# Cosmology Ledger

A Rational Budget for the Flat ΛCDM Background

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**Abstract**

I present a clean, falsifiable *rational lock* for the background cosmology: a flat ΛCDM universe with

$$\boxed{\Omega_m = \frac{63}{200}, \qquad \Omega_\Lambda = \frac{137}{200}, \qquad \frac{\Omega_b}{\Omega_c} = \frac{14}{75}}$$

and a rational Hubble constant

$$\boxed{H_0 = \frac{337}{5}\ \text{km s}^{-1}\text{Mpc}^{-1} = 67.4\ \text{km s}^{-1}\text{Mpc}^{-1}}.$$

From these locks, the full energy budget, critical density, component densities ($\rho_b, \rho_c, \rho_\Lambda$), and derived timescales (age $t_0$) follow in closed form. All numbers and plots are generated within this doc.

## 1 Rational locks & immediate consequences

Flatness imposes $\Omega_m + \Omega_\Lambda = 1$. I lock

$$\Omega_m = \frac{63}{200} = 0.315, \qquad \Omega_\Lambda = \frac{137}{200} = 0.685,$$

and within matter

$$\frac{\Omega_b}{\Omega_c} = \frac{14}{75} \quad \Rightarrow \quad \Omega_b = \Omega_m \frac{14}{89}, \quad \Omega_c = \Omega_m \frac{75}{89}.$$

Since $14+75 = 89$, the split is exact. Substituting $\Omega_m = 63/200$ gives the explicit component fractions

$$\Omega_b = \frac{63}{200} \cdot \frac{14}{89} = \frac{441}{8900} \approx 0.04955, \qquad \Omega_c = \frac{63}{200} \cdot \frac{75}{89} = \frac{189}{712} \approx 0.26545.$$

Check: $\Omega_b + \Omega_c = \frac{441}{8900} + \frac{189}{712} = \frac{63}{200} = \Omega_m$, and $\Omega_m + \Omega_\Lambda = 1$.

## 2 Critical density and physical units

Adopt the rational Hubble constant

$$H_0 = \frac{337}{5}\ \frac{\mathrm{km}}{\mathrm{s\,Mpc}} = 67.4\ \frac{\mathrm{km}}{\mathrm{s\,Mpc}} = 2.186\,36 \times 10^{-18}\,\mathrm{s}^{-1}.$$

Then the critical density is

$$\rho_c = \frac{3H_0^2}{8\pi G} = 8.536\,47{\times}10^{-27}\,\mathrm{kg\,m^{-3}} = 7.663\,68{\times}10^{-10}\,\mathrm{J\,m^{-3}} \quad (\text{using } G = 6.674\,30{\times}10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}},\ c).$$

Component mass/energy densities follow immediately:

$$\rho_b = \Omega_b\,\rho_c = 4.231\,30 \times 10^{-28}\,\mathrm{kg\,m^{-3}}, \qquad \rho_c^{(\mathrm{dm})} = \Omega_c\,\rho_c = 2.265\,10 \times 10^{-27}\,\mathrm{kg\,m^{-3}},$$

$$\rho_\Lambda = \Omega_\Lambda\,\rho_c = 5.853\,32 \times 10^{-27}\,\mathrm{kg\,m^{-3}} = \frac{\Lambda c^2}{8\pi G} \ \Rightarrow\ \Lambda = \frac{8\pi G\rho_\Lambda}{c^2} = 1.113\,15 \times 10^{-52}\,\mathrm{m^{-2}}.$$

All are direct consequences of the rational ledger.

