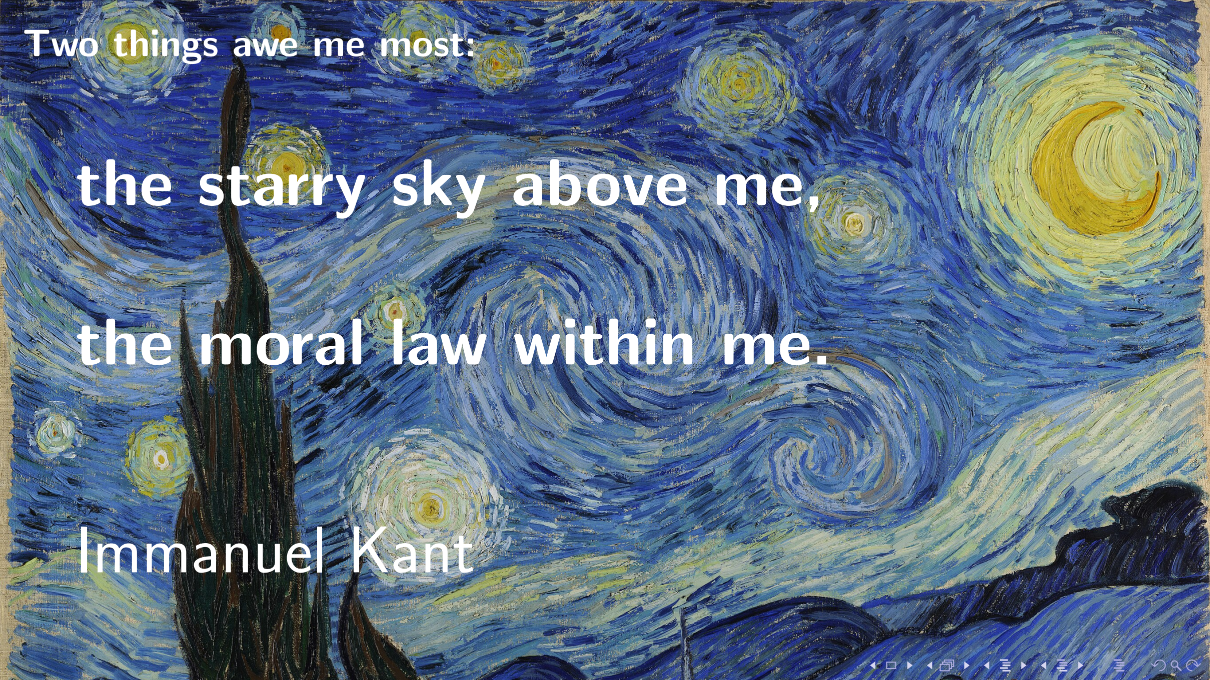


Two things awe me most:

the starry sky above me,

the moral law within me.

Immanuel Kant

Discovering the Mechanics of the Solar System

Jan Plaza

github.com/plazajan

SUNY Plattsburgh Totality Conference

April 6, 2024

Prehistoric times: Earth, Sun and Moon

Prehistoric times: stars, Milky Way

Prehistoric times: comets

Prehistoric times: solar eclipses

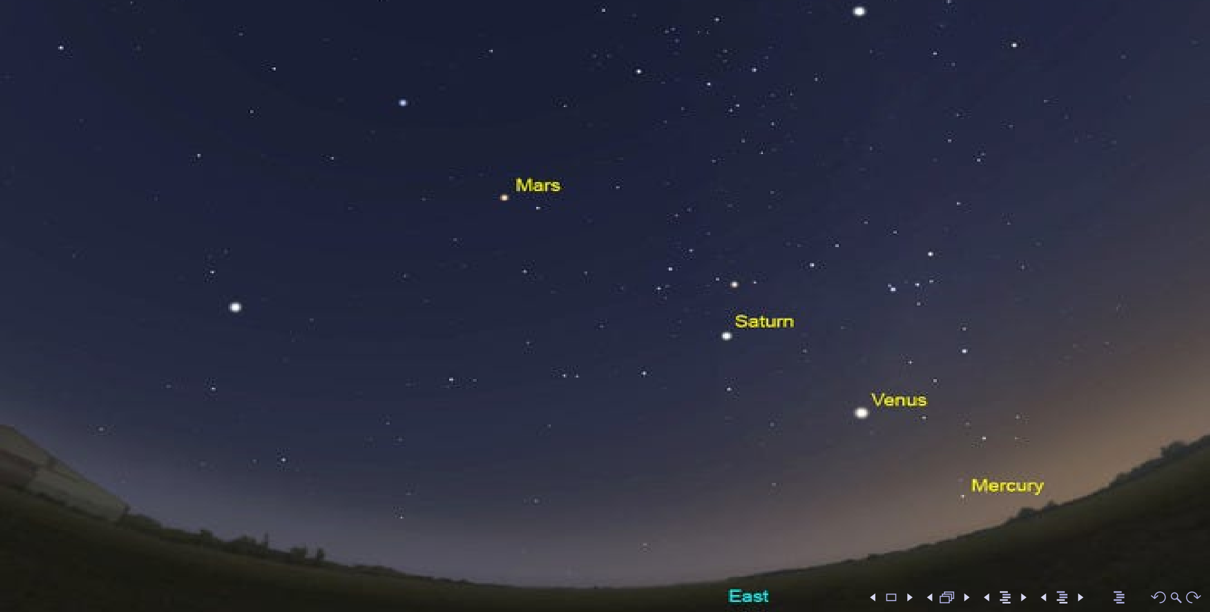
First records:

