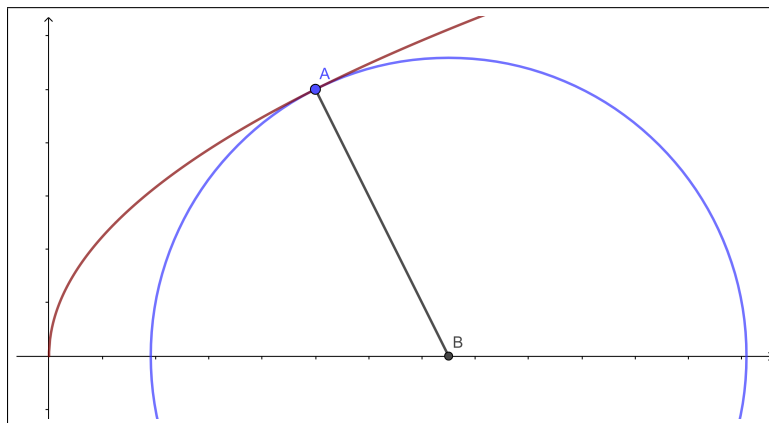


**Problem 1.** Let  $f(x) = \sqrt{x}$ . Then the point  $(1,1)$  is a point on the graph of  $f$ . There is a unique line which passes through  $(1,1)$  and otherwise lies above the graph of  $f$ . This is called the *tangent line*.



Descartes found the slope of this line by first finding the unique circle, centered on the  $x$ -axis, which is tangent to the graph of  $f$ . Let  $(h,0)$  be the center of this circle. In the diagram above, point  $A = (1,1)$  is the point of tangency, and point  $B = (h,0)$  is the center of the circle.

- (a) Find the distance  $r$  from  $A$  to  $B$ , written in terms of  $h$ .
- (b) Write the equation of the circle centered at  $B$  with radius  $r$ .
- (c) Plug  $y = \sqrt{x}$  into the equation of the circle. You now have a quadratic equation in variable  $x$  and constant  $h$ . Put this equation in standard form  $ax^2 + bx + c = 0$ .

(d) Write the  $a$ ,  $b$ , and  $c$  you found in part (c). Plug  $a$ ,  $b$ , and  $c$  into the discriminant  $b^2 - 4ac$ .

(e) We want the discriminant to be zero (why?). Set  $b^2 - 4ac = 0$  and solve for  $h$ .

(f) Find the slope of the line through  $A$  and  $B$ .

(g) Find the slope of the tangent line.