Rational Flavor Double Ledger:
CKM & PMNS Side-by-Side with Exact Fractions and Closed-Form CP Geometry

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Abstract

A tiny set of exact fractions locks the entire quark and lepton flavor geometry. On the CKM side, four rationals in a Wolfenstein-like basis,

$$\lambda = \frac{2}{9}, \quad A = \frac{21}{25}, \quad \bar{\rho} = \frac{3}{20}, \quad \bar{\eta} = \frac{7}{20},$$

produce closed forms for the unitarity triangle, including

$$\tan\beta = \frac{7}{17}, \qquad \sin 2\beta = \frac{119}{169}, \qquad \delta_{\text{CKM}} = \arctan\frac{7}{3}, \qquad J_{\text{CKM}} = A^2\lambda^6\bar{\eta}.$$

On the PMNS side, a compact Dirac-neutrino ledger with maximal CP,

$$\sin^2\theta_{12} = \frac{7}{23}, \quad \sin^2\theta_{13} = \frac{2}{89}, \quad \sin^2\theta_{23} = \frac{9}{16}, \quad \delta_{\text{PMNS}} = -\frac{\pi}{2},$$

yields an exact first-row probability vector $(|U_{e1}|^2, |U_{e2}|^2, |U_{e3}|^2) = \left(\frac{1392}{2047}, \frac{609}{2047}, \frac{2}{89}\right)$ and $J_{\text{PMNS}} = c_{12}s_{12}c_{23}s_{23}c_{13}^2 s_{13}$. The Cabibbo–solar complementarity check $\theta_C + \theta_{12}^{(\nu)} \approx 46.2^\circ$ lands strikingly near 45°. Every statement here is falsifiable to the next digit.

1. CKM ledger in exact fractions

Take

$$\lambda = \frac{2}{9} = 0.222222\ldots, \qquad A = \frac{21}{25} = 0.84, \qquad \bar{\rho} = \frac{3}{20} = 0.15, \qquad \bar{\eta} = \frac{7}{20} = 0.35.$$

The unitarity-triangle apex is $(\bar{\rho}, \bar{\eta})$. The base angles are

$$\gamma = \arg(\bar{\rho} + i\bar{\eta}) = \arctan\left(\frac{7}{3}\right), \qquad \beta = \arctan\left(\frac{\bar{\eta}}{1-\bar{\rho}}\right) = \arctan\left(\frac{7}{17}\right),$$

so

$$\sin 2\beta = \frac{2\tan\beta}{1+\tan^2\beta} = \frac{119}{169} = 0.704142\ldots, \qquad \delta_{\text{CKM}} \simeq \gamma = \arctan\frac{7}{3} \approx 66.801^\circ,$$

$$\sin\gamma = \frac{7}{\sqrt{58}}, \quad \cos\gamma = \frac{3}{\sqrt{58}}, \quad \alpha = 180^\circ - \beta - \gamma \approx 90.8^\circ.$$

Leading CKM magnitudes follow:

$$|V_{us}| = \lambda = \frac{2}{9} = 0.222222, \qquad |V_{ud}| \simeq 1 - \frac{\lambda^2}{2} = \frac{79}{81} = 0.975309,$$

$$|V_{cb}| = A\lambda^2 = \frac{28}{675} = 0.0414815, \qquad |V_{ub}| \simeq A\lambda^3\sqrt{\bar{\rho}^2 + \bar{\eta}^2} = \frac{21}{25}\cdot\frac{8}{729}\cdot\frac{\sqrt{58}}{20} = 3.512\times 10^{-3},$$

$$|V_{td}| \simeq A\lambda^3\sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2} = \frac{21}{25}\cdot\frac{8}{729}\cdot\frac{\sqrt{338}}{20} = 8.476\times 10^{-3}.$$

The universal CP measure is exact from the lock:

$$J_{\text{CKM}} = A^2\lambda^6\bar{\eta} = \left(\tfrac{21}{25}\right)^2\left(\tfrac{2}{9}\right)^6\left(\tfrac{7}{20}\right) = \frac{197{,}568}{6{,}643{,}012{,}500} = 2.973\times 10^{-5}.$$

2. PMNS ledger in exact fractions

Lock the angles and Dirac phase by

$$\sin^2\theta_{12} = \frac{7}{23}, \quad \sin^2\theta_{13} = \frac{2}{89}, \quad \sin^2\theta_{23} = \frac{9}{16}, \quad \delta_{\text{PMNS}} = -\frac{\pi}{2}.$$

Then

$$s_{12} = \sqrt{\tfrac{7}{23}}, \quad c_{12} = \sqrt{\tfrac{16}{23}}, \qquad s_{13} = \sqrt{\tfrac{2}{89}}, \quad c_{13} = \sqrt{\tfrac{87}{89}}, \qquad s_{23} = \frac{3}{4}, \quad c_{23} = \frac{\sqrt{7}}{4}.$$

