Name:

**Algebra II**
**Examination 3**

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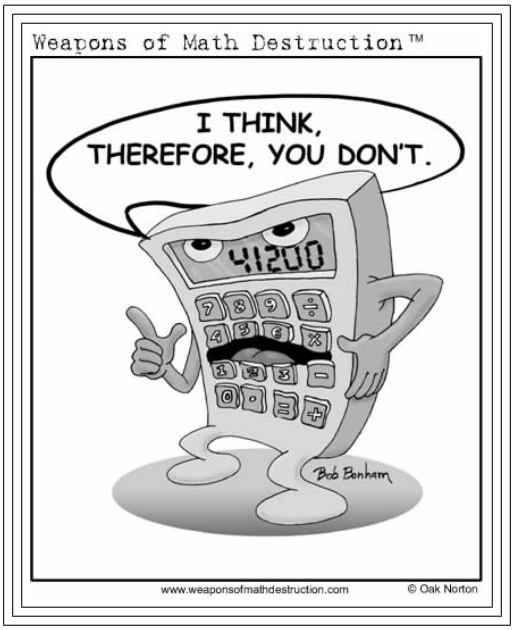
Thursday, October 28, 2021

The examination contains five problems which are worth 20 points each, and two bonus problems worth an additional 20 points each, for a maximum of 100 points. Calculators and all other electronic devices are prohibited.

- *ALL* answers must the justified with appropriate words, sentences, and/or computations.
- *DO NOT* write a negative number inside a square root.
  Make appropriate use of the symbol $i$ if necessary.
- *Standard Form* of a complex number is $x + yi$. Always write complex numbers in standard form.
- *Standard Form* of a polynomial is $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$; that is, with like terms combined, and in decreasing power order. Always write polynomials in standard form, unless otherwise indicated.




| Prob 1 | Prob 2 | Prob 3 | Prob 4 | Prob 5 | Bonus 1 | Bonus 2 | Total Score |
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**Problem 1. (Definitions)**
State the precise definition, as given in class, of the following concepts.

**(a)** Parabola

**(b)** Complex Number

**(c)** Monomial

**(d)** Degree of a Polynomial

**(e)** $\mathbb{R}^2$

**Problem 2. (Solving Equations)**
Find all real numbers $x$ which satisfy the following equations. Using correct set notation, write the solution set.

**(a)** $7x + 5 = 3x + 17$

**(b)** $4x^2 = 121$

**(c)** $x^2 - 22x + 121 = 0$

**(d)** $x^2 - 6x + 12 = 0$

**(e)** $x^3 - 8x^2 + 15x = 0$

**Problem 3. (Complex Numbers)**
Let $z = 4 - 7i$ and $w = 3 + 5i$. Compute the following.

**(a)** $z + w$

**(b)** $3z - 2w$

**(c)** $|w|$

**(d)** $zw$

**(e)** $z/w$

**Problem 4. (Polynomials)** Compute the following polynomials. Write the result in standard form.

**(a)** $(7x^3) \cdot (5x^2)$

**(b)** $(3x + 2)(x - 5)$

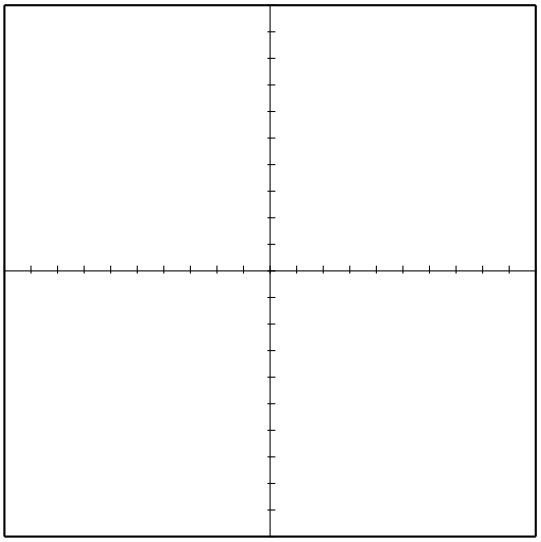
**(c)** $(3x^4 + 2x^2 - 3x + 7 + 6x^3) + (x^5 - x + 7x^2 - 3)$

**(d)** $(x + 2)(x - 3)(x - 5)$

**(e)** $(x^3 - 4x^2 + 5x + 9)(x + 3)$

**Problem 5. (Graphing)** Fill out the charts, and sketch the graph.

**(a)** Consider the linear function $f(x) = 2(x + 5) - 7$. Find the slope-intercept form $f(x) = mx + b$ of the function, and identify the numbers $m$ and $b$. Find the slope, the $y$-intercept, and the $x$-intercept (if any) of the line. Graph the line and label these points.



**Linear Function:** $f(x) = 2(x + 5) - 7$

**Standard Form:**

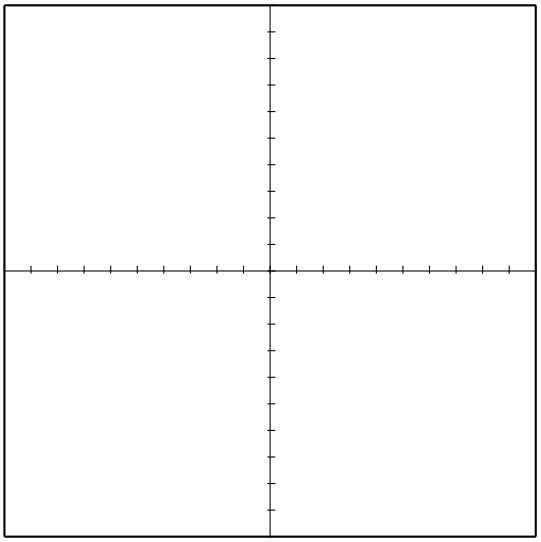
**m:** **b:**

**Slope:**

**$y$-intercept:**

**$x$-intercept:**

**(b)** Consider the quadratic function $f(x) = x^2 - 3x - 10$. Find the standard form $f(x) = ax^2 + bx + c$ and the shifted form $f(x) = a(x - h)^2 + k$. Identify the constants $a$, $b$, $c$, $h$, and $k$. Find the zeros, intercepts, and vertex. Graph the function and label these points.



**Quadratic Function:** $f(x) = x^2 - 3x - 10$

**Standard Form:**

**Shifted Form:**

**a:** **b:** **c:** **h:** **k:**

**Discriminant:**

**Zeros:**

**$y$-intercept:**

**$x$-intercept(s):**

**Vertex:**

**Problem 6. (Bonus - Polynomial Division)**
Let $f(x) = x - 5$ and $g(x) = x^3 - 6x^2 + 3x + 10$.

**(a)** Divide $f$ into $g$. Find the quotient and the remainder.

**(b)** Using part **(a)**, find the zeros of $g$.

**Problem 7. (Bonus - Conjugate Pairs)**
Let $z = 4 + 3i$.

**(a)** Let $f(x) = x^2 + bx + c$ be a quadratic function with real coefficients such that $f(z) = 0$. Find $b$ and $c$.

**(b)** Let $g(x) = ax^2 + bx + c$ be a quadratic function with real coefficients such that $g(z) = 0$ and $g(0) = 75$. Find $a$, $b$, and $c$.

**(c)** What did the supercomputer Deep Thought compute, after seven and a half million years of thought, to be the answer to the Ultimate Question of Life, the Universe, and Everything?