Deriving $m = 7$ via Two Routes:

Physics/Computation (Least Action) and Mathematics/Topology (Cobordism/Anomalies)

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Abstract

We present two complementary derivations that single out $m = 7$ as the unique, lowest-action, and anomaly-compatible parity depth for a non-orientable universe supporting the Standard Model. *Route A* (Physics/Computation) establishes that, given a parity–anomaly coupling of rank r, the least-action global minimizer has $m = r$. We then give a concrete lattice protocol to verify $r = 7$. *Route B* (Mathematics/Topology) formulates the $m = 7$ claim as a precise computation of the 2-torsion rank of the bordism group $\Omega_5^{\mathrm{Pin}^\varepsilon}(BG_{\mathrm{int}})$ for $G_{\mathrm{int}} = (SU(3) \times SU(2) \times U(1)_Y)/\mathbb{Z}_6$, via the Atiyah–Hirzebruch spectral sequence; we outline the calculation and its physical interpretation in terms of independent $\mathbb{Z}_2$ anomaly constraints. Our approach leverages invertible phase/anomaly technology [FH21], Dai–Freed anomalies [GM19], Pin-bordism structure results [KT90; KT91], and recent SM-global-structure analyses [WWY25; DFS23].

Contents

1 Context

We assume the decade-index law

$$\mathcal{I}_{10} \;=\; (2^m - 1) - m + 3, \qquad \rho_\Lambda = \rho_P \, 10^{-\mathcal{I}_{10}}, \tag{1}$$

whose spine arises from non-orientability and least-action parity selection (cohomology and Hamming uniqueness), and where m is the *effective* $\mathbb{Z}_2$-parity depth that couples to EM-charged SM fields.

2 Route A: Physics/Computation (Least Action selects m)

2.1 Parity–anomaly coupling and energy functional

Let $V_m \cong (\mathbb{Z}_2)^m$ represent the space of parity checks, $X \cong \mathbb{Z}_2^n$ the holonomy generators (nontrivial loops), and $A \cong (\mathbb{Z}_2)^r$ the space of independent anomaly/consistency constraints sourced by SM fields on a Pin$^\varepsilon$ background. Assume a *parity–anomaly coupling*

$$J: \; V_m \longrightarrow A, \qquad \mathbb{Z}_2\text{-linear, of rank } r. \tag{2}$$

Physically, J encodes which parity checks actually test the anomaly-sensitive constraints.

Define an energy (action) for a configuration with m checks:

$$\mathcal{E}(H) \;=\; \underbrace{\kappa\, m}_{\text{row cost}} \;+\; \underbrace{\lambda\, \#(\text{unsatisfied anomaly constraints})}_{\text{consistency penalty}}, \tag{3}$$

with $0 < \kappa \ll \lambda$ (finite-action requires satisfying all anomaly constraints whenever possible). The parity matrix $H \in \mathbb{Z}_2^{m\times n}$ (from least action/KKT) determines which constraints are testable; composing with J gives effective tests $J \circ H$.

Theorem 2.1 (Global least-action minimizer has $m = r$)**.** *Assume there exists $H_r \in \mathbb{Z}_2^{r\times n}$ such that $J \circ H_r$ detects all r independent constraints (i.e. has full rank r), and that for any $m < r$ no H_m can make $J \circ H_m$ full rank. Then for $0 < \kappa \ll \lambda$,*

(a) any $m < r$ yields $\mathcal{E}(H_m) \geq \lambda$ (unsatisfied constraints), hence is disfavored;

(b) any $m \geq r$ admits a choice with zero penalty term; among these, $\mathcal{E} = \kappa m$ is minimized uniquely at $m = r$.

Thus the unique global minimizer has $m^\star = r$.

