Cosmic Bit Balance Sheet

Normalizing the Universe's Entropy to Bits (S_{bits})

Evan Wesley, Rosetta Stone of Physics — Reality Encoded (build: September 2, 2025)

Abstract. We present a self-contained, executable-style ledger of cosmic entropy cast in *bits*. Every sector—black holes, radiation (CMB photons, cosmic neutrinos), baryonic gas (IGM/ICM), compact objects, and Hawking channels—has (i) a single-line formula, (ii) a normalization to bits, and (iii) a plug-in worksheet to evaluate *per cubic Mpc* and *per Hubble volume*. The balance sheet is designed to be falsifiable, updatable, and useful: it exposes which observables dominate the budget and where better data matter.

1 Normalization and cosmology anchors

We keep $G, \hbar, k_{\mathrm{B}}, c$ explicit. Bit units: $S_{\text{bits}} \equiv S/\ln 2$. Our fiducial cosmology is the Rosetta ledger (flat ΛCDM):

$$\Omega_m = \tfrac{63}{200} = 0.315, \quad \Omega_\Lambda = \tfrac{137}{200} = 0.685, \quad \frac{\Omega_b}{\Omega_c} = \tfrac{14}{75}.$$

We adopt the rational Hubble constant $H_0 = \frac{337}{5}\,\mathrm{km\,s^{-1}\,Mpc^{-1}} = 67.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (convert with $1\,\mathrm{Mpc} = 3.085\,677\,581\,491\,367\,3 \times 10^{22}\,\mathrm{m}$). From this:

$$H_0 = \frac{337}{5}\,\frac{10^3}{3.085\,677\,581\,491\,367\,3 \times 10^{22}}\,\mathrm{s^{-1}} = 2.184\,285\,241\,085\,5 \times 10^{-18}\,\mathrm{s^{-1}}, \tag{1}$$

$$\rho_c = \frac{3H_0^2}{8\pi G} = 8.532\,855\,163\,739\,3 \times 10^{-27}\,\mathrm{kg\,m^{-3}}, \tag{2}$$

$$\Omega_c = \Omega_m \frac{75}{89} = 0.265\,45, \qquad \Omega_b = \Omega_m - \Omega_c = 0.049\,55. \tag{3}$$

The Hubble radius and volume are

$$R_H = \frac{c}{H_0} = 4447.959\,\mathrm{Mpc}, \qquad V_H = \frac{4\pi}{3} R_H^3 = 3.686\,133\,233\,074\,354 \times 10^{11}\,\mathrm{Mpc}^3. \tag{4}$$

All sector totals are reported both per Mpc^3 and scaled to V_H.

2 Radiation sectors: CMB photons and cosmic neutrinos

For a relativistic species with temperature T and internal degrees g, the entropy density is

$$s = \frac{2\pi^2}{45}\, g\, k_{\mathrm{B}} \left(\frac{k_{\mathrm{B}} T}{\hbar c}\right)^3, \qquad s_{\text{bits}} = \frac{s}{k_{\mathrm{B}} \ln 2}. \tag{5}$$

Photons (CMB). With $g_\gamma = 2$ and $T_\gamma = 2.7255\,\mathrm{K}$:

$$s_{\gamma,\text{bits}} = 2.134\,123\,646 \times 10^9\,\mathrm{bits\,m^{-3}} = 6.270\,053\,023 \times 10^{76}\,\mathrm{bits\,Mpc^{-3}}. \tag{6}$$

Neutrinos (CNB). Treating three flavors as effectively relativistic with $g_\nu = (7/8) \times 6 = 21/4$ and $T_\nu = (4/11)^{1/3} T_\gamma$:

$$s_{\nu,\text{bits}} = 2.037\,118\,026 \times 10^9\,\mathrm{bits\,m^{-3}} = 5.985\,050\,613 \times 10^{76}\,\mathrm{bits\,Mpc^{-3}}. \tag{7}$$

Radiation total. $s_{\text{rad, bits}} = 1.225\,510\,364 \times 10^{77}$ bits Mpc^{-3}, which integrates over the Hubble volume to

$$S_{\text{rad, bits}}(V_H) = 4.517\,394\,479 \times 10^{88}\ \text{bits}. \tag{8}$$

Note. If massive neutrinos are partially nonrelativistic today, the exact s_ν decreases modestly; this ledger keeps the clean relativistic form and flags a neutrino-mass correction as an optional refinement (Appendix B).

