Rosetta Stone Physics — Volume 1

The Smart–Idiot Guide to Doing Real Physics with Fractions

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Abstract

This booklet teaches a practical way to do real physics with basic fractions. Pick a single ruler (v), keep a small table of exact ratios (no messy units), and push everything through with ordinary arithmetic. We show the method and work through concrete examples: weak masses, a collider decay ($H \to \tau\tau$), a quick sanity pass on $H \to b\bar{b}$, the hydrogen ground state, the custodial test, and Koide's relation. No code. Just fractions, calculators, and verifiable numbers.

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How to read this

You need only three ideas. First, choose $v = 246.219\,652\,\text{GeV}$ as the ruler. Second, store physical inputs as exact ratios relative to v or as pure dimensionless numbers. Third, compute everything with those ratios; convert to GeV or eV at the very end. That is all.

1 The Registry (exact ratios at $\mu_0 = M_Z$)

This table is the Rosetta key. Each entry is an exact fraction p/q; the decimal is just for orientation.

1.1 Couplings and electroweak mass ratios

Symbol	Definition	Exact p/q	Approx
α_{em}	electromagnetic at M_Z	2639/361638	0.00729735
$\alpha_s(M_Z)$	strong at M_Z	9953/84419	0.11790
$\sin^2\theta_W$	weak mixing	7852/33959	0.23122
M_W/v	boson ratio	17807/54547	0.32645
M_Z/v	boson ratio	18749/50625	0.37035
M_H/v	boson ratio	22034/43315	0.50808

Table 1: Dimensionless seeds at the reference scale $\mu_0 = M_Z$.

1.2 Fermion mass ratios m_f/v

Field	Definition	Exact p/q	Approx
e	m_e/v	43/20719113	2.075×10^{-6}
μ	m_μ/v	421/981072	4.291×10^{-4}
τ	m_τ/v	2561/354878	1.040×10^{-2}
u	m_u/v	83/9461218	8.775×10^{-6}
d	m_d/v	111/5852330	1.897×10^{-5}
s	m_s/v	411/1088132	3.776×10^{-4}
c	m_c/v	1687/327065	5.158×10^{-3}
b	m_b/v	3268/192499	1.698×10^{-2}
t	m_t/v	24087/34343	7.015×10^{-1}

Table 2: Store masses as m_f/v. Multiply by v at the end if you need GeV.

1.3 CKM seed sines and phase over π

Symbol	Definition	Exact p/q	Approx
s_{12}	$\sin\theta_{12}$	13482/60107	0.2243
s_{23}	$\sin\theta_{23}$	6419/152109	0.0422
s_{13}	$\sin\theta_{13}$	1913/485533	0.00394
δ/π	CP phase ratio	6869/17983	0.3818

Table 3: Flavor seeds as bare dimensionless numbers.

Two handy fingerprints

$$\frac{M_W}{M_Z} = \frac{17807/54547}{18749/50625}, \qquad \frac{m_\tau}{m_\mu} = \frac{2561/354878}{421/981072}.$$

We'll use these in quick checks later.

2 The Translator: how to compute with fractions

Everything flows from a tiny set of templates. Keep them in mind, and you can do most "expert" calculations with a four-function calculator.

Mass from ratio. If $m_f/v = p/q$, then $m_f = (p/q)\,v$. That's it.

Yukawa from mass. $y_f = \sqrt{2}\,m_f/v$. If you have m_f/v, just multiply by $\sqrt{2}$.

Two-body scalar decay to a Dirac fermion.

$$\Gamma(H \to f\bar{f}) = N_c\,\frac{M_H}{8\pi}\left(\frac{m_f}{v}\right)^2\left(1-\frac{4m_f^2}{M_H^2}\right)^{3/2},$$

with $N_c = 1$ for leptons, 3 for quarks. The β^3 factor is pure kinematics.

Branching ratio. $\mathrm{BR} = \Gamma/\Gamma_{\mathrm{tot}}$. When $M_H \approx 125.25\,\mathrm{GeV}$, a good reference total width is $\Gamma_{\mathrm{tot}} \approx 4.07\,\mathrm{MeV}$.

Hydrogen ground state in natural units.

$$E_1 = -\frac{\alpha_{\mathrm{em}}^2}{2}\,m_e, \qquad \text{so in eV: } E_1[\mathrm{eV}] = -\frac{\alpha_{\mathrm{em}}^2}{2}\,m_e[\mathrm{GeV}] \times 10^9.$$

Custodial check.

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 \quad \Rightarrow \quad \left(\frac{M_W}{M_Z}\right)^2 \stackrel{?}{=} 1 - \sin^2\theta_W.$$

3 Worked examples (start to finish, no code)

All numbers below are computed *only* from the registry plus $v = 246.219\,652\,\mathrm{GeV}$.

3.1 Electroweak anchor

$M_W/v = 17807/54547$ and $M_W = 80.379\,\mathrm{GeV} \Rightarrow\ v = M_W/(M_W/v) = 246.219\,652\,\mathrm{GeV}$. Then $M_Z = (18749/50625)\,v = 91.1876\,\mathrm{GeV}$ and $M_H = (22034/43315)\,v = 125.2500\,\mathrm{GeV}$.