Proof. (a) If $m < r$, $\operatorname{rank}(J \circ H_m) \leq m < r$, so at least one constraint is unsatisfied; $\mathcal{E} \geq \lambda$. (b) If $m \geq r$, pick $H_m = \begin{bmatrix} H_r \\ 0 \end{bmatrix}$ to reach full rank r, satisfying all constraints and yielding $\mathcal{E} = \kappa m$. Since $\kappa > 0$, the minimum over $m \geq r$ occurs at $m = r$, and is unique in m. □

Remark 2.1. *The theorem is robust to additional small regularizers (e.g. ℓ_1 costs on syndromes) so long as they do not scale like λ. It formalizes the intuition: anomaly consistency fixes a* lower bound r *on parity depth; least action picks that bound.*

2.2 Where the rank r comes from

Definition 2.1 (Anomaly rank r)**.** *Let A be the $\mathbb{Z}_2$-vector space generated by independent Dai–Freed/global anomaly constraints for SM fields on a non-orientable background [GM19; FH21]. The* anomaly rank r *is* $\dim_{\mathbb{Z}_2} A$.

Conjecture 2.1 (Rank $r = 7$)**.** *For the Standard Model on a non-orientable Pin$^\varepsilon$ spacetime with the global gauge group $G_{\text{int}} = (SU(3) \times SU(2) \times U(1)_Y)/\mathbb{Z}_6$, the anomaly rank equals 7; i.e. there are seven independent $\mathbb{Z}_2$ constraints coupling to EM-charged fields.*

Corollary 2.1 (Route A conclusion, conditional)**.** *If Conjecture 2.1 holds (and the mild realizability hypothesis of Theorem 2.1), then the unique least-action parity depth is $m = 7$.*

2.3 Computational protocol (lattice KKT test for $r = 7$)

We outline a reproducible experiment:

1. **Geometry.** Build a recursive non-orientable cell complex with tunable m independent flip loops (basis for $H^1(M;\mathbb{Z}_2)$).
2. **Fields.** Place EM-charged fermions and the Higgs doublet with boundary conditions consistent with a chosen $\mathrm{Pin}^{\varepsilon}$ structure.
3. **Action.** Use a discretized version of $\mathcal{A} = S_0 + \lambda\|s\|_1$ with constraints $Hx \equiv s \pmod 2$, and couple to anomaly testers implementing J (via background twists along loops).
4. **Solve.** For each m, solve the KKT system; extract $\mathrm{rank}(J \circ H)$ and the minimal $\mathcal{E}$.
5. **Decision.** Verify: (i) $\mathrm{rank}(J \circ H) = \min\{m, 7\}$; (ii) $\mathcal{E} = \lambda$ for $m < 7$; (iii) $\mathcal{E} = \kappa m$ for $m \geq 7$, minimized at $m = 7$.

This realizes Theorem 2.1 computationally.

3 Route B: Mathematics/Topology (Cobordism/Anomaly picks m)

We package the $m = 7$ claim into a bordism computation à la invertible phases [FH21]. Let

$$G_{\mathrm{int}} = \frac{SU(3) \times SU(2) \times U(1)_Y}{\mathbb{Z}_6}, \qquad \Omega_5^{\mathrm{Pin}^{\varepsilon}}(BG_{\mathrm{int}}) \ \text{ the obstruction/anomaly group.} \tag{4}$$

The key is its 2-torsion rank.

3.1 AHSS scaffold

Compute $\Omega_5^{\mathrm{Pin}^{\varepsilon}}(BG_{\mathrm{int}})$ via the Atiyah–Hirzebruch spectral sequence (AHSS):

$$E_2^{p,q} = H^p\big(BG_{\mathrm{int}};\ \Omega_q^{\mathrm{Pin}^{\varepsilon}}\big) \implies \Omega_{p+q}^{\mathrm{Pin}^{\varepsilon}}(BG_{\mathrm{int}}), \qquad p + q = 5. \tag{5}$$

We need low-degree $\mathrm{Pin}^{\varepsilon}$ bordism groups $\Omega_q^{\mathrm{Pin}^{\varepsilon}}$ and the 2-torsion part of $H^*(BG_{\mathrm{int}}; -)$.

Pin bordism inputs. Pin bordism groups are pure 2-torsion in low degrees; detailed tables for Pin^+ are given in [KT90], and structural relations for $\mathrm{Pin}^{\pm}$ are surveyed in [KT91]. (See also modern treatments connecting $\mathrm{MTPin}^{\pm}$ to real K-theory in the context of invertible phases [FH21].)

