

# Category Theory

## Logic and Set Summary

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### Logic Notation Summary

Symbol	Abbrev	Name	Format
$\neg$	NOT	Negation	$\neg p$
$\wedge$	AND	Conjunction	$p \wedge q$
$\vee$	OR	Disjunction	$p \vee q$
$\Rightarrow$	IMP	Implication	$p \Rightarrow q$
$\Leftrightarrow$	IFF	Equivalence	$p \Leftrightarrow q$
$\Uparrow$	XOR	Exclusion	$p \Uparrow q$
$\Downarrow$	NOR	Alternate Denial	$p \Downarrow q$
$\Uparrow$	NAND	Joint Denial	$p \Uparrow q$

Table 1: Logical Operators

$p$	$q$	$\neg p$	$p \wedge q$	$p \vee q$	$p \Rightarrow q$	$p \Leftrightarrow q$	$p \Uparrow q$	$p \Downarrow q$	$p \Uparrow q$
<b>T</b>	<b>T</b>	<b>F</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>F</b>	<b>F</b>	<b>F</b>
<b>T</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>T</b>	<b>F</b>	<b>F</b>	<b>T</b>	<b>F</b>	<b>T</b>
<b>F</b>	<b>T</b>	<b>T</b>	<b>F</b>	<b>T</b>	<b>T</b>	<b>F</b>	<b>T</b>	<b>F</b>	<b>T</b>
<b>F</b>	<b>F</b>	<b>T</b>	<b>F</b>	<b>F</b>	<b>T</b>	<b>T</b>	<b>F</b>	<b>T</b>	<b>T</b>

Table 2: Truth Tables

#### Precedence of Operators

1. NOT
2. AND, OR
3. XOR, NOR, NAND
4. IMP
5. IFF

Symbol	Abbrev	Meaning
$\forall$	FORALL	for every (for all)
$\exists$	EXISTS	there exists (for some)
$\exists!$	UNIQUE	there exists uniquely
$\ni$	ST	such that

Table 3: Quantifiers

Set Notation Summary

Symbol	Meaning	Definition
$\in$	is an element of	Example: $\pi \in \mathbb{R}$
$\notin$	is not an element of	Example: $\pi \notin \mathbb{Q}$
$\subset$	is a subset of	$A \subset B \Leftrightarrow (a \in A \Rightarrow a \in B)$
$\cap$	intersection	$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
$\cup$	union	$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
$\setminus$	complement	$A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$
$\times$	cartesian product	$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$

Table 4: Set Operations

Set	Name	Definition
$\mathbb{N}$	Natural Numbers	$\{1, 2, 3, \dots\}$
$\mathbb{Z}$	Integers	$\{\dots, -2, -1, 0, 1, 2, \dots\}$
$\mathbb{Q}$	Rational Numbers	$\{p/q \mid p, q \in \mathbb{Z}\}$
$\mathbb{R}$	Real Numbers	$\{\text{“Dedekind Cuts”}\}$
$\mathbb{C}$	Complex Numbers	$\{a + ib \mid a, b \in \mathbb{R} \text{ and } i^2 = -1\}$
$\mathbb{R}^2$	Euclidean Plane	$\{(a, b) \mid a, b \in \mathbb{R}\}$
$\mathbb{R}^3$	Euclidean Space	$\{(a, b, c) \mid a, b, c \in \mathbb{R}\}$

Table 5: Standard Sets