GROUP THEORY

STUDENT NAME HERE

1. Groups

Definition 1. A *group* $(G, \cdot)$ is a set G together with a binary operation

$$\cdot : G \times G \to G$$

such that

- **(G1)** $(g_1 g_2) g_3 = g_1 (g_2 g_3)$ for every $g_1, g_2, g_3 \in G$;
- **(G2)** there exists $1 \in G$ such that $g \cdot 1 = 1 \cdot g = g$ for every $g \in G$;
- **(G3)** for every $g \in G$ there exists $g^{-1} \in G$ such that $gg^{-1} = g^{-1}g = 1$.

Definition 2. A group G is *abelian* if $g_1 g_2 = g_2 g_1$ for every $g_1, g_2 \in G$.

Proposition 1. *Let G be a group such that $g^2 = 1$ for every $g \in G$. Then G is abelian.*

Proof. Type proof here. □

Type examples here.

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2. Subgroups

Definition 3. Let G be a group and let $H \subset G$. We say that H is a *subgroup* of G, and write $H \leq G$, if

- **(S0)** H is nonempty;
- **(S1)** $h_1, h_2 \in H$ implies $h_1 h_2 \in H$;
- **(S2)** $h \in H$ implies $h^{-1} \in H$.

Problem 1. Let G be a group and let $\mathcal{H}$ be a collection of subgroups of G. The *intersection* of $\mathcal{H}$ is

$$\cap\mathcal{H} = \{h \in G \mid h \in H \text{ for every } H \in \mathcal{H}\}.$$

Then $\cap\mathcal{H} \leq G$.

Problem 2. Let G be a group. The *center* of G is

$$Z(G) = \{h \in G \mid hg = gh \text{ for all } g \in G\}.$$

Then $Z(G) \leq G$.

Problem 3. Let G be a group, $g \in G$, and $H \leq G$. The *centralizer* of G in H is

$$C_H(g) = \{h \in H \mid hg = gh\}.$$

Then $C_H(g) \leq G$.

Problem 4. Let G be a group and $H, K \leq G$. The *normalizer* of K in H is

$$N_H(K) = \{h \in H \mid hK = Kh\}.$$

Then $N_H(K) \leq G$.

3. Cyclic Groups

Type definition and theory here.

4. Next Section

Continue building all significant theory, peppered with examples.

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