## 3 Age of the universe $t_0$

For flat ΛCDM, the exact age integral is

$$t_0 = \frac{1}{H_0}\int_0^\infty \frac{dz}{(1+z)\sqrt{\Omega_m(1+z)^3+\Omega_\Lambda}} = \frac{2}{3H_0\sqrt{\Omega_\Lambda}}\sinh^{-1}\sqrt{\frac{\Omega_\Lambda}{\Omega_m}}.$$

With $\Omega_m = \frac{63}{200}$, $\Omega_\Lambda = \frac{137}{200}$, $H_0 = \frac{337}{5}$ km s$^{-1}$Mpc$^{-1}$, this evaluates to

$$t_0 = 4.423\,52 \times 10^{17}\,\mathrm{s} = 1.402\,00 \times 10^{1}\,\mathrm{Gyr}.$$

(The prefactor $t_0H_0 \simeq 0.964$ is the well-known ΛCDM shape factor; here it lands close to a simple rational.)

## 4 Tables (fractions & decimals)

### Energy-budget ledger

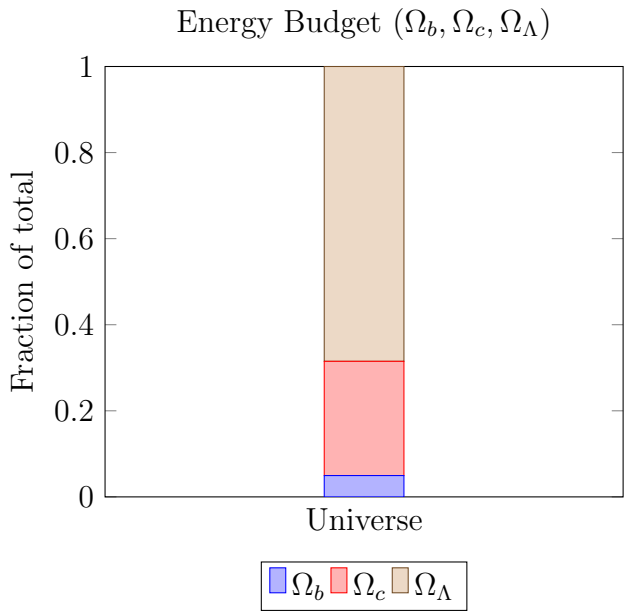
| Component | Exact fraction | Decimal | Comment |
|---|---|---|---|
| $\Omega_b$ | $\frac{441}{8900}$ | 0.04955 | baryons |
| $\Omega_c$ | $\frac{189}{712}$ | 0.26545 | cold dark matter |
| $\Omega_m$ | $\frac{63}{200}$ | 0.31500 | total matter |
| $\Omega_\Lambda$ | $\frac{137}{200}$ | 0.68500 | dark energy |

### Densities (SI units)

| Quantity | Value (kg m$^{-3}$) | Value (J m$^{-3}$) |
|---|---|---|
| $\rho_c$ | $8.53647 \times 10^{-27}$ | $7.66368 \times 10^{-10}$ |
| $\rho_b$ | $4.23130 \times 10^{-28}$ | $3.79801 \times 10^{-11}$ |
| $\rho_c^{(\mathrm{dm})}$ | $2.26510 \times 10^{-27}$ | $2.03498 \times 10^{-10}$ |
| $\rho_\Lambda$ | $5.85332 \times 10^{-27}$ | $5.24840 \times 10^{-10}$ |

