Black Hole Thermodynamics in Bits — Part III: Kerr/Kerr–Newman, Measurement Limits, and Observational Hooks

Evan Wesley, Rosetta Stone of Physics — Reality Encoded
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Abstract. We extend the previous Schwarzschild-bit identities to rotating and charged holes, derive the one-bit energy law from the full first law, and formalize the integer-bit "mass ladder" in the Kerr/Kerr–Newman family. We then quantify measurement limits: for astrophysical black holes the step in M^2 implied by one bit is a Planck-scale constant, utterly unresolvable by current gravitational-wave or EHT instruments. Finally we outline observational hooks (ringdown quantization and area-inference) and connect horizon thermodynamics to a bit-rate bound on entropy/knowledge flux.

1 Conventions

We keep G, $\hbar$, k_{B}, c explicit until stated otherwise. The Bekenstein–Hawking entropy and horizon area are

$$S = \frac{\mathcal{A}}{4\,\ell_{\mathrm{P}}^2}\,, \qquad S_{\mathrm{bits}} \equiv \frac{S}{\ln 2} = \frac{\mathcal{A}}{4\,\ell_{\mathrm{P}}^2\,\ln 2}\,. \tag{1}$$

The one-bit area quantum and Landauer energy are

$$\Delta\mathcal{A}_{1\ \mathrm{bit}} = 4\,\ell_{\mathrm{P}}^2\,\ln 2\,, \qquad \Delta E_{1\ \mathrm{bit}} = k_{\mathrm{B}}\ \ln 2\,. \tag{2}$$

Numerically, $4\ell_{\mathrm{P}}^2\ln 2 = 7.242\,778\,905\,692\,047 \times 10^{-70}\,\mathrm{m}^2$.

2 Schwarzschild recap and the integer-bit ladder

For Schwarzschild mass M,

$$\mathcal{A} = \frac{16\pi G^2 M^2}{c^4}\,, \qquad = \frac{\hbar c^3}{8\pi k_{\mathrm{B}} G M}\,. \tag{3}$$

The bit count and the mass ladder follow immediately:

$$N_{\mathrm{bits}} = \frac{\mathcal{A}}{4\ell_{\mathrm{P}}^2\ln 2} = \frac{4\pi G M^2}{\hbar c\,\ln 2}\,, \qquad M_N = m_{\mathrm{P}}\sqrt{\frac{\ln 2}{4\pi}\,N}\,. \tag{4}$$

A uniform step in N corresponds to a *constant* step in M^2:

$$\Delta(M^2) = \frac{\hbar c\,\ln 2}{4\pi G} = m_{\mathrm{P}}^2\,\frac{\ln 2}{4\pi} \approx 2.61\times 10^{-17}\,\mathrm{kg}^2\,. \tag{5}$$

At $M \sim M_\odot$, the corresponding mass increment is $\Delta M \approx \Delta(M^2)/(2M) \sim 6.6\times 10^{-48}\,\mathrm{kg}$, far beyond any conceivable direct resolution.

3 Kerr and Kerr–Newman in bits

Work now in geometric units ($G = c = \hbar = k_{\mathrm{B}} = 1$) for clarity, restoring constants at the end. For Kerr–Newman with mass M, charge Q, and spin parameter $a \equiv J/M$, the horizons are $r_+, r_- = M \pm \sqrt{M^2 - a^2 - Q^2}$ and the area and surface gravity are

$$\mathcal{A} = 4\pi\,(r_+^2 + a^2) = 4\pi\,(2Mr_+ - Q^2)\,, \tag{6}$$

$$\kappa_{\mathrm{H}} = \frac{r_+ - r_-}{2(r_+^2 + a^2)} = \frac{\sqrt{M^2 - a^2 - Q^2}}{r_+^2 + a^2}\,. \tag{7}$$

The angular velocity and electric potential at the horizon are

$$\Omega_{\rm H} = \frac{a}{r_+^2 + a^2}\,, \qquad \Phi_{\rm H} = \frac{Q\,r_+}{r_+^2 + a^2}\,. \tag{8}$$

