Name:

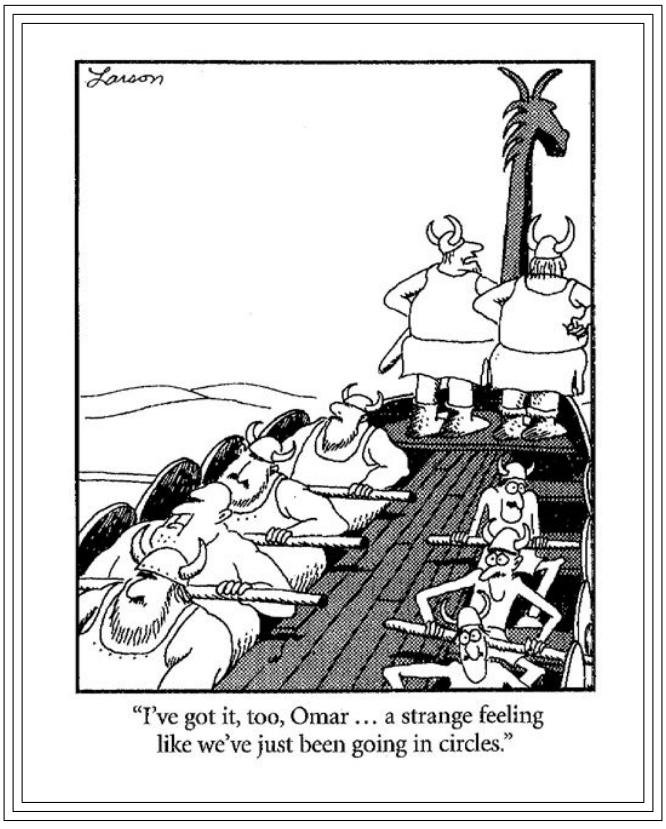
Algebra II
Examination 15 (Test)

Dr. Paul Bailey
Thursday, March 31, 2022

The examination contains ten problems which are worth 10 points each, and two bonus problems worth ten points each. All answers must be justified. An appropriate amount of work must be shown to receive credit.

The *wrapping function* is

$$W : \mathbb{R} \to \mathbb{R}^2 \quad \text{given by} \quad W(t) = (\cos(t), \sin(t)).$$



"I've got it, too, Omar ... a strange feeling like we've just been going in circles."

Prob1	Prob2	Prob3	Prob4	Prob5	Prob6	Prob7	Prob8	Prob9	Prob10	Bonus1	Bonus2	Total

Problem 1. (Solving Logarithmic/Rational Equations)
Solve the equation

$$\log_7(x) + \log_7(x-10) - \log_7(x-6) = 1.$$

Correctly write the solution set. (Hint: $1 = \log_7(7)$.)

Problem 2. (Trigonometric Equations)
Find all $x \in [0, 2\pi]$ such that

$$2\sin(x) = 1.$$

Correctly write the solution set. (Here, x is measured in radians.)

Problem 3. (Finding the Equation of a Line)
Let $A = (7, -7)$ and $B = (-3, 13)$. Find slope-intercept form of the equation of the line through A and B.

Problem 4. (Finding the Equation of a Circle)
Let $A = (7, -7)$ and $B = (-3, 13)$. Find the equation of a circle which has $\overline{AB}$ as a diameter.

Problem 5. (Finding Inverses)
Let

$$f(x) = x^2 - 6x - 1.$$

Let g be an inverse function of f. Compute a formula for $g(x)$. State it's domain and range.

Problem 6. (Finding the Domain)
Let

$$f(x) = \frac{\sqrt{x^2 - 8x + 15}}{x^2 - 16}.$$

Find the domain of f. Write your answer as the union of disjoint intervals.

Problem 7. (Factoring Cubics)
Let

$$f(x) = x^3 - 7x^2 + 11x + 3.$$

Notice that $f(3) = 0$. Use this to find the other two zeros of f. Write the solution set for the equation $f(x) = 0$.

Problem 8. (Trigonometry)
Suppose that $t \in [0, \pi/2]$ and that $\tan(t) = 2$. Find $\sin(t)$ and $\cos(t)$.
(Hint: tangent is opposite/adjacent; draw a triangle.)
Is t less than or greater than $60°$? Justify your answer.

Problem 9. (Reference Angles)
For each of the following angle measures (given in radians), convert the angle to degrees, draw the angle with that measure in standard position, and state the reference angle (in radians).

(a) $\dfrac{10\pi}{3}$

(b) $-\dfrac{11\pi}{6}$

Problem 10. (Wrapping Function)
Find $W\left(\dfrac{20\pi}{3}\right)$.

Problem 11. (Bonus - Lines and Circles)
A line through the origin of slope 2 is tangent to a circle centered at $(4, 4)$.

(a) Find the point of tangency.

(b) Write equation of the circle.

Problem 12. (Bonus - Clocks)
Solve these clock problems.

(a) What is $2 + 2$?

(b) If it were 10 AM now, what time would it be in 30 hours?

(c) Find the *exact* angle between the hands of a clock at 1:20 pm. (Hint: it isn't $90°$.)