CATEGORY THEORY
CATEGORY III - GRAPHS

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1. Graphs

Definition 1 (Objects)**.** Let *graph* $(V, \mathcal{E})$ consists of a set V together with a collection of subsets $\mathcal{E} \subset \mathcal{P}(V)$ such that each member of $\mathcal{E}$ contains exactly two elements. The members of V are called *vertices* and the members of $\mathcal{E}$ are called *edges*.

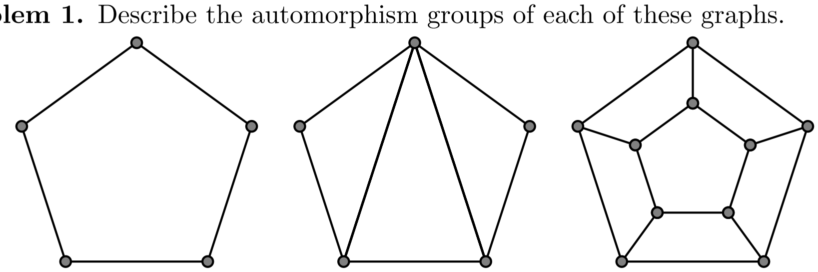
Definition 2 (Subobjects)**.** Let $(V, \mathcal{E})$ be a graph. A *subgraph* of V consists of a set $W \subset V$ and a set $\mathcal{F} \subset \mathcal{E}$ such that $\{w_1, w_2\} \in \mathcal{F}$ implies $w_1, w_2 \in H$.

Definition 3 (Morphisms)**.** Let $(G, \mathcal{E})$ and $(H, \mathcal{F})$ be graphs. A function $f : G \to H$ is called *edge preserving* if

$$\{v_1, v_2\} \in \mathcal{E} \quad \Rightarrow \quad \{f(v_1), f(v_2)\} \in \mathcal{F}.$$

The identity map on V is edge preserving, and the composition of edge preserving functions is edge preserving. Thus, graphs with edge preserving maps form a category.

Problem 1. Describe the automorphism groups of each of these graphs.



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