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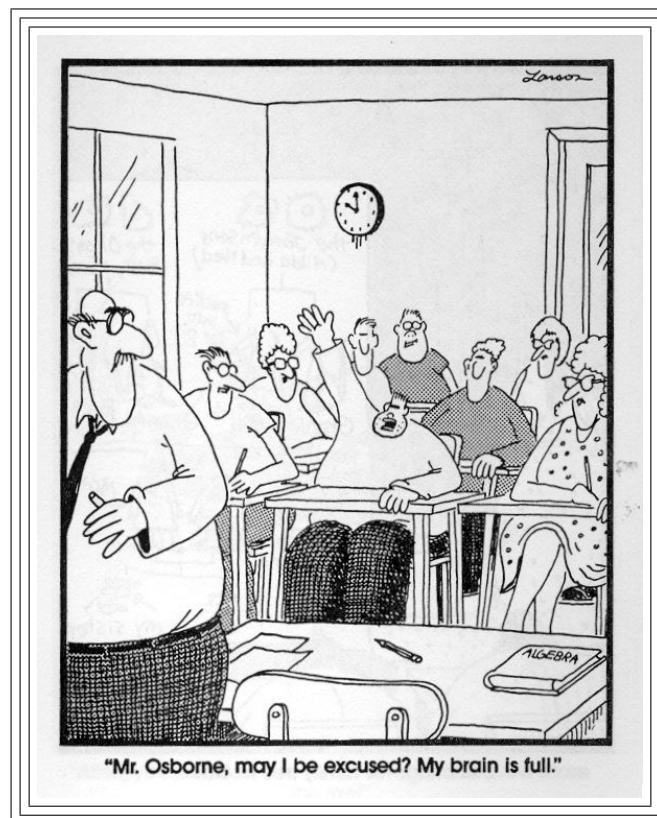
**Algebra II**  
**Examination 13 (Test)**

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The examination contains ten problems which are worth 10 points each, and two bonus problems worth ten points each. All answers must be justified. An appropriate amount of work must be shown of receive credit.

The formula for the area  $A$  of a triangle with base  $b$  and height  $h$  is

$$A = \frac{1}{2}bh.$$

[illegible]

**Problem 1. (Solving Logarithmic Equations)**

Find all  $x \in \mathbb{R}$  such that

$$\log_{343} \left( \frac{1}{49} \right) = 5x.$$

**Problem 2. (Solving Exponential Functions)**

Find all  $x \in \mathbb{R}$  such that

$$8^{(5x+1)} = \frac{1}{64^{(x-2)}}.$$

**Problem 3. (Finding the Equation of a Line)**

Find slope-intercept form of the equation of the line through  $(5, -2)$  and  $(12, 12)$ .

**Problem 4. (Finding the Equation of a Circle)**

Find the equation of the circle which passes through  $(0, 0)$ ,  $(8, 0)$ , and  $(0, 2)$ .

**Problem 5. (Finding Inverses)**

Let

$$f(x) = x^2 - 4x - 28.$$

Let  $g$  be an inverse function of  $f$ . Compute a formula for  $g(x)$ . State its domain and range.

**Problem 6. (Finding the Domain)**

Let

$$f(x) = \frac{\sqrt{25 - x^2}}{x^2 - 4}.$$

Find the domain of  $f$ . Write your answer as the union of disjoint intervals.

**Problem 7. (Factoring Cubics)**

Let

$$f(x) = x^3 - 8x^2 + x + 42.$$

Notice that  $f(3) = 0$ . Use this to find the other two zeros of  $f$ . Write the solution set for the equation  $f(x) = 0$ .

**Problem 8. (Set Operations)**

Compute the following sets. Write your answer using correct set notation.

Let  $A = [5, 12]$ ,  $B = (7, 14]$ , and  $C = \{1, 4, 9, 16\}$ .

(a)  $A \cup B$

(b)  $A \cap B$

(c)  $A \setminus B$

(d)  $B \setminus A$

(e)  $A \setminus C$

**Problem 9. (Classifying Triangles)**

Let  $A, B, C \in \mathbb{R}^2$  be given by  $A = (0, 0)$ ,  $B = (5, 2)$ , and  $C = (-4, 10)$ . Show that  $\triangle BAC$  is a right triangle.

**Problem 10. (Area of a Triangle)**

Let  $A, B, C \in \mathbb{R}^2$  be given by  $A = (0, 0)$ ,  $B = (5, 2)$ , and  $C = (-2, 5)$ . Find the area of  $\triangle BAC$ .

**Problem 11. (Bonus - Lines and Circles)**

A line through the origin of slope  $\frac{1}{2}$  is tangent to a circle centered at  $(0, 10)$ .

(a) Sketch this situation.

(b) Write the equation of the line.

(c) Find the point of tangency.

(d) Write equation of the circle.

**Definition 1.** Let  $f : A \rightarrow B$ , and let  $D \subset B$ . The *preimage* of  $B$  under  $f$  is

$$f^{-1}(D) = \{a \in A \mid f(a) \in D\}.$$

That is, the preimage of  $D$  is the set of all points in the domain which are mapped by  $f$  into  $D$ .

**Problem 12. (Bonus - Preimage)**

Let  $A = \mathbb{R}$  and  $B = [0, \infty)$ . Let  $f : A \rightarrow B$  be given by  $f(x) = x^2$ . Let  $D \subset B$  be given by  $D = [1, 3)$ . Find  $f^{-1}(D)$ .