CALCULUS II
DR. PAUL L. BAILEY

Homework 0326
Wednesday, March 24, 2021

Name: KEY?

The solutions to these problems are due Friday, March 26, 2021. Please figure out how to solve these integrals, and then write your solutions neatly onto this page.

Problem 1. Compute (divide, then partial fractions):

$$\int \frac{x^4}{x^2-1}\,dx. = \int \frac{x^4-1}{x^2-1}dx + \int \frac{dx}{x^2-1}$$

$$\frac{1}{x^2-1} = \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$$

$$= \int x^2+1\,dx + \int \frac{dx}{x^2-1}$$

$$= \frac{x^3}{3} + x + \frac{1}{2}\int \frac{1}{x-1} - \frac{1}{x+1}\,dx$$

$$= \frac{x^3}{3} + x + \frac{1}{2}\ln(x-1) - \frac{1}{2}\ln(x+1) + C$$

Problem 2. Compute (factor, then partial fractions):

$$\int \frac{dx}{x^3-2x^2+4x-8}.$$

$$x^3-2x^2+4x-8$$
$$= x^2(x-2) + 4(x-2)$$
$$= (x-2)(x^2+4)$$

$$\boxed{\int \frac{dx}{a^2+x^2} = \frac{\arctan(\frac{x}{a})}{a} + C}$$

$$\frac{1}{(x-2)(x^2+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+4}$$

$$1 = A(x^2+4) + (Bx+C)(x-2)$$

$x=2$: $1 = 8A + 0 \Rightarrow A = \frac{1}{8}$

x^2: $0 = A + B \Rightarrow B = -\frac{1}{8}$

x^0: $1 = 4A - 2C \Rightarrow 2C = 4A - 1 = -\frac{1}{2} \Rightarrow C = -\frac{1}{4}$

$$\int = \int \frac{1}{8}\frac{1}{x-2} - \frac{1}{16}\frac{2x}{x^2+4} - \frac{1}{4}\frac{1}{x^2+4}\,dx$$

$$= \frac{1}{8}\ln(x-2) - \frac{1}{16}\ln(x^2+4) - \frac{1}{8}\arctan(\tfrac{x}{2}) + C$$

$\sin^2\theta + \cos^2\theta = 1$

$1 + \cot^2\theta = \csc^2\theta$

Problem 3. Compute (substitute $x = 3\sin\theta$):

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx.$$

$x = 3\sin\theta$
so $dx = 3\cos\theta\, d\theta$

$$= \int \frac{\sqrt{9-9\sin^2\theta}}{9\sin^2\theta}\, 3\cos\theta\, d\theta$$

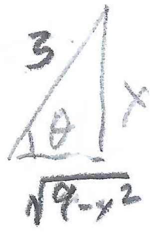
$$= \int \frac{\cos\theta}{\sin^2\theta} \cdot \cos\theta\, d\theta$$

$$= \int \cot^2\theta\, d\theta$$

$$= \int \csc^2\theta - 1\, d\theta$$

$$= -\cot\theta - \theta + C$$

$$= -\frac{\sqrt{9-x^2}}{x} - \arcsin\left(\frac{x}{3}\right) + C$$



Problem 4. Compute (substitute $u = e^t$, then $u = 3\tan\theta$):

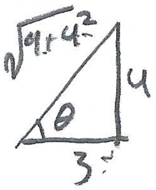
$$\int_0^{\ln 4} \frac{e^t\,dt}{\sqrt{e^{2t}+9}}.$$

Let $u = e^t$ so $du = e^t\,dt$

$$\int \frac{du}{\sqrt{u^2+9}}$$

Let $u = 3\tan\theta$ so $du = 3\sec^2\theta\, d\theta$

$$= \int \frac{3\sec^2\theta\, d\theta}{3\sqrt{\tan^2\theta + 1}}$$



$$= \int \sec\theta\, d\theta$$

$$= \ln(\sec\theta + \tan\theta) + C$$

$$= \ln\left(\frac{\sqrt{u^2+9}}{3} + \frac{u}{3}\right) + C$$

$$= \ln\left(\frac{\sqrt{e^{2t}+9}}{3} + \frac{e^t}{3}\right)\Bigg|_0^{\ln 4}$$

$$= \ln\left(\frac{\sqrt{16+9}}{3} + \frac{4}{3}\right) - \left(\ln\left(\frac{\sqrt{10}}{3} + \frac{1}{3}\right)\right)$$

$$\cancel{= \ln\left(\frac{9}{3}\right) - \ln(3)}$$

$$= \ln\left(\frac{9}{3}\right) - \ln\left(\frac{\sqrt{10}+1}{3}\right)$$

$$= \ln(9) - \ln(\sqrt{10}+1)$$

$$= \ln\left(\frac{\sqrt{10}^2 - 1}{\sqrt{10}+1}\right) = \ln(\sqrt{10}-1)$$