

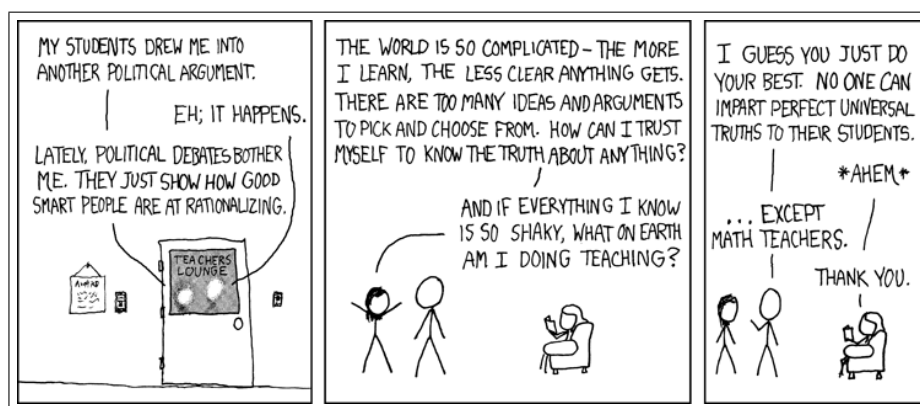
Name:

Algebra II Examination 4

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WEDNESDAY, NOVEMBER 10, 2021

The examination contains five problems which are worth 20 points each, and two bonus problems worth an additional 20 points each, for a maximum of 100 points. Calculators and all other electronic devices are prohibited.

- *ALL* answers must be justified with appropriate words, sentences, and/or computations.
- *DO NOT* write a negative number inside a square root.
Make appropriate use of the symbol i if necessary.
- *Standard Form* of a complex number is $x + yi$. Always write complex numbers in standard form.
- *Standard Form* of a polynomial is $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$; that is, with like terms combined, and in decreasing power order. Always write polynomials in standard form, unless otherwise indicated.



Prob 1	Prob 2	Prob 3	Prob 4	Prob 5	Bonus 1	Bonus 2	Total Score

Problem 1. (Matching)

Match the terms or phrases on the left with the descriptions on the right. Write the number of the matching description in the blank next to each term. Use each description exactly once.

(a) _____ Circle

(1) The set of decimal expansions.

(b) _____ Division Algorithm

(2) If r is the remainder when g is divided by $x - a$, then $g(a) = r$.

(c) _____ \mathbb{R}^2

(3) The set of fractions.

(d) _____ Conjugate Pairs Theorem

(4) There exist q and r such that $g = fq + r$ and $\deg(r) < \deg(f)$.

(e) _____ Line

(5) The set of points in a plane equally distant to two different points.

(f) _____ \mathbb{R}

(6) The set of ordered pairs of real numbers.

(g) _____ Remainder Theorem

(7) If $g(a) = 0$, then $x - a$ divides g .

(h) _____ \mathbb{Q}

(8) The set of points in a plane equally distant to a point and a line.

(i) _____ Parabola

(9) If $f(z) = 0$, then $f(\bar{z}) = 0$.

(j) _____ Factor Theorem

(10) The set of points in a plane equidistant to a given point.

Problem 2. (Solving Equations)

Find all real numbers x which satisfy the following equations. Using correct set notation, write the solution set.

(a) $8x + 3 = 2x - 5$

(b) $121x^2 = 4$

(c) $x^2 - 24x + 144 = 0$

(d) $x^2 + 3x - 40 = 0$

(e) $x^3 - 7x^2 + 12x = 0$

Problem 3. (Polynomials) Compute the following polynomials. Write the result in standard form.

(a) $x^3 + 3x^2 - 6x^5 + 10 + 2x + 7x^3 - 5x^2 + 7x^5 + x + x + 3$

(b) $(2x^3) \cdot (5x^7)$

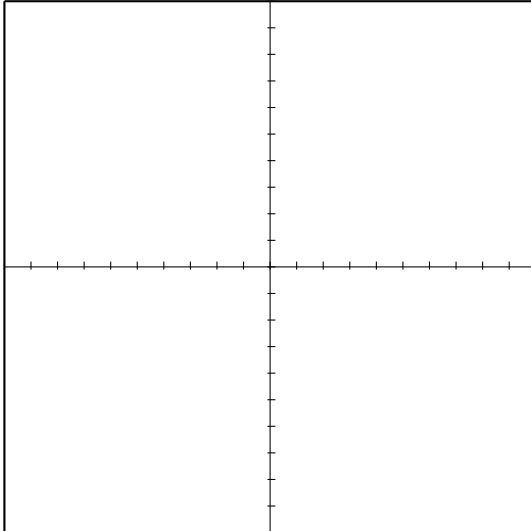
(c) $(2x - 4)(3x - 6)$

(d) $(x - 1)(2x - 3)(5x - 7)$

(e) $(\odot + 2)(\odot - 5)$

Problem 4. (Graphing) Fill out the charts, and sketch the graph.

