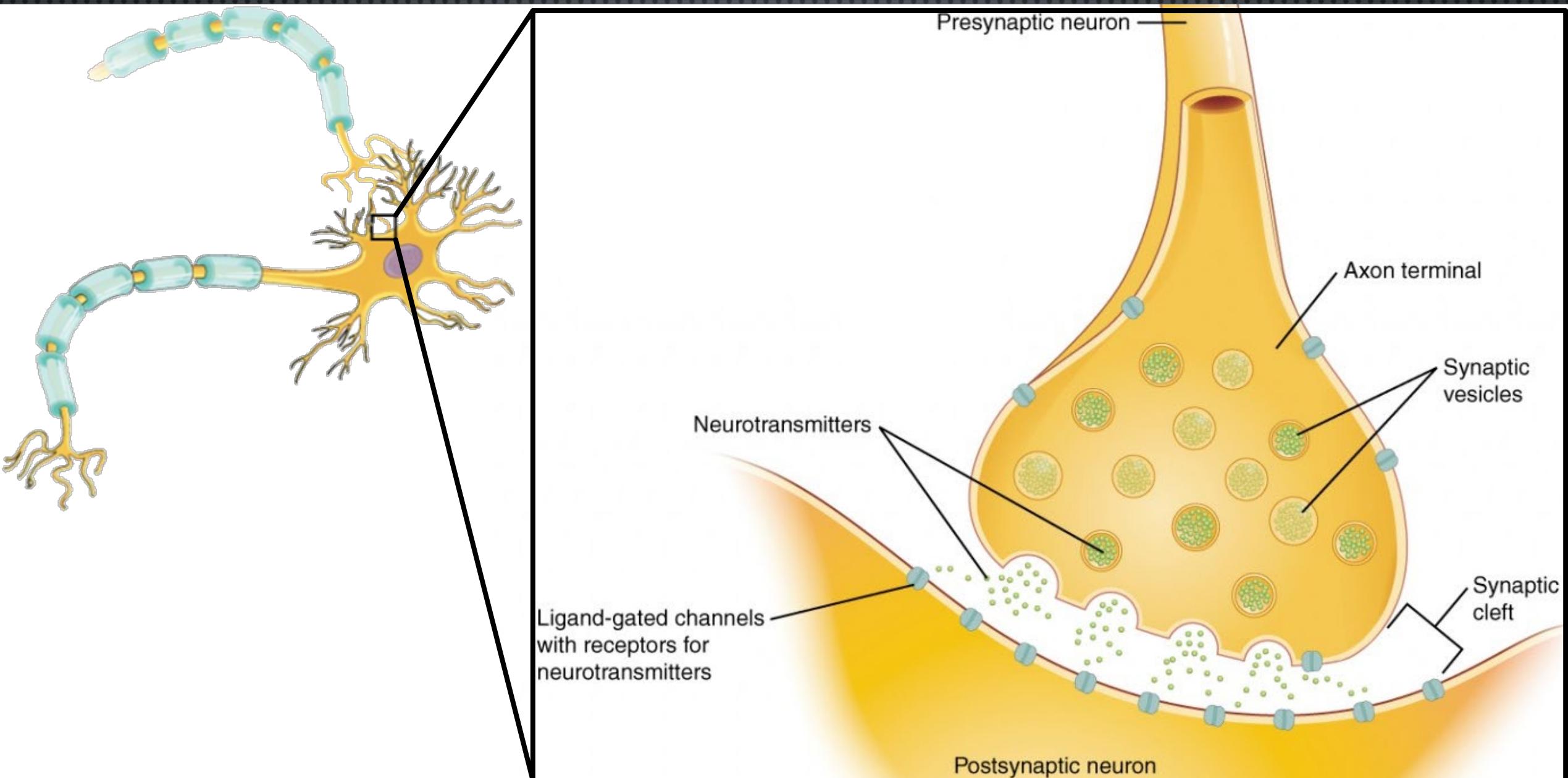
Neurons collect electrical signals (called **action potentials**) from other neurons via **synapses** and transmit them to other neurons via the axon

Synapses bridge neurons together

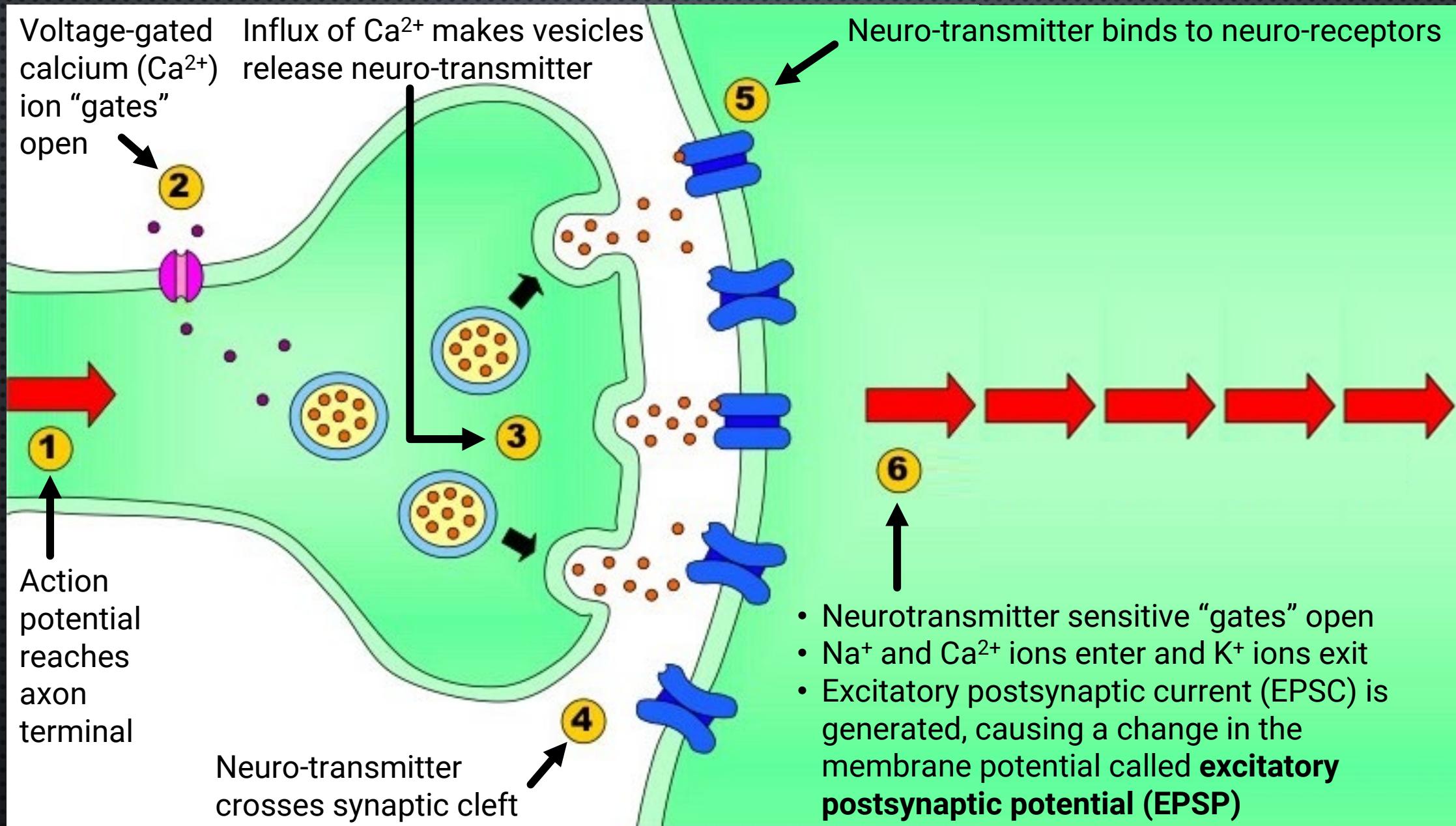


A **synapse** is a structure that permits a neuron to pass an **electrical or chemical signal** to another neuron.

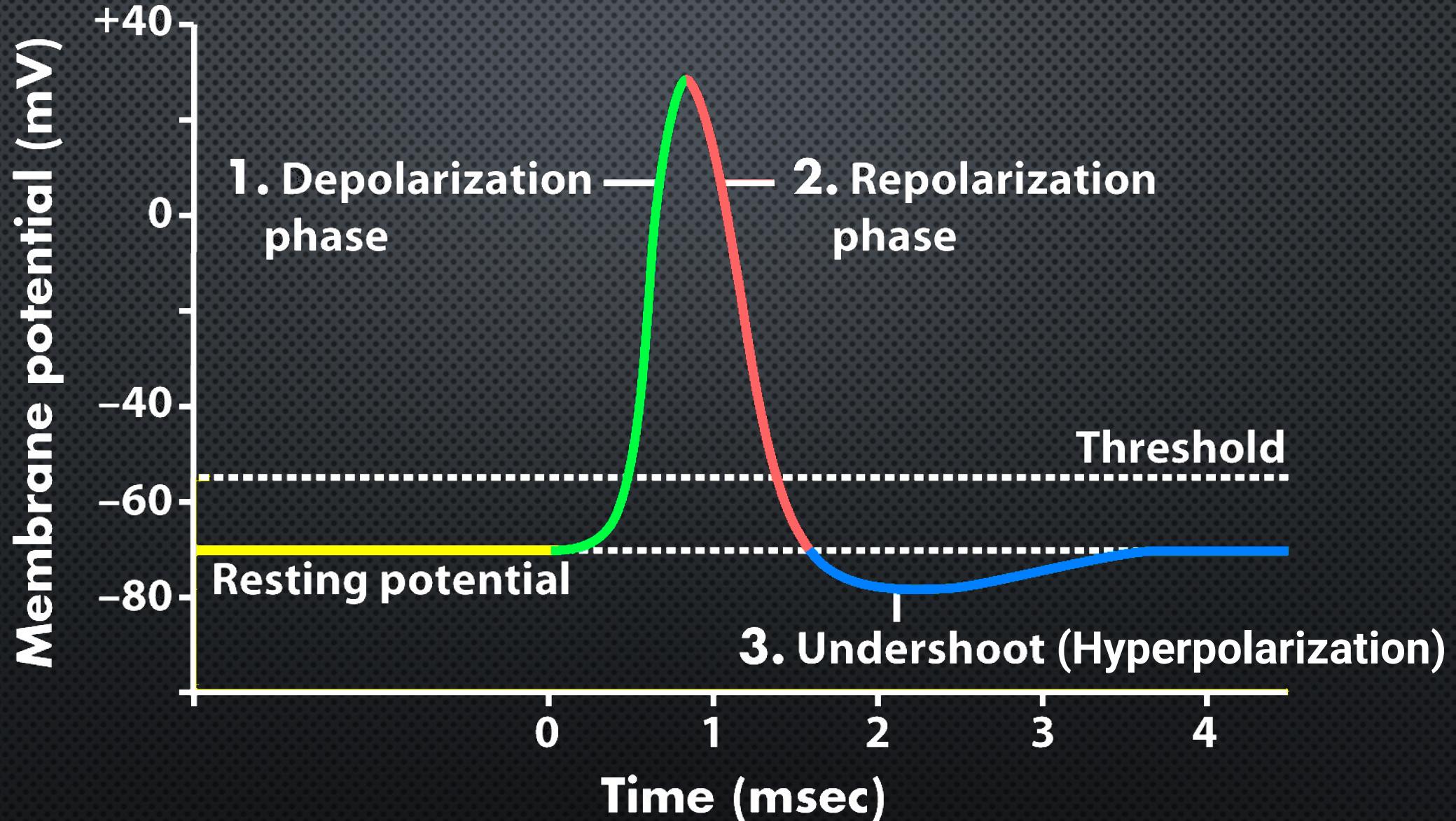
Structure of a chemical synapse



Signal transmission in synapses

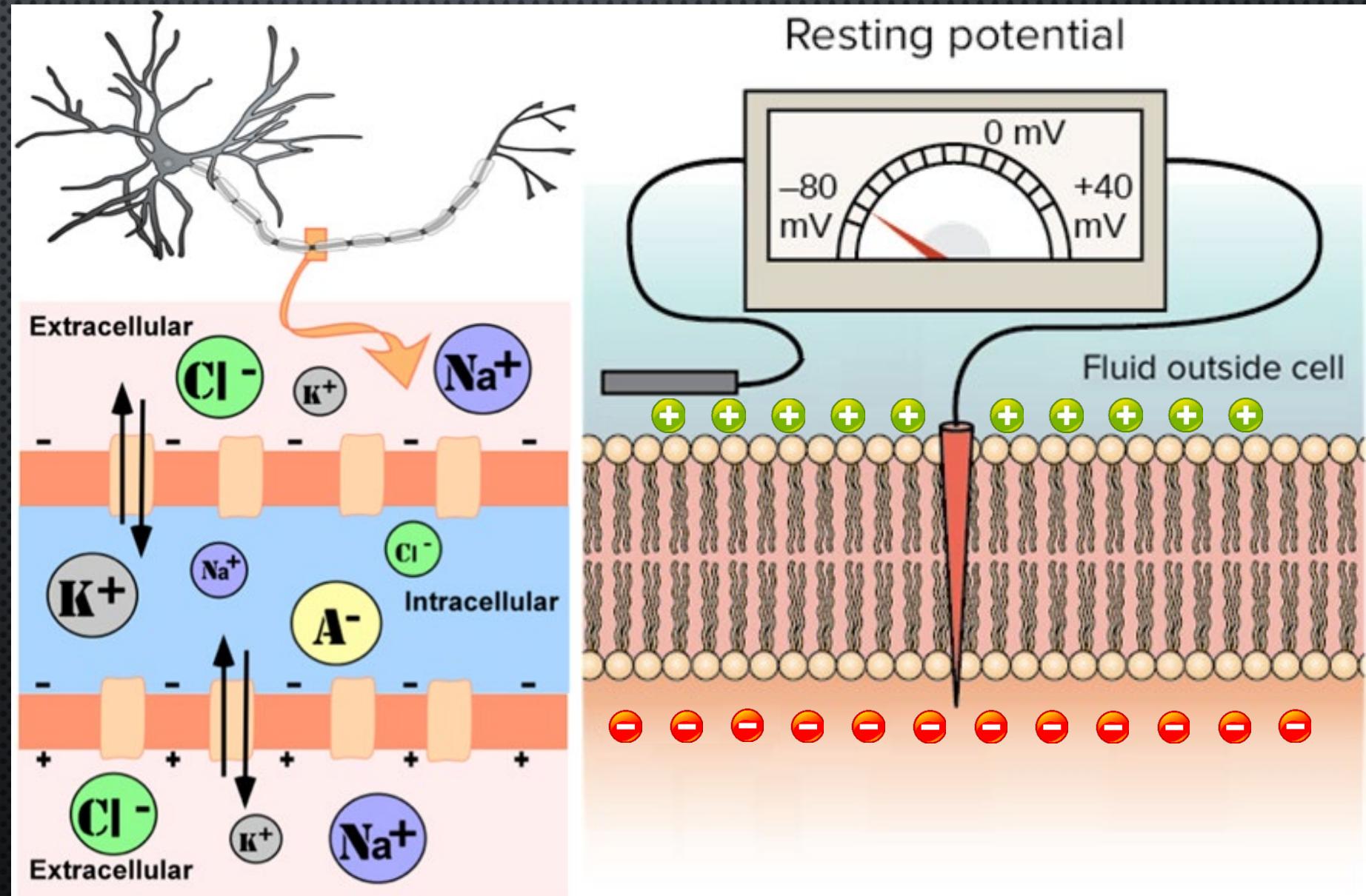


Action potential

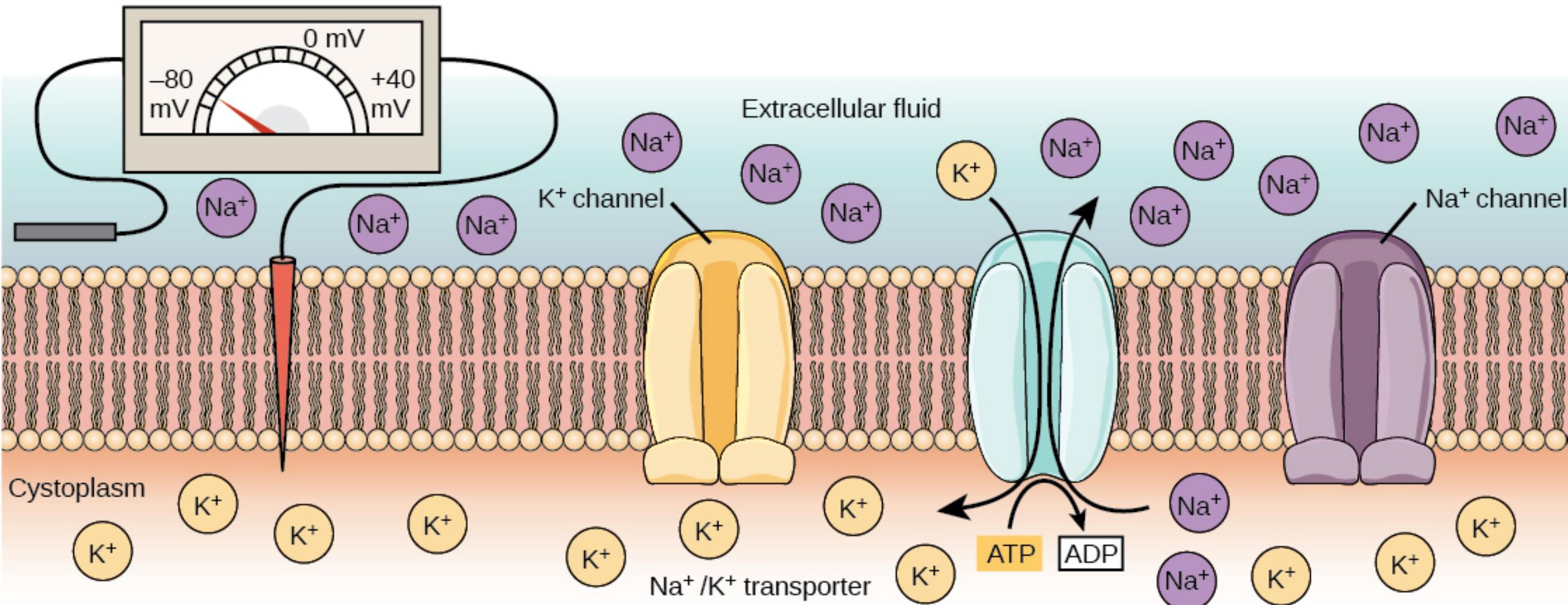


Resting membrane potential: default state of a neuron

- Difference between electric potential inside and outside a neuron, due to different ion concentration
- Potassium (K^+) ions have higher concentration inside
- Sodium (Na^+) ions have higher concentration outside
- Leakage of K^+ to outside leaves excess negative charge inside and excess positive charge outside
- Ion leakage sets resting membrane potential to around -65 mV to -70 mV

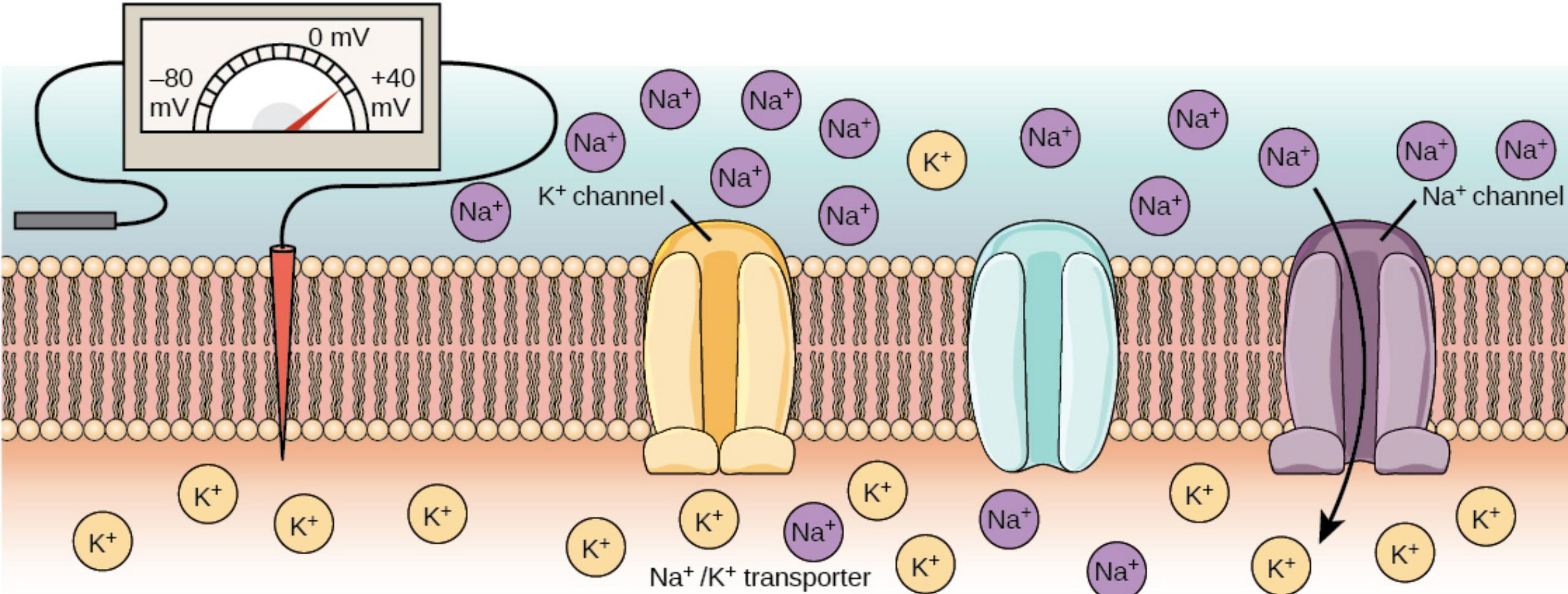


Resting membrane potential: default state of a neuron



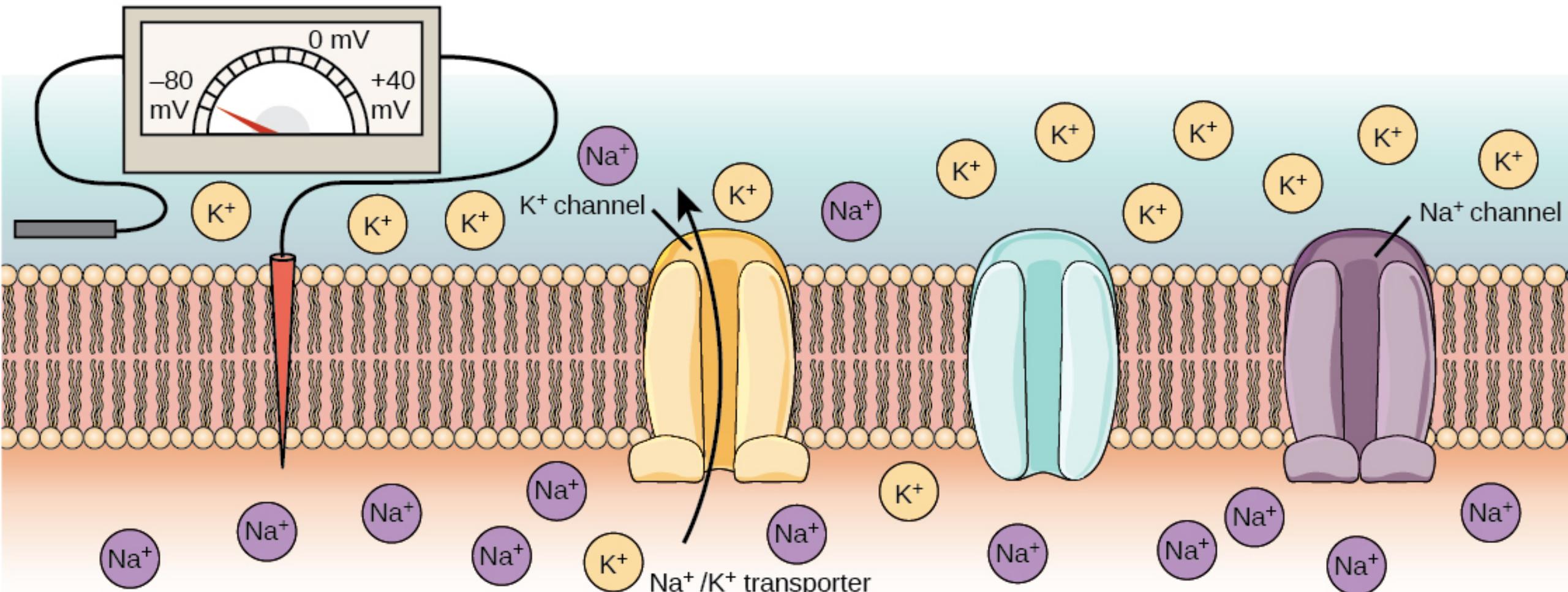
At the resting potential, all voltage-gated Na^+ channels and most voltage-gated K^+ channels are closed. The Na^+/K^+ transporter pumps K^+ ions into the cell and Na^+ ions out.

De-polarisation



In response to a depolarization, some Na^+ channels open, allowing Na^+ ions to enter the cell. The membrane starts to depolarize (the charge across the membrane lessens). If the threshold of excitation is reached, all the Na^+ channels open.

Hyper-polarisation

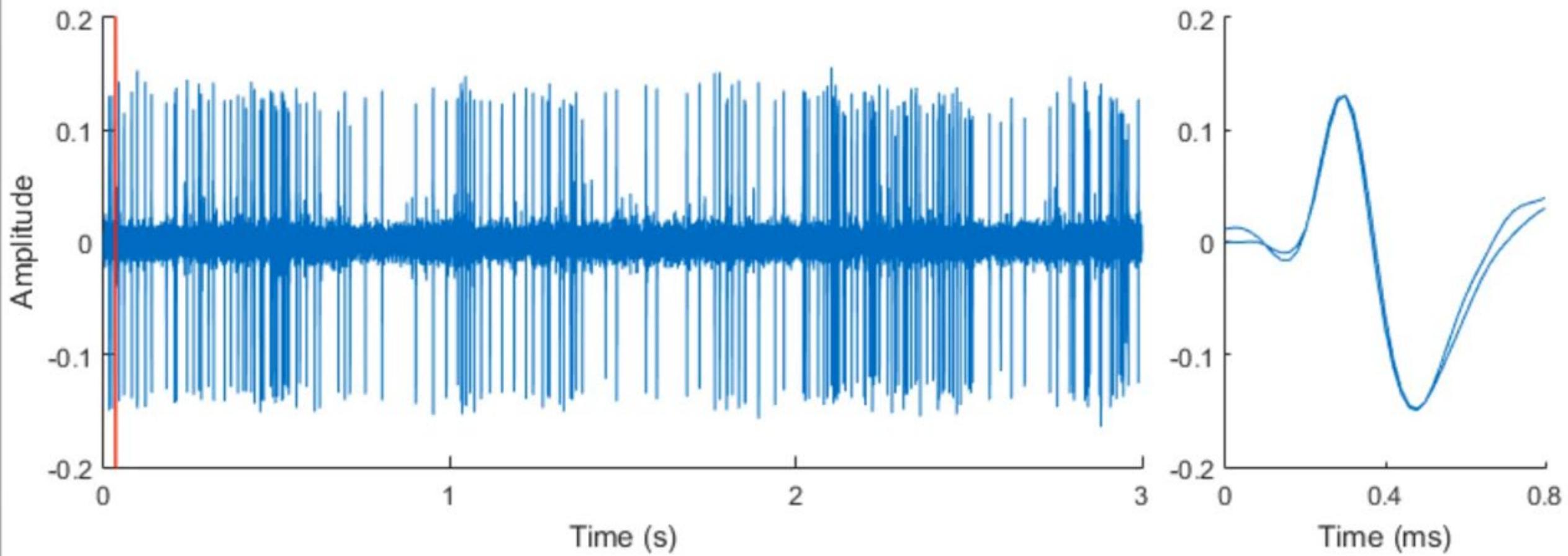


At the peak action potential, Na^+ channels close while K^+ channels open. K^+ leaves the cell, and the membrane eventually becomes hyperpolarized.

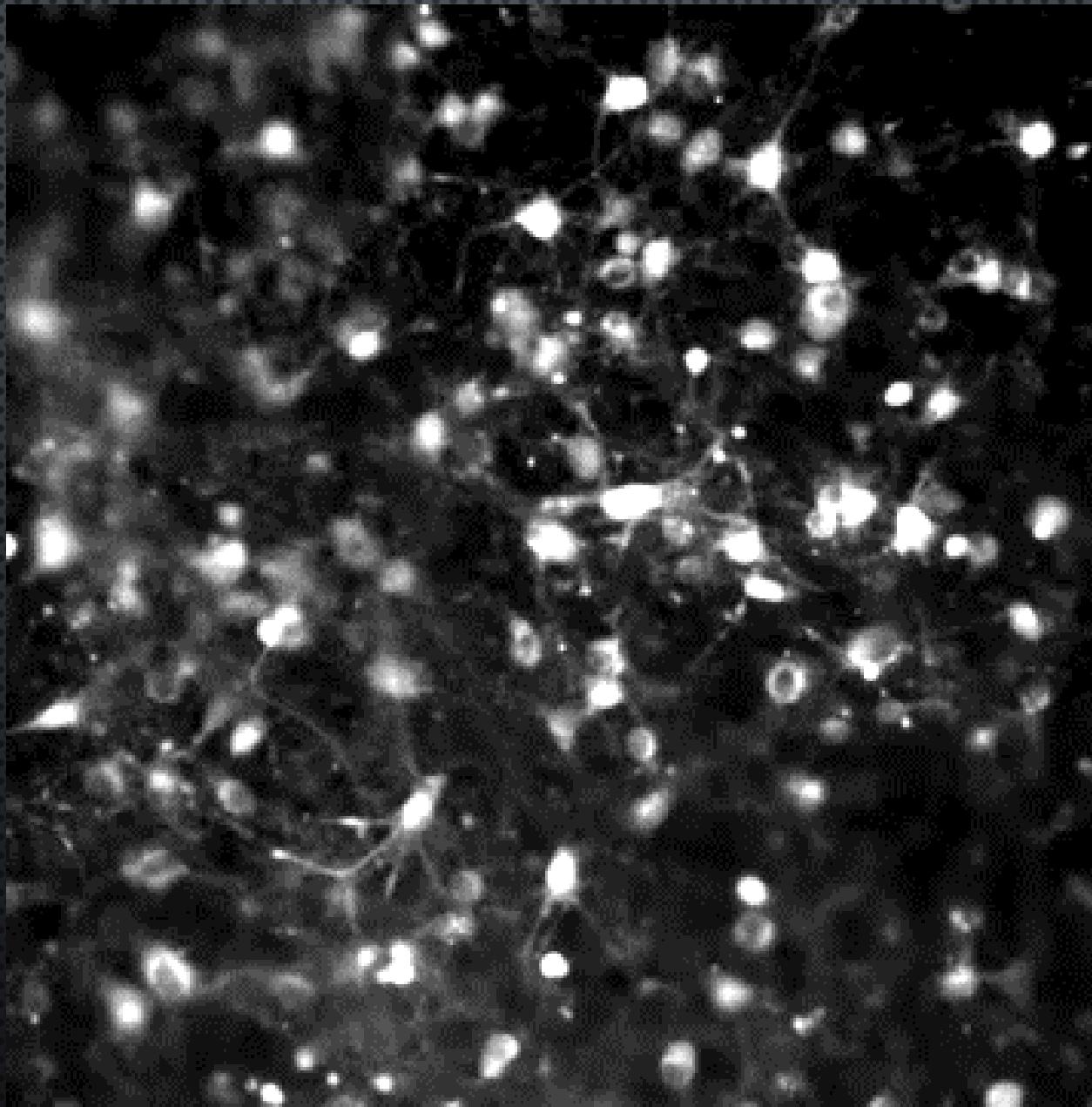
How is action potential actually measured?



Sights and sounds of neuronal recordings

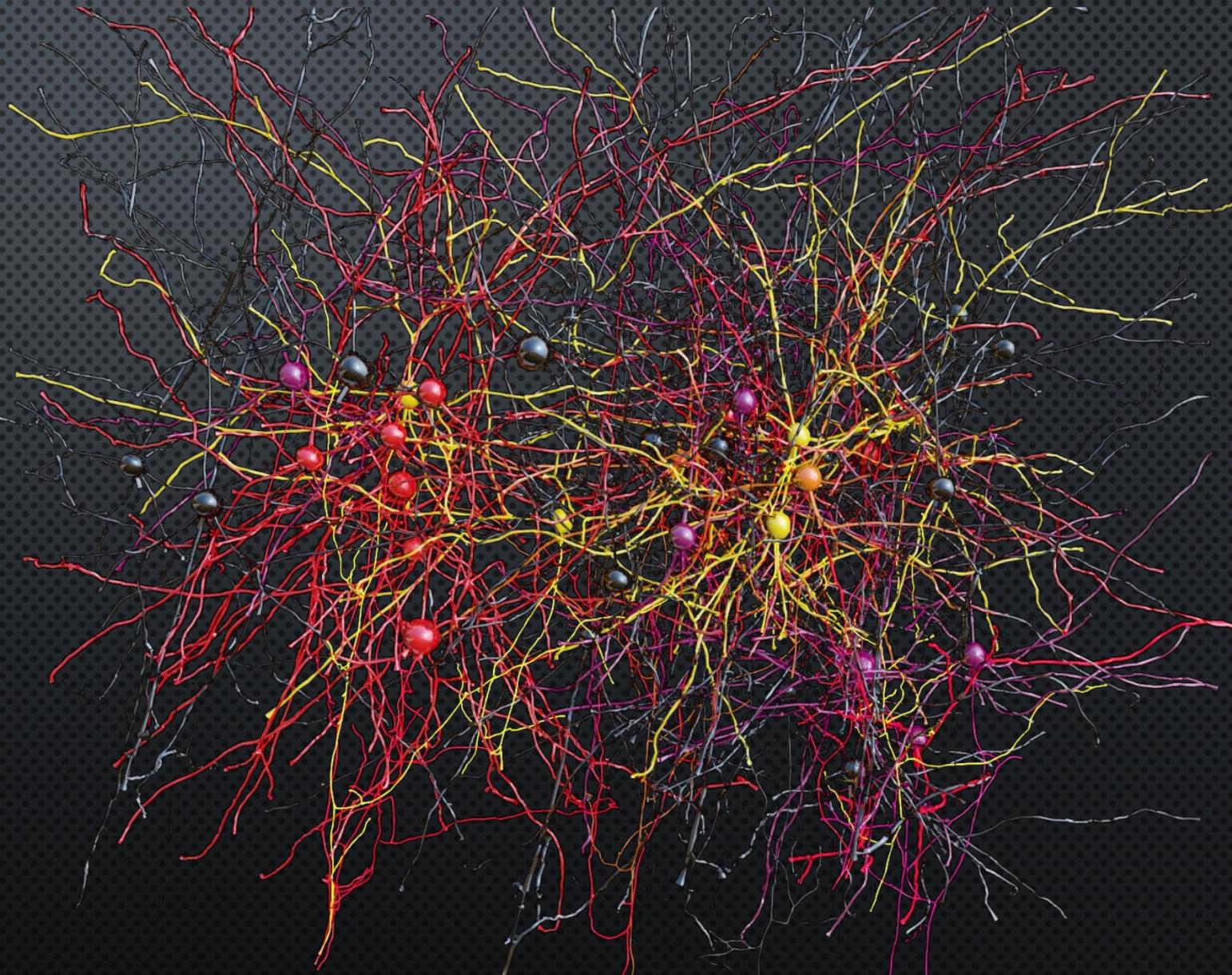


Neuronal firing

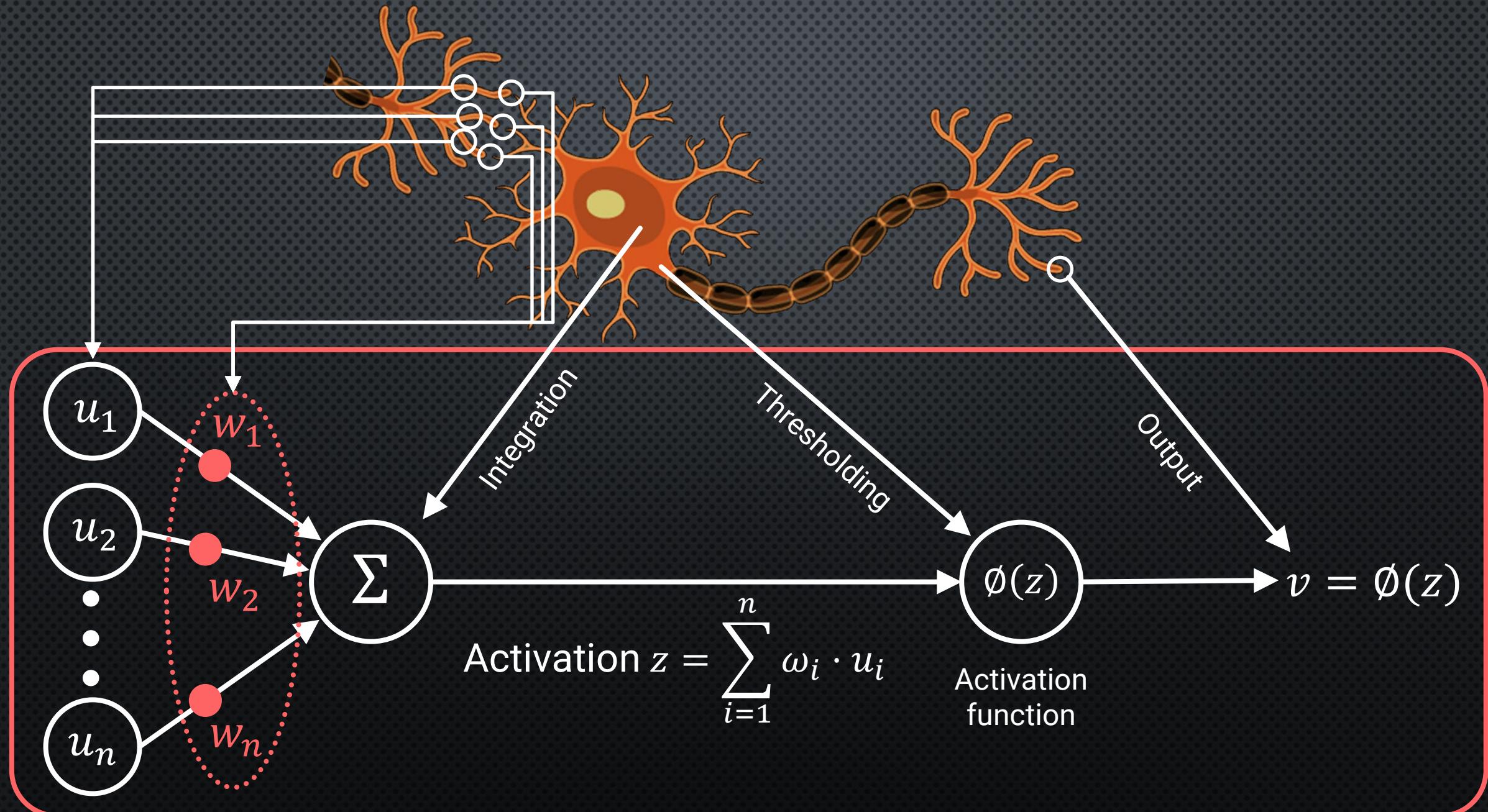


Neural connectivity is very complex!

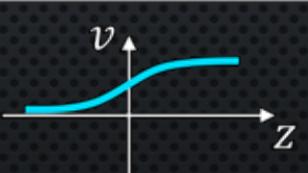
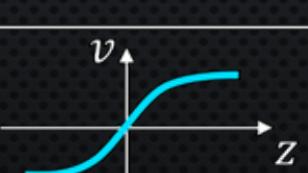
- A typical neuron may receive inputs from on the order of 10,000 neurons
- Pyramidal cells in the hippocampus can receive on the order of 50,000 inputs
- There are an estimated **1000 trillion** synapses in the human brain
- Although neural connectivity is highly dense, **the brain is not a fully connected network**



Perceptrons: from real to artificial neurons

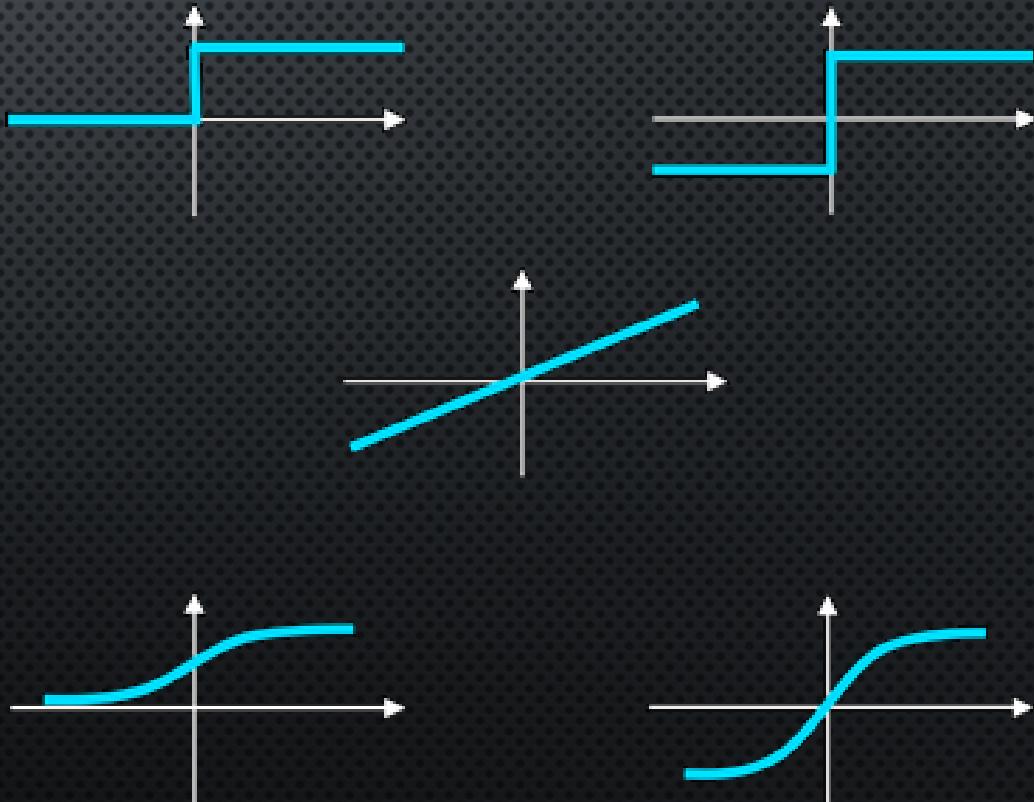


Common activation functions

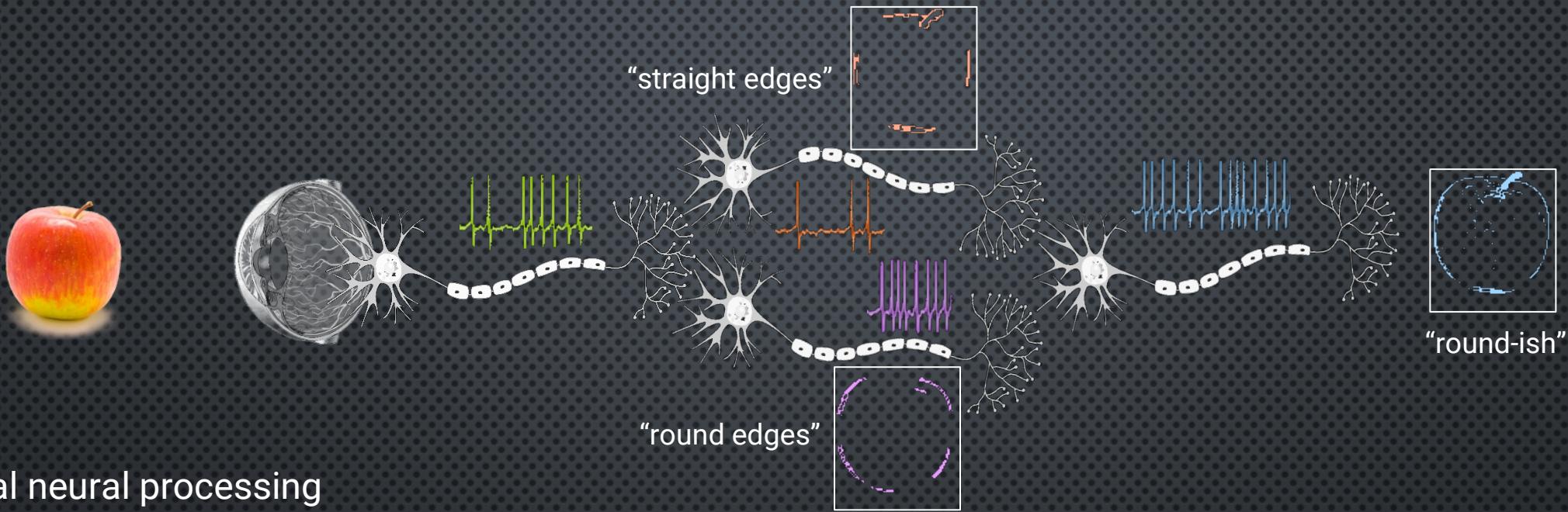
Activation function	Equation	Example	1D Graph
Unit step (Heaviside) Threshold Logic Unit (TLU)	$\phi(z) = \begin{cases} 0, & z < 0, \\ 0.5, & z = 0, \\ 1, & z > 0, \end{cases}$	Perceptron variant	
Sign (Signum)	$\phi(z) = \begin{cases} -1, & z < 0, \\ 0, & z = 0, \\ 1, & z > 0, \end{cases}$	Perceptron variant	
Linear	$\phi(z) = z$	Adaline, linear regression	
Piece-wise linear	$\phi(z) = \begin{cases} 1, & z \geq \frac{1}{2}, \\ z + \frac{1}{2}, & -\frac{1}{2} < z < \frac{1}{2}, \\ 0, & z \leq -\frac{1}{2}, \end{cases}$	Support vector machine	
Logistic (sigmoid)	$\phi(z) = \frac{1}{1 + e^{-z}}$	Logistic regression, Multi-layer NN	
Hyperbolic tangent	$\phi(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$	Multi-layer NN	

Which activation function to choose?

