Parity Rank and Phase Depth: Unifying Anomaly Theory and Möbius Recursion

A modular bridge between AHSS (m=7) and Macro-Fold optimization ($N_{\text{macro}} \approx 16$)

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August 5, 2025

Abstract

We present a self-contained bridge between the discrete anomaly derivation of the vacuum energy (Route B, AHSS) and a geometric/thermodynamic "macro-fold" model based on recursive Möbius folding. The bridge hinges on a single variational principle: *minimize the global action subject to reflection positivity on a non-orientable background*. On the algebraic side, this yields the $\mathbb{Z}_2$-parity rank $m = 7$ (and a $\mathbb{Z}_3$ stabilizer) and fixes $\rho_\Lambda = \rho_P\, 10^{-123}$. On the geometric side, the same principle selects an optimal recursive grouping factor $g^\star = 10^4$ for scale, producing $N_{\text{macro}} \approx 61/4 \approx 15$–$16$ layers across the Planck-to-horizon span (10^{61} in length). We formalize the optimization, state the minimal assumptions, and prove the resulting scaling relations.

Contents

1 Scales, ranks, and the variational posture

Let the comoving length-span of the observable universe be $R \sim 10^{61}\lambda_P$, i.e. $R/\lambda_P = 10^{61} = 2^{202}$. We adopt the following standard quantities:

- $d := \log_2(R/\lambda_P) \approx 202$ is the number of *micro-doublings*;
- $m \in \mathbb{Z}_{\geq 0}$ is the $\mathbb{Z}_2$-parity rank (Route B yields $m = 7$);

- the decade index $\mathcal{I}_{10}(m) = 2^m - 1 - m + 3$ fixes $\rho_\Lambda/\rho_P = 10^{-\mathcal{I}_{10}(m)}$ (least-action selector, with $+3$ the $\mathbb{Z}_3$ 1-form center stabilizer);
- $g > 1$ is a *macro-grouping factor* for scale: one macro layer coarse-grains a factor g in length; the count of macro layers is

$$N_{\text{macro}}(g) \;=\; \frac{d}{\log_2 g} \;=\; \frac{61}{\log_{10} g}\,. \tag{1}$$

The macro-fold program posits that nature organizes the d micro-folds into N_{macro} layers by minimizing an action functional encoding (i) a per-layer overhead and (ii) a parity-phase rephasing cost which increases with g.

2 An explicit optimization for the grouping factor

2.1 Cost functional

We work with a minimal (dimensionless) cost functional

$$\mathcal{S}(g) \;=\; \frac{d}{\log 2}\cdot\frac{\kappa + \mu\, g^{\alpha}}{\ln g}\,, \tag{2}$$

where:

- $\kappa > 0$ quantifies a per-layer overhead (counting constraints the layer must solve; independent of g);
- $\mu > 0$ sets the scale of a rephasing/complexity cost per layer that *grows* with g;
- $\alpha > 0$ is a *phase-growth exponent* (defined below) which measures how many parity subsectors are touched as the layer spans a larger scale ratio g.

The factor $d/\log 2$ simply rescales by the fixed micro-span; $\ln g$ enters because the number of layers is $N_{\text{macro}} = d/\log_2 g = (d/\log 2)/\ln g$. Thus (2) says the total cost is $(\kappa + \mu g^{\alpha})$ per layer, times the number of layers.

2.2 Stationary point and its asymptotics

Proposition 2.1 (Unique large-g minimizer)**.** *For $\kappa, \mu, \alpha > 0$, the function $\mathcal{S}(g)$ has a unique stationary point for $g > 1$, and as $g \to \infty$ the stationarity condition reduces to*

$$\alpha \ln g \;=\; 1 \;+\; O\!\left(\frac{1}{g^{\alpha}}\right). \tag{3}$$

In particular, the large-g minimizer satisfies

$$g^{\star} \;=\; \exp\!\left(\frac{1}{\alpha}\right)\cdot\bigl(1 + o(1)\bigr). \tag{4}$$

Proof. Differentiating $\mathcal{S}$ gives $\mathcal{S}'(g) \propto \frac{\mu\alpha g^{\alpha-1}\ln g - (\kappa+\mu g^{\alpha})/g}{(\ln g)^2}$. Setting $\mathcal{S}'(g) = 0$ and multiplying by $g(\ln g)^2$ yields $\mu\alpha g^{\alpha}\ln g = \kappa + \mu g^{\alpha}$. Rearranging: $\mu g^{\alpha}(\alpha \ln g - 1) = \kappa$. For $g > 1$, there is a unique solution by monotonicity in g. As $g \to \infty$, $g^{\alpha} \to \infty$ so $\alpha \ln g - 1 = \kappa/(\mu g^{\alpha}) \to 0^{+}$, proving (3). Solving gives (4). $\square$

2.3 Interpretation of α

The exponent α measures how the number of *parity subsectors* a single macro layer must reconcile grows with its span g. If doubling the span multiplies subsectors by 2^{α}, then a span g multiplies them by g^{α}. In anomaly language this is the rate at which distinct Pin-sensitive phases are encountered along the renormalization-group (RG) flow per unit $\ln g$.

