Rare-Decay Ledger from Tiny Exact Fractions: $K \to \pi\nu\bar{\nu}$ and $B_{s,d} \to \mu^+\mu^-$ from a Rational CKM Lock

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Abstract

I show that a handful of tiny exact fractions for the CKM parameters

$$\boxed{\lambda = \frac{2}{9}, \quad A = \frac{21}{25}, \quad \bar{\rho} = \frac{3}{20}, \quad \bar{\eta} = \frac{7}{20}}$$

compress the clean rare decays $K \to \pi\nu\bar{\nu}$ and $B_{s,d} \to \mu^+\mu^-$ into closed forms with almost no moving parts. In the standard Buras normalizations,

$$\mathrm{BR}(K_L \to \pi^0\nu\bar{\nu}) = \kappa_L \left(\frac{\mathrm{Im}\,\lambda_t}{\lambda^5} X_t\right)^2, \qquad \mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) = \kappa_+ \left[\left(\frac{\mathrm{Im}\,\lambda_t}{\lambda^5} X_t\right)^2 + \left(\frac{\mathrm{Re}\,\lambda_c}{\lambda} P_c + \frac{\mathrm{Re}\,\lambda_t}{\lambda^5} X_t\right)^2\right],$$

while $B_q \to \mu^+\mu^-$ scale like $|V_{tb}V_{tq}^*|^2\, Y_t{}^2$. With the exact fractions above the CKM pieces collapse to

$$\frac{\mathrm{Im}\,\lambda_t}{\lambda^5} = A^2\bar{\eta} = \left(\tfrac{21}{25}\right)^2\left(\tfrac{7}{20}\right), \qquad \frac{\mathrm{Re}\,\lambda_t}{\lambda^5} = -A^2(1-\bar{\rho}) = -\left(\tfrac{21}{25}\right)^2\left(\tfrac{17}{20}\right), \qquad \frac{\mathrm{Re}\,\lambda_c}{\lambda} \simeq -1,$$

so the kaon modes become two *quadratics in small rationals.* Adopting compact rational benchmarks $X_t = \frac{37}{25}$ and $P_c = \frac{2}{5}$, the dimensionless cores evaluate to $A^4\bar{\eta}^2X_t^2 = 1.335\,91 \times 10^{-1}$ and $\left[P_c + A^2(1-\bar{\rho})X_t\right]^2 = 1.658\,03$, giving

$$\boxed{\mathrm{BR}(K_L \to \pi^0\nu\bar{\nu}) = \kappa_L \times 0.1335908348\ , \qquad \mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) = \kappa_+ \times 1.791619966}.$$

For B-leptonic decays, the clean CKM ratio

$$\boxed{\frac{|V_{td}|^2}{|V_{ts}|^2} = \lambda^2\left[(1-\bar{\rho})^2 + \bar{\eta}^2\right] = \frac{169}{4050} = 0.0417283950}$$

drives the null-hadronic headline

$$\boxed{\frac{\mathrm{BR}(B_d \to \mu^+\mu^-)}{\mathrm{BR}(B_s \to \mu^+\mu^-)} = \frac{\tau_{B_d}}{\tau_{B_s}}\frac{m_{B_d}}{m_{B_s}}\frac{f_{B_d}^2}{f_{B_s}^2} \times \frac{169}{4050}}.$$

Using standard lattice/PDG ratios $(\tau_{B_d}/\tau_{B_s}) \approx 0.99$, $(m_{B_d}/m_{B_s}) \approx 0.983$, $(f_{B_s}/f_{B_d}) \approx 1.206$, this lands at $\approx 2.790\,00 \times 10^{-2}$. Everything here is falsifiable to the next digit: change the fractions, the predictions move in one line.

1. Exact CKM pieces that matter

Work in the Wolfenstein–Buras language with barred parameters and keep only the clean leading structures (higher-order $\mathcal{O}(\lambda^2)$ refinements do not change the point). With the exact lock

$$\lambda = \frac{2}{9}, \quad A = \frac{21}{25}, \quad \bar{\rho} = \frac{3}{20}, \quad \bar{\eta} = \frac{7}{20},$$

the kaon CKM factors reduce to

$$\frac{\operatorname{Im}\lambda_t}{\lambda^5} = A^2\bar\eta = \frac{441}{625}\cdot\frac{7}{20} = \frac{3087}{12500}, \qquad \frac{\operatorname{Re}\lambda_t}{\lambda^5} = -A^2(1-\bar\rho) = -\frac{441}{625}\cdot\frac{17}{20} = -\frac{7497}{12500},$$

$$\frac{\operatorname{Re}\lambda_c}{\lambda} = -1 + \mathcal{O}(\lambda^2) = -1 + \mathcal{O}\Big(\frac{4}{81}\Big).$$

That's the entire CKM dependence for $K\to\pi\nu\bar\nu$.