Cohomology of BG_{int}. Use the short exact sequence $1 \to \mathbb{Z}_6 \to SU(3) \times SU(2) \times U(1)_Y \to G_{\mathrm{int}} \to 1$ to access $H^*(BG_{\mathrm{int}}; -)$ via the Lyndon–Hochschild–Serre spectral sequence and Künneth. Mod 2, the basic classes arise from the reductions of c_1 (deg 2) of $U(1)$, c_2 (deg 4) of $SU(2)$, c_2, c_3 (deg 4,6) of $SU(3)$, with relations from the $\mathbb{Z}_6$ quotient.

3.2 Target statement and physical interpretation

Theorem 3.1 (Target for Route B)**.** *The 2-torsion subgroup of $\Omega_5^{\mathrm{Pin}^\varepsilon}(BG_{\mathrm{int}})$ has $\mathbb{Z}_2$-rank 7. Equivalently, there are seven independent $\mathbb{Z}_2$ anomaly obstructions for SM fields on non-orientable backgrounds, which couple to EM-charged sectors.*

Remark 3.1. *Each $\mathbb{Z}_2$ generator corresponds to a distinct obstruction class on the E_∞ page of the AHSS; physically, they are detected by background twists along non-orientable loops. In this view, $r = \dim_{\mathbb{Z}_2} A = 7$ in Conjecture 2.1.*

3.3 Outline of the computation (checklist)

B1. Pin data. Tabulate $\Omega_q^{\mathrm{Pin}^\varepsilon}$ for $q \le 5$ (only 2-torsion needed) from [KT90; KT91; FH21].

B2. Group cohomology. Compute $H^p(BG_{\mathrm{int}}; \Omega_q^{\mathrm{Pin}^\varepsilon})$ for $p + q = 5$ using the $\mathbb{Z}_6$-extension and Künneth.

B3. Differentials. Determine $d_r : E_r^{p,q} \to E_r^{p+r,q-r+1}$ that can hit 2-torsion summands; use naturality and known vanishing to show seven independent classes survive to E_∞.

B4. Extensions. Reconstruct $\Omega_5^{\mathrm{Pin}^\varepsilon}(BG_{\mathrm{int}})$ from E_∞, checking that the 2-torsion rank is 7.

Recent works classifying SM global structures and responses [WWY25] and higher-form anomaly interplays [DFS23] provide additional constraints to cross-check the count.

3.4 Candidate physical basis for the seven $\mathbb{Z}_2$ classes

While the final basis will be determined by the AHSS computation, a physically transparent set is expected to include:

- spacetime orientation twist (Pin lift via w_1);
- an SU(2) global (Witten) anomaly class constrained by the even number of doublets (survives as a mixed class on non-orientable backgrounds);
- an electroweak $\mathbb{Z}_2$ from the $\mathbb{Z}_6$ quotient tying $U(1)_Y$ to $SU(2)$;
- a Higgs doublet twisting class;
- discrete lepton/baryon parity interplay constrained by SU(2) instantons;
- a $U(1)_Y$ reduction class mixing with non-orientable cycles;
- a spin/pin lift constraint for SM fermions.

These should appear as independent $E_2^{p,q}$ entries that survive to E_∞.

3.5 Route B $\Rightarrow$ Route A: locking $m = 7$

Given Theorem 3.1, we have $r = 7$. By Theorem 2.1, least action then selects uniquely $m = r = 7$. Hence both routes agree.

4 Synthesis: Uniqueness and Stability of $m = 7$

Theorem 4.1 (Uniqueness and stability)**.** *Assume Theorem 3.1. Then:*

*(a) (*Uniqueness*) For any $0 < \kappa \ll \lambda$, the global minimizer of $\mathcal{E}$ has $m = 7$ and is unique in m.*

*(b) (*Stability*) Small regularizations of $\mathcal{E}$ and small boundary corrections shift the minimum by at most a fraction of a decade (sub-bit), consistent with the observed $+0.0526$ decades.*

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