Blackhole Ledger: Budgets for Area, Bits, and Energy Flows

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Purpose. A compact, event-ready accounting system for black-hole processes in the *bits* language. Each section is a plug-and-play worksheet: drop in masses, spins, or powers and read off bounds and balances.

1 Normalization and one-bit units

We keep G, $\hbar$, $k_{\rm B}$, c explicit. Entropy and bit count are

$$S = \frac{\mathcal{A}}{4\,\ell_{\rm P}^2}\,, \qquad S_{\rm bits} = \frac{S}{\ln 2} = \frac{\mathcal{A}}{4\,\ell_{\rm P}^2\,\ln 2}\,. \tag{1}$$

One-bit area and energy costs:

$$\Delta\mathcal{A}_{\rm 1bit} = 4\,\ell_{\rm P}^2\,\ln 2\,, \qquad \Delta E_{\rm 1bit} = k_{\rm B}\;\ln 2\,. \tag{2}$$

For Kerr–Newman, the differential first law

$$dM = \frac{\kappa_{\rm H}}{8\pi}\,d\mathcal{A} + \Omega_{\rm H}\,dJ + \Phi_{\rm H}\,dQ \tag{3}$$

implies the one-bit cost at fixed (J,Q) is universal: $(\Delta M)_{J,Q} = \ln 2$.

2 Merger ledger (area theorem prior and ε bound)

For two holes (M_1, a_{*1}), (M_2, a_{*2}) merging to (M_f, a_{*f}) with radiated fraction ε, the Kerr area is

$$\mathcal{A}(M, a_*) = 8\pi M^2\Big(1+\sqrt{1-a_*^2}\Big), \tag{4}$$

so the **area theorem** requires $\mathcal{A}(M_f, a_{*f}) \geq \mathcal{A}(M_1, a_{*1}) + \mathcal{A}(M_2, a_{*2})$. With $M_f = (1-\varepsilon)(M_1 + M_2)$ this yields

$$\varepsilon \;\leq\; 1 - \sqrt{\frac{\sum_{i=1}^{2} M_i^2\Big(1+\sqrt{1-a_{*i}^2}\Big)}{(M_1+M_2)^2\,\Big(1+\sqrt{1-a_{*f}^2}\Big)}}\;. \tag{5}$$

Bits budget:

$$\Delta S_{\rm bits} = \frac{\mathcal{A}(M_f, a_{*f}) - \mathcal{A}(M_1, a_{*1}) - \mathcal{A}(M_2, a_{*2})}{4\,\ell_{\rm P}^2\,\ln 2} \;\geq\; 0\,. \tag{6}$$

Equal-mass, nonspinning gives $\varepsilon \leq 1 - 1/\sqrt{2} \approx 0.293$ and $\Delta S_{\rm bits} \to 0^+$ at saturation.

3 Accretion ledger (bits per baryon)

Let a hole of mass M accrete rest mass δm from a disk of efficiency η (radiation to infinity). The horizon mass increase is $\delta M = (1-\eta)\,\delta m$. For Schwarzschild,

$$\mathcal{A} = 16\pi G^2 M^2/c^4 \quad\Rightarrow\quad \delta\mathcal{A} = 32\pi G^2 M\,\delta M/c^4\,. \tag{7}$$

The **bits gained** per accreted mass is

$$\frac{\delta N_{\rm bits}}{\delta m} = \frac{\delta\mathcal{A}}{4\,\ell_{\rm P}^2\,\ln 2\,\delta m} = \frac{8\pi G M}{\hbar c\,\ln 2}\,(1-\eta)\,. \tag{8}$$

At fixed η, accretion onto more massive holes deposits more bits per kilogram. At the Eddington rate $\dot{m}_{\rm Edd}$ this defines a *bit influx* $\dot{N}_{\rm bits}$.

4 Jet ledger (bit–joule bound)

Given a jet/mechanical power $P_{\rm jet}$ and an efficiency factor $\epsilon_{\rm gb} \in (0,1)$ linking horizon thermodynamics to asymptotic power, the maximum horizon-mediated bit rate is

$$\dot{N_{\rm bits max}} = \frac{\epsilon_{\rm gb}\, P_{\rm jet}}{k_{\rm B}\ \ln 2}\,. \tag{9}$$

Usage: insert bands for $P_{\rm jet}$ (e.g., M87*, Sgr A*) and an $\epsilon_{\rm gb}$ prior to draw a forbidden region for claims that exceed the bound.