772 BCE, China

750 BCE, Babylon

Prehistoric times: noon, seasons, solstices, year

Prehistoric times: wandering stars, ecliptic



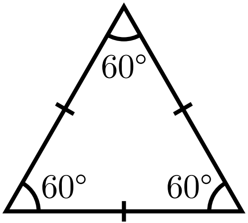
Prehistoric times: constellations

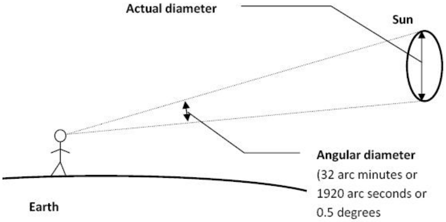
Prehistoric times: star trails, pole star
Phoenicia: navigation by the pole star

Prehistoric times: lunar eclipse.

Mesopotamia, 8th century BCE: 18-year cycle.

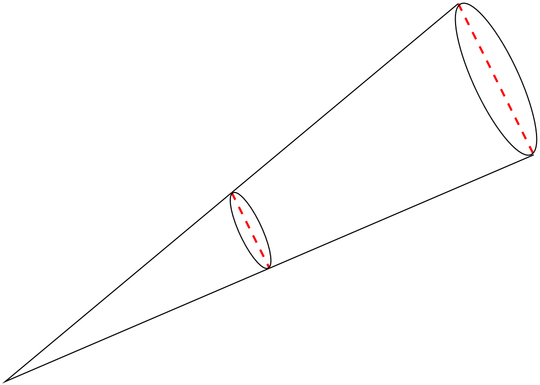
Early civilization, Sumeria and Babylon: angular measures

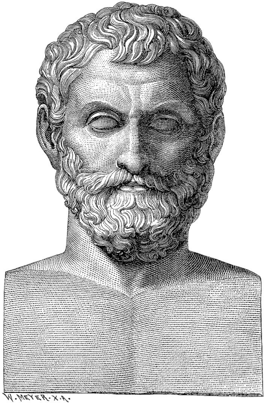




- 1° (degree) $\approx$ the pinky at arm's length.
- $1^\circ = 60'$ (arc minutes).
- $1' = 60''$ (arc seconds).
- $1'$ (arc second) $\approx$ human eye resolution limit.
- Angular diameter of the Moon $\approx 0.5^\circ$.
- Angular diameter of the Sun $\approx 0.5^\circ$.
- Angular diameters of planets – too small to measure without telescopes.

Thales of Miletus, 6^{th} century BCE



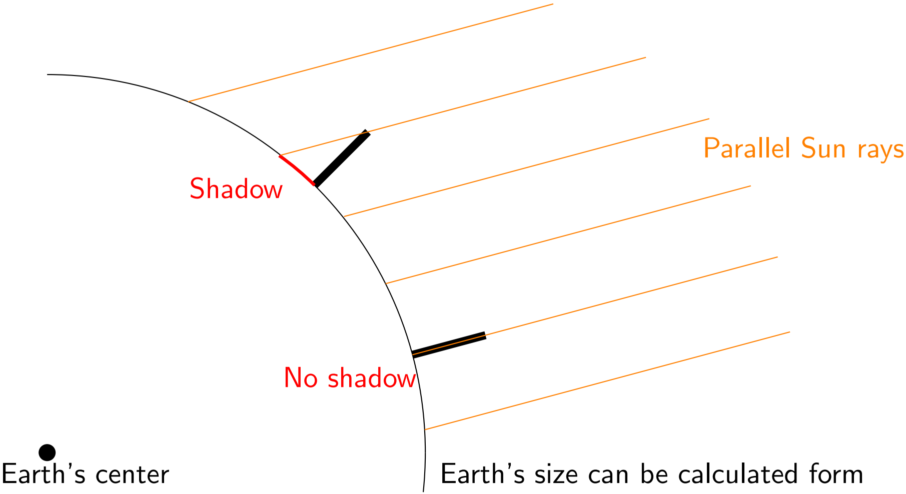


- A Greek mathematician and philosopher.
- A theorem on similar triangles.
- Later used for celestial objects with equal angular measures: the ratio of diameters equals the ratio of distances.

