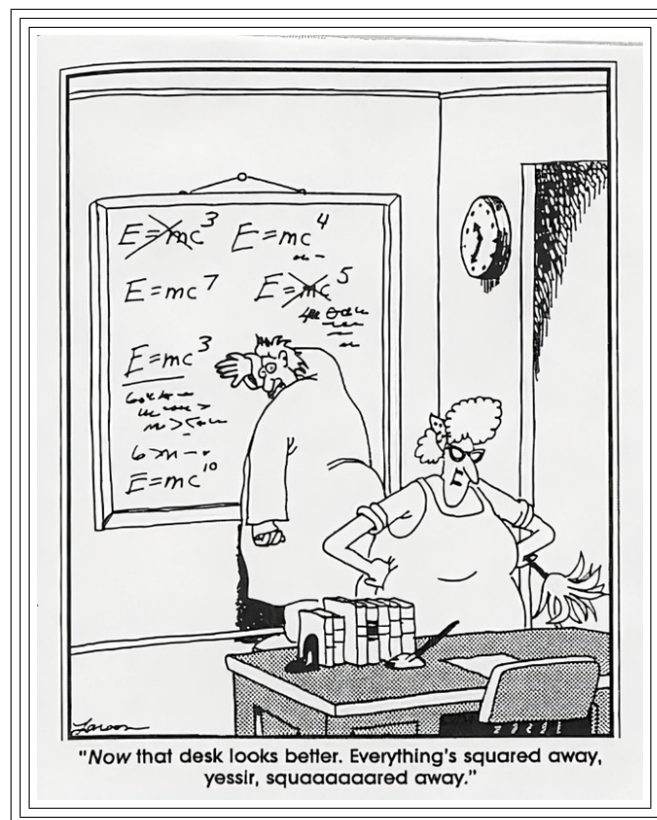


Name:

Algebra II
Examination 10

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The examination contains ten problems which are worth 10 points each, and two bonus problems worth ten points each.

[illegible]

Problem 1. (Solving Radical Equations)

Solve the equation

$$\sqrt[5]{x^2 - 3x + 4} = 2.$$

Correctly write the solution set.

Problem 2. (Simplifying Exponential Expressions)

Simplify

$$343^{2/3} + 625^{3/4}.$$

The answer should be an integer.

Problem 3. (Solving Exponential Equations)

Find $x \in \mathbb{R}$ such that

$$9^{3x+7} = 27^{5x-12}.$$

Correctly write the solution set.

Problem 4. (Solving Logarithmic Equations)

Find all $x \in \mathbb{R}$ such that

$$2 \log_7(x) = 2 \log_7(2) + \log_7(x + 15).$$

Correctly write the solution set.

Problem 5. (Finding the Equation of a Circle)

Find the center and radius of a circle with equation $x^2 - 8x + y^2 - 10y = 3$. (Hint: complete the square.)

Problem 6. (Factoring Cubics)

Let $f(x) = x^3 - 6x^2 + 7x + 2$. Notice that $f(2) = 0$. Divide $f(x)$ by $(x - 2)$ to find the other two zeros of f . Write the solution set for the equation $f(x) = 0$.

Problem 7. (Solving Rational Equations)

Let $f(x) = \frac{x^2 - 3x - 6}{x + 4}$. Solve the equation $f(x) = 2$. Correctly write the solution set.

Problem 8. (Finding the Domain)

Let $f(x) = \frac{\sqrt{2x - 10}}{x^2 - 10x + 21}$. Find the domain of f . Write your answer as the union of disjoint intervals.

Problem 9. (Set Operations)

Compute the following sets. Write your answer using correct set notation.

Let $A = \{1, 3, 5, 7, 9\}$, $B = \{3, 4, 5, 6, 7\}$, and $C = [0, 3]$.

(a) $A \cup B$

(b) $A \cap B$

(c) $A \setminus B$

(d) $B \setminus A$

(e) $C \setminus A$

Problem 10. (Finding Inverses)

Let $A = [4, \infty)$ and $B = [6, \infty)$. Let $f : A \rightarrow B$ be given by $f(x) = x^2 - 8x + 22$, and let $g : B \rightarrow A$ be its inverse. Find $g(x)$. (Hint: complete the square.)

Problem 11. (Bonus - Finding a Fun Line)

If P and Q are points in \mathbb{R}^2 , let $d(P, Q)$ denote the distance from P to Q .

Let $A, B \in \mathbb{R}^2$ be given by $A = (3, 1)$ and $B = (5, 5)$. Let L denote the set of all points $P \in \mathbb{R}^2$ such that $d(P, A) = d(P, B)$. Then L is a line. Find the equation for this line. (Hint: first draw a picture of this.)

Definition 1. Let $f : A \rightarrow B$. We say that f is *injective* (or *one-to-one*) on A if, for every $a_1, a_2 \in A$, we have

$$f(a_1) = f(a_2) \quad \Rightarrow \quad a_1 = a_2.$$

Problem 12. (Bonus - Injectivity)

Find an interval A such that the given function is injective on A .

(a) $f(x) = 3x + 3$

(b) $f(x) = x^2 - 10x + 25$

(c) $f(x) = \sqrt{x-3} + 5$

(d) $f(x) = |x|$

(e) $f(x) = x^3 + 2x^2$