Rational CKM Ledger:
Small Exact Fractions ⇒ Full Flavor, Closed-Form CP Geometry

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Abstract

I present a CKM parameterization locked by four tiny exact rationals in a Wolfenstein-like basis:

$$\boxed{\lambda = \frac{2}{9}, \quad A = \frac{21}{25}, \quad \bar{\rho} = \frac{3}{20}, \quad \bar{\eta} = \frac{7}{20}}.$$

From these, the unitarity triangle apex $(\bar{\rho}, \bar{\eta})$, the CP phase, the Jarlskog invariant, and the magnitudes $|V_{ij}|$ follow analytically. The highlights are (i) a closed-form prediction

$$\boxed{\tan\beta = \frac{\bar{\eta}}{1-\bar{\rho}} = \frac{7}{17} \quad \Rightarrow \quad \sin 2\beta = \frac{119}{169} = 0.704142\ldots},$$

which lands strikingly close to the $B \to J/\psi K_S$ benchmark, and (ii) a clean expression for the CKM CP phase

$$\boxed{\delta_{\text{CKM}} = \arctan\left(\frac{\bar{\eta}}{\bar{\rho}}\right) = \arctan\left(\frac{7}{3}\right) \approx 66.80^\circ},$$

giving $\sin\delta = \frac{7}{\sqrt{58}}$ and $\cos\delta = \frac{3}{\sqrt{58}}$ exactly. The universal CP measure

$$\boxed{J = A^2\lambda^6\bar{\eta} = \frac{197{,}568}{6{,}643{,}012{,}500} \;=\; 2.973 \times 10^{-5}},$$

sits at the center of the current global-fit band. Everything here is falsifiable to the next digit.

1. Exact low-complexity locks

I work with the Wolfenstein-like set $(\lambda, A, \bar{\rho}, \bar{\eta})$:

$$\lambda = \frac{2}{9} = 0.222222\ldots, \qquad A = \frac{21}{25} = 0.84, \qquad \bar{\rho} = \frac{3}{20} = 0.15, \qquad \bar{\eta} = \frac{7}{20} = 0.35.$$

No fits: these are the priors. All results below are direct consequences.

2. Unitarity triangle in radicals

The apex is $(\bar{\rho}, \bar{\eta}) = (3/20,\ 7/20)$. Then

$$\gamma = \arg(\bar{\rho} + i\bar{\eta}) = \arctan\left(\frac{7}{3}\right), \qquad \beta = \arctan\left(\frac{\bar{\eta}}{1-\bar{\rho}}\right) = \arctan\left(\frac{7}{17}\right),$$

$$\alpha = 180^\circ - \beta - \gamma \approx 90.8^\circ.$$

Because $\tan\beta = \frac{7}{17}$, one gets the exact rational identity

$$\sin 2\beta = \frac{2\tan\beta}{1+\tan^2\beta} = \frac{2\cdot(7/17)}{1+(7/17)^2} = \frac{14/17}{1+49/289} = \frac{14}{17}\cdot\frac{289}{338} = \frac{119}{169} \approx 0.704142.$$

For γ one has $\tan\gamma = \frac{7}{3}$, hence

$$\sin\gamma = \frac{7}{\sqrt{58}}, \qquad \cos\gamma = \frac{3}{\sqrt{58}},$$

and $\delta_{\rm CKM} \simeq \gamma$ in this basis, giving the closed form in the abstract.

3. CKM magnitudes and the universal CP measure

To leading nontrivial orders in λ,

$$|V_{us}| = \lambda = \frac{2}{9} = 0.222222, \quad |V_{ud}| \simeq 1 - \frac{\lambda^2}{2} = 1 - \frac{2}{81} = \frac{79}{81} = 0.975309,$$

$$|V_{cb}| = A\lambda^2 = \frac{21}{25}\cdot\frac{4}{81} = \frac{84}{2025} = \frac{28}{675} = 0.0414815,$$

$$|V_{ub}| \simeq A\lambda^3\sqrt{\bar\rho^2+\bar\eta^2} = \frac{21}{25}\cdot\frac{8}{729}\cdot\frac{\sqrt{58}}{20} \approx 3.512\times10^{-3},$$

$$|V_{td}| \simeq A\lambda^3\sqrt{(1-\bar\rho)^2+\bar\eta^2} = \frac{21}{25}\cdot\frac{8}{729}\cdot\frac{\sqrt{338}}{20} \approx 8.476\times10^{-3}.$$