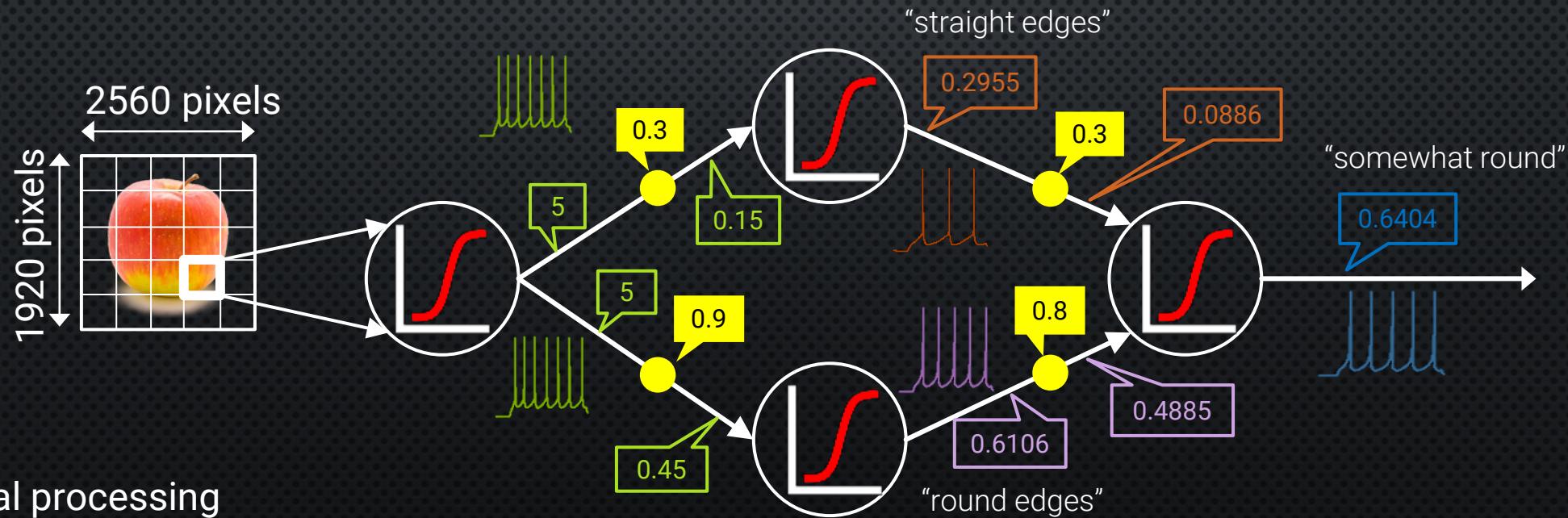
- Activation function decides what output the perceptron produces
- Choose an AF based on what you want your output to look like. For example, if your output:
 - can only be 0 or 1 (-1 or 1), choose TLU or sign
 - Good for binary classification problems but bad for everything else
 - can be any value between $-\infty$ to $+\infty$, choose linear
 - Almost always a bad choice so try to avoid using it
 - can be any value between 0 or 1 (-1 or 1), choose logistic or hyperbolic tangent
 - Almost always the best choice for most ML problems



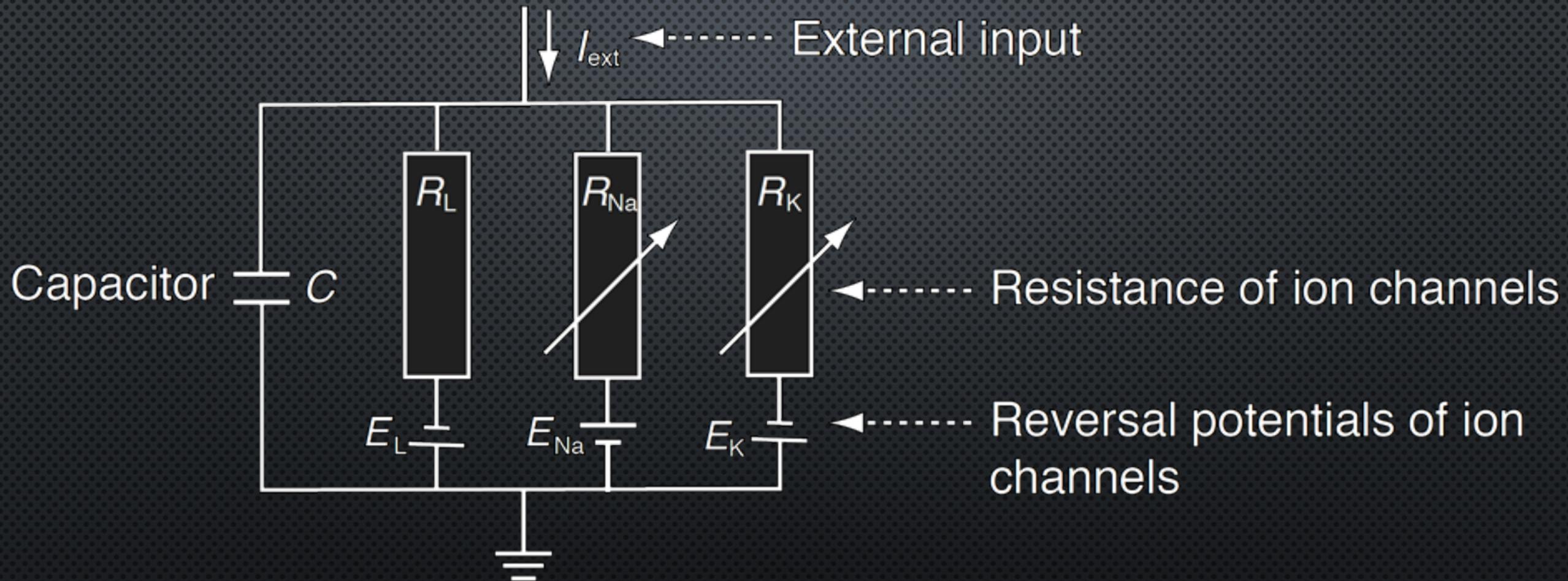
Biologically-inspired computation



Biological neural processing



A biologically realistic neuron model: Hodgkin-Huxley model



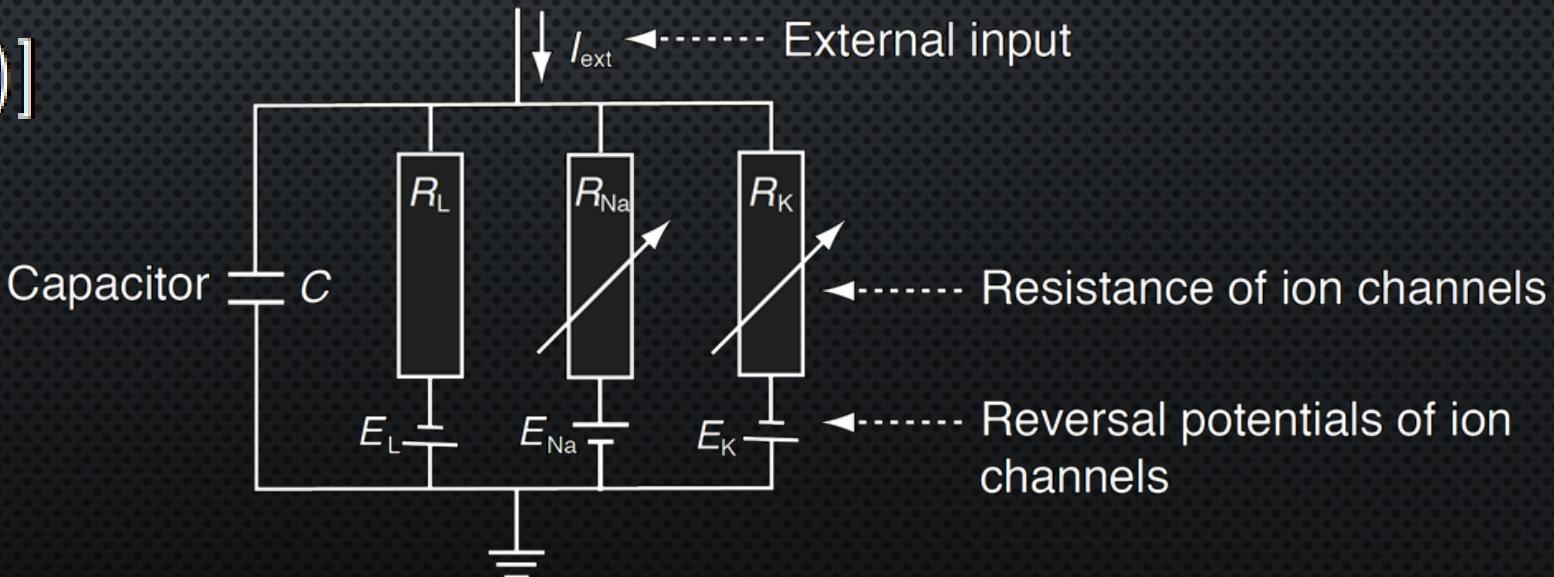
A biologically realistic neuron model: Hodgkin-Huxley model

$$C \frac{dV}{dt} = -g_K n^4 (V - E_K) - g_{Na} m^3 h (V - E_{Na}) - g_L (V - E_L) + I(t)$$

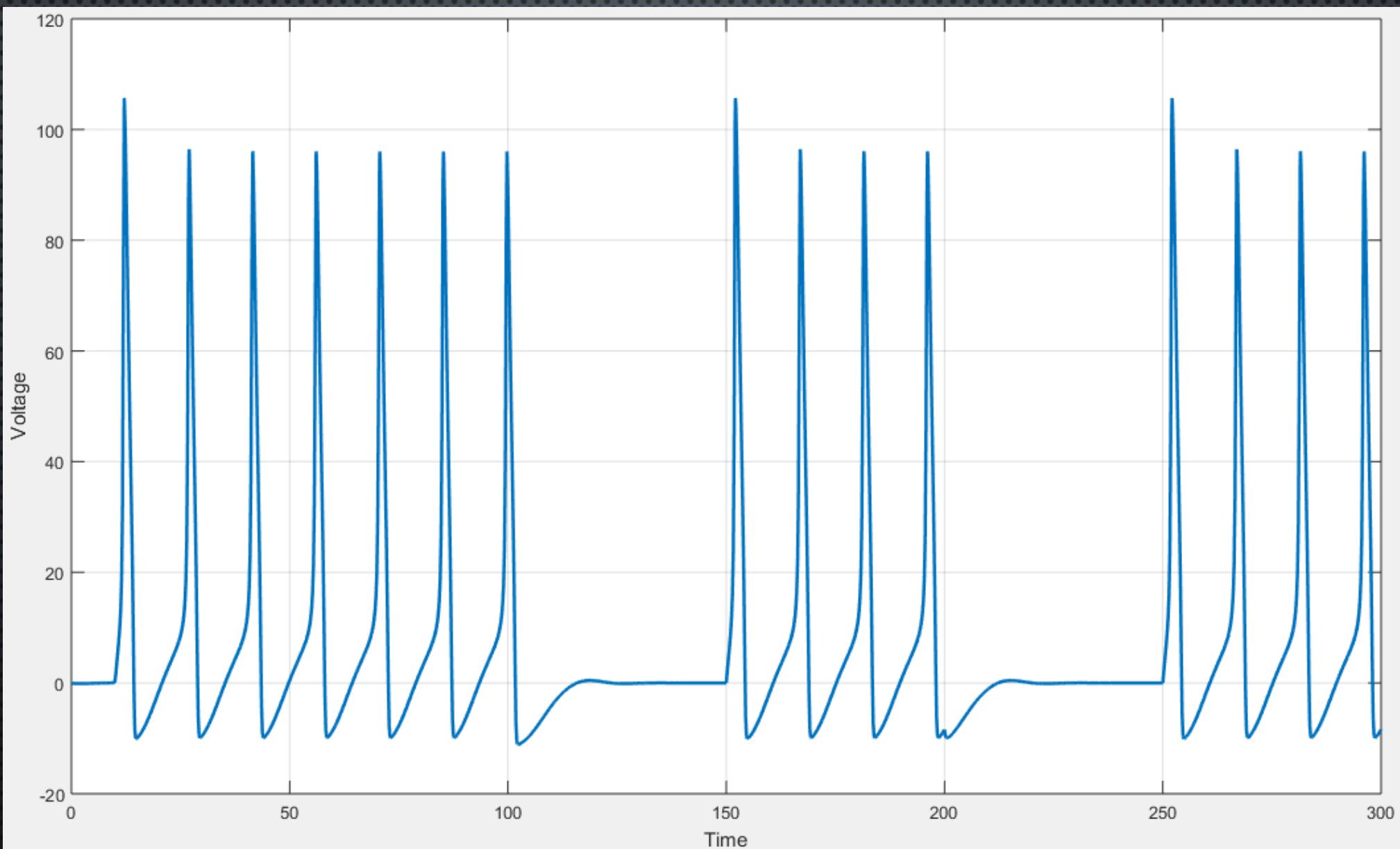
$$\tau_n(V) \frac{dn}{dt} = -[n - n_0(V)]$$

$$\tau_m(V) \frac{dm}{dt} = -[m - m_0(V)]$$

$$\tau_h(V) \frac{dh}{dt} = -[h - h_0(V)]$$



A biologically realistic neuron model: Hodgkin-Huxley model



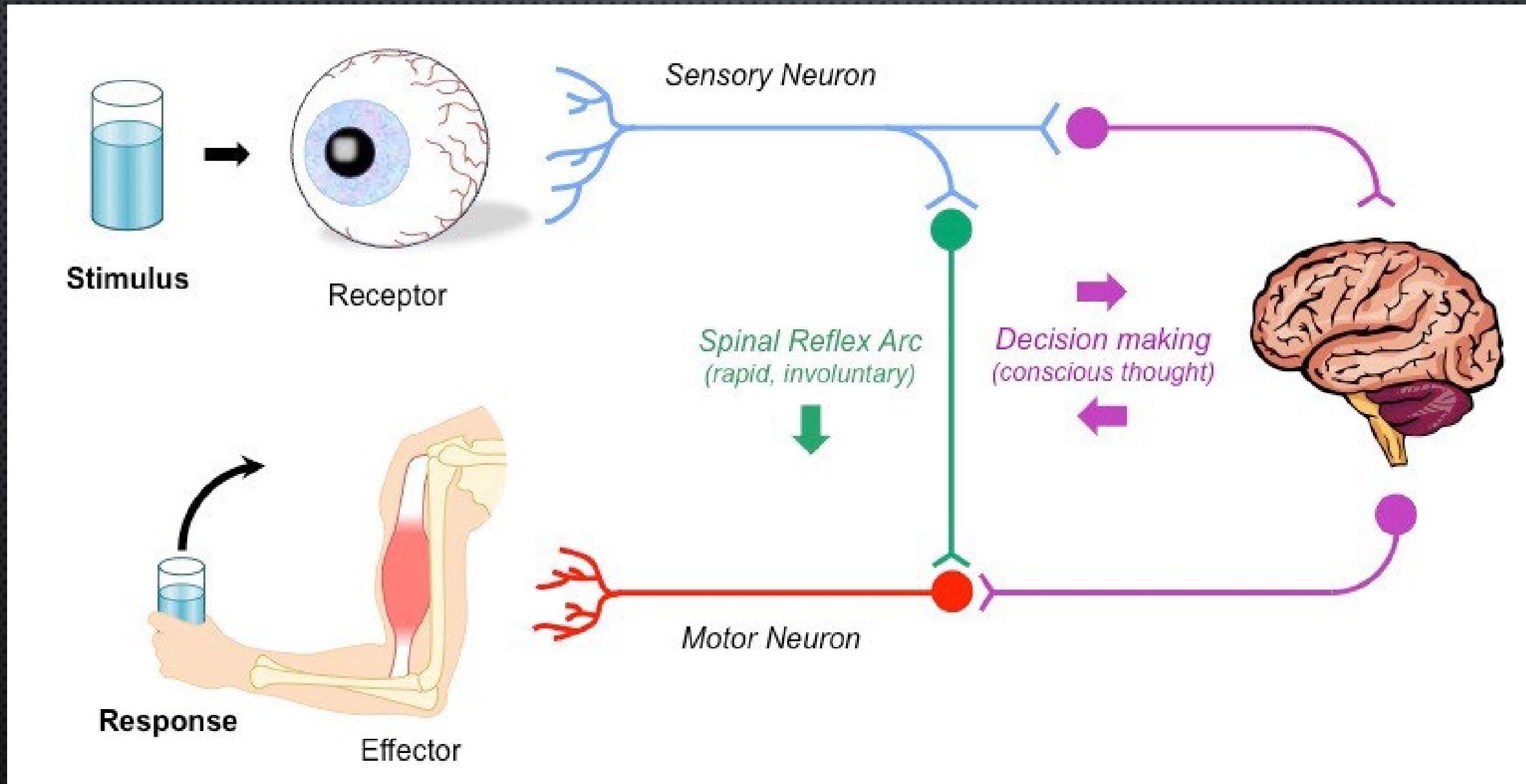
Matlab exercises

- Download “Assignment 1.pdf” from Itslearning and follow the instructions in it.



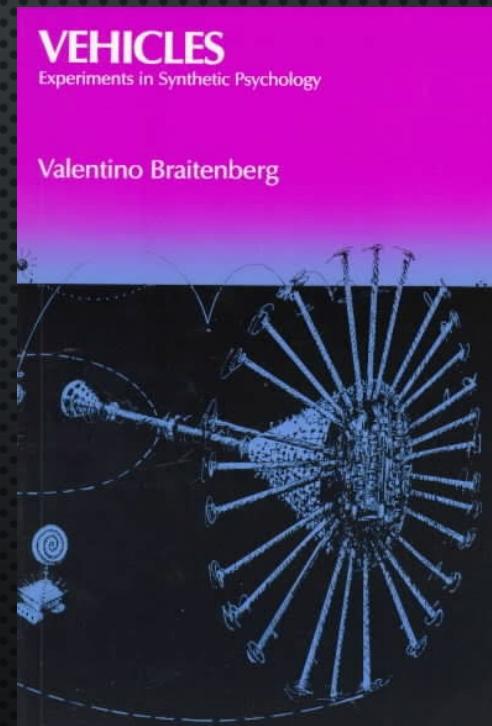
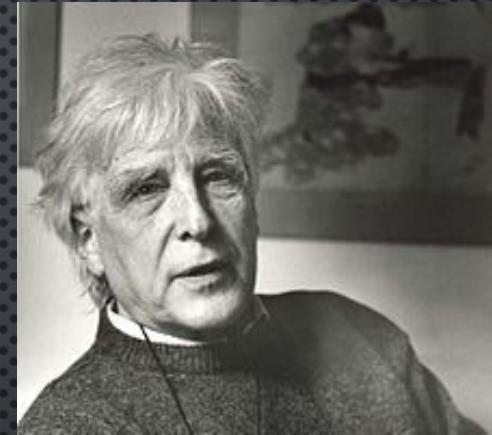
3 Simple artificial neural brains for sensing and control

A simple template for an artificial brain



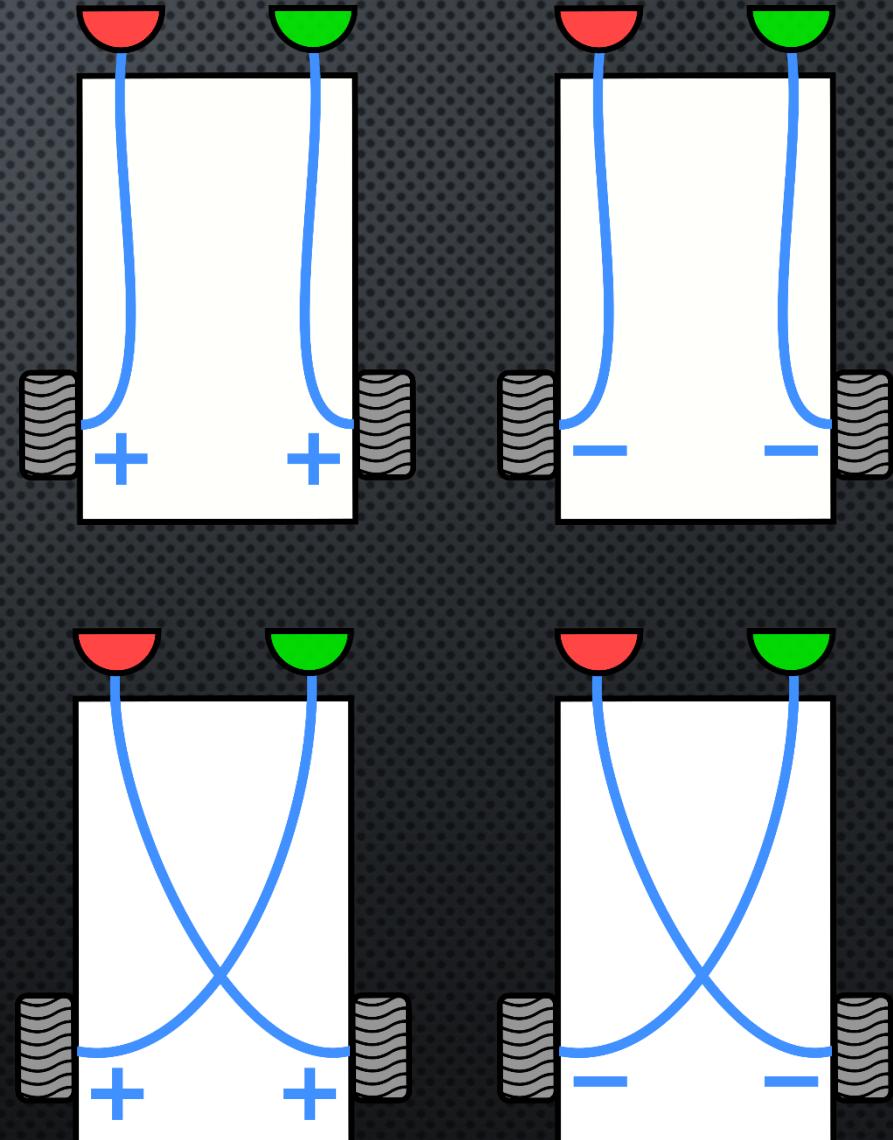
Braitenberg vehicles: a simple neurorobot

- Proposed by Valentino Braitenberg in 1984 book “Vehicles: Experiments in synthetic psychology”
- Exploring relation between structures and functions of the brain
- Hypothetical analogue vehicles (a combination of sensors, actuators and their interconnections)
- Vehicles displayed behaviours akin to aggression, love, fear, and exploration



Braitenberg vehicles: a simple neurorobot

- A sensor is directly connected to an actuator (e.g. light sensor → wheel motor)
- Sensorimotor connections can be ipsilateral or contralateral and excitatory or inhibitory
- Depending on how sensors and wheels are connected, the vehicle exhibits different movement behaviours
- In a complex environment with several sources of stimulus, vehicles exhibit complex and dynamic behaviour
- Functioning of the vehicle is purely mechanical, without any information processing or other apparently cognitive processes.



Some basic Braitenberg vehicles

$$v_{left} = k \times \frac{1}{s_{right}}$$

$$v_{right} = k \times \frac{1}{s_{left}}$$

“Exploration”

OR

$$v_{left} = k \times (s_{max} - s_{right})$$

$$v_{right} = k \times (s_{max} - s_{left})$$

$$v_{left} = \frac{1}{k} \times \frac{1}{s_{left}}$$

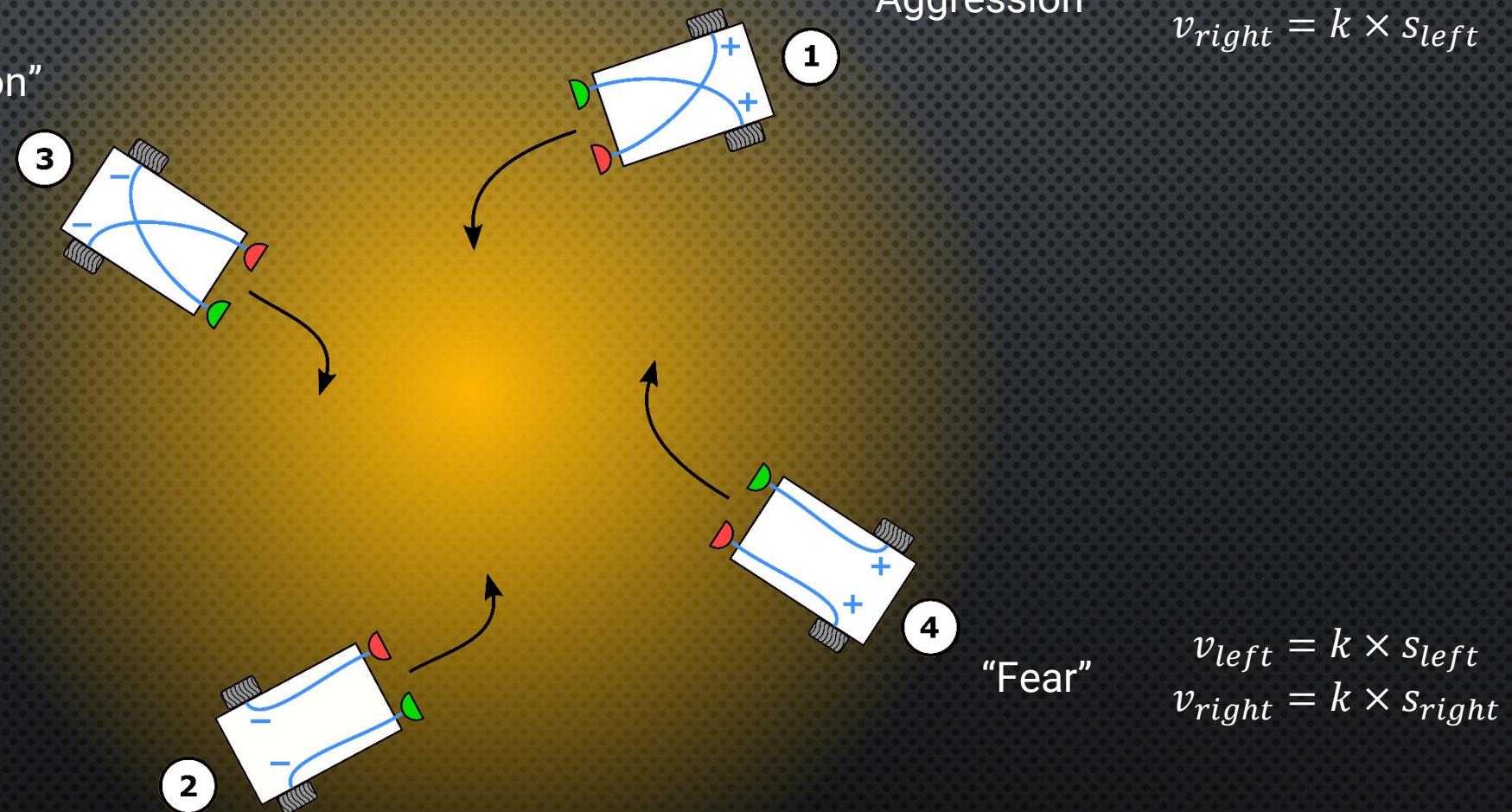
$$v_{right} = \frac{1}{k} \times \frac{1}{s_{right}}$$

OR

$$v_{left} = k \times (s_{max} - s_{left})$$

“Love”

$$v_{right} = k \times (s_{max} - s_{right})$$



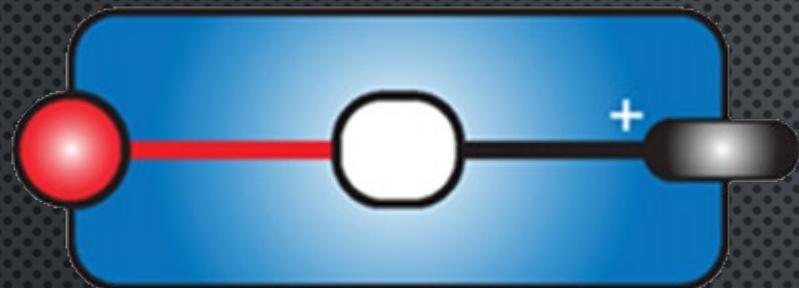
$$v_{left} = k \times s_{right}$$

$$v_{right} = k \times s_{left}$$

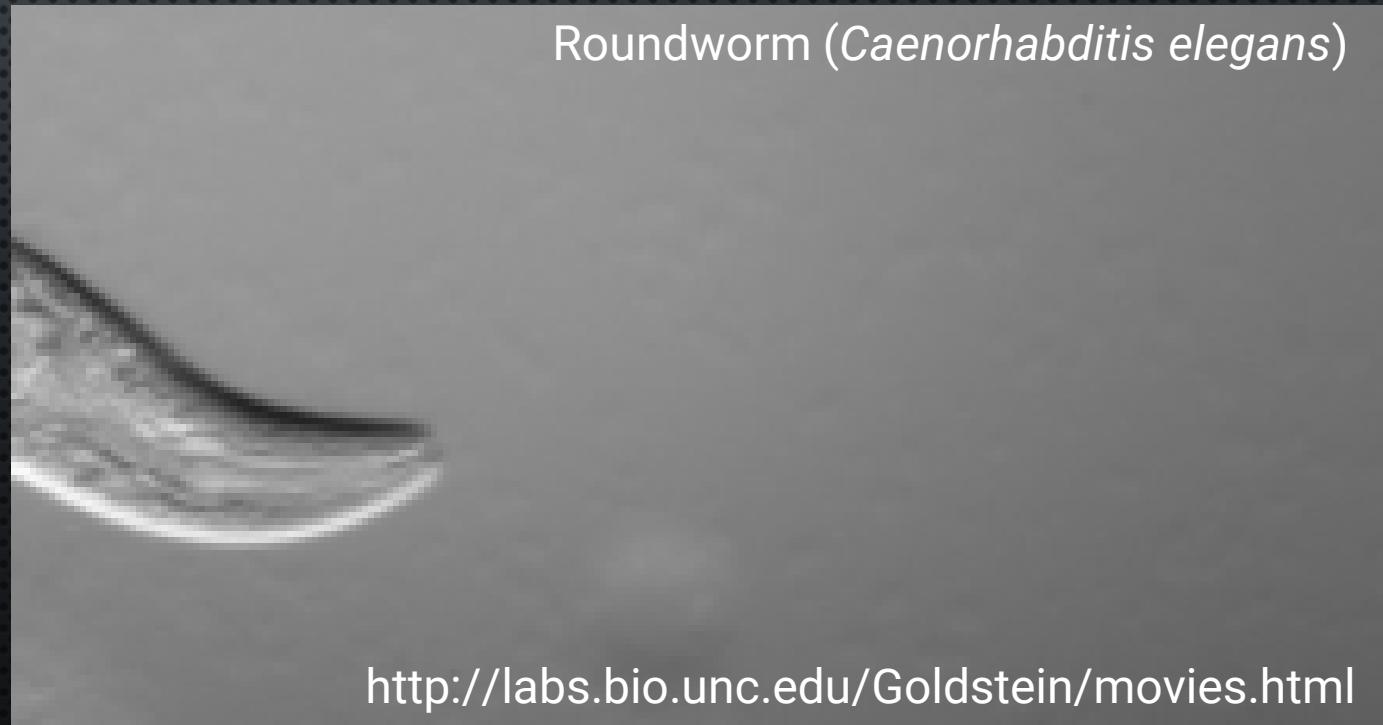
$$v_{left} = k \times s_{left}$$

$$v_{right} = k \times s_{right}$$

Braitenberg vehicles as animal models

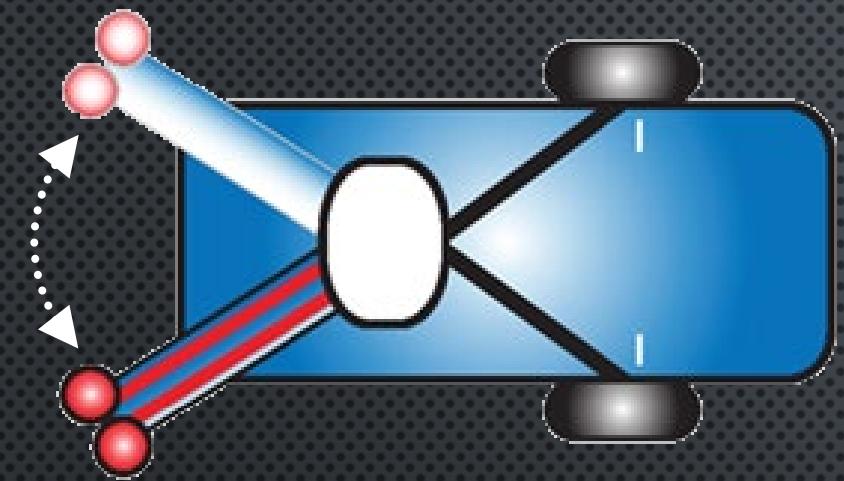


Alex Gomez-Marin et al., 2010



Roundworm (*Caenorhabditis elegans*)

Braitenberg vehicles as animal models



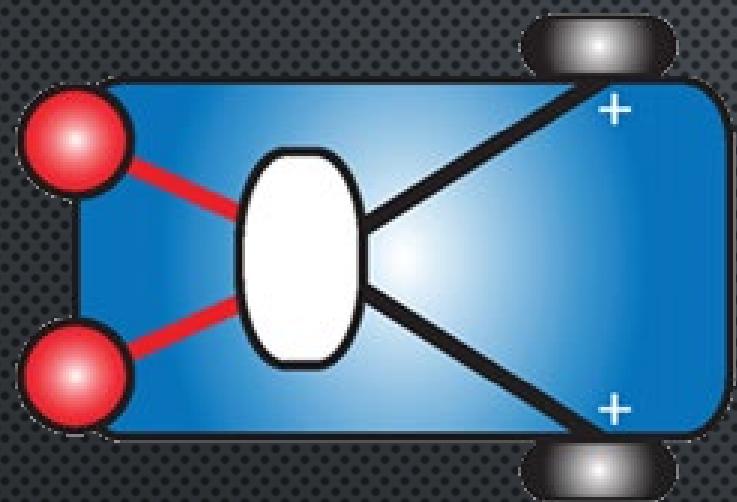
Alex Gomez-Marin et al., 2010



Drosophila melanogaster or common fruit fly, larval stage



Braitenberg vehicles as animal models

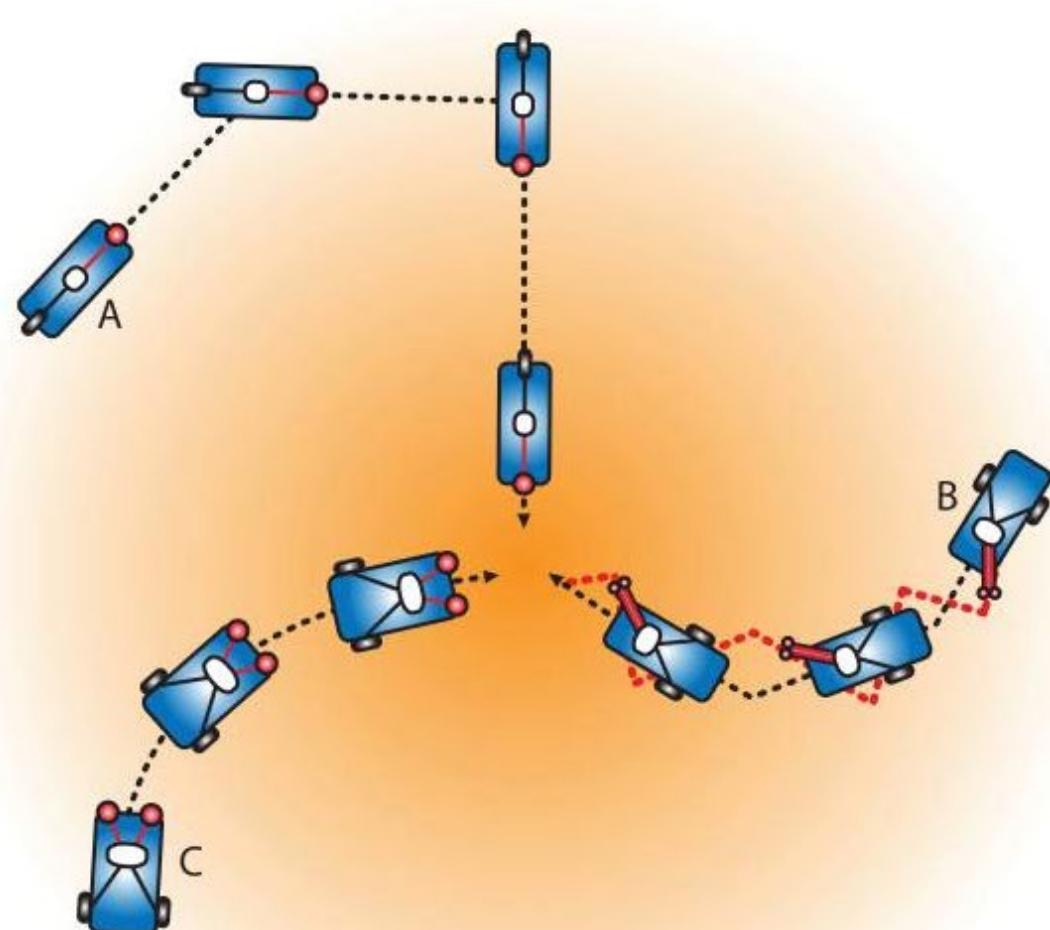
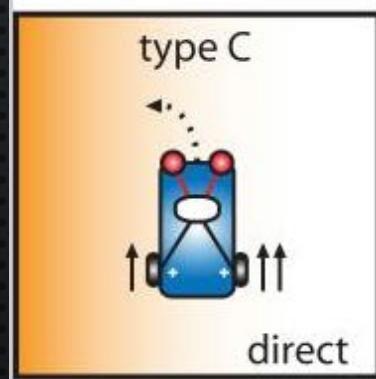
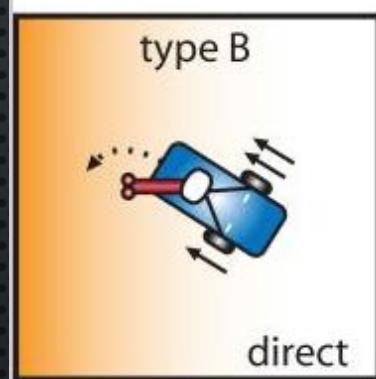
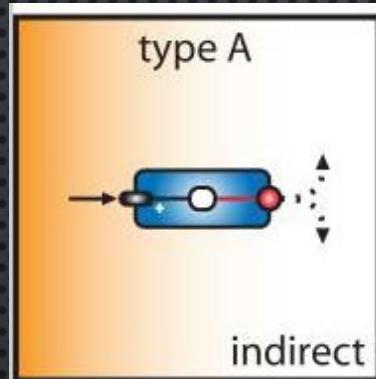


Alex Gomez-Marin et al., 2010



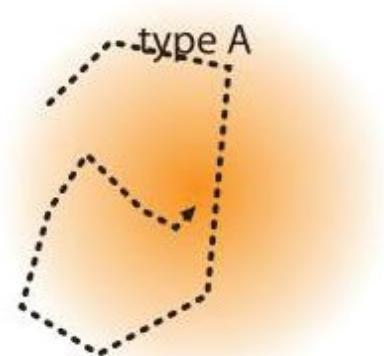
Drosophila melanogaster or
common fruit fly, adult stage

Braitenberg vehicles as animal models



low high odor

Alex Gomez-Marin et al., 2010



type B



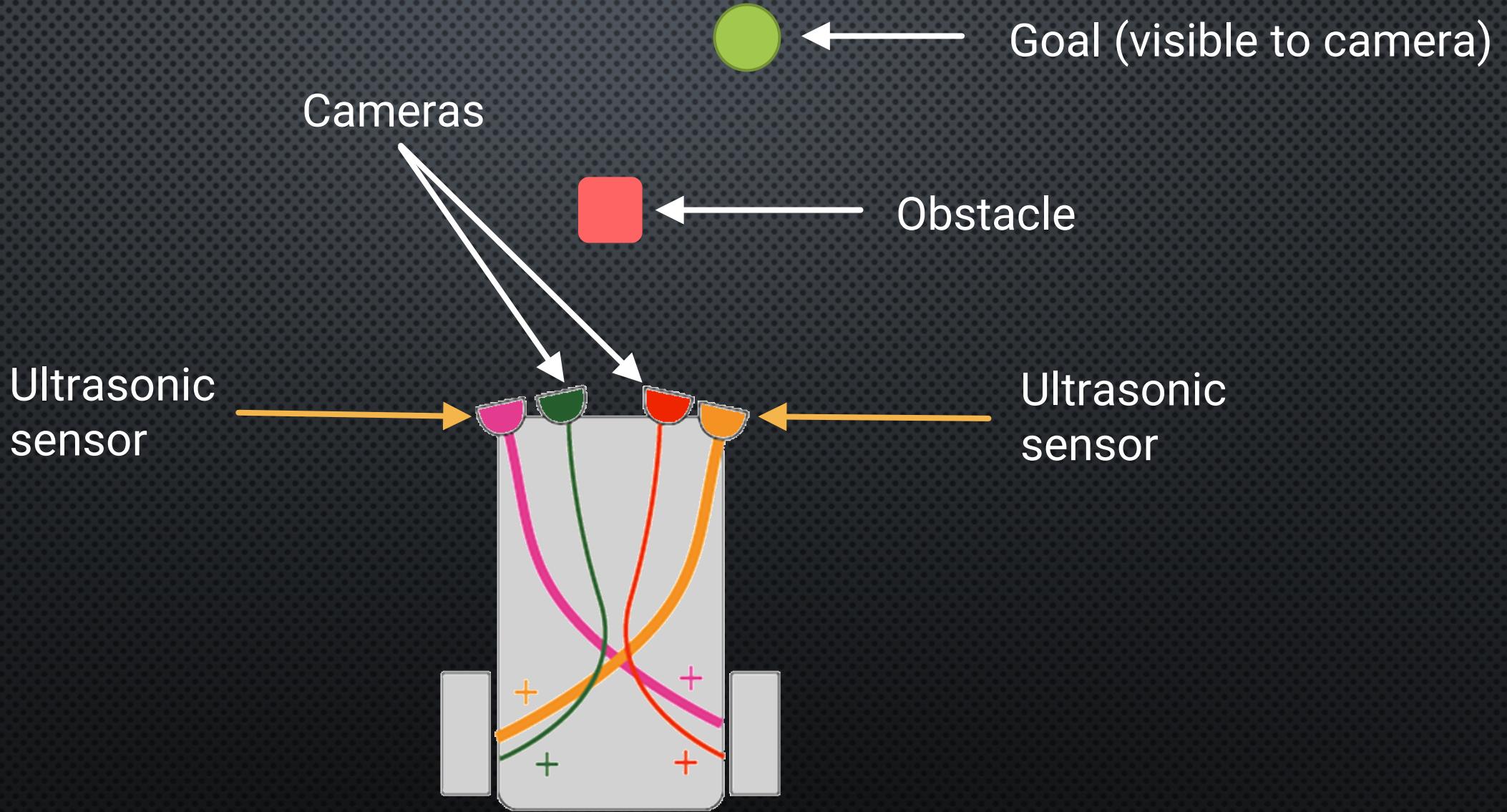
type C

Some examples

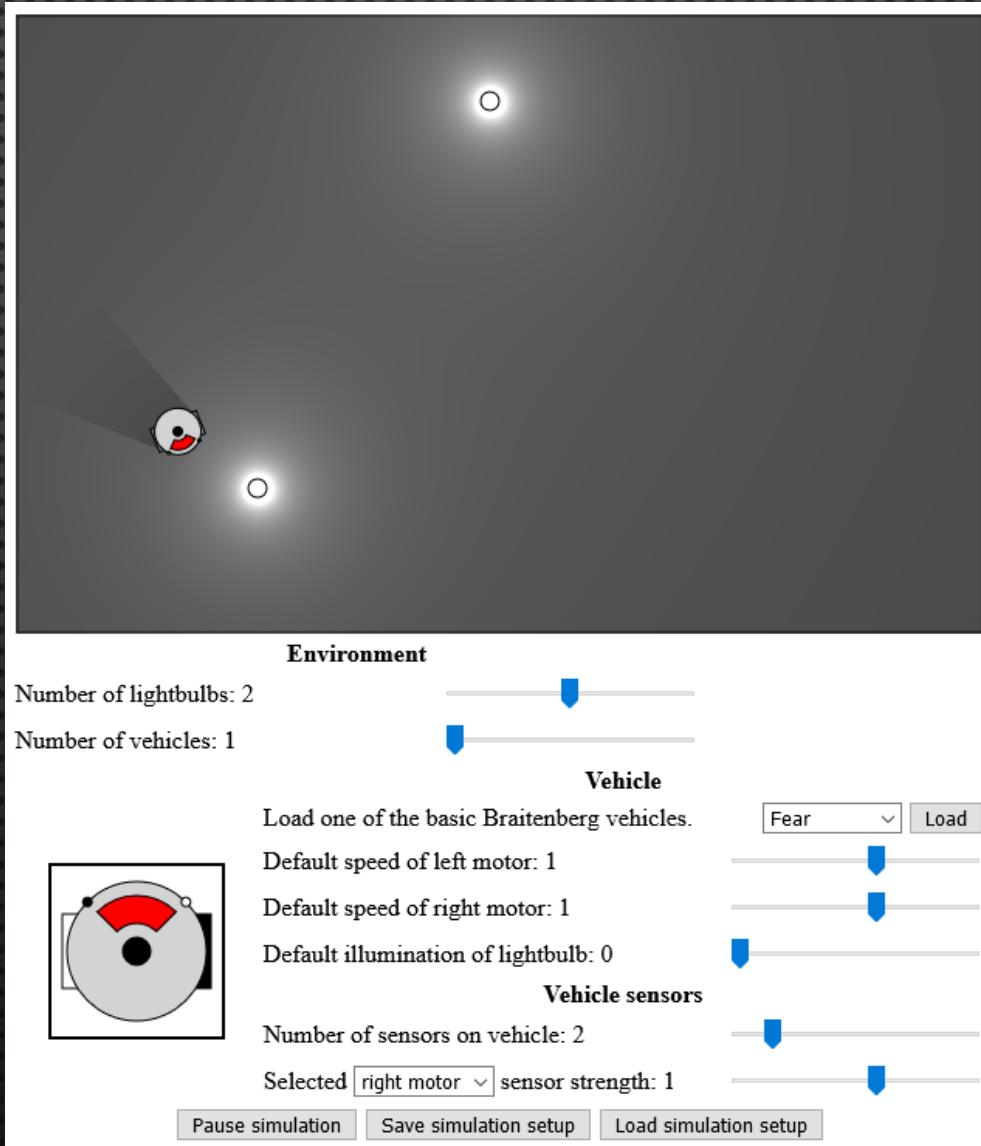
LEGO NXT Braitenberg Vehicle 2

From Bricks to Brains:
Chapter 4: "Braitenberg's Vehicle 2"

How will this vehicle behave?



