Fractional Action Principle: A Rational Standard Model

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Abstract

We present a formulation of the Standard Model in which *all seed parameters are exact rationals*. Fields, operators, and symmetries are conventional, but couplings, mixing angles, and mass ratios are specified by a rational registry. The action is therefore specified over $\mathbb{Q}$ (with explicit π retained symbolically), enabling auditably exact predictions at *anchor scales*. We formalize (i) a *selection rule* that treats anomaly equalities as preconditions on field content (thereby requiring three right-handed neutrinos when $B-L$ is gauged), (ii) a *rational running* scheme between anchor scales via Möbius/spline maps that preserve exactness at nodes, and (iii) a *rational path-sum* discretization. The framework doubles as a proof-of-principle and a teaching guide with unit tests (CKM geometry, rare-decay ratios, $g-2$, EW identities) implemented as exact checks.

Contents

1 Axioms of the $\mathbb{Q}$-First Program

A1. Rational seeds. Every dimensionless input is fixed as $p/q \in \mathbb{Q}$. Transcendentals (e.g., π) are left as symbols.

A2. Field content. $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ with three families; include three sterile ν_R to satisfy anomaly preconditions if $B-L$ is gauged.

A3. Renormalizable operator basis. The canonical SM Lagrangian; only the weights (couplings) are exact rationals.

A4. Exact constraints as preconditions. Anomaly equalities and custodial identities imposed *exactly.*

A5. Rational running. Between anchors, couplings follow rational maps preserving node exactness.

A6. Audit discipline. Each prediction carries its exact fraction and MDL cost $\mathrm{MDL}(p/q) = \lceil \log_2 p \rceil + \lceil \log_2 q \rceil$.

2 Seed Registry and Immediate Consequences

2.1 Electroweak and QCD seeds

$$\alpha_{\mathrm{em}} := \frac{2639}{361638}, \tag{2.1}$$
$$\alpha_s(M_Z) := \frac{23}{195}, \tag{2.2}$$
$$\sin^2\theta_W := \frac{25}{108}, \tag{2.3}$$
$$\cos^2\theta_W := 1 - \frac{25}{108}, \tag{2.4}$$
$$\lambda_H := \frac{9}{40}. \tag{2.5}$$

Then $e^2 = 4\pi\,\frac{2639}{361638}$, $g^2 = 4\pi\,\frac{2639}{361638}/\frac{25}{108}$, $g'^2 = 4\pi\,\frac{2639}{361638}/(1-\frac{25}{108})$, $g_3^2 = 4\pi\,\frac{23}{195}$.

2.2 Flavor and mixing seeds

For CKM we adopt Wolfenstein-like seeds

$$\lambda = \frac{2}{9}, \quad A = \frac{21}{25}, \quad \bar{\rho} = \frac{3}{20}, \quad \bar{\eta} = \frac{7}{20}. \tag{2.6}$$

For PMNS,

$$\sin^2\theta_{12} = \frac{7}{23}, \quad \sin^2\theta_{13} = \frac{2}{89}, \quad \sin^2\theta_{23} = \frac{9}{16}, \quad \delta_{\mathrm{PMNS}} = -\frac{\pi}{2}. \tag{2.7}$$

3 The Fractional Action

Let $\Phi = \{Q_L, u_R, d_R, L_L, e_R, \nu_R, H;\ G^a_\mu, W^i_\mu, B_\mu\}$. The action is

$$\mathcal{S}_{\mathbb{Q}}[\Phi] = \int d^4x \left(\mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{kin}} + \mathcal{L}_H + \mathcal{L}_Y\right). \tag{3.1}$$

3.1 Gauge sector

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4g_3^2} G^a_{\mu\nu} G^{a\,\mu\nu} - \frac{1}{4g^2} W^i_{\mu\nu} W^{i\,\mu\nu} - \frac{1}{4g'^2} B_{\mu\nu} B^{\mu\nu}. \tag{3.2}$$

3.2 Matter kinetics

$$\mathcal{L}_{\text{kin}} = \sum_{\psi} \bar{\psi} i\gamma^\mu D_\mu \psi + (D_\mu H)^\dagger (D^\mu H), \quad D_\mu = \partial_\mu - igT^i W^i_\mu - ig' Y B_\mu - ig_3 T^a G^a_\mu. \tag{3.3}$$

3.3 Higgs potential

$$\mathcal{L}_H = -V(H), \qquad V(H) = \lambda_H \left(H^\dagger H - \tfrac{1}{2} v^2\right)^2, \qquad \lambda_H = \frac{9}{40}. \tag{3.4}$$

3.4 Yukawa sector

$$\mathcal{L}_Y = -\left(\bar{Q}_L \tilde{H} Y_u u_R + \bar{Q}_L H Y_d d_R + \bar{L}_L H Y_e e_R + \bar{L}_L \tilde{H} Y_\nu \nu_R\right) + \text{h.c.}, \tag{3.5}$$

$$Y_f = \sqrt{2}\,\text{diag}(m_f^{(1)}, m_f^{(2)}, m_f^{(3)})/v, \quad m_f^{(i)}/v \in \mathbb{Q}. \tag{3.6}$$

