Name:

Algebra II
Examination 1

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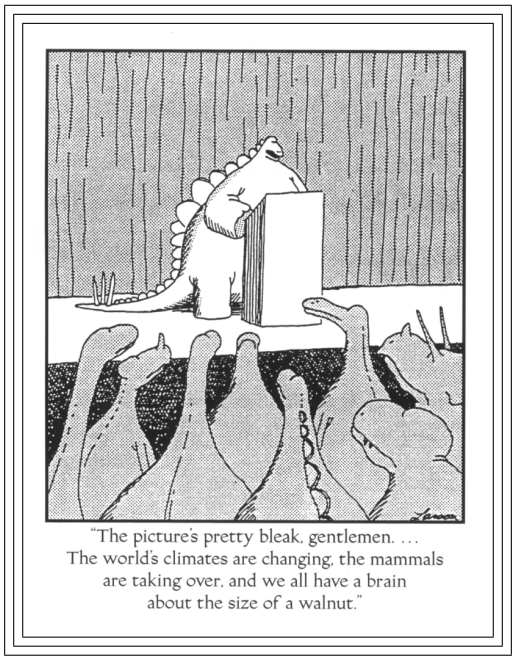
The examination contains five problems which are worth 20 points each, and two bonus problems worth an additional 20 points each, for a maximum of 100 points.

The *quadratic formula* to solve $ax^2 + bx + c = 0$ is

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.}$$



Prob 1	Prob 2	Prob 3	Prob 4	Prob 5	Bonus1	Bonus 2	Total Score

Problem 1. (Sets of Numbers)
Recall our familiar sets of numbers:

$$\begin{aligned}
\text{Natural Numbers:} \quad & \mathbb{N} = \Big\{1, 2, 3, \dots\Big\} \\
\text{Integers:} \quad & \mathbb{Z} = \Big\{\dots, -2, -1, 0, 1, 2, \dots\Big\} \\
\text{Rational Numbers:} \quad & \mathbb{Q} = \left\{\frac{p}{q} \,\middle|\, p, q \in \mathbb{Z}, q \neq 0\right\} \\
\text{Real Numbers:} \quad & \mathbb{R} = \Big\{ \text{ numbers given by decimal expansions } \Big\} \\
\text{Complex Numbers:} \quad & \mathbb{C} = \Big\{a + ib \mid a, b \in \mathbb{R} \text{ and } i^2 = -1\Big\}
\end{aligned}$$

Note that

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}.$$

Of these sets of numbers, state the smallest set which contains all of the solutions to the given equation. Write the symbol of the appropriate set in the blank next to the equation.

(a) _____ $x - 3 = 0$

(b) _____ $x + 3 = 0$

(c) _____ $3x - 1 = 0$

(d) _____ $x^2 - 3 = 0$

(e) _____ $x^2 + 3 = 0$

(f) _____ $2x - 2 = 3x + 1$

(g) _____ $x = 2 + i$

(h) _____ $x = \dfrac{2i}{3}$

(i) _____ $x = \sqrt{17}$

(j) _____ $x^2 - 6x + 10 = 0$

Problem 2. (True/False)
Circle the letter corresponding to the best answer.

Every real number is rational.

(T) True **(F)** False

The integers are closed under division.

(T) True **(F)** False

$0.\overline{9} = 1.\overline{0}$.

(T) True **(F)** False

$3 \in \{1, 2, 4\}$.

(T) True **(F)** False

$2 \subset \{1, 2, 4\}$.

(T) True **(F)** False

$\mathbb{Z} \subset \mathbb{R}$

(T) True **(F)** False

The natural numbers are closed under addition.

(T) True **(F)** False

A linear equation with integer coefficient has a solution in $\mathbb{Z}$.

(T) True **(F)** False

A quadratic equation with real coefficients has a solution in $\mathbb{C}$.

(T) True **(F)** False

A rational number is real if and only if its decimal expansion terminates or repeats.

(T) True **(F)** False

Problem 3. (Repeating Decimals)
Answer the following questions.

(a) Find the decimal expansion of $\frac{13}{37}$.

(b) Find a fraction which equals $0.8\overline{573}$.

(c) State the reason that we learned about **(a)** and **(b)**?

Problem 4. (Solving Quadratics) Solve the quadratic equation using the indicated method. Justify your answer. Show all work.

(a) $8x^2 - 7 = 13$ (Extracting Roots)

(b) $x^2 - 7x - 60 = 0$ (Factoring)

Problem 5. (Solving Quadratics) Solve the quadratic equation using the indicated method. Justify your answer. Show all steps.

(a) $x^2 - 12x - 5 = 0$ (Completing the Square)

(b) $3x^2 + 8x - 7 = x^2 + 5x + 1$ (Quadratic Formula)

Problem 6. (Real Bonus)
Solve the problem.

(a) Find two numbers whose product is 3 and whose difference is 6.

(b) The equation $7x^2 - 5x + c = 0$ has exactly one solution. Find c.

Problem 7. (Complex Bonus)
Every complex number can be written in the form $a + bi$, where $a, b \in \mathbb{R}$ and $i^2 = -1$.
Suppose $a, b, c, d \in \mathbb{R}$. Then $a + bi = c + di$ if and only if $a = c$ and $b = d$.

(a) Let $a + bi = i^{231}$, where $a, b \in \mathbb{R}$. Find a and b. Justify your answer.

(b) Let $(a + bi)^2 = i$. Find a and b.