

Name:

**Algebra II**  
**Examination 16 (Test)**

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The examination contains ten problems which are worth 10 points each, and two bonus problems worth ten points each. All answers must be justified. An appropriate amount of work must be shown of receive credit.

[illegible]

**Problem 1. (Solving Logarithmic/Rational Equations)**

Solve the equation

$$2\ln(x-1) - \ln(x-5) = 2\ln(3) + \ln(2).$$

Correctly write the solution set.

**Problem 2. (Solving Trigonometric Equations)**

Find all  $x \in [0, 2\pi]$  such that

$$2\cos(x) = 1.$$

Correctly write the solution set. (Here,  $x$  is measured in radians.)

**Problem 3. (Finding the Equation of a Line)**

Let  $A = (-1, 5)$  and  $B = (2, 11)$ . Find slope-intercept form of the equation of the line through  $A$  and  $B$ .

**Problem 4. (Finding the Equation of a Circle)**

Let  $A = (-1, 5)$  and  $B = (2, 11)$ . Find the equation of a circle which has  $\overline{AB}$  as a diameter.

**Problem 5. (Finding Inverses)**

Let

$$f(x) = x^2 + 10x + 32.$$

Let  $f^{-1}$  be an inverse function of  $f$ . Compute a formula for  $f^{-1}(x)$ . State its domain and range.

**Problem 6. (Finding the Domain)**

Let

$$f(x) = \frac{\ln(x^2 - 3x - 40)}{x^2 - 49}.$$

Find the domain of  $f$ . Write your answer as the union of disjoint intervals.

**Problem 7. (Factoring Cubics)**

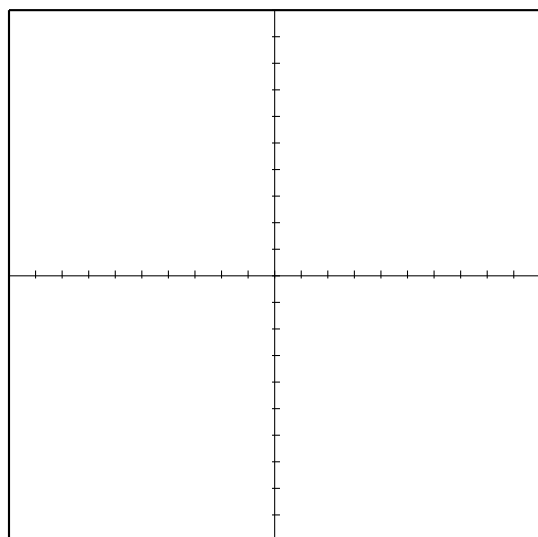
Let

$$f(x) = x^3 - 6x^2 + 2x + 24.$$

Notice that  $f(4) = 0$ . Use this to find the other two zeros of  $f$ . Write the solution set for the equation  $f(x) = 0$ .

**Problem 8. (Graphing)**

Consider the rational function  $f(x) = \frac{8}{x^2 - 4}$ . Find its degree, zeros, and poles. Find its intercepts and asymptotes. Graph the function and label these features.



**Rational Function:**  $f(x) = \frac{8}{x^2 - 4}$

**Degree:**

**Zeros:**

**Poles:**

**$y$ -intercept:**

**$x$ -intercepts:**

**Vertical Asymptotes:**

**Polynomial Asymptote:**

**Problem 9. (Reference Angles)**

For each of the following angle measures (given in radians), convert the measure to degrees, draw the angle with that measure in standard position, and state the reference angle (in degrees).

(a)  $\frac{16\pi}{3}$

(b)  $-\frac{13\pi}{6}$

**Problem 10. (Wrapping Function)**

Find  $W(87\pi/4)$ .

**Problem 11. (Bonus - Equation of a Circle via Perpendicular Bisectors)**

Given two points  $A$  and  $B$ , the *perpendicular bisector* of  $\overline{AB}$  is the line through the midpoint of  $\overline{AB}$  and perpendicular to  $\overleftrightarrow{AB}$ .

Let  $A = (1, 5)$ ,  $B = (5, 1)$ , and  $C = (9, 3)$ . The perpendicular bisector of  $\overline{AB}$  is  $y = x$ .

(a) Find the midpoint of  $\overline{BC}$ .

(b) Find the slope of  $\overleftrightarrow{BC}$ , and the slope of any line perpendicular to  $\overleftrightarrow{BC}$ .

(c) Find the equation of the perpendicular bisector of  $\overline{BC}$ .

(d) Find the center of the unique circle which passes through  $A$ ,  $B$ , and  $C$ .

(e) Find the distance from the center of the circle to point either  $A$ ,  $B$ , or  $C$ .

(f) Find the equation of the unique circle which passes through  $A$ ,  $B$ , and  $C$ .

**Problem 12. (Bonus - Clocks)**

Solve the following clock problems.

- (a) It was 7 am when little Billy was teasing the pit bull. Find the time 83 hours later, when he was discharged from the hospital.

- (b) Find the *exact* angle between the hands of a clock at 2:24 pm. *Thoroughly* justify your answer, using complete sentences.