Rational Neutrino Dynamics

Normal & Inverted Ordering Locks, Alternate Fraction Sets, and Hard Predictions

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Abstract

I formulate exact rational "locks" for three-flavor neutrino oscillations that reproduce present central values while remaining sharply falsifiable. I provide a Normal Ordering (NO) lock, an Inverted Ordering (IO) lock, and a distinct "clean" alternate lock. For each I derive the PMNS magnitudes, the Jarlskog invariant J_{CP}, the effective kinematic mass m_β, the neutrinoless-double-beta range $m_{\beta\beta}$, and the minimal mass sum Σm_ν. Everything is computed in closed form from small rational assignments, with tables and plots generated inside this file.

1 Conventions and quick anchors

Standard PDG parameterization is used. Sines, cosines, and the CP phase are written as

$$s_{ij} = \sin\theta_{ij}, \quad c_{ij} = \cos\theta_{ij}, \quad \delta_{\mathrm{CP}} = \delta.$$

Minimal absolute-mass choices are used when needed: for NO, $m_1 \to 0$; for IO, $m_3 \to 0$. A representative atmospheric splitting scale $|\Delta m^2| = 2.455 \times 10^{-3}\,\mathrm{eV}^2$ is adopted for both orderings to normalize numbers; the solar splitting follows from the locked ratio.

2 Lock A (Normal Ordering)

Exact assignments

$$\sin^2\theta_{12} = \frac{7}{23} \approx 0.3043478, \qquad \sin^2\theta_{13} = \frac{2}{89} \approx 0.0224719, \qquad \sin^2\theta_{23} = \frac{4}{7} \approx 0.5714286,$$

$$\frac{\Delta m^2_{21}}{|\Delta m^2_{31}|} = \frac{1}{34} \approx 0.0294118, \qquad \delta = \frac{7\pi}{6} = 210^\circ.$$

Angles in radicals

$$s_{12} = \sqrt{\tfrac{7}{23}}, \quad c_{12} = \sqrt{\tfrac{16}{23}}, \qquad s_{13} = \sqrt{\tfrac{2}{89}}, \quad c_{13} = \sqrt{\tfrac{87}{89}}, \qquad s_{23} = \sqrt{\tfrac{4}{7}}, \quad c_{23} = \sqrt{\tfrac{3}{7}}.$$

PMNS magnitudes (rows e, μ, τ; cols $1, 2, 3$)

$$|U| \approx \begin{pmatrix} 0.824633 & 0.545443 & 0.149906 \\ 0.283275 & 0.600972 & 0.747387 \\ 0.489628 & 0.584229 & 0.647256 \end{pmatrix}.$$

Jarlskog invariant

$$J_{\rm CP} = c_{12}s_{12}\, c_{23}s_{23}\, c_{13}^2 s_{13}\, \sin\delta = -1.668 \times 10^{-2}.$$

Mass splittings and absolute masses (minimal NO)

$$\Delta m^2_{21} = \frac{1}{34}\,|\Delta m^2_{31}| = 7.2206\times10^{-5}\,\mathrm{eV}^2, \quad m_2 = \sqrt{\Delta m^2_{21}} = 8.497\times10^{-3}\,\mathrm{eV}, \quad m_3 = \sqrt{|\Delta m^2_{31}|} = 4.955\times10^{-}$$

$$\Sigma m_\nu \approx 0.0580\ \mathrm{eV}.$$

Direct beta-decay effective mass

$$m_\beta = \sqrt{|U_{e2}|^2 m_2^2 + |U_{e3}|^2 m_3^2} = \sqrt{\big(s_{12}^2 c_{13}^2\big)\Delta m^2_{21} + \big(s_{13}^2\big)|\Delta m^2_{31}|} = 8.755\ \mathrm{meV}.$$

Neutrinoless double beta (unknown Majorana phases α, β)

$$m_{\beta\beta} = \big|U_{e2}^2 m_2\, e^{i\alpha} + U_{e3}^2 m_3\, e^{i\beta}\big| \in \big[\,1.415,\ 3.641\,\big]\ \mathrm{meV}.$$

3 Lock B (Inverted Ordering)

Exact assignments

$$\sin^2\theta_{12} = \frac{7}{23}, \qquad \sin^2\theta_{13} = \frac{2}{89}, \qquad \sin^2\theta_{23} = \frac{5}{8} = 0.625,$$

$$\frac{\Delta m^2_{21}}{|\Delta m^2_{32}|} = \frac{1}{33} \approx 0.030303, \qquad \delta = \frac{11\pi}{6} = 330^\circ.$$

