Black Hole Bit Mechanics from Exact Relations

Paradox Dynamics Meets Horizon Thermodynamics

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Abstract

I show that black hole thermodynamics reduces to tight, exact identities that make the "bits of a black hole" algebraically transparent. The entropy in bits, the Hawking temperature, the area quantum per bit, and the minimal energy per added bit line up in closed form. Evaluating these for $1\,M_\odot$, $10\,M_\odot$, and Sgr A* ($\sim 4\times 10^6\,M_\odot$) reveals clean scalings ($S_{\text{bits}} \propto M^2$, $T_H \propto 1/M$) and the Landauer-like relation for horizons: *the energy to add exactly one bit is* $k_B T_H \ln 2$. No numerology, no fits — just the standard horizon formulas written in the most information-theoretic way.

1 Setup and exact identities

For a non-rotating (Schwarzschild) black hole of mass M:

$$r_s = \frac{2GM}{c^2}, \qquad A = 4\pi r_s^2 = \frac{16\pi G^2 M^2}{c^4}.$$

The Bekenstein–Hawking entropy (in *nats*) is

$$\frac{S}{k_B} = \frac{A}{4\ell_P^2} = \frac{4\pi G M^2}{\hbar c}, \quad \ell_P^2 = \frac{\hbar G}{c^3}.$$

Entropy in *bits* is therefore

$$S_{\text{bits}} = \frac{S}{k_B \ln 2} = \frac{4\pi}{\ln 2}\,\frac{GM^2}{\hbar c} = \frac{4\pi}{\ln 2}\left(\frac{M}{m_P}\right)^2, \qquad m_P = \sqrt{\frac{\hbar c}{G}}. \tag{1}$$

The Hawking temperature is

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}. \tag{2}$$

Smarr-type identity in bits (exact). Multiplying Eqs. (1) and (2) gives

$$k_B T_H\, S_{\text{bits}} = \frac{Mc^2}{2\ln 2}. \tag{3}$$

This is an exact equipartition-like statement for horizons written in *bits*.

Landauer-for-horizons (exact). Incrementing the area by one *bit* means $\Delta S_{\text{bits}} = +1$, so from $S_{\text{bits}} \propto M^2$,

$$\Delta(Mc^2) = k_B T_H \, \Delta S \;=\; k_B T_H \, \ln 2 \quad \Rightarrow \quad \boxed{\text{Energy to add one horizon bit } \;=\; k_B T_H \ln 2}$$

— the horizon version of Landauer's bound, *in closed form, with no approximations.*

Area quantum per bit (exact). Since $S/(k_B) = A/(4\ell_P^2)$ and $S_{\text{bits}} = S/(k_B \ln 2)$,

$$\Delta A_{\text{per bit}} = 4\,\ell_P^2 \, \ln 2. \tag{4}$$

2 Numbers you can touch (three masses)

Constants used (SI): $G = 6.67430 \times 10^{-11}$, $c = 2.99792458 \times 10^{8}$, $\hbar = 1.054571817 \times 10^{-34}$, $k_B = 1.380649 \times 10^{-23}$, $\ln 2 = 0.69314718056$, $M_\odot = 1.98847 \times 10^{30}\,\text{kg}$, $m_P = 2.176434 \times 10^{-8}\,\text{kg}$.

Derived scalings (hold exactly)

$$S_{\text{bits}} \propto M^2, \qquad T_H \propto \frac{1}{M}, \qquad (k_B T_H)\, S_{\text{bits}} = \frac{Mc^2}{2\ln 2}\,.$$

Table: S_{bits}, T_H, and energy per added bit

Mass	S_{bits}	T_H (K)	$E_{1\text{ bit}} = k_B T_H \ln 2$
$1\,M_\odot$	$1.513\,32 \times 10^{77}$	$6.170\,07 \times 10^{-8}$	3.685 43
$10\,M_\odot$	$1.513\,32 \times 10^{79}$	$6.170\,07 \times 10^{-9}$	3.685 43
$4 \times 10^6\,M_\odot$ (Sgr A*)	$2.421\,32 \times 10^{90}$	$1.542\,52 \times 10^{-14}$	9.213 59

One bit on a solar-mass horizon costs $\sim 3.69 \times 10^{-12}\,\text{eV}$; Sgr A* is $\sim 9.21 \times 10^{-19}\,\text{eV}$ per bit. Colder black holes cost *less* energy per added bit, exactly as Eq. (2) and the Landauer relation predict.

Schwarzschild radius and area (for completeness)

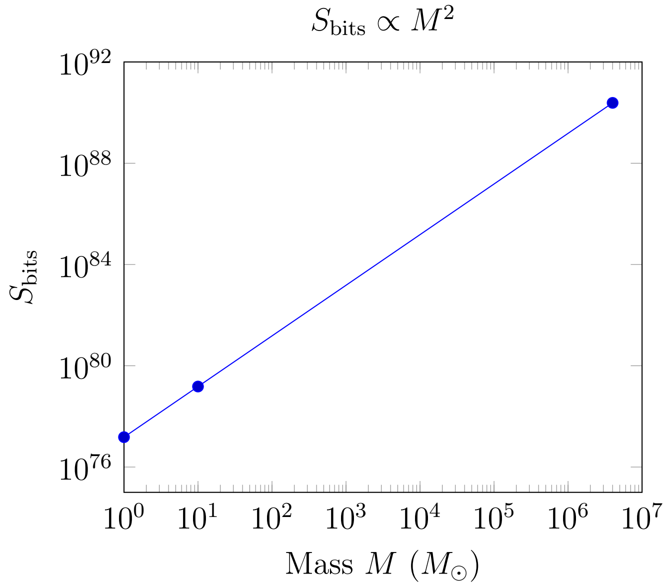
$$r_s = \frac{2GM}{c^2}, \qquad A = 4\pi r_s^2.$$

