Name:

Algebra II
Examination 5

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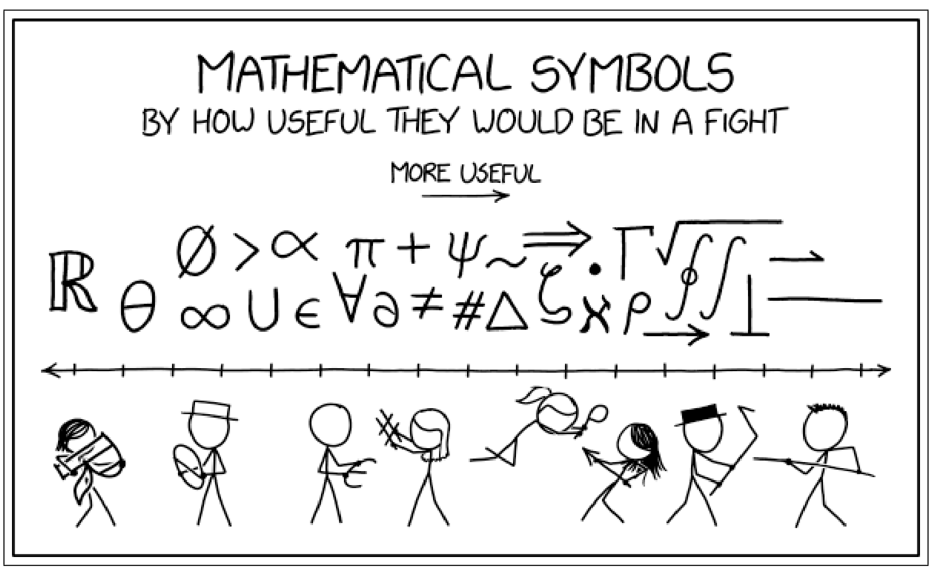
Thursday, December 2, 2021

The examination contains five problems which are worth 20 points each, and two bonus problems worth an additional 20 points each, for a maximum of 100 points. Calculators and all other electronic devices are prohibited.

- *ALL* answers must the justified with appropriate words, sentences, and/or computations.
- *DO NOT* write a negative number inside a square root.
  Make appropriate use of the symbol i if necessary.
- *Standard Form* of a complex number is $x + yi$. Always write complex numbers in standard form.
- *Standard Form* of a polynomial is $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$; that is, with like terms combined, and in decreasing power order. Always write polynomials in standard form, unless otherwise indicated.



Prob 1	Prob 2	Prob 3	Prob 4	Prob 5	Bonus 1	Bonus 2	Total Score

Problem 1. (Matching)
Match the terms or phrases on the left with the descriptions on the right. Write the number of the matching description in the blank next to each term. Use each description exactly once.

(a) ______ Connected	**(1)**	The set of decimal expansions.
(b) ______ Remainder Theorem	**(2)**	There exist q and r such that $g = fq + r$ and $\deg(r) < \deg(f)$.
(c) ______ $\mathbb{Q}$	**(3)**	A description of how a polynomial behaves away from the origin.
(d) ______ Multiplicity	**(4)**	If $f(z) = 0$, then $f(\overline{z}) = 0$.
(e) ______ Division Algorithm	**(5)**	For every $a, b \in A$, if $a < x < b$, then $x \in A$.
(f) ______ $\mathbb{R}^2$	**(6)**	If r is the remainder when g is divided by $x - a$, then $g(a) = r$.
(g) ______ Conjugate Pairs Theorem	**(7)**	The set of fractions.
(h) ______ End Behavior	**(8)**	If $g(a) = 0$, then $x - a$ divides g.
(i) ______ $\mathbb{R}$	**(9)**	The maximum power of $(x - a)$ which divides a polynomial.
(j) ______ Factor Theorem	**(10)**	The set of ordered pairs of real numbers.

Problem 2. (Solving Equations)
Find all real numbers x which satisfy the following equations. Simplify all fractions and radicals where possible. Using correct set notation, write the solution set.

(a) $5 - 11x = 7x + 18$

(b) $169x^2 + 5 = 21$

(c) $x^2 - 26x + 169 = 0$

(d) $x^2 - 5x - 14 = 0$

(e) $x^3 + 21 = 7x^2 + 3x$

Problem 3. (Intervals)
Write the following subsets of $\mathbb{R}$ using correct set notation.

(a) The set of real numbers greater than or equal to 5.

(b) The set of real numbers which are strictly greater than 2 and less than or equal to 13.