Play time: Braitenberg vehicle playground



<http://www.harmendeweerd.nl/braitenberg-vehicles/>

A case for Braitenberg controllers

The artificial solution



<https://www.youtube.com/watch?v=umRdt3zGgpU>

The natural solution



https://www.youtube.com/watch?v=p-_RHRAzUHM

Case study 1

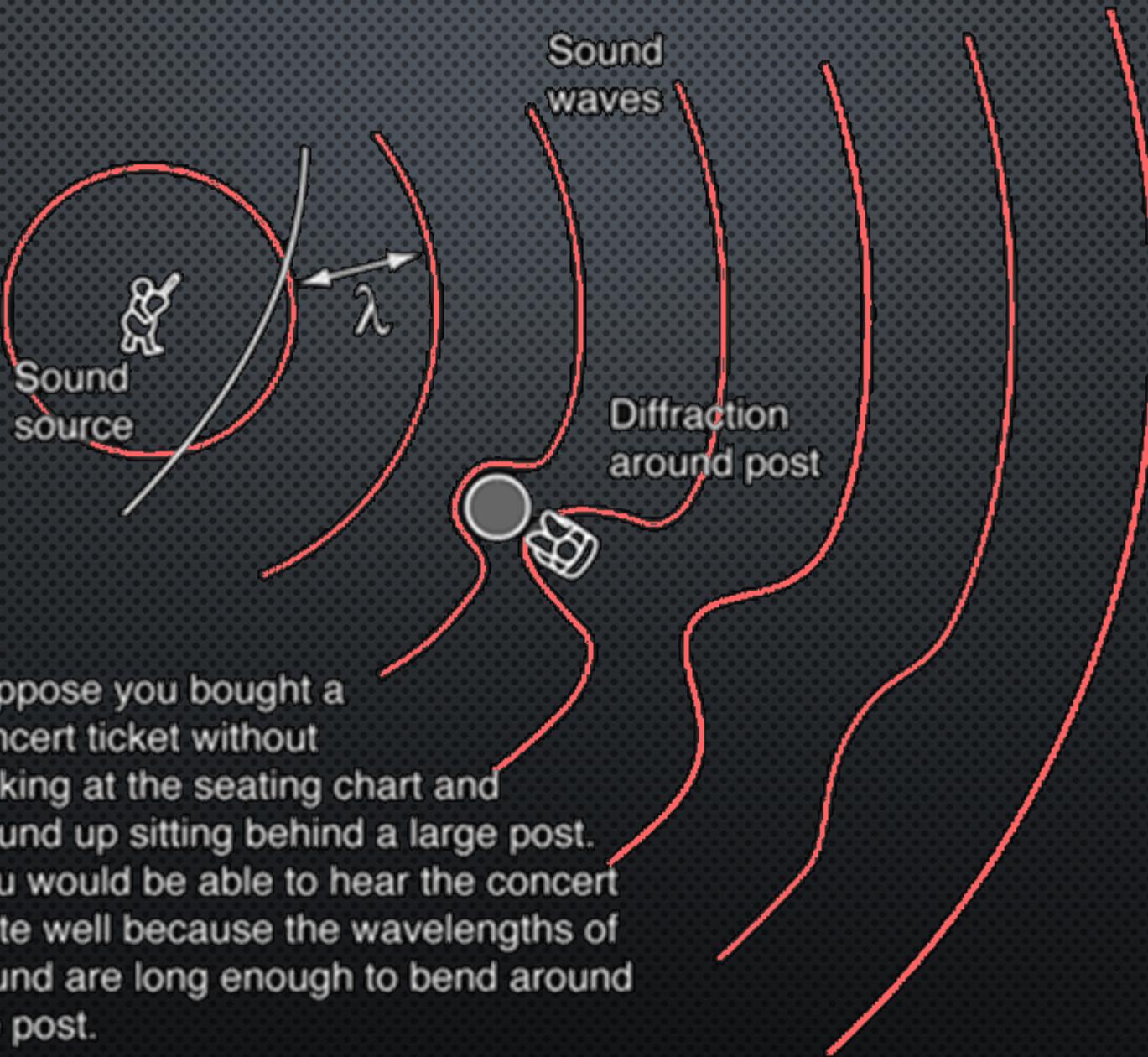
A lizard-inspired sound localising robot

Lizard audition



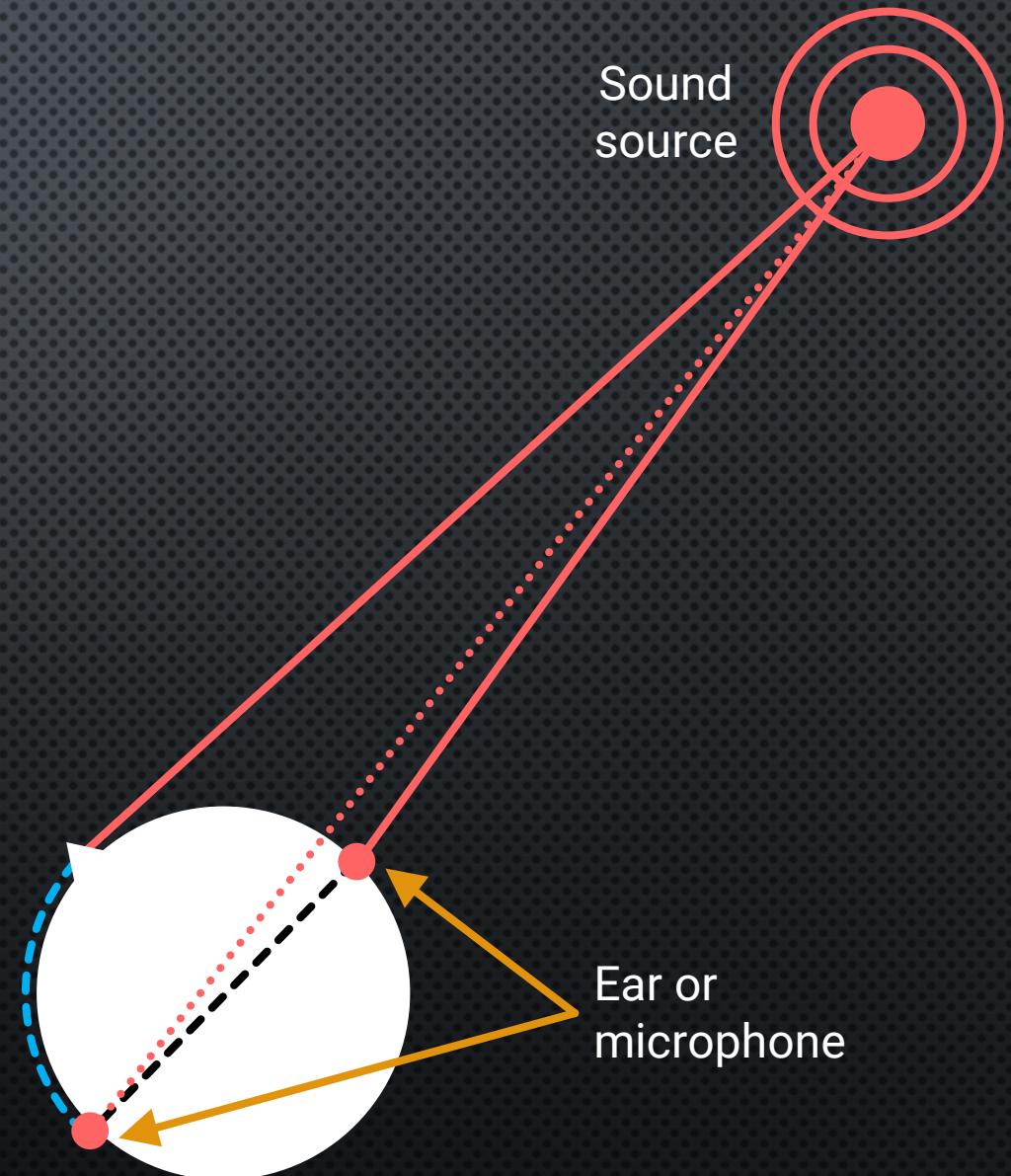
wildlife
singapore
www.wildsingapore.per.sg

Ecological niche: physics of sound propagation



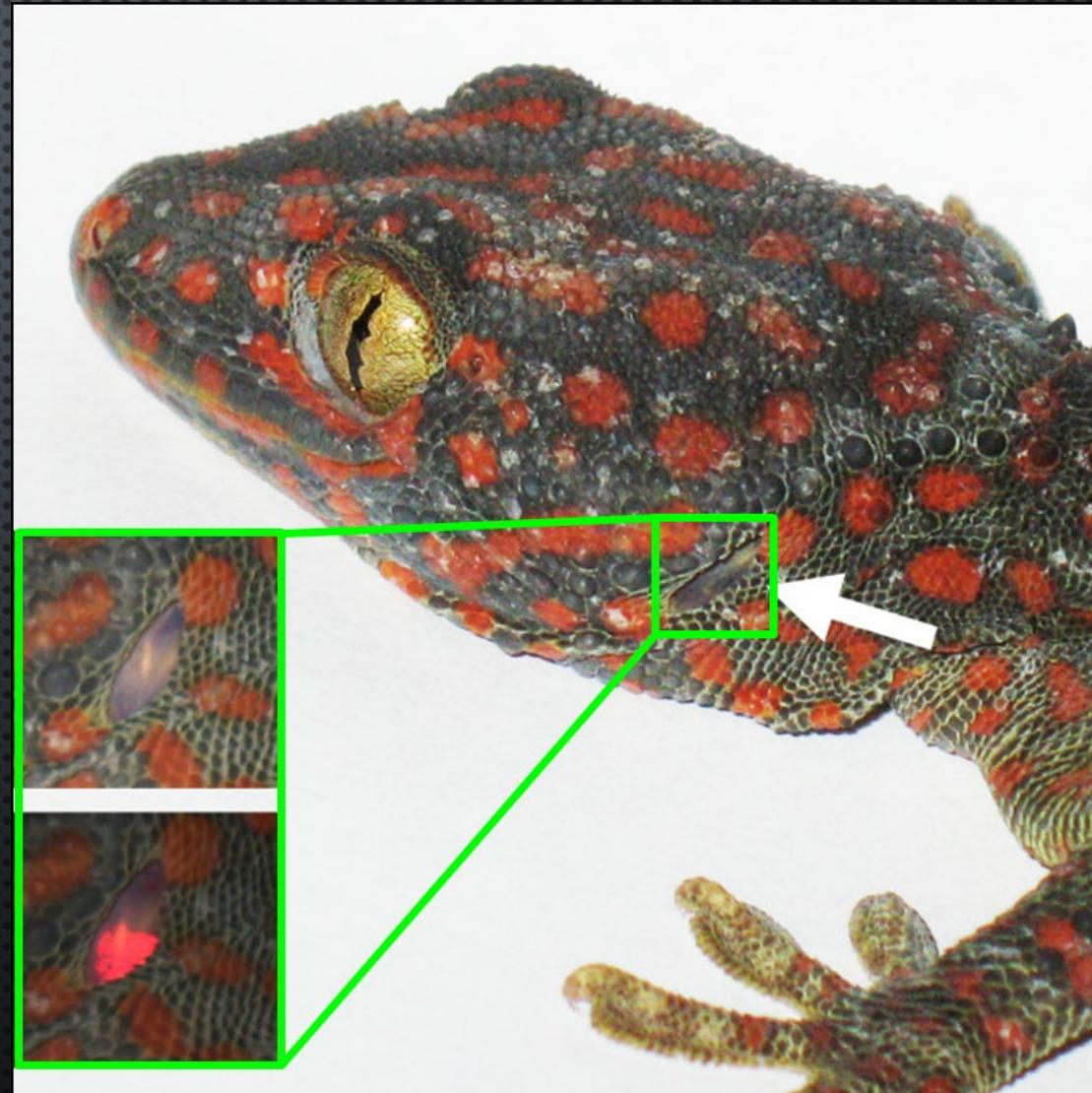
Basics of acoustics: diffraction

- Two ways to determine sound direction
 - Inter-aural **level difference** (ILD)
 - Inter-aural **time difference** (ITD)
- ILD = difference in sensed loudness (in dB) of sound at each ear
- ITD = difference in time of arrival (in μs) of sound at each ear
- If sound frequency is high (**wavelength > distance between ears**), ILD is significant
- If sound frequency is low (**wavelength < distance between ears**) ITD is significant



Characteristics of lizard hearing

- Small size (10–20 mm) w.r.t sound wavelengths (85–340 mm) → **sound diffracts around the head**
- Diffraction leads to small inter-aural level difference (ILD) (typically 1-2 dBs → **too small to measure without special, big, heavy and expensive equipment**)
- Tiny (μ s scale) inter-aural time difference (ITD) information available to determine sound direction

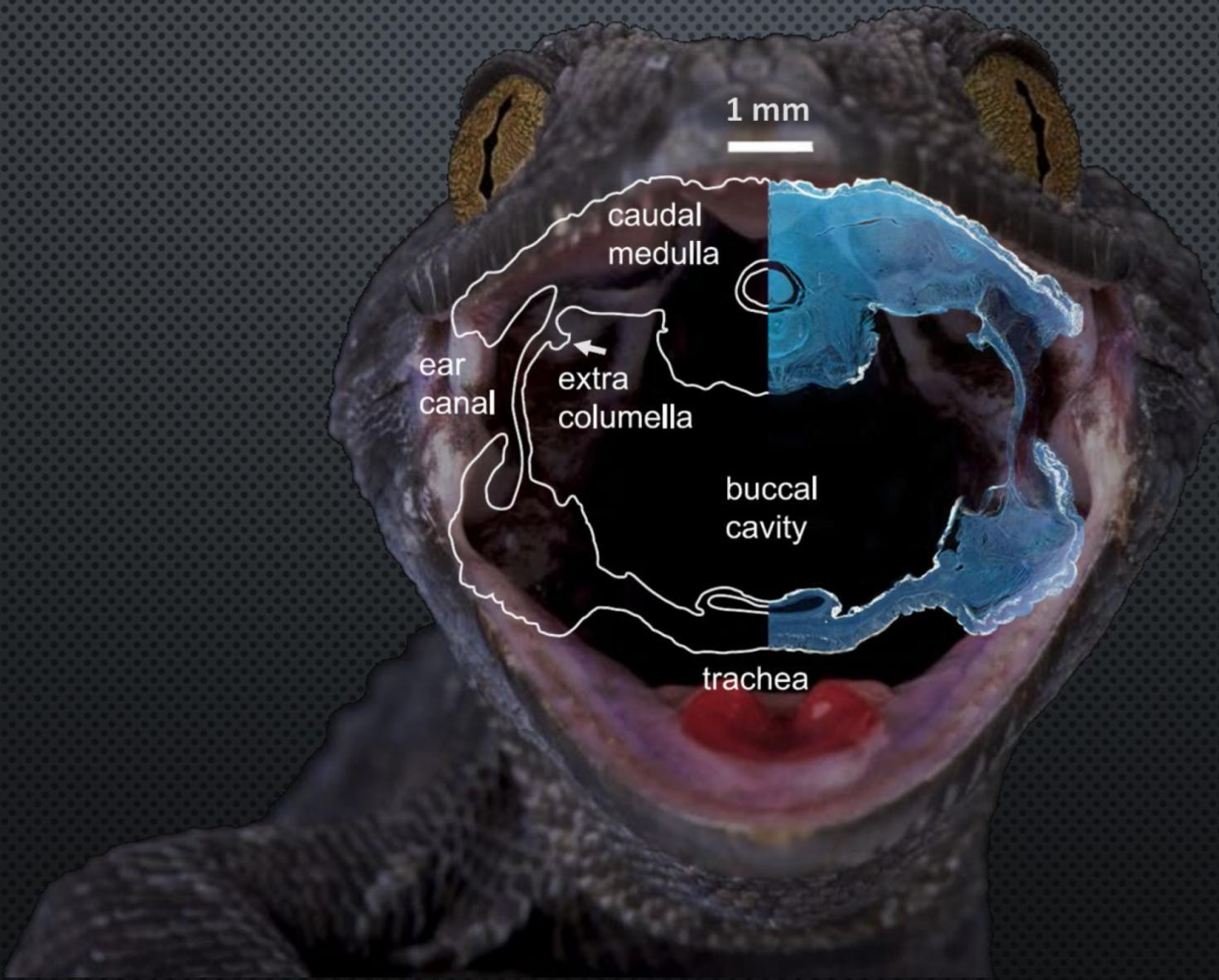


Auditory directional cues

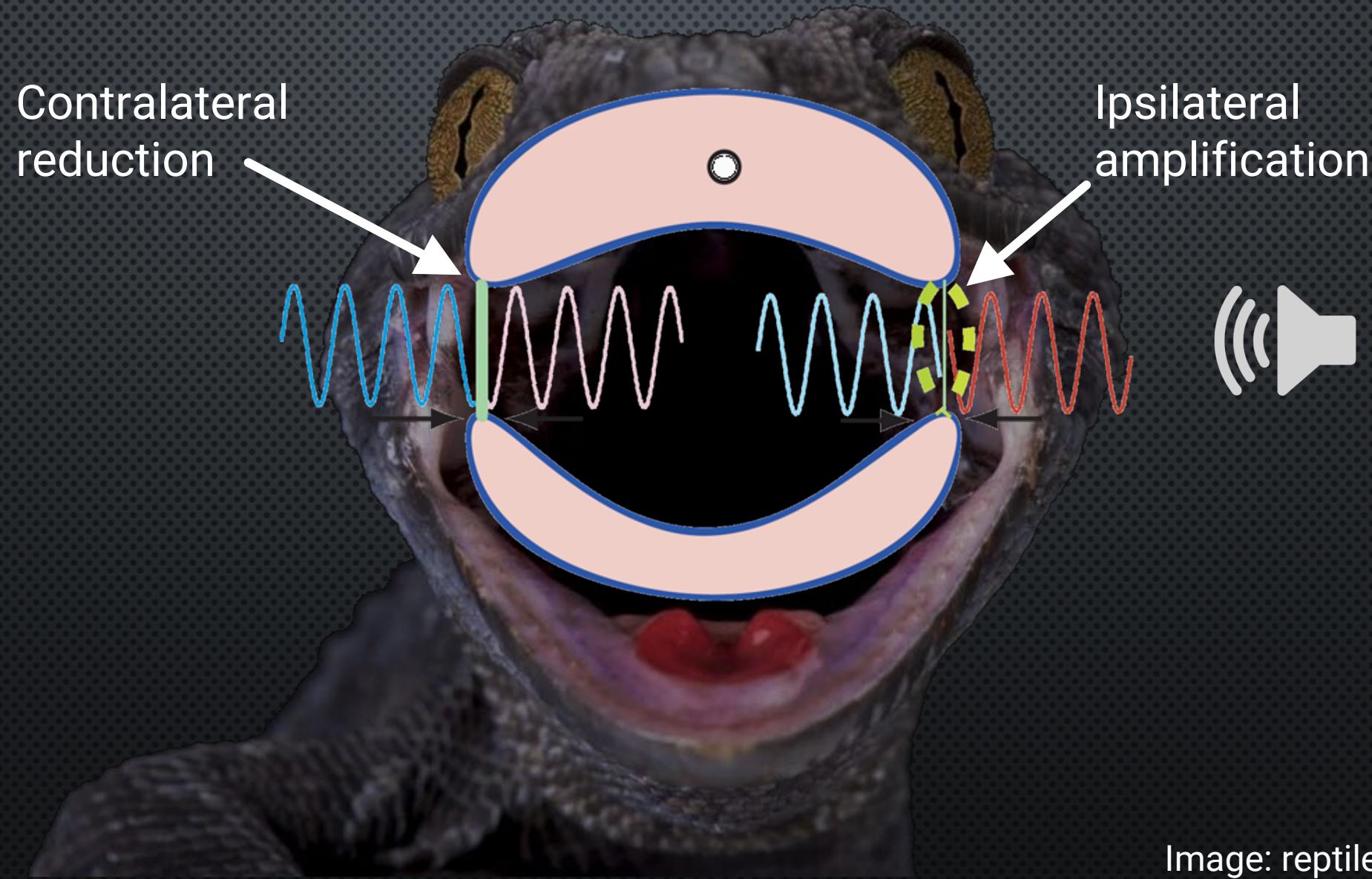


Image: reptilepark.com.au

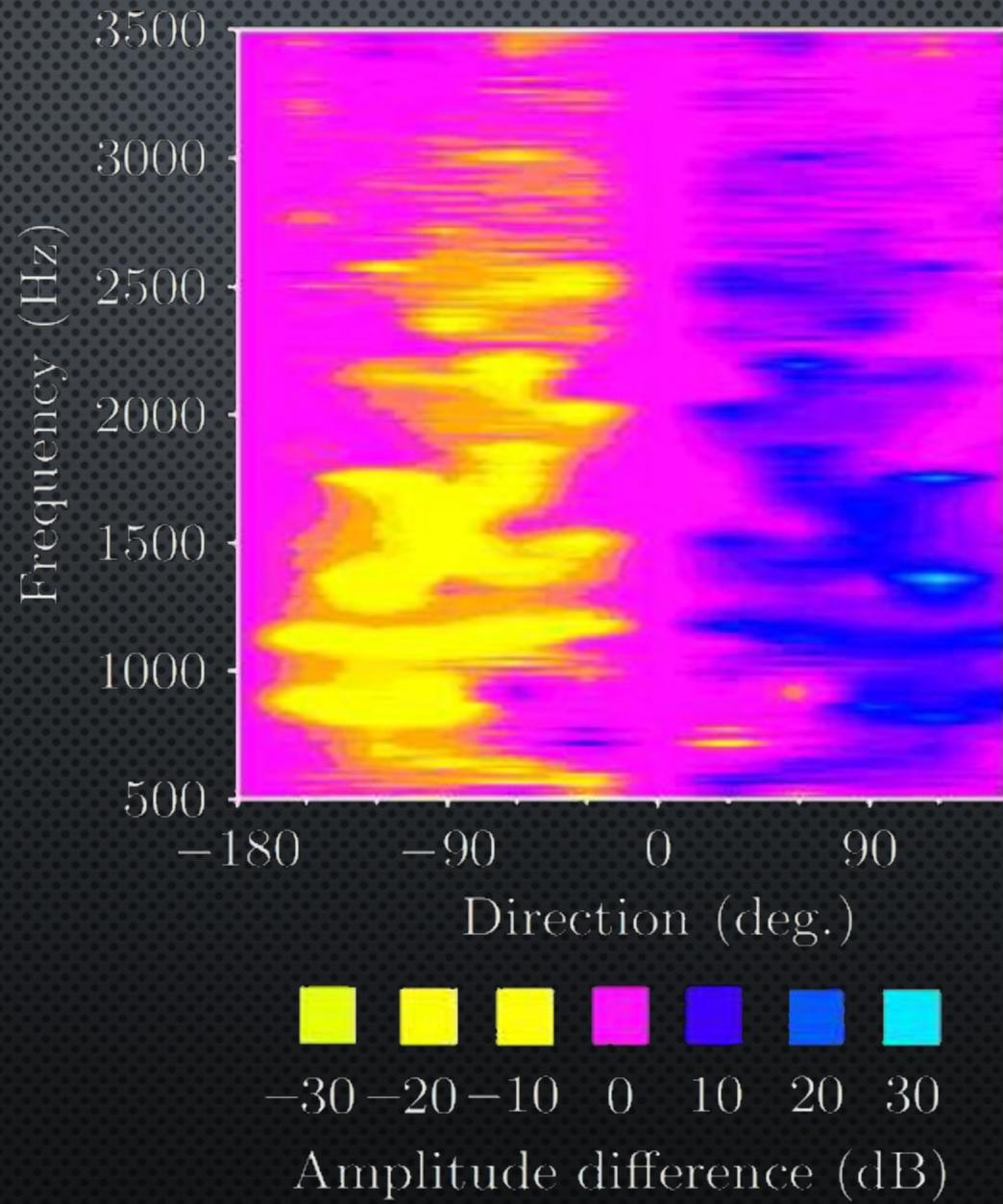
Auditory directional cues



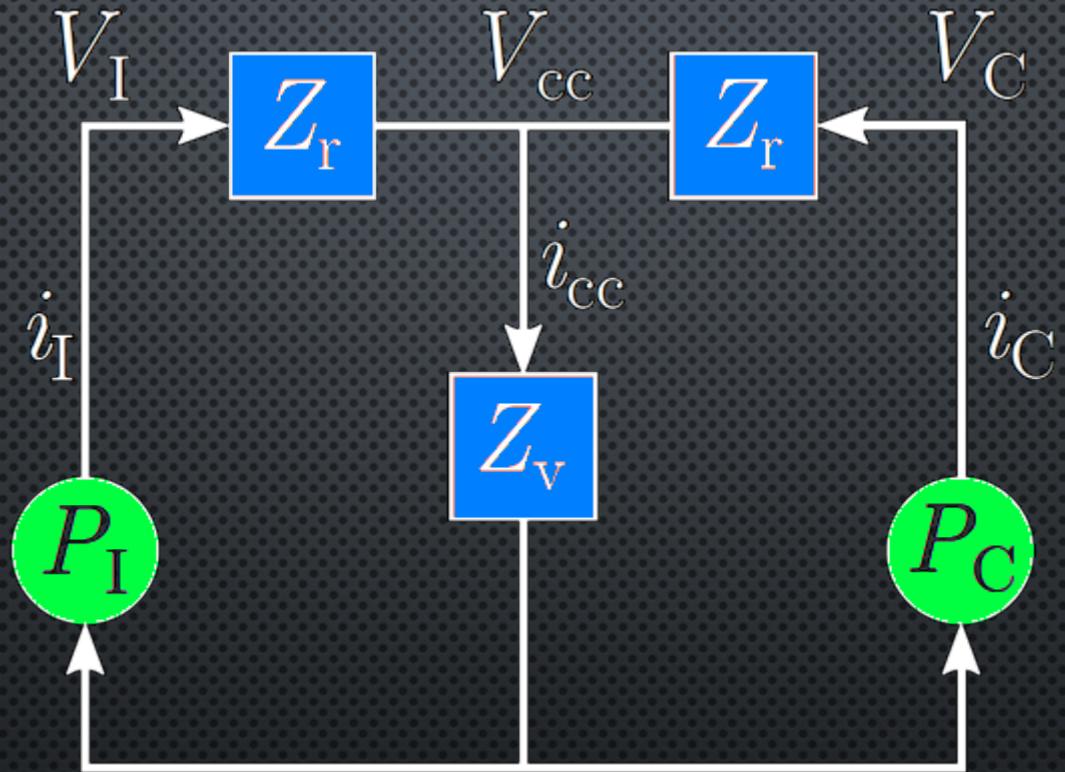
Auditory directional cues



Sound direction information

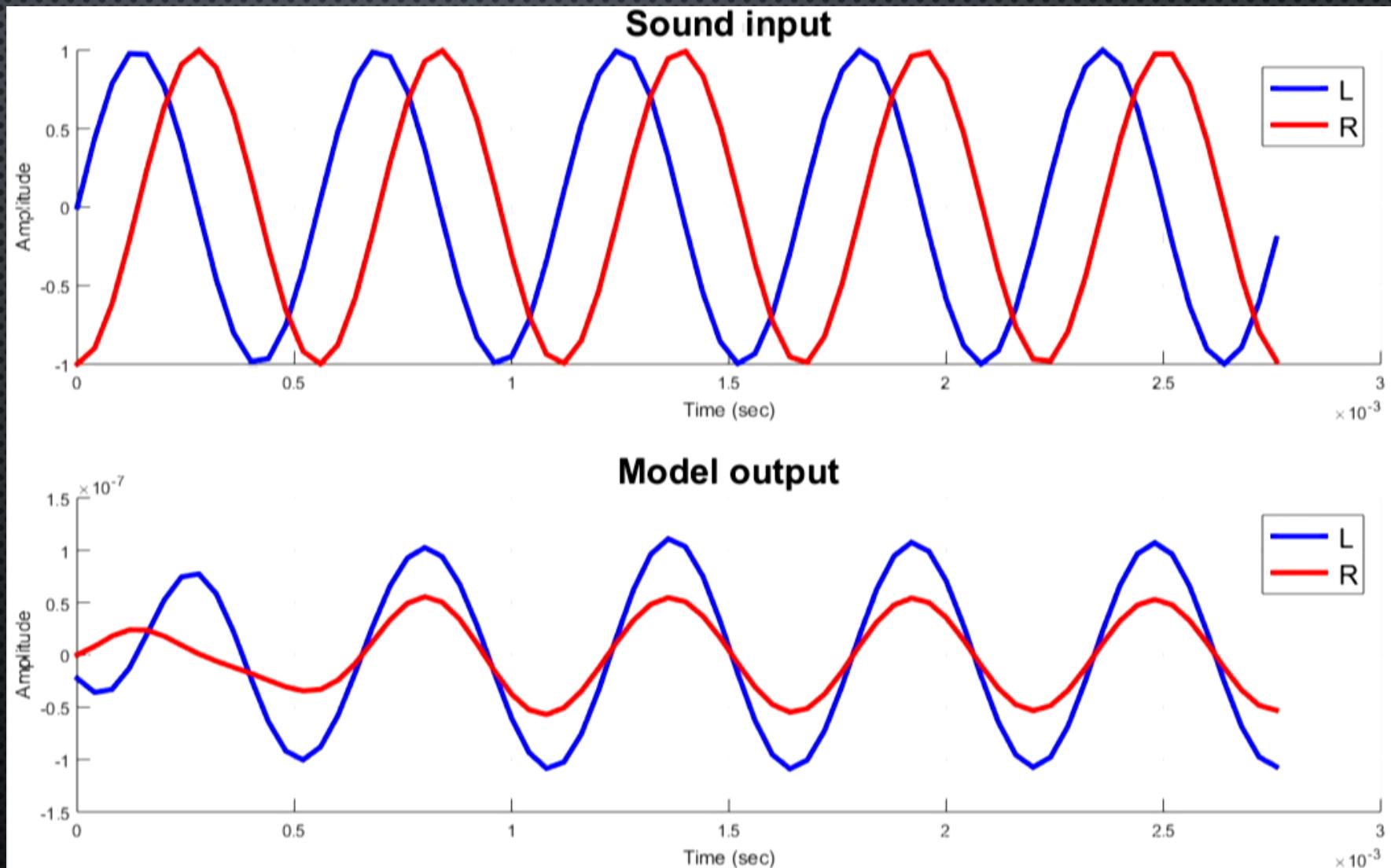
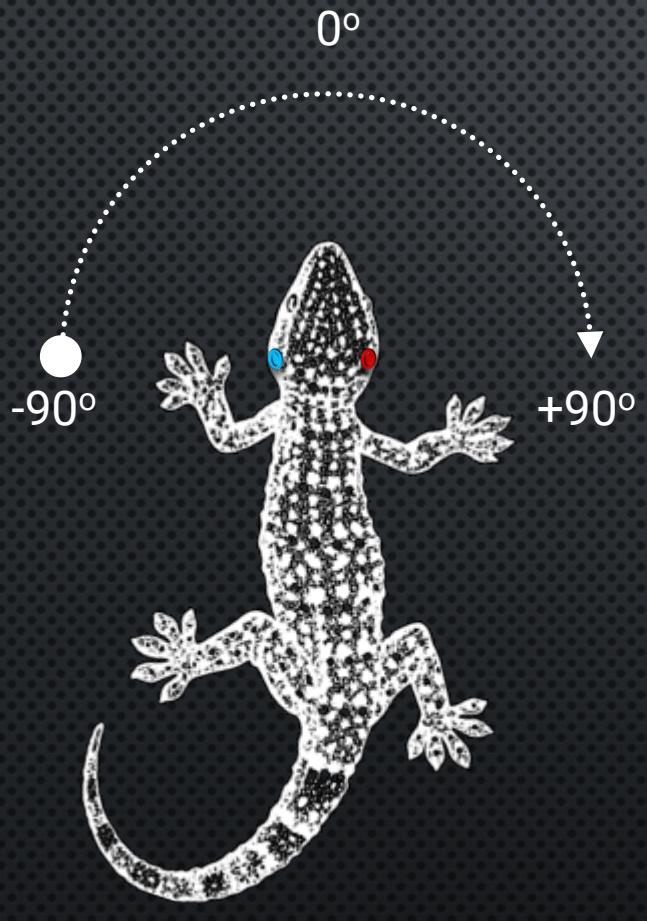


Sensor model

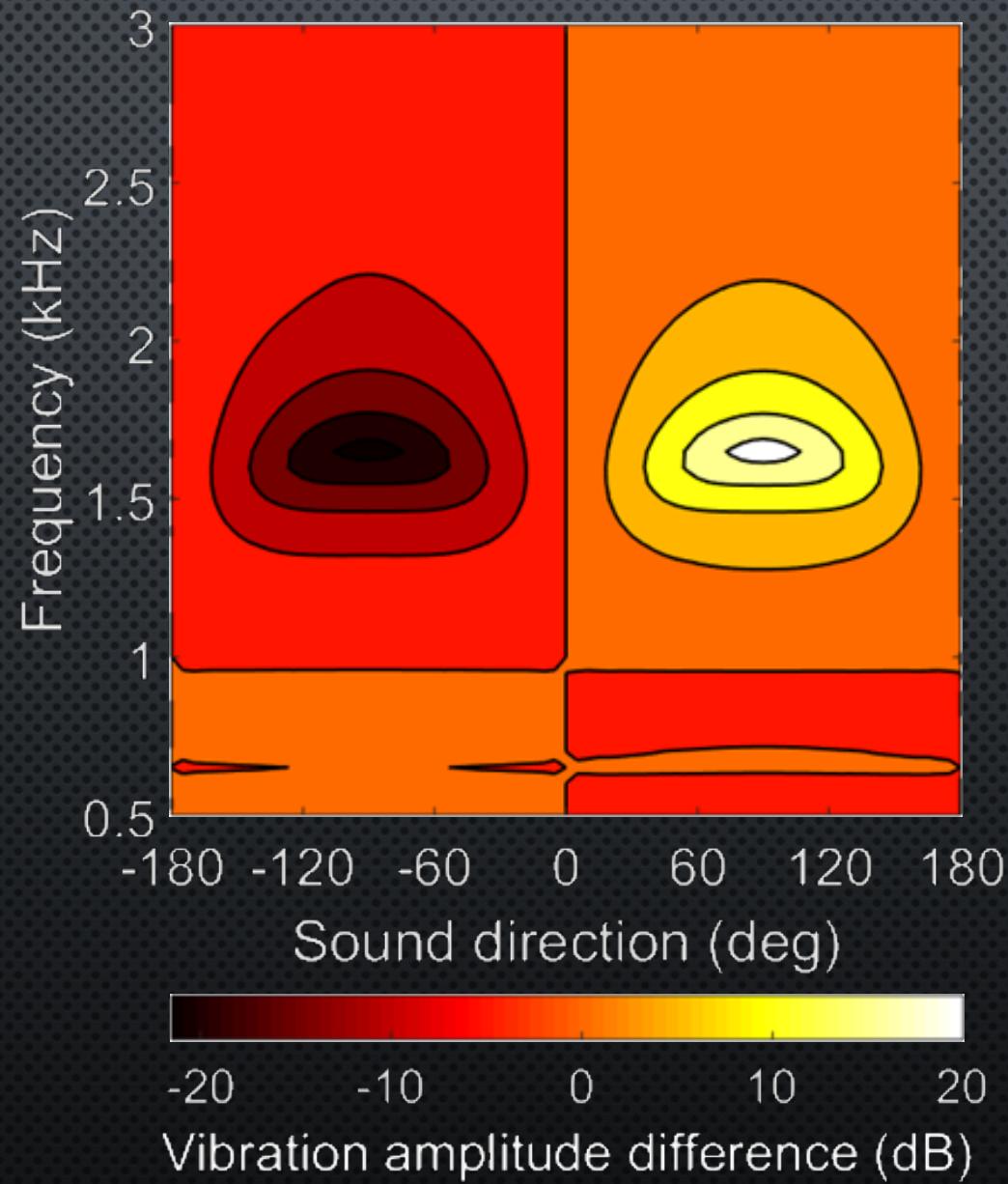


$$\left| \frac{i_I}{i_C} \right| = \left| \frac{G_I \cdot V_I + G_C \cdot V_C}{G_C \cdot V_I + G_I \cdot V_C} \right| \equiv 20 (\log |i_I| - \log |i_C|) \text{ dB}$$

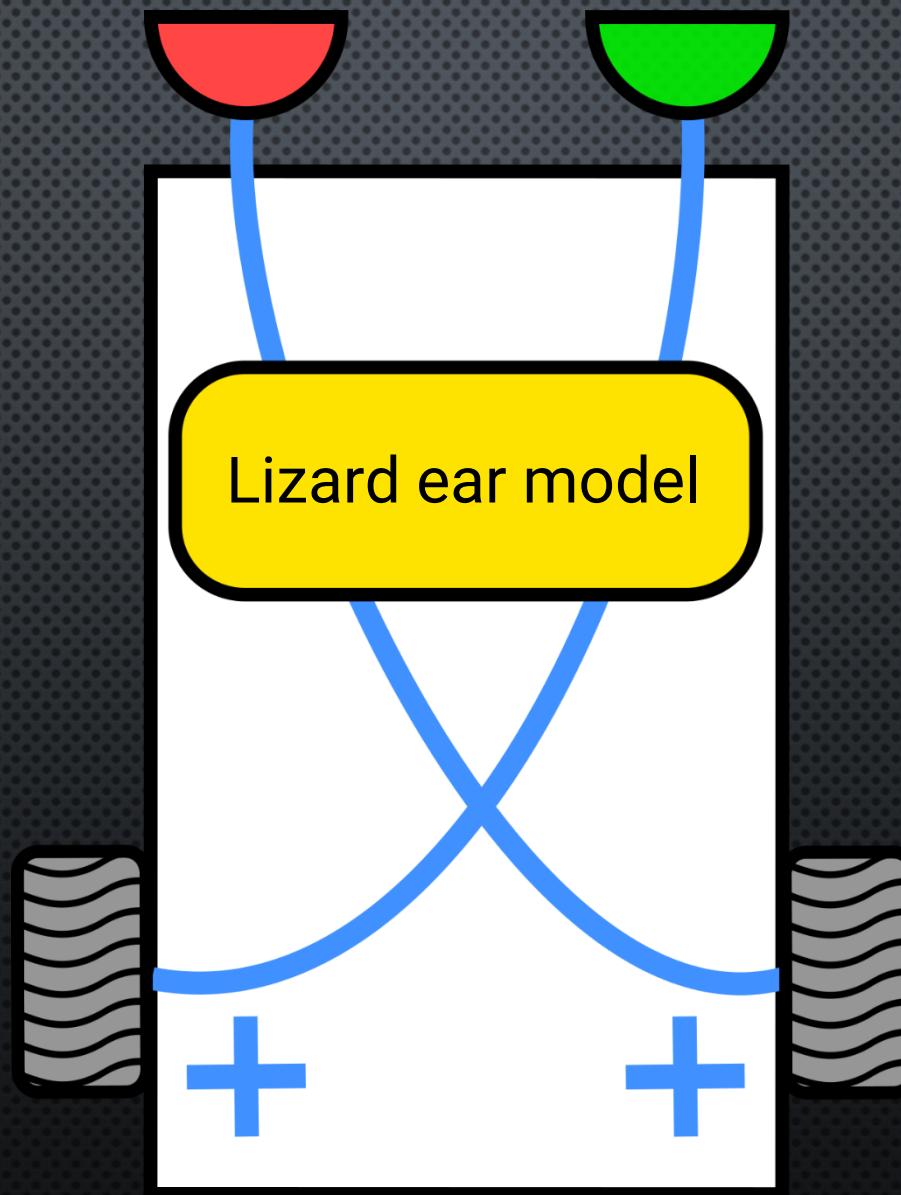
Model response in the azimuth



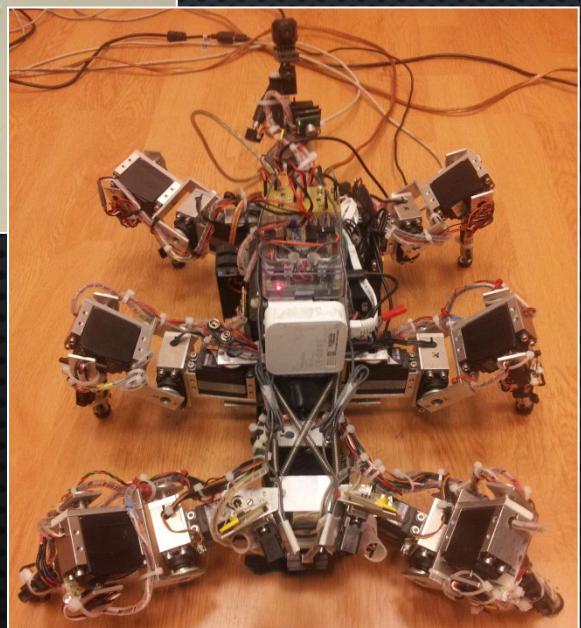
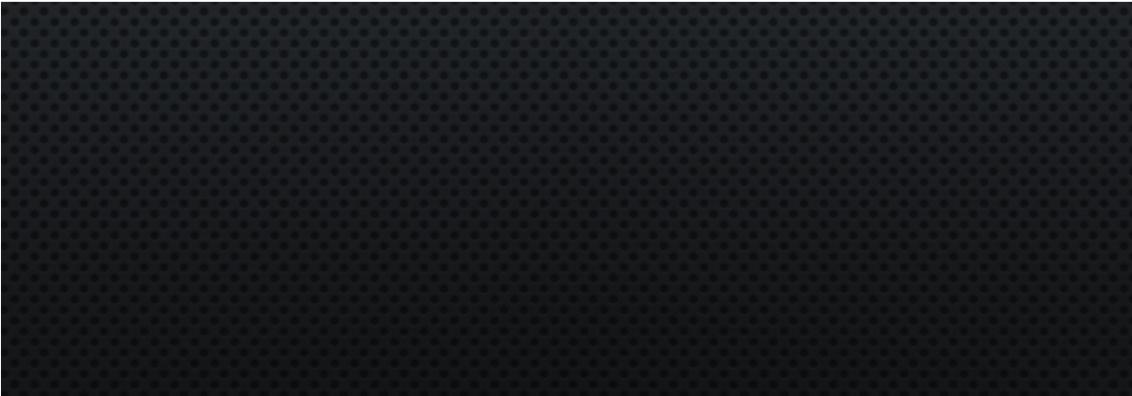
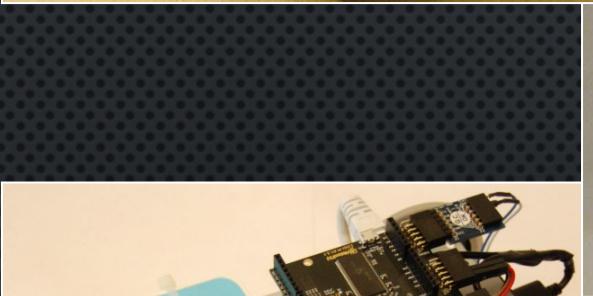
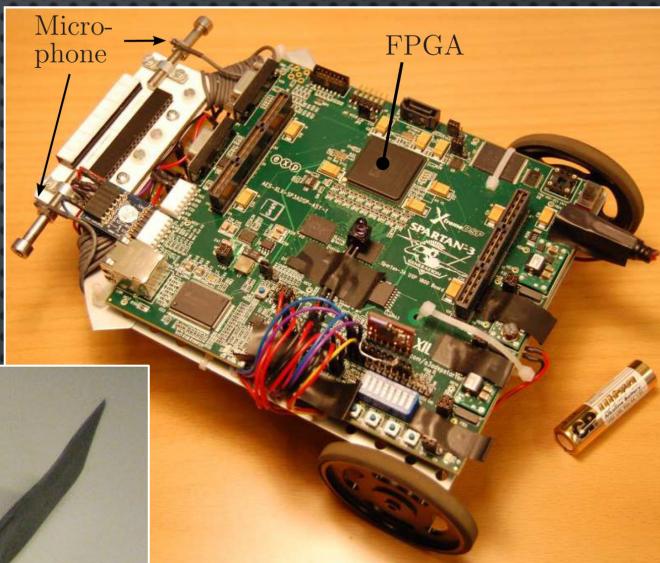
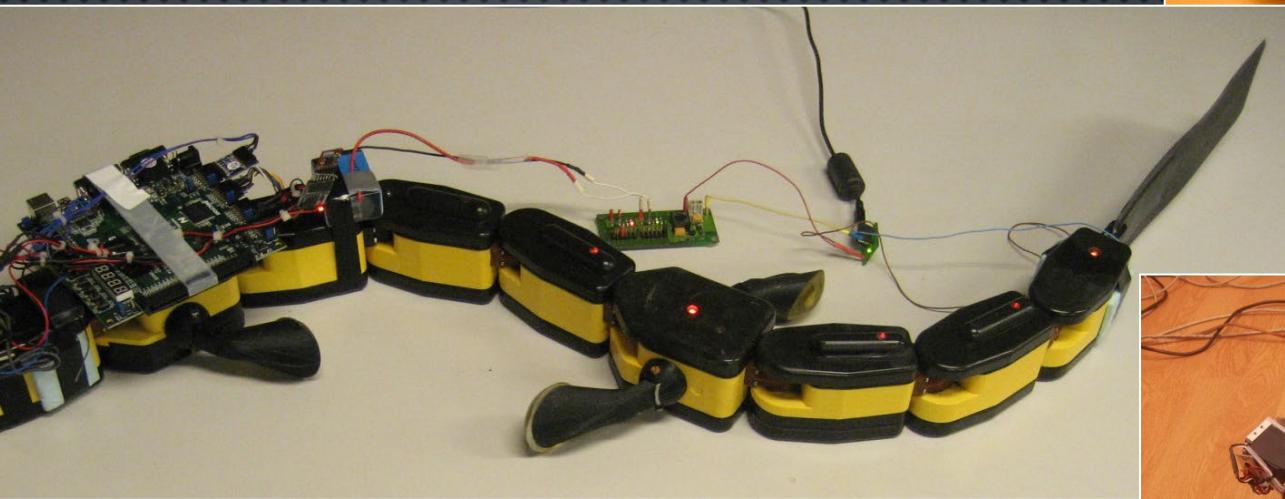
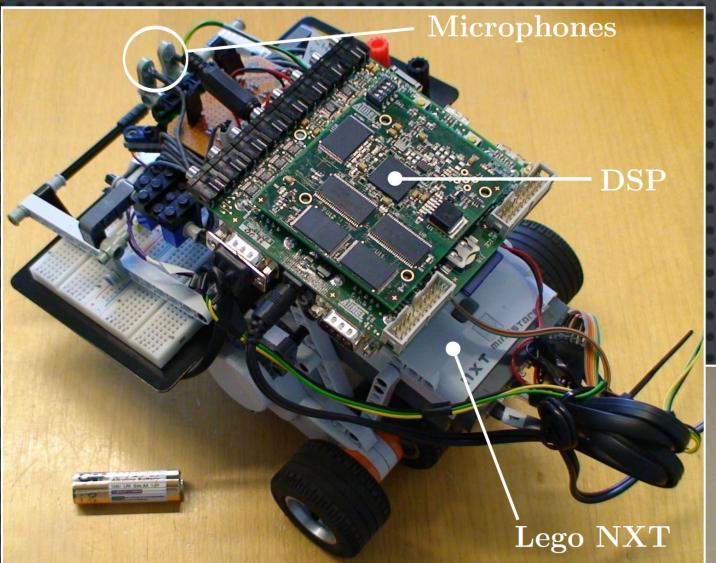
Model response in the azimuth



