Name:

Student ID:

History of Mathematics (Math 4123)
Practice Final Exam

Professor Paul Bailey
May 13, 2004

The examination contains 50 multiple choice problems which are worth 2 points each. Circle the letter corresponding to the best answer. If you have any questions about the meaning of any of the words or notation on the test, please ask.

Final Date and Time: Thursday, May 13, 2004, from 1:10 PM to 3:00 PM

On the actual final, 25 of the questions will be from this practice final, and 25 will be different but similar.

Problem 1. The ancient Greeks used a

(a) simple grouping numeral system;

(b) multiplicative numeral system;

(c) ciphered numeral system;

(d) positional numeral system.

Problem 2. The ancient Chinese used a

(a) simple grouping numeral system;

(b) multiplicative numeral system;

(c) ciphered numeral system;

(d) positional numeral system.

Problem 3. The ancient Egyptians used a

(a) simple grouping numeral system;

(b) multiplicative numeral system;

(c) ciphered numeral system;

(d) positional numeral system.

Problem 4. In Mayan notation, :|| ⁝| + ⁝ :||| equals

(a) 265;

(b) 325;

(c) 562;

(d) 625.

Problem 5. Which of the following is different from the rest?

(a) 1001011 (base 2);

(b) 2211 (base 3);

(c) 300 (base 5);

(d) 135 (base 7).

Problem 6. Which of the following is odd?

(a) 22222 (base 3);

(b) 21312 (base 4);

(d) 12321 (base 5);

(c) 23432 (base 7).

Problem 7. The Egyptians approximated the area of a circle to be that of a square on eight ninths of the diameter. To the nearest tenth, what value does this imply for π?

(a) 3.0;

(b) 3.1;

(c) 3.3;

(d) 3.6.

Problem 8. None of the following are regular Babylonian integers except

(a) 33;

(b) 34;

(c) 35;

(d) 36.

Problem 9. Which ancient culture used a rope with 12 knots to form right angles?

(a) Egyptian;

(b) Babylonian;

(c) Mayan;

(d) Incan.

Problem 10. Papyrus is made from

(a) reeds;

(b) bamboo;

(c) hide;

(d) wood.

Problem 11. The flawed Pythagorean proofs which assumed commensurability were repaired by

(a) Thales' principle of inscription;

(b) Zeno's paradoxes;

(c) Eudoxus' theory of proportion;

(d) Archimedes' method of exhaustion.

Problem 12. The first 4 Pythagorean pentagonal numbers are 1, 5, 12, and

(a) 16;

(b) 19;

(c) 22;

(d) 25.

Problem 13. The angle between adjacent sides of a regular pentagon

(a) 98°;

(b) 105°;

(c) 108°;

(d) 120°.

Problem 14. Which of the following mathematicians did not visit Alexandria?

(a) Euclid

(b) Archimedes

(c) Pythagoras

(d) Diophantus

Problem 15. Given two points in a plane, what is the minimum number of lines and circles which can be constructed from them to produce their midpoint?

(a) 0;

(b) 2;

(c) 4;

(d) 6.

Problem 16. Apollonius wrote a book about

(a) prime numbers;

(b) plane geometry;

(c) solid geometry;

(d) conic sections.

Problem 17. Let $m \in \mathbb{Q}$ and let $P = (\frac{1-m^2}{1+m^2}, \frac{2m}{1+m^2})$. Then P is the intersection of the unit circle with a line of slope m through the point

(a) $(0, 0)$;

(b) $(1, 0)$;

(c) $(-1, 0)$;

(d) $(1, 1)$.

Problem 18. *Prime numbers are more than any assigned multitude of prime numbers.* Who wrote this?

(a) Pythagoras;

(b) Eudoxus;

(c) Euclid;

(d) Archimedes.

Problem 19. *The area of a circle is equal to a right-angled triangle in which one of the sides about the right angle is equal to the radius, and the other is equal to the circumference, of the circle.* Who wrote this?

(a) Pythagoras;

(b) Eudoxus;

(c) Euclid;

(d) Archimedes.

Problem 20. The ancient Chinese estimated that a circular segment with base b and height s has area $s(b + s)/2$. To the nearest tenth, what value does this give for π?

(a) 3.0;

(b) 3.2;

(c) 3.4;

(d) 3.6.

Problem 21. The Euclidean algorithm may be used to solve:
find an integer x between 0 and 29 such that $7x \equiv 1 \pmod{30}$.

(a) 2;

(b) 13;

(c) 17;

(d) 29.

Problem 22. The Chinese Remainder Theorem may be used to solve:
find an integer x between 0 and 1456 such that $x \equiv 11 \pmod{47}$ and $y \equiv 17 \pmod{31}$.

(a) 187;

(b) 575;

(c) 882;

(d) 1117.

Problem 23. The word "algorithm" comes from the author's name of a book containing which phrase?

(a) al-jabr;

(b) in-gor;

(c) calcu-latin;

(d) kompu-rituni.

Problem 24. The method of casting out nines succeeds because

(a) $10 \equiv 1 \pmod{9}$;

(b) $9 \equiv -1 \pmod{10}$;

(c) $3^4 \equiv 1 \pmod{10}$;

(d) $100 - 99 = 1$.

Problem 25. *I swear to you by the Sacred Gospel, and on my faith as a gentleman, not only never to publish your discoveries, if you tell them to me, but I also promise and pledge my faith as a true Christian to put them down in cipher so that after my death, no one shall be able to understand them.* Who wrote this?

(a) Fibonacci;

(b) del Ferro;

(c) Cardano;

(d) Galileo.

Problem 26. Cardano learned how to depress a cubic. How does one depress a quadratic?

