NUMERICAL ANALYSIS
TAYLOR SERIES FOR ARCTAN

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Let $f(x) = \arctan(x)$. Then $f'(x) = \frac{1}{1+x^2}$; view this as a geometric series. This produces

$$\begin{aligned} f'(x) &= \frac{1}{1+x^2} \\ &= \frac{1}{1-(-x^2)} \\ &= \sum_{n=0}^{\infty} (-x^2)^n \\ &= \sum_{n=0}^{\infty} (-1)^n x^{2n} \\ &= 1 - x^2 + x^4 - x^6 + x^8 + \cdots . \end{aligned}$$

Now

$$\begin{aligned} f(x) &= \int \frac{1}{1+x^2}\,dx \\ &= \int \Big(\sum_{n=0}^{\infty} (-1)^n x^{2n} \Big)\,dx \\ &= \sum_{n=0}^{\infty} (-1)^n \int x^{2n}\,dx \\ &= \sum_{n=1}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \\ &= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots . \end{aligned}$$

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Date: October 15, 2003.