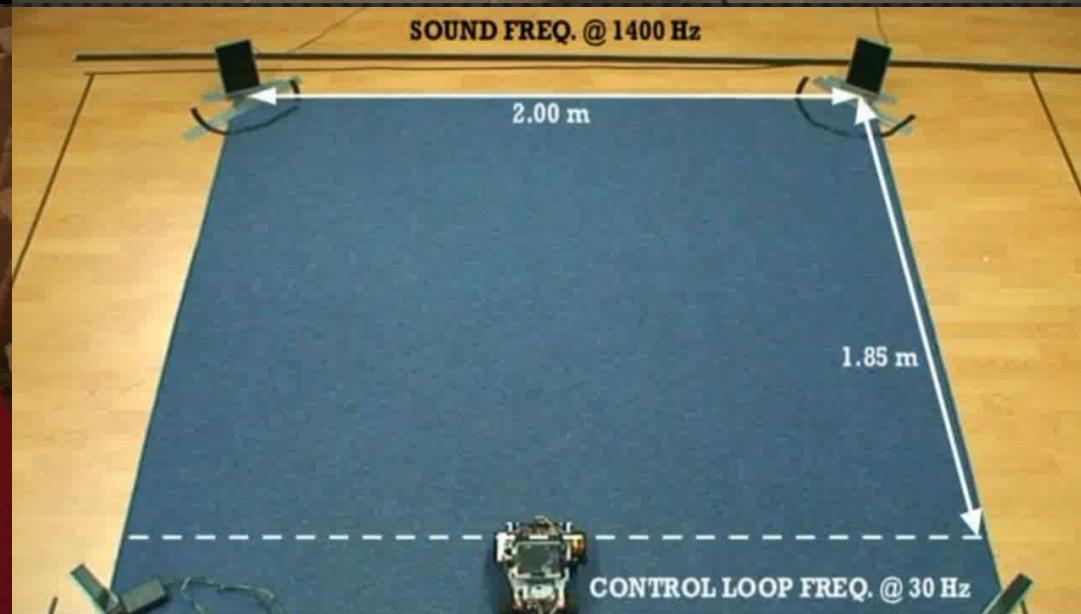
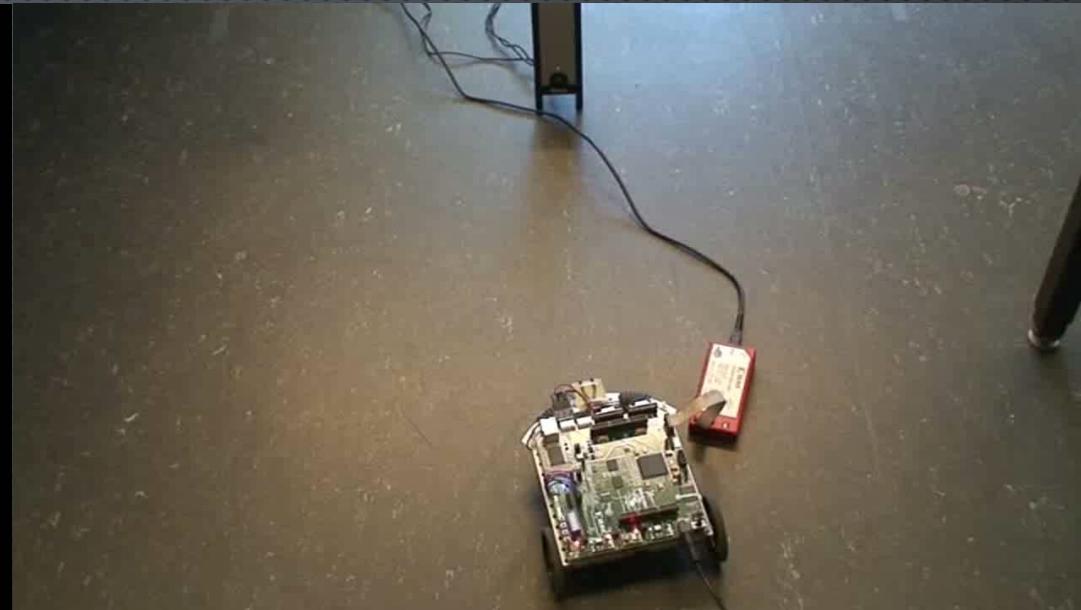
A Braatenberg model for sound localisation



Biologically-inspired sound localising robots (by yours truly)



Biologically-inspired sound localising robots (by yours truly)



Case study 2

A cockroach-inspired gas leak localising robot

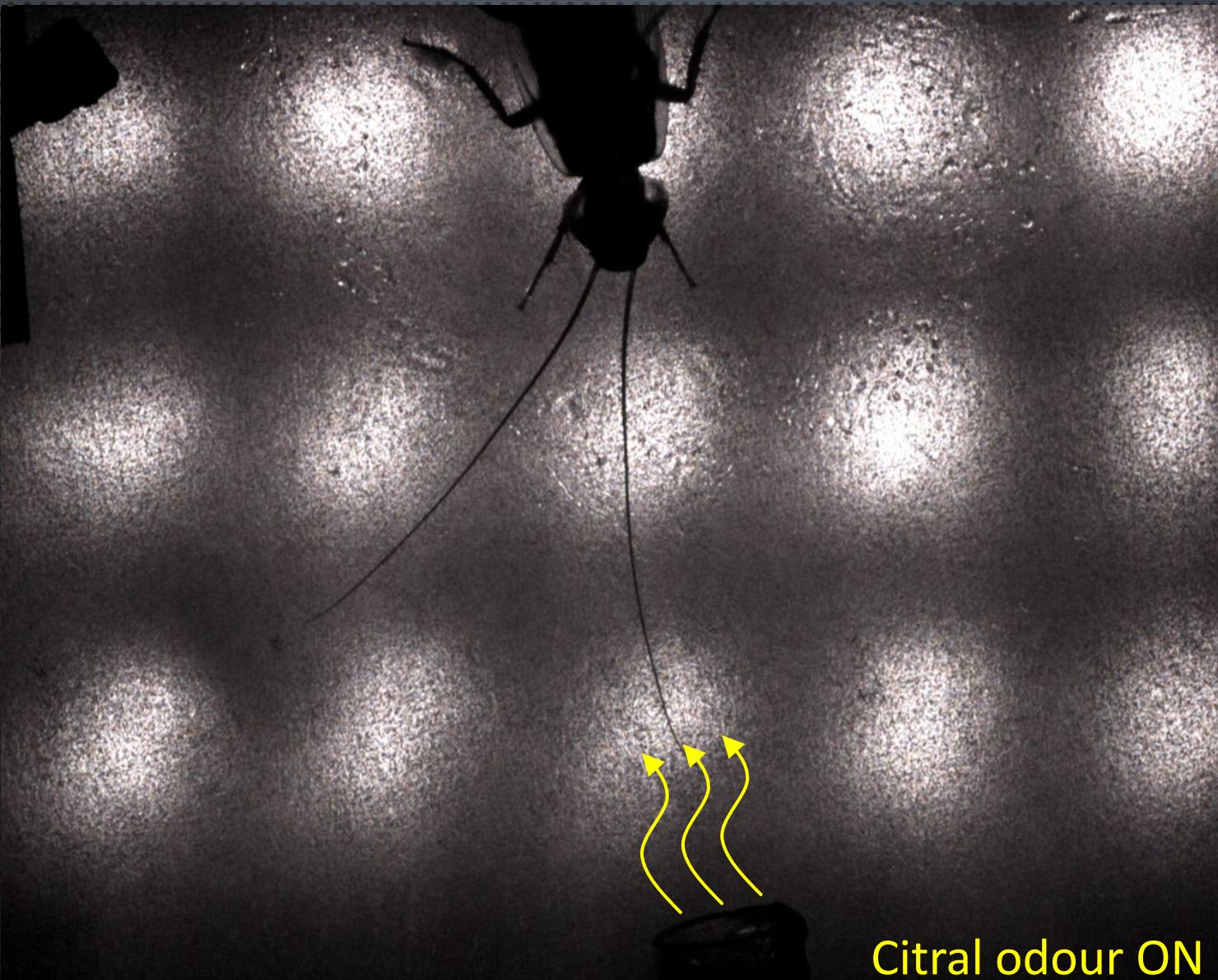
Cockroach olfaction



Observing the animal

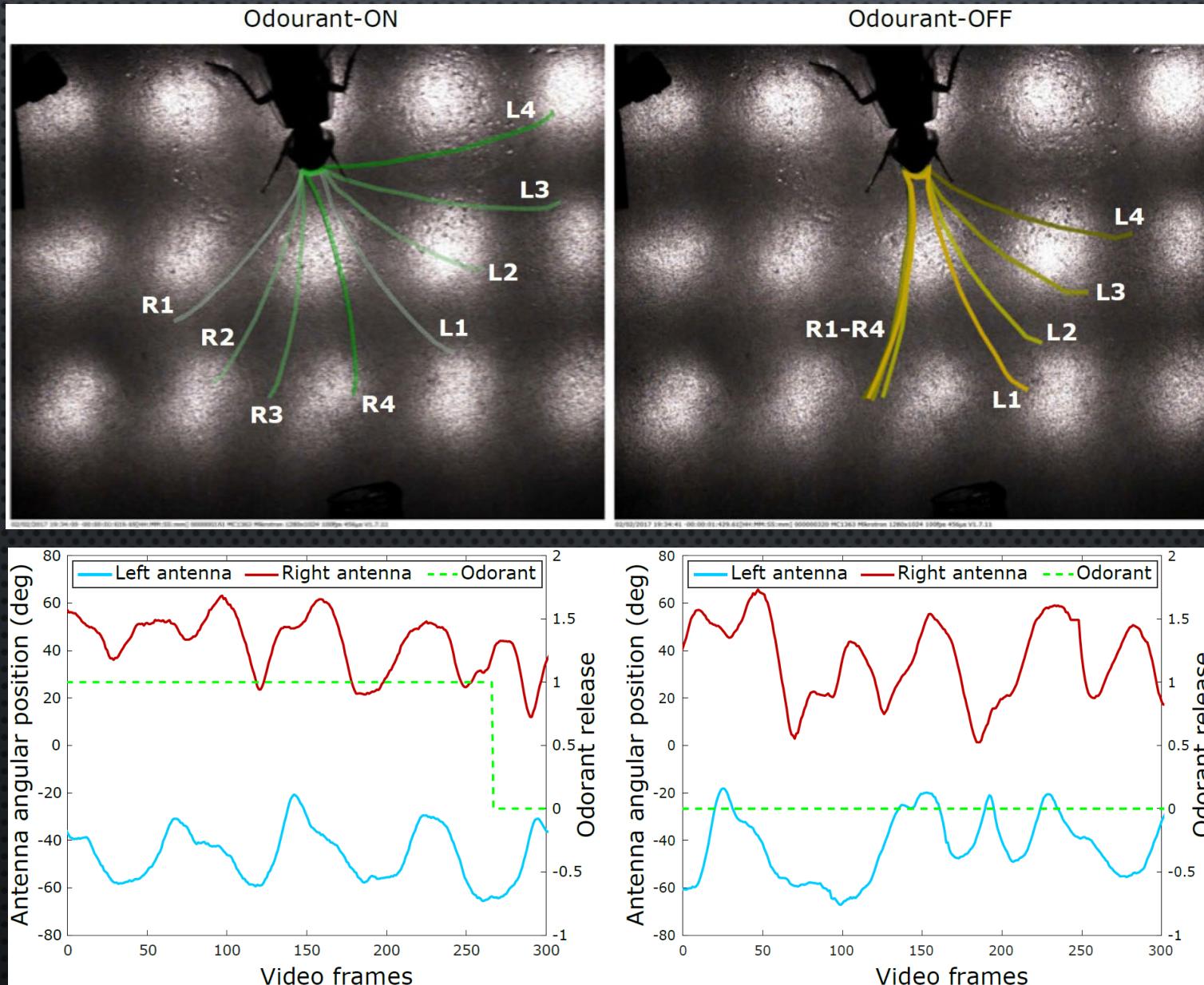


Antennal response to chemical odours

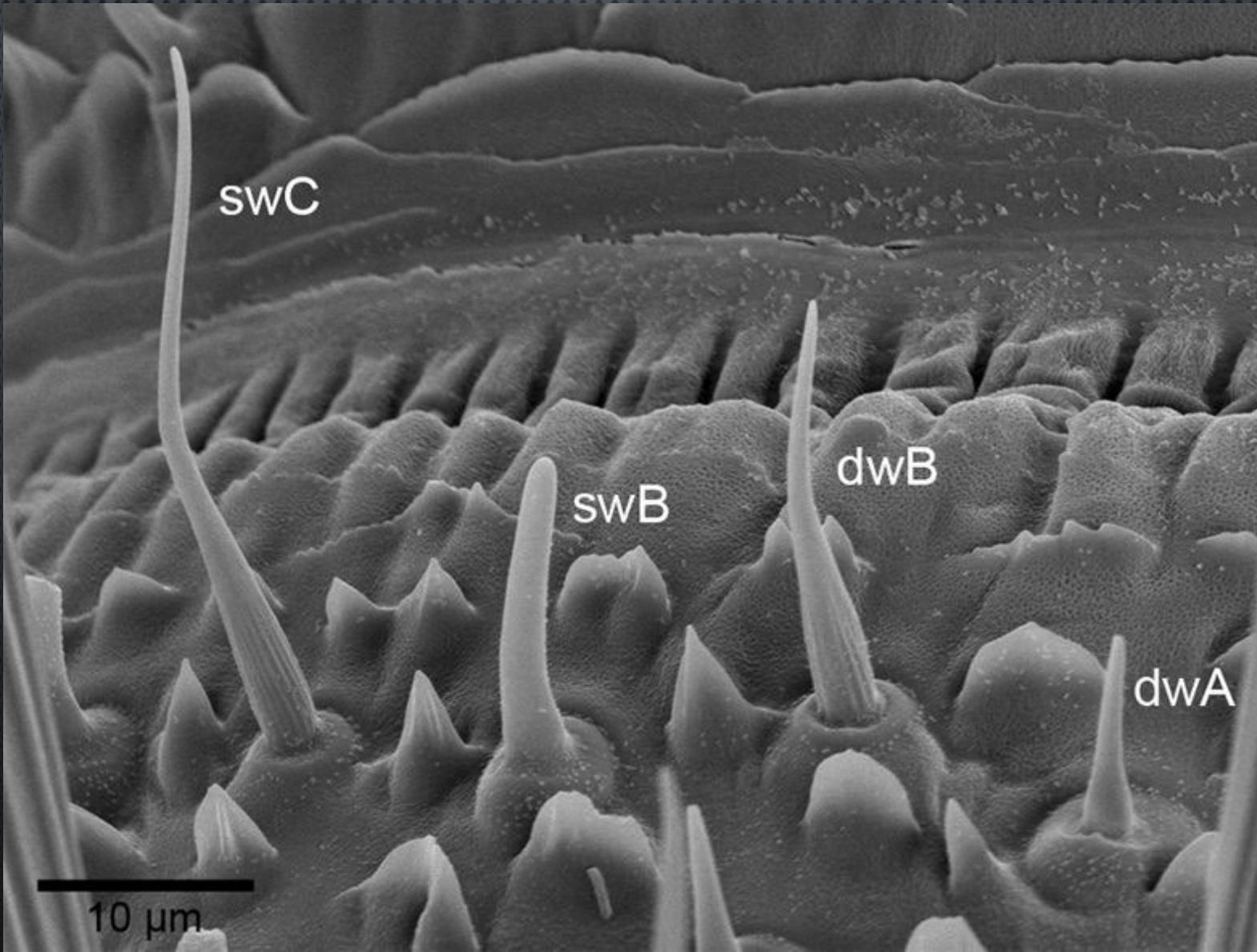


Citral odour ON

Antennal response to chemical odours

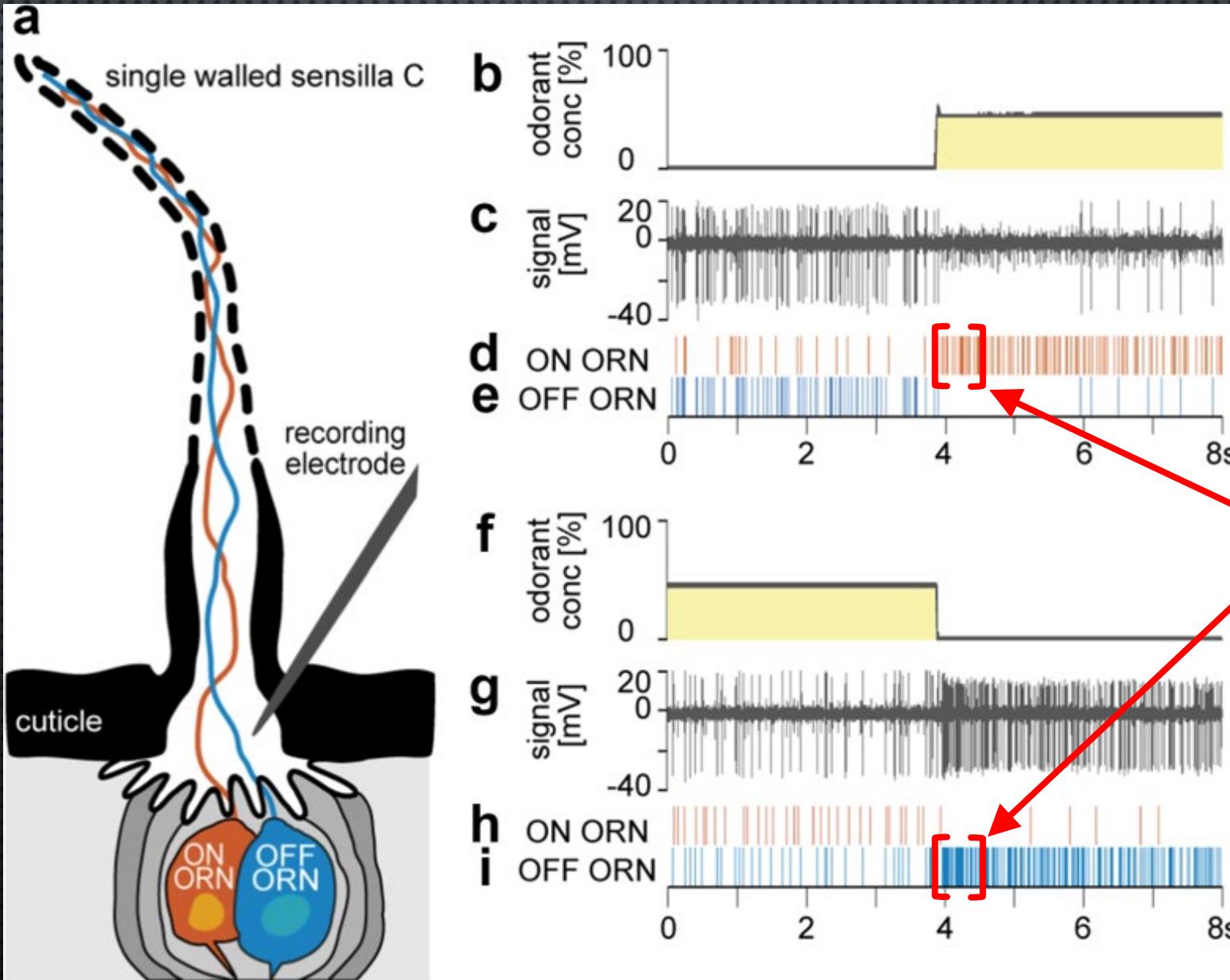


How do cockroaches smell?

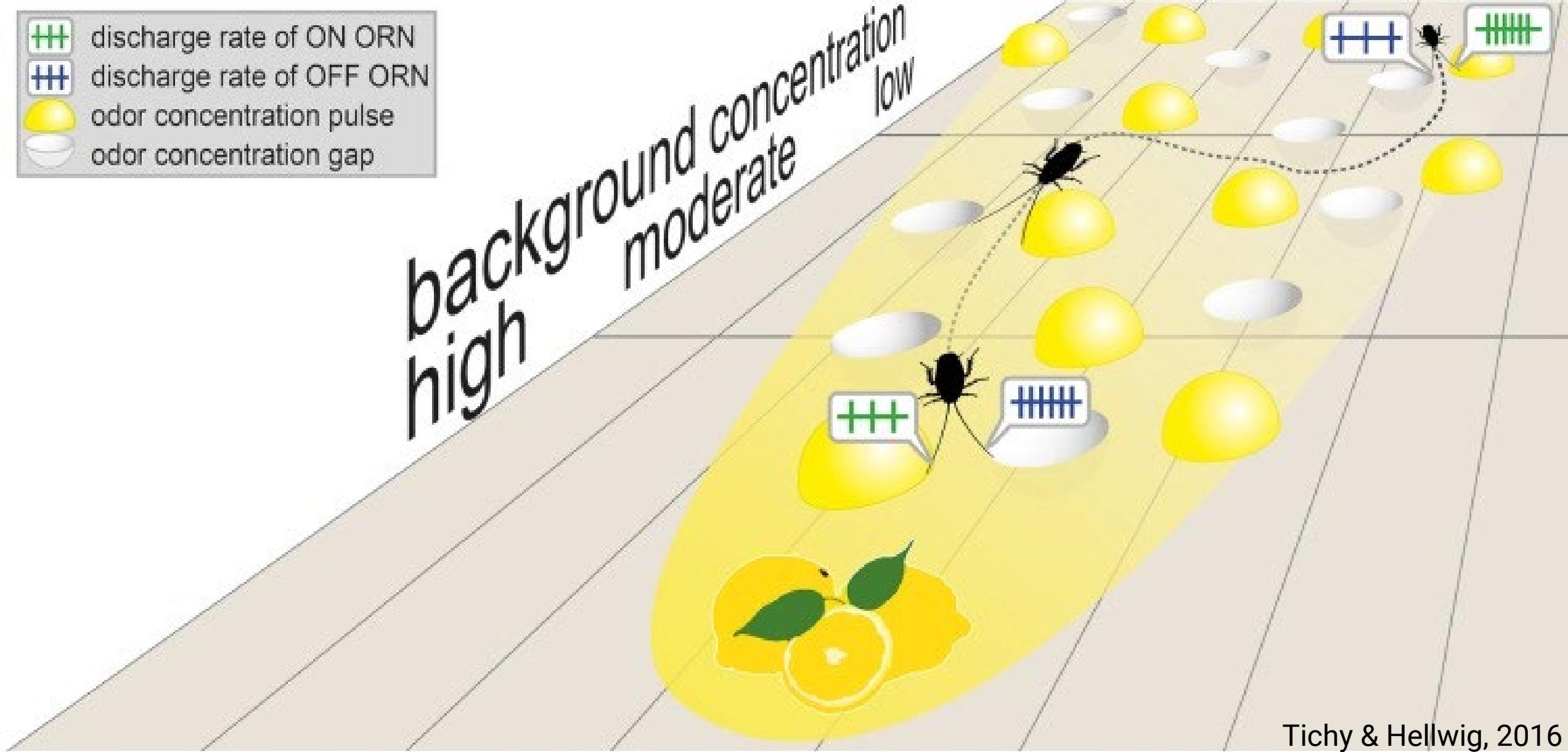


Tichy & Hellwig, 2018

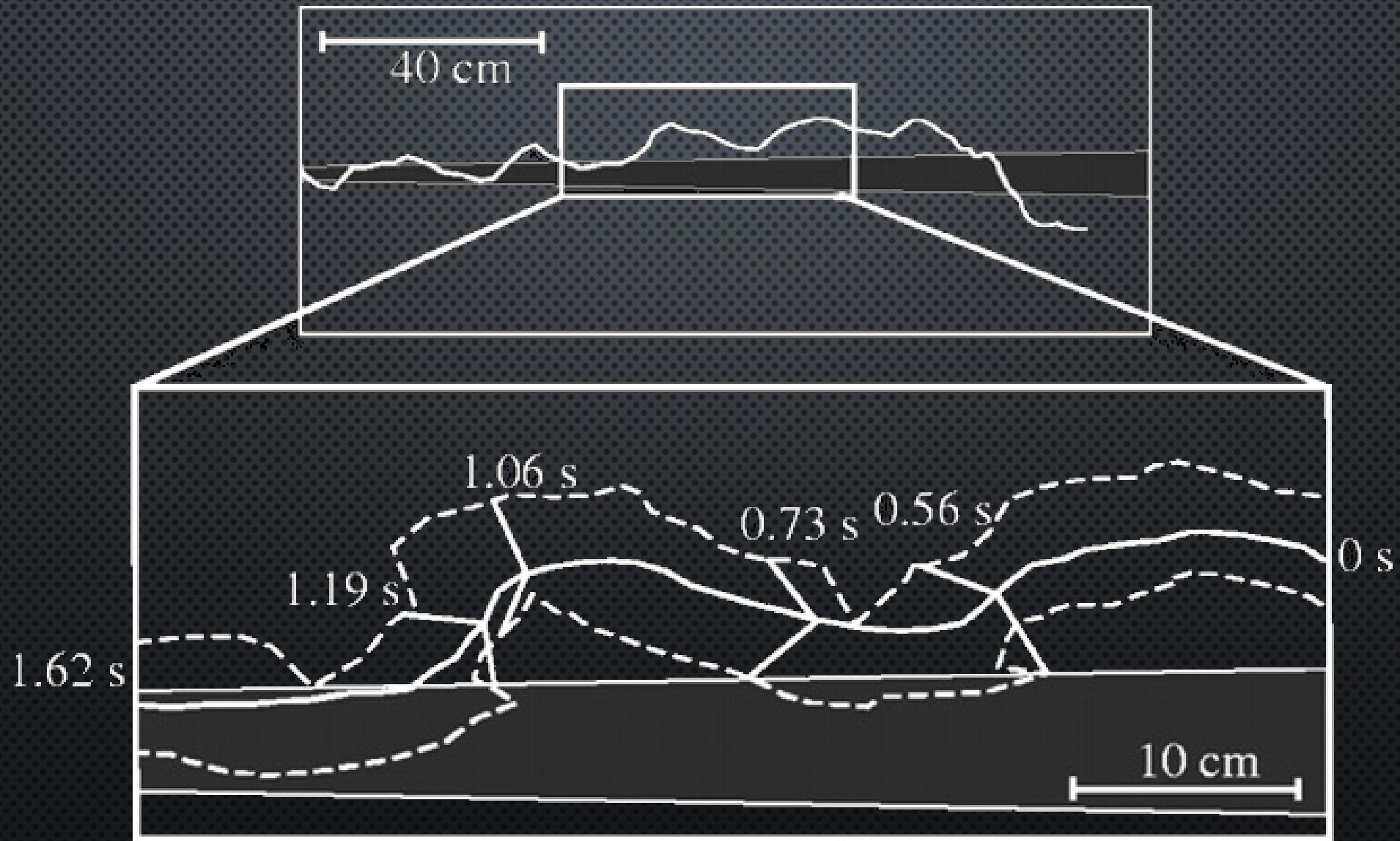
Olfactory neurons (ORNs) encode odour concentration gradient



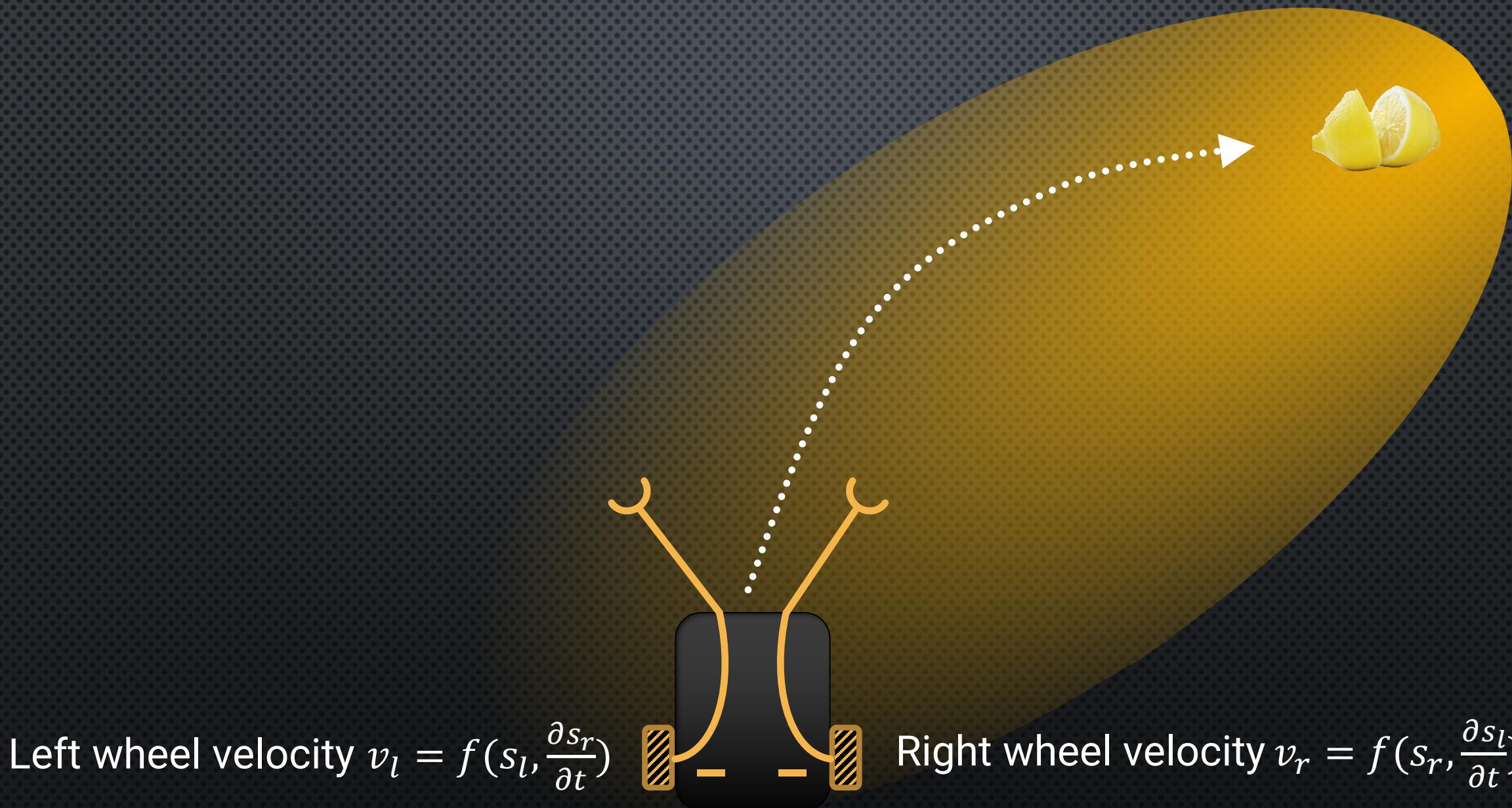
From neurons to behaviour: a hypothesis



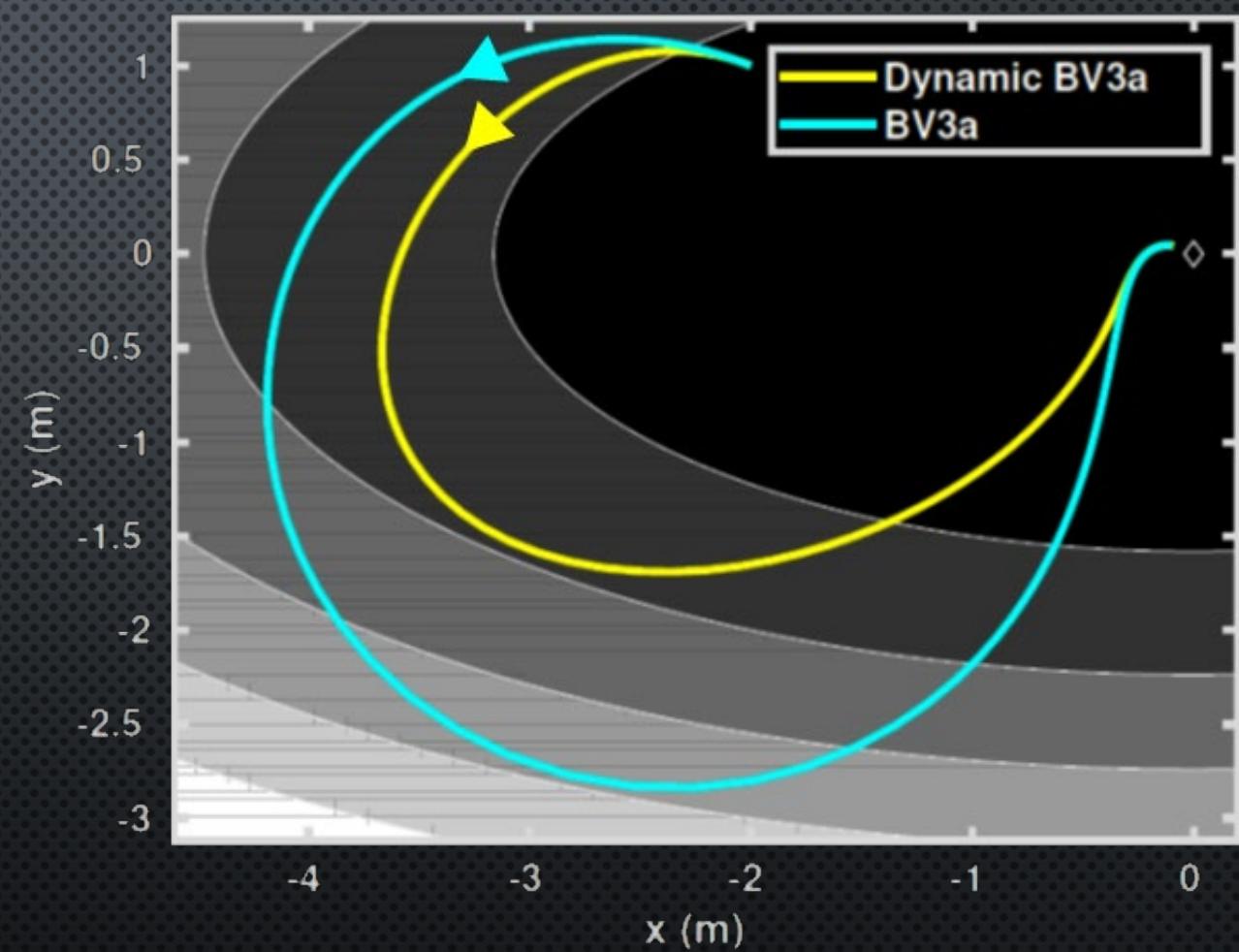
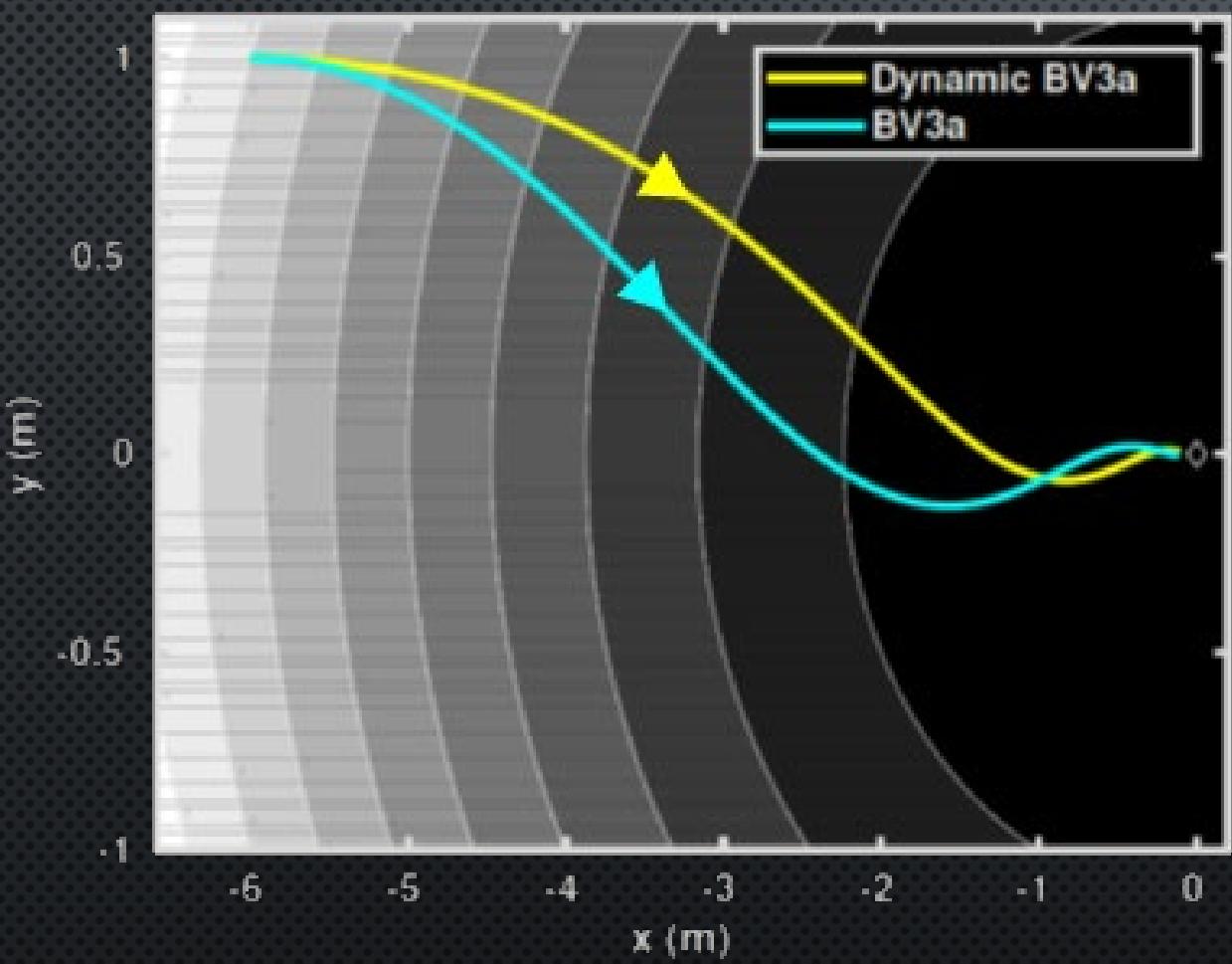
Biological cockroach trajectories



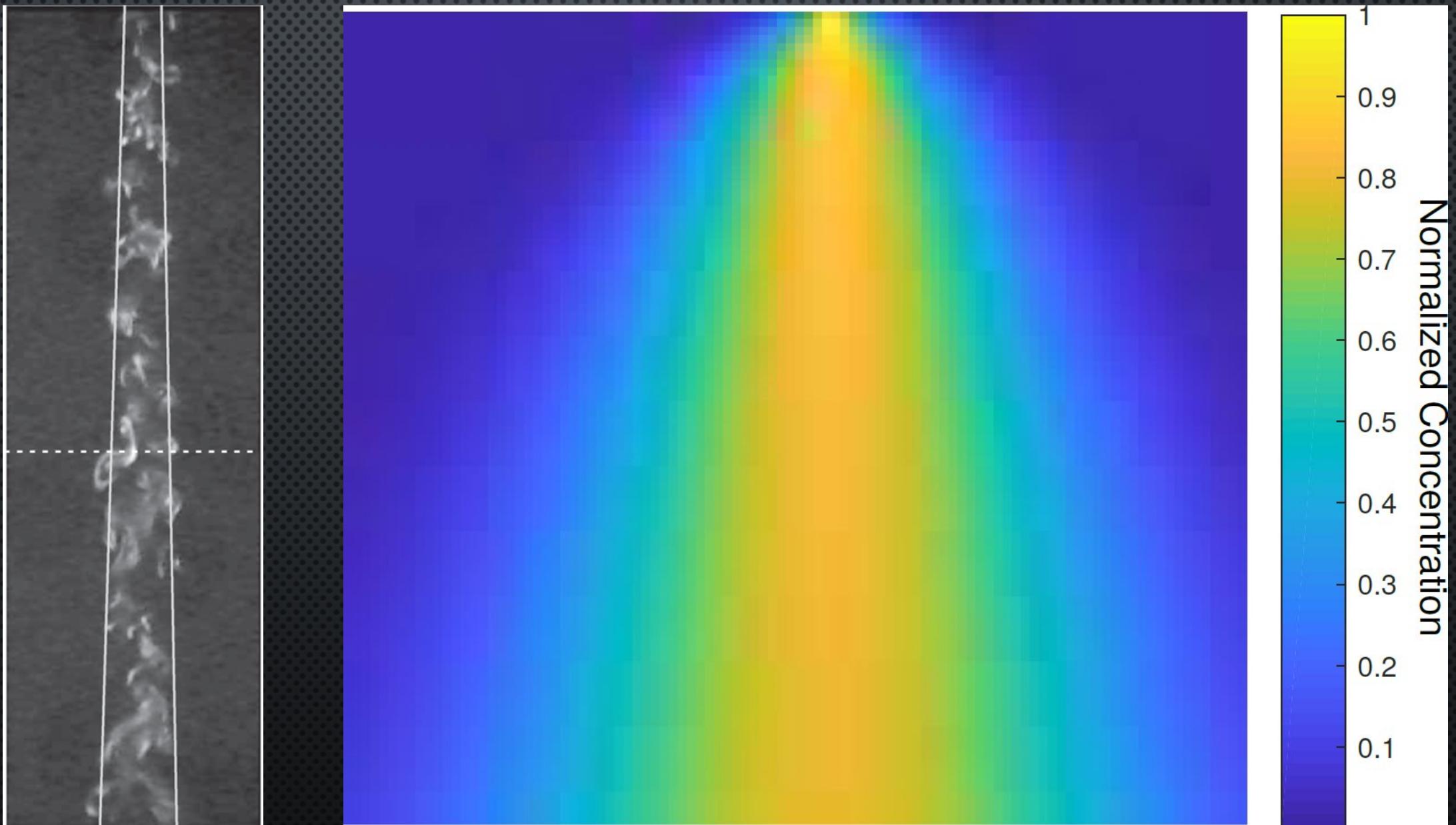
A Braatenberg model for odour localisation (static antennae)



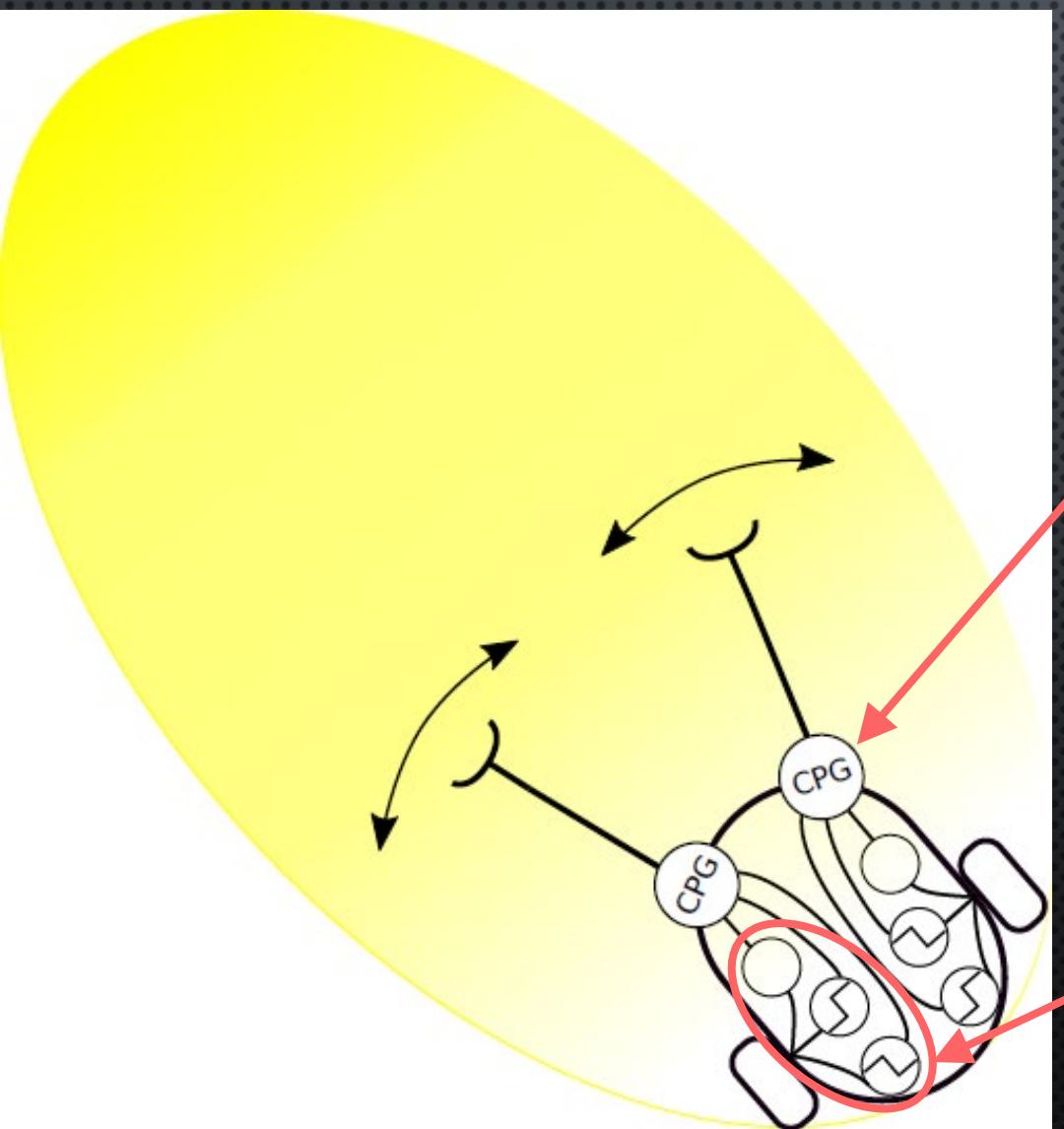
Taxis with and without stimulus dynamics (static antennae)



Task environment



A Braatenberg model for odour localisation (moving antennae)



$$\begin{aligned}\dot{u} &= (\mu^2 - r) \cdot x + \omega y \\ \dot{v} &= (\mu^2 - r) \cdot y + \omega x \\ r &= (x^2 + y^2)\end{aligned}$$

Antenna CPG parameters:
 μ = steady state amplitude
 $\omega = 2\pi f$ = angular velocity

$c_{l/r}$ = instantaneous odour concentration
 $s_{l/r}$ = total stimulus
 $v_{l/r}$ = wheel velocity

