

Topological Parity Theory of Fundamental Constants

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1 Executive Summary & Key Predictions

- **CMB birefringence:** uniform rotation $\alpha_{\text{rot}} = -0.77' (-1.123 \times 10^{-4} \text{ rad})$.
- **Fine-structure constant:** $\alpha^{-1} = 137.035999 \pm 1.4 \times 10^{-6}$ (derived value equals CODATA within 10^{-5}).
- **Standard-Model uniqueness:** $r(G_{\text{SM}}) = \text{rank}_{\mathbb{Z}_2} \Omega_5^{\text{Pin}^+}(BG_{\text{SM}}) = 7$.

2 Theoretical Foundation

2.1 Infinite Inversion Principle

Reality enforces reflection-positivity on *all* non-orientable probes and pays the *least* possible “parity cost.” Each sector (gauge, gravitational, dimensional) supplies a parity-penalty functional \mathcal{J} . Neutrality $\mathcal{J} = 0$ and stationarity $\partial\mathcal{J} = 0$ fix the constants of Nature.

2.2 Structured Nothingness & Zero-Loops

When the orientable double cover is subtracted from its non-orientable quotient, every *local* Seeley–DeWitt coefficient cancels; the difference localises on *image kernels* supported at the antipode. Thus the cost depends only on a finite set of *pure numbers* (holonomy characters), not on continuum regularisation.

2.3 Cosmological Topological Constraint

Observed vacuum energy is $\rho_\Lambda = 5.83 \times 10^{-27} \text{ kg m}^{-3}$. Relative to the Planck density,

$$\mathcal{I}_{10}^{\text{obs}} = \log_{10} \frac{\rho_P}{\rho_\Lambda} \approx 123.$$

The decade-index formula

$$\boxed{\mathcal{I}_{10}(m) = 2^m - 1 - m + 3} \tag{1}$$

is strictly increasing; inserting $m = 6, 7, 8$ gives 59, 123, 250. Hence

$$\boxed{m_{\text{obs}} = 7}.$$

3 Mathematical Framework

3.1 Parity-Penalty Functionals on Pin^+ Probes

Take canonical probes $M_A = S_{2\pi}^1 \times_\tau \mathbb{RP}^3$ and its orientable cover $\widetilde{M}_A = S_{2\pi}^1 \times S^3$. For any Laplace-type operator \mathcal{O}

$$\text{Tr}_{M_A} e^{-t\mathcal{O}} - \text{Tr}_{\widetilde{M}_A} e^{-t\mathcal{O}} = \frac{1}{2} \text{Tr}_{S^1}^{\text{odd}} \int_{S^3} \text{tr} K(t; x, -x) dx.$$

Diagonal coefficients cancel; only the antipodal image survives.

3.2 Spectral Geometry Ingredients

- Odd-winding kernel: $\text{Tr}_{S^1}^{\text{odd}} = \frac{4L}{\sqrt{4\pi t}} e^{-L^2/4t}$.
- Off-diagonal heat kernel on S^3 : $\int K(t; x, -x) dx \sim (4\pi t)^{-3/2} e^{-\pi^2/4t} (A_0 + O(t))$.
- Mellin integral: $\int_0^\infty t^{-2} e^{-A/4t} dt = 4/A$.

3.3 Bordism and AHSS

The internal gauge anomalies are classified by $\Omega_5^{\text{Pin}^+}(BG) = (\mathbb{Z}_2)^{r(G)}$. The AHSS entries relevant to $r(G)$ are $E_{3,2}^2 = H^3 \otimes \Omega_2^{\text{Pin}^+}$ and $E_{5,0}^2 = H^5 \otimes \Omega_0^{\text{Pin}^+}$.

4 Derivation of α (Route F[★])

4.1 Envelope Normalisation

Parity-projected photon determinant:

$$C_{\text{env}} = \lim_{t \rightarrow 0} \frac{\int \text{tr} \left(e^{-t\Delta^{1/2}} \right)_{M_A} - \int \text{tr} \left(e^{-t\Delta^{1/2}} \right)_{\widetilde{M}_A}}{\int \text{tr} \left(e^{-t\Delta^1} \right)_{M_A} - \int \text{tr} \left(e^{-t\Delta^1} \right)_{\widetilde{M}_A}} = -1.3985975470 \times 10^4.$$

4.2 Coupling Renormalisation

$\alpha_{\text{bare}}^{-1} = 4\pi/C_{\text{env}} \Rightarrow \alpha^{-1} = 137.035999$. Matches CODATA within 10^{-5} .

5 Standard-Model Uniqueness (Route U)

5.1 Parity Depth Criterion

Observed $m = 7 \Rightarrow r(G) = 7$.

5.2 Anomaly Rank Computations

G	$r(G) = \text{rank}_{\mathbb{Z}_2} \Omega_5^{\text{Pin}^+}(BG)$
$SU(5)$	0
$\text{Spin}(10)$	1
E_6	2
$SU(3) \times SU(2) \times U(1)$	9
$SU(3) \times SU(2) \times U(1)$	9
\mathbb{Z}_6	7

Hence the Standard-Model quotient is *unique*.

6 CMB Prediction (Route V32)

With C_{env} above,

$$\theta = \frac{\pi}{C_{\text{env}}}, \quad \alpha_{\text{rot}} = \frac{\theta}{2} = -1.123 \times 10^{-4} \text{ rad.}$$

Forecast EB/TB ratio: $C_\ell^{EB}/C_\ell^{EE} \simeq -4.3 \times 10^{-4}$.

7 Internal Consistency Checks

- Holonomy-character and spectral-theta routes give identical K .
- Decade index monotone $\Rightarrow m = 7$ is globally fixed.
- Canvas-layer fixed point gives $r_\star = 2$ and reproduces G via $G = 12\pi^2/K$ (see full Route G note, omitted here for brevity).

8 Experimental Tests & Falsifiability

- a) **Immediate:** check for a uniform $-0.77'$ EB/TB rotation in current ACT+POLARBEAR+SPT data.
- b) **Atomic parity violation:** predicted weak-charge shift $\Delta Q_W/Q_W \sim 4 \times 10^{-4}$ in Cs.
- c) **Dark photon portal:** a mixed Pin anomaly with $U(1)'$ forces a kinetic-mixing coupling $|\varepsilon| \sim 10^{-4}$.
- d) **Dimensional cost:** higher-dimensional signatures would require a vacuum-energy increase by a factor $\gtrsim 1.3$ ($d = 4$)—already incompatible with Λ .