Eratosthenes of Cyrene, 3rd century BCE

- A Greek polymath: mathematician, geographer, poet, astronomer, music theorist, chief librarian at the Library of Alexandria.
- The first to calculate Earth's circumference.
- Sun's diameter relative to to that of the Earth.
- The distance between the Earth and the Sun.
- How? By the Thales' theorem!

Earth's circumference, 3rd century BCE



Earth's size can be calculated form
the distance between the poles, pole height, and the size of the shadow.

Aristarchus of Samos, 3rd century BCE

- A Greek astronomer and mathematician.
- The first known heliocentric model
  soon forgotten due to the influence of Aristotle.
- Sizes and distances of the Earth, Moon and Sun.
- How? By observation/measurement of Sun and Moon eclipses, Moon phases, and Thales' theorem.

And now,

And now, we all will conduct

And now, we all will conduct
the great experiment of

And now, we all will conduct
the great experiment of

The Thumb Parallax

And now, we all will conduct
the great experiment of

The Thumb Parallax

which dates back to prehistoric times

And now, we all will conduct
the great experiment of

The Thumb Parallax

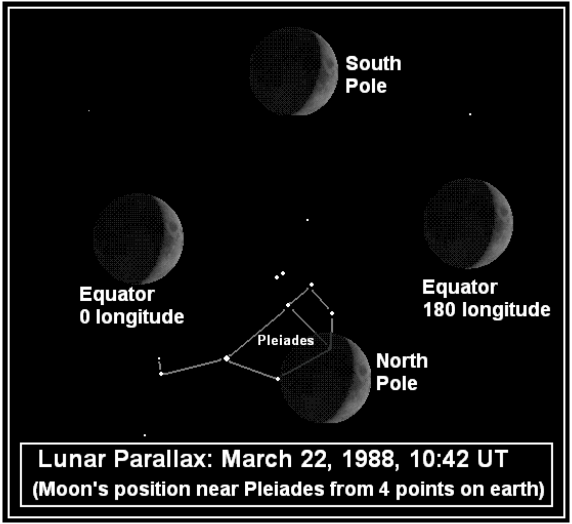
which dates back to prehistoric times
or early childhood!

The Thumb Parallax Experiment

1. Outstretch one hand with the thumb up. Hold it still.
   Close one eye.
   Observe the relative position of the thumb with respect to the distant background.
   Switch eyes.
   Observe the relative position of the thumb with respect to the distant background.
   Does the thumb seem to move?
   The angular measure of the movement is called parallax.
2. Bring the thumb closer, half of the previous distance. Hold it still.
   Close one eye.
   Observe the relative position of the thumb with respect to the distant background.
   Switch eyes.
   Observe the relative position of the thumb with respect to the distant background.
   Does the thumb seem to move more or less than in the previous experiment?

The closer the thumb the more it seems to move – the bigger the parallax!
From the parallax one can calculate the distance!

Greece, 3rd century BCE: lunar parallax



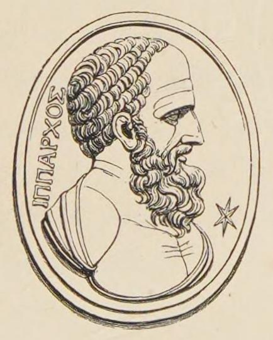
Lunar parallax – the angle of the apparent shift of the Moon while viewed form different places.

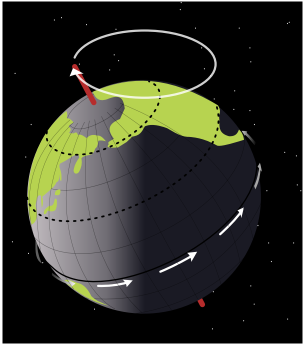
Measuring the distance to the Moon:

- Aristarchus, Greece, 3rd century BCE.
- Hipparchus, Greece, 2nd century BCE.
- Ptolemy, Alexandria, 2nd century CE.
- Hipparchus and Ptolemy used trigonometry.
- 2007 amateur measurement, from places 1600 miles apart, has shown a $0.5'$ shift with respect to the background star Regulus; 11% error in the Moon's distance.

