Evan Wesley, September 17th 2025

Locking the theory to experiment by a single parameter a

We use the fixed-point closure

$$\alpha^{-1} \;=\; 137 \;+\; \frac{c}{137}, \qquad c \equiv c_{\text{ledger}} + c_{\text{Pauli}}, \tag{1}$$

and require $\alpha^{-1} = \alpha_{\text{exp}}^{-1}$ (CODATA 2022).

Target c. From (1),

$$c_{\text{target}} \;\equiv\; 137\big(\alpha_{\text{exp}}^{-1} - 137\big) \;=\; 4.931887249. \tag{2}$$

Ledger side (from `5shell.pdf` with universal wedge and two-corner Berry).

$$c_{\text{ledger,base}} = 3.154, \tag{3}$$

$$c_{\text{wedge}} = -\alpha \simeq -0.00729735256433, \tag{4}$$

$$c_{\text{Berry}} = +0.073, \tag{5}$$

$$\Rightarrow \quad c_{\text{ledger,eff}} = c_{\text{ledger,base}} + c_{\text{wedge}} + c_{\text{Berry}} = 3.21970264744. \tag{6}$$

Introduce a single locking parameter a on the Pauli block. Choose a Pauli baseline $c_{\text{Pauli}}^{(0)}$ (here we take the midpoint baseline to minimize modification):

$$c_{\text{Pauli}}^{(0)} = 1.38. \tag{7}$$

Promote it to a one-parameter family

$$c_{\text{Pauli}}(a) \;=\; a\, c_{\text{Pauli}}^{(0)}. \tag{8}$$

Solve a by locking to experiment. Impose $c_{\text{ledger,eff}} + c_{\text{Pauli}}(a) = c_{\text{target}}$. Using (2)–(8):

$$a^\star \;=\; \frac{c_{\text{target}} - c_{\text{ledger,eff}}}{c_{\text{Pauli}}^{(0)}} \;=\; \frac{4.931887249 - 3.21970264744}{1.38} \;=\; \boxed{1.2407134793944420}. \tag{9}$$

Equivalently, in additive form,

$$\Delta_{\text{Pauli}} \;\equiv\; c_{\text{Pauli}}(a^\star) - c_{\text{Pauli}}^{(0)} = \boxed{0.3321846015643314}. \tag{10}$$

Locked Pauli and total c.

$$c_{\text{Pauli,req}} = a^\star\, c_{\text{Pauli}}^{(0)} = \boxed{1.7121846015643314}, \tag{11}$$

$$c_{\text{theory}} = c_{\text{ledger,eff}} + c_{\text{Pauli,req}} = \boxed{4.931887249}. \tag{12}$$

Consistency check (by construction).

$$\alpha_{\text{pred}}^{-1} = 137 + \frac{c_{\text{theory}}}{137} = 137 + \frac{4.931887249}{137} = \boxed{\alpha_{\text{exp}}^{-1}}. \tag{13}$$

Notes and alternative normalizations (optional)

If one prefers a different Pauli baseline $c_{\mathrm{Pauli}}^{(0)}$, the same one-liner

$$a^{\star} = \frac{c_{\mathrm{target}} - c_{\mathrm{ledger,eff}}}{c_{\mathrm{Pauli}}^{(0)}}$$

applies. Numerically,

$$\begin{aligned} c_{\mathrm{Pauli}}^{(0)} &= 0.293250241962737662 \text{ (continuum)} &\Rightarrow\quad a^{\star} = 5.838646850227976, \\ c_{\mathrm{Pauli}}^{(0)} &= 0.332945182852222606 \text{ (ψ-avg lattice)} &\Rightarrow\quad a^{\star} = 5.142542045199920. \end{aligned}$$

These yield the same $c_{\mathrm{Pauli,req}} = 1.7121846015643314$ and hence the same closure $c_{\mathrm{theory}} = 4.931887249$.