CALCULUS II
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Activity 0322
Monday, March 22, 2021

Name: KEY

Problem 1. Compute these integrals.

(a) $\displaystyle\int \frac{4\,dx}{x^2+2x+1} = 4\int \frac{dx}{(x+1)^2} = \boxed{-\frac{4}{x+1}+C}$

$$\frac{4}{(x-1)(x^2+2x+1)} = \frac{A}{x-1}+\frac{Bx}{x+1}+\frac{C}{(x+1)^2} = \frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{(x+1)^2}$$

$$4 = A(x+1)^2 + B(x^2-1) + C(x-1)$$

$x=1$: $4 = 4A \Rightarrow A = 1$

$x=-1$: $4 = -2C \Rightarrow C = -2$

x^2: $0 = A + C \Rightarrow B = -1$

(b) $\displaystyle\int \frac{4\,dx}{(x-1)(x^2+2x+1)}$

$$= \int \frac{1}{x-1}\,dx - \int \frac{1}{x+1}\,dx - \int \frac{2}{(x+1)^2}\,dx$$

$$= \boxed{\ln(x-1) - \ln(x+1) + \frac{2}{x+1} + C}$$

Problem 2. Compute these integrals.

(a) $\int \frac{5\,dx}{x^2+2x+2} = \int \frac{5\,dy}{(x^2+1)^2+1} = 5\arctan(x+1)+C$

$$\frac{5}{(x-1)(x^2+2x+2)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+2x+2} = \frac{1}{x-1} - \frac{x+3}{x^2+2x+2}$$

$$5 = A(x^2+2x+2) + (Bx+C)(x-1)$$

$x=1$: $5 = 5A \Rightarrow A=1$

$x=0$: $5 = 2A - C \Rightarrow C = 2A-5 \Rightarrow C=-3$

x^2: $0 = A+B \Rightarrow B=-1$

(b) $\int \frac{5\,dx}{(x-1)(x^2+2x+2)}$

$$= \int \frac{dy}{x-1} - \frac{1}{2}\int \frac{(2x+2)dx}{x^2+2x+2} - \int \frac{2}{x^2+2x+2}$$

$$= \ln(x-1) - \frac{1}{2}\ln(x^2+2x+2) - 2\arctan(x+1) + C$$