Vector Calculus — Responses 03/19
Dr. Paul L. Bailey — Wednesday, March 18, 2020

Problem 1 (Thomas §8.1 # 20)**.** Let $f(x,y) = \dfrac{x+y^2}{\sqrt{1+x^2}}$ and $C : y = \dfrac{x^2}{2}$ from $(1,1/2)$ to $(0,0)$. Integrate f over C.

Solution. We parameterize the path by $\vec{r}(t) = \left\langle t, \dfrac{t^2}{2} \right\rangle$. Then $\vec{v}(t) = \langle 1, t\rangle$, so $|\vec{v}(t)| = \sqrt{1+t^2}$. Also, $f(\vec{r}(t)) = \dfrac{t + \frac{t^4}{4}}{\sqrt{1+t^2}}$. Thus

$$\int_C f\,ds = \int_0^1 f(\vec{r}(t))|\vec{v}(t)|\,dt = \int_0^1 \frac{t+\frac{t^4}{4}}{\sqrt{1+t^2}}\sqrt{1+t^2}\,dt = \int_0^1 t + \frac{t^4}{4}\,dt = t^2 + \frac{t^5}{20}\Big|_0^1 = \frac{11}{20}.$$

□

Problem 2 (Thomas §8.2 # 5)**.** Give a formula $\vec{F} = M(x,y)\vec{i} + N(x,y)\vec{j}$ for the vector field in the plane that has the property that $\vec{F}$ points toward the origin with magnitude inversely proportional to the square of the distance from (x,y) to the origin. (The field is not defined at the origin.)

Solution. The vector field $\vec{F}$ attaches a vector to each point in the plane. The vector represented by an arrow starting at (x,y) points toward the origin, so a unit vector in that direction is $-\dfrac{\langle x,y\rangle}{\sqrt{x^2+y^2}}$. The length of the vector is $\dfrac{k}{x^2+y^2}$, where k is a constant. So, the vector itself is the product of the unit vector and its length. Thus

$$\vec{F}(x,y) = \left\langle \frac{-kx}{(x^2+y^2)^{3/2}}, \frac{-ky}{(x^2+y^2)^{3/2}} \right\rangle.$$

□

Question 1. I'm not sure how to find mass, center of mass, moment of inertia, radius of gyration.

Answer. We skipped this. You may read it if you want, but we will not use it. □

Question 2. How they line integrals computationally different from normal integrals?

Answer. Once in the form $\int_a^b f(\vec{r}(t))|\vec{v}(t)|\,dt$, it is not computationally different. It is only conceptually different. □

Question 3. How do the limits and the domain of integration manifest the curve needed?

Answer. Yes, it takes practice to be able to look at and expression $\vec{r}(t) = \langle x(t), y(t)\rangle$, and imagine the curve, or even more, to see a curve and express it as a path. You will get more practice, but if you wish, review §13.1 and §13.3. □