3 Baryonic gas sectors (IGM/ICM): Sackur–Tetrode worksheet

For a monatomic ideal gas the entropy density follows Sackur–Tetrode. For a fully ionized primordial plasma with mean molecular weight $\mu \simeq 0.59$ and baryon density $\rho_b = \Omega_b \rho_c$, the particle density is $n = \rho_b/(\mu m_p)$. The entropy density is

$$s \approx n\,k_\text{B} \left[\ln\!\left(\frac{5/2}{n} \Big(\frac{2\pi m k_\text{B} T}{h^2}\Big)^{3/2} \right) \right], \qquad s_\text{bits} = s/(k_\text{B} \ln 2), \tag{9}$$

with $m \approx \mu m_p$. **Worked anchor (cosmic mean, $T = 2 \times 10^4$ K):**

$$n \approx 4.284\,431\,071 \times 10^{-1}\ \text{m}^{-3}, \quad s/(nk_\text{B}) \approx 77.9204, \quad s_\text{bits} \approx 1.415\,045\,404 \times 10^{69}\ \text{bits Mpc}^{-3}. \tag{10}$$

This sits *eight orders of magnitude* below the radiation bit density; clusters at higher T move the number but not the hierarchy.

4 Black holes: area → bits, per-object and populations

For a hole of mass M,

$$S_\text{bits}(M) = \frac{\mathcal{A}}{4\ell_\text{P}^2 \ln 2} = \frac{4\pi G}{\hbar c \ln 2} M^2 \equiv K\,M^2, \qquad K = \frac{4\pi G}{\hbar c \ln 2}. \tag{11}$$

Per-object anchors. $S_\text{bits}(1\,M_\odot) = 1.513\,322\,012 \times 10^{77}$, $S_\text{bits}(10\,M_\odot) = 1.513\,322\,012 \times 10^{79}$, $S_\text{bits}(4 \times 10^6\,M_\odot) = 2.421\,315\,220 \times 10^{90}$.

Population ledger. For a mass function $\mathrm{d}n/\mathrm{d}M$ (objects per volume per mass), the bit density is

$$s_{\text{BH, bits}} = K \int M^2 \frac{\mathrm{d}n}{\mathrm{d}M}\,\mathrm{d}M \ = \ K\,n_\text{BH} \frac{\langle M^2 \rangle}{}, \tag{12}$$

where n_BH is the number density. Equivalently, with a
emphmass density $\rho_\text{BH} = \int M\,\mathrm{d}n$ and an *effective mass* $M_\text{eff} \equiv \langle M^2\rangle/\langle M\rangle$, one may write

$$s_{\text{BH, bits}} = K\,\rho_\text{BH}\,M_\text{eff}. \tag{13}$$

Because of the M^2 scaling, entropy is maximized by concentrating fixed mass into the most massive holes: supermassive BHs dominate the cosmic bit budget for any reasonable ρ_BH.

Dominance condition (back-of-envelope). Radiation bits per volume are $s_{\text{rad, bits}} \approx 4.171\,241\,672 \times 10^9$ bits m^{-3}. Black holes dominate when

$$K\,\rho_\text{BH}\,M_\text{eff} \ \gtrsim \ s_{\text{rad, bits}}. \tag{14}$$

For a tiny mass fraction $f_\text{BH} \equiv \rho_\text{BH}/\rho_c = 10^{-5}$ the threshold is $M_\text{eff} \gtrsim$ kg ($\sim 6.4 \times 10^{-7}\,M_\odot$)—far *below* typical astrophysical BH masses. Thus even a minute cosmic BH mass density yields a BH entropy that dwarfs the CMB/CNB.

Illustrative population (*example numbers*). If one adopts a fiducial $\rho_{\rm BH} = 5{\times}10^5\, M_\odot\,{\rm Mpc}^{-3}$ concentrated at $M_{\rm eff} = 10^8\, M_\odot$, then

$$s_{\rm BH,\,bits} \approx 7.566\,610\,062 \times 10^{90}\,{\rm bits\,Mpc^{-3}} \quad \Rightarrow \quad S_{\rm BH,\,bits}(V_H) \approx \text{ bits.} \tag{15}$$

Caveat: the inputs are placeholders for illustration; plug your preferred mass function or mass-density measurement into the worksheet below.

5 Compact objects (stars, WDs, NSs): where they sit

Stellar interiors have large *specific* entropies but a tiny cosmic mass fraction and do not compete with BHs or radiation totals. **White dwarfs** and **neutron stars** have lower specific entropies (degenerate equations of state); despite large number counts, their contribution remains subdominant to radiation and negligible vs BHs in any realistic census. A worksheet for degenerate equations of state (polytropic approximations) is provided in Appendix C for completeness.

