Calculus II
Dr. Paul L. Bailey

Quiz 0324
Wednesday, March 24, 2021

Name: Key

They are worth 4 points each, for a maximum of ten points, so try to do at least three of them.

Problem 1. (Substitution)
Compute

$$\int x\sqrt{x^2+1}\,dx.$$

$= \frac{1}{2}\int 2x\sqrt{x^2+1}\,dx$

$= \boxed{\frac{1}{2}\cdot\frac{2}{3}(x^2+1)^{3/2} + C}$

$= \frac{1}{3}(x^2+1)^{3/2} + C$

Jacob:

Let $u = \sqrt{x^2+1}$

So $du = \frac{2x}{2\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}} = \frac{x}{u}\,dx$

Then $u\,du = x\,dx$

So $\int = \int u\,x\,dx = \int u^2\,du = \frac{u^3}{3}$

$= \frac{(x^2+1)^{2/3}}{3} + C$

Problem 2. (Integration by Parts)
Compute

$$\int x\sqrt{x+1}\,dx.$$

Let $u = x$ $\quad v = \frac{2}{3}(x+1)^{3/2}$

$du = dx$ $\quad dv = \sqrt{x+1}$

$= \frac{2}{3}x(x+1)^{3/2} - \int \frac{2}{3}(x+1)^{3/2}\,dx$

$= \frac{2}{3}x(x+1)^{3/2} - \frac{2}{3}\cdot\frac{2}{5}(x+1)^{5/2} + C$

$= \boxed{\frac{2}{3}x(x+1)^{3/2} - \frac{4}{15}(x+1)^{5/2} + C}$

Problem 3. (Improper Fraction)
Compute
$$\int \frac{x^3 - 7x^2 + 9x + 5}{x - 2}\, dx.$$

$$= \int x^2 - 5x - 1 + \frac{3}{x-2}\, dx$$

$$\begin{array}{r|rrrr} 2 & 1 & -7 & 9 & 5 \\ & & 2 & -10 & -2 \\ \hline & 1 & -5 & -1 & 3 \end{array}$$

$$= \boxed{\frac{x^3}{3} - \frac{5}{2}x^2 - x + 3\ln(x-2) + C}$$

Problem 4. (Partial Fractions)
Compute
$$\int \frac{dx}{x^4 - 1}.$$

$$\frac{1}{x^4-1} = \frac{1}{(x^2-1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$$

So. $1 = A(x+1)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x^2-1)$

$x=1$: $1 = 4A \Rightarrow A = \frac{1}{4}$

$x=-1$: $1 = -4B \Rightarrow B = -\frac{1}{4}$

x^3 : $0 = A + B + C \Rightarrow C = 0$

$x=0$: $1 = A - B - D \Rightarrow 1 = \frac{1}{2} - D \Rightarrow D = -\frac{1}{2}$

$$\int = \int \frac{1}{4}\frac{1}{x-1} - \frac{1}{4}\frac{1}{x+1} - \frac{1}{2}\frac{1}{x^2+1}\, dx = \boxed{\frac{1}{4}\ln(x-1) - \frac{1}{4}\ln(x+1) - \frac{1}{2}\arctan(x) +}$$