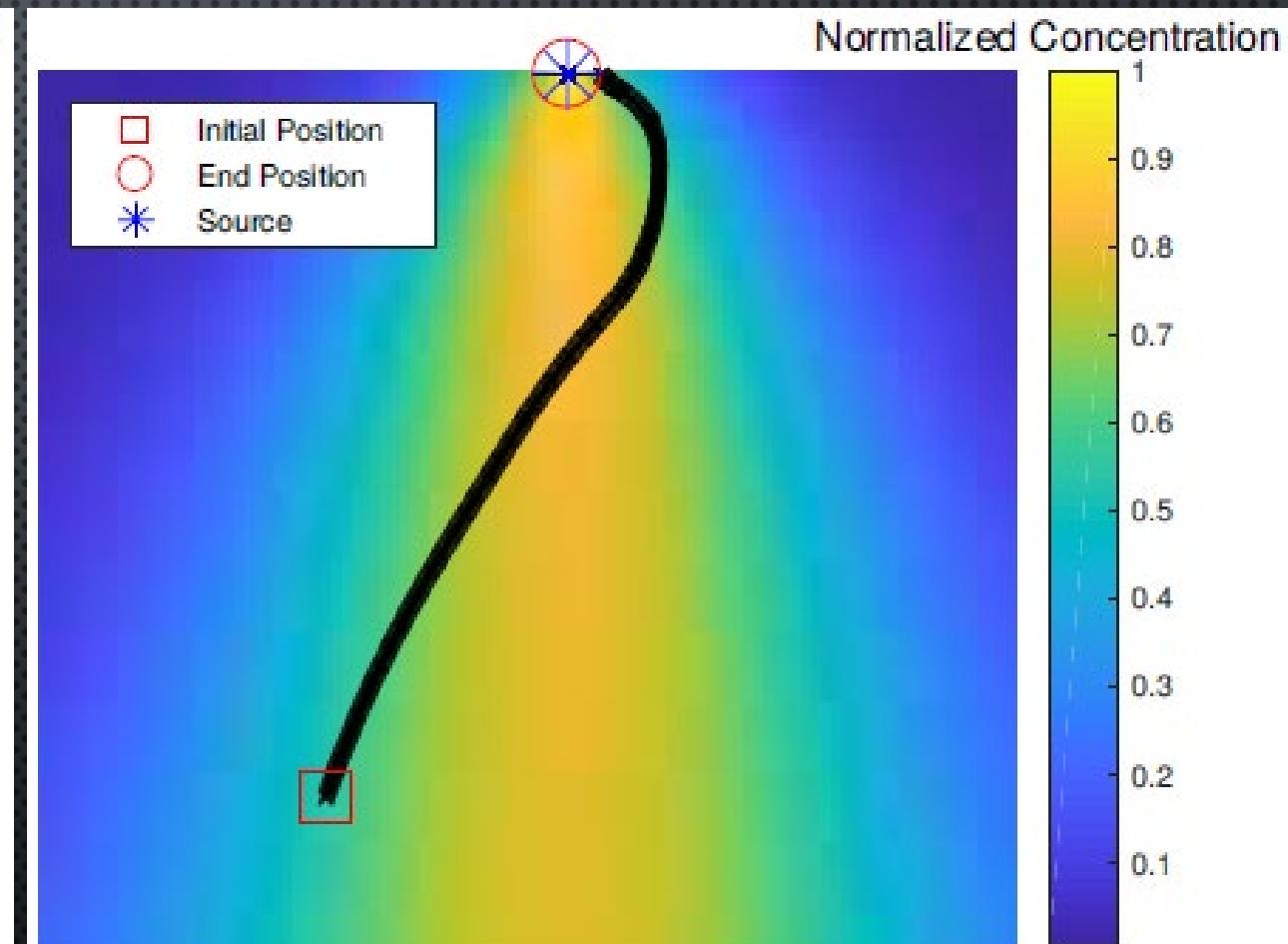
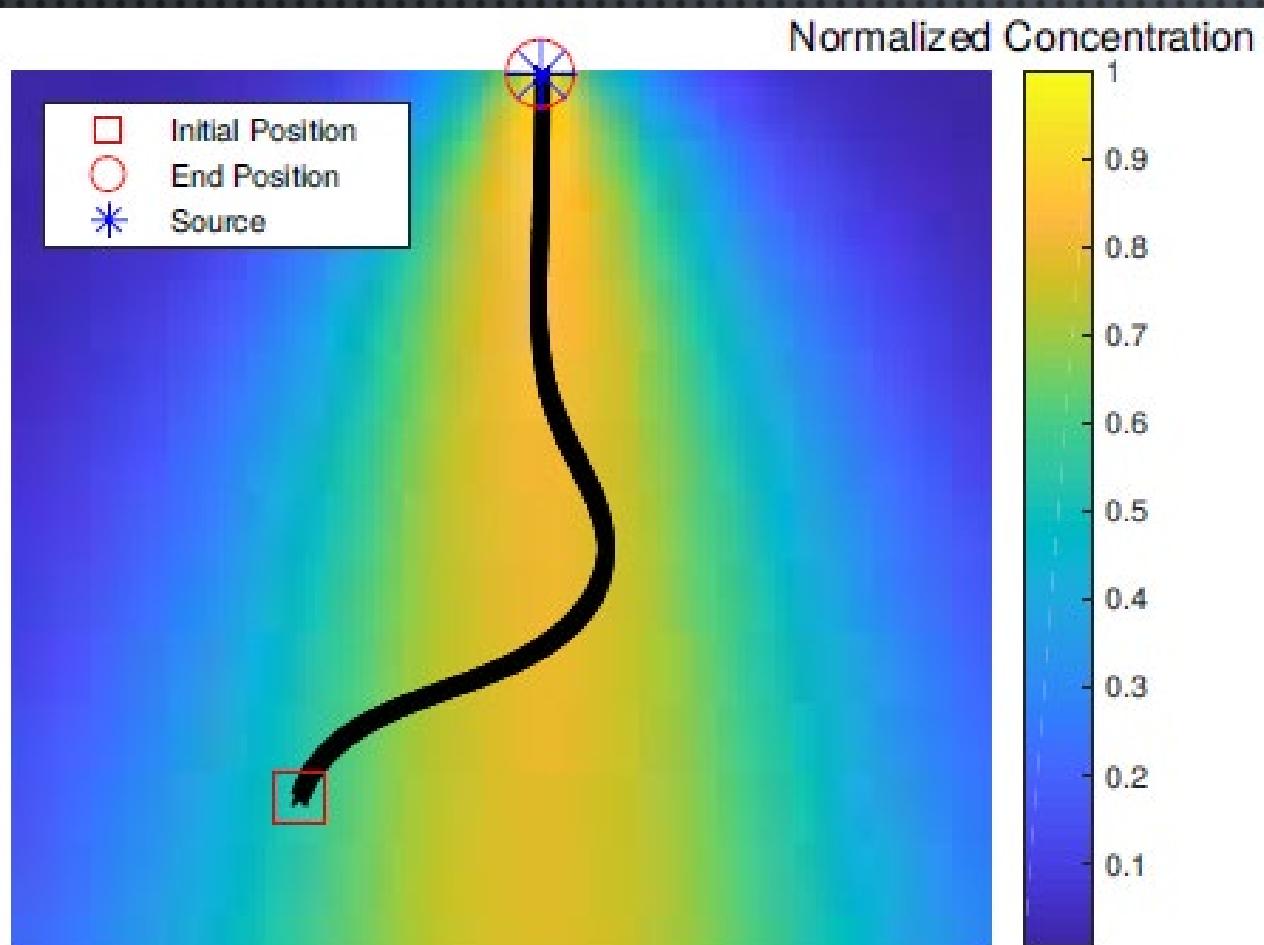
$f(\dot{c}_{r/l})$ = Heaviside step function
(ON neuron model)

$f(-\dot{c}_{r/l})$ = Heaviside step function
(OFF neuron model)

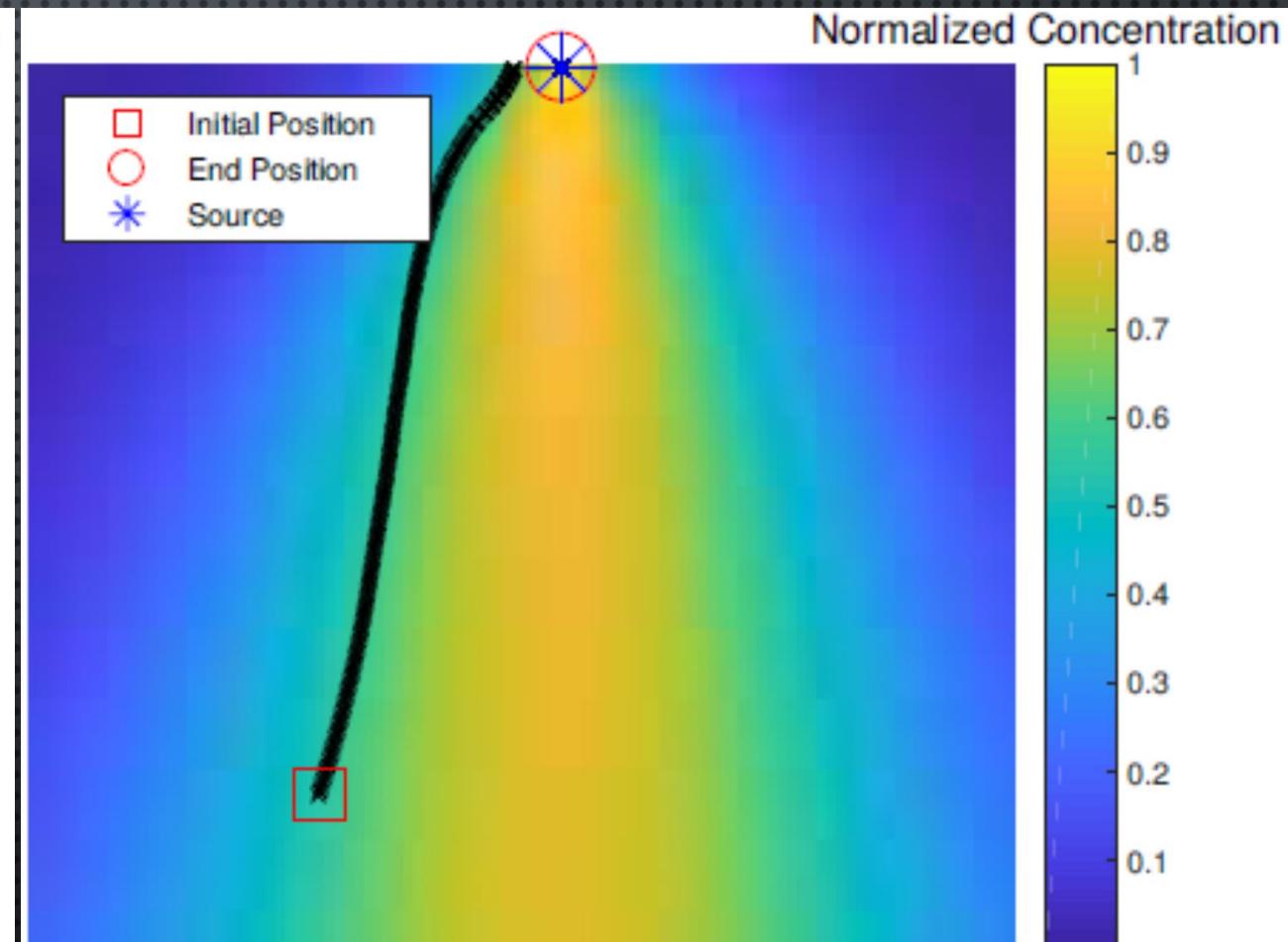
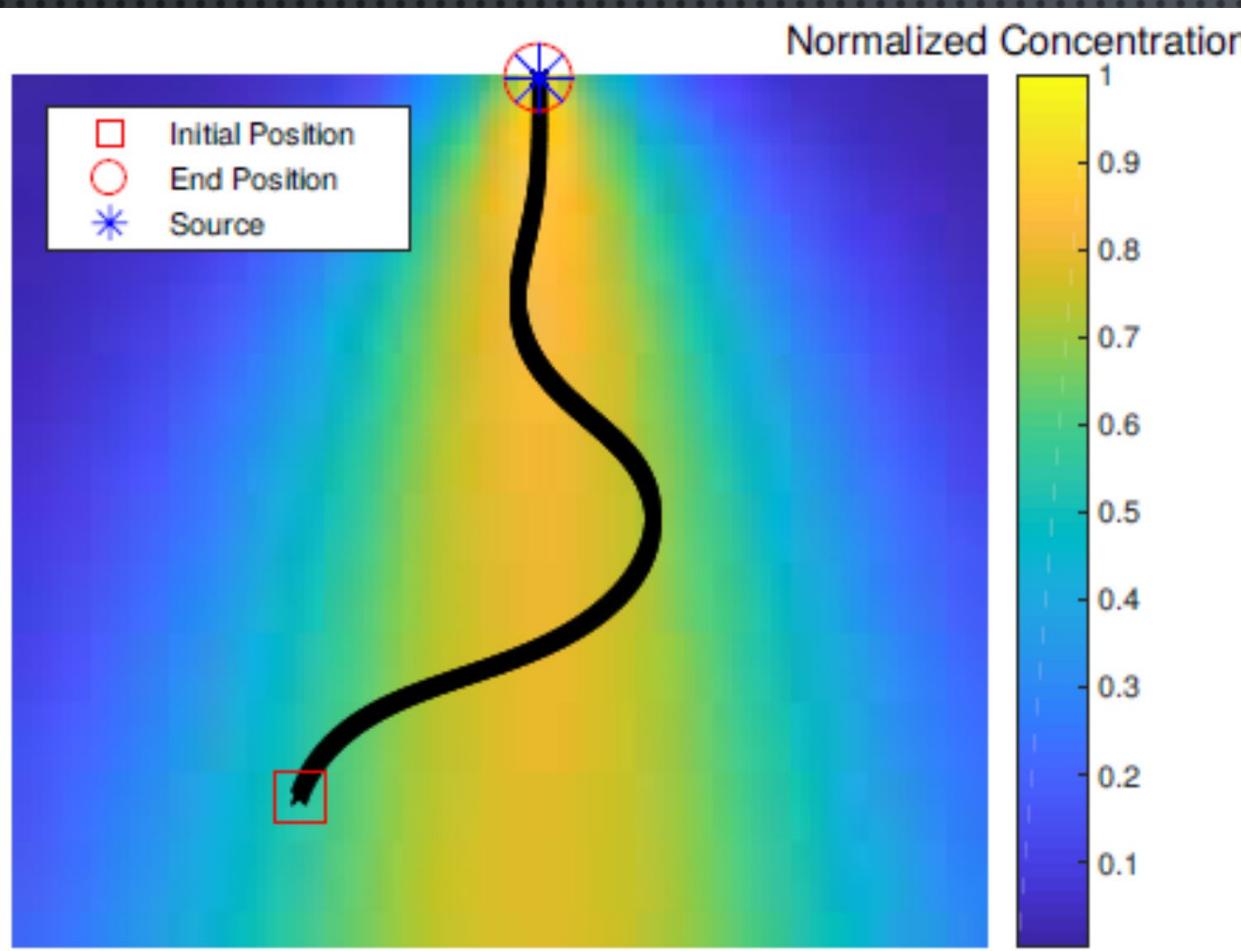
$$\begin{aligned}c_{l/r} &= C(x_{l/r}, y_{l/r}) \\ s_{l/r} &= K_n c_{l/r} + K_d f(-\dot{c}_{r/l}) + K_i f(\dot{c}_{r/l}) \\ v_{l/r} &= \alpha(s_M - s_{l/r})\end{aligned}$$

Sensorimotor coupling parameters:
 α = gain term
 s_M = maximum stimulus
 K_n, K_d, K_i = gain terms

Localising odour source (static antennae)

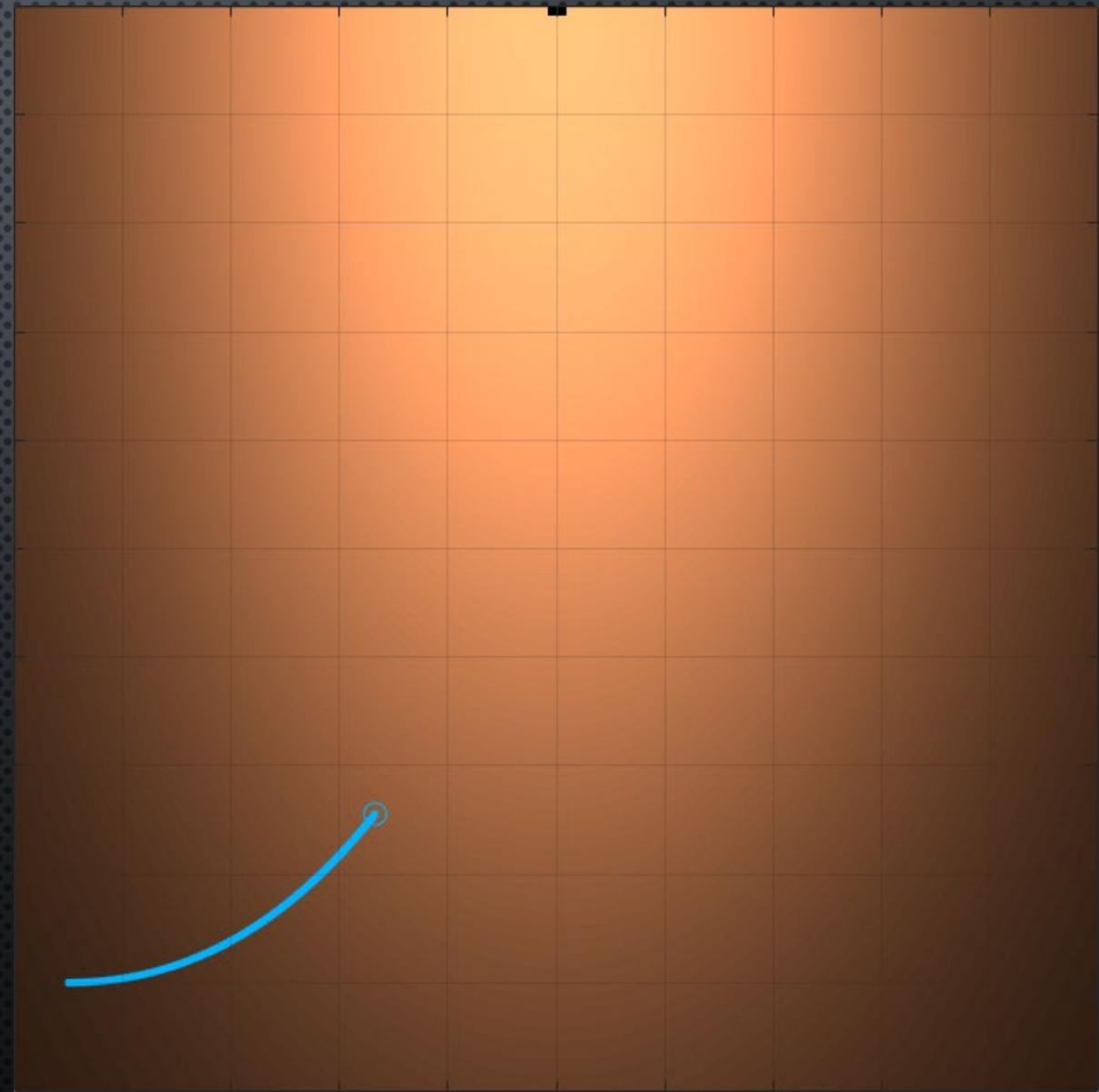


Localising odour source (moving antennae)



Matlab exercises

- Download “braitenberg_gradient.m”
- Implement various light following Braitenberg vehicles and observe their behaviour
- Vary parameters and observe behaviour
 - *sensor_width*
 - *add_noise* and *snr_db*
 - *robot_pose*





4 Learning mechanisms in biological neurons and their functional engineering models

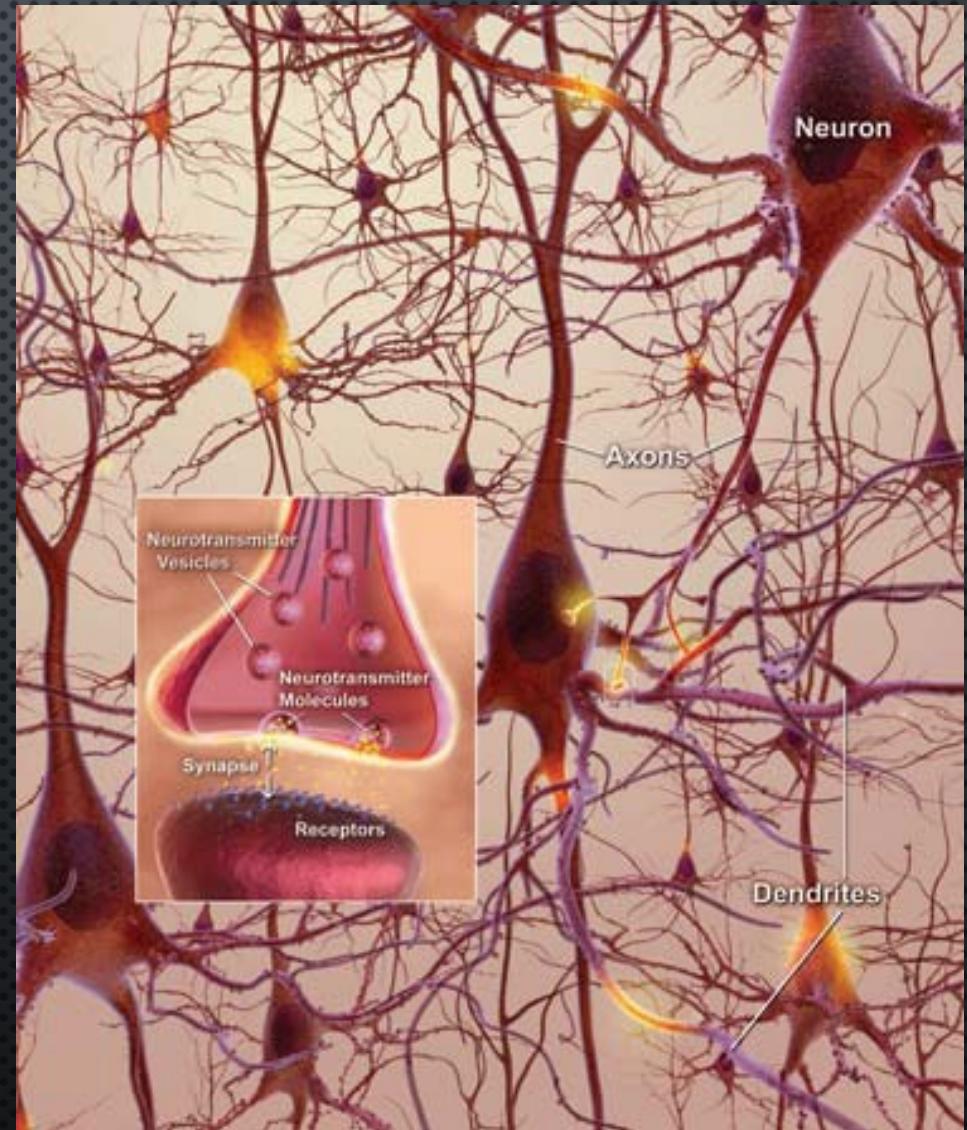


Neurobiology ahead
(but now you love it 😊)

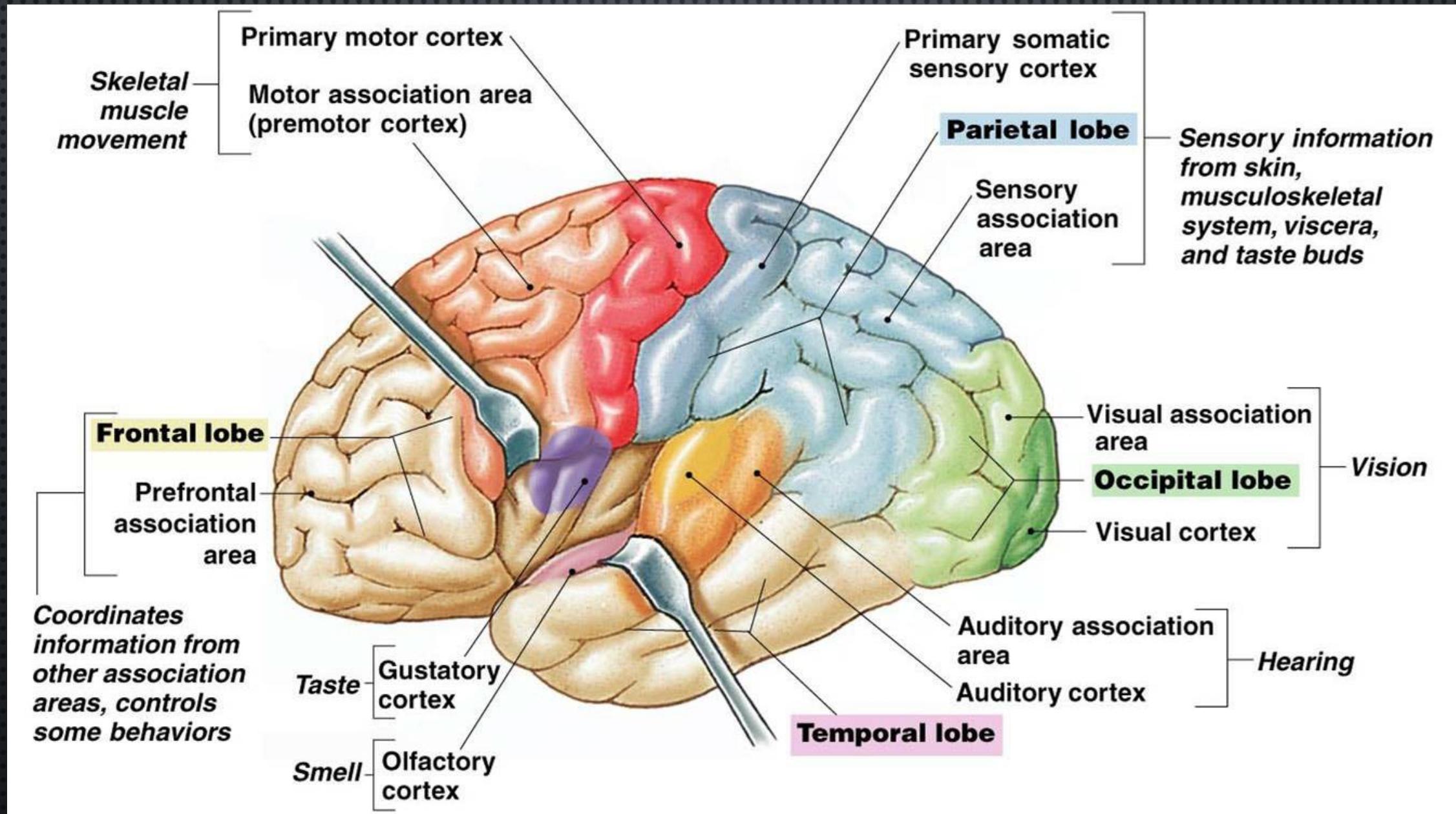


Neuroplasticity: a mechanism for biological learning

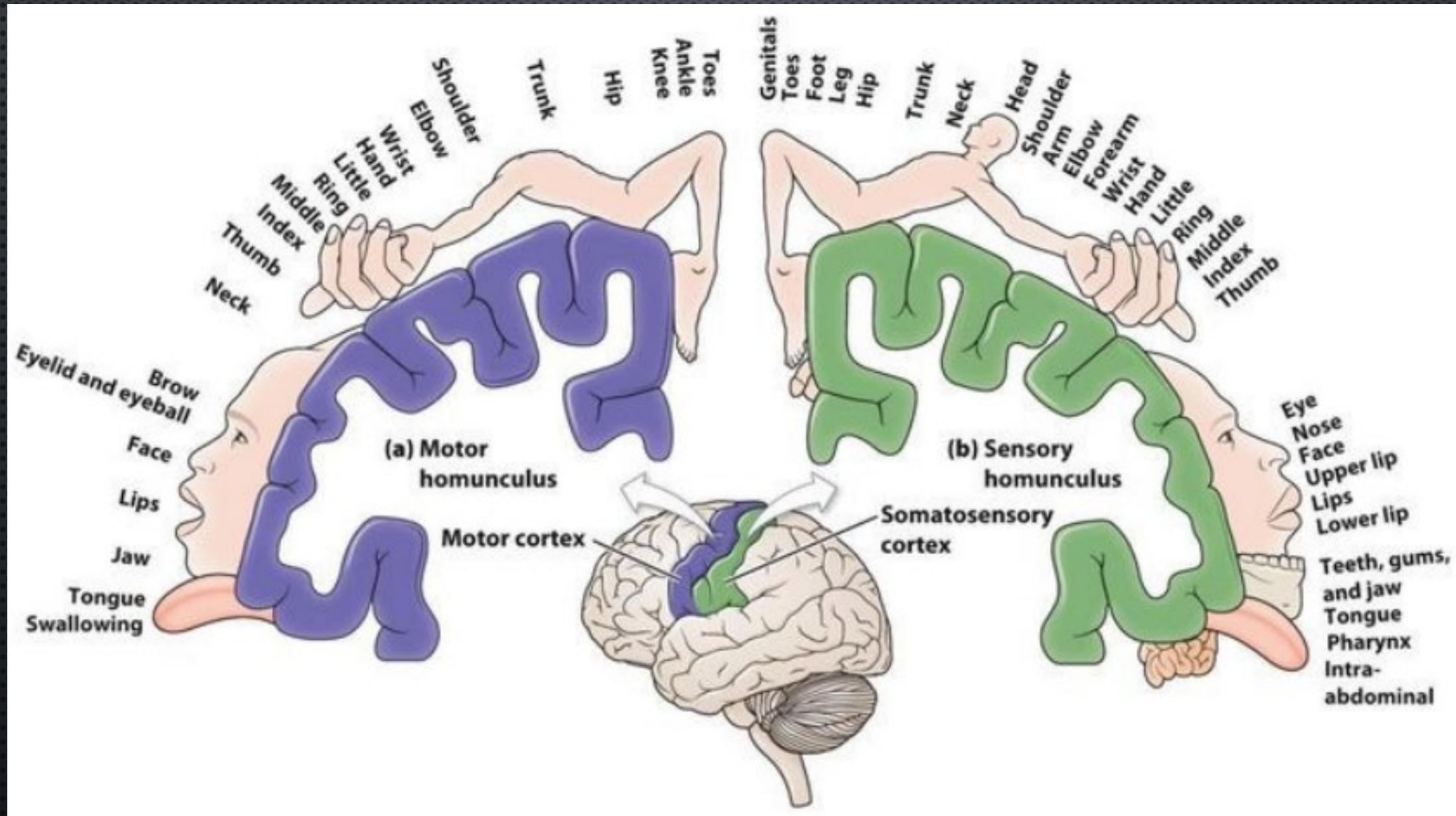
- Neuroplasticity is the ability of the brain to **reorganise** itself in both **structure** and **function** over time due to external and internal events
- Reorganisation can occur throughout the neural hierarchy – from single neurons to entire cortical areas
- Neuroplasticity is a continuous process that occurs at multiple timescales from milliseconds to days
- It is the underlying mechanism behind learning and memory formation



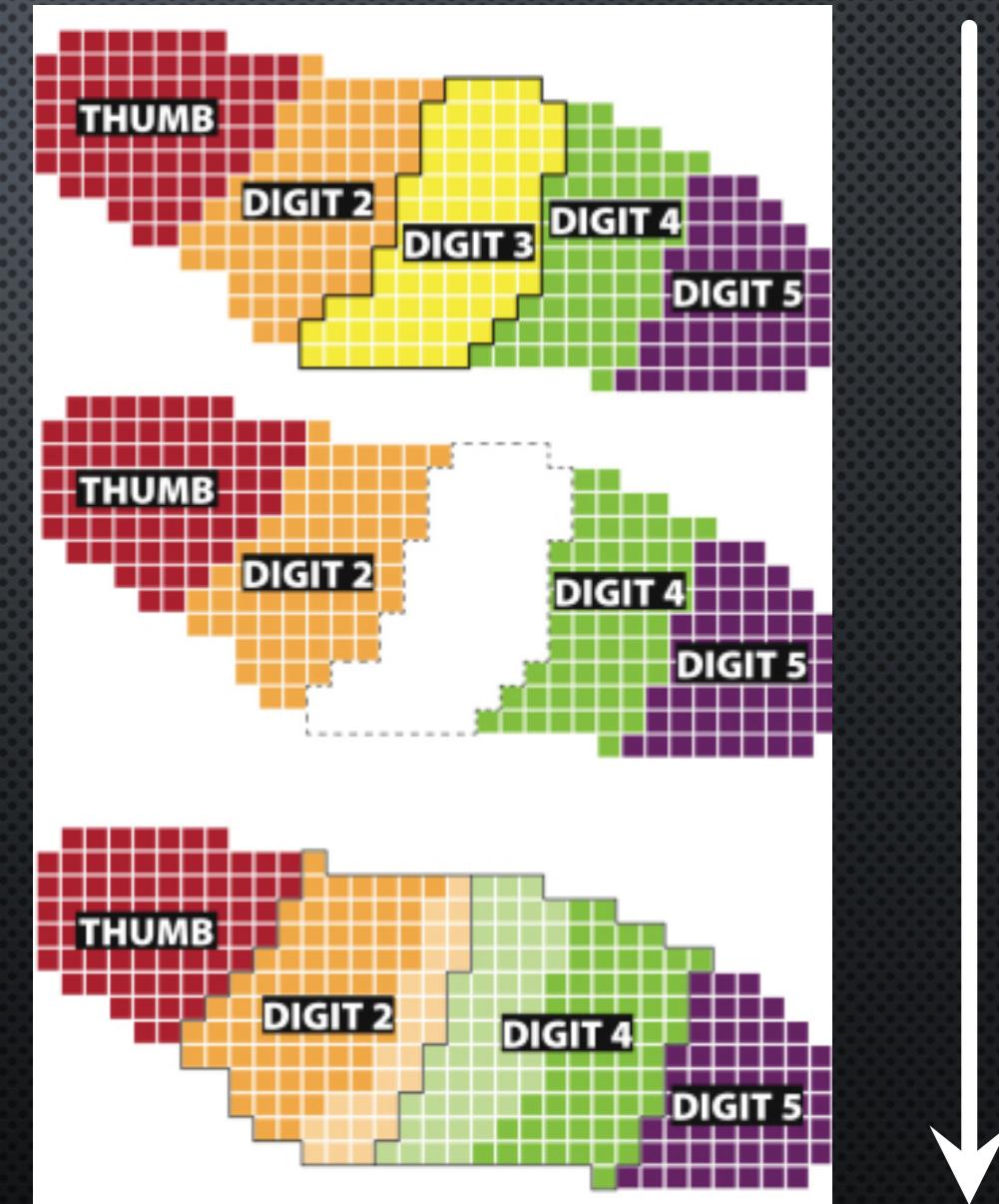
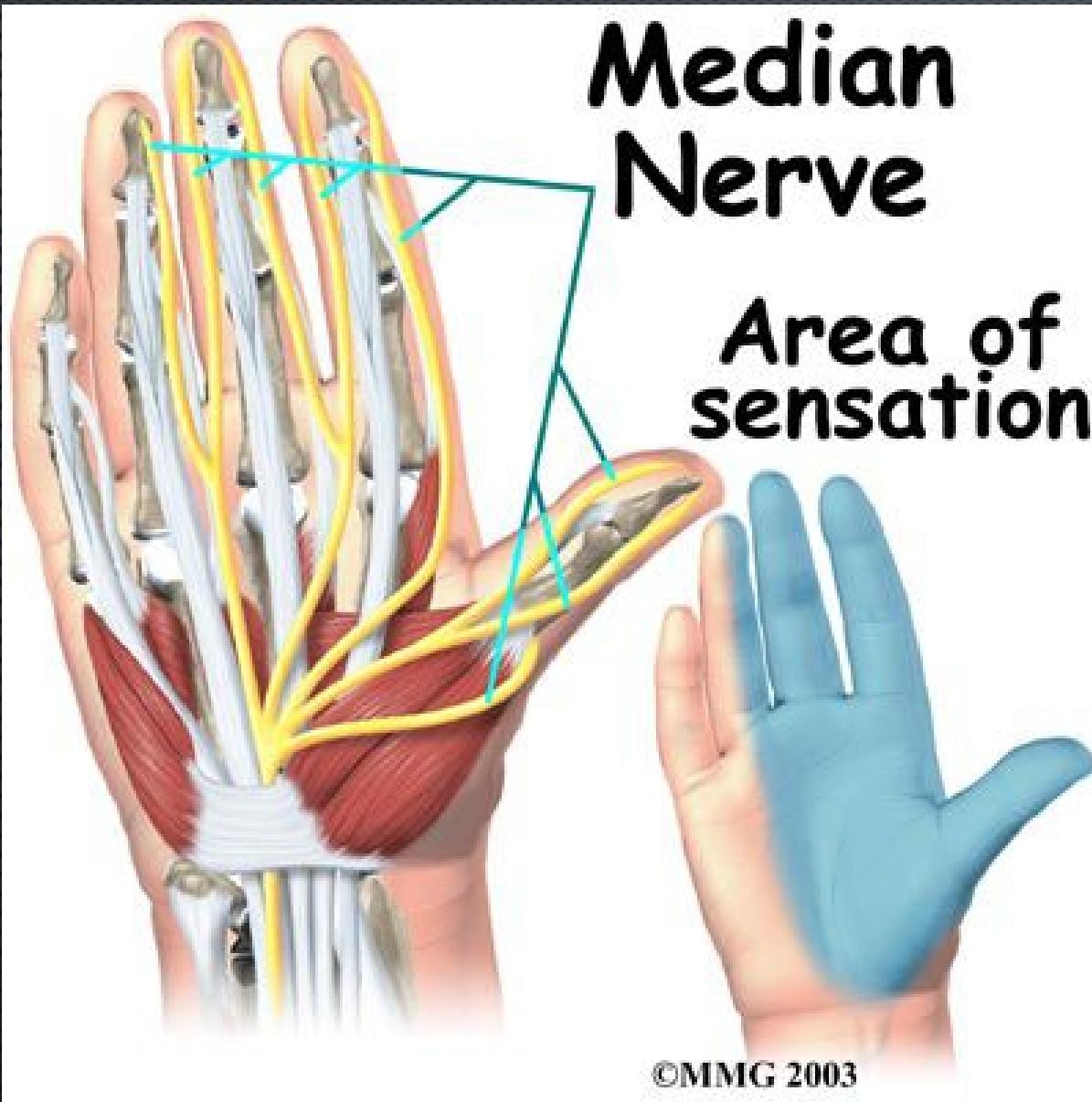
Brain areas



Neuroplasticity at the cortical level



Neuroplasticity at the cortical level



Types of neuroplasticity

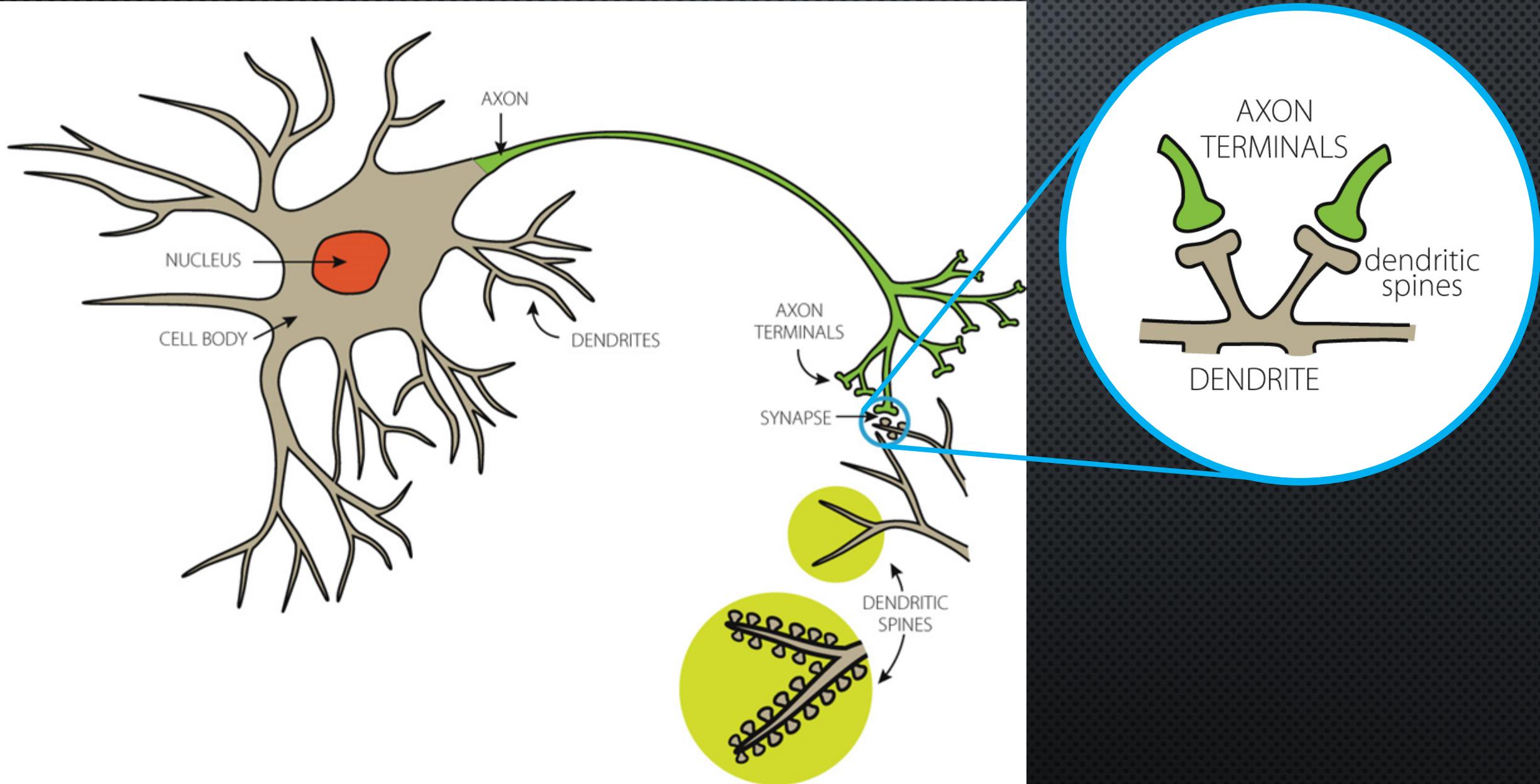
Structural plasticity

- mechanism describing the generation of new neural connections
- long-term changes
- Biological mechanisms:
 - Synapses – changes in number
 - Synaptic receptors – changes in density
 - Dendritic arborization (complexity of the dendritic tree)
 - Dendritic spines – changes in density
 - Axonal arborization (sprouting)
 - Glial and neuron interactions
 - Vascular processes and angiogenesis (new blood vessel growth)
 - Cell proliferation (including neurogenesis)

Functional plasticity

- mechanism of changing the strength of existing neural connections
- short term changes
- Biological mechanisms:
 - Synapses – changes in strength. Widely known as synaptic plasticity

Recap: neuron structure



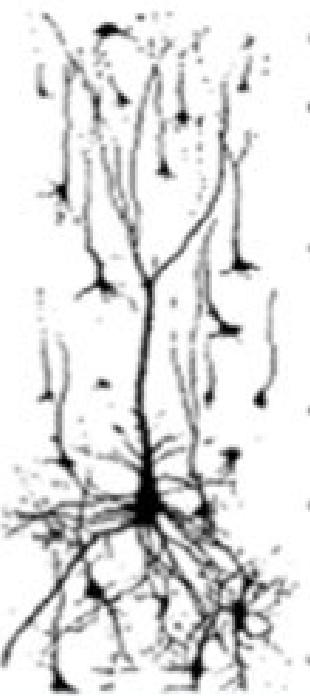
Structural plasticity: number of synapses

