MATH 2753 | LINEAR ALGEBRA | Project 3 | Name:
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Complete these problems and turn in the solution by Friday, March 23, 2007. Attach this page to the front of the solution. Solutions should be self explanatory and written in complete sentences.

Gaussian Elimination

Problem 1. Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ which rotates by $90°$ around the z-axis, then stretches the x-axis by 2, the y-axis by 3, and the z-axis by 5, then rotates by $180°$ around the x-axis, then reflects through the plane $x = y$.

(a) Find the matrix A of T.

(b) Find A^{-1}.

(c) Find the unique vector $\vec{x} \in \mathbb{R}^3$ such that $T(\vec{x}) = \langle 1, 1, 1 \rangle$.

Problem 2. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation whose corresponding matrix is

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 6 \\ 0 & 0 & 1 \end{bmatrix}.$$

Write A are a product of elementary invertible matrices, and describe the geometric effect of T by describing the geometric effect of each of these elementary invertible matrices.

Problem 3. Consider the four hyperplanes in $\mathbb{R}^5$ given by the equations

$$\begin{aligned} x_1 - x_2 - x_3 + x_4 + 5x_5 &= 1 \\ 2x_1 - 2x_2 - 2x_3 + x_4 + 4x_5 &= 1 \\ 3x_1 - 2x_2 + 2x_4 + 14x_5 &= 1 \\ 4x_1 - 3x_2 - x_3 + 4x_4 + 15x_5 &= 1 \end{aligned}$$

Find a parametric function $P : \mathbb{R}^2 \to \mathbb{R}^5$ whose image is the intersection of these hyperplanes.