Sunday, March 29, 2020

Solutions coming soon to a theater near you.

Problem 1. Use the fact that exp and log are injective to solve these problems. That is, if $\exp a = \exp b$, then a = b, and if $\log a = \log b$, then a = b.

(a) Find all $x \in \mathbb{R}$ such that $7^{x^2-4x+1} = 49^{x-2}$.

(b) Find all $x \in \mathbb{R}$ such that $\ln(x+1) + \ln(x+2) = \ln(x+3)$.

Problem 2. Solve the initial value problem

$$\frac{dy}{dx} = 3x^2 - 4 \quad \text{where } y(2) = 5.$$

Problem 3. Let f and g be functions that are differentiable everywhere, such that g is the inverse function of f. Suppose that g(-2) = 5 and $f'(5) = -\frac{1}{2}$. Find g'(-2).

Problem 4. Compute

$$\int \frac{\sec^2 y \, dy}{\sqrt{1 - \tan^2 x}}.$$