Evan Wesley - Black Hole Thermodynamics in Bits — Part III

Appendices C–H (Gas Pack)

(This file is a supplement to `Blackhole3.tex`.)

Appendix C: Jet Bit–Joule Bound (AGN Horizons as Computers)

Let P_{jet} denote the mechanical/EM power launched by a black–hole engine and $\epsilon_{\text{gb}} \in (0,1)$ an effective transmission efficiency from the horizon to infinity (proxy for greybody/magnetospheric losses). The one–bit energy cost at fixed (J, Q) implies an instantaneous bound on any horizon–mediated information flux

$$\dot{N_{\text{bits max}}} = \frac{\epsilon_{\text{gb}} \, P_{\text{jet}}}{k_{\text{B}} \; \ln 2} \,. \tag{1}$$

Example (symbolic): For Sgr A* with $\approx 1.54 \times 10^{-14}\,\text{K}$, one–bit energy is $k_{\text{B}} \ln 2 \approx 9.21 \times 10^{-19}$ eV, giving

$$\dot{N_{\text{bits max}}} \approx 6.77 \times 10^{69} \, \epsilon_{\text{gb}} \left(\frac{P_{\text{jet}}}{10^{33}\,\text{W}} \right) \text{bits s}^{-1} \,. \tag{2}$$

Usage: choose targets (M87*, Sgr A*), adopt conservative P_{jet} and ϵ_{gb} to plot a forbidden region where any claimed horizon computation would violate the bound.

Appendix D: Ringdown Log–Constant Stack (Search for $\ln 2$)

Hypothesis. If the horizon area is quantized in steps of $4 \ln 2 \, \ell_{\text{P}}^2$, asymptotic QNM spectra may encode a universal logarithmic factor.

Pipeline (simulation first):

1. Generate damped–sine ringdowns with overtones $n = 0, 1, \ldots, N$ at random SNRs; add Gaussian noise.
2. Fit frequencies $\{\omega_n\}$ via maximum likelihood.
3. Test null $\mathcal{H}_0$ (no constant) vs $\mathcal{H}_1$ (spacings follow $\Delta \operatorname{Re} \omega_n \propto \kappa_{\text{H}}$ with a universal factor compatible with $\ln 2$).
4. Power analysis: estimate SNR and N needed to discriminate $\mathcal{H}_1$ from $\mathcal{H}_0$ at fixed FAP.

Deliverable: a small Python stacker that outputs Bayes factor vs SNR.

Appendix E: Cosmic Bit Balance Sheet

Goal. A ledger of cosmic entropy in bits with the same normalization as horizon bits: $S_{\text{bits}} = \mathcal{A}/(4\ell_{\text{P}}^2 \ln 2)$.

Entries:

- Supermassive BHs: $S_{\text{bits}}^{\text{SMBH}} = \sum_i \mathcal{A}_i/(4\ell_{\text{P}}^2 \ln 2)$ using the BH mass function $\mathrm{d}n/\mathrm{d}M$.
- Stellar–mass BHs/NSs/WDs: area contributions (subdominant) included for completeness.
- Relics: CMB and cosmic neutrinos (thermodynamic entropies, distinct from area entropy), IGM, stars/dust.

Output: one bar chart (orders of magnitude) + a sensitivity table highlighting the dominant astrophysical uncertainties.

Appendix F: Analogue Horizon One–Bit Engine

Objective. Test $\Delta E_{1\mathrm{bit}} = k_\mathrm{B} \ln 2$ in a tunable analogue horizon.
Design sketch:

1. Choose platform (optical fibre / BEC / water tank) with calibrated Hawking temperature T_H^{eff}.
2. Define a single–bit memory coupled to the horizon (two metastable states with reset pulse).
3. Protocol: prepare maximally mixed state; erase to a standard state while in contact with the analogue bath at T_H^{eff}; measure work.
4. Prediction: minimal work per operation $\geq k_\mathrm{B} T_H^{\mathrm{eff}} \ln 2$, up to known inefficiencies.

Knobs: vary T_H^{eff} to observe linear scaling of work/bit.

Appendix G: Kerr Bit Chemical Potentials

With $N \equiv \mathcal{A}/(4\ell_\mathrm{P}^2 \ln 2)$, the first law yields

$$dM = \underbrace{\frac{\kappa_\mathrm{H} \ln 2}{2\pi}}_{\mu_N} dN + \Omega_\mathrm{H}\, dJ + \Phi_\mathrm{H}\, dQ\,. \tag{3}$$

Thus μ_N acts as a *bit chemical potential*. In SI units, $\mu_N c^2 = k_\mathrm{B} \ln 2$. Near extremality ($a_* \to 1$ or large Q) one has $\kappa_\mathrm{H} \to 0$ and $\mu_N \to 0$: rotation/charge discount the energy cost per bit while the area–per–bit remains fixed.

Appendix H: Kerr Spin Map (Numerics at $M = 10\, M_\odot$)

For a Kerr black hole of mass $M = 10\, M_\odot$ the table shows Hawking temperature, one–bit energy, horizon angular velocity, and bit count versus dimensionless spin a_*. Values scale as $T_H \propto f(a_*)/M$, $\Delta E_{1\mathrm{bit}} \propto T_H$, and $S_\mathrm{bits} \propto \mathcal{A}$.

a_*	T_H (K)	$\Delta E_{1\mathrm{bit}}$ (eV)	Ω_H (rad/s)	S_bits
0.00	6.17×10^{-9}	3.69×10^{-13}	0	1.51×10^{79}
0.50	5.73×10^{-9}	3.42×10^{-13}	2.72×10^{3}	1.41×10^{79}
0.90	3.75×10^{-9}	2.24×10^{-13}	6.36×10^{3}	1.09×10^{79}
0.99	1.53×10^{-9}	9.11×10^{-14}	8.81×10^{3}	8.63×10^{78}

Notes. (i) $\Omega_H = a/(r_+^2 + a^2)\, c$; (ii) $r_+ = M + \sqrt{M^2 - a^2}$ (geometric units); (iii) $S_\mathrm{bits} = \mathcal{A}/(4\ell_\mathrm{P}^2 \ln 2)$ with $\mathcal{A} = 4\pi(r_+^2 + a^2)$.