3.2 $H \to \tau^+\tau^-$ from pure ratios

Use $m_\tau/v = 2561/354878$ and $M_H/v = 22034/43315$.

$$m_\tau = \frac{2561}{354878}\,v = 1.776\,860\,\mathrm{GeV}, \quad M_H = \frac{22034}{43315}\,v = 125.250\,001\,\mathrm{GeV}.$$

Kinematics $\beta = \sqrt{1 - 4m_\tau^2/M_H^2} = 0.999597405$. Width

$$\Gamma_{\tau\tau} = \frac{M_H}{8\pi}\left(\frac{m_\tau}{v}\right)^2\beta^3 = 0.259\,223\,\mathrm{MeV}.$$

Branching ratio with $\Gamma_{\mathrm{tot}} = 4.07\,\mathrm{MeV}$:

$$\mathrm{BR}(H \to \tau\tau) = \frac{0.259223}{4.07} = 0.06369 = 6.369\%.$$

That is a collider observable recovered directly from the fraction table.

3.3 $H \to b\bar{b}$: a sanity check that teaches a lesson

Take $m_b/v = 3268/192499 \Rightarrow m_b = 4.1800\,\text{GeV}$. Naively at leading order,

$$\Gamma^{\text{LO}}_{b\bar{b}} = 3 \cdot \frac{M_H}{8\pi}\left(\frac{m_b}{v}\right)^2 = 4.280\,\text{MeV} \;\Rightarrow\; \text{BR} \approx 105\%.$$

That overshoots because quark Yukawas *run* with energy; at the Higgs scale the effective m_b is smaller. If you instead use a running mass near $3.0\,\text{GeV}$ as a simple correction,

$$\Gamma_{b\bar{b}} \approx 2.212\,\text{MeV} \;\Rightarrow\; \text{BR} \approx 54.3\%.$$

Moral: the fraction method is exact, but for quarks you should evaluate m_q at the relevant scale. The recipe stays the same; just pick the scale-appropriate ratio.

3.4 Hydrogen ground state from α_{em} and m_e/v

From the registry, $\alpha_{\text{em}} = 2639/361638$ and $m_e/v = 43/20719113$. Multiply once: $m_e = (43/20719113)\,v = 0.000\,510\,999\,\text{GeV}$. Then

$$E_1 = -\frac{\alpha_{\text{em}}^2}{2}\,m_e = -13.6057\ \text{eV},$$

the classic number, recovered purely from the ratios.

3.5 Custodial test: $(M_W/M_Z)^2$ vs $1 - \sin^2\theta_W$

Compute both sides with the registry:

$$\left(\frac{M_W}{M_Z}\right)^2 = 0.77698678, \qquad 1 - \sin^2\theta_W = 0.76878000.$$

The small difference ($\approx 8.21 \times 10^{-3}$) is a snapshot effect of scheme and radiative corrections; the tree identity is visible in the numbers.

3.6 Koide's relation for charged leptons

Using m_e, m_μ, m_τ from the registry,

$$Q_\ell = \frac{m_e + m_\mu + m_\tau}{\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2} = 0.6666605,$$

within 6×10^{-6} of $2/3$. It is an empirical regularity reproduced by the same inputs.

4 Beyond the SM (ratio playcards)

These are *templates* you can use without software. The inputs remain fractions; the outputs are physical predictions.

Axion quicklook

Mass and photon coupling from a single scale f_a:

$$m_a \approx 5.7\,\mu\text{eV}\left(\frac{10^{12}\ \text{GeV}}{f_a}\right), \qquad g_{a\gamma\gamma} \approx C\frac{\alpha_{\text{em}}}{2\pi f_a},\ C \sim 0.75.$$

Misalignment abundance:

$$\Omega_a h^2 \approx 0.12\left(\frac{f_a}{5 \times 10^{11}\ \text{GeV}}\right)^{7/6}\left(\frac{\theta_i}{\pi}\right)^2.$$

Choose f_a/E_P and θ_i/π as small fractions and evaluate.

Type-I seesaw

For one flavor,

$$m_\nu \simeq \frac{y_\nu^2\, v^2}{M_R}.$$

Pick y_ν and M_R as rational handles (you can even tie M_R to a unification scale chosen as a fraction of E_P) and get m_ν directly.

Freeze-in sketch

For a feeble Yukawa y_χ,

$$\Omega_\chi h^2 \propto y_\chi^2 \left(\frac{m_\chi}{100\ \text{keV}}\right).$$

Tiny fractions for y_χ (e.g. $1/10^{11}$) and m_χ/v yield back-of-envelope abundances without a single loop integral on paper.

5 Compression scoreboard

Count information in bits by adding the binary lengths of numerators and denominators across the registry. The 19 base fractions plus a few derived ones total roughly ~ 350 bits; the same numbers as 64-bit floats would be about ~ 1200 mantissa bits. You are not just simplifying aesthetics; you are compressing the universe's "source" while keeping it exact.

Glossary of symbols

v: Higgs vacuum modulus ($246.219\,652\,\text{GeV}$). M_W, M_Z, M_H: weak boson and Higgs masses. $\alpha_{\text{em}}, \alpha_s$: electromagnetic and strong couplings. $\sin^2\theta_W$: weak mixing. $G_F = 1/(\sqrt{2}\,v^2)$: Fermi constant in this scheme. BR: branching ratio. N_c: color factor (1 leptons, 3 quarks).

What to do next

Take any observable you care about, rewrite its definition in terms of ratios from the registry, and run the arithmetic. If a quark is involved, remember the "$\text{H}\to b\bar{b}$" lesson: evaluate its effective mass at the relevant scale. Everything else is just the same three moves: look up the fraction, multiply by v, and apply the template. Boom. You are a physicist now.