Current measurement by laser ranging, Moon elliptical orbit's semi-major axis: 384,399 km

Hipparchus of Nicea, 2nd century BCE: trigonometry



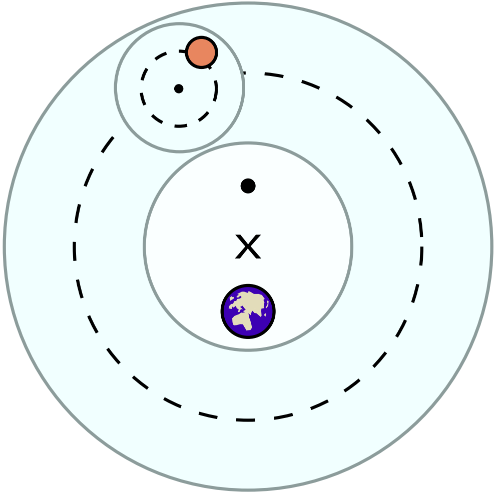


- A Greek mathematician, the founder of trigonometry.
- Accurate quantitative model of sizes, distances and motion of the Sun and Moon; and of eclipses. The Moon and Sun revolve around the Earth.
  (Notice that mathematically, geocentric $\equiv$ heliocentric, if there are no planets.)
- Precession of Earth's axis (period 26,000 years).

Claudius Ptolemy, 2nd century

- An Alexandrian mathematician, astronomer, astrologer, geographer, music theorist
- 13-volume Almagest
  - a compendium of ancient astronomy,
  with astronomical tables spanning 800 years.
- Geocentric model of the universe.

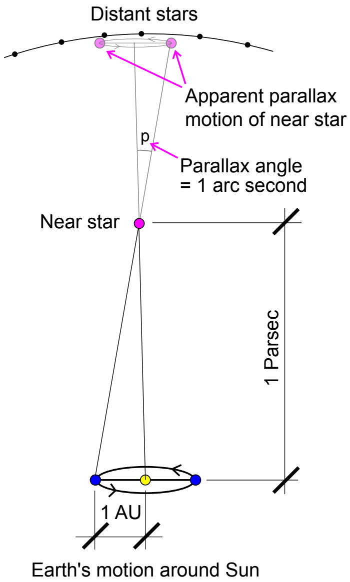
Ptolemaic geocentric model



Spherical Earth at the “center”.
Moon, planets, Sun, each revolving around separate equant points,
on a small circle (epicycle)
whose center moves on a big circle (deferent).

Later, second-order epicycles were added
to improve predictions of positions of planets, and of eclipses.

Stellar Parallax



Stellar parallax of a near star – the angle of apparent shift when viewed from two points on the Earth's (hypothetical?) orbit around the Sun.

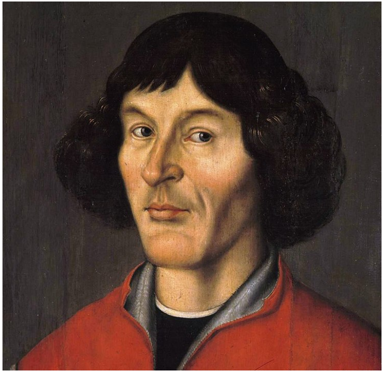
The ancients looked for a stellar parallax, but it was unnoticeable without telescopes – the main scientific argument against the heliocentric system.

Ptolemaic geocentric model dominates for 13 centuries

- Both in Europe and in the Islamic world.
- "If the Earth were moving, we would be blown off its surface."
- "We are the most important
  – the Earth must be the center of the universe."
- Pythagoreanism considers mathematics to be the essence of reality, implying mathematical perfection of the universe:
  perfect circles, revolving "crystal spheres" creating "heavenly music";
  a blurred distinction between mathematics and nature.
- Ptolemaic geocentric model, rooted in Aristotle, was supported by religious institutions.
- Without telescopes (till the 17^{th} century), stellar parallax was unnoticeable, leading Tycho Brahe and most thinkers before him to believe Earth does not move.
- Current view: tolerable predictions, but fitting a square peg into a round hole.

