Category Theory | Homework 4 | Name:
Dr. Paul L. Bailey | Wednesday, September 18, 2019

Due Tuesday, September 25, 2019. Write *neatly* on separate white $8\frac{1}{2} \times 11$ blank printer paper. Write with words, in complete sentences and paragraphs.

Read Chapters 2, 3, and 4 of the book. The hardcopy (5^{th} edition) and the pdf on my webpage (8^{th} edition) have essentially the same material. The following problems are from these sections of the book.

We will use multiplicative notation for all non-specific groups. For $\mathbb{Z}_n$, we write the members without bars (even though they are residue classes).

Problem 1. A subgroup of $\mathbb{Z}_{91}$ contains $\{1, 9, 16, 22, 53, 74, 79, 81, x\}$. Find x.

Problem 2. Let G be a group such that $g^2 = 1$ for every $g \in G$. Show that G is abelian.

Problem 3. Let G be a finite group. Show that the number of elements $g \in G$ such that $g^3 = 1$ is odd.

Problem 4. Let G be a group such that, for all $a, b, c, d, x \in G$, we have

$$axb = cxd \quad \Rightarrow \quad ab = cd.$$

Show that G is abelian.

Definition 1. Let G be a group and let $h \in G$. The *centralizer* of h in G is

$$C_G(h) = \{g \in G \mid gh = hg\}.$$

Problem 5. Let G a group and let $h \in G$. Show that $C_G(h)$ is a subgroup of G.