Category Theory

Course Information 2019-2020

Course Description

Category Theory builds a framework that allows mathematicians to study isolated aspects of potentially multifaceted, complicated mathematical structures. For example, the set of real numbers has order, distance, and arithmetic. To better understand the real numbers, we may study all objects that have order only, or distance only, or arithmetic only, and find what these properties contribute to the reals.

Our study of set-based categories proceeds approximately as follows. First we identify and axiomatize the structure to be studied; for example, order, distance, or binary operations. An *object* is a set with such an additional structure; for example, an ordered set. A *morphism* between objects is a function which preserves the order; for example, an order preserving map between ordered sets. A *category* is the collection of objects which a type of structure, together with their morphisms; for example, the category of ordered sets.

We start with the algebraic categories of groups, rings, and fields, and proceed to the geometric categories such as ordered sets, metric spaces, and topological spaces.

Course Information

Teacher: Dr. Paul L. Bailey

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Website: `http://plbailey79.github.io/portal`

Book: *Contemporary Abstract Algebra*, 5^{th} edition, Joseph Gallian, ISBN: 978-0618122141

Book: *Introduction to Topology*, 3^{rd} edition, Bert Mendelson, ISBN: 978-0486663524

Grade Components

Classwork:	10%
Homework:	10%
Quizzes:	20%
Exams:	60%

Classwork consists of participation in discussion, and activities such as team quizzes, worksheets, and other group work. Classwork activities are normally be graded on a scale of zero to ten.

Homework exercises from the textbook will be routinely, to be due at the beginning of the next class period. Homework assignments will be graded on a scale of zero to ten.

Quizzes are about twenty minutes long and occur almost every week, normally on Wednesday, covering the previous week's worth of material. These will be graded on a scale of zero to ten.

Exams are hour long assessments and are cumulative in nature. We will have about three exams per trimester. These will be graded on a scale of zero to one hundred points.

Homework is normally computational in nature; more complex problems may be assigned as take-home quizzes or exams.

Course Outline

Below we list an approximate sequence and pacing of topics. Please note that this information is subject to change by the instructor in the best interests of the class.

Week	Unit	Topic	Gallian
Week 1	Set Theory	Sets, collections, functions	Chapter 0
Week 2	Set Theory	Relations, binary operations	Chapter 0
Week 3	Set Theory	Integers, modular arithmetic	Chapter 0
Week 4	Set Theory	Permutations	Chapter 1, 5
Week 5	Group Theory	Groups, subgroups, cyclic groups	Chapter 2, 3, 4
Week 6	Group Theory	Group homomophisms, kernels	Chapter 6
Week 7	Group Theory	Cosets	Chapter 7
Week 8	Group Theory	Normal subgroups, factors groups	Chapter 9
Week 9	Ring Theory	Rings, domains, fields	Chapter 12, 13
Week 10	Ring Theory	Polynomial rings	Chapter 16
Week 11	Ring Theory	Ring homomorphisms, kernels	Chapter 15
Week 12	Ring Theory	Ideals, factor rings	Chapter 14
Week 13	Field Theory	Vector spaces	Chapter 19
Week 14	Field Theory	Field extensions	Chapter 20, 21
Week 15	Field Theory	Finite fields	Chapter 22
Week 16	Category Theory	Categories, graphs, posets, esets	
Week 17	Category Theory	Automorphisms, functors	
Week 18	Topology	Metrics, neighborhoods	
Week 19	Topology	Isometries, continuity	
Week 20	Topology	Topologies, subspaces, bases	
Week 21	Topology	Connectedness, compactness	
Week 22	Covers	Group actions	
Week 23	Covers	Even covers	
Week 24	Covers	Fundamental group	