**36 weeks
gestation**

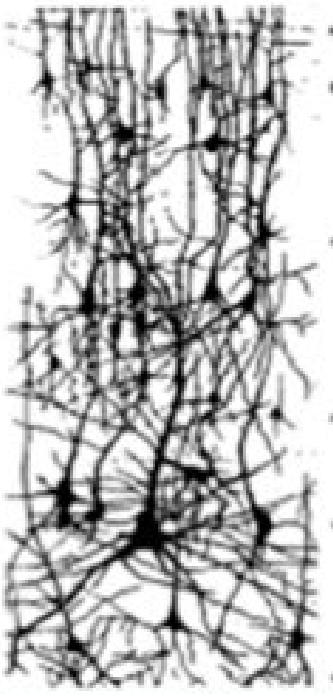
Newborn



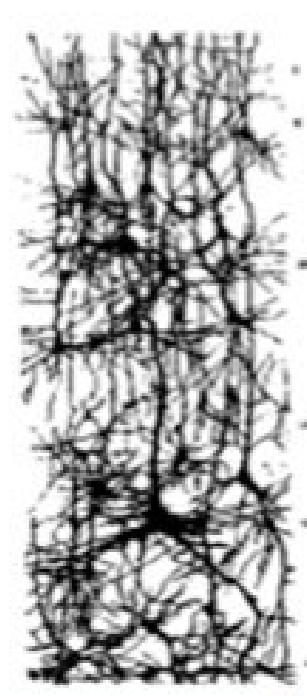
3 months



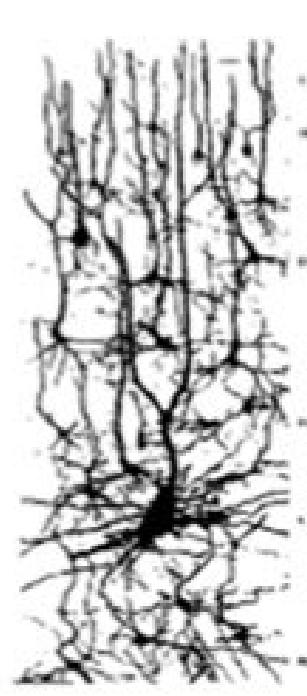
6 months



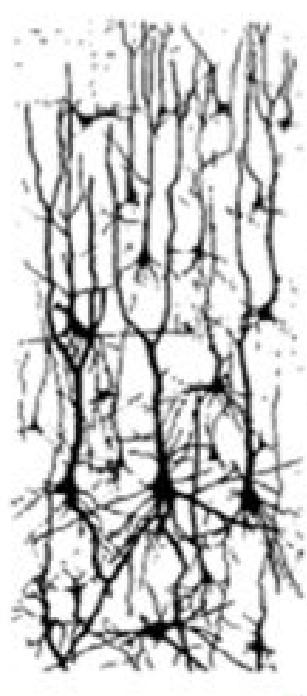
2 years



4 years



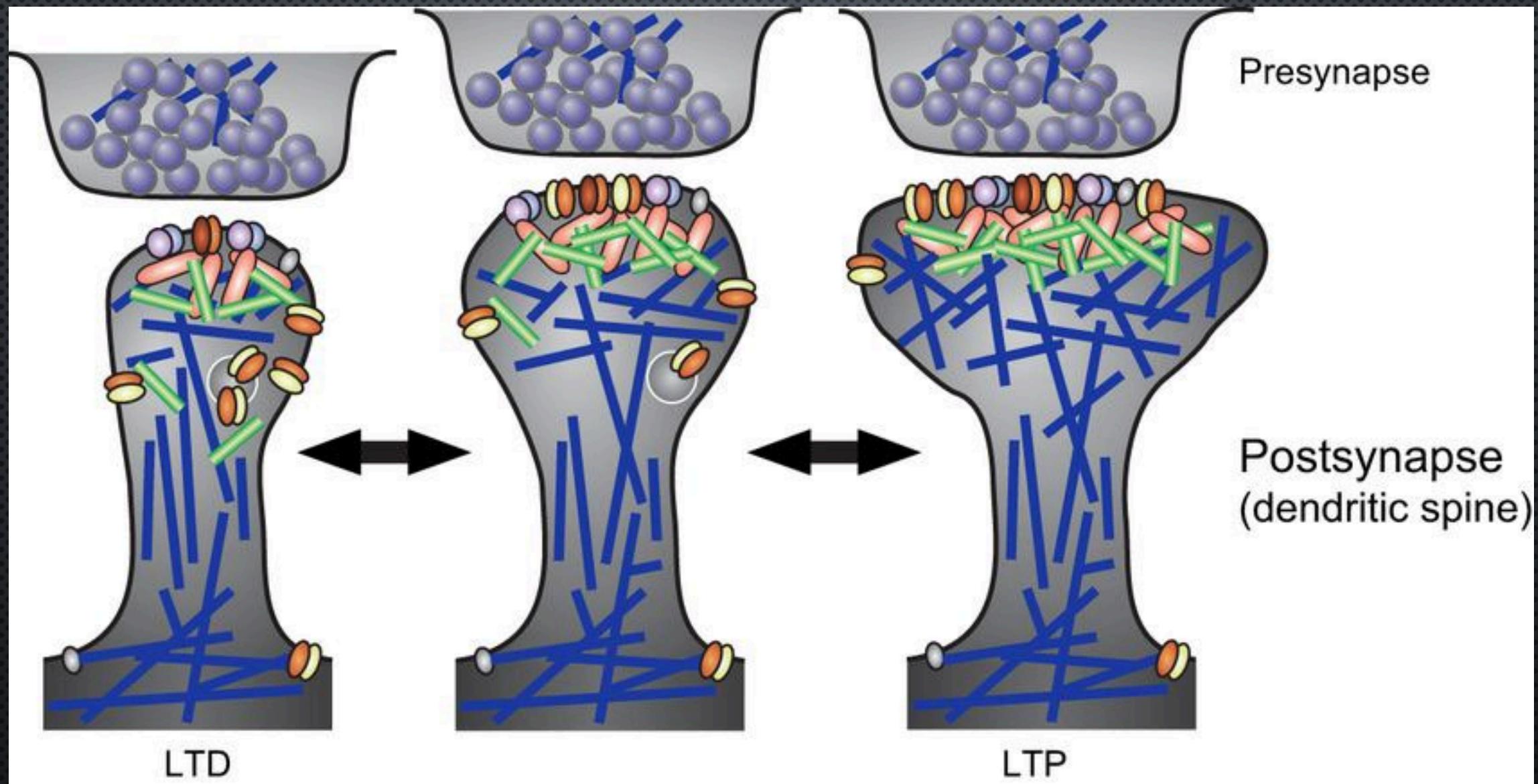
6 years



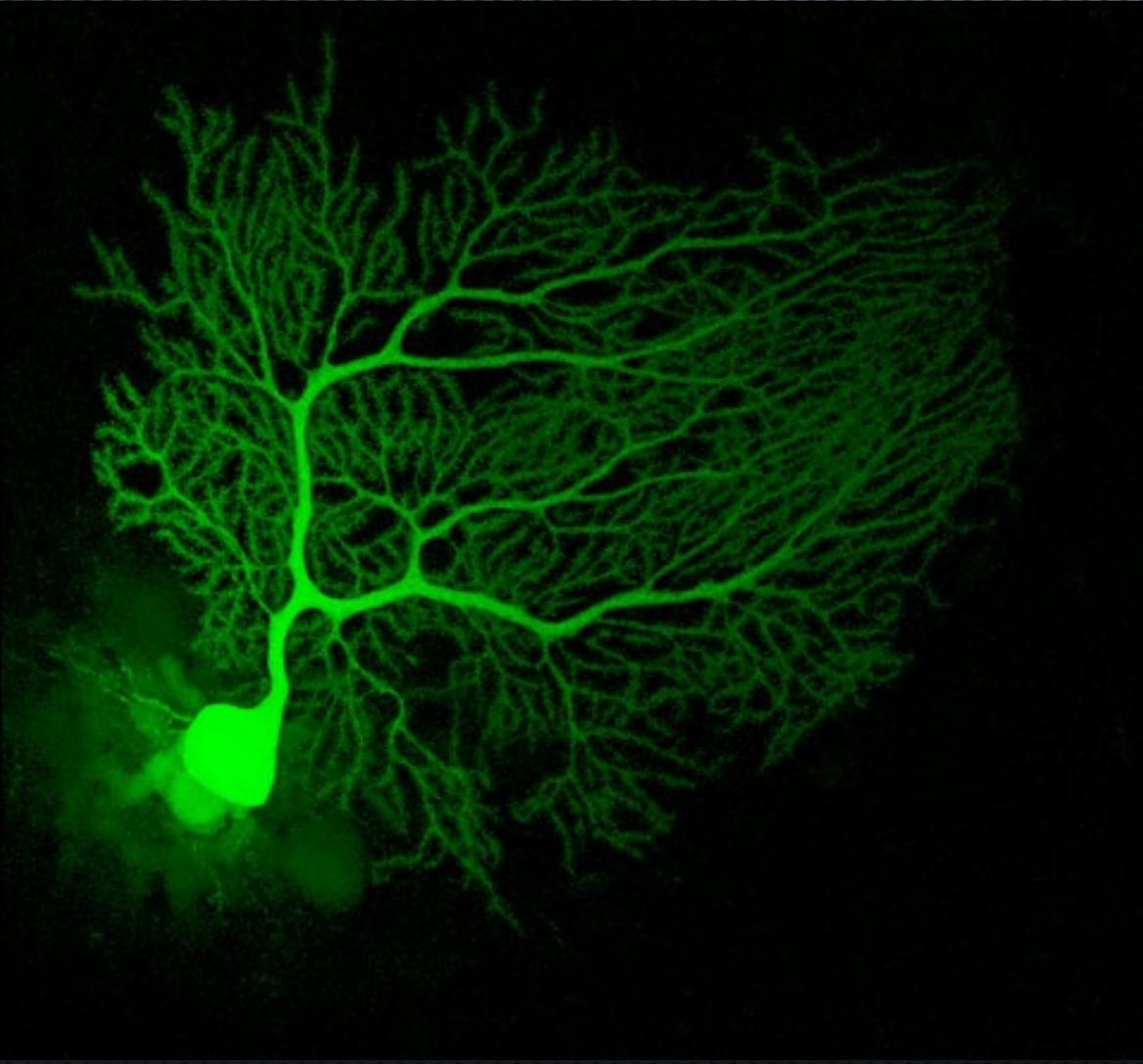
Synapse formation

Synapse pruning

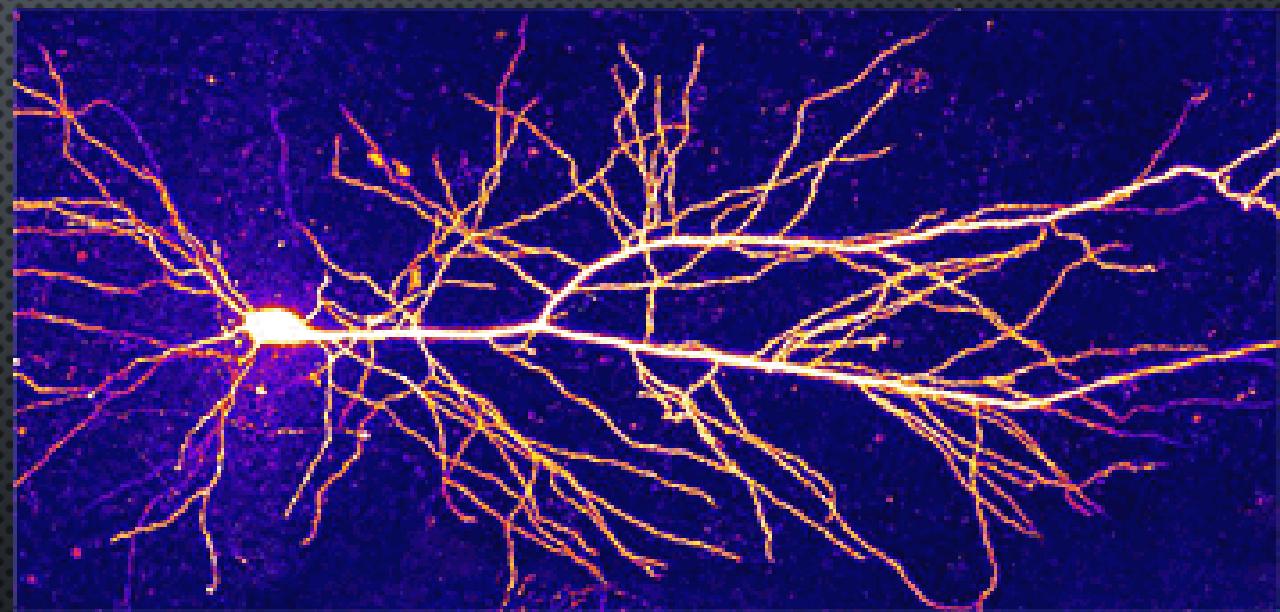
Structural plasticity: synaptic receptor density



Structural plasticity: dendritic arborization

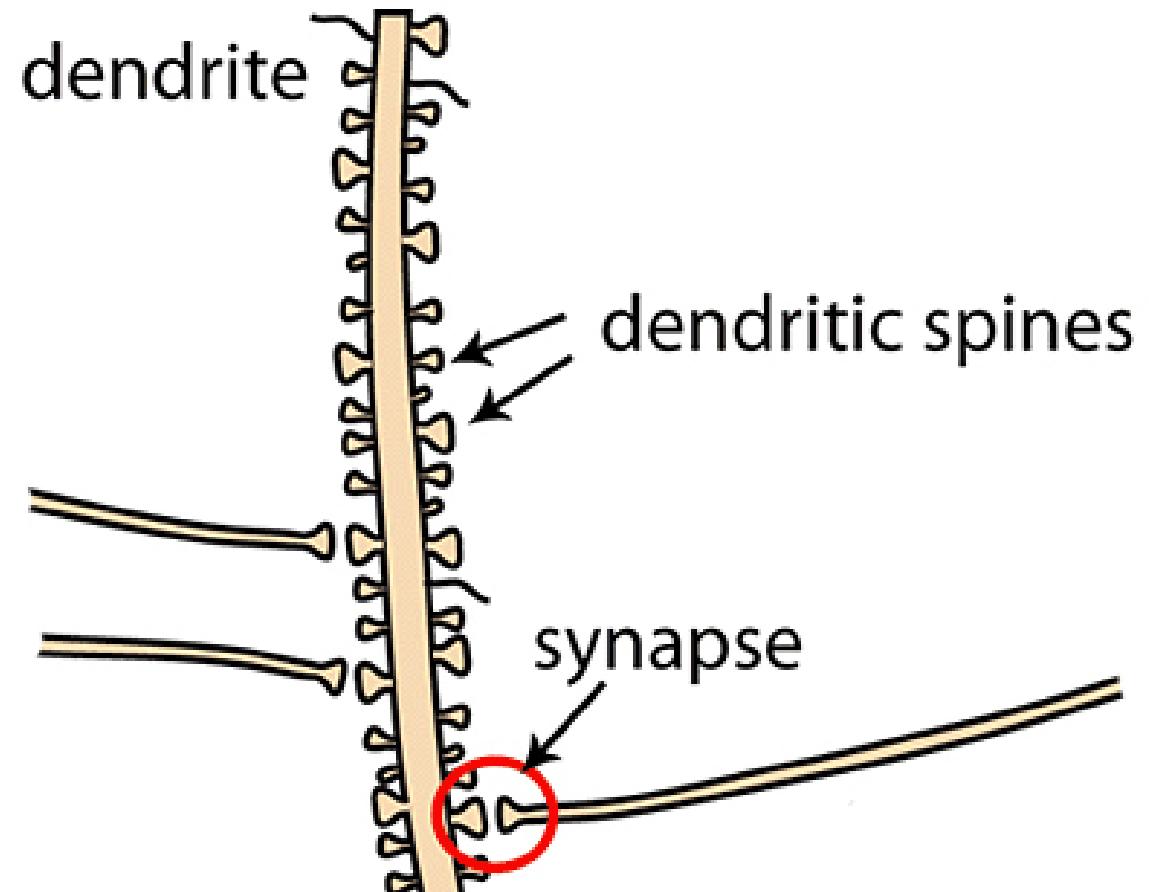


Purkinje neuron in the cerebellum

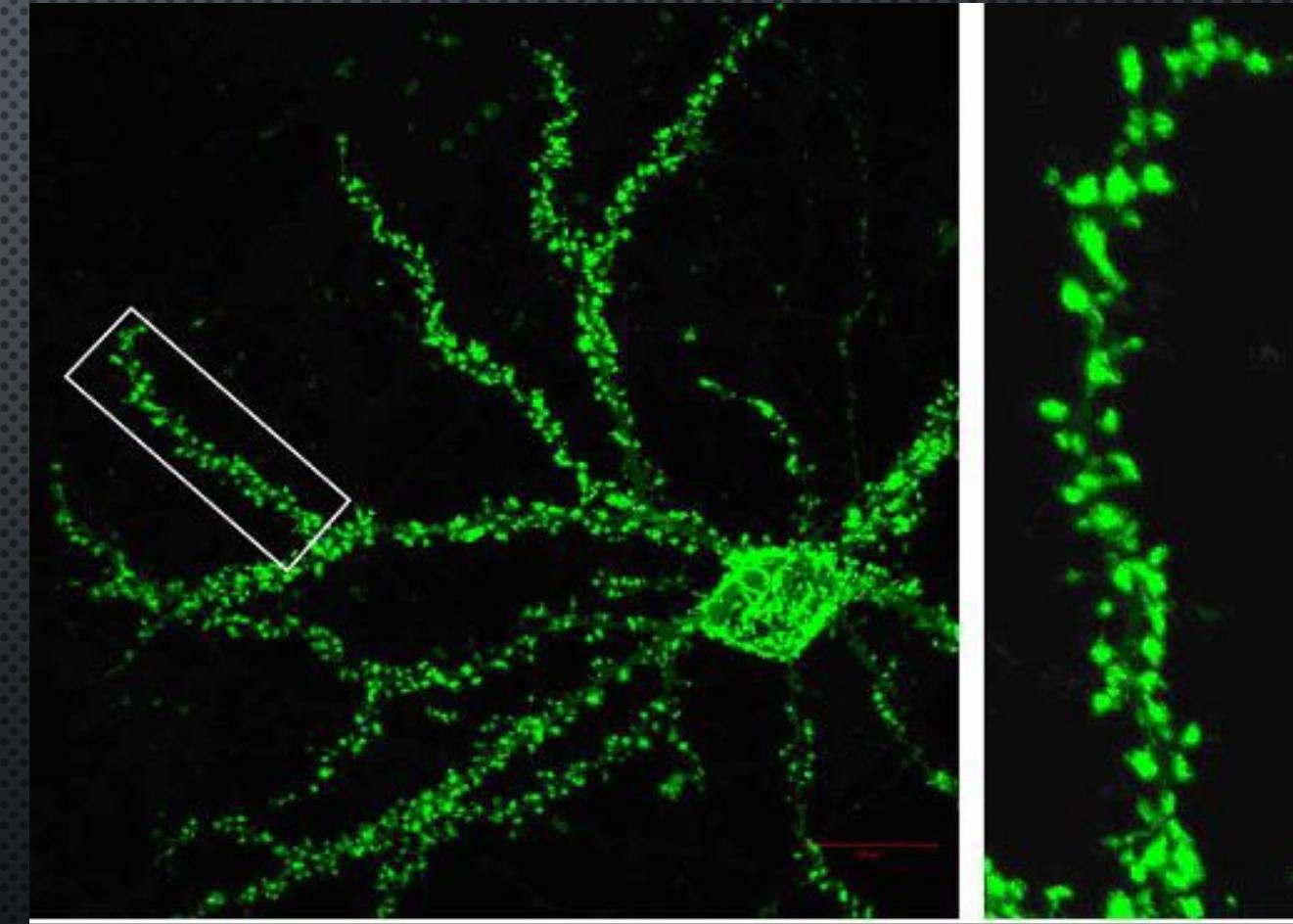


Hippocampal CA1 pyramidal neuron

Structural plasticity: dendritic spine density

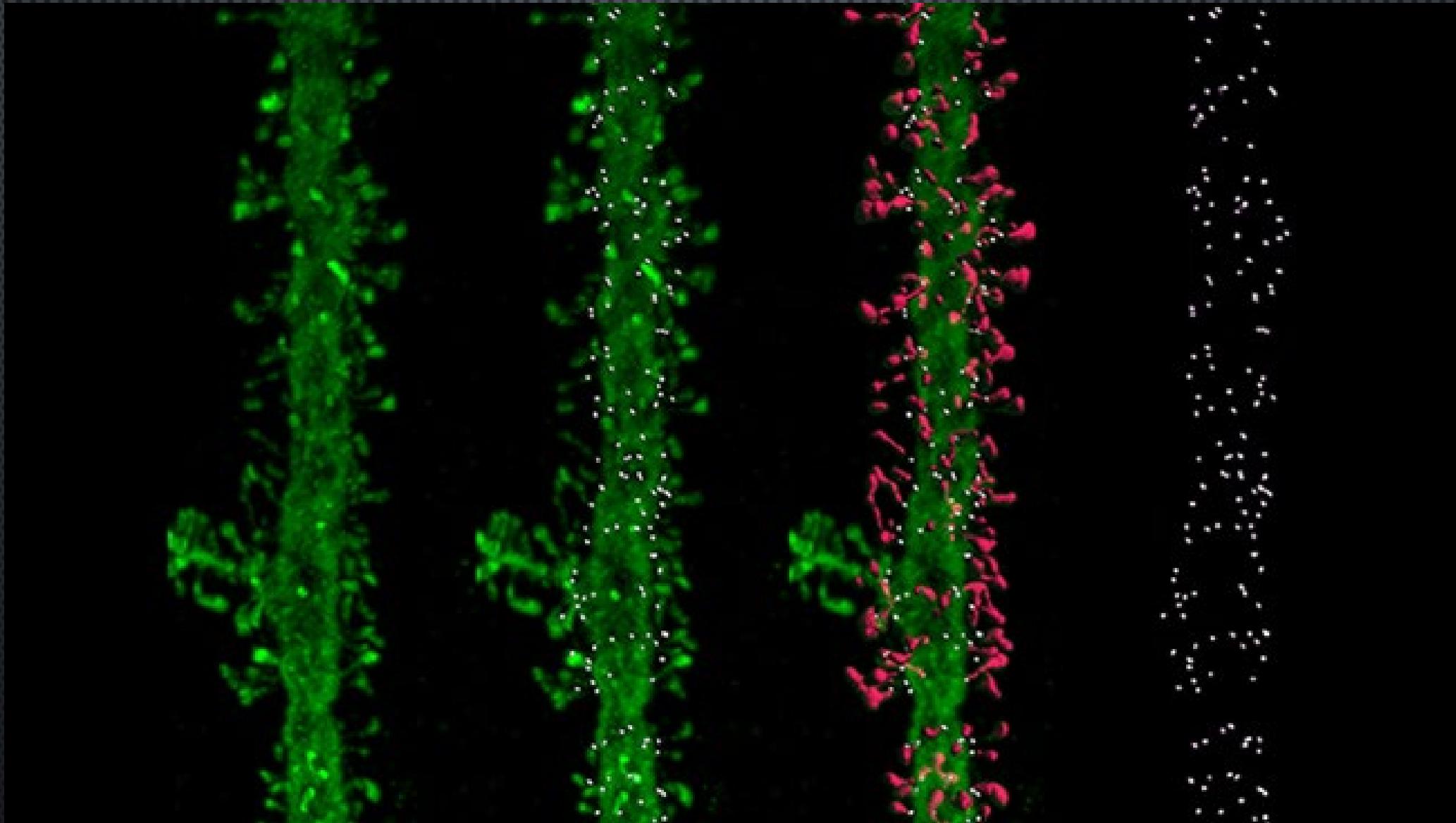


<https://neuwritesd.org/2017/03/16/mind-the-gap-spaced-learning-and-dendritic-spines/>

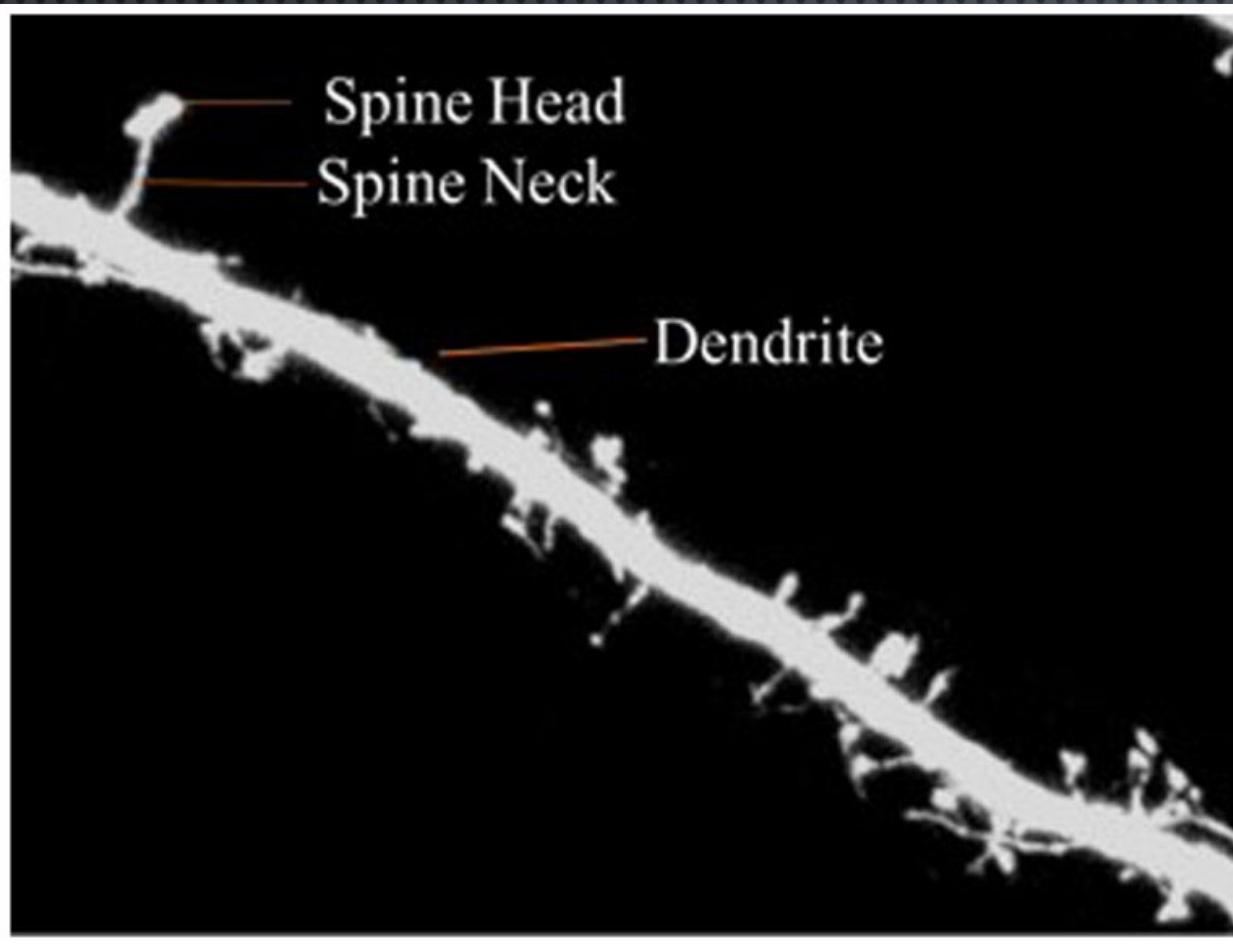


<http://www.lacasamorett.com/foxgallery/dendritic-spines.html>

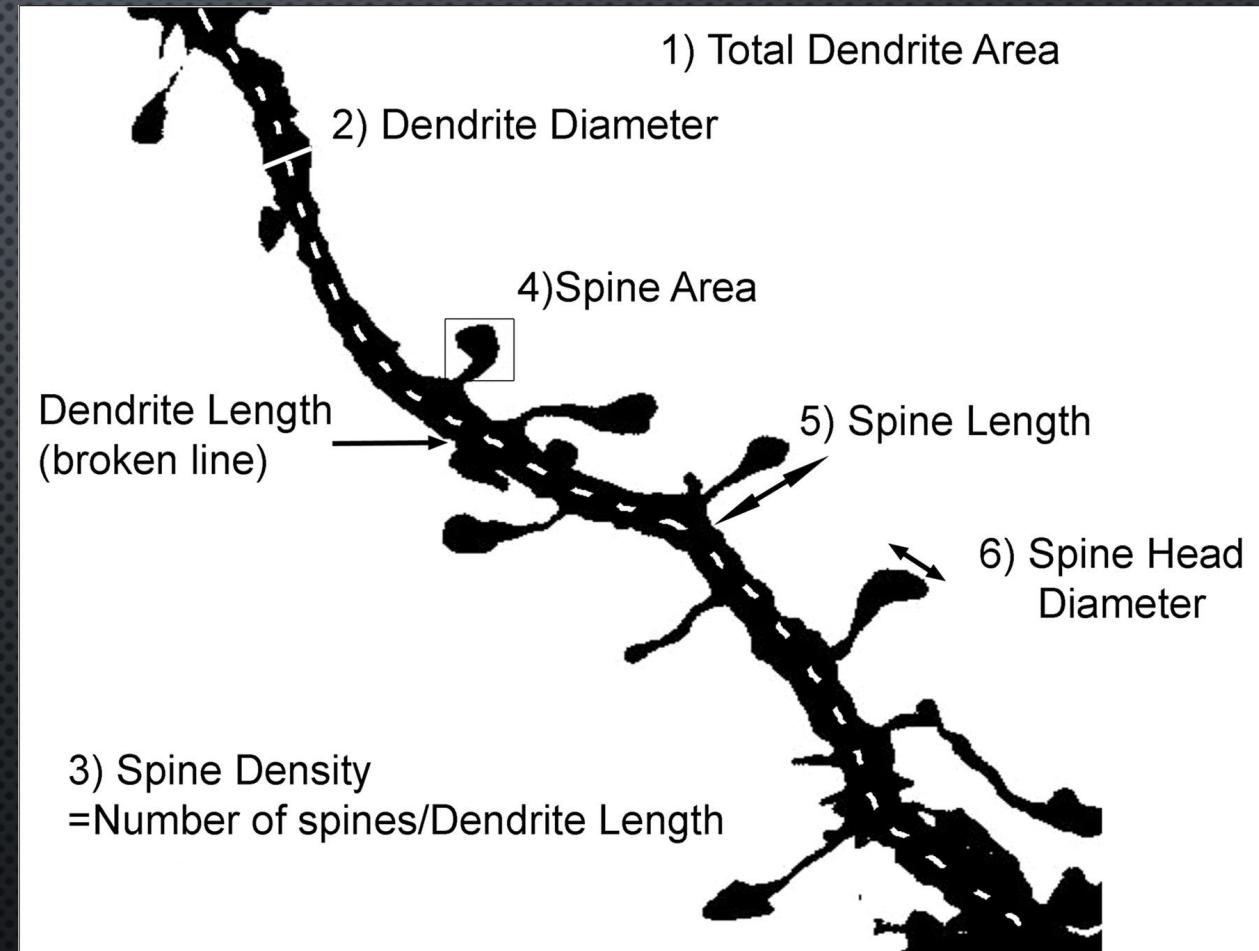
Structural plasticity: dendritic spine density



Structural plasticity: dendritic spine density

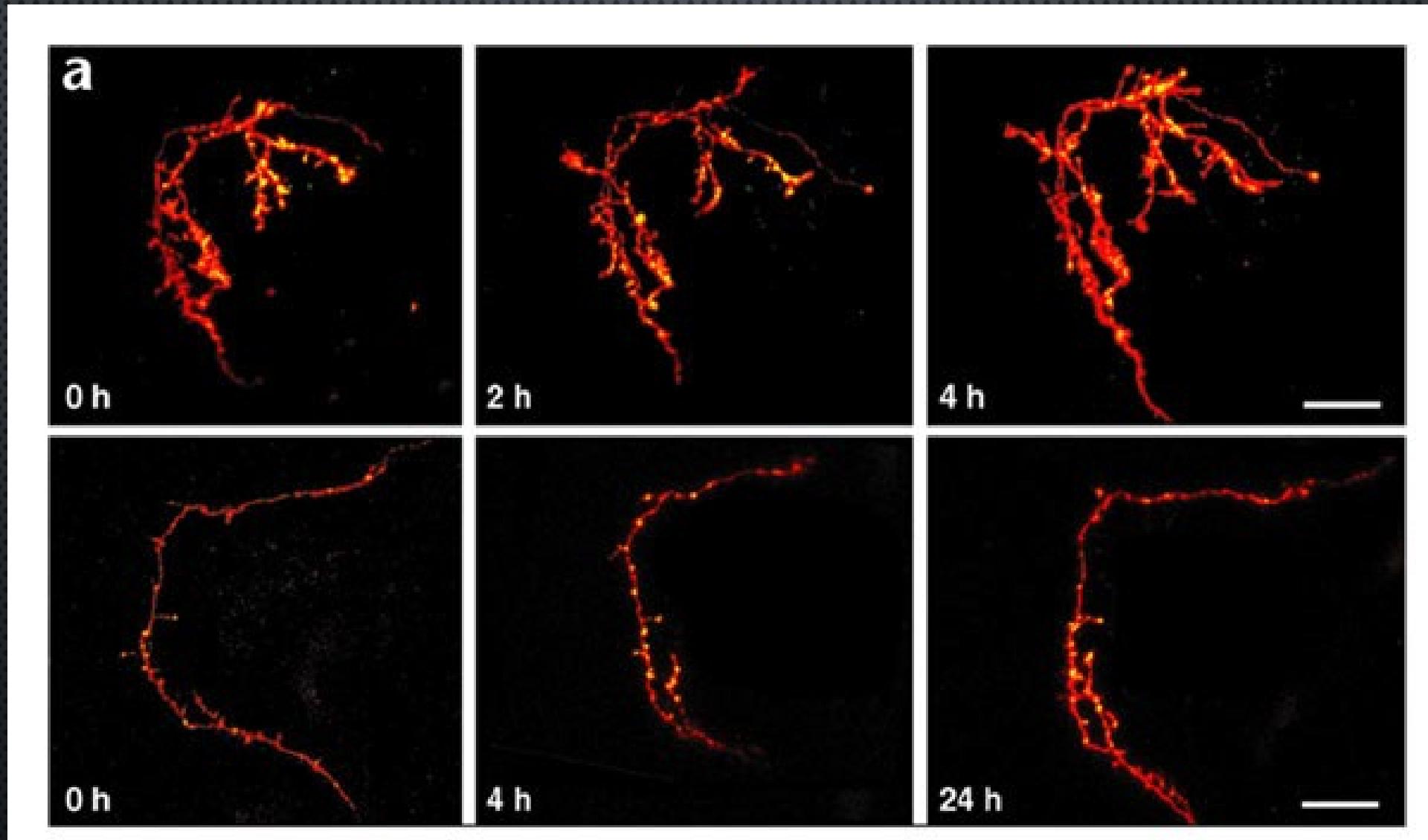


Wang et al., PeerJ, 2016

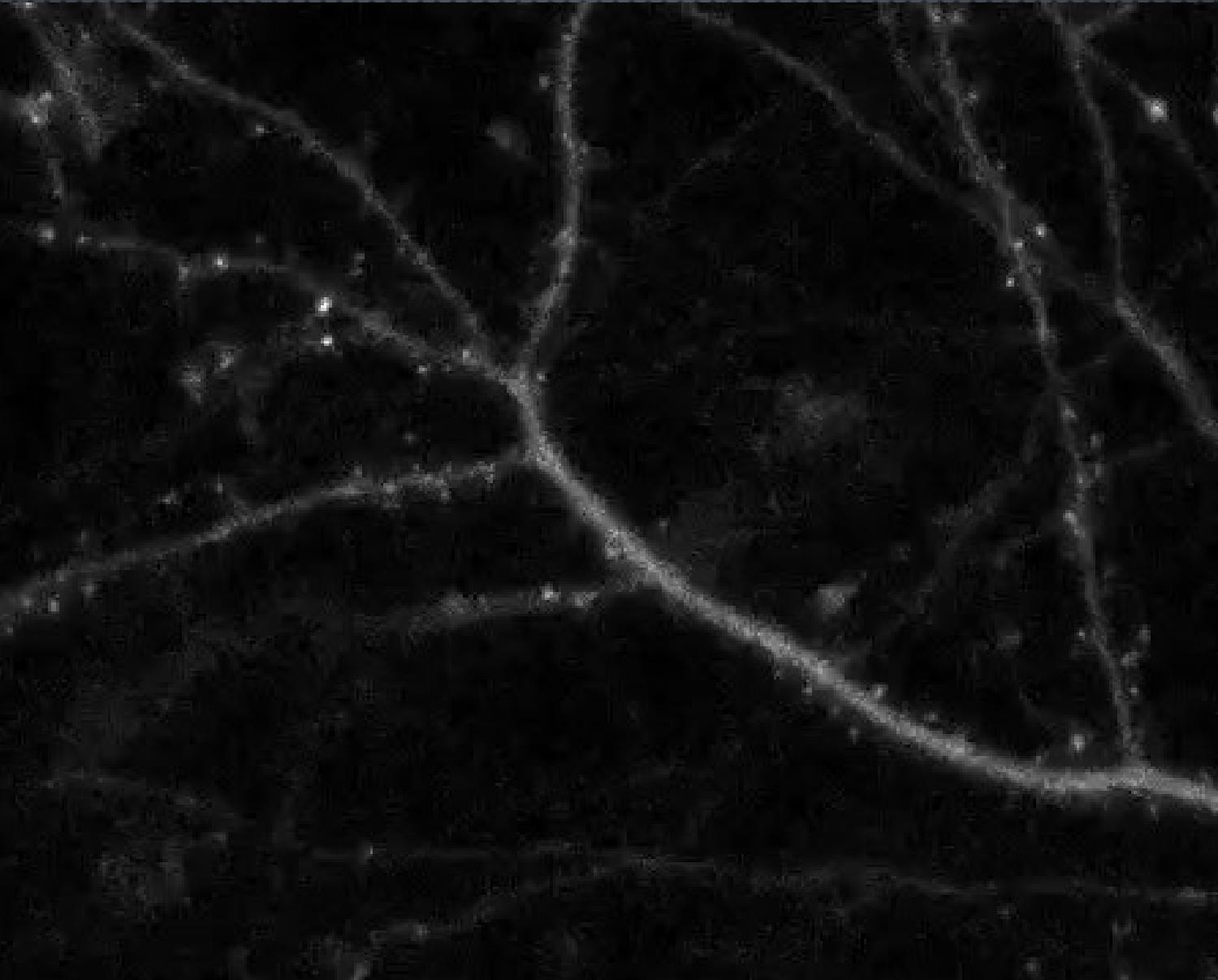


Smith et al., PNAS, 2009

Structural plasticity: axonal arborization

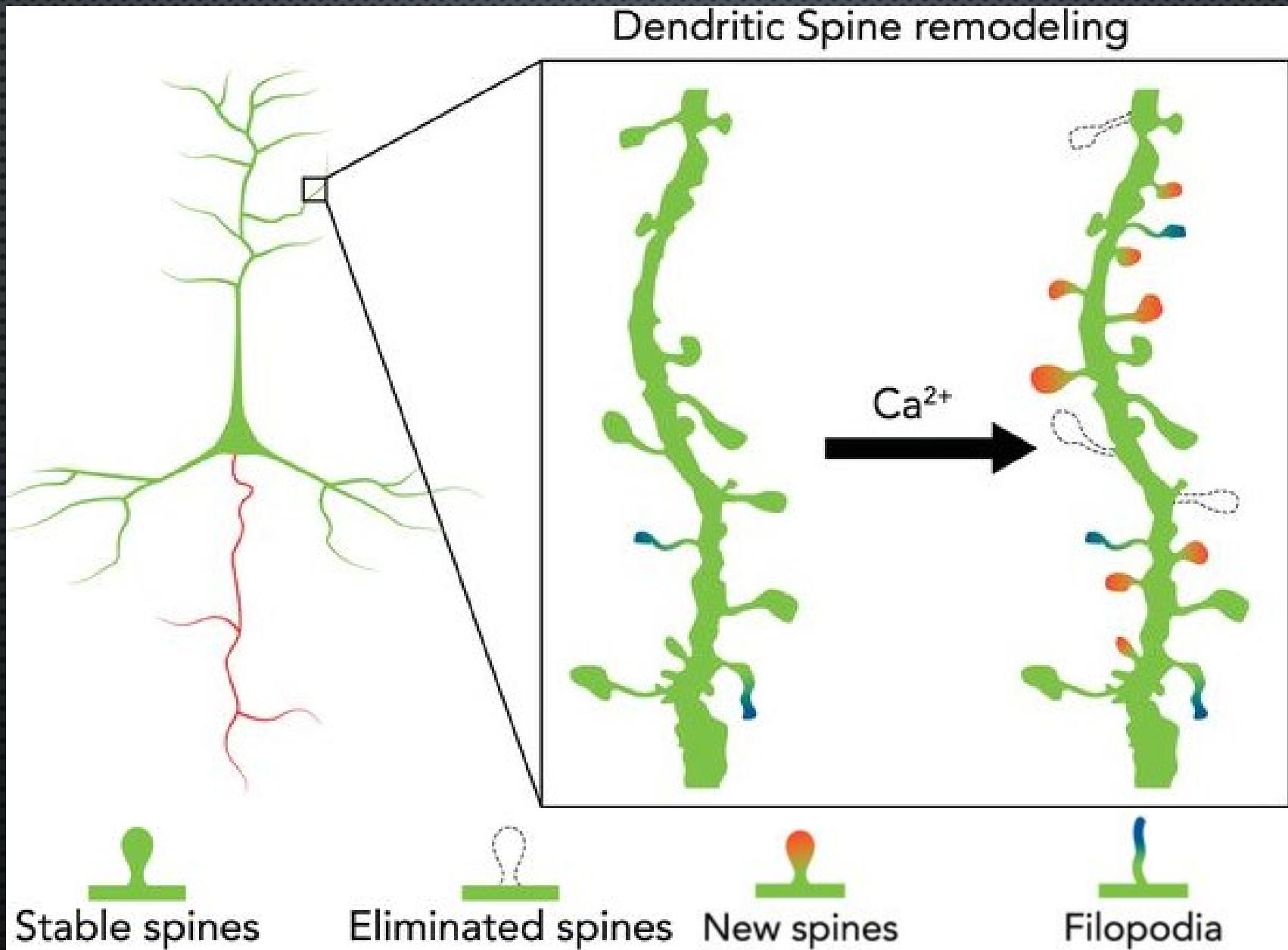


Structural plasticity in action

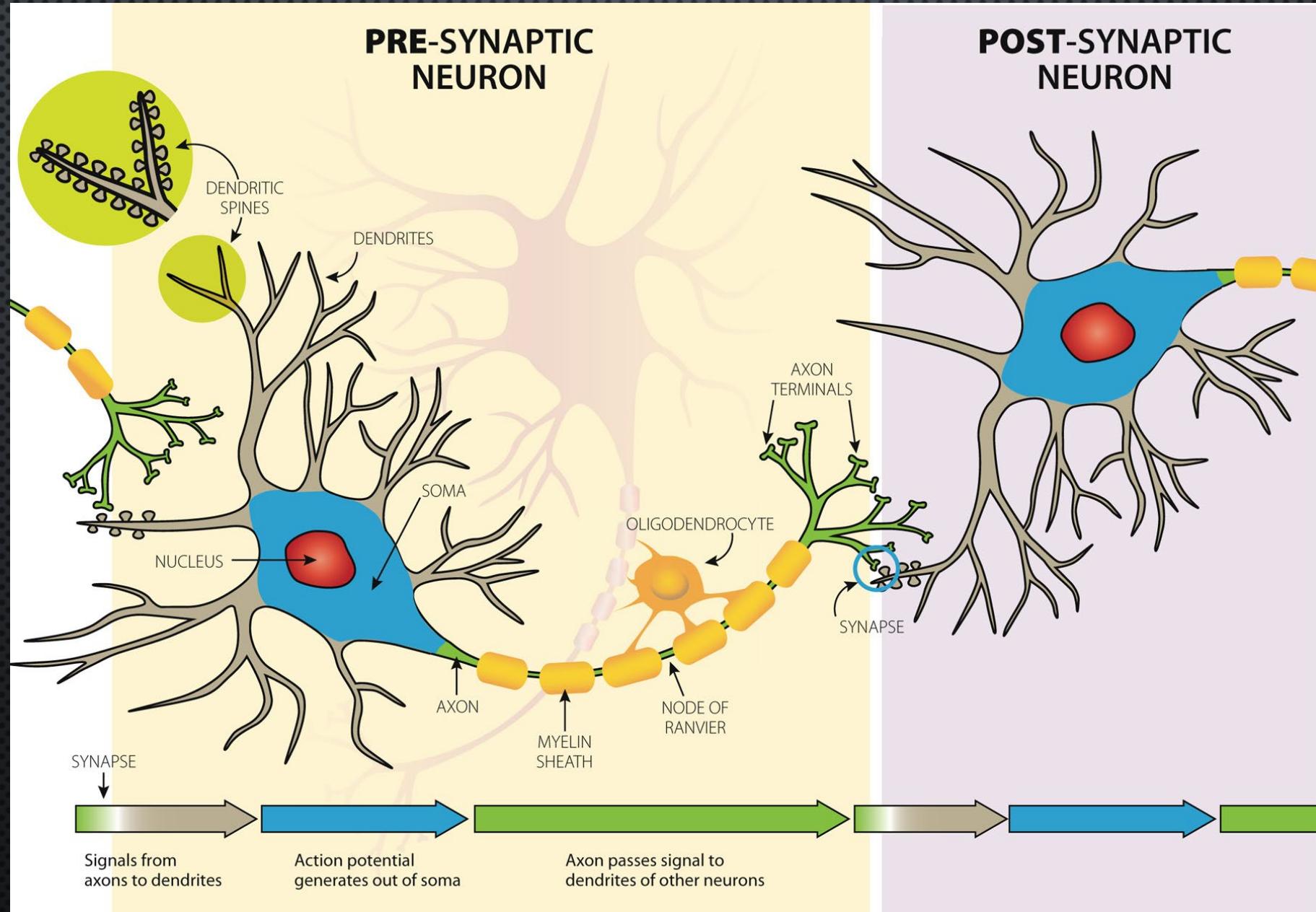


Minerbi et al., *PLOS Biol.*, 2009

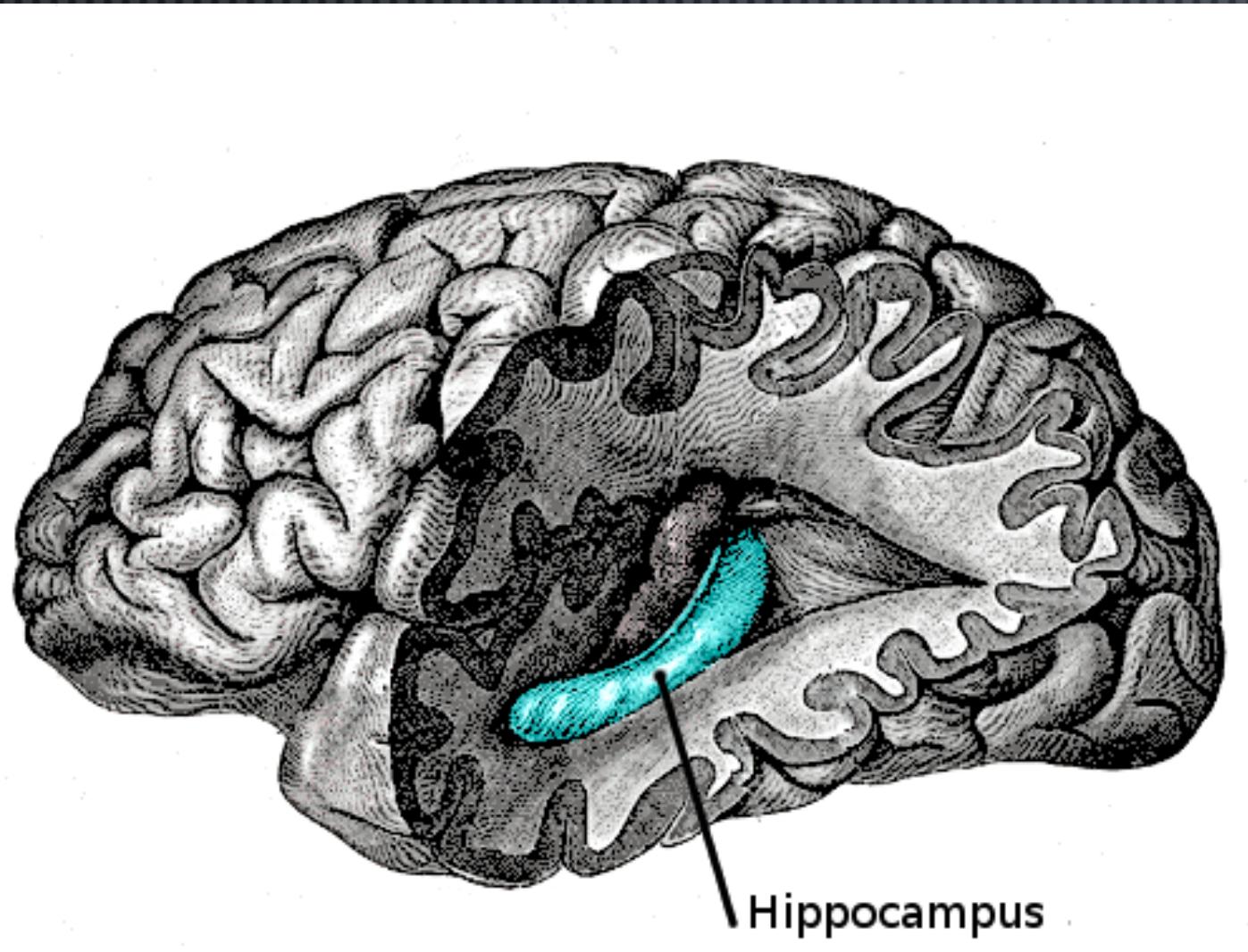
Structural plasticity in the hippocampus