6 Why a bit ledger? (what this buys you)

- **GSL as a budget:** Cast the Generalized Second Law as $\Delta(S_{\rm out} + S_{\rm BH}) \geq 0$ in bits; mergers/accretion/radiation become entry-by-entry transactions.
- **Decision power:** The ledger shows *which measurements matter*. Improving CMB T shifts $\sim 10^{88}$ bits; tuning an SMBH mass function shifts $\sim 10^{100}$–10^{103} bits.
- **Rosetta unification:** Ties the (Howtotransduceme file) EKTL (energy $\to$ knowledge) to the cosmic budget; bit–joule bounds at horizons become global constraints when integrated over populations.
- **Communication:** One unit (bits) across quantum, thermo, and gravity lowers the barrier for new collaborators.

7 How to use this sheet (plug-and-play)

1. **Pick a volume:** per Mpc^3 for density-style arguments or V_H for a Hubble-sphere snapshot.
2. **Radiation:** use the closed forms above for $s_{\gamma,\rm bits}$ and $s_{\nu,\rm bits}$ (or rerun with your T_γ and $N_{\rm eff}$).
3. **IGM/ICM:** choose T and overdensity Δ; scale $n \to \Delta n$ in Sackur–Tetrode; add cluster components if desired.
4. **Black holes:** provide either ${\rm d}n/{\rm d}M$ and integrate $K \int M^2 {\rm d}n$ or specify $(\rho_{\rm BH}, M_{\rm eff})$ and multiply.
5. **Report:** write totals in bits; show a bar chart and a sensitivity table (which parameter moves the most bits?).

Summary tables

Sector	Formula (bits density)	Value / Mpc^3	Notes
CMB photons	$\frac{2\pi^2}{45}\,\frac{2}{\ln 2}(\frac{k_\mathrm{B}T_\gamma}{\hbar c})^3$	$6.270\,053\,023 \times 10^{76}$	exact at $T_\gamma = 2.7255$
CNB neutrinos	$\frac{2\pi^2}{45}\,\frac{(7/8)6}{\ln 2}(\frac{k_\mathrm{B}T_\nu}{\hbar c})^3$	$5.985\,050\,613 \times 10^{76}$	$T_\nu = (4/11)^{1/3}T_\gamma$
IGM (mean, 2×10^4 K)	ST formula	$1.415\,045\,404 \times 10^{69}$	depends on T, overdensity
BHs (illustrative)	$K\,\rho_\mathrm{BH}M_\mathrm{eff}$	$7.566\,610\,062 \times 10^{90}$	choose $\rho_\mathrm{BH}, M_\mathrm{eff}$

A Constants and conversions

$k_\mathrm{B} = 1.380\,649 \times 10^{-23}\,\mathrm{J\,K^{-1}}$, $\hbar = 1.054\,571\,817 \times 10^{-34}\,\mathrm{J\,s}$, $c = 2.997\,924\,58 \times 10^{8}\,\mathrm{m\,s^{-1}}$, $G = 6.674\,30 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$, $= 3.085\,677\,581\,491\,367\,3 \times 10^{22}\,\mathrm{m}$, $M_\odot = 1.988\,47 \times 10^{30}\,\mathrm{kg}$. One bit equals $k_\mathrm{B}\ln 2$ of thermodynamic entropy.

B Neutrino refinement (optional)

If light neutrinos are partially nonrelativistic today, replace the relativistic s_ν with the exact Fermi–Dirac expression at finite mass. The effect is a mild suppression relative to the relativistic value and can be bounded with your favorite $\sum m_\nu$ prior.

C Compact-object entropy (WDs and NSs)

For degenerate objects, use $s \sim \pi^2 n k_B (k_B T/\epsilon_F)$ for a Fermi gas (electrons in WDs, neutrons in NSs). Even at stellar-core temperatures the small T/ϵ_F keeps s subdominant cosmologically.

D BH population worksheet (mass function route)

Provide $\mathrm{d}n/\mathrm{d}M$ for SMBHs (e.g., a Schechter or lognormal fit) and evaluate $K\int M^2\mathrm{d}n$. Report both $s_{\mathrm{BH,\,bits}}$ and $S_{\mathrm{BH,\,bits}}(V_H)$.

E EKTL link (energy $\rightarrow$ knowledge)

At any horizon, the maximum bit throughput obeys $\dot{N_\mathrm{bits}} \le P/(k_\mathrm{B}\ln 2)$ (greybody factors aside). Integrated over cosmic populations, this provides a global constraint on energy-to-information transduction.