The temperature is $= \kappa_{\rm H}/(2\pi)$ and the entropy $S = \mathcal{A}/4$. The differential first law and Smarr relation read

$$dM = \frac{\kappa_{\rm H}}{8\pi}\, d\mathcal{A} + \Omega_{\rm H}\, dJ + \Phi_{\rm H}\, dQ\,, \tag{9}$$

$$M = 2\,S + 2\Omega_{\rm H} J + \Phi_{\rm H} Q\,. \tag{10}$$

One-bit law at fixed (J, Q). If an interaction changes the horizon area by exactly one bit while holding J and Q fixed, then from (9)

$$(\Delta M)_{J,Q} = \frac{\kappa_{\rm H}}{8\pi}\,\Delta\mathcal{A} = \Delta S = \ln 2 \equiv \Delta E_{1\ \rm bit}\,, \tag{11}$$

precisely the Landauer cost. Thus the one-bit energy law is *universal* across Schwarzschild, Kerr, and Kerr–Newman.

Area-per-bit is invariant. Since $\Delta\mathcal{A}$ depends only on $\ell_{\rm P}$ and $\ln 2$, the area quantum does not care about rotation or charge. What changes with (J, Q) is the local surface gravity $\kappa_{\rm H}$ and hence ; as one approaches extremality $(a^2 + Q^2 \to M^2)$, $\kappa_{\rm H} \to 0$ and the energy-per-bit tends to zero even though each bit still occupies area $4\ell_{\rm P}^2 \ln 2$.

Bit-labeled families. Define the integer bit label N by $\mathcal{A}(N) = 4\ell_{\rm P}^2 \ln 2\, N$. For Kerr the map $(M, a) \mapsto N$ is many-to-one; given (N, a) the mass solves $4\pi(r_+^2 + a^2) = 4\ell_{\rm P}^2 \ln 2\, N$ with $r_+ = M + \sqrt{M^2 - a^2}$. The Schwarzschild ladder M_N is recovered at $a = 0$. For Kerr–Newman the family extends with $Q \neq 0$ under the cosmic censorship bound $a^2 + Q^2 \leq M^2$.

4 Measurement limits for the mass ladder

4.1 Gravitational-wave masses

Detector posteriors resolve component masses at $\mathcal{O}(\%)$ in favorable events. The Schwarzschild ladder predicts a *constant* $\Delta(M^2)$ at the Planck scale; for any stellar or supermassive black hole,

$$\frac{\Delta M}{M} \sim \frac{m_{\rm P}^2 (\ln 2/4\pi)}{2M^2} \ll 10^{-60} \quad (M \gtrsim M_\odot)\,. \tag{12}$$

Hence direct step-resolving tests in the mass spectrum are intractable. The same conclusion holds for EHT-scale masses.

4.2 Ringdown and area inference

While $\Delta(M^2)$ is unresolvably small, *asymptotic* properties of quasinormal modes (QNMs) might encode area quantization through their spacing constants. Different proposals correspond to different area quanta (e.g. $\propto \ln k$ factors). The bit quantum here is $4 \ln 2\, \ell_{\rm P}^2$. A practical program is to (i) stack high-SNR ringdowns to calibrate the dependence of overtone spacing on surface gravity, then (ii) test for a universal logarithmic factor compatible with $\ln 2$. This is a long-term target; present instruments likely lack the required SNR and mode counts.

5 Greybody flux and a bit-rate bound

Let P be the total Hawking power at infinity and let $\epsilon_{\rm gb} \in (0,1)$ denote the greybody efficiency (frequency- and spin-dependent). The maximum bit emission rate consistent with the one-bit cost is

$$\dot{N_{\rm bits}} \le \frac{\epsilon_{\rm gb}\, P}{k_{\rm B}\, \ln 2}\,. \tag{13}$$

This also bounds any channel that treats the horizon as a thermodynamic transducer: to communicate or erase information at the horizon you must pay $k_{\rm B} \ln 2$ per bit (at fixed J, Q). For astrophysical holes $\propto 1/M$ is minuscule, making the energy-per-bit tiny *but* the absolute flux $P \propto 1/M^2$ is tinier still; net bit throughput is effectively zero on observational timescales.