Nicolaus Copernicus, 16th century

Nicolaus Copernicus, 1473 - 1543

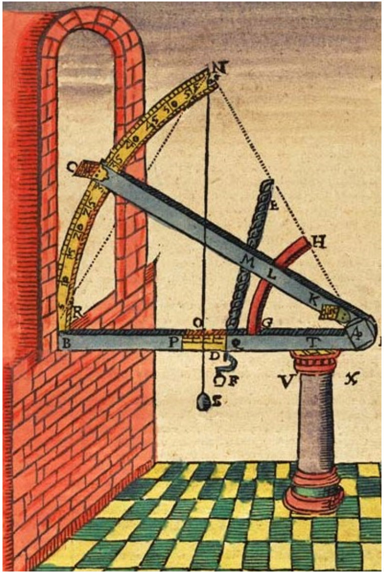


- A Polish polymath, mathematical astronomer, medical practitioner, church administrator, economist.
- Must have studied astrology but did not practice it – unique in his age.
- Formulated quantitative theory of money and Gresham's law in economy.
- 1532, Heliocentric system
  (independently of Aristarchus);
  circular orbits, epicycles but no equants.
- **1543**, *De Revolutionibus Orbium Coellestium* published – the beginning of the Scientific Revolution in Europe.

Galileo Galilei, 1564 - 1642

- An Italian astronomer, physicist and engineer.
- Improved the optical telescope.
- Experimental and theoretical science, classical physics, applying the scientific method.
- The father of modern science.
- Confirmed the Copernican heliocentric model through telescope observations: Phases of Venus, moons of Jupyter.
- Imprisoned for his work.

Tycho Brahe, 1546 - 1601



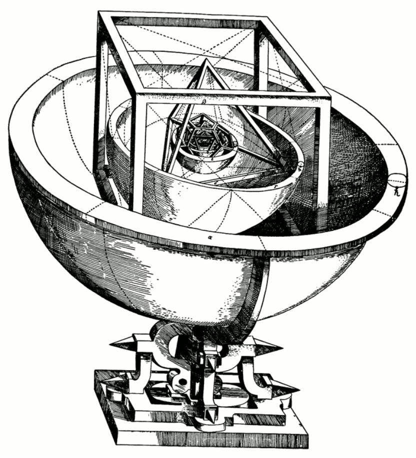
- A Danish astronomer.
- The most advanced and biggest observatory in the pre-telescope era, Uraniborg, on the island of Hven.
- Very accurate observations of positions of planets by naked eye, over a period of 30 years, **c. 1570 - 1601**. $\frac{1}{4}$ of the error of earlier observations.
- Chose Johannes Kepler to publish the results; *Rudolphine Tables* published by Kepler in **1627**.

Tycho Brahe, 1546 - 1601

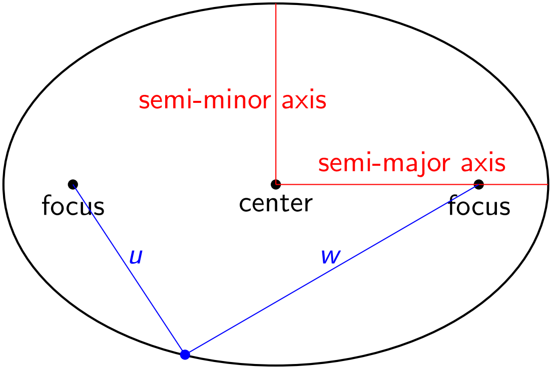
A colorful persona. Studied in Denmark and Germany. Lost his nose in a duel over math prowess. A bronze nose, and a gold-silver one for special occasions. Astrologer and alchemist. Nobleman, friend of King Frederick II of Denmark. Harsh landlord of peasantry. Angered the new king Christian IV by not maintaining a royal chapel; left Denmark for Prague. Very secretive and stingy. Appointed imperial mathematician by an even more stingy Emperor Rudolf II. Referred to Kepler as "dear friend", absolving himself from paying his assistant. Tycho's pet elk died after falling down the stairs while drunk.

Johannes Kepler, 1571 - 1630

A German mathematician, astronomer, astrologer, first science fiction writer.

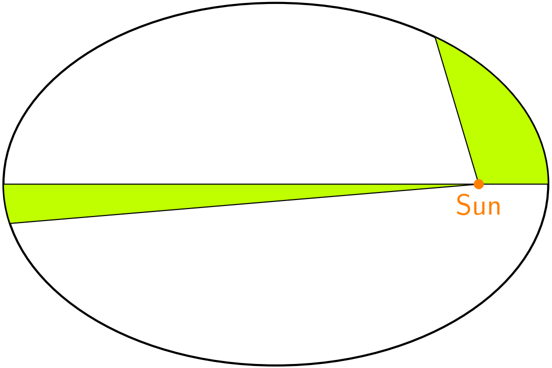


Ancient Greece: ellipse



- Draw a circle on a rubber sheet, and stretch it horizontally – you get an ellipse.
- Draw a circle on a vertically stretched rubber sheet, and let it shrink – you get an ellipse.
- $u + w =$ constant
- When the foci coincide, the ellipse is a circle.

