Name:

**Vector Calculus**
**Examination 1**

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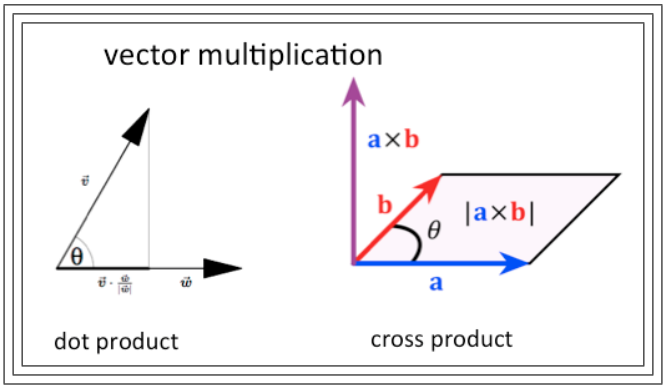
Wednesday, September 6, 2017

The examination contains five problems which are worth 20 points each, and two bonus problem worth an additional 20 points each, for a maximum of 100 points. No books, notes, calculators, laptop computers, cell phones, or other electronic devices may be used.




| Prob 1 | Prob 2 | Prob 3 | Prob 4 | Prob 5 | Bonus 1 | Bonus 2 | Total Score |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

**Problem 1. (Matching)**
Match the terms or phrases on the left with the descriptions on the right. Write the number of the matching description in the blank next to each term. Use each description exactly once.

**(a)** _____Cross product

**(b)** _____Sphere in $\mathbb{R}^3$

**(c)** _____Vector

**(d)** _____Line in $\mathbb{R}^2$

**(e)** _____Locus

**(f)** _____Plane in $\mathbb{R}^3$

**(g)** _____$\mathbb{R}^n$

**(h)** _____Dot product

**(i)** _____Linear combination

**(j)** _____Line in $\mathbb{R}^3$

**(1)** $ax + by = c$

**(2)** The set of points which satisfy an equation.

**(3)** $\langle v_2w_3 - v_3w_2, v_3w_1 - v_1w_3, v_1w_2 - v_2w_1 \rangle$

**(4)** The set of all ordered $n$-tuples.

**(5)** $\langle x_0 + tv_1, y_0 + tv_2, z_0 + tv_3 \rangle$

**(6)** $v_1w_1 + v_2w_2 + v_3w_3$

**(7)** An equivalence class of arrows.

**(8)** $a\vec{v} + b\vec{w}$

**(9)** $ax + by + cz = d$

**(10)** $x^2 + y^2 + z^2 = d$

**Problem 2. (Computation)**
Let $\vec{v} = \langle 2, -3, 5 \rangle$ and $\vec{w} = \langle -5, 1, 4 \rangle$. Compute the following.

**(a)** $\vec{v} + \vec{w}$

**(b)** $3\vec{v} - 2\vec{w}$

**(c)** $\vec{v} \cdot \vec{w}$

**(d)** $\vec{v} \times \vec{w}$

**(e)** $\text{proj}_{\vec{w}}\vec{v}$

**Problem 3. (Points in $\mathbb{R}^2$)**
Consider the points $A(2,5)$, $B(6,1)$, and $C(-7,3)$.
Let $\vec{v}$ be the vector from $A$ to $B$, and let $\vec{w}$ be the vector from $A$ to $C$.

**(a)** Compute $\vec{v}$ and $\vec{w}$.

**(b)** Find the cosine of the angle $\angle BAC$.

**(c)** Find the area of the triangle $\triangle ABC$.

**(d)** Find the general equation of the line $\overleftrightarrow{AB}$.

**(e)** Find the distance from the point $C$ to the line $\overleftrightarrow{AB}$.

**Problem 4. (Parallelepipeds in $\mathbb{R}^3$)**
Consider the points $P(0,1,2)$, $Q(3,7,5)$, $R(-1,0,1)$, and $S(6,2,8)$.
Please answer the questions below, with complete justification.

**(a)** Do these four points lie on the same plane?

**(b)** Can one change the answer to **(a)** by changing the $y$-coordinate of $Q$?

**(c)** What does this tell you?

**Problem 5. (Lines, Planes, and Spheres in $\mathbb{R}^3$)**
Let $A$ be the locus of the equation

$$x^2 + y^2 + z^2 = 4x + 6y + 12z.$$

Let $B$ be the locus of the equation

$$x - 4y + 8z = 11.$$

Let $C = A \cap B$.

**(a)** $A$ is a sphere. Find its center and radius.

**(b)** $B$ is a plane. Find a normal vector for $B$. Find the vector equation of a line through the center of $A$ and perpendicular to $B$.

**(c)** $C$ is a circle. Find its center.

**Problem 6. (Bonus 1)**
Let $A = [1, 2) = \{x \in \mathbb{R} \mid 1 \leq x < 2\}$.
Let $B = [3, 5] = \{x \in \mathbb{R} \mid 3 \leq x \leq 5\} \cup \{7, 8\}$.
Let $C = A \times A \times B$.

**(a)** Sketch the set $A \times A$ in $\mathbb{R}^2$.

**(b)** Count the set $A \times A \cap \mathbb{Z}^2$.

**(c)** Sketch the set $A \times A \times B$ in $\mathbb{R}^3$

**(d)** Count the set $A \times A \times B \cap \mathbb{Z}^3$.

**Problem 7. (Bonus 2)**
Let $A$ be the plane given by $x + 2y + 3z = 6$ and $B$ be the plane given by $3x + 2y + z = 6$.
Let $L = A \cap B$ be the line of intersection of $A$ and $B$. Let $P_0 = (1, 1, 1)$ and note that $P_0 \in L$.
Find the equation of the plane which is perpendicular to $L$ and passes through the point $P_0$, expressed in the form $ax + by + cz = d$.