Name:

Algebra II
Examination 11

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The examination contains ten problems which are worth 10 points each, and two bonus problems worth ten points each. All answers must be justified. An appropriate amount of work must be shown to receive credit.

You can do the problems introduced on this test involving e and ln using the following facts:

- e is a positive real number, which happens to be approximately equal to 2.72.
- The "natural logarithm" is $\ln(x) = \log_e(x)$.

Prob1	Prob2	Prob3	Prob4	Prob5	Prob6	Prob7	Prob8	Prob9	Prob10	Bonus1	Bonus2	Total

Problem 1. (Solving Radical Equations)
Solve the equation

$$\sqrt[4]{x^2 - x + 9} = 3.$$

Correctly write the solution set.

Problem 2. (Solving Rational Equations)
Let

$$f(x) = \frac{7x - 11}{2x - 5}.$$

Solve the equation $f(x) = 3$. Correctly write the solution set.

Problem 3. (Solving Logarithmic Equations)
Find all $x \in \mathbb{R}$ such that

$$2\ln(x+1) = 2\ln(3) + \ln(x+5).$$

Correctly write the solution set.

Problem 4. (Solving Exponential Functions)
Find the unique real number y such that $x = \ln(y)$, where

$$e^{2x} - 3e^x - 8 = 20.$$

Problem 5. (Finding Inverses)
Let

$$f(x) = \sqrt{x-4} + 7.$$

Let $g(x)$ be the inverse of f.
Find the domain and range of g, and find a formula for g.

Problem 6. (Finding the Equation of a Circle)
Find the center and radius of a circle with equation

$$x^2 - 12x + y^2 + 2y = 0.$$

Problem 7. (Finding the Domain)
Let

$$f(x) = \frac{\sqrt{3x - 21}}{x^2 - 10x + 9}.$$

Find the domain of f. Write your answer as the union of disjoint intervals.

Problem 8. (Simplifying Exponential Expressions)
Simplify

$$121^{3/2} + 125^{2/3}.$$

The answer should be an integer.

Problem 9. (Factoring Cubics)
Let

$$f(x) = x^3 - 4x^2 + 2x + 3.$$

Notice that $f(3) = 0$. Divide $f(x)$ by $(x - 3)$ to find the other two zeros of f. Write the solution set for the equation $f(x) = 0$.

Problem 10. (Set Operations)
Compute the following sets. Write your answer using correct set notation.

Let $A = [1, 4)$, $B = (5, 11]$, and $C = \{1, 4, 9, 16\}$.

(a) $A \cup B$

(b) $A \cap B$

(c) $A \setminus B$

(d) $B \setminus A$

(e) $B \setminus C$

Problem 11. (Bonus - Lines and Circles)
A line through the origin of slope 2 is tangent to a circle centered at $(0, 7)$.

(a) Sketch this situation.

(b) Write the equation of the line.

(c) Find the point of tangency.

(d) Write equation of the circle.

Definition 1. Let $f : A \to B$, and let $D \subset B$. The *preimage* of B under f is

$$f^{-1}(D) = \{a \in A \mid f(a) \in D\}.$$

That is, the preimage of D is the set of all points in the domain which are mapped by f into D.

Problem 12. (Bonus - Preimage)
Let $A = \mathbb{R}$ and $B = [0, \infty)$. Let $f : A \to B$ be given by $f(x) = x^2 - 4$. Let $D \subset B$ be given by $D = [2, 5)$. Find $f^{-1}(D)$.