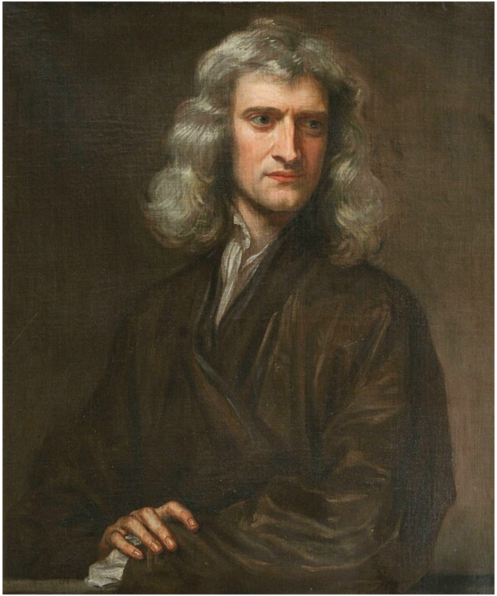
Kepler's laws, 1609 - 1619



Empirical laws of planetary motion
(based on Tycho's observations):

1. The orbit of a planet is an ellipse with the Sun in a focus.
2. The line joining a planet and the Sun sweeps out equal areas during equal intervals of time. (Based on observations of Mars.)
3. The square of the orbital period is proportional to the cube of the semi-major axis.

Sir Isaac Newton, 1642 - 1727



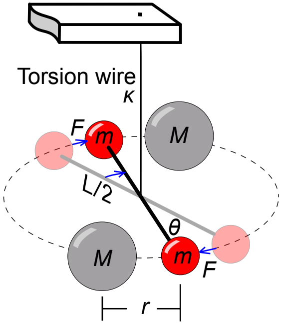
- An English natural philosopher: mathematician, physicist, astronomer, alchemist, theologian.
- Classical mechanics.
- Optics, theory of color.
- Built the first practical reflective telescope.
- Master of the Royal Mint.
- President of the Royal Society.
- **1687**, *Principia Mathematica* can be viewed as the start of the Age of Enlightenment.

Newton, Philosophiae Naturalis Principia Mathematica: 1687

- Calculus (independently of Leibniz).
- Newton's Laws of Motion:
  1. A body not subject to forces, is at rest or moves straight at a constant speed.
  2. $F = ma$
  3. To every action, there is an opposed and equal reaction.
- Law of gravity: $F = G\frac{Mm}{r^2}$
- Newtonian mechanics.
- Mathematical derivation of Kepler's Laws.

Henry Cavendish, 1731 - 1810

1798, experimental measurement of the gravitational constant G. Equivalently - of the mass of the Earth.



Current measurement:
$G = 6.67430 \cdot 10^{-11} \ \ \mathrm{m}^3/(\mathrm{kg} \cdot \mathrm{s}^2)$

Enhancements to Kepler's laws

Kepler's 2nd law, enhanced, vis viva equation:

$$v^2 = G(M+m)\left(\frac{2}{r} - \frac{1}{a}\right)$$

Kepler's 3rd law, enhanced:

$$\frac{T^2}{a^3} = \frac{4\pi^2}{G(M+m)}$$

where:
T – orbital period,
a – semi-major axis,
G – gravitational constant,
$M,\ m$ – masses,
v – speed,
r – radius i.e. distance M to m.

Weighing the Sun and planets with moons

Kepler's 3rd law under the assumption that $M \gg m$:

$$\frac{T^2}{a^3} \approx \frac{4\pi^2}{GM} \quad \Rightarrow \quad M \approx \frac{4\pi^2}{G} \cdot \frac{a^3}{T^2}$$

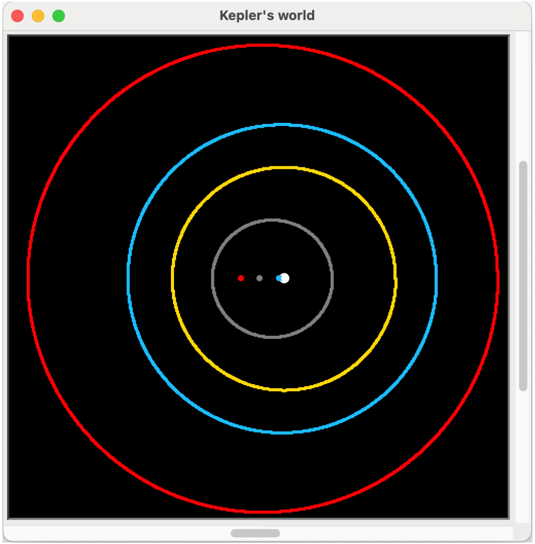
The orbital period T and semi-major axis a are observable.
The mass M is not observable, but it can be calculated from this equation!

