

# Trigonometry Summary

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### Definitions

The *radian measure* of an angle is the arclength of the arc on a circle of radius 1 which subtends the angle when the vertex is placed at the center of the circle. We can convert between degrees and radians by using the fact that

$$360^\circ = 2\pi \text{ radians.}$$

The *wrapping function*  $P : \mathbb{R} \rightarrow \mathbb{R}^2$  is defined to be the point  $P(\theta)$  on the unit circle  $x^2 + y^2 = 1$  obtained by moving counterclockwise along the circle an arclength of  $\theta$ . If  $\theta < 0$ , this is interpreted as moving clockwise by an arclength of  $|\theta|$ .

The *sine* and *cosine* functions as defined by

$$\sin \theta = \text{the } y\text{-coordinate of } P(\theta) \quad \text{and} \quad \cos \theta = \text{the } x\text{-coordinate of } P(\theta).$$

It is clear that

$$\sin^2 \theta + \cos^2 \theta = 1.$$

The *tangent*, *cotangent*, *secant*, and *cosecant* are defined by

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \cot \theta = \frac{\cos \theta}{\sin \theta}, \quad \sec \theta = \frac{1}{\cos \theta}, \quad \csc \theta = \frac{1}{\sin \theta}.$$

### Trigonometric Identities

#### Identities that Come from Geometry

Description	Identity
<b>Pythagorean Identity</b>	$\cos^2 \theta + \sin^2 \theta = 1$
<b>Symmetry</b>	$\sin(-\theta) = -\sin \theta$ $\cos(-\theta) = \cos \theta$
<b>Periodicity</b>	$\sin(\theta + 2\pi) = \sin \theta$ $\cos(\theta + 2\pi) = \cos \theta$
<b>Rotation by <math>180^\circ</math></b>	$\sin(\theta + \pi) = -\sin \theta$ $\cos(\theta + \pi) = -\cos \theta$
<b>Rotation by <math>90^\circ</math></b>	$\sin(\theta + \frac{\pi}{2}) = \cos \theta$ $\cos(\theta - \frac{\pi}{2}) = \sin \theta$
<b>Reflection across <math>45^\circ</math></b>	$\sin(\frac{\pi}{2} - \theta) = \cos \theta$ $\cos(\frac{\pi}{2} - \theta) = \sin \theta$
<b>Difference of Angles Formula</b>	$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$

#### Identities that Come from Algebra

Description	Identity
<b>Sum and Difference Formulas</b>	$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$ $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$ $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$ $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$
<b>Double Angle Formulas</b>	$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$ $\sin(2\theta) = 2 \sin \theta \cos \theta$
<b>Half Angle Formulas</b>	$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}}$ $\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}}$
<b>Pythagorean Formulae</b>	$1 + \tan^2 \theta = \sec^2 \theta$ $\cot^2 \theta + 1 = \csc^2 \theta$

### Trigonometric Values

$\deg(\theta)$	$\text{rad}(\theta)$	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\cot(\theta)$	$\sec(\theta)$	$\csc(\theta)$
$0^\circ$	0	0	1	0	$\infty$	1	$\infty$
$15^\circ$	$\frac{\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{2-\sqrt{3}}{2}$	$\frac{2+\sqrt{3}}{2}$	$\sqrt{6}-\sqrt{2}$	$\sqrt{6}+\sqrt{2}$
$18^\circ$	$\frac{\pi}{10}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{5-2\sqrt{5}}{2}$	$\sqrt{5+2\sqrt{5}}$	$\sqrt{2\sqrt{5}-5}$	$\sqrt{5}+1$
	$\frac{\pi}{8}$	$\frac{\sqrt{2}-\sqrt{2}}{2}$	$\frac{\sqrt{2}+\sqrt{2}}{2}$	$\sqrt{3-2\sqrt{2}}$	$\sqrt{3+2\sqrt{2}}$	$\sqrt{4-2\sqrt{2}}$	$\sqrt{4+2\sqrt{2}}$
$30^\circ$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
$36^\circ$	$\frac{\pi}{5}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{1+\sqrt{5}}{4}$	$\sqrt{5-2\sqrt{5}}$	$\frac{\sqrt{25+10\sqrt{5}}}{5}$	$\sqrt{5}-1$	$\frac{10+2\sqrt{5}}{5}$
$45^\circ$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
$54^\circ$	$\frac{3\pi}{10}$	$\frac{1+\sqrt{5}}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{25+10\sqrt{5}}}{5}$	$\sqrt{5-2\sqrt{5}}$	$\frac{10+2\sqrt{5}}{5}$	$\sqrt{5}-1$
$60^\circ$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$
	$\frac{3\pi}{8}$	$\frac{\sqrt{2}+\sqrt{2}}{2}$	$\frac{\sqrt{2}-\sqrt{2}}{2}$	$\sqrt{3+2\sqrt{2}}$	$\sqrt{3-2\sqrt{2}}$	$\sqrt{4+2\sqrt{2}}$	$\sqrt{4-2\sqrt{2}}$
$72^\circ$	$\frac{2\pi}{5}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{5+2\sqrt{5}}$	$\frac{5-2\sqrt{5}}{2}$	$\sqrt{5}+1$	$\sqrt{2\sqrt{5}-5}$
$75^\circ$	$\frac{5\pi}{12}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{2+\sqrt{3}}{2}$	$\frac{2-\sqrt{3}}{2}$	$\sqrt{6}+\sqrt{2}$	$\sqrt{6}-\sqrt{2}$
$90^\circ$	$\frac{\pi}{2}$	1	0	$\infty$	0	$\infty$	1