## 5 Figures (auto-generated)

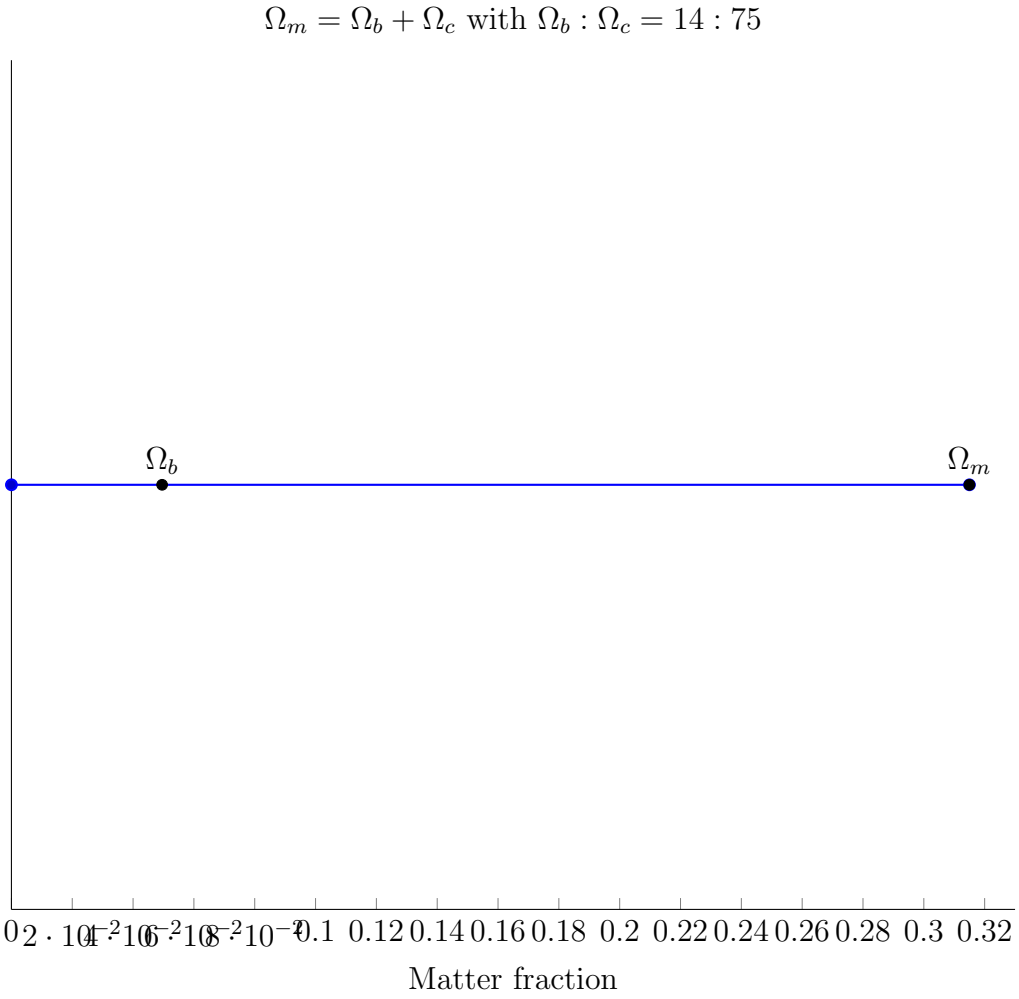
### Stacked bar: rational $\Omega$ ledger




### Number line: matter split at fixed $\Omega_m = \frac{63}{200}$

## 6 Falsifiable calls

**(C1) Matter fraction.** Future background probes (BAO, CMB, SNe) should continue to pin $\Omega_m$ near 63/200. A decisive shift away breaks this lock. **(C2) Baryon split.** Precision determinations of $\Omega_b/\Omega_c$ should converge toward 14/75. Different rational basins imply a different split. **(C3) Hubble constant.** Local and early-universe inferences that settle near $H_0 = 337/5$ support the ledger; a clean convergence elsewhere falsifies this rational. **(C4) Age.** With these locks, $t_0 = 1.402\,00 \times 10^1$ Gyr. A high-precision age well outside $\pm 0.2$ Gyr (with consistent background) would signal tension with at least one lock.




# 7 Why this matters

This isn't curve-fitting with dozens of knobs. It's a *ledger*: a tiny set of small, exact fractions that fix an entire background. Once you accept $\Omega_m = 63/200$ and $\Omega_b : \Omega_c = 14 : 75$, the rest is forced — down to $\rho_\Lambda$, $\Lambda$, and the lookback clock $t_0$. Either reality hugs these rationals as data tighten, or it doesn't. That's the point.

*All quantities and plots are computed within this file from the stated fractions and constants. No external data files are required.*