Energy $\to$ Knowledge Transduction Law (EKTL)

A Fraction Physics Standalone Teaching Module

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Abstract

I formalize a practical law that connects *exergy power* to *knowledge gain* in bits, bounded by Landauer's principle and organized by a factored efficiency pipeline. The module is: (i) **why it matters** (ceiling and design knobs), (ii) **how to use it** (measurement and fitting recipes), (iii) **closed forms** for planning time/energy budgets, and (iv) **worked examples** at room temperature and cryogenic regimes.

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Conventions and Philosophy

- Units: k_B=1, $\hbar$=c=1 when convenient; otherwise SI is explicit. Transcendentals (e.g. $\ln 2$, π) carry no MDL charge; **rational locks** p/q are scored by $L(p/q) = \lceil \log_2 p \rceil + \lceil \log_2 q \rceil$.
- **Knowledge** K is measured in *bits* (e.g. cumulative *mutual information* gained by the agent about the task/source, or predictive log-loss improvement divided by $\ln 2$).
- EKTL separates the **thermodynamic ceiling** (Landauer) from **pipeline efficiencies** and an **epistemic saturator**. All three must be reported to compare systems.

1 Variables and Base Law

1.1 Definitions (units in brackets)

$$\begin{aligned} &K(t): \text{ knowledge in } \textbf{bits} \text{ [bit]},\\ &P(t): \text{ available } \textit{exergy} \text{ power to the learning pipeline [J s}^{-1}\text{]},\\ &T_{\text{eff}}(t): \text{ effective temperature of erasure/sensing reservoirs [K]},\\ &\Gamma(T) \equiv \frac{1}{k_B T \ln 2}: \text{ Landauer gain [bit J}^{-1}\text{]}. \end{aligned}$$

1.2 Ceiling (Landauer-limited gain)

For any logically irreversible stage at temperature T,

$$\left.\frac{dK}{dt}\right|_{\max} = \Gamma(T)\, P(t). \tag{1}$$

At T=300 K, $k_B T \ln 2 \approx 2.87 \times 10^{-21}$ J/bit so $\Gamma \approx 3.48 \times 10^{20}$ bit/J.

2 Real Pipelines: Factorized Efficiency and Epistemics

2.1 Efficiency product

We factor the pipeline's efficiency into measurable terms:

$$\eta(t) \equiv \eta_{\text{cap}}\, \eta_{\text{conv}}\, \eta_{\text{sense}}\, \eta_{\text{comp}}(\rho)\, \eta_{\text{learn}}(\pi)\, \eta_{\text{val}} \in [0, 1], \tag{2}$$

where $\rho \in [0, 1]$ is the fraction of *logically irreversible* compute; π encodes the learning architecture/rule.

2.2 Epistemic saturator

Define $\Phi \in [0, 1]$ to capture novelty/identifiability (diminishing returns as knowledge accumulates): we use a Hill/logistic family

$$\Phi(K, t) = \left(1 - \tfrac{K(t)}{C(t)}\right)^{\alpha}, \qquad \alpha \in [1, 2], \tag{3}$$

with capacity $C(t)$ (bits) depending on task/instrument/prior.

2.3 Energy → knowledge transduction

Putting ceiling, efficiency, and epistemics together:

$$\boxed{\frac{dK}{dt} = \Gamma\big(T_{\text{eff}}(t)\big)\,\eta(t)\,P(t)\,\Phi\big(K,t\big)} \tag{4}$$

A **dimensionless performance score** (proximity to Landauer):

$$\boxed{\kappa(t) \equiv \frac{1}{\Gamma(T_{\text{eff}})P}\frac{dK}{dt} = \eta(t)\,\Phi(K,t) \in [0,1].} \tag{5}$$

3 Closed Forms and Budgets (constant params)

Assume $T_{\text{eff}}, \eta, P, C$ are constant on a planning window. Let $A \equiv \Gamma(T_{\text{eff}})\,\eta\,P$.

3.1 Case $\alpha = 1$ (exponential approach)

$$\frac{dK}{dt} = A\Big(1 - \frac{K}{C}\Big), \tag{6}$$

$$K(t) = C - \big(C - K_0\big)\,\exp\Big[-\frac{A}{C}(t - t_0)\Big], \tag{7}$$

$$\Delta t\ (K_0 \to K_1) = \frac{C}{A}\ln\frac{C - K_0}{C - K_1}. \tag{8}$$

Energy-for-knowledge. Using $\frac{dE}{dK} = \frac{k_B T_{\text{eff}}\ln 2}{\eta\,(1-K/C)}$,

$$E(K) = \frac{k_B T_{\text{eff}}\ln 2}{\eta}\Big[-C\ln\Big(1 - \frac{K}{C}\Big)\Big]. \tag{9}$$

3.2 Case $\alpha = 2$ (hyperbolic tail)

$$\frac{dK}{dt} = A\Big(1 - \frac{K}{C}\Big)^2, \tag{10}$$

$$K(t) = C - \frac{C^2}{A(t - t_0) + \dfrac{C^2}{C - K_0}}, \tag{11}$$

$$E(K) = \frac{k_B T_{\text{eff}}\ln 2}{\eta}\,\frac{C\,K}{C - K}. \tag{12}$$

3.3 Growing capacity $C(t)$

Normalize $s \equiv K/C$. Then

$$\dot{s} = \frac{A}{C}(1 - s)^{\alpha} - s\,\frac{\dot{C}}{C}, \tag{13}$$

cleanly separating "learn faster" (increase A) from "grow the bucket" ($\dot{C} > 0$).

