CATEGORY THEORY
CATEGORY II - EQUISETS

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1. Equisets

Definition 1 (Objects)**.** An *equiset* $(E, \sim)$ consists of a set E together with a relation $\sim$ on E with the following properties:

(E1)	$a \sim a$	(Reflexivity)
(E2)	$a \sim b$ implies $b \sim a$	(Symmetry)
(E3)	$a \leq b$ and $b \leq c$ implies $a \leq c$	(Transitivity)

The relation $\sim$ is called an *equivalence relation* on E.

Definition 2 (Subobjects)**.** Let $(E, \sim)$ be an equiset. If $F \subset E$, the restriction of $\sim$ to F satisfies the properties of an equivalence relation on F, making $(F, \sim)$ an equiset. We may call $(F, \sim)$, or just F, a *subequiset*.

Definition 3 (Morphisms)**.** Let $(E, \sim)$ and $(F, \approx)$ be equisets. A function $f : E \to F$ is called *equivalence preserving* if

$$e_1 \sim e_2 \quad \Rightarrow \quad f(e_1) \approx f(e_2).$$

The identity map on P is equivalence preserving, and the composition of equivalence preserving functions is equivalence preserving. Thus, equisets with equivalence preserving maps form a category.

Problem 1. Discuss when a function $f : (\mathbb{Z}, \equiv_n) \to (\mathbb{Z}, \equiv_m)$ is equivalence preserving, where

$$a \equiv_n b \quad \Leftrightarrow \quad n \mid b - a.$$

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