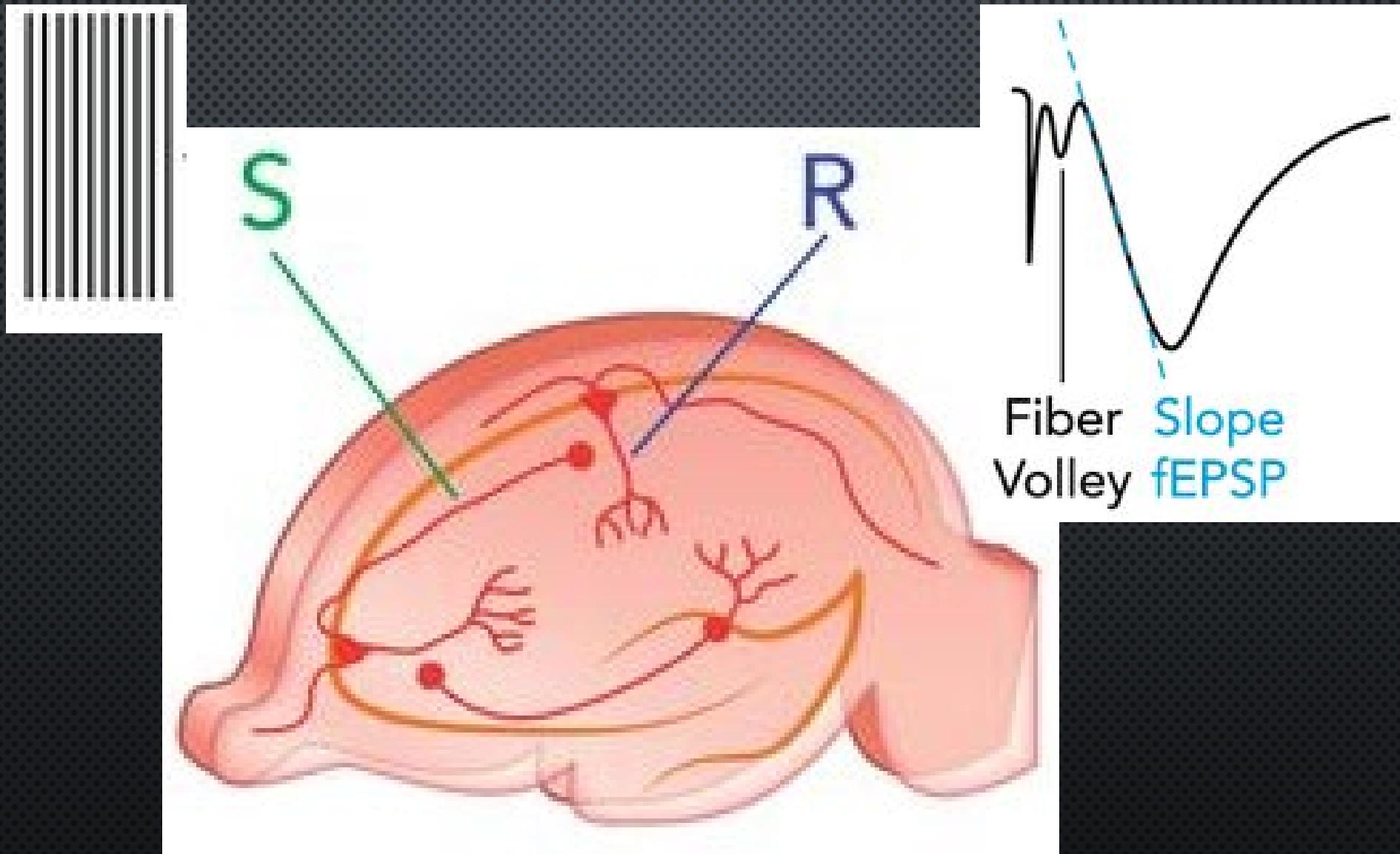
Functional (synaptic) plasticity



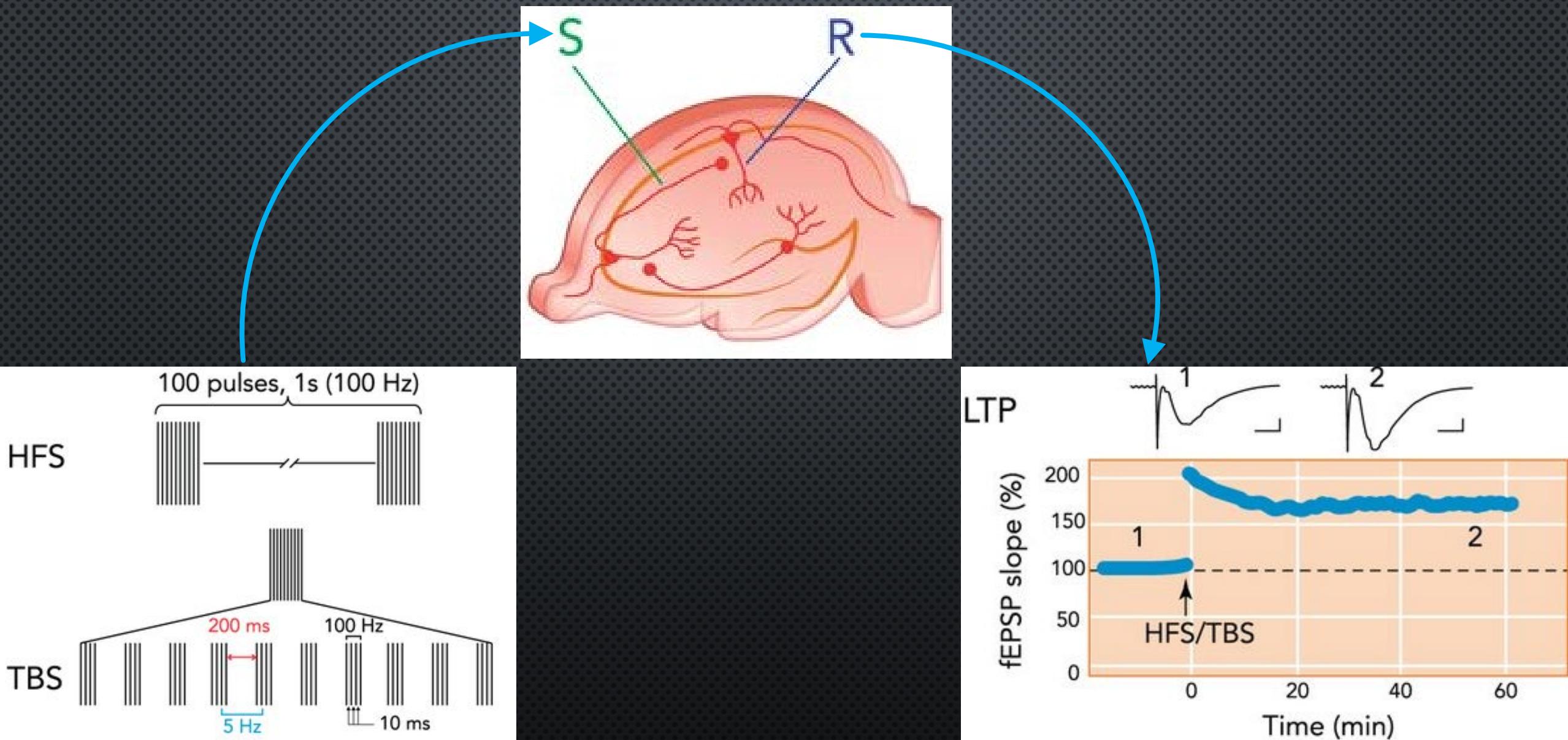
Synaptic plasticity in the hippocampus



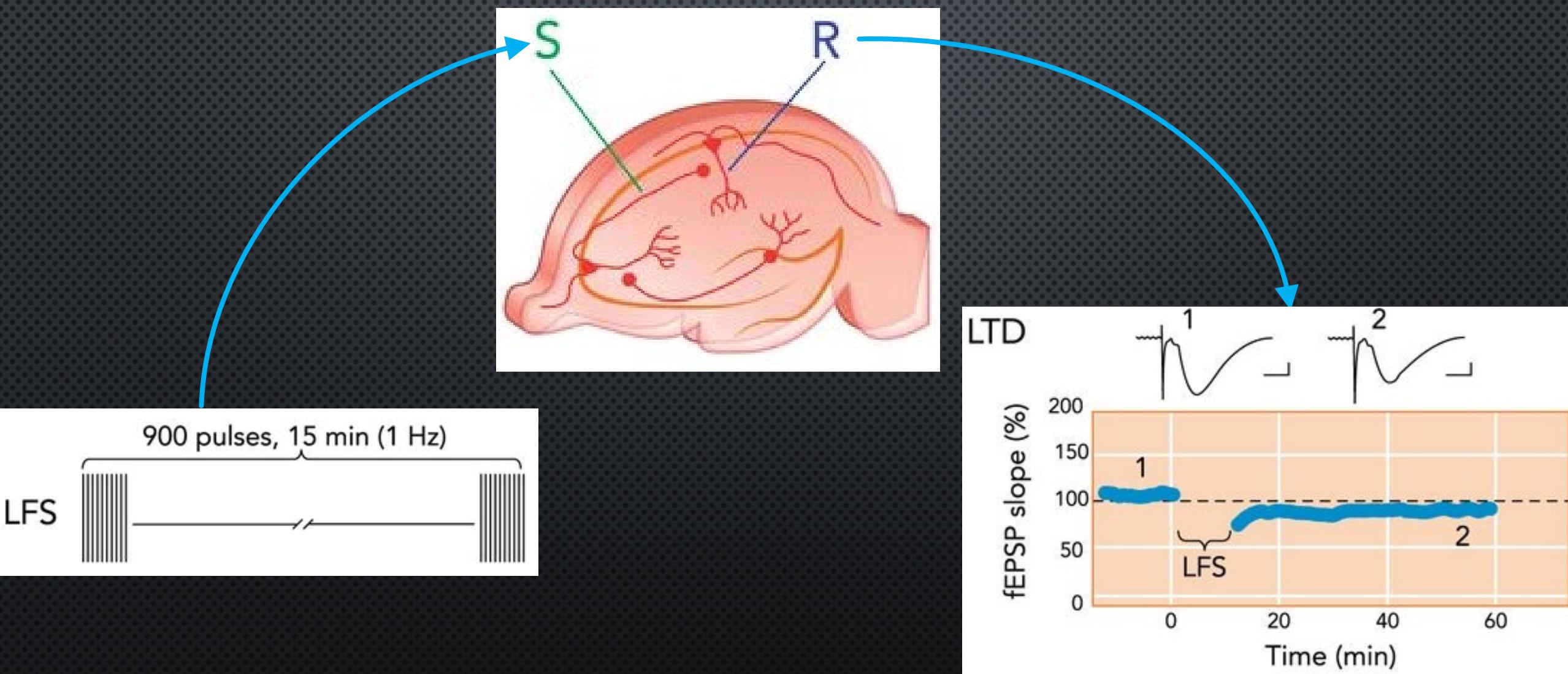
Mechanisms underlying synaptic plasticity



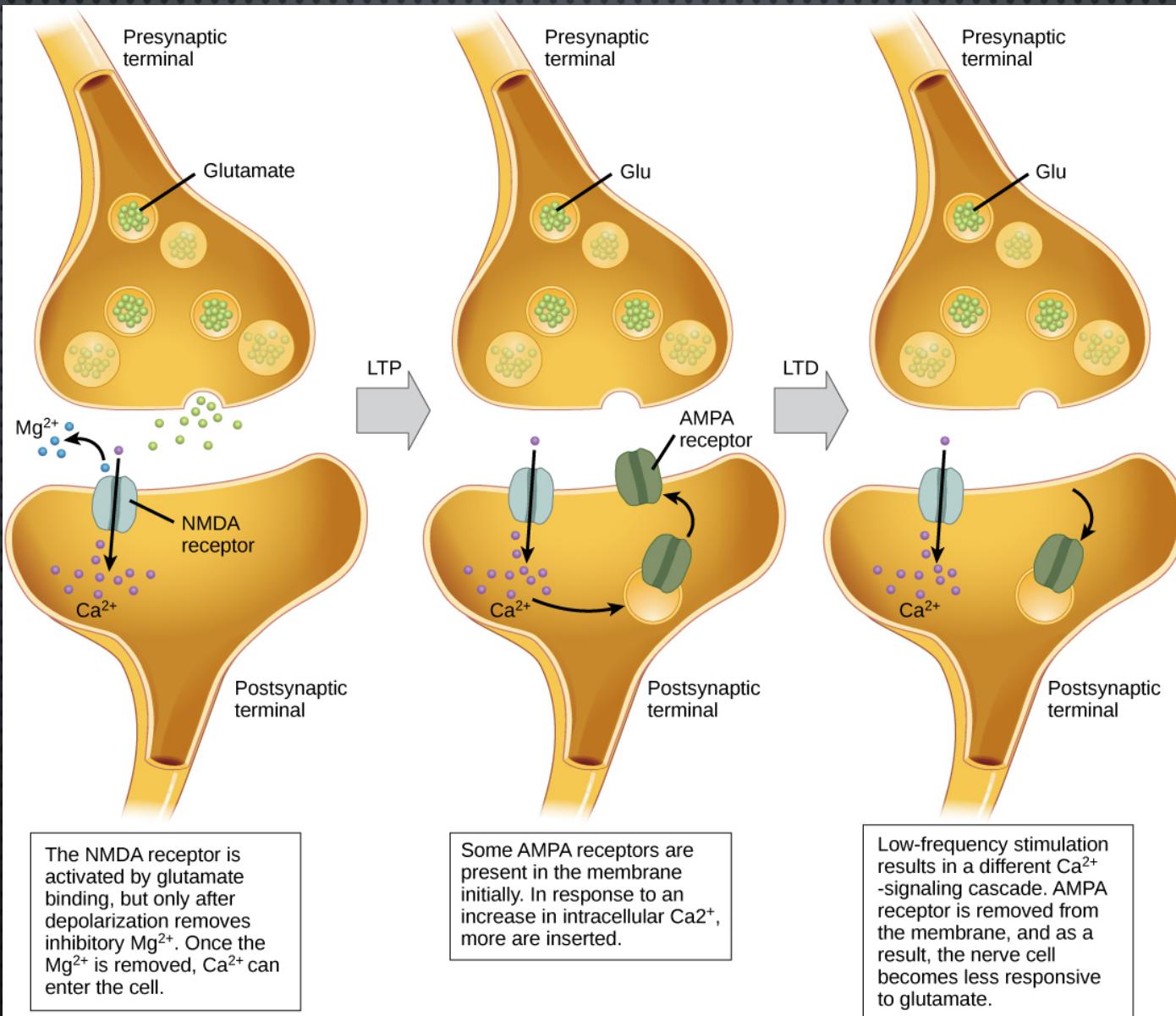
Long Term Potentiation (LTP)



Long Term Depression (LTD)

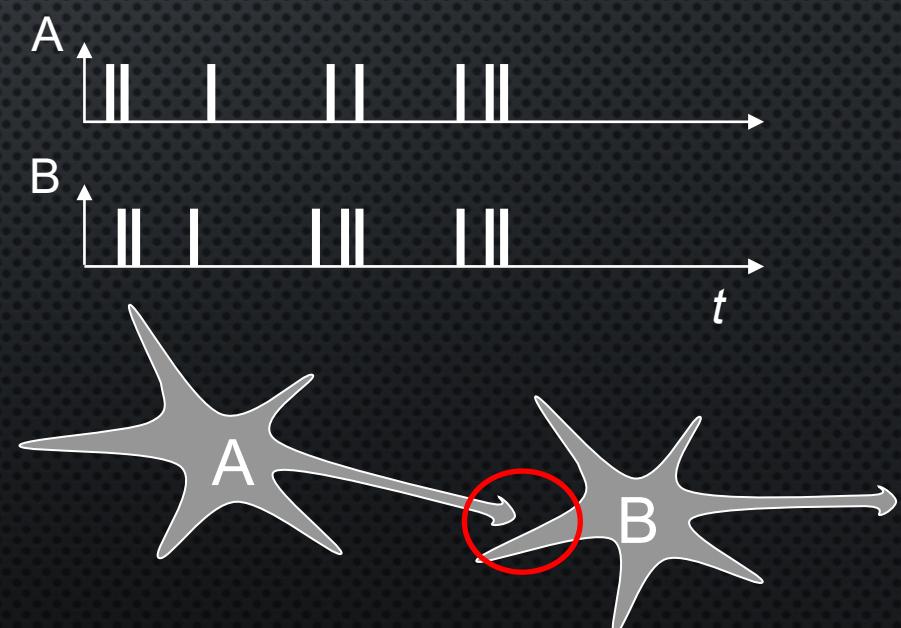


Chemical basis for LTP and LTD

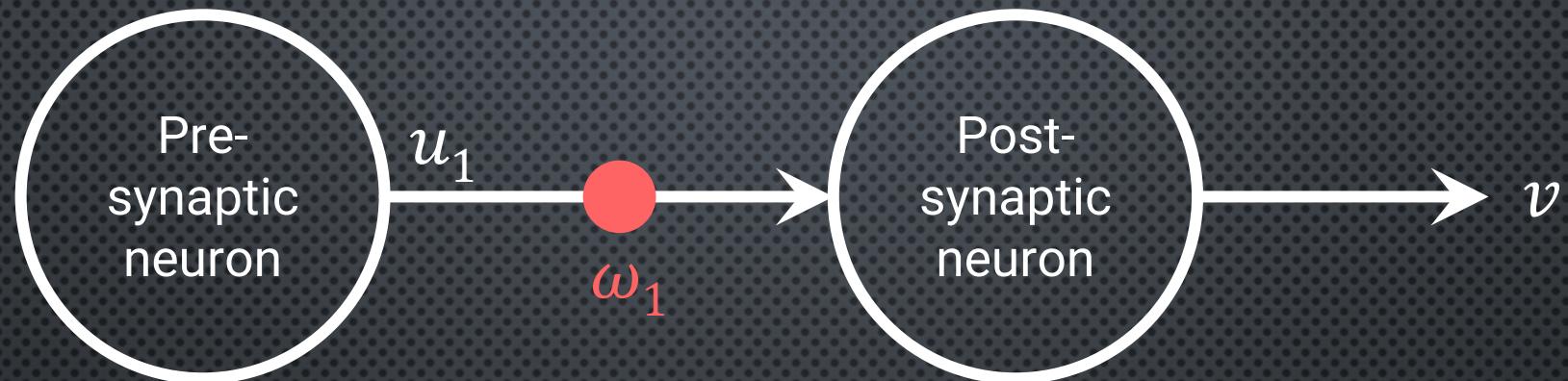


Hebbian learning

- Postulated in 1949 by Canadian psychologist Donald O. Hebb in his book “The organization of behaviour”
- *“When an axon of a cell A is near enough to excite cell B or repeatedly or persistently takes part in firing it, some growth or metabolic change takes place in both cells such that A’s efficiency, as one of the cells firing B, is increased.”*
- The functional basis for this change is correlation between presynaptic and postsynaptic activity – “neurons that fire together, wire together”
- Hebbian learning is temporal correlation-based, associative, unsupervised learning



Hebbian learning: a simple model



ω_1 always grows → **UNSTABLE!**

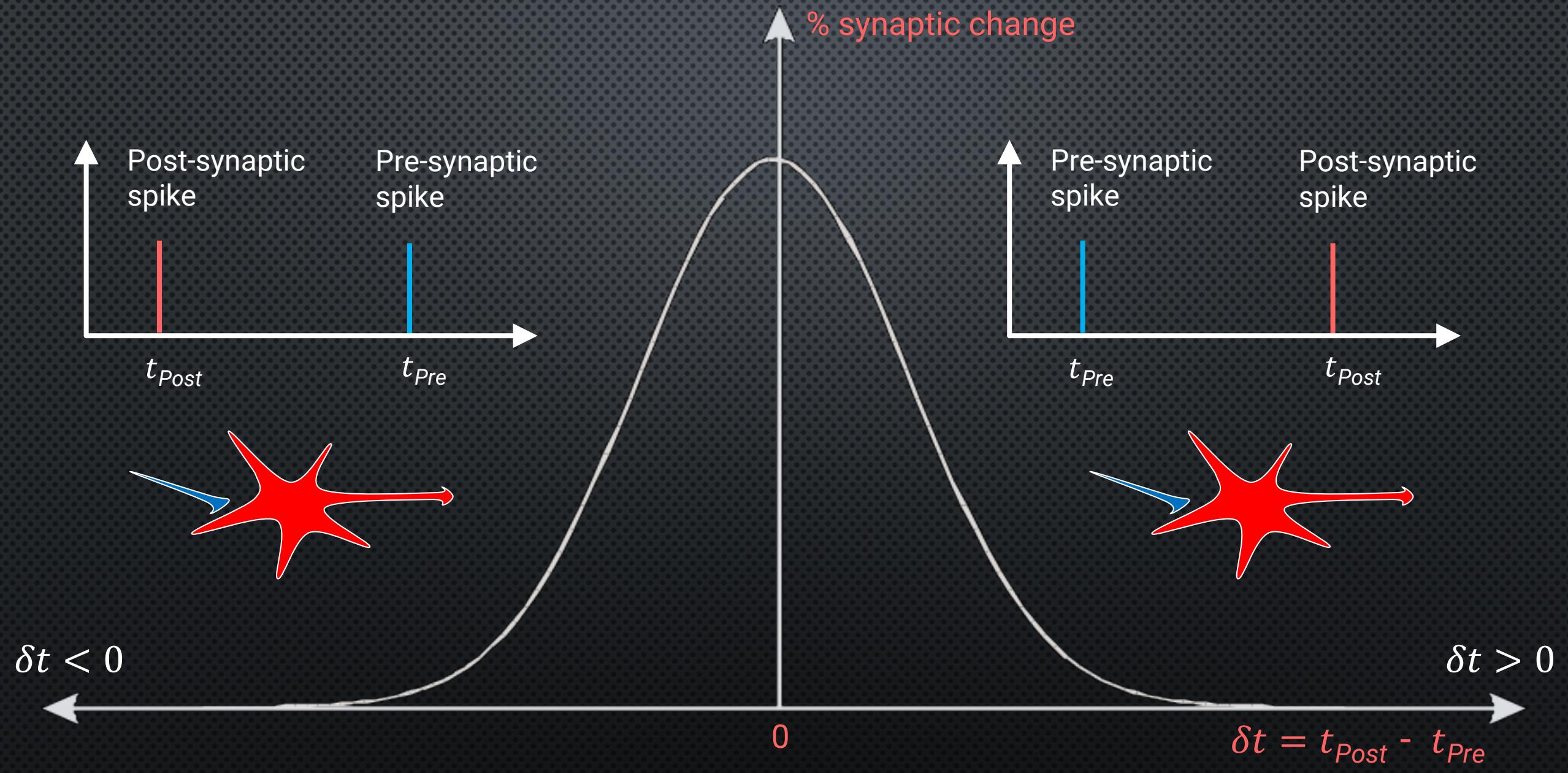
$$\frac{d\omega_1}{dt} = \mu \cdot v \cdot u_1$$

Change in
synaptic weight

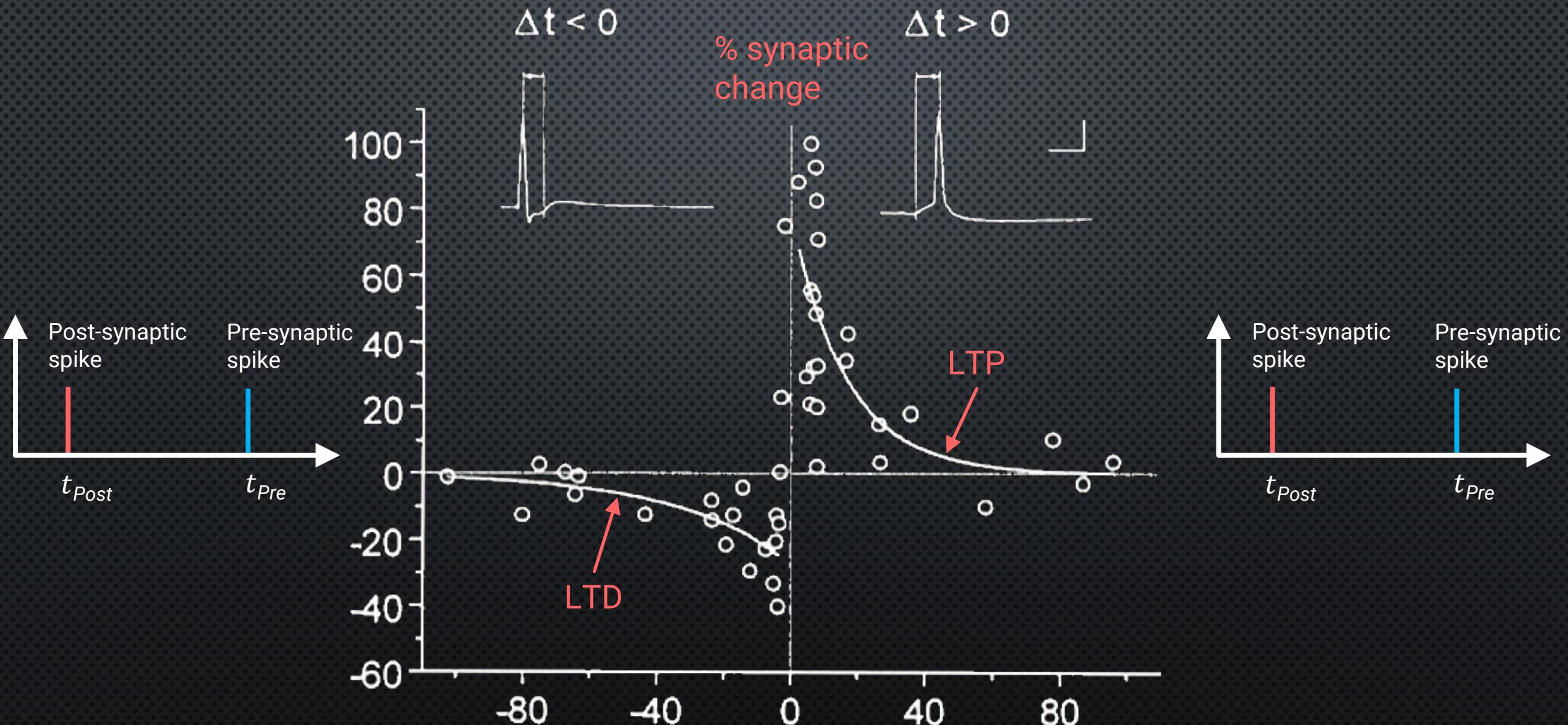
Output of post-synaptic neuron
Learning rate ($\mu \ll 1$)

Output of pre-synaptic neuron / input to post-synaptic neuron

Hebbian learning: LTP



Spike-timing dependent plasticity (STDP): LTP and LTD



Other forms of functional plasticity

Homosynaptic plasticity

changes in synapse strength occur only at post-synaptic targets that are specifically stimulated by a pre-synaptic target

Heterosynaptic plasticity

activity of a third neuron can releases chemical neuromodulators that induce changes in synaptic strength between two other neurons

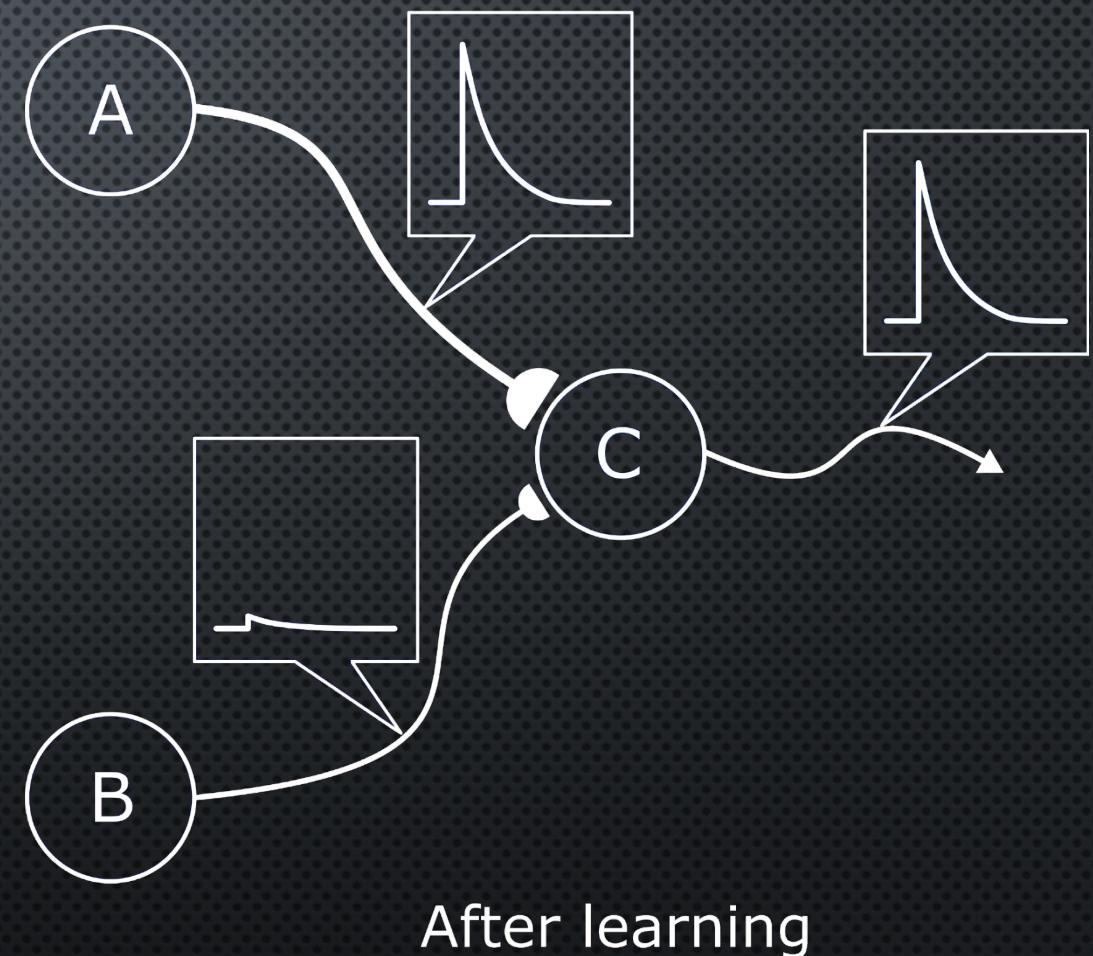
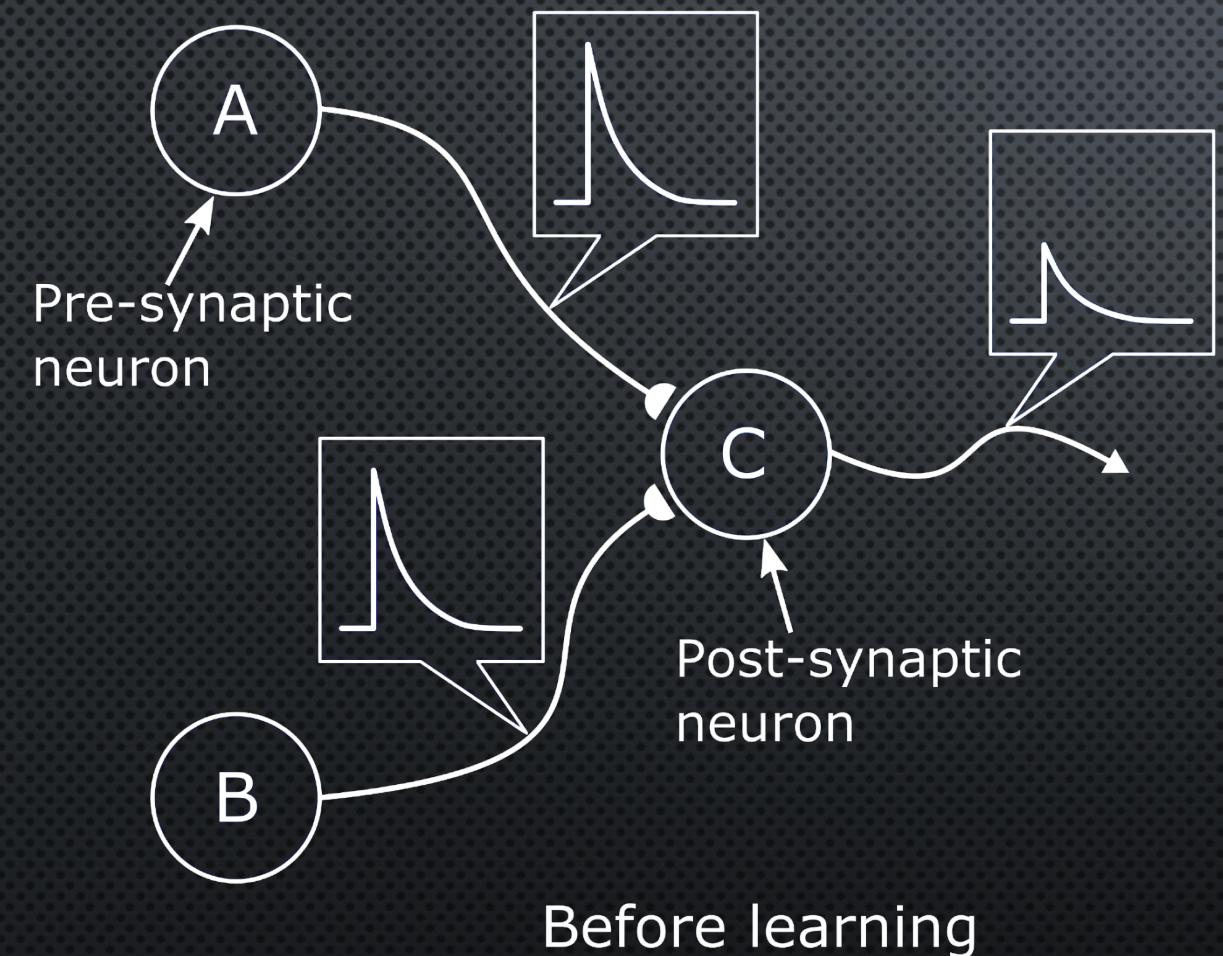
Non-synaptic plasticity

intrinsic excitability, i.e. sensitivity to synaptic input, of neurons can be altered and is manifested as changes in the firing characteristics of the neuron itself

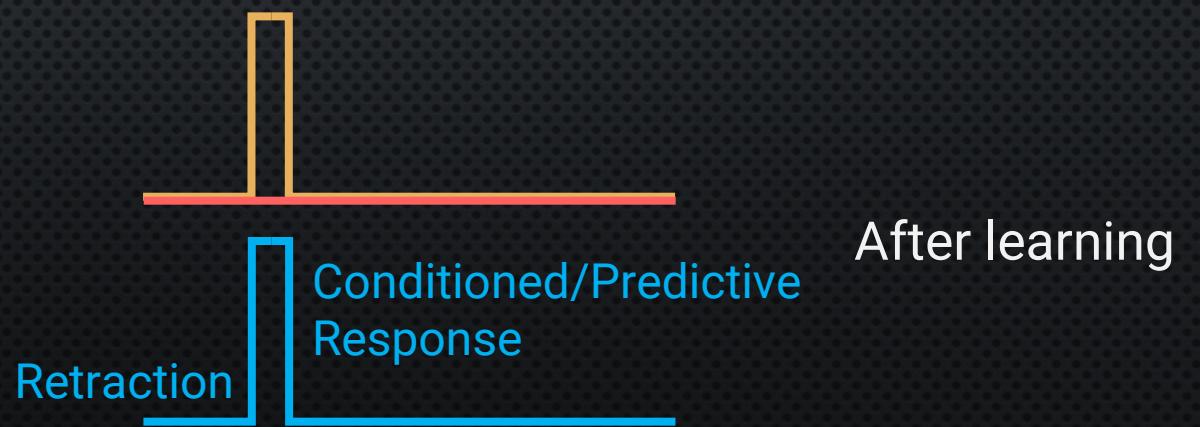
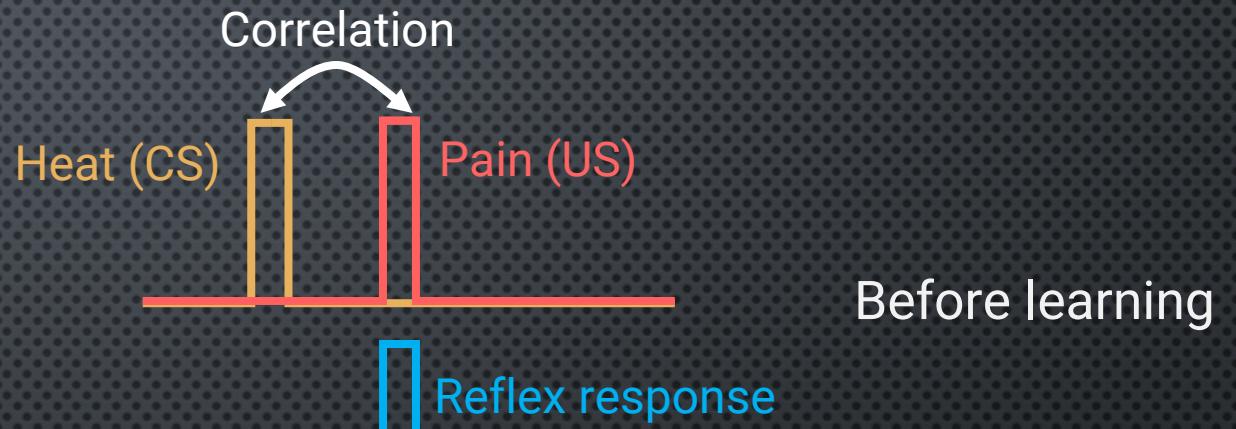
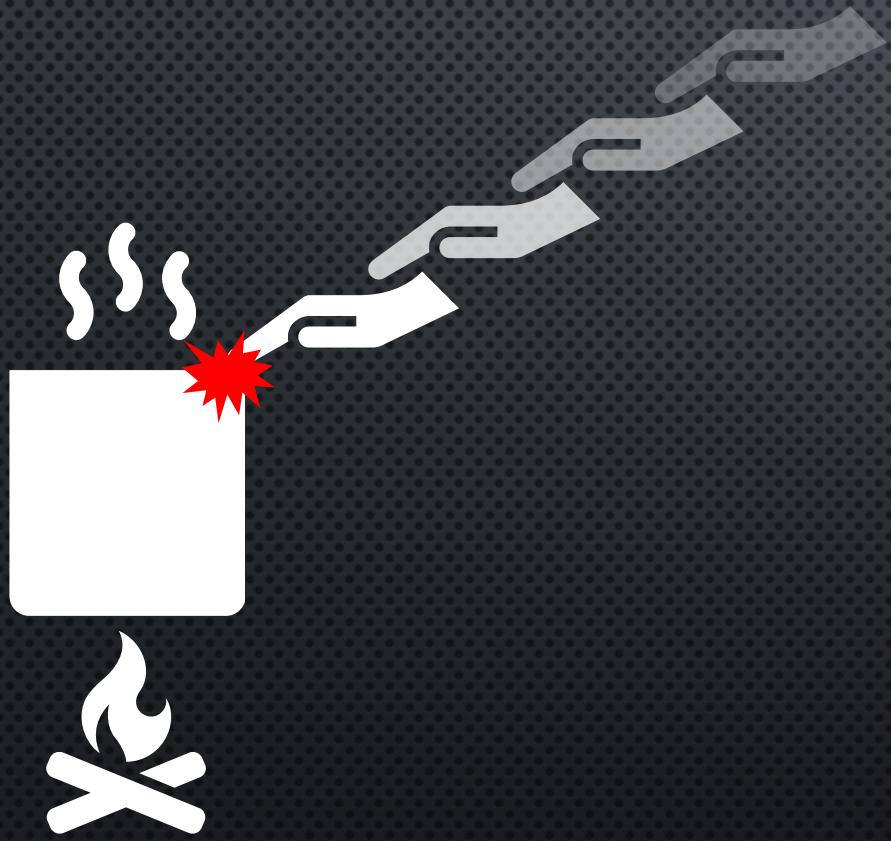
Homeostatic plasticity

capacity of neurons to regulate their own excitability relative to network activity, a compensatory adjustment that occurs over the timescale of days

Heterosynaptic plasticity



Learning quickly from temporal correlations



Input correlation learning (ICO learning)

Predictive signal

Learning rule:

$$\frac{\delta w_a}{\delta t} = \eta \cdot f(A, t) \otimes \frac{\delta f(B, t)}{\delta t}$$

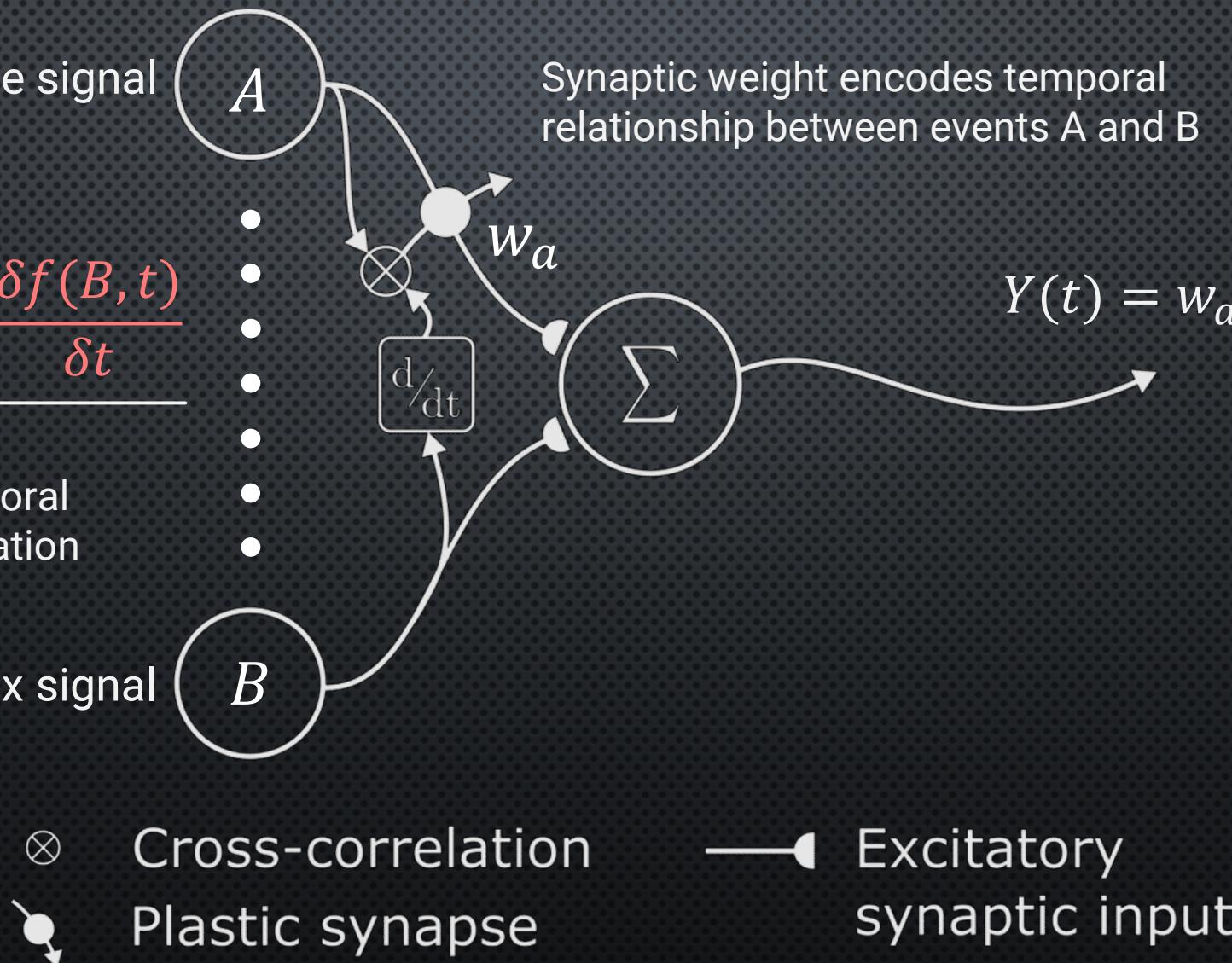
η
Learning rate

$\frac{d}{dt}$
Temporal correlation

Reflex signal

Synaptic weight encodes temporal relationship between events A and B

$$Y(t) = w_a \cdot f(A, t) + f(B, t)$$



Artificial brain with learning

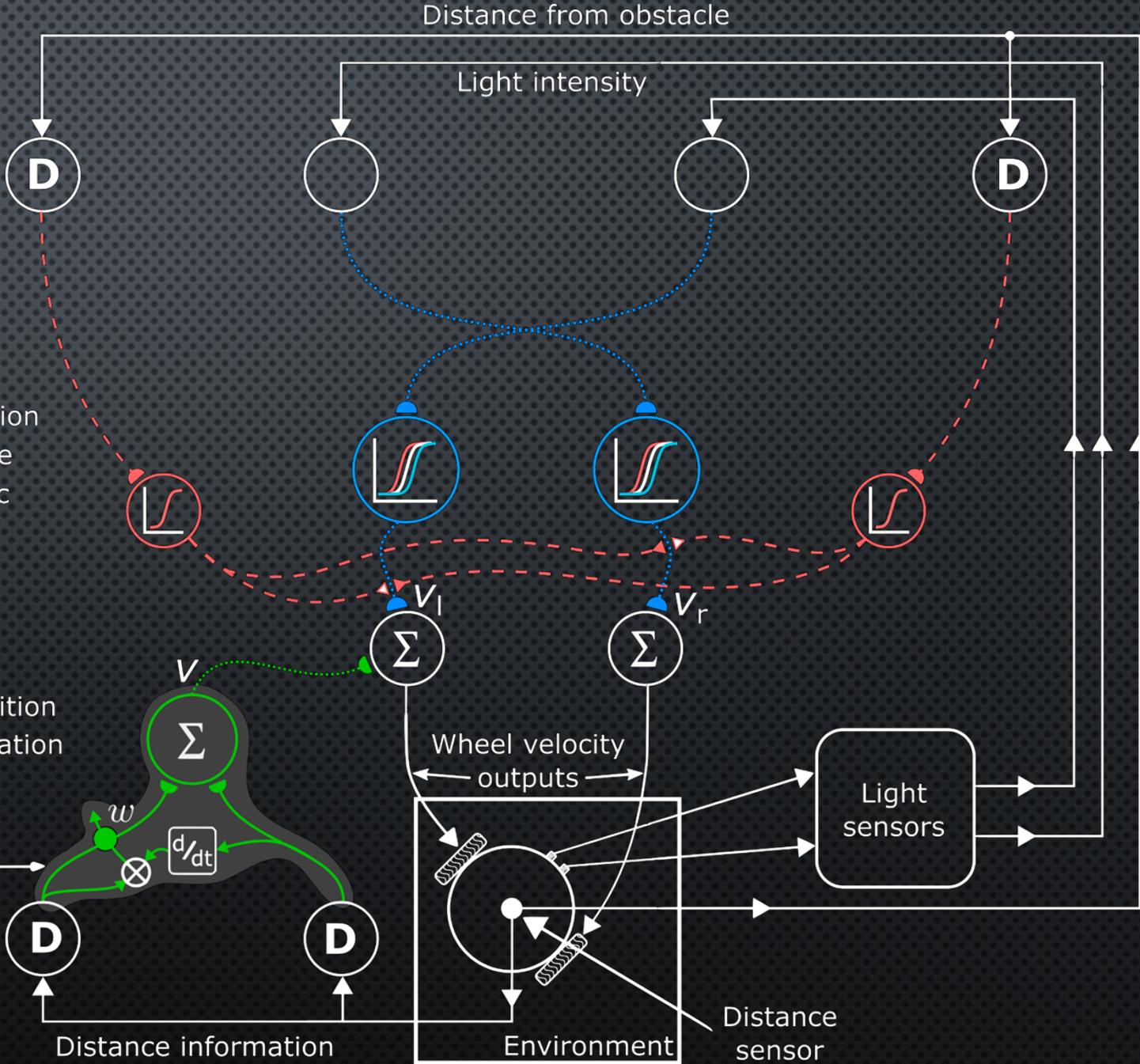
Learning rule:

$$\frac{dw}{dt} = \mu \cdot D \cdot \frac{Dt}{dt}$$

$$v = \omega \cdot D(t) + D(t - 1)$$

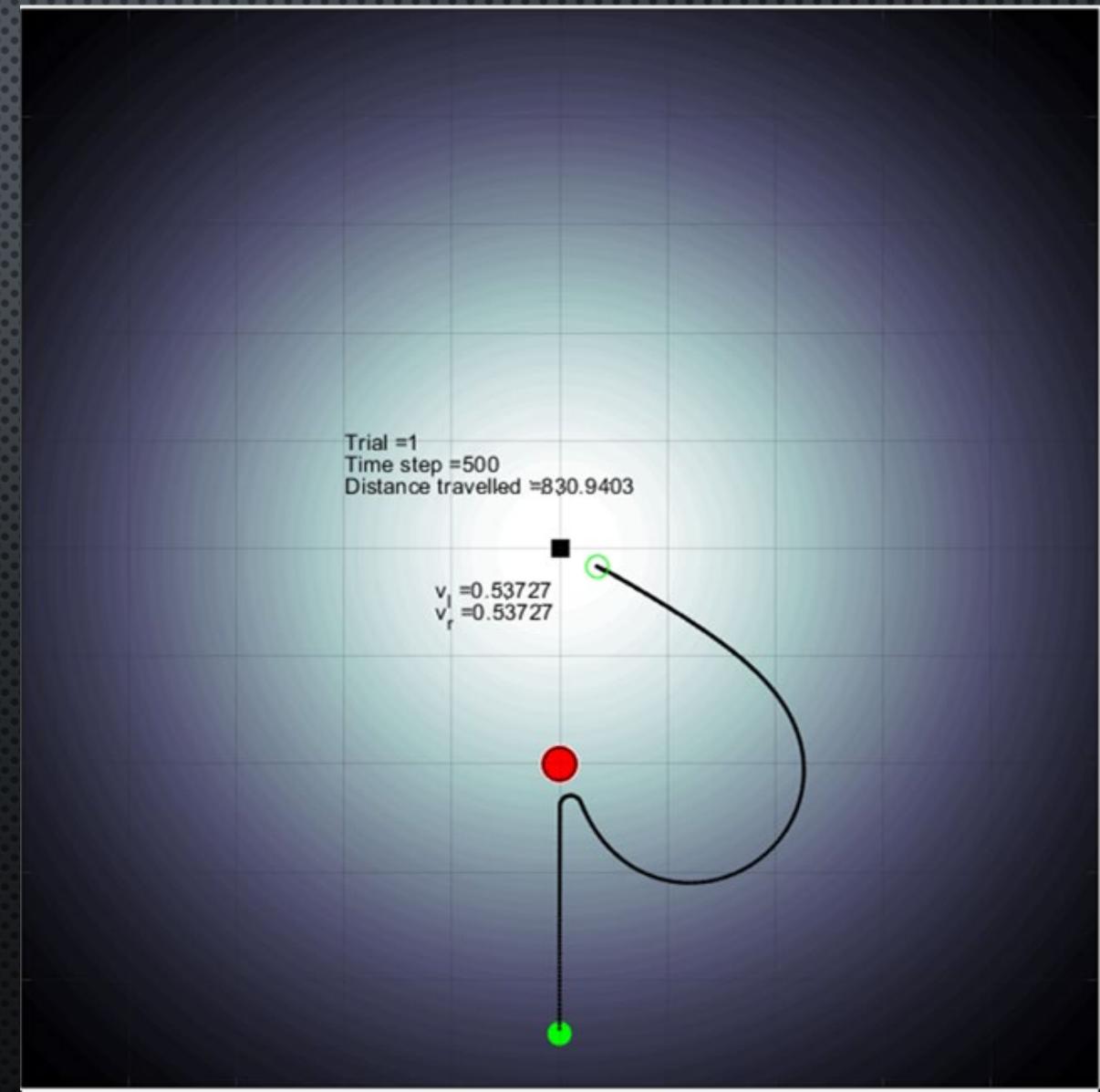
- ⊗ Cross-correlation
- ↘ Plastic synapse
- Heterosynaptic plasticity
- Non-synaptic plasticity
- Excitatory synaptic input
- ← Synaptic inhibition
- Synaptic excitation

ICO learning mechanism



Matlab exercise: obstacle avoidance learning

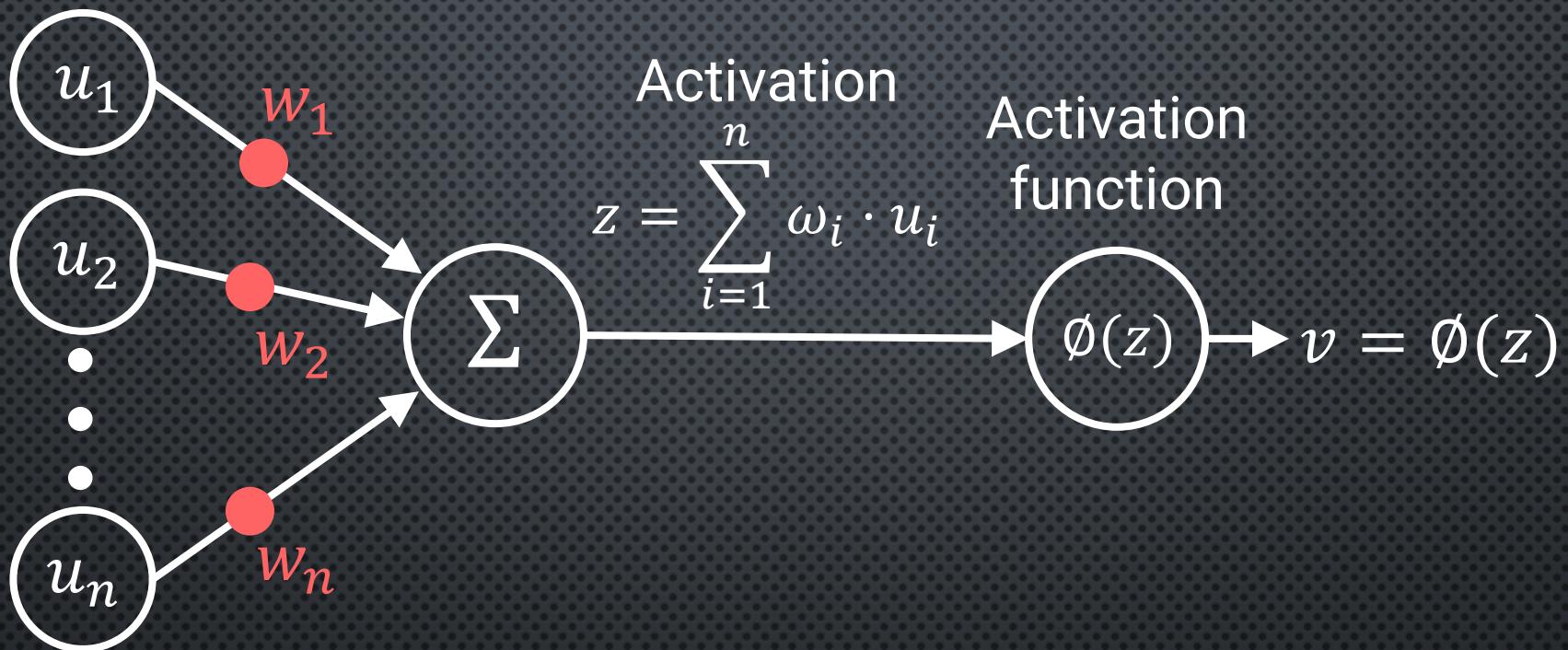
- Download “Assignment 3.pdf” and “obstacle_avoidance_learning.m”
- Follow instructions in PDF file





5 Supervised and unsupervised learning in artificial neural brains

Learning in perceptrons



Perceptrons have 2 sets of parameters - weights and activation function, and both affect the output.