(a) factoring;

(b) completing the square;

(c) quadratic formula;

(d) graphing.

Problem 27. Cardano wrote that complex numbers are:

(a) "obscured by the mists of imagination."

(b) "a pathway to actual solutions."

(c) "as subtle as they are useless."

(d) "the only logical next step."

Problem 28. Viete realized that the coefficients of a polynomial are symmetric functions of the roots. If the coefficient of x^3 in a polynomial of degree 5 is written as a sum of products of its roots, how many terms are in the sum?

(a) 5;

(b) 10;

(c) 15;

(d) 20.

Problem 29. The invention of logarithms is credited to

(a) Werner;

(b) Napier;

(c) Briggs;

(d) Kepler.

Problem 30. Who proposed the heliocentric model?

(a) Eratosthenes;

(b) Aristotle;

(c) Aristarchus;

(d) Ptolemy.

Problem 31. In Desargues' projective plane, there are how many points at infinity?

(a) zero;

(b) one;

(c) two;

(d) infinitely many.

Problem 32. Desargues' classified conic sections by the number of points at infinity they had. Ellipsess have this many points at infinity:

(a) zero;

(b) one;

(c) two;

(d) infinitely many.

Problem 33. Desargues' classified conic sections by the number of points at infinity they had. Hyperbolas have this many points at infinity:

(a) zero;

(b) one;

(c) two;

(d) infinitely many.

Problem 34. What happens to the graph of $tx^2 + y^2 = 1$ as t varies from 1 to -1?

(a) ellipse $\to$ lines $\to$ hyperbola;

(b) hyperbola $\to$ lines $\to$ ellipe;

(c) ellipse $\to$ parabola $\to$ hyperbola;

(d) hyperbola $\to$ parabola $\to$ ellipse.

Problem 35. Descartes considered the placement of the roots of the polynomial on the real line to obtain his rule of signs. Suppose that $f(x) = x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$ has equally spaced distinct roots, and that $a_0 < 0$ and $a_4 > 0$. How many positive roots does $f(x)$ have?

(a) 1;

(b) 2;

(c) 3;

(d) 4.

Problem 36. Pascal formulated the special case of the binomial theorem, which implies that if $\sum_{i=0}^{n} \frac{n!}{i!(n-i)!} = 128$, then n must be

(a) 5;

(b) 6;

(c) 7;

(d) 8.

Problem 37. Who described Descartes' efforts in analytic geometry as "groping in shadows"?

(a) Desargues;

(b) Pascal;

(c) Fermat;

(d) Cavalieri.

Problem 38. Which of the following is a direct application of Fermat's Little Theorem?

(a) $7^2 \equiv 1 \pmod 3$;

(b) $4^2 \equiv 1 \pmod 5$;

(c) $6^2 \equiv 1 \pmod 7$;

(c) $5^8 \equiv 1 \pmod 9$.

Problem 39. Which of the following is implied by Fermat's Last Theorem?

(a) $10^{10} - 1$ is divisible by 11;

(b) all Pythagorean triples are of the form $(u^2 - v^2, 4uv, u^2 + v^2)$;

(c) tangent lines to $x^3 + y^3 = 1$ are computable;

(d) there are exactly two rational points on the curve $x^3 + y^3 = 1$.

Problem 40. *To divide a given square number into two squares, to divide a cube into two cubes, a fourth power, or in general any power whatever into two powers of the same denomination above the second is impossible.* Who wrote this?

(a) Diophantus;

(b) Descartes;

(c) Fermat;

(d) Euler.

Problem 41. *If two solids are included between a pair of parallel planes, and if the areas of the two sections cut by them on any plane parallel to the including planes are always in a given ratio, then the volumes of the two solids are also in this ratio.* Who wrote this?

(a) Archimedes;

(b) Pascal;

(c) Cavalieri;

(d) Wallis.

Problem 42. Newton used his law of gravitation $F = G\frac{m_1 m_2}{r^2}$ to verify whose model of the solar system?

(a) Ptolemy;

(b) Copernicus;

(c) Kepler;

(d) Inquisition.

Problem 43. The word "function" was introduced by

(a) Descartes;

(b) Newton;

(c) Leibnitz;

(d) Euler.

Problem 44. The notation $f(x)$ was introduced by

(a) Descartes;

(b) Newton;

(c) Leibnitz;

(d) Euler.

Problem 45. Euler defined complex exponents using

(a) trigonometry;

(b) integration;

(c) power series;

(d) differentiation.

Problem 46. The Euler characteristic $\chi = F - E + V$ of a torus is

(a) 3;

(b) 2;

(c) 1;

(d) 0.

Problem 47. Which of the following is not attributable to Gauss?

(a) constructibility of polygons;

(b) congruence modulo n;

(c) geometry of the complex plane;

(d) probability integrals.

Problem 48. Which of the following is not Cauchy's theorem?

(a) G has an element of order p if and only if p divides the order of G;

(b) z is constructible if and only if a tower of quadratic extensions reaches z;

(c) f is analytic in a disk D if and only if $\int_\gamma f = 0$ for every γ in D;

(d) A has an eigenvalue of λ if and only if $\det(A - \lambda I) = 0$.

Problem 49. Which of the following regular n-gons is not constructible?

(a) $n = 5$;

(b) $n = 6$;

(c) $n = 7$;

(d) $n = 8$.

Problem 50. Galois coined the word "group" to mean

(a) a set of matrices acting on the complex plane;

(b) a set of permutations of the roots of a polynomial;

(c) a set of nonzero integers modulo a prime number;

(d) a set of complex numbers closed under algebraic operations.