Name:

Algebra II
Examination 12 (Project)

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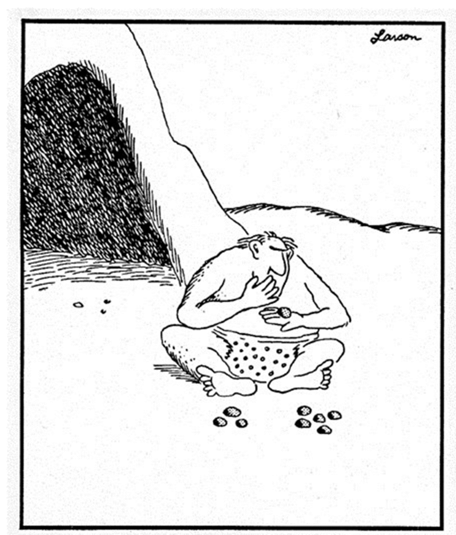
Thursday, February 23, 2022

The examination contains five problems which are worth 20 points each, and two bonus problems worth an additional 20 points each, for a maximum of 100 points. The problems have been selected from the first three exams of the first semester.



Early stages of math anxiety

Prob 1	Prob 2	Prob 3	Prob 4	Prob 5	Bonus1	Bonus 2	Total Score

Problem 1. (Matching)
Match the terms or phrases on the left with the descriptions on the right. Write the number of the matching description in the blank next to each term. Use each description exactly once.

(a) ______ Multiplicity	**(1)** If $f(z) = 0$, then $f(\overline{z}) = 0$.
(b) ______ Division Algorithm	**(2)** If r is the remainder when g is divided by $x - a$, then $g(a) = r$.
(c) ______ $\mathbb{R}$	**(3)** The set of fractions.
(d) ______ Poles	**(4)** A removable discontinuity of a rational function.
(e) ______ $\mathbb{R}^2$	**(5)** There exist q and r such that $g = fq + r$ and $\deg(r) < \deg(f)$.
(f) ______ Remainder Theorem	**(6)** If $g(a) = 0$, then $x - a$ divides g.
(g) ______ $\mathbb{Q}$	**(7)** The maximum power of $(x - a)$ which divides a polynomial.
(h) ______ Holes	**(8)** The set of decimal expansions.
(i) ______ Conjugate Pairs Theorem	**(9)** A nonremovable discontinuity of a rational function.
(j) ______ Factor Theorem	**(10)** The set of ordered pairs of real numbers.

Problem 2. (Solving Equations)
Find all real numbers x which satisfy the following equations. Simplify all fractions and radicals where possible. Using correct set notation, write the solution set.

(a) $7x + 5 = 3x - 13$

(b) $121x^2 - 26 = 10$

(c) $x^2 - 18x + 81 = 0$

(d) $3x^2 - x = 10$

(e) $x^3 + 5x^2 = 2x + 10$

Problem 3. (Intervals)
Write the following subsets of $\mathbb{R}$ using correct interval notation.

(a) The set of real numbers greater than or equal to 13.

(b) The set of real numbers which are strictly greater than 5 and less than or equal to 7.

(c) The set of real numbers whose square is less than or equal to 25.

(d) The set of real numbers such that $x^2 \geq 2x + 15$

(e) The set of real numbers such that $x^3 + 5x^2 \leq 2x + 10$.

Problem 4. (Polynomial Division)
Let $f(x) = x^3 - 9x^2 + 13x + 2$.

(a) Find $f(2)$.

(b) Find q and r such that $f(x) = (x - 2)q(x) + r$.

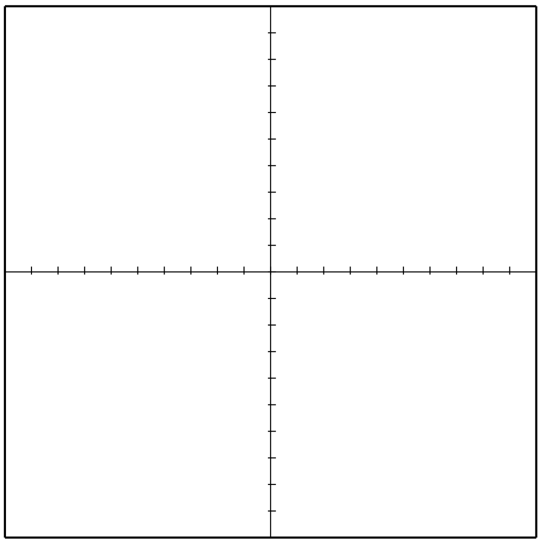
(c) Solve $q(x) = 0$.

