

Due Wednesday, November 3, 2021. Write all complex number and polynomials in standard form.

**Definition 1.** Recall that a polynomial is *monic* if its leading coefficient is 1.

**Problem 1.** Let  $z = 5 + 3i$ .

(a) Compute  $z + \bar{z}$ .

(b) Compute  $z\bar{z}$ .

(c) Find a monic polynomial  $f$  such that  $f(z) = 0$ .

**Problem 2.** Solve the following quadratic equations. Write the solution set.

(a)  $x^2 + 9 = 0$  (extract the square root)

(b)  $x^2 + 8x + 16 = 0$  (its a perfect square)

(c)  $x^2 - 5x - 15 = 0$  (factor)

(d)  $3x^2 = 7x - 11$  (quadratic formula)

**Definition 2.** Let  $f$  and  $g$  be polynomials. We say that  $f$  *divides*  $g$ , and write  $f \mid g$ , if there exists a polynomial  $q$  such that  $g = fq$ .

If  $f \mid g$ , we may say that  $f$  is a *factor* of  $g$ , or that  $g$  is a *multiple* of  $f$ .

**Proposition 1. (Factor Theorem)**

Let  $g(x)$  be a polynomial and let  $a \in \mathbb{C}$ . Let  $f(x) = x - a$ . Then  $f$  is a factor of  $g$  if and only if  $g(a) = 0$ .

**Problem 3.** Write a monic polynomial  $f$  with real coefficients such that  $f(1) = 0$  and  $f(i) = 0$

**Problem 4.** Write a polynomial  $g$  with integer coefficients such that  $g(5) = 0$  and  $g\left(\frac{2}{3}\right) = 0$ .

**Problem 5.** Let  $f(x) = x^5 + 4x^3 - 8x + 3$ .

(a) Find the remainder when  $f(x)$  is divided by 2.

(b) Find the remainder when  $f(x)$  is divided by  $-1$ .

(c) Find the remainder when  $f(x)$  is divided by  $x^2$ .