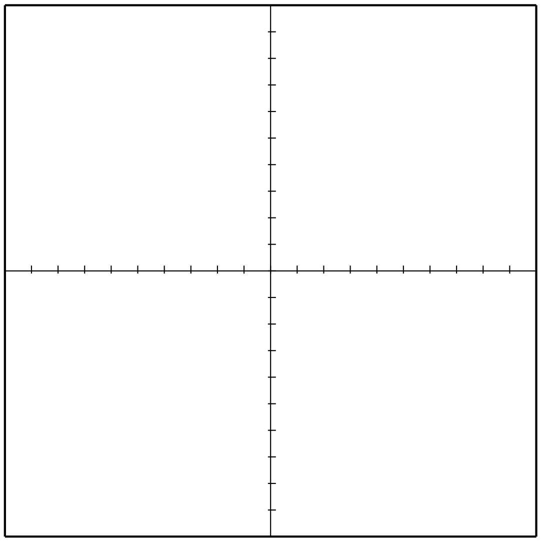
(c) The set of real numbers whose square is less than or equal to 7.

(d) The set of real numbers such that $x^2 - 5x < 14$.

(e) The set of real numbers such that $x^3 + 21 \leq 7x^2 + 3x$.

Problem 4. (Graphing) Fill out the charts, and sketch the graph.

(a) Consider the quadratic function $f(x) = x^2 - 2x - 7$. Find the standard form $f(x) = ax^2 + bx + c$ and the shifted form $f(x) = a(x-h)^2 + k$. Identify the constants a, b, c, h, and k. Find the zeros, intercepts, and vertex. Graph the function and label these points.



Quadratic Function: $f(x) = x^2 - 2x - 7$

Standard Form:

Shifted Form:

a: **b:** **c:** **h:** **k:**

Discriminant:

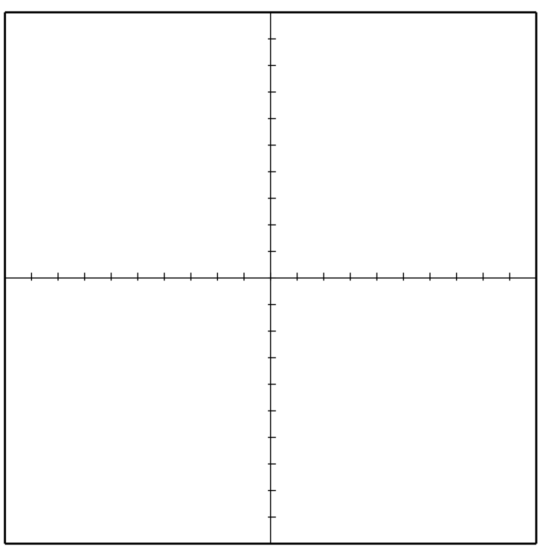
Zeros:

y**-intercept:**

x**-intercept(s):**

Vertex:

(b) Consider the polynomial function $f(x) = x^3 - 2x^2 - 4x + 8$. Find its degree, leading coefficient, constant coefficient, zeros, and end behavior. Find the y-intercept and x-intercepts. Graph the function and label these points.



Polynomial: $f(x) = x^3 - 2x^2 - 4x + 8$

Degree:

Leading Coefficient:

Constant Coefficient:

Zeros:

y**-intercept:**

x**-intercepts:**

End Behavior:

Problem 5. (Polynomial Division)
Let $f(x) = x^3 - 8x^2 + x + 42$.

(a) Find $f(3)$.

(b) Find the quotient and remainder when $f(x)$ is divided by $x - 3$.

(c) Factor the quotient from part **(b)**.

(d) Solve $f(x) = 0$. Correctly write the solution set.

(e) Draw the sign chart for $f(x)$.

Problem 6. (Bonus - Remember your Theorems)
Compute the following.

(a) Let $f(x) = x^5 - 8x^4 + 12x^3 - 25x^2 - 72x + 30$. Find $f(7)$.

(b) Let $p(x) = x^4 - 6x^2 - 40$. Solve $p(x) = 0$. Write the solution set, including all complex solutions.

(c) Let $p(x) = x^4 - 6x^2 - 40$. Find the range of p. Write the range using correct interval notation.

Definition 1. A *binary set operation* is a way of taking two sets and creating a third. The three set operations we consider today are *union*, *intersection*, and *complement*.

$$\begin{aligned} \text{Union:} &\quad A \cup B = \{x \mid x \in A \text{ or } x \in B\} \\ \text{Intersection:} &\quad A \cap B = \{x \mid x \in A \text{ and } x \in B\} \\ \text{Complement:} &\quad A \smallsetminus B = \{x \mid x \in A \text{ and } x \notin B\} \end{aligned}$$

We read $A \smallsetminus B$ as "A remove B".

The *empty set* is the set which contains no elements, and is denoted $\varnothing$.

Two sets are called *disjoint* if their intersection is empty.

Problem 7. (Bonus - Set Operations)
Compute the following sets. Write each as an interval or as the union of disjoint intervals.

(a) $[1, 6] \cup [3, 9]$

(b) $[1, 6] \cap [3, 9]$

(c) $[1, 6] \smallsetminus [3, 9]$

(d) $[3, 9] \smallsetminus [1, 6]$

(e) $[0, 10] \smallsetminus (4, 7]$

(f) $\mathbb{R} \smallsetminus \{2, 7\}$

(g) $[2, 9] \smallsetminus \{0, 3, 6, 9\}$