- (a) Consider the linear function $f(x) = (5x + 4) - 3(x + 2)$. Find the slope-intercept form $f(x) = mx + b$ of the function, and identify the numbers m and b . Find the slope, the y -intercept, and the x -intercept (if any) of the line. Graph the line and label these points.



Linear Function: $f(x) = (5x + 4) - 3(x + 2)$

Standard Form:

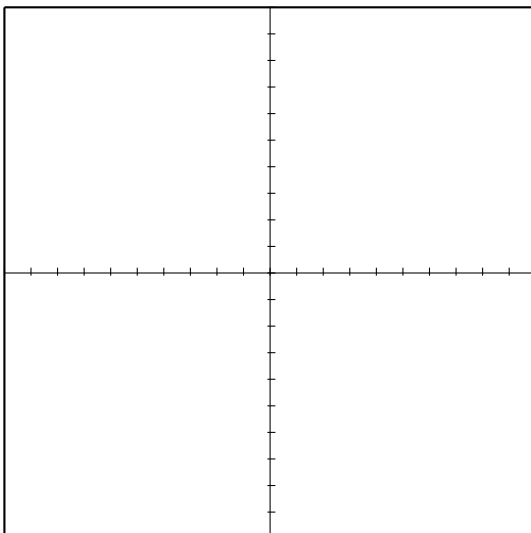
m: **b:**

Slope:

y -intercept:

x -intercept:

- (b) Consider the quadratic function $f(x) = x^2 - 8x + 7$. Find the standard form $f(x) = ax^2 + bx + c$ and the shifted form $f(x) = a(x - h)^2 + k$. Identify the constants a , b , c , h , and k . Find the zeros, intercepts, and vertex. Graph the function and label these points.



Quadratic Function: $f(x) = x^2 - 8x + 7$

Standard Form:

Shifted Form:

a: **b:** **c:** **h:** **k:**

Discriminant:

Zeros:

y -intercept:

x -intercept(s):

Vertex:

Problem 5. (Polynomial Division)

Let $f(x) = x^3 - 8x^2 + x + 42$.

(a) Find $f(3)$.

(b) Find the quotient and remainder when $f(x)$ is divided by $x - 3$.

(c) Factor the quotient from part (b).

(d) Solve $f(x) = 0$. Correctly write the solution set.

(e) Draw the sign chart for $f(x)$.

Problem 6. (Bonus - Remember your Theorems)

Compute the following.

(a) Let $f(x) = x^5 - 11x^4 + 20x^3 - 17x^2 - 7x - 3$. Find $f(9)$.

(b) Let $g(x) = (x - 5)(x + k)$. Suppose that the remainder is 8 when $g(x)$ is divided by $x - 3$. Find k .

(c) Let $p(x) = x^4 - 5x^2 - 36$. Solve $p(x) = 0$. Write the solution set.
(Hint: first, find two numbers whose product is 36 and whose difference is 5.)

Problem 7. (Bonus - Smiley Planets)

If you smile at me, I will understand, for that is something everybody everywhere does in the same language.

- Crosby, Stills, and Nash, *Wooden Ships*, 1968

- (a) If you smile at me, that will take away n frowns, where n is a positive integer. That is,

$$\ominus = -n\ominus.$$

Given that

$$\odot^2 - 2\odot\ominus = 15\odot^2,$$

find n .

- (b) The planets and their symbols, in order from the sun, are

Mercury	☿
Venus	♀
Earth	♁
Mars	♂
Jupiter	♃
Saturn	♄
Uranus	♅
Neptune	♆

Suppose

$$f = 2♂ - 3♀ + 7☿ + 2♄ + ♁, \quad g = 11♅ - 3☿ + ♂ - ♁, \quad \text{and} \quad h = 8☿ + ♀ - ♁ - ♄.$$

Find $f + g + h$. Combine like terms, and write them in increasing order from the sun.