(d) Solve $f(x) = 0$. Correctly write the solution set.

(e) Solve $f(x) \geq 0$. Write the solution using correct interval notation.

Problem 5. (Graphing) Fill out the charts, and sketch the graph.

(a) Consider the polynomial function $f(x) = (x+3)(x-1)(x-2)$. Find its degree, leading coefficient, constant coefficient, zeros, and end behavior. Find the y-intercept and x-intercepts. Graph the function and label these points.



Polynomial: $f(x) = (x+3)(x-1)(x-2)$

Degree:

Leading Coefficient:

Constant Coefficient:

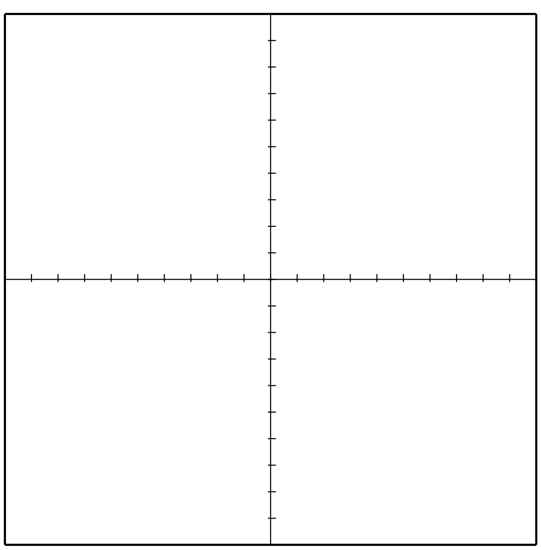
Zeros:

y-intercept:

x-intercepts:

End Behavior:

(b) Consider the rational function $f(x) = \dfrac{2x-8}{x+2}$. Find its degree, zeros, and poles. Find its intercepts and asymptotes. Graph the function and label these features.



Rational Function: $f(x) = \dfrac{2x-8}{x+2}$

Degree:

Zeros:

Poles:

y-intercept:

x-intercepts:

Vertical Asymptotes:

Polynomial Asymptote:

Problem 6. (Bonus - Exponential Word Problem)
If a radioactive material has a half-life of h, and the amount at time zero is A_0, then the amount at time t is

$$A(t) = A_0 \left(\frac{1}{2}\right)^{t/h}.$$

In 1992, an evil dictator smuggled in 9.6 kilograms of radioactive delusium. After 12 years (in 2004), conquering armies found that only 0.3 kilograms remained. How much delusium actually existed when the intelligence report was forged in 2001?

(a) Find h; how long does it take for half of the material to decay?

(b) Write the function $A(t)$ using the values for A_0 and h.

(c) Use $A(t)$ to answer the question.

Problem 7. (Bonus - Abstract Algebra)
The *field of integers modulo seven* is

$$\mathbb{F}_7 = \{0, 1, 2, 3, 4, 5, 6\}.$$

You can add, subtract, multiply, and divide the members of this set (except you can't divide by zero), with the understanding that two numbers are equal if they have the same remainder when divided by 7. Thus, for example:

- $7 = 14 = 21 = 28 = 700 = 0$
- $4 = 11 = -3 = 74$
- $5 + 3 = 8 = 1$
- $7 - 3 = 7 + 4 = 11 = 4$
- $4 \cdot 5 = 20 = 14 + 6 = 6$
- Since $2 \cdot 4 = 8 = 1$, we have $1/2 = 4$, so $3/2 = 3 \cdot 4 = 12 = 5$

Find all x in $\mathbb{F}_7$ such that the equation is true. Write the solution set.

(a) $x - 4 = 6$

(b) $x^2 - 5x + 1 = 0$

(c) $3x = 5$

(d) $5x - 3 = 6 - 4x$

(e) $x^3 - 6 = 0$