4 Selection Rule: Anomalies as Preconditions

Definition 4.1 (Admissibility)**.** A field content is *admissible* iff the non-abelian cubic, abelian cubic, mixed abelian–non-abelian, and mixed gravitational anomalies satisfy the exact equalities

$$\sum_{\text{LH Weyl}} \text{tr}\left(\{T^a, T^b\} T^c\right) = 0, \qquad \sum Y = 0, \qquad \sum Y^3 = 0, \tag{4.1}$$

$$\sum Y \ \text{tr}(T^a T^a) = 0, \qquad \sum Y = 0 \ (\text{grav}). \tag{4.2}$$

Proposition 4.2 (Necessity of ν_R for gauged $B{-}L$)**.** *With three generations, the above equalities hold exactly for hypercharge without ν_R; gauging $B-L$ makes inclusion of a sterile ν_R per generation* necessary *so that linear and cubic $B{-}L$ sums vanish.*

5 Electroweak Symmetry Breaking and Exact Mass Relations

At tree level,

$$M_W = \tfrac{1}{2} g v, \qquad M_Z = \tfrac{1}{2}\sqrt{g^2 + g'^2}\, v, \qquad \sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}. \tag{5.1}$$

Proposition 5.1 (Rational closure of boson masses)**.** *If $\sin^2\theta_W \in \mathbb{Q}$ and $\alpha_{\text{em}} \in \mathbb{Q}$, then $g^2, g'^2 \in \pi\mathbb{Q}$ and $(M_W/M_Z)^2 = 1 - \sin^2\theta_W \in \mathbb{Q}$.*

6 Exact Mixing Geometry

6.1 CKM

With $(\lambda, A, \bar{\rho}, \bar{\eta})$ rational, the unitarity triangle has apex $(\bar{\rho}, \bar{\eta})$ and

$$\tan\beta = \frac{\bar{\eta}}{1 - \bar{\rho}} = \frac{7}{17}, \qquad \sin 2\beta = \frac{2\tan\beta}{1 + \tan^2\beta} = \frac{119}{169}. \tag{6.1}$$

The Jarlskog is $J = A^2 \lambda^6 \bar{\eta} \in \mathbb{Q}$ to the chosen order.

6.2 PMNS

Take $\sin^2\theta_{ij}$ rational and $\delta_{\mathrm{PMNS}} = -\pi/2$. Then $|U_{e1}|^2, |U_{e2}|^2, |U_{e3}|^2$ are exact rationals; the leptonic Jarlskog $J_{\mathrm{PMNS}} = c_{12}s_{12}c_{23}s_{23}c_{13}^2 s_{13}\,\sin\delta$ inherits exactness.

7 Rational Running Between Anchor Scales

Definition 7.1 (Rational map)**.** Let $\{\mu_k\}$ be anchor scales with node values $\alpha_i(\mu_k) \in \mathbb{Q}$. A *rational running* is a piecewise map on intervals $[\mu_k, \mu_{k+1}]$ of the form

$$\alpha_i(\mu) = \frac{a_i\mu + b_i}{c_i\mu + d_i}, \qquad a_i, b_i, c_i, d_i \in \mathbb{Q}, \tag{7.1}$$

(or a rational spline in $\ln\mu$) chosen to interpolate node values (and optionally slopes) exactly.

8 Path-Sum over Rational Configurations

Definition 8.1 (Rational mode ledger)**.** Choose a finite rational basis (e.g., box modes on a rational lattice). Replace the continuum functional integral by a sum over coefficients in a bounded subset of $\mathbb{Q}$.

9 Unit Tests (Exact Checks)

- **U1. CKM geometry:** compute $\sin 2\beta = 119/169$ and $J = A^2\lambda^6\bar{\eta}$ from seeds.
- **U2. Rare-decay ratio:** using $|V_{td}|^2/|V_{ts}|^2 = 169/4050$, predict $\mathrm{BR}(B_d \to \mu^+\mu^-)/\mathrm{BR}(B_s \to \mu^+\mu^-)$ up to hadronic factors; record the exact CKM fraction.
- **U3. EW identity:** verify $\sin^2\theta_W + (M_W/M_Z)^2 = 1$ or snap variant.
- **U4. Higgs mass lock:** with $\lambda_H = 9/40$ and an EW anchor for v, check $M_H^2 = 2\lambda_H v^2$.
- **U5. Muon $g-2$ (QED core):** evaluate $a_\mu^{\mathrm{QED}} = \sum_{\ell=1}^{5} C_\ell(\alpha_{\mathrm{em}}/\pi)^\ell$ with exact α_{em}.

10 Appendices (Selected)

A: Explicit CKM/PMNS blocks

$\sin\delta = \dfrac{7}{\sqrt{58}}$, $\cos\delta = \dfrac{3}{\sqrt{58}}$; first-row PMNS probabilities example $(\dfrac{1392}{2047}, \dfrac{609}{2047}, \dfrac{2}{89})$.

B: Anomaly ledgers

Left-handed basis tables show exact cancellation of $[\mathrm{SU}(3)]^2\mathrm{U}(1)_Y$, $[\mathrm{SU}(2)]^2\mathrm{U}(1)_Y$, $[\mathrm{U}(1)_Y]^3$, and mixed gravitational anomalies; gauged $B-L$ requires ν_R.

C: Running templates

Two-node Möbius or rational spline in $\ln\mu$ with node exactness.

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