$$M_{\text{sun}} \approx \frac{4\pi^2}{G} \cdot \frac{a^3_{\text{earth}}}{T^2_{\text{earth}}} \approx 1.9885 \cdot 10^{30}\text{kg}$$

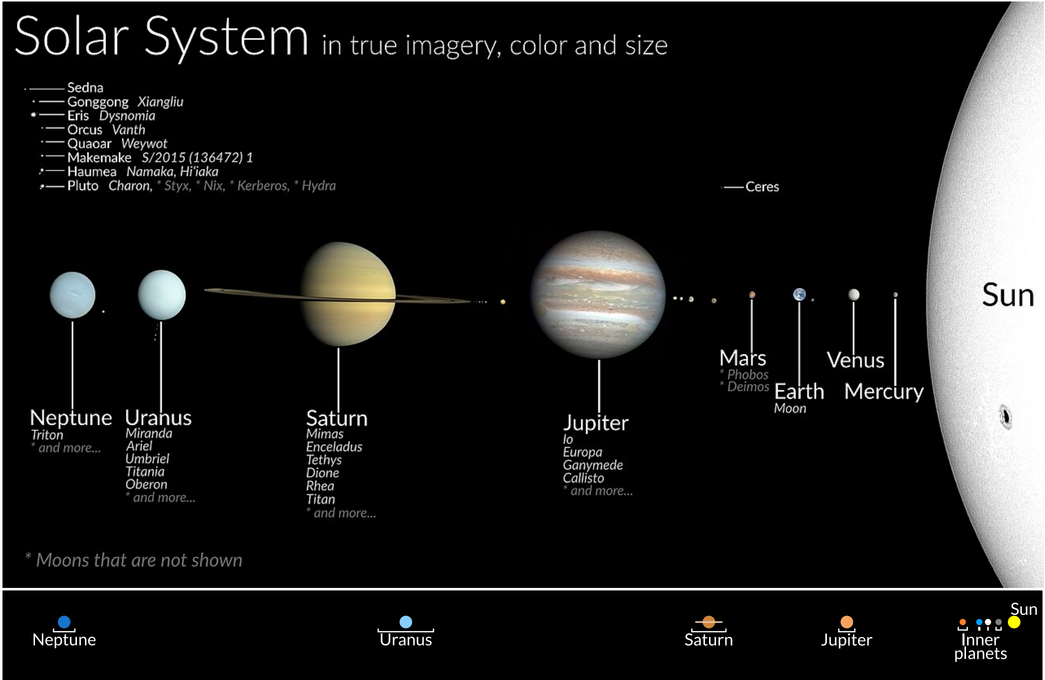
$$M_{\text{mars}} \approx \frac{4\pi^2}{G} \cdot \frac{a^3_{\text{phobos}}}{T^2_{\text{phobos}}} \approx 6.4171 \cdot 10^{23}\text{kg}$$

$$M_{\text{jupiter}} \approx \frac{4\pi^2}{G} \cdot \frac{a^3_{\text{io}}}{T^2_{\text{io}}} \approx 1.8982 \cdot 10^{27}\text{kg}$$

Kepler's world - a computer simulation



- A computer simulation of orbits resulting from the continuing local effect of the Newton's law of gravity $F = G\frac{Mm}{r^2}$
- Tests if the simulated planets obey (global) Kepler's laws 1 and 3.
- The screenshot on the left:
  Mercury, Venus, Earth, Mars.
- Verified ellipses with the Sun in one focus.
- Orbital periods in days,
  simulation vs. actual:
  Mercury: 88.77 vs. .87.97 (0.91% error)
  Venus: 225.46 vs. 224.7 (0.34% error)
  Earth: 365.97 vs 365.26 (0.2% error)
  Mars: 687.85 vs 686.98 (0.13% error)
- Available at `github.com/plazajan`


Solar System in true imagery, color and size
Sedna
Gonggong Xiangliu
Eris Dysnomia
Orcus Vanth
Quaoar Weywot
Makemake S/2015 (136472) 1
Haumea Namaka, Hi'iaka
Pluto Charon, * Styx, * Nix, * Kerberos, * Hydra
Ceres
Sun
Neptune
Triton
* and more...
Uranus
Miranda
Ariel
Umbriel
Titania
Oberon
* and more...
Saturn
Mimas
Enceladus
Tethys
Dione
Rhea
Titan
* and more...
Jupiter
Io
Europa
Ganymede
Callisto
* and more...
Mars
* Phobos
* Deimos
Earth
Moon
Venus
Mercury
* Moons that are not shown
Neptune
Uranus
Saturn
Jupiter
Inner planets
Sun

Other discoveries

Recall: the ecliptic is the plane of Earth's orbit around the Sun.

