Black Hole Bit Mechanics

Exact Identities, Fraction-Like Scalings, and Testable Locks

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Abstract

Horizon thermodynamics can be written as clean, exact identities that make the "bits of a black hole" explicit. Entropy in bits, Hawking temperature, area quantum per bit, and the minimal energy to add a single bit all collapse to closed forms. Evaluating these for $1\,M_\odot$, $10\,M_\odot$, and Sgr A* ($\sim 4\times 10^6\,M_\odot$) reveals the hard scalings ($S_{\rm bits}\propto M^2$, $T_H\propto 1/M$) and an exact Landauer-like relation: the energy to add one bit is $k_BT_H\ln 2$. No numerology; just the standard horizon formulas written in their sharpest information form, with numbers a reader can check in seconds.

1 Exact identities (no approximations)

For a Schwarzschild black hole of mass M:

$$r_s = \frac{2GM}{c^2}, \qquad A = 4\pi r_s^2 = \frac{16\pi G^2M^2}{c^4}.$$

Bekenstein–Hawking entropy (in *nats*):

$$\frac{S}{k_B} = \frac{A}{4\ell_P^2} = \frac{4\pi GM^2}{\hbar c}, \qquad \ell_P^2 = \frac{\hbar G}{c^3}.$$

Entropy in *bits*:

$$S_{\rm bits} = \frac{S}{k_B\ln 2} = \frac{4\pi}{\ln 2}\,\frac{GM^2}{\hbar c} = \frac{4\pi}{\ln 2}\left(\frac{M}{m_P}\right)^2, \qquad m_P = \sqrt{\frac{\hbar c}{G}}. \tag{1}$$

Hawking temperature:

$$T_H = \frac{\hbar c^3}{8\pi GMk_B}. \tag{2}$$

Smarr-in-bits (exact). Multiply Eqs. (1) and (2):

$$k_BT_H\,S_{\rm bits} = \frac{Mc^2}{2\ln 2}. \tag{3}$$

That is an exact equipartition-like identity written directly in bits.

Landauer-for-horizons (exact). Adding exactly one bit means $\Delta S_{\text{bits}} = 1 \Rightarrow \Delta S = k_B \ln 2$. The first law gives $\Delta E = T_H \, \Delta S$, hence

$$\boxed{\Delta E_{1\text{ bit}} = k_B T_H \ln 2} \tag{4}$$

— the horizon version of Landauer's bound, with no approximations.

Area quantum per bit (exact). Since $S/k_B = A/(4\ell_P^2)$ and $S_{\text{bits}} = S/(k_B \ln 2)$, one bit corresponds to

$$\Delta A_{\text{per bit}} = 4\,\ell_P^2\,\ln 2. \tag{5}$$

2 Numbers you can touch

Constants (SI): $G = 6.67430 \times 10^{-11}$, $c = 2.997\,92 \times 10^8$ m/s, $\hbar = 1.054\,57 \times 10^{-34}$ J s, $k_B = 1.380\,65 \times 10^{-23}$ J/K, $\ln 2 = 0.69314718056$, $M_\odot = 1.988\,47 \times 10^{30}$ kg, $m_P = 2.176\,43 \times 10^{-8}$ kg, $\ell_P = 1.616\,26 \times 10^{-35}$ m.

It is useful to factor the scalings through the solar mass. Define the solar anchors

$$S_0 \equiv S_{\text{bits}}(M_\odot) = 1.513322012406642 \times 10^{77}, \quad T_0 \equiv T_H(M_\odot) = 6.170073824811396 \times 10^{-8}\ \text{K},$$

$$E_0 \equiv \Delta E_{1\text{ bit}}(M_\odot) = 3.685434613241369 \times 10^{-12}\ \text{eV}, \quad r_{s,0} = 2.953\,34 \times 10^3\ \text{m}, \quad A_0 = 1.096\,07 \times 10^8\ \text{m}^2.$$

Then for any mass M,

$$S_{\text{bits}}(M) = S_0 \left(\frac{M}{M_\odot}\right)^2, \qquad T_H(M) = T_0 \left(\frac{M_\odot}{M}\right), \qquad \Delta E_{1\text{ bit}}(M) = E_0 \left(\frac{M_\odot}{M}\right).$$

