PRINCIPLES OF ANALYSIS
PROBLEM SET D

PAUL L. BAILEY

ABSTRACT. The problems are taken from the book and are due Thursday, November 20, 2003.

Problem 1 (Exercise 3.19)**.** Let $f, g : D \to \mathbb{R}$ be uniformly continuous. Show that the function $f + g : D \to \mathbb{R}$ is uniformly continuous. What can be said about the function $fg : D \to \mathbb{R}$? Justify.

Problem 2 (Exercise 3.20)**.** Let $f : A \to B$ and $g : B \to C$ be uniformly continuous. What can be said about the function $g \circ f : A \to C$? Justify.

Problem 3 (Exercise 3.23)**.** A function $f : \mathbb{R} \to \mathbb{R}$ is *periodic* if there exists $h \in \mathbb{R}$ with $h > 0$ such that $f(x + h) = f(x)$ for all $x \in \mathbb{R}$. Show that if $f : \mathbb{R} \to \mathbb{R}$ is periodic and continuous, then it is uniformly continuous.

Problem 4 (Exercise 3.31)**.** Suppose $f : [a, b] \to \mathbb{R}$ and $g : [a, b] \to \mathbb{R}$ are continuous. Let $T = \{x \in [a, b] \mid f(x) = g(x)\}$. Show that T is closed.

Problem 5 (Exercise 3.44)**.** Suppose that $f : [a, b] \to [a, b]$ is continuous. Show that f has a *fixed point*, that is, there exists $x \in [a, b]$ such that $f(x) = x$.

DEPARTMENT OF MATHEMATICS AND CSCI, SOUTHERN ARKANSAS UNIVERSITY
E-mail address: `plbailey@saumag.edu`

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