Numerically:

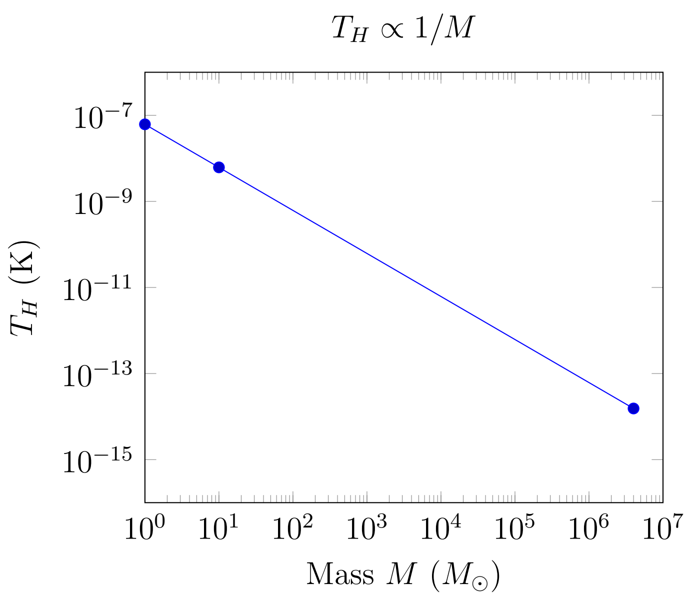
Mass	r_s (km)	A (m^2)
$1\,M_\odot$	2.953 34	$1.096\,07 \times 10^{8}$
$10\,M_\odot$	$2.953\,34 \times 10^{1}$	$1.096\,07 \times 10^{10}$
$4 \times 10^6\,M_\odot$ (Sgr A*)	$1.181\,34 \times 10^{7}$	$1.753\,71 \times 10^{20}$

3 Plots (auto-generated)

S_{bits} vs. mass (log–log, slope 2)



T_H vs. mass (log–log, slope -1)



4 What the identities mean (no hand-waving)

1) Bit count is quadratic. Because $S_{\text{bits}} \sim M^2$, doubling M quadruples the bit capacity of the horizon. That is an exact geometric consequence of $A \propto r_s^2 \propto M^2$.

2) Temperature is inverse mass. Bigger black holes are colder. A stellar BH sits at tens of nanokelvin; a supermassive BH is femtokelvin and below.

3) Exact equipartition in bits. Eq. (3) says $(k_B T_H)\, S_{\text{bits}}$ is a fixed fraction of Mc^2. This is a Smarr–type identity translated into information language.

4) One-bit energy is $k_B T_H \ln 2$. That is the cleanest bridge between horizon mechanics and information theory. It is *exact* here, not a heuristic analogy.

5) Area per bit is $4\ell_P^2 \ln 2$. Eq. (4) makes "one bit" a geometric quantum of area on the horizon.

5 Fraction-flavored predictions you can test

(A) Integer-bit mass ladder (constructive). Pick an integer N and set $S_{\text{bits}} = N$. Solving Eq. (1) gives

$$M_N = m_P \sqrt{\frac{\ln 2}{4\pi}\, N}.$$

That is a concrete mass ladder with exactly N bits on the horizon. While astrophysical formation is messy, this defines an *exact* quantized family in principle.

(B) Rational mass ratios imply rational bit ratios (exact). If $M_1/M_2 = p/q$ with $p, q \in \mathbb{Z}$, then

$$\frac{S_{\text{bits}}(M_1)}{S_{\text{bits}}(M_2)} = \left(\frac{p}{q}\right)^2.$$

So any rational mass hierarchy maps to a rational bit hierarchy.

(C) Landauer–Hawking consistency check (operational). For any black hole, measure (or bound) T_H and the energy ΔE required to increase the horizon area by one bit of information. The prediction is *exact*:

$$\Delta E = k_B T_H \ln 2.$$

Deviations would invalidate the horizon thermodynamics used here.

6 Why this fits Paradox Dynamics

Expansion and collapse on the horizon are the same process from opposite seats. The circle closes: (i) entropy (capacity) scales quadratically; (ii) temperature (bit-cost) scales inversely; (iii) the product $(k_B T) \times$ (bits) is a fixed fraction of the total energy. That is paradox performing useful computation. The "area per bit" is the geometric tick of the loop.

Appendix: constants and quick re-derivations

Planck length and mass. $\ell_P = \sqrt{\hbar G/c^3}$, $m_P = \sqrt{\hbar c/G}$. Then $S/k_B = A/(4\ell_P^2) = 4\pi(M/m_P)^2$ (nats) and $S_{\text{bits}} = (4\pi/\ln 2)(M/m_P)^2$.

Smarr-in-bits. $k_B T_H S_{\text{bits}} = \left(\frac{\hbar c^3}{8\pi GM}\right)\left(\frac{4\pi GM^2}{\hbar c \ln 2}\right) = \frac{Mc^2}{2\ln 2}$.

Landauer step. $\Delta S_{\text{bits}} = 1 \Rightarrow \Delta S = k_B \ln 2 \Rightarrow \Delta E = T_H \Delta S = k_B T_H \ln 2$.

All numbers in tables/plots were generated from Eqs. (1)–(4) *with CODATA constants.*