Name:

Algebra II
Examination 9

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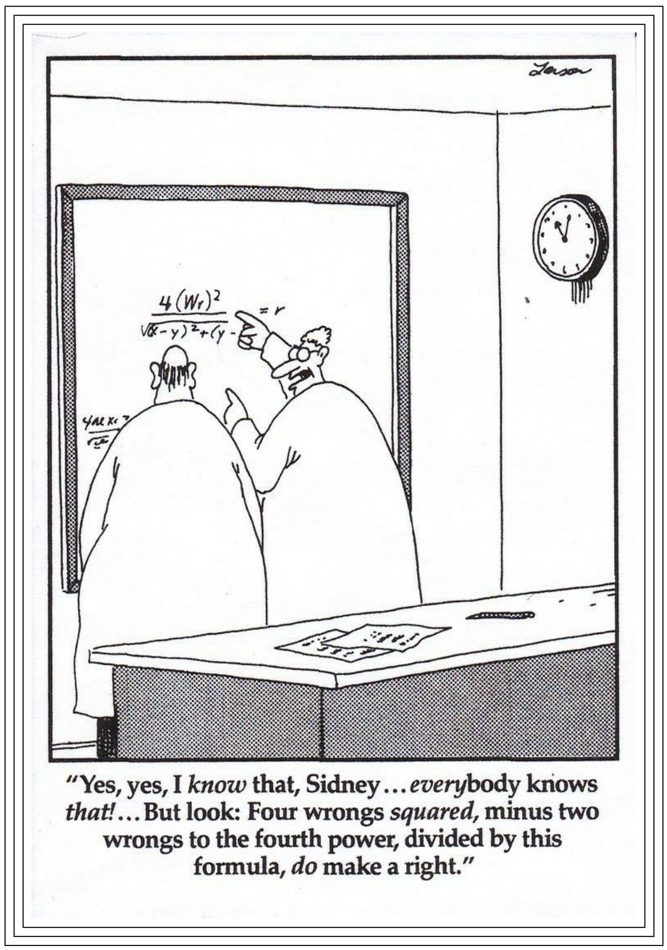
Thursday, February 3, 2022

The examination contains ten problems which are worth 10 