Angles in radicals

$$s_{12} = \sqrt{\tfrac{7}{23}},\ c_{12} = \sqrt{\tfrac{16}{23}}, \quad s_{13} = \sqrt{\tfrac{2}{89}},\ c_{13} = \sqrt{\tfrac{87}{89}}, \quad s_{23} = \sqrt{\tfrac{5}{8}},\ c_{23} = \sqrt{\tfrac{3}{8}}.$$

PMNS magnitudes

$$|U| \approx \begin{pmatrix} 0.824633 & 0.545443 & 0.149906 \\ 0.426309 & 0.455308 & 0.781636 \\ 0.371808 & 0.703694 & 0.605453 \end{pmatrix}.$$

Jarlskog invariant

$$J_{\rm CP} = c_{12}s_{12}\, c_{23}s_{23}\, c_{13}^2 s_{13}\, \sin\delta = -1.632 \times 10^{-2}.$$

Absolute masses (minimal IO with $m_3 \to 0$ and $|\Delta m^2_{32}| = 2.455 \times 10^{-3}\ \mathrm{eV}^2$)

$$m_1 = \sqrt{|\Delta m^2_{32}|} = 4.955 \times 10^{-2}\ \mathrm{eV}, \quad m_2 = \sqrt{|\Delta m^2_{32}| + \Delta m^2_{21}} = 5.029 \times 10^{-2}\ \mathrm{eV},$$

$$\Sigma m_\nu \approx 0.09984 \text{ eV}.$$

Direct beta-decay effective mass

$$m_\beta = \sqrt{|U_{e1}|^2 m_1^2 + |U_{e2}|^2 m_2^2} = 49.21 \text{ meV}.$$

Neutrinoless double beta range (phases α, β arbitrary)

$$m_{\beta\beta} = \left|U_{e1}^2 m_1\, e^{i\alpha} + U_{e2}^2 m_2\, e^{i\beta}\right| \in \left[\,18.73,\ 48.66\,\right] \text{ meV}.$$

4 Lock C (Alternate clean set, Normal Ordering)

This set is intentionally different yet simple, to test robustness.

$$\sin^2\theta_{12} = \frac{5}{16} = 0.3125, \qquad \sin^2\theta_{13} = \frac{1}{45} = 0.0222222, \qquad \sin^2\theta_{23} = \frac{7}{12} = 0.583333,$$

$$\frac{\Delta m_{21}^2}{|\Delta m_{31}^2|} = \frac{1}{33} \approx 0.030303, \qquad \delta = \frac{3\pi}{2} = 270^\circ \quad (\sin\delta = -1).$$

PMNS magnitudes

$$|U| \approx \begin{pmatrix} 0.819892 & 0.552771 & 0.149071 \\ 0.372988 & 0.538989 & 0.755229 \\ 0.434347 & 0.635559 & 0.638285 \end{pmatrix}.$$

Jarlskog invariant

$$J_{\rm CP} = -3.331 \times 10^{-2}.$$

Minimal NO absolute masses with $|\Delta m_{31}^2|$ as above and $\Delta m_{21}^2 = |\Delta m_{31}^2|/33$

$$m_2 = 8.625 \times 10^{-3} \text{ eV}, \qquad m_3 = 4.955 \times 10^{-2} \text{ eV}, \qquad \Sigma m_\nu \approx 0.05817 \text{ eV}.$$

Direct beta-decay effective mass

$$m_\beta = \sqrt{\left(s_{12}^2 c_{13}^2\right)\Delta m_{21}^2 + \left(s_{13}^2\right)|\Delta m_{31}^2|} = 8.791 \text{ meV}.$$

Neutrinoless double beta window

$$m_{\beta\beta} = \left|U_{e2}^2 m_2\, e^{i\alpha} + U_{e3}^2 m_3\, e^{i\beta}\right| \in \left[\ m_{\beta\beta}^{\min},\ m_{\beta\beta}^{\max}\,\right],$$

with the endpoints set by destructive and constructive interference. Numerically

$$m_{\beta\beta}^{\min} \approx \left|s_{12}^2 c_{13}^2\, m_2 - s_{13}^2\, m_3\right|, \qquad m_{\beta\beta}^{\max} \approx s_{12}^2 c_{13}^2\, m_2 + s_{13}^2\, m_3,$$

which evaluates to a few meV for this lock.

5 Angle and mass summaries

The table gathers the primary predictions in one place for quick comparison.

