Problem 1. Fermat found the slope of a tangent line by computing the slope from point $A = (x, f(x))$ to point $B = (x + h, f(x + h))$, simplifying to cancel an h, and then setting $h = 0$ and simplifying again. Try this for the case Descartes considered; let $f(x) = \sqrt{x}$, and find the slope of the tangent line at $(1, 1)$ as follows:

(a) The average rate of change of $f(x)$ from $x = a$ to $x = b$ is $\dfrac{\Delta y}{\Delta x} = \dfrac{f(b) - f(a)}{b - a}$. Compute the average rate of change of $f(x) = \sqrt{x}$ from $a = 1$ to $b = 1 + h$; you get a “difference quotient”.

(b) You want to set $h = 0$ but you cannot divide by zero. So, multiply the top and the bottom by the “conjugate” of the top. Simplify until the h on the bottom cancels with one on the top.

(c) Now plug in $h = 0$ everywhere else. What is the slope?

Problem 2. Repeat the process above, but in more generality. Let $f(x) = \sqrt{x}$, and find the slope of the tangent line at $(x, f(x))$ using Fermat’s method:

(a) Compute the average rate of change of $f(x) = \sqrt{x}$ from x to $x + h$.

(b) Multiply the top and the bottom by the “conjugate” of the top. Simplify until h cancels.

(c) Plug in $h = 0$. What is the slope? (It is a function of x.)

Problem 3. Repeat Problem 2 with the function $f(x) = x^2$.

Problem 4. Repeat Problem 2 with the function $f(x) = x^3$. Note that $(x-h)^3 = x^3 - 3hx^2 + 3h^2x - h^3$.

Problem 5. Repeat Problem 2 with the function $f(x) = \dfrac{1}{x}$.