The first row probabilities are *exact* rationals:

$$|U_{e1}|^2 = c_{12}^2 c_{13}^2 = \frac{16}{23} \cdot \frac{87}{89} = \frac{1392}{2047} = 0.6800195,$$
$$|U_{e2}|^2 = s_{12}^2 c_{13}^2 = \frac{7}{23} \cdot \frac{87}{89} = \frac{609}{2047} = 0.2975080,$$
$$|U_{e3}|^2 = s_{13}^2 = \frac{2}{89} = 0.0224719, \qquad \frac{1392+609}{2047} + \frac{2}{89} = 1.$$

The Dirac-leptonic Jarlskog is

$$J_{\text{PMNS}} = c_{12}s_{12}\, c_{23}s_{23}\, c_{13}^2 s_{13}\, \sin\delta_{\text{PMNS}} = -\,0.03345 \quad \text{(from the exact factors above)}.$$

The headline angles are

$$\theta_{12}^{(\nu)} = \arcsin\sqrt{\tfrac{7}{23}} \approx 33.45^\circ, \quad \theta_{13}^{(\nu)} = \arcsin\sqrt{\tfrac{2}{89}} \approx 8.60^\circ, \quad \theta_{23}^{(\nu)} = \arcsin\tfrac{3}{4} = 48.59^\circ,$$

with $\delta_{\text{PMNS}} = -90^\circ$.

3. Quark–lepton complementarity and shared structure

The Cabibbo angle from the CKM lock is $\theta_C = \arcsin(\lambda) = \arcsin(2/9) = 12.78^\circ$. The solar leptonic angle from the PMNS lock is $\theta_{12}^{(\nu)} \approx 33.45^\circ$. Their sum is

$$\theta_C + \theta_{12}^{(\nu)} \approx 46.23^\circ,$$

sitting close to the folklore 45° complementarity without being hard-wired. The atmospheric angle $\theta_{23}^{(\nu)} = 48.59^\circ$ likewise sits just above 45°, while the quark CP phase $\delta_{\text{CKM}} = \arctan(7/3)$ and the leptonic phase $\delta_{\text{PMNS}} = -\pi/2$ are both large in magnitude, giving sizable and clean J measures on both sides.

4. Tables you can audit with a four-function calculator

CKM summary from the exact lock

Quantity	Exact from fractions	Numeric
$\lvert V_{us}\rvert$	$2/9$	0.222222
$\lvert V_{ud}\rvert$	$1-\lambda^2/2 = 79/81$	0.975309
$\lvert V_{cb}\rvert$	$28/675$	0.0414815
$\lvert V_{ub}\rvert$	$\frac{21}{25}\frac{8}{729}\frac{\sqrt{58}}{20}$	3.512×10^{-3}
$\lvert V_{td}\rvert$	$\frac{21}{25}\frac{8}{729}\frac{\sqrt{338}}{20}$	8.476×10^{-3}
$\tan\beta$	$7/17$	0.411765
$\sin 2\beta$	$119/169$	0.704142
δ_{CKM}	$\arctan(7/3)$	66.801°
J_{CKM}	$A^2\lambda^6\bar\eta$	2.973×10^{-5}

PMNS summary from the exact lock

Quantity	Exact from fractions	Numeric
$\lvert U_{e1}\rvert^2$	$(16/23)\,(87/89) = 1392/2047$	0.680020
$\lvert U_{e2}\rvert^2$	$(7/23)\,(87/89) = 609/2047$	0.297508
$\lvert U_{e3}\rvert^2$	$2/89$	0.022472
$\theta_{12}^{(\nu)}$	$\arcsin\sqrt{7/23}$	33.45°
$\theta_{13}^{(\nu)}$	$\arcsin\sqrt{2/89}$	8.60°
$\theta_{23}^{(\nu)}$	$\arcsin(3/4)$	48.59°
δ_{PMNS}	$-\pi/2$	-90°
J_{PMNS}	$c_{12}s_{12}\,c_{23}s_{23}\,c_{13}^2 s_{13}(-1)$	-0.03345

5. Immediate falsification targets

The CKM lock predicts the exact identity $\sin 2\beta = 119/169$. The PMNS lock predicts $|U_{e1}|^2, |U_{e2}|^2$ as the exact rationals 1392/2047 and 609/2047 given the small $|U_{e3}|^2 = 2/89$. The quark phase is tethered to $\arctan(7/3)$; the lepton phase is exactly $-\pi/2$. Any decisive, stable deviation from these will break the locks cleanly. If the data keep hugging them as uncertainties shrink, the case for a fraction-structured flavor sector strengthens.

6. Why professionals should care

This is not a sprawling numerology game. It is extreme compression: four tiny rationals on the quark side and four on the lepton side generating closed-form angles, an exact $\sin 2\beta$, exact row-probability rationals for PMNS, and realistic J values on both sectors. Either nature respects these integers or it doesn't. There isn't much room to hide.