PRINCIPLES OF ANALYSIS
PROBLEM SET E

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ABSTRACT. The problems are due Thursday, December 4, 2003.

Problem 1 (Exercise 4.25)**.** Let $f : (a, b) \to \mathbb{R}$ be differentiable on (a, b). Suppose that there exists $M > 0$ such that for every $x \in (a, b)$, we have $|f'(x)| \leq M$.

- **(a)** Show that if $x, y \in (a, b)$, then $|\frac{f(x)-f(y)}{x-y}| \leq M$.
- **(b)** Show that f is uniformly continuous on (a, b).

Hints.
(a) use MVT.
(b) Start like this:
Let $\epsilon > 0$. Set $\delta =$ (an appropriate quantity). We show that if $x, y \in (a, b)$ and $|x - y| < \delta$, then $|f(x) - f(y)| < \epsilon$. □

Problem 2 (Exercise 4.35)**.** Let $f(x) = x^3 + 2x^2 - x + 1$. Find an equation for the line tangent to the graph of f^{-1} at the point $(3, 1)$.

Observation 1 (Alternate Definition)**.** Let $f : \mathbb{R} \to \mathbb{R}$ and let $x_0 \in \mathbb{R}$. Define

$$Q : \mathbb{R} \smallsetminus \{0\} \to \mathbb{R} \quad \text{by} \quad Q(h) = \frac{f(x_0 + h) - f(x_0)}{h}.$$

Then f is differentiable at x_0 if and only if $\lim_{h\to 0} Q(h)$ exists, in which case $f'(x_0) = \lim_{h\to 0} Q(h)$.

Problem 3 (Exercise 4.39)**.** Let $f : \mathbb{R} \to \mathbb{R}$ be a function satisfying

- **(1)** $f(0) = 1$;
- **(2)** f is differentiable at 0 and $f'(0) = 1$;
- **(3)** $f(x + y) = f(x)f(y)$.

Show that f is differentiable on $\mathbb{R}$ and that $f'(x) = f(x)$ for every $x \in \mathbb{R}$.

Hint. Use the preceding alternate definition of differentiable. □

Definition 1. A function $f : [-b, b] \to \mathbb{R}$ is called *odd* if $f(x) = -f(x)$ for every $x \in [-b, b]$.

Problem 4 (Exercise 5.14)**.** Let $f : [-b, b] \to \mathbb{R}$ be an odd function which is integrable on $[-b, b]$. Show that $\int_{-b}^{b} f\, dx = 0$.

Problem 5 (Exercise 5.27)**.** Let $f, g : [a, b] \to \mathbb{R}$ be integrable on $[a, b]$. Define $h : [a, b] \to \mathbb{R}$ by $h(x) = \max\{f(x), g(x)\}$. Show that h is integrable on $[a, b]$.

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