4 How To Use EKTL (Recipe Cards)

4.1 Measure and report

1. **Pick** $K(t)$: cumulative MI gain or predictive log-loss improvement (divide by $\ln 2$ to get bits).
2. **Log** $P(t)$: exergy power into compute+sense; note duty cycle.
3. **Assign** $T_{\text{eff}}(t)$: power-weighted effective temperature of erasure/sensing reservoirs.
4. **Compute** $\kappa(t)$: $\kappa = [\Gamma(T_{\text{eff}})P]^{-1}\, dK/dt$.
5. **Publish an η budget**: $(\eta_{\text{cap}}, \eta_{\text{conv}}, \eta_{\text{sense}}, \eta_{\text{comp}}, \eta_{\text{learn}}, \eta_{\text{val}})$.

4.2 Fit C and α from data

Collect (t_i, K_i) and fit using linearizations:

- $\alpha = 1$: plot $\ln(1 - K/C)$ vs t; slope $-A/C$ (try candidate C and pick the best straightness by R^2).
- $\alpha = 2$: plot $1/(C - K)$ vs t; slope A/C^2.

Then back out $A = \Gamma\eta P$ to infer an *effective* η for the platform.

4.3 Budgeting to a target

Given target fraction $f \in (0, 1)$ of capacity: $K_\star = fC$.

$$\alpha = 1: \quad E(f) = \frac{k_B T_{\text{eff}} \ln 2}{\eta}\Big[-C\ln(1-f)\Big], \qquad T(f) = \frac{C}{A}\ln\frac{1}{1-f}. \tag{14}$$

$$\alpha = 2: \quad E(f) = \frac{k_B T_{\text{eff}} \ln 2}{\eta}\Big[\frac{Cf}{1-f}\Big], \qquad T(f) = \frac{C}{A}\Big(\frac{f}{1-f}\Big). \tag{15}$$

Rule of thumb: The last 10% (from f=0.9 to 1.0) is disproportionately expensive.

5 Worked Examples

5.1 Room temperature platform

T_{eff}=300 K, P=100 W, $\eta\Phi$=10^{-6} (i.e. κ=10^{-6}). Then

$$\frac{dK}{dt} = \Gamma P\,\kappa \approx (3.48 \times 10^{20}) \times 100 \times 10^{-6} = 3.48 \times 10^{16} \text{ bit/s}. \tag{16}$$

If C=10^{15} bits and α=1, time to reach f=0.9 is

$$T(0.9) = \frac{C}{A}\ln 10 = \frac{10^{15}}{\Gamma\eta P}\ln 10 \approx \frac{10^{15}}{(3.48 \times 10^{22})}\, 2.303 \text{ s} \approx 6.6 \times 10^{-8} \text{ s}. \tag{17}$$

This shows the *ceiling is huge*; practical limits are elsewhere (typically Φ and η_{comp}).

5.2 Cryogenic improvement

At T_{eff}=30 K, Γ improves by $\times 10$. With identical P and κ, all times/energies shrink by $\times 10$. Alternatively, hold dK/dt fixed and reduce power by $\times 10$.

5.3 Reversibility knob

If architectural changes halve the irreversible fraction ρ so that $\eta_{\text{comp}} \to 2\eta_{\text{comp}}$, then $A \to 2A$ and all timeshalve at fixed P and T_{eff}.

6 Engineering Guidance (Knobs and Tradeoffs)

1. **Lower** T_{eff} (cryogenic sensing/logic, error-corrected reversible steps) to reduce $E_{\text{bit}} = k_B T_{\text{eff}} \ln 2/[\eta\,\Phi]$.
2. **Increase reversibility** (reduce ρ) to raise η_{comp}.
3. **Raise novelty** (active experiment design, curriculum, de-duplication) to increase Φ.
4. **Grow capacity** C (new sensors/tasks/priors) to avoid early saturation.
5. **Publish budgets**: report $\kappa(t)$, $E_{\text{bit}}(t)$, and an η table per system for apples-to-apples comparisons.

7 FAQ and Pitfalls

- **Is K subjective?** Use a task-tied definition (e.g., MI between model and labeled source, or predictive log-loss improvement). Consistency beats perfection.
- **Multiple reservoirs?** Use a power-weighted T_{eff} for the erasure-dominant stages.
- **Non-stationary data?** Let $C(t)$ grow; track s=K/C and $\dot{C}/C$.
- **Hitting the ceiling?** Report κ; most systems are far below 1 due to η_{comp} and low Φ.

Staging Table (this pack)

Module	Observable(s) / Content	Frozen value(s) / Locks	Bit-cost	Sector	Status
M–EKTL–01	Energy→Knowledge law	Eqs. (4) and κ def.	– (no new rationals)	Methods	Ready

Copy/Use Checklist

1. Decide how you will measure $K(t)$ (bits).
2. Log $P(t)$ and assign $T_{\text{eff}}(t)$.
3. Compute $\Gamma(T_{\text{eff}})$, $\kappa(t)$, and $E_{\text{bit}}(t)$.
4. Fit C, α from (t, K) traces; choose planning window.
5. Publish the η budget, κ curve, and a pre-registered performance target.