2. Golden kaons $K\to\pi\nu\bar\nu$

Keep the short-distance constants explicit:

$$X_t \equiv X(x_t), \qquad P_c \equiv P_c(X,Y), \qquad \kappa_+,\ \kappa_L \text{ (known phase-space \& isospin factors).}$$

Then

$$\mathrm{BR}(K_L\to\pi^0\nu\bar\nu) = \kappa_L\left(A^2\bar\eta\,X_t\right)^2, \qquad \mathrm{BR}(K^+\to\pi^+\nu\bar\nu) = \kappa_+\left[\left(A^2\bar\eta\,X_t\right)^2 + \left(-P_c - A^2(1-\bar\rho)X_t\right)^2\right].$$

Adopt compact rational benchmarks for the short-distance numbers (close to standard NNLO values but human-auditable):

$$X_t = \frac{37}{25} = 1.48, \qquad P_c = \frac{2}{5} = 0.40.$$

Then the two core pieces evaluate to

$$A^4\bar\eta^2 X_t^2 = \underbrace{\left(\tfrac{21}{25}\right)^4}_{=\frac{194{,}481}{390{,}625}}\cdot\underbrace{\left(\tfrac{7}{20}\right)^2}_{=\frac{49}{400}}\cdot\underbrace{\left(\tfrac{37}{25}\right)^2}_{=\frac{1369}{625}} = \frac{13{,}045{,}979{,}961}{97{,}656{,}250{,}000} \approx 0.1335908348,$$

$$\Big[P_c + A^2(1-\bar\rho)X_t\Big]^2 = \Big[\tfrac{2}{5} + \tfrac{441}{625}\cdot\tfrac{17}{20}\cdot\tfrac{37}{25}\Big]^2 = \Big(\frac{402{,}389}{312{,}500}\Big)^2 \approx 1.658029131.$$

Hence the rare-kaon predictions condense to the one-liners

$$\boxed{\mathrm{BR}(K_L\to\pi^0\nu\bar\nu) = \kappa_L\times 0.1335908348\ , \qquad \mathrm{BR}(K^+\to\pi^+\nu\bar\nu) = \kappa_+\times(0.1335908348 + 1.658029131)}\,.$$

If you insert the standard κ normalizations used in the literature, the numbers land in the usual SM ballpark (order 10^{-11}), but now they are driven by tiny exact fractions with a two-parameter short-distance front end (X_t, P_c).

3. Leptonic B decays $B_{s,d}\to\mu^+\mu^-$

The absolute branching fractions depend on decay constants and Wilson coefficients, but the *ratio* is almost purely CKM:

$$\frac{\mathrm{BR}(B_d\to\mu^+\mu^-)}{\mathrm{BR}(B_s\to\mu^+\mu^-)} = \frac{\tau_{B_d}}{\tau_{B_s}}\frac{m_{B_d}}{m_{B_s}}\frac{f_{B_d}^2}{f_{B_s}^2}\times\frac{|V_{td}|^2}{|V_{ts}|^2}.$$

Your fractions give the CKM factor in closed form:

$$\frac{|V_{td}|^2}{|V_{ts}|^2} = \lambda^2\big[(1-\bar\rho)^2 + \bar\eta^2\big] = \Big(\frac{2}{9}\Big)^2\Big[\Big(\frac{17}{20}\Big)^2 + \Big(\frac{7}{20}\Big)^2\Big] = \frac{169}{4050} \approx 0.0417283950.$$

Taking representative lattice/PDG ratios $\tau_{B_d}/\tau_{B_s} \approx 0.99$, $m_{B_d}/m_{B_s} \approx 0.983$, $f_{B_s}/f_{B_d} \approx 1.206$ gives

$$\frac{\mathrm{BR}(B_d\to\mu^+\mu^-)}{\mathrm{BR}(B_s\to\mu^+\mu^-)} \approx 0.99\times 0.983\times\Big(\frac{1}{1.206}\Big)^2\times\frac{169}{4050} \ \approx\ 2.79\times 10^{-2}.$$

That is a one-equation, one-digit falsification target: the hadronic prefactor can (and does) sharpen, but the CKM core is exactly 169/4050.

4. Audit table (everything in one glance)

Object	Exact closed form	Numeric
$\dfrac{\mathrm{Im}\,\lambda_t}{\lambda^5}$	$A^2\bar{\eta} = \dfrac{3087}{12500}$	0.24696
$\dfrac{\mathrm{Re}\,\lambda_t}{\lambda^5}$	$-A^2(1-\bar{\rho}) = -\dfrac{7497}{12500}$	−0.59976
X_t (benchmark)	$\dfrac{37}{25}$	1.48
P_c (benchmark)	$\dfrac{2}{5}$	0.40
Core for K_L	$A^4\bar{\eta}^2X_t^2$	0.1335908348
Core add-on for K^+	$\big[P_c + A^2(1-\bar{\rho})X_t\big]^2$	1.658029131
$\lvert V_{td}\rvert^2/\lvert V_{ts}\rvert^2$	$\dfrac{169}{4050}$	0.0417283950

5. What to test (clean falsifiers)

Fix (X_t, P_c) to the latest NNLO determinations and (κ_+, κ_L) to the standard chiral/phase-space normalizations. Then the kaon modes above are *parameter-free* consequences of the tiny CKM fractions. Likewise, the B-leptonic ratio is pushed almost entirely by the exact CKM factor 169/4050 with a well-measured lattice prefactor. If future fits drift decisively away, the ledger breaks. If data keep hugging these forms as errors shrink, the "small exact fractions" hypothesis gains force.

6. Notes

I kept the structure deliberately tight: no baroque numerology, just four tiny CKM rationals and two short-distance numbers (X_t, P_c) written as simple fractions for human audit. Replacing the rational benchmarks with your preferred NNLO values is one line; the algebraic scaffolding does not change.