- Orbital planes of the planets are close to,
  but not identical with the ecliptic.
- Moon's orbit is close to, but not in the ecliptic
  – so eclipses are rare.
- There is an asteroid belt between Mars and Jupyter.
  Asteroids are not spherical.
- Comets are lumps of ice and dust, a few km across,
  on highly eccentric orbits, with orbital periods of the order of 100 years.
- Kuiper belt – icy objects past Neptune.

The Sun and the Solar System are still evolving
– fascinating, but not in the scope of this talk.

Methodology of science

- Abstraction.
- Measurements: relative vs. absolute.
- Statements: qualitative vs. quantitative.
- Approximation (and estimation of error.)
- Continuous vs. discrete.
- Models: local vs. global.
- Areas of science:
  - experimental,
  - theoretical,
  - computational.
- The scientific method:
  observe/experiment, construct a model, make predictions, test, repeat.
- Ockham's razor - given two models with the same predictions, choose the simpler one.

This is the end

This is the end of Chapter I

This is the end of Chapter I
of the great book of science.

This is the end of Chapter I
of the great book of science.

There is more ...

This is the end of Chapter I
of the great book of science.

There is more ...

Stay curious!

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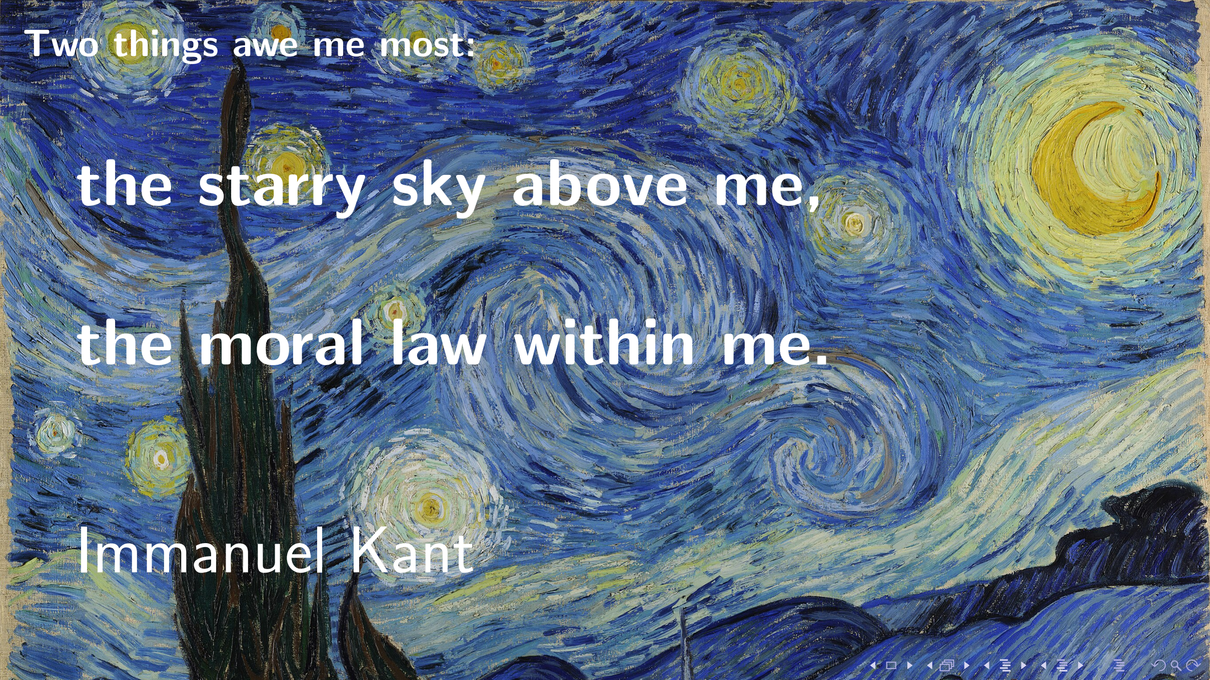

Two things awe me most:

the starry sky above me,

the moral law within me.

Immanuel Kant