AP Calculus AB | Activity 0828 | Name:
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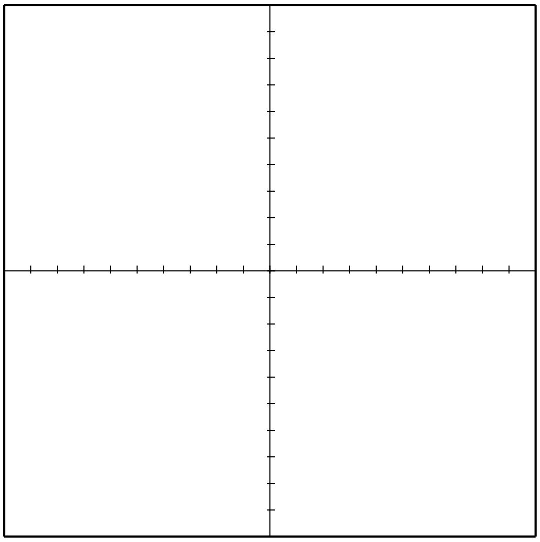
Problem 8. Consider the polynomial function $f(x) = x^4 - 5x^3 - 3x^2 + 17x - 10$. Find its degree, leading coefficient, constant coefficient, zeros, and end behavior. Find the y-intercept and x-intercepts. Graph the function and label these points.



Polynomial: $f(x) = x^4 - 5x^3 - 3x^2 + 17x - 10$

Degree:

Leading Coefficient:

Constant Coefficient:

Zeros:

y-intercept:

x-intercepts:

End Behavior:

Solution. The degree is 4, the leading coefficient is 1, and the constant coefficient is -10.

Since this is a polynomial of even degree with a positive leading coefficient, the end behavior is $+|+$.

The constant coefficient gives that y-intercept as $(0, -10)$. This tells us already that the entire graph will not fit in the box above.

We need to find the zeros. We do this by factoring. We factor by guess and check. We guess that perhaps 1 is a zero. We try it:

$$f(x) = 1 - 5 - 3 + 17 - 10 = 0.$$

So, by the Factor Theorem, $(x - 1)$ is a factor of f, so $f(x) = (x - 1)q(x)$ for some cubic polynomial q. We use synthetic division to find q:

```
1    |     1  -5  -3  17  -10
     |         1  -4  10   10
     -----------------------
           1  -4  -7  10    0
```

This $q(x) = x^3 - 4x - 7x + 10$, so $f(x) = (x - 1)(x^3 - 4x - 7x + 10)$. Now we need to factor q. We check if 1 is a zero:

$$q(1) = 1 - 4 - 7 + 10 = 0.$$

So $(x - 1)$ is a factor of q, and we again use synthetic division to factor it out:

```
1    |     1  -4  -7  10
     |         1  -3 -10
     ------------------
           1  -3 -10   0
```

So $q(x) = (x - 1)(x^2 - 3x - 10)$, and $f(x) = (x - 1)^2(x^2 - 3x - 10)$. Two numbers that multiply to 10 whose difference is 3 are 2 and 5; we see that

$$f(x) = (x - 1)^2(x - 5)(x + 3).$$

So, the zeros are 1, 5, and -3, where 1 is a zero of multiplicity 2. $\square$

Geogebra gives the following graph. Note that the y scale is ten times the x scale.