Transformation Summary

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Given a subset of $\mathbb{R}^2$, we may modify the position or shape of the set by transforming each of the coordinates in each point of the set, according to the chart below.

Shift right h units	$x \mapsto x + h$
Shift left h units	$x \mapsto x - h$
Shift up k units	$y \mapsto y + k$
Shift down k units	$y \mapsto y - k$
Stretch horizontally by a factor of a	$x \mapsto ax$
Shrink horizontally by a factor of a	$x \mapsto \frac{x}{a}$
Stretch vertically by a factor of b	$y \mapsto by$
Shrink vertically by a factor of b	$y \mapsto \frac{y}{b}$
Reflect across the y-axis	$x \mapsto -x$
Reflect across the x-axis	$y \mapsto -y$

`Transformations of Sets`

Given an equation, we may transform the locus of the equation by modifying the original equations according to the rules laid out in the chart below.

Shift right h units	$x \mapsto x - h$
Shift left h units	$x \mapsto x + h$
Shift up k units	$y \mapsto y - k$
Shift down k units	$y \mapsto y + k$
Stretch horizontally by a factor of a	$x \mapsto \frac{x}{a}$
Shrink horizontally by a factor of a	$x \mapsto ax$
Stretch vertically by a factor of b	$y \mapsto \frac{y}{b}$
Shrink vertically by a factor of b	$y \mapsto by$
Reflect across the y-axis	$x \mapsto -x$
Reflect across the x-axis	$y \mapsto -y$

`Transformations of Loci`

Given a function f , we consider the equation $y = f(x)$ and obtain its locus. We may transform the graph of this locus by modifying the function according to the chart below.

Shift right h units	$f(x - h)$
Shift left h units	$f(x + h)$
Shift up k units	$f(x) + k$
Shift down k units	$f(x) - k$
Stretch horizontally by a factor of a	$f(\frac{x}{a})$
Shrink horizontally by a factor of a	$f(ax)$
Stretch vertically by a factor of b	$bf(x)$
Shrink vertically by a factor of b	$\frac{f(x)}{b}$
Reflect across the y-axis	$f(-x)$
Reflect across the x-axis	$-f(x)$

`Transformations of Functions` $f(x)$