5 Penrose/superradiance ledger (discounted μ_N)

Define the bit "chemical potential" $\mu_N \equiv \kappa_{\rm H}\ \ln 2/(2\pi)$ so that

$$dM = \mu_N\, dN + \Omega_{\rm H}\, dJ + \Phi_{\rm H}\, dQ\,. \tag{10}$$

For reversible Penrose-like extraction ($d\mathcal{A} \geq 0$ with $dJ < 0$), the extracted energy per added bit is *discounted* near extremality where $\kappa_{\rm H} \to 0$. A track a_*: $a_{*i} \to a_{*f}$ yields the ledger

$$\Delta E_{\rm ext} = \int (\Omega_{\rm H}\, {\rm d}J + \Phi_{\rm H}\, {\rm d}Q) - \int \mu_N\, {\rm d}N\,, \qquad \Delta N \geq 0\,. \tag{11}$$

6 Evaporation ledger (Hawking outflows)

For an isolated Schwarzschild hole, $= \hbar c^3/(8\pi k_{\rm B} G M)$ and the luminosity scales $P \propto 1/M^2$. The **bit emission rate** obeys

$$\dot{N_{\rm bits}} \lesssim \frac{P}{k_{\rm B}\ \ln 2} \ \propto\ \frac{1/M^2}{(1/M)} \ \propto\ \frac{1}{M}\,. \tag{12}$$

Thus stellar and supermassive holes emit negligible bits per second; the ledger is dominated by mergers and accretion for any realistic timeframe.

7 Worked anchors (plug-and-play)

For quick use, the table shows scalings and example anchors; substitute your system and scale accordingly.

Process	Core relation	Scaling in M	Example anchor
Merger bound	$\varepsilon \leq 1 - \sqrt{\frac{\sum M_i^2(1+\sqrt{1-a_{*i}^2})}{(\sum M_i)^2(1+\sqrt{1-a_{*f}^2})}}$	–	eq.-mass, $a_* = 0$: $\varepsilon \lesssim 0.293$
Accretion bits	$\delta N/\delta m = \frac{8\pi G M}{\hbar c\, \ln 2}(1-\eta)$	$\propto M$	$\eta = 0.1$: $\delta N/\delta m \approx \frac{8\pi G M}{\hbar c\, \ln 2} \times 0.9$
Jet bit rate	$\dot{N}_{\max} = \frac{\epsilon_{\rm gb} P_{\rm jet}}{k_{\rm B} \ln 2}$	$\propto M$ (via $1/$)	Sgr A*: tiny $\Rightarrow$ huge formal $\dot{N}_{\max}$
Penrose track	$dM = \mu_N dN + \Omega_{\rm H} dJ$	$\mu_N \downarrow$ as $a_* \to 1$	Energy/bit is discounted near extremal
Evaporation	$\dot{N} \lesssim P/(k_{\rm B} \ln 2)$	$\propto 1/M$	negligible for astrophysical BHs

8 How to use this ledger with data

1. **GW events:** draw posterior samples for $(M_1, a_{*1}), (M_2, a_{*2}), \varepsilon, a_{*f}$; reject any sample violating the area theorem; report the induced prior on ε and the distribution of $\Delta S_{\rm bits}$.
2. **AGN/XRB accretion:** adopt an efficiency prior η and an accretion rate; compute $\dot{N}$ and compare to jet $\dot{N}_{\max}$ to form a closed bit budget.
3. **Analogue horizons:** set $^{\rm eff}$ from the platform; test $\Delta E_{1\rm bit} = k_{\rm B}^{\rm eff} \ln 2$.

Constants (for convenience)

$m_{\rm P} = \sqrt{\hbar c/G}$, $\ell_{\rm P} = \sqrt{\hbar G/c^3}$, $\Delta(M^2) = m_{\rm P}^2 \frac{\ln 2}{4\pi}$, $\Delta\mathcal{A}_{1\rm bit} = 4\ell_{\rm P}^2 \ln 2$.