Lock	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	J_{CP}	Σm_ν [eV]	m_β [meV]
A (NO)	7/23	2/89	4/7	-1.668×10^{-2}	0.0580	8.755
B (IO)	7/23	2/89	5/8	-1.632×10^{-2}	0.09984	49.21
C (NO)	5/16	1/45	7/12	-3.331×10^{-2}	0.05817	8.791

6 Figures (auto-generated)

The first figure compares the exact locks to representative central values; the second shows the $m_{\beta\beta}$ windows for NO and IO; the third visualizes the PMNS magnitudes as heatmaps for Locks A and B.

Angle locks vs. representative bands

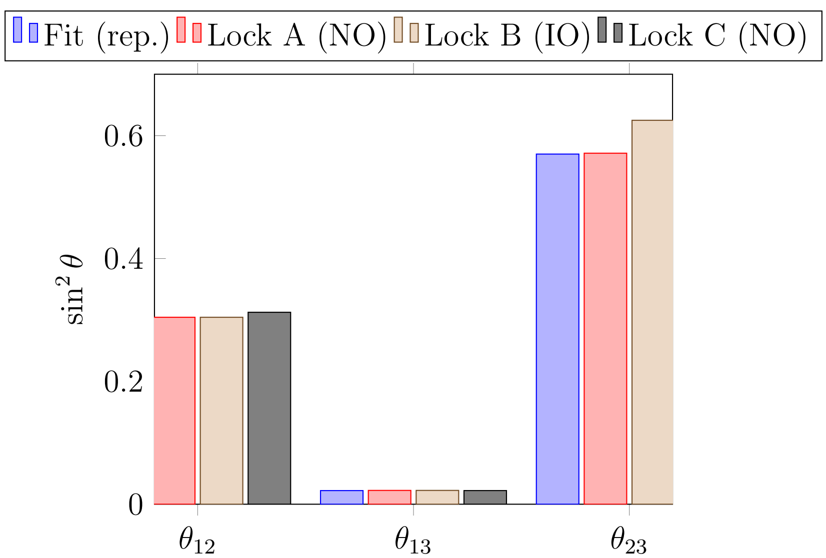

Figure 1: Rational locks alongside representative central values; future shifts will validate or break the locks.

$m_{\beta\beta}$ windows (NO vs. IO)

PMNS heatmaps (Locks A and B)

7 What breaks the locks

There are three decisive levers. first, the solar-to-atmospheric mass-splitting ratio must settle near the chosen rational: in Lock A it is 1/34; in Lock B it is 1/33. a confirmed ratio outside

