Name:

AP Calculus AB
PRACTICE Examination 1

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Tuesday, September 10, 2024

The examination contains ten problems which are worth 10 points each, and two bonus problems worth ten points each. All answers must be justified. An appropriate amount of work must be shown to receive credit.

Problem 1. (Sets)
Let $A = [1, 8]$ and $B = (5, 11]$. Find the indicated set.
Express your answer in interval notation.

(a) $A \cup B$

(b) $A \cap B$

(c) $A \smallsetminus B$

Problem 2. (Domain and Range)
Find the domain and range of $f(x) = \sqrt{x^2 - 1}$.

Problem 3. (Absolute Value Inequalities)
Solve the inequality $|2x + 8| \leq 20$. Correctly write the solution set using interval notation.

Problem 4. (Quadratic Inequalities)
Solve the inequality $x^2 - 2x - 15 \leq 0$. Correctly write the solution set using interval notation.

Problem 5. (Equation of a Line)
Consider the points $A = (-2, 9)$ and $B = (12, 2)$.
Find the slope-intercept form of the equation of the line through A and B.

Problem 6. (Equation of a Circle)
Find the center and radius of a circle with equation

$$x^2 + 12x + y^2 - 14y = 21.$$

Problem 7. (Wrapping Function)

Let $W : \mathbb{R} \to \mathbb{R}^2$ be the wrapping function. Find $W\left(\dfrac{53\pi}{6}\right)$.

Problem 8. (Trigonometric Values)

If $\sin\theta = \dfrac{5}{13}$, what is $\cos\theta$?

Problem 9. (Average Rate of Change)

Let $f(x) = x^2 - 3x + 1$. Find the average rate of change of f on the interval $[1, 3]$.

Problem 10. (Remainder Theorem)

Let $f(x) = x^4 - 9x^3 + 17x - 11$. Find $f(7)$.

Problem 11. (Bonus - Tangent Circles)

Consider the circles given by the equations $x^2 + y^2 = 9$ and $(x - 5)^2 + (y - 12)^2 = r^2$. Find r such that the circles are tangent.

Problem 12. (Bonus - Descartes' Method)

Let $f(x) = c - x^2$ and $g(x) = (x - 1)^2$. Find c such that graphs of f and g are tangent.