6 Worked numbers (Schwarzschild anchors)

For quick reference (using the constants defined above):

Mass	(K)	$\Delta E_{1\ \rm bit}$ (eV)	$S_{\rm bits}$
$1\,M_\odot$	$\approx 6.17 \times 10^{-8}$	$\approx 3.69 \times 10^{-12}$	$\approx 1.50 \times 10^{77}$
$10\,M_\odot$	$\approx 6.17 \times 10^{-9}$	$\approx 3.69 \times 10^{-13}$	$\approx 1.50 \times 10^{79}$
$4 \times 10^6\,M_\odot$	$\approx 1.54 \times 10^{-14}$	$\approx 9.21 \times 10^{-19}$	$\approx 2.40 \times 10^{90}$

Values scale as $\propto 1/M$, $\Delta E_{1\ \rm bit} \propto 1/M$, and $S_{\rm bits} \propto M^2$.

7 Falsification checklist

1. **One-bit law under spin/charge.** Verify $\Delta E_{1\ \rm bit} = k_{\rm B} \ln 2$ experimentally in any analogue horizon with controlled $\Omega_{\rm H}$ (rotation) or $\Phi_{\rm H}$ (effective charge).

2. **Area-per-bit invariance.** Independent of (J, Q) all horizon area changes in single-bit increments of $4\ell_{\rm P}^2 \ln 2$.

3. **Ringdown spacing test.** Look for a universal logarithmic factor compatible with $\ln 2$ in high-overtone QNM stacks.

4. **Entropy flux bound.** In any process that couples to the horizon, the bit-rate satisfies $\dot{N_{\rm bits}} \le P/(k_{\rm B} \ln 2)$ up to greybody factors; any persistent violation rules out the framework.

8 Discussion

The key structural facts survive generalization: (i) the *energy per bit* is set by the local Hawking temperature, (ii) the *area per bit* is Planck-locked, and (iii) the Schwarzschild mass ladder becomes a family of (N, J, Q)-labeled states with the same $\Delta\mathcal{A}$ but suppressed by rotation/charge. Observationally, direct resolution of $\Delta(M^2)$ is impossible; the promising path is spectral—through precision ringdown physics and analogue systems where and P are tunable.

Outlook. Two natural follow-ups: (1) a Kerr numerical appendix that tabulates $(\kappa_{\rm H}, \Omega_{\rm H}, \Phi_{\rm H})$ and $\Delta E_{1\ \rm bit}$ across spin a_* at fixed M, and (2) a ringdown data challenge that tests for a universal logarithmic constant in overtone spacing.

Appendix A: Restoring constants

The geometric-units expressions can be mapped back to SI by $= \hbar\kappa_{\mathrm{H}}/(2\pi k_{\mathrm{B}} c)$ and $S = \mathcal{A}c^3/(4G\hbar)$. The first law reads

$$d(Mc^2) = \frac{\kappa_{\mathrm{H}} c^2}{8\pi G}\, d\mathcal{A} + \Omega_{\mathrm{H}}\, dJ + \Phi_{\mathrm{H}}\, dQ\,. \tag{14}$$

At fixed (J, Q) and for a single-bit area change $\Delta\mathcal{A} = 4\ell_{\mathrm{P}}^2 \ln 2$ one obtains $\Delta E_{1\ \mathrm{bit}} = k_{\mathrm{B}} \ln 2$.

Appendix B: Useful constants

$$m_{\mathrm{P}} = \sqrt{\frac{\hbar c}{G}} = \quad 4\ell_{\mathrm{P}}^2 \ln 2 = 7.242\,778\,905\,692\,047 \times 10^{-70}\,\mathrm{m}^2\,, \qquad \Delta(M^2) = m_{\mathrm{P}}^2\,\frac{\ln 2}{4\pi} \approx 2.61 \times 10^{-17}\,\mathrm{kg}^2\,. \tag{15}$$