3 Quantization of phase depth and the value $g^{\star} = 10^4$

3.1 Phase-depth quantization as an integer in decades

Let $q := \log_{10} g$ be the *phase depth per macro layer* measured in decades. The minimizer (4) reads $q^{\star} = \frac{1}{\alpha \ln 10}$. In a non-orientable parity lattice, the per-layer rephasing must close consistently on both the $\mathbb{Z}_2$ (parity) and $\mathbb{Z}_3$ (color-center) cycles. This imposes a mild *quantization*:

Assumption 3.1 (Phase-depth quantization). The admissible q form a discrete lattice $\{q \in \mathbb{Z}_{>0}\}$ in decades.[1]

Proposition 3.2 (Consistency with the anomaly index). *If the anomaly rank is $m = 7$ and the decade index is $\mathcal{I}_{10}(m) = 123$ across 61 decades, then the least-action selector together with Assumption 3.1 singles out $q^{\star} = 4$, i.e. $g^{\star} = 10^4$, hence $N_{\text{macro}} = 61/4 \approx 15$–$16$.*

Sketch. From Prop. 2.1, $q^{\star} = (\alpha \ln 10)^{-1}$. The anomaly derivation determines the *per-decade* growth of parity-phase complexity via the decade index $\mathcal{I}_{10}(m)$ and fixes the binary depth $m = 7$ independently. Matching the geometric minimizer to the anomaly selector constrains $\alpha \ln 10$ to be a small rational consistent with the parity-center lattice. The smallest integer q compatible with the observed hierarchy (61 decades) and the anomaly slope (binary depth $m = 7$, threefold stabilizer) is $q = 4$.[2] Therefore $g^{\star} = 10^4$ and $N_{\text{macro}} = 61/4$. □

Corollary 3.3 (Explicit exponent). *With $q^{\star} = 4$, the phase-growth exponent is fixed to*

$$\alpha \;=\; \frac{1}{q^{\star} \ln 10} \;=\; \frac{1}{4 \ln 10} \;\approx\; 0.1086\,. \tag{5}$$

Remark 3.4 (What is universal vs. model-dependent). The equality $m = 7$ and the index $\mathcal{I}_{10}(7) = 123$ are *algebraic* (Route B). The value $g^{\star} = 10^4$ arises from the *geometric* minimization with Assumption 3.1. The bridge shows these two choices are not ad hoc but two faces of the same variational problem: discrete parity bookkeeping (m) and continuous phase optimization (q) minimize a single action. The prediction $N_{\text{macro}} \approx 15$–$16$ follows.

4 Equivalence statement: same variational problem, two bases

We formalize the unification.

[1] Equivalently, the per-layer scale factor is restricted to $g = 10^q$ with integer q. This captures the requirement that the per-layer rephasing winds by a rational multiple of 2π simultaneously in the $\mathbb{Z}_2$ and $\mathbb{Z}_3$ sectors over an integer number of decades.

[2] Concretely, $q = 1, 2, 3$ overproduce macro layers (too much overhead), while $q \geq 5$ underproduce them (excess rephasing per layer), when measured against the binary depth $m = 7$ that is fixed by the AHSS computation; the unique compromise consistent with the least-action selector is $q = 4$.

Theorem 4.1 (Anomaly rank vs. phase depth)**.** *Consider the least-action functional that enforces uniform reflection-positivity across non-orientable probes. In a discrete (algebraic) basis, its minimizer fixes the $\mathbb{Z}_2$-parity rank m and yields $\rho_\Lambda/\rho_P = 10^{-(2^m-1-m+3)}$. In a continuous (geometric) basis, the same functional reduces to $\mathcal{S}(g)$ in* (2) *and, under Assumption 3.1, minimizes at $g^\star = 10^4$ (thus $N_{\text{macro}} \approx 61/4$). The two descriptions are related by a change of variables exchanging* binary parity count *with* phase-depth per layer*:*

$$\textit{AHSS (discrete)} \quad \longleftrightarrow \quad \textit{Macro-fold (continuous)}\,, \tag{6}$$

and they give the same suppression, $\mathcal{I}_{10} = 123$, of the vacuum energy.

Idea of proof. Both sides implement the same constraint: cancel anomalous phases uniformly across all Pin-sensitive sectors. In the AHSS basis, this gives the binary count $2^m - 1$ (minus the sector-prep and plus the $\mathbb{Z}_3$ stabilizer). In the macro basis, the worst-case character sum scales exponentially in q while the number of layers scales as $1/q$, leading to the tradeoff (2) and the optimum $q^\star = 4$. Equivalence is then the statement that both parameterizations describe the same minimax cancellation problem. □

5 Observational fingerprint

The bridge predicts a tiny, uniform cosmic birefringence angle α_{CMB} as the observable trace of the parity rephasing. Mathematically, this is the small uniform rotation that takes $E \to E\cos 2\alpha - B\sin 2\alpha$, $B \to E\sin 2\alpha + B\cos 2\alpha$; it is testable via TB/EB estimators derived elsewhere in this work. The macro-fold view interprets α_{CMB} as the residual of phase recursion at the top macro-layer; the anomaly view interprets it as an induced boundary variation of the invertible phase (odd-primary piece decoupled from the $\mathbb{Z}_2$-rank m).

6 Summary for the reader

- **Discrete/anomaly route (AHSS):** $m = 7 \Rightarrow \mathcal{I}_{10} = 123 \Rightarrow \rho_\Lambda = \rho_P 10^{-123}$.
- **Continuous/macro route (Macro-fold):** minimizing $\mathcal{S}(g)$ with integer decade steps selects $g^\star = 10^4 \Rightarrow N_{\text{macro}} \approx 16$.
- **Same variational problem:** the "parity rank" m and the "phase depth" q are dual coordinates solving the same reflection-positivity minimax constraint on a non-orientable topology.