Trigonometric Identities
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Definition

The *wrapping function* $P : \mathbb{R} \to \mathbb{R}^2$ is defined to be the point $P(\theta)$ on the unit circle $x^2 + y^2 = 1$ obtained by moving counterclockwise along the circle an arclength of θ. Then

$$\sin\theta = \text{ the } y\text{-coordinate of } P(\theta) \quad \text{and} \quad \cos\theta = \text{ the } x\text{-coordinate of } P(\theta).$$

Identities that come from Geometry

(1) Pythagorean Identity
$\cos^2\theta + \sin^2\theta = 1$

(2) Symmetry
$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$

(3) Periodicity
$\sin(\theta + 2\pi) = \sin\theta$
$\cos(\theta + 2\pi) = \cos\theta$

(4) Rotation by $180°$
$\sin(\theta + \pi) = -\sin\theta$
$\cos(\theta + \pi) = -\cos\theta$

(5) Rotation by $90°$
$\sin(\theta + \frac{\pi}{2}) = \cos\theta$
$\cos(\theta - \frac{\pi}{2}) = \sin\theta$

(6) Reflection across $45°$
$\sin(\frac{\pi}{2} - \theta) = \cos\theta$
$\cos(\frac{\pi}{2} - \theta) = \sin\theta$

(7) Difference of Angles Formula
$\cos(\alpha - \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

Description	Identity
Pythagorean Identity	$\cos^2\theta + \sin^2\theta = 1$