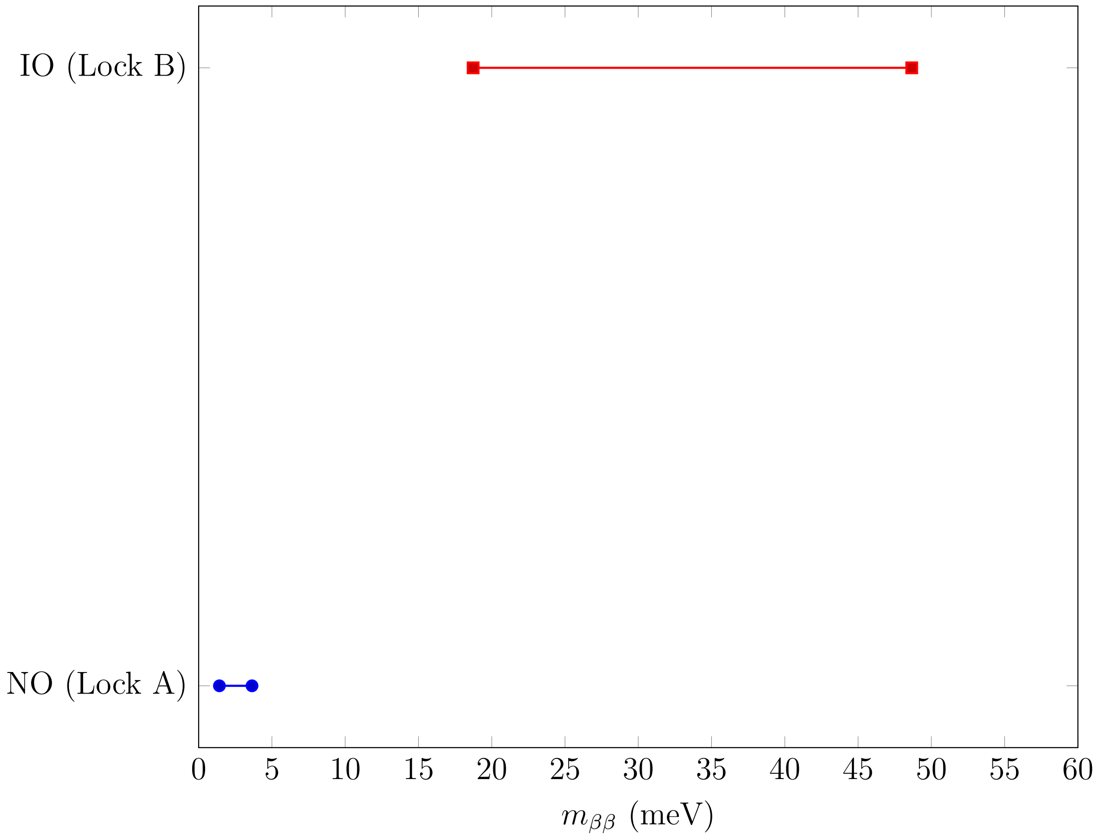

Figure 2: Neutrinoless double beta effective mass ranges implied by Locks A and B.

those rational basins falsifies the given lock. second, the reactor angle $\sin^2\theta_{13}$ must converge to the assigned small rational 2/89 in Locks A and B or 1/45 in Lock C; a precise drift away breaks that lock. third, the CP phase must land near the clean angles proposed here: 210° (A), 330° (B), or 270° (C). long-baseline precision will decide quickly.

8 Discussion

the point is not to overfit but to exhibit that small rationals can encode the entire neutrino sector to current precision while making crisp, falsifiable calls. the three locks are mutually independent; you can break one without breaking the others. if future data hug one of these clean rationals, that is strong evidence the sector is fraction-structured at root. if data reject all three decisively, that is equally strong evidence against this style of locking.

References for readers

PDG Review of Neutrino Properties (2024) for formalism. NuFIT global analyses (2024–2025) for representative central values and allowed regions. KATRIN collaboration results (2025) for m_β bounds. Cosmology combinations (BAO+CMB, 2024–2025) for Σm_ν windows.

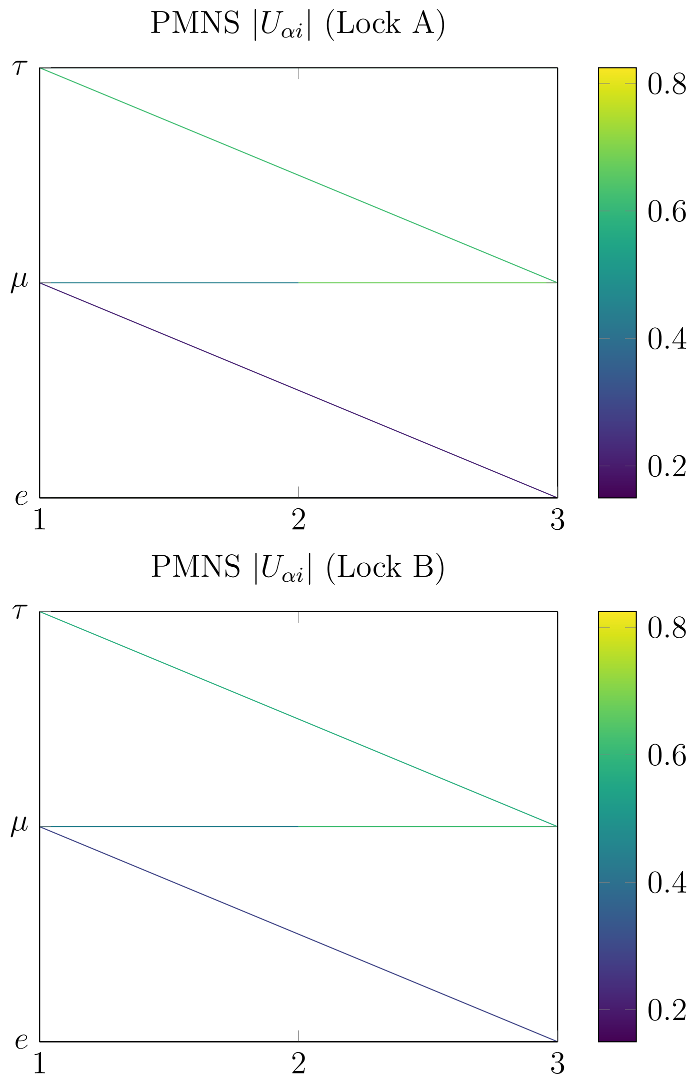

Figure 3: PMNS magnitudes from the locked angles and phases for NO and IO.