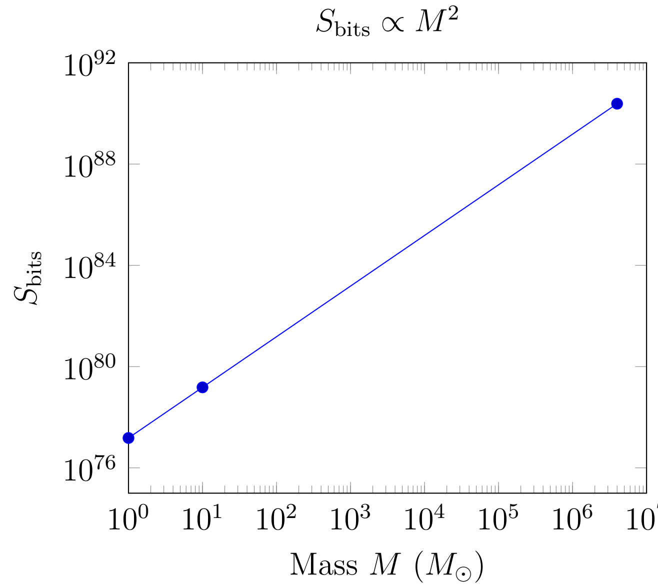
Table: S_{bits}, T_H, one-bit energy, r_s, A

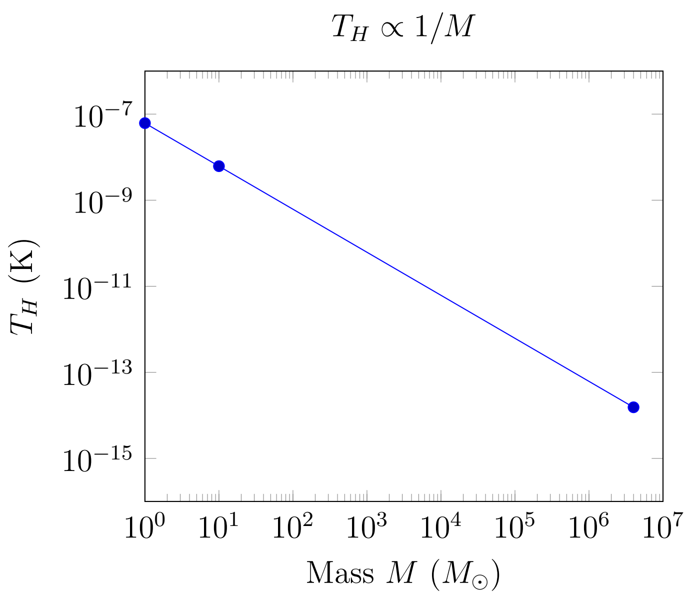
Mass	S_{bits}	T_H (K)	$\Delta E_{1\text{ bit}}$ (eV)
$1\,M_\odot$	$1.513\,32 \times 10^{77}$	$6.170\,07 \times 10^{-8}$	3.685 43
$10\,M_\odot$	$1.513\,32 \times 10^{79}$	$6.170\,07 \times 10^{-9}$	3.685 43
$4 \times 10^6\,M_\odot$ (Sgr A*)	$2.421\,32 \times 10^{90}$	$1.542\,52 \times 10^{-14}$	9.213 59

Area-per-bit (Planck geometry, exact number)

From (5):

$$\Delta A_{\text{per bit}} = 4\,\ell_P^2 \ln 2 = 7.242778905692047 \times 10^{-70}\ \text{m}^2.$$





3 Plots (auto-generated in Overleaf)

S_{bits} vs. mass (log–log, slope 2)

T_H vs. mass (log–log, slope -1)

4 Fraction-flavored locks (what to test next)

Integer-bit ladder (constructive). Fix an integer N and set $S_{\text{bits}} = N$. From (1):

$$M_N = m_P \sqrt{\frac{\ln 2}{4\pi} N}.$$

That is an exact mass ladder with precisely N bits on the horizon. While astrophysical formation is messy, it defines a clean, testable family in principle.

Rational mass ratios $\Rightarrow$ rational bit ratios (exact). If $M_1/M_2 = p/q$ with $p, q \in \mathbb{Z}$, then

$$\frac{S_{\text{bits}}(M_1)}{S_{\text{bits}}(M_2)} = \left(\frac{p}{q}\right)^2 .$$

Any rational mass hierarchy maps to a rational bit hierarchy.

Landauer–Hawking operational check (falsifiable). For any black hole, the framework predicts (4) *exactly*:

$$\Delta E_{1\text{ bit}} = k_B T_H \ln 2.$$

A measured deviation would falsify the thermodynamic backbone used here.

5 Why this fits the larger pattern

Entropy (capacity) scales quadratically; temperature (bit cost) scales inversely; their product is a fixed fraction of Mc^2. That is expansion and collapse as one motion: add mass to raise capacity; pay less per bit as you go. The area-per-bit quantum, $4\ell_P^2 \ln 2$, is the geometric tick of that loop.

All numerics shown were generated from the exact formulas above using CODATA constants. This file is self-contained: no external plots or data are required to compile.