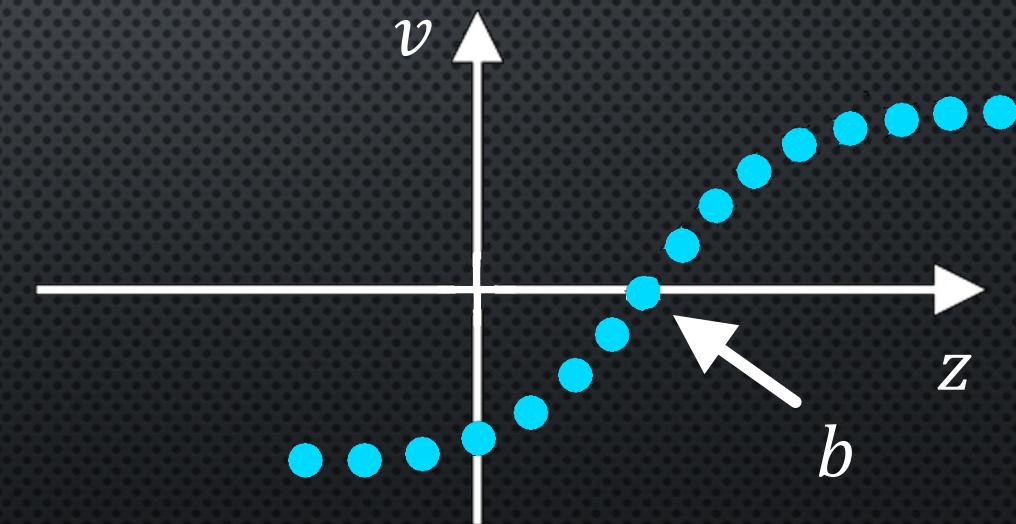
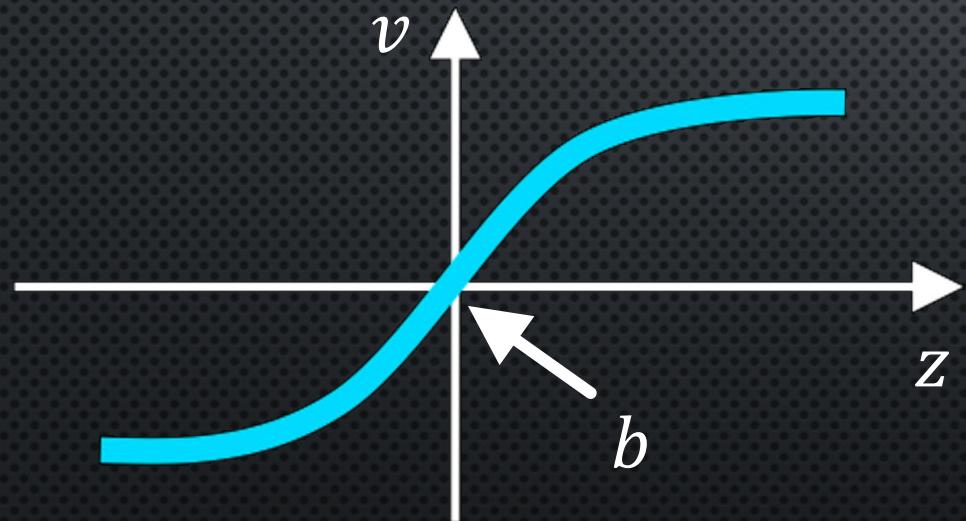
Question: So which one should we modify to learn?

Answer: Ideally, both should be modified.

Introducing bias: a way to describe the activation function

- Bias is the point on the z -axis at which $v = 0$
- By changing bias, one can shift the activation function to the left or right.

Example: Sigmoid activation function $v = \frac{1}{1 + e^{-S(z-b)}}$ where b is the bias and S determines the slope of linear part.



Question: So how do we decide what should be the value of bias b ?

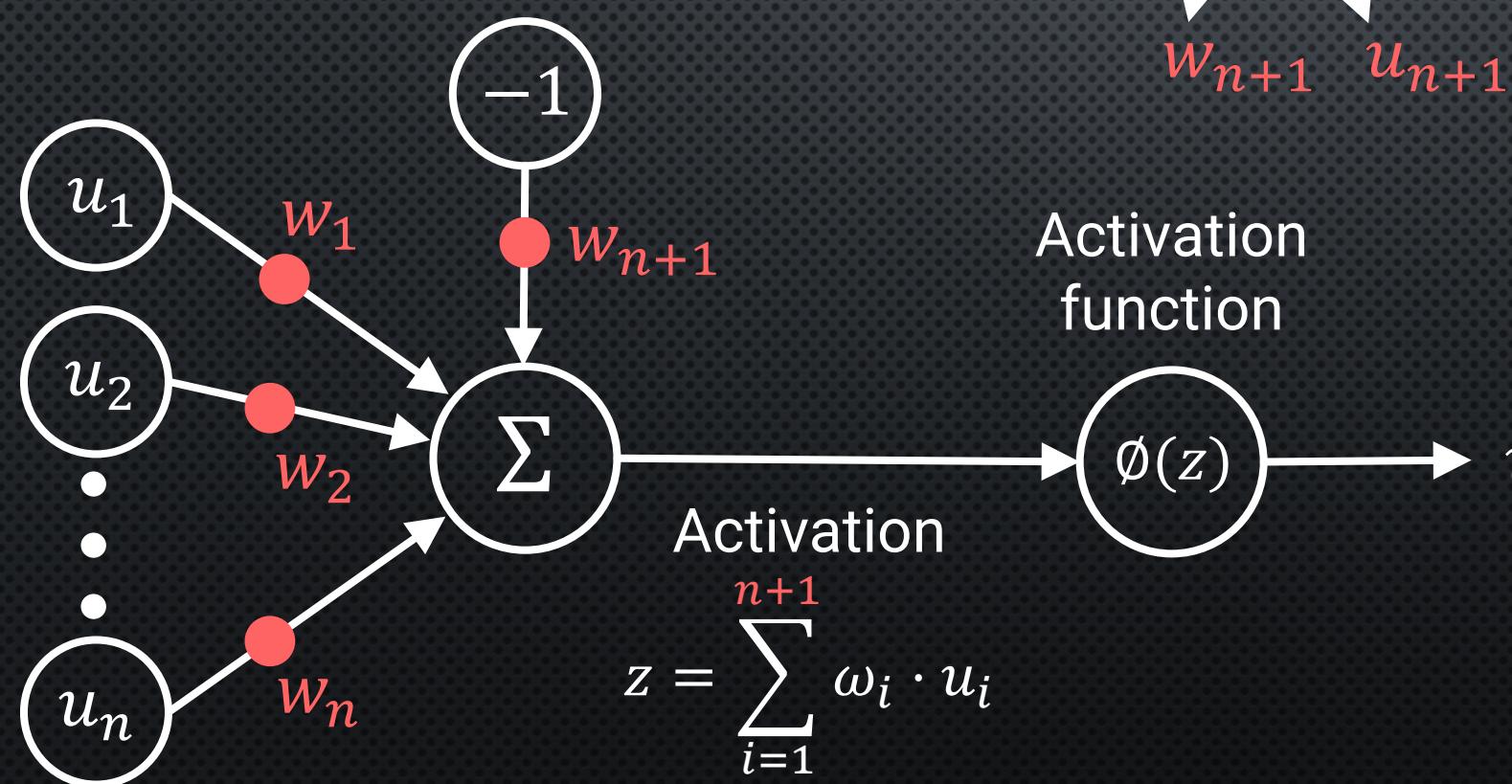
Answer: We could learn it by considering b as another “weight”.

Bias b as a weight

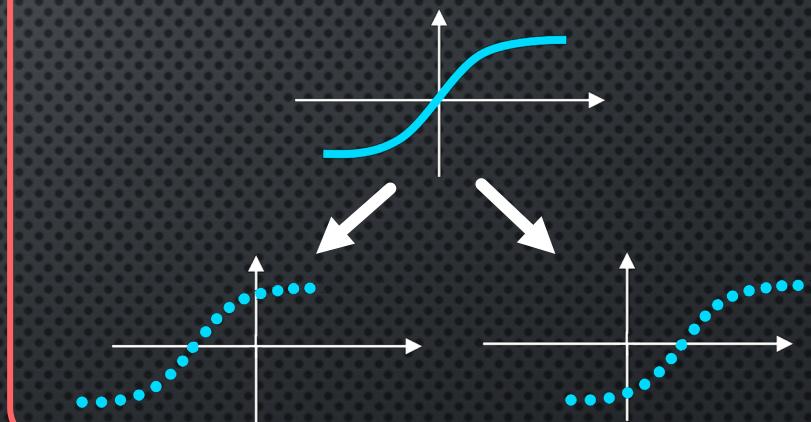
$$z = w_1 u_1 + w_2 u_2 + \cdots + w_n u_n$$

$$\rightarrow (z - b) = w_1 u_1 + w_2 u_2 + \cdots + w_n u_n - b$$

$$\rightarrow (z - b) = w_1 u_1 + w_2 u_2 \dots + w_n u_n + b (-1)$$

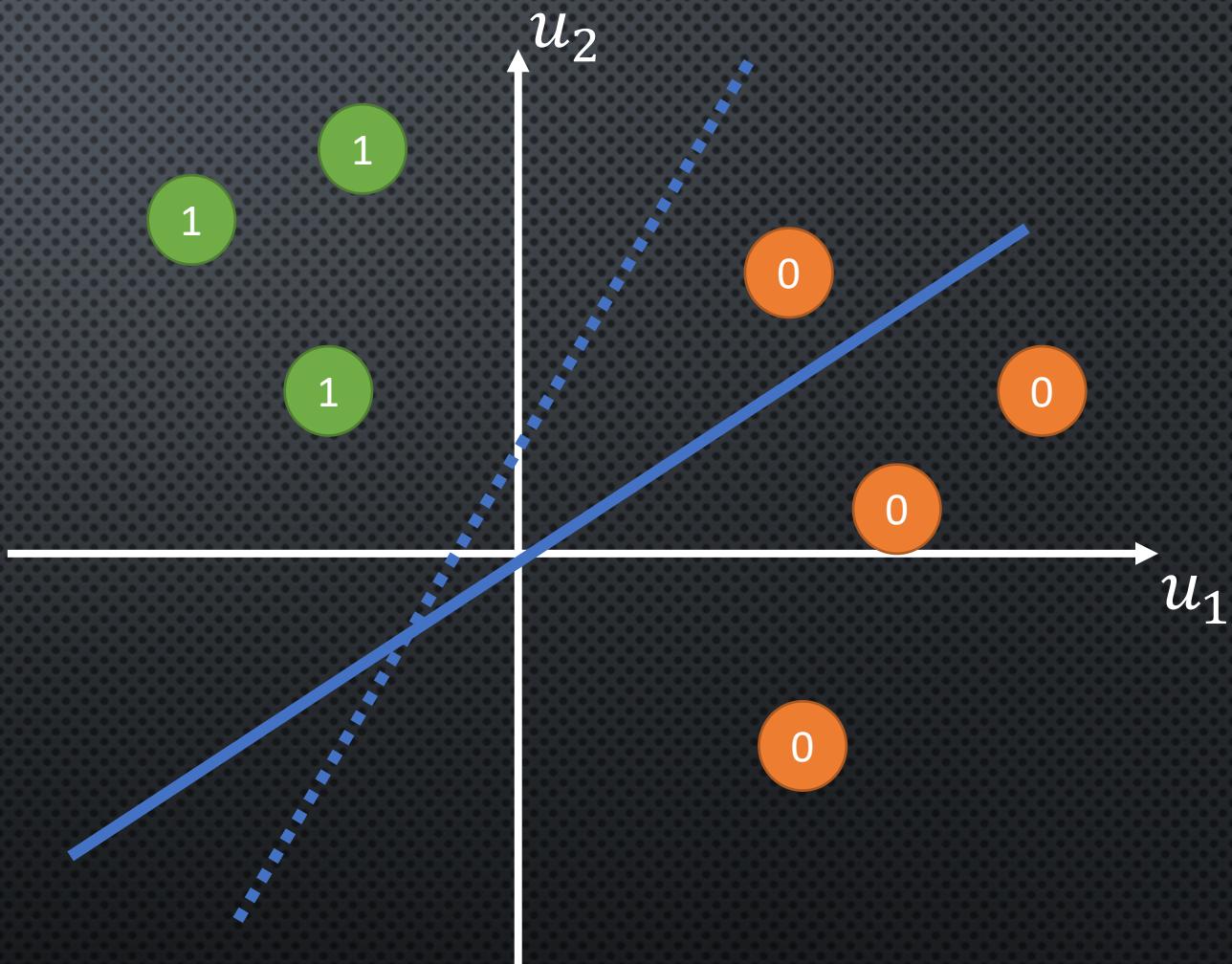


This small change sets the AF in the centre before learning. By learning w_{n+1} , we can shift the activation function automatically.



Why shift the activation function?

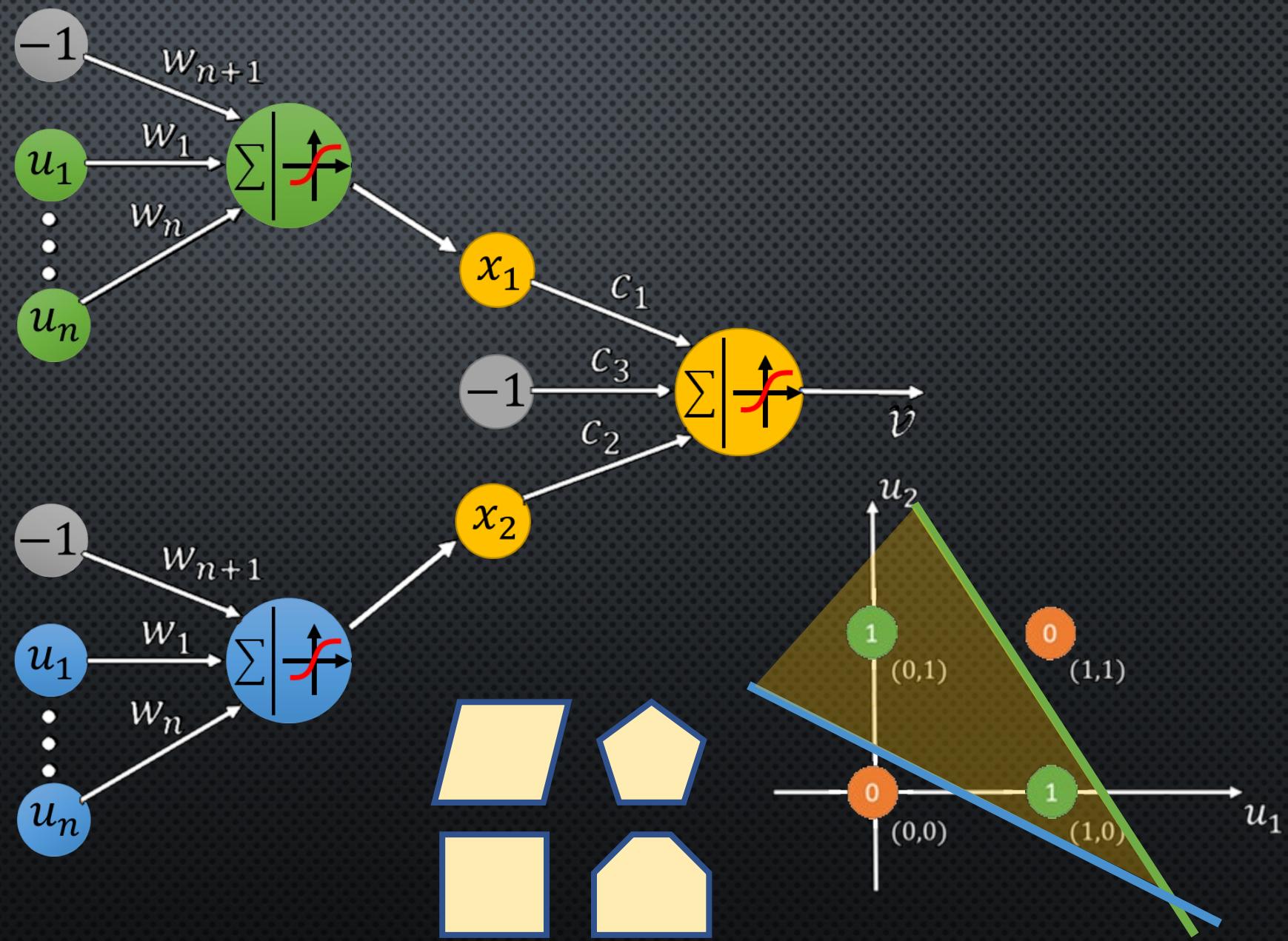
- The blue line is called the **decision boundary**.
- Decision boundary separates inputs into different classes (for a classification problem). Here the classes are “0” and “1”, but it can also be labels such as “cat”, “dog” etc.
- By changing $w_1, w_2, \dots, w_n, w_{n+1}$ by learning, the decision boundary can be shifted to adapt to new input data.



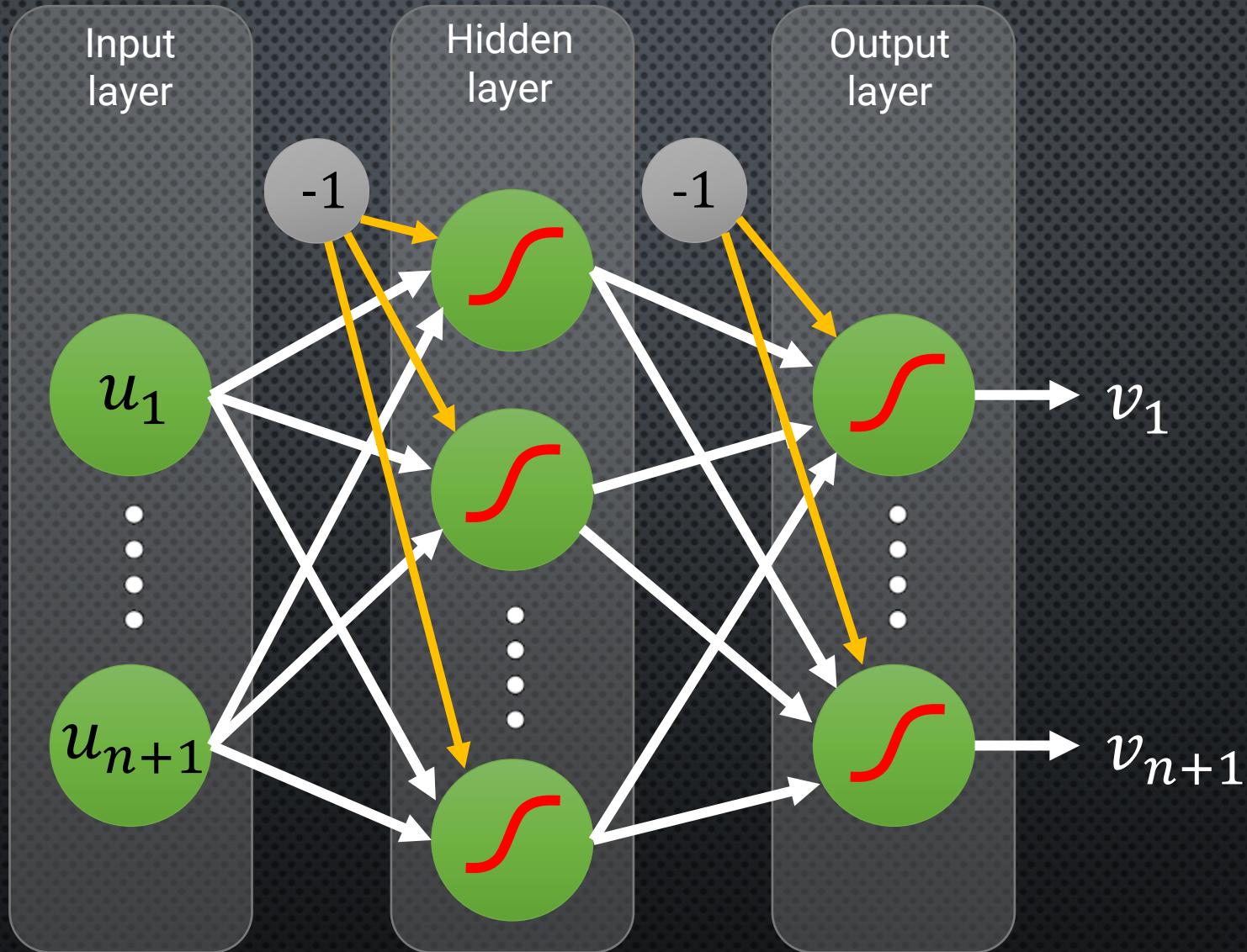
Equation for blue line:
 $w_1 u_1 + w_2 u_2 \dots + w_n u_n + b(-1) = (z - b)$

From decision boundaries to decision surfaces

- By adding a third perceptron as the next layer, we get a 2-dimensional **decision surface**.
- By adding more perceptrons in the first layer, we can draw more decision boundaries to enclose more complex **decision surfaces**.

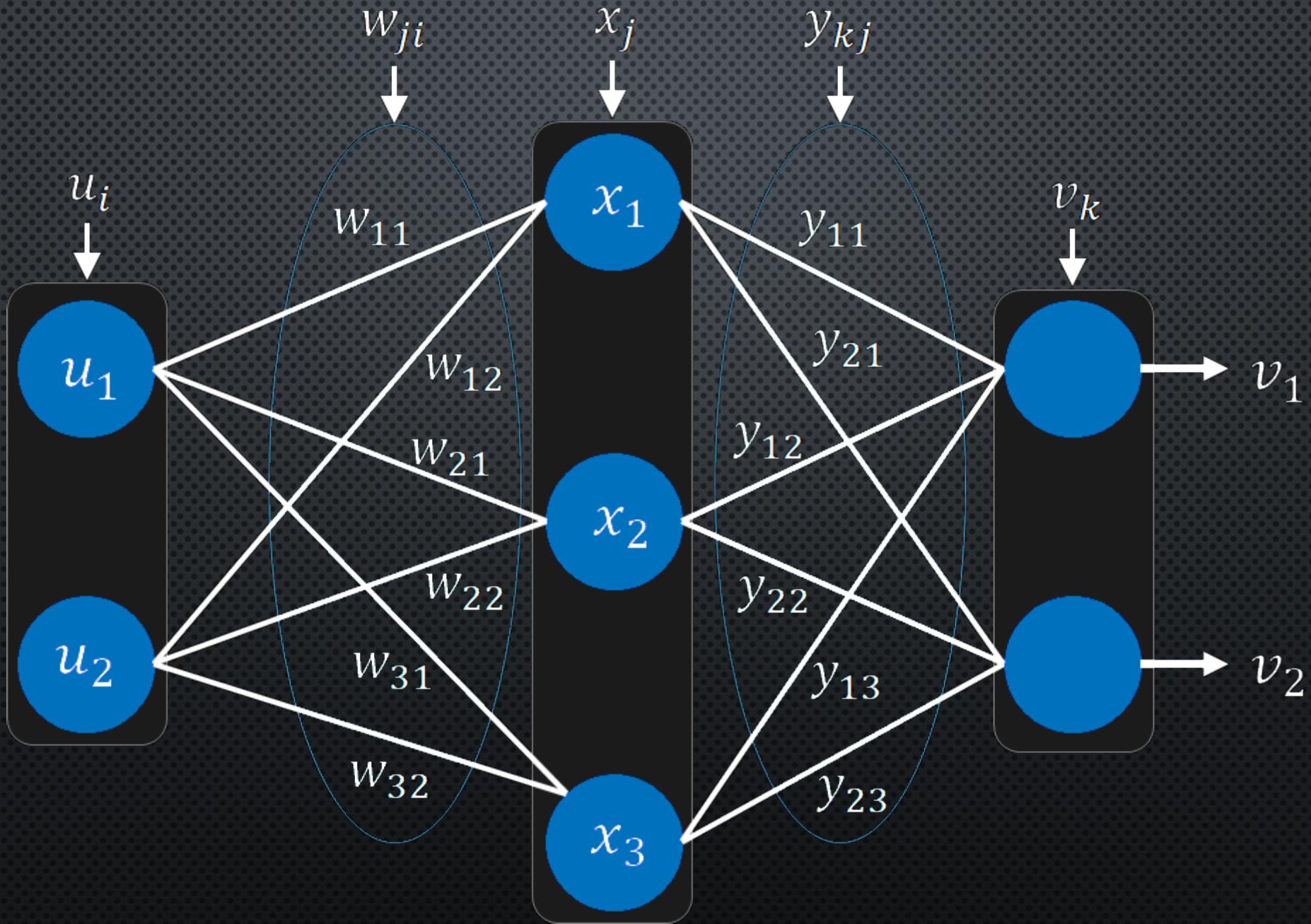


Multi-Layer Perceptron (MLP)



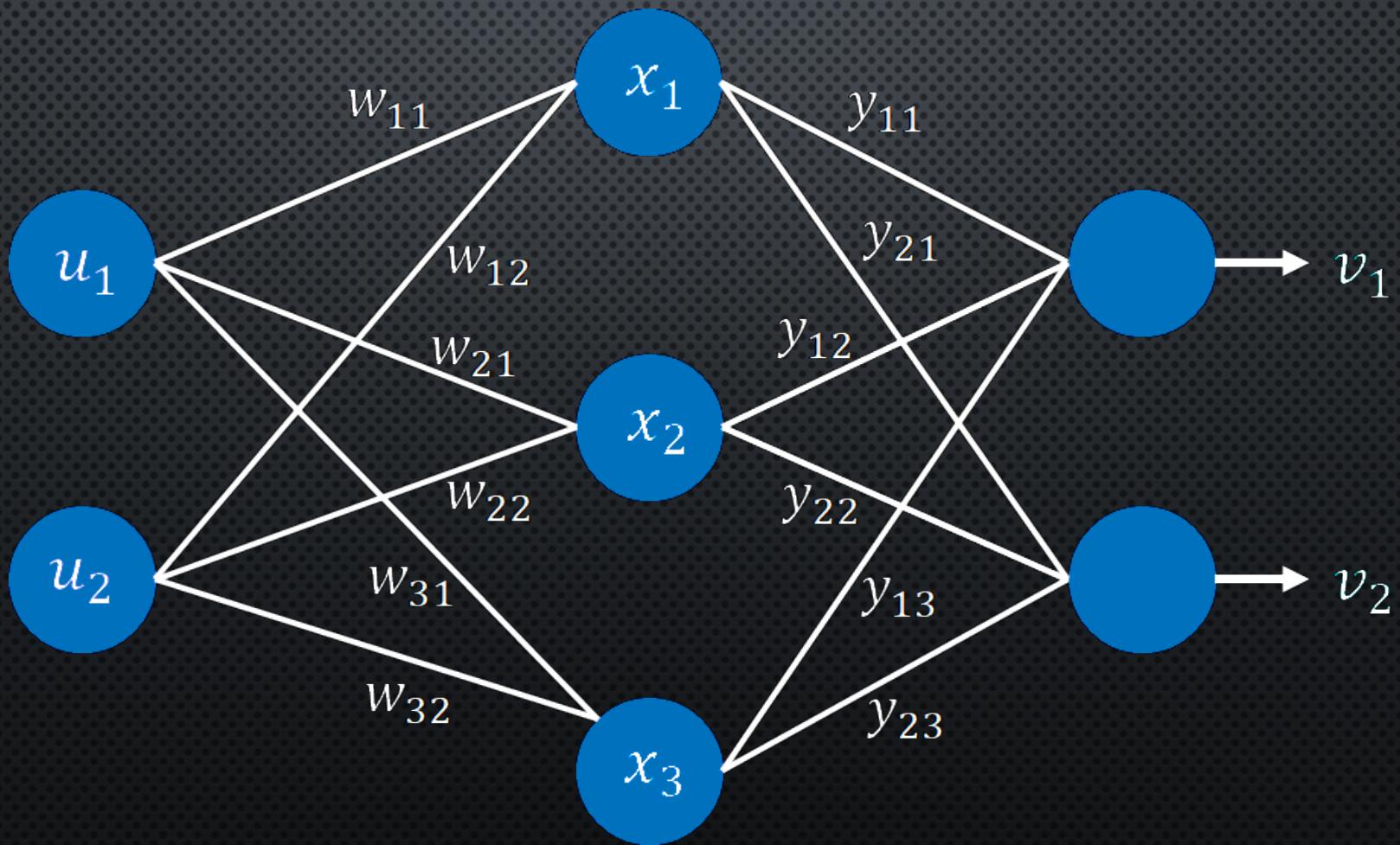
In deep learning neural networks, there is **more than one** hidden layer

Training a MLP: Notation



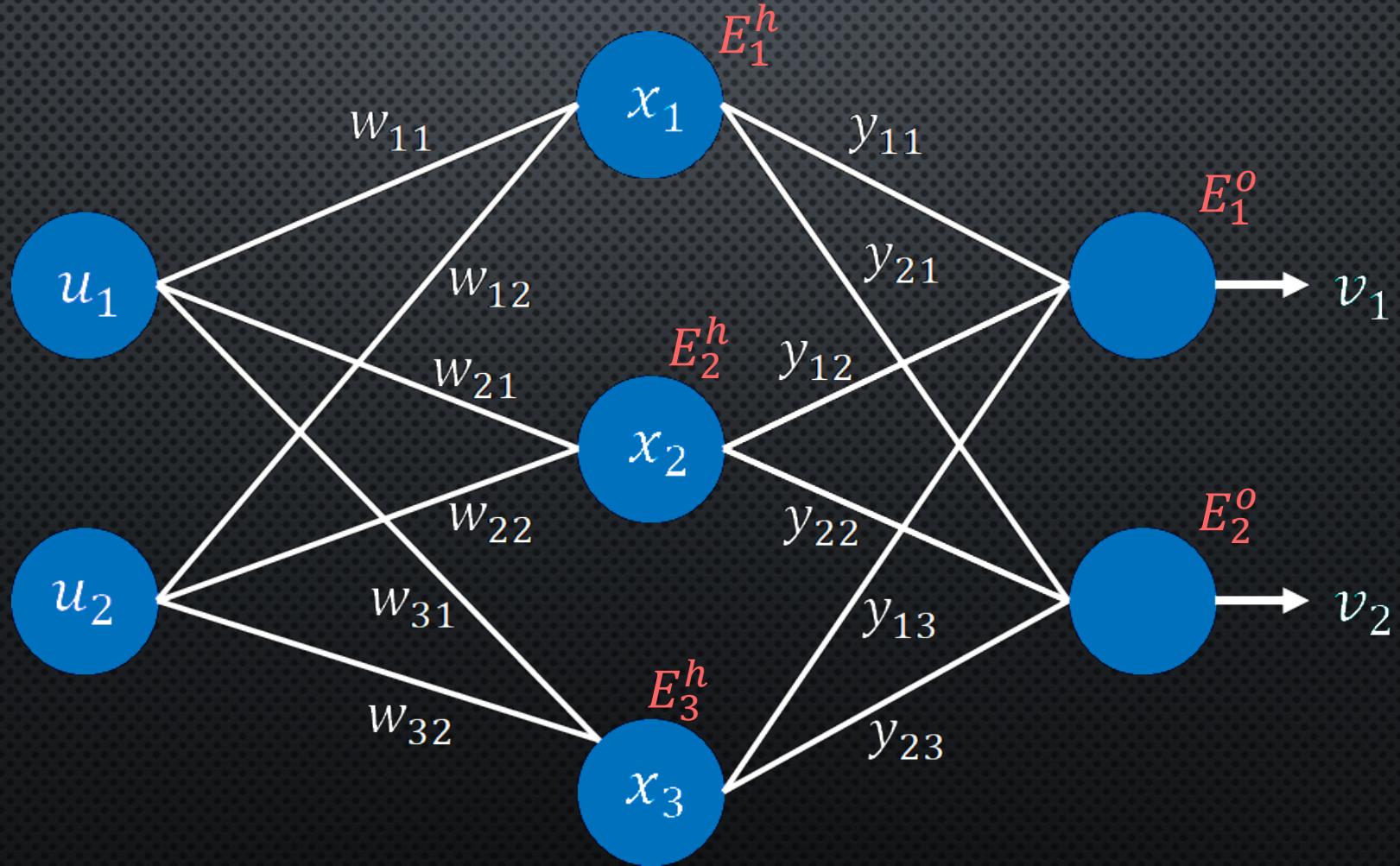
Step 1: Forward propagation

- Calculate output $x_j = \sum_i u_i w_{ji}$ for all hidden neurons
- Calculate output $v_k = \sum_j x_j y_{kj}$ for all output neurons



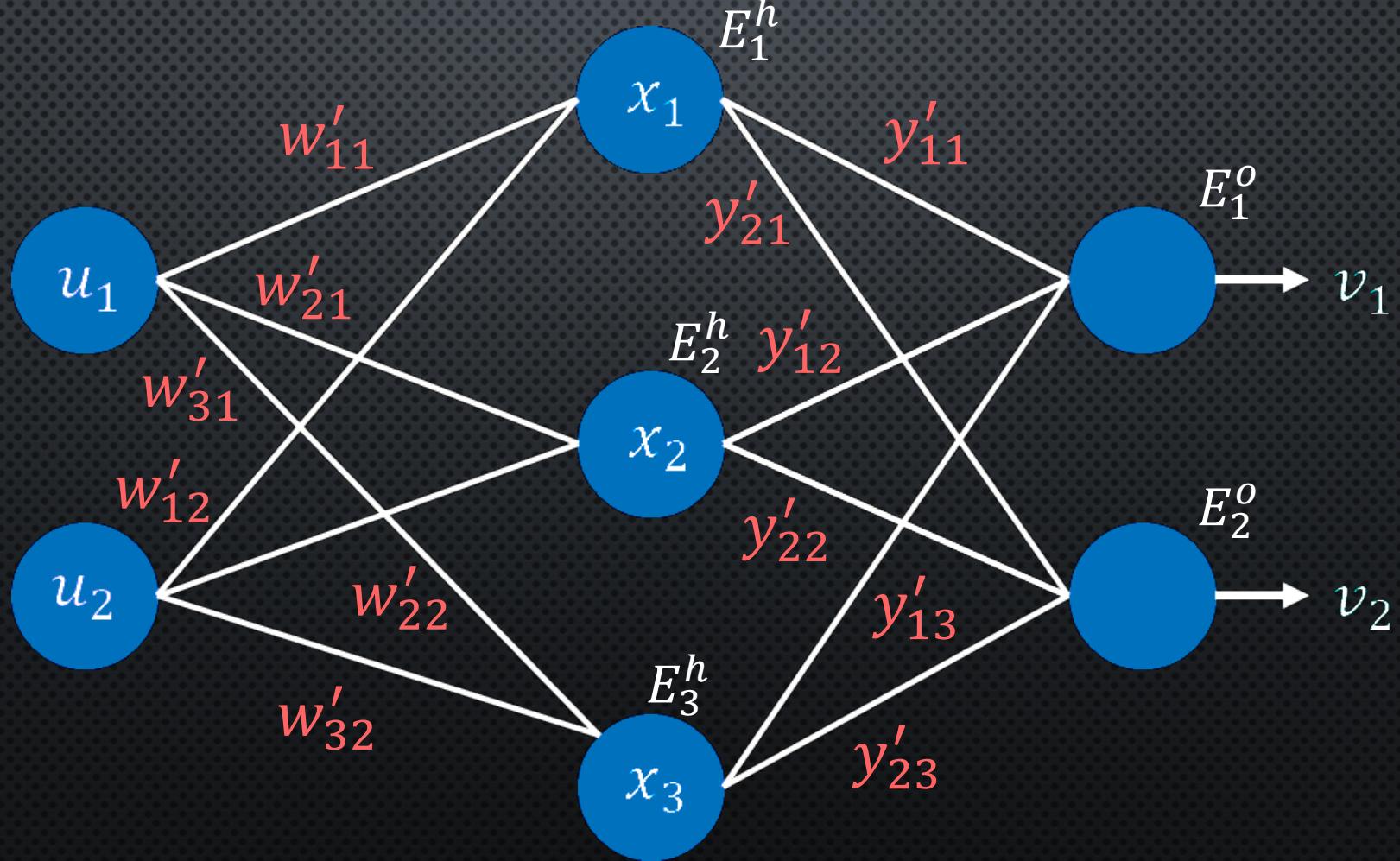
Step 2: Backpropagation

- Calculate error gradient for all output neurons, $E_k^o = v_k(1 - v_k)(t_k - v_k)$
- Calculate error gradient for all hidden neurons, $E_j^h = x_j(1 - x_j) \sum_k E_k^o y_{kj}$



Step 2: Backpropagation

- Update weights for the output neurons, $y'_{kj} = y_{kj} + \mu E_k^o x_j$
- Update weights for the hidden neurons, $w'_{ji} = w_{ji} + \mu E_j^h u_i$



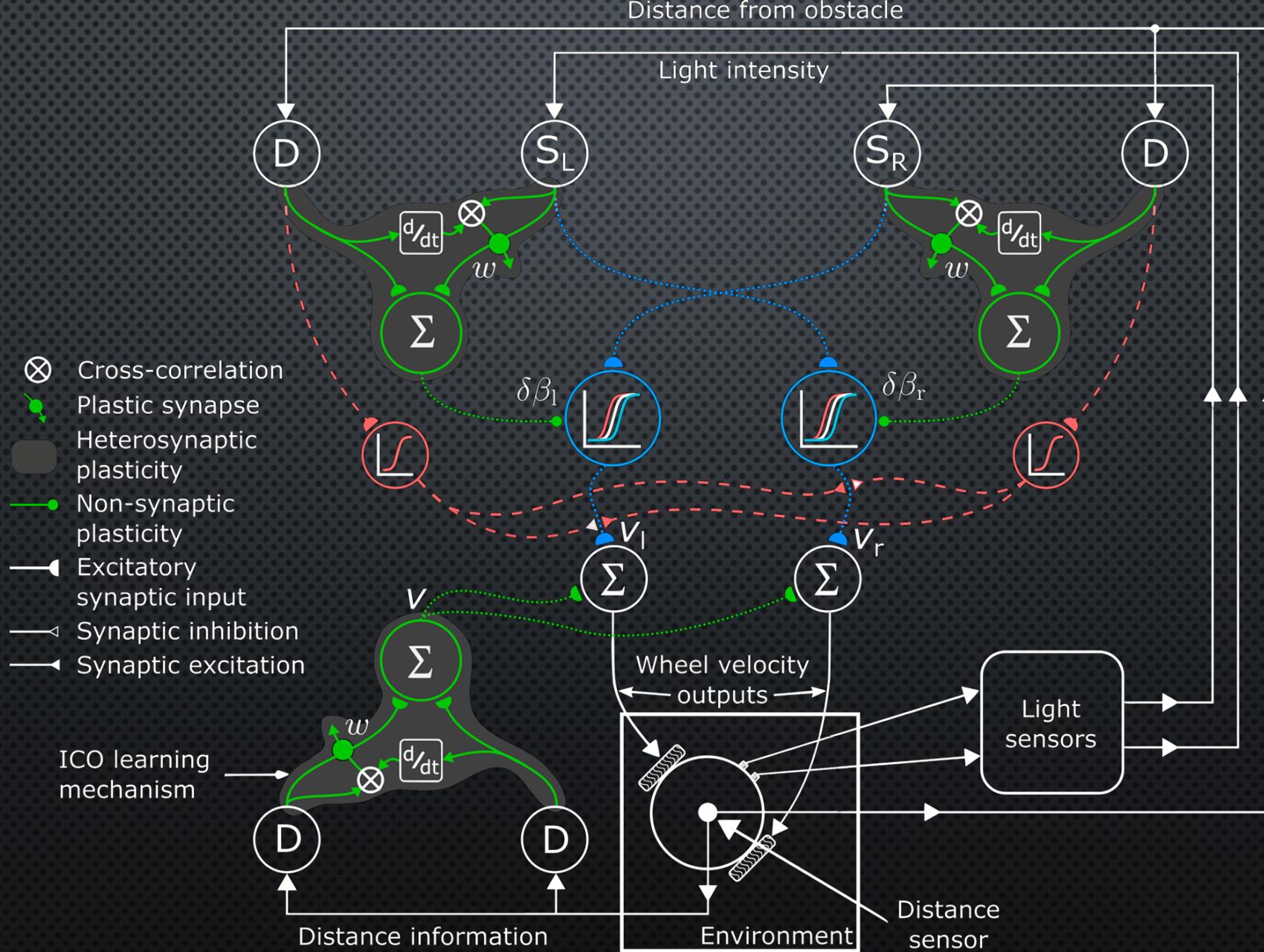
Training a MLP: backpropagation algorithm

- Divide dataset into two sets – training dataset (70% of total input data points) and testing dataset (30% of total input data points)
- Initialise all weights to random values between 0 and 1 (or -1 and +1)
- Step 1: Feed forward
 1. Randomly shuffle training dataset and select a (new) randomly chosen input data point
 2. Calculate output $x_j = \sum_j \sum_i u_i w_{ji}$ for all hidden neurons, and $v_k = \sum_k \sum_j x_j y_{kj}$ for all output neurons
- Step 2: Backpropagation
 1. Calculate error gradient for all output neurons, $E_k^o = v_k(1 - v_k)(t_k - v_k)$
 2. Calculate error gradient for all hidden neurons, $E_j^h = x_j(1 - x_j) \sum_k E_k^o y_{kj}$
 3. Update weights for the output neurons, $y'_{kj} = y_{kj} + \mu E_k^o x_j$
 4. Update weights for the hidden neurons, $w'_{ji} = w_{ji} + \mu E_j^h u_i$
- Repeat Step 1 and Step 2 until the error is very low or max. number of epochs is reached
- Test your trained network on testing dataset by repeating Step 1

Additional notes on training

- **Training set:** Used to adjust weights of the neural network
- **Validation set:** Used to minimize overfitting
- **Testing set:** Used only for testing the final solution
- **Monte Carlo cross validation**
 - Sub-sample data randomly into training and test sets (e.g., 70% and 30%)
- **K-fold cross validation**
 - Divide data into k subsets
 - Each time (in total k times) one of the subsets is used for testing and the rest $k-1$ subsets are joined and used as a training set
- **Leave-p-out cross validation**
 - Use p data samples as training samples and the rest ($n-p$) data samples as test samples
 - Train and test $\frac{n!}{p! \cdot (n-p)!}$ times

Learning the activation function



Hungry for more?

VEHICLES
Experiments in Synthetic Psychology

Valentino Braitenberg

