Vector Calculus — Responses 0403
Dr. Paul L. Bailey — Thursday, April 2, 2020

Here are the "answers".
Decide whether divergence at you is positive, negative, or zero in the following cases.

- You are standing at the entrance to baseball stadium ten minutes before the game begins. Zero
- You are sitting in the stands at a baseball game five minutes before the first pitch. Positive
- You are sitting in the stands at a baseball game in the middle of the fourth inning. Zero
- You are sitting in the stands at Dodger's Stadium in the eighth inning just after the other team hit a grand slam to go ahead 11 to 2. Negative
- You threw a rock at a hornet's nest and they know you did it. Negative
- Your friend just broke wind in a crowded shopping mall. Positive
- You yell "fire" in a crowded movie theater. Positive
- You yell "free beer here" in the middle of Ted Nugent concert. Negative
- You are standing in the middle of a Ted Nugent concert (covering your ears) when the intercom announces "free beer outside". Zero
- You stand in a zephyr. Zero

Question 1. For divergence to be negligible, is the fluid remaining in the same location?

Answer. The divergence is zero if the incoming equals the outgoing. So no, it doesn't have to be standing still. □

Question 2. I understand the concept but I also read that divergence is a scalar ... if negative divergence indicates that the fluid is compressing, how is that scalar? It's including information about the direction of the fluid, isn't it?

Answer. Given a vector field, its divergence is a scalar valued function.

Let $\vec{F} : D \to \mathbb{R}^2$, where $D \subset \mathbb{R}^2$. Then $\vec{F} = \langle M, N \rangle$, where M and N are the component functions. Define

$$\operatorname{div} \vec{F} : D \to \mathbb{R} \quad \text{given by} \quad \operatorname{div} \vec{F}(x,y) = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}.$$

So the value of divergence is a real number.

A real number can be negative. Can we view real numbers as vectors? Perhaps, vectors on a line. They can point in the positive direction, or in the negative direction.

If the divergence is positive at a point, the vector field is tending away from that point. If the divergence is negative at a point, the vector field is tending toward that point. If the divergence is zero, the extent to which the vector field is tending toward the point is cancelled by the extent to which it is tending away from the point.

It is not including information about the direction of the flow. If you are standing in an eastern flowing river, the divergence will be zero if the incoming equals the outgoing. Divergence will not detect the easterly direction. □

Question 3. In the page 1770, it is said that the rate at which fluid leaves the rectangle across the bottom edge is approximately F(x,y)*(-j)*dx=-N(x,y)dx. Why would this be the rate of leaving fluid? Is F(x,y) the velocity of a fluid flow? Honestly, I can't understand how the book to reach the flux density through these seemingly easy but conceptually obscure derivations.

Answer. You can think of $\vec{F}(x,y)$ as being the velocity of the flow at the point (x,y). Fluid is not the best model for visualizing this, though, since our experience of water is the it is relatively difficult to compress. Think of a gas instead, or, of a force field (gravitational or magnetic).

The flux along a curve is the integral of the projection of the vector field onto a unit normal vector to the curve, at a given point on the curve. It measures the component of the vector field in the direction perpendicular to the curve.

The horizontal line segment which is the bottom edge of the rectangle is parameterized by a path whose derivative is $\rangle 1, 0\langle$. The unit normal vector is $\vec{n} = \langle 0, -1\rangle$. We dot this with the vector field to get

$$\vec{F} \cdot \vec{n} = \langle M, N\rangle \cdot \langle 0, -1\rangle = \langle 0, -N\rangle = -N\vec{j}.$$

So in the derivation, what they are doing (in effect) is to compute the flux across this rectangle, and then letting the lengths of the sides of the rectangle go to zero, and taking the limit. Maybe I can write up that approach formally over the weekend.

I hope that helps. $\square$