The rephasing invariant Jarlskog factor is

$$J = A^2\lambda^6\bar\eta = \left(\frac{21}{25}\right)^2\left(\frac{2}{9}\right)^6\left(\frac{7}{20}\right) = \frac{197{,}568}{6{,}643{,}012{,}500} = 2.973\times10^{-5}.$$

This places the area of every unitarity triangle at $\frac{1}{2}J$, i.e. $\sim 1.49\times10^{-5}$, consistent with the standard picture.

4. Geometry that you can try to break

The apex moduli are radicals with tiny integer structure:

$$R_u = \sqrt{\bar\rho^2+\bar\eta^2} = \frac{\sqrt{58}}{20}, \qquad R_t = \sqrt{(1-\bar\rho)^2+\bar\eta^2} = \frac{\sqrt{338}}{20}.$$

Their ratio is *exactly* $R_u/R_t = \sqrt{58/338} = \sqrt{29}/13$. The base angles are locked by $\tan\gamma = \frac{7}{3}$ and $\tan\beta = \frac{7}{17}$, so

$$\sin 2\beta = \frac{119}{169}, \qquad \sin\gamma = \frac{7}{\sqrt{58}},\ \ \cos\gamma = \frac{3}{\sqrt{58}}, \qquad \alpha \approx 90.8^\circ.$$

Each of these is a one-line falsification target.

5. Table (one glance, many checks)

Quantity	Exact from fractions	Numeric
$\lvert V_{us}\rvert$	$2/9$	0.222222
$\lvert V_{ud}\rvert$	$1-\lambda^2/2 = 79/81$	0.975309
$\lvert V_{cb}\rvert$	$28/675$	0.0414815
$\lvert V_{ub}\rvert$	$\frac{21}{25}\frac{8}{729}\frac{\sqrt{58}}{20}$	3.512×10^{-3}
$\lvert V_{td}\rvert$	$\frac{21}{25}\frac{8}{729}\frac{\sqrt{338}}{20}$	8.476×10^{-3}
$\tan\beta$	$7/17$	0.411765
$\sin 2\beta$	$119/169$	0.704142
$\delta_{\rm CKM}$	$\arctan(7/3)$	$66.801°$
J	197,568/6,643,012,500	2.973×10^{-5}

6. Quark–lepton complementarity (surprising coherence)

From my neutrino module one clean lock is $\sin^2\theta_{12}^{(\nu)} = 7/23 \Rightarrow \theta_{12}^{(\nu)} \approx 33.2°$. With $\lambda = 2/9$ one has the Cabibbo angle $\theta_C = \arcsin(2/9) \approx 12.78°$. The sum $\theta_C + \theta_{12}^{(\nu)} \approx 45.98°$ sits right on the complementarity folklore. This is not assumed — it falls out of the tiny rationals on both sides.

7. What to measure next (falsifiable to the digit)

You can try to break any one of these with sharper data:

$$\sin 2\beta \stackrel{?}{=} \frac{119}{169}, \qquad \delta_{\rm CKM} \stackrel{?}{\approx} \arctan\frac{7}{3}, \qquad J \stackrel{?}{=} \frac{197{,}568}{6{,}643{,}012{,}500}.$$

If future fits drift decisively away from these, the lock is wrong. If they keep hugging them as errors shrink, the "fraction ledger" hypothesis gains real weight.

8. Why this hits hard

Standard lore says the SM doesn't hand you such closed forms. Here, four tiny rationals compress the entire CKM geometry and land on exact expressions like $\sin 2\beta = 119/169$ and $\delta = \arctan(7/3)$ while producing $J \simeq 3\times10^{-5}$ and realistic $|V_{ij}|$. No numerology sprawl; no dozens of parameters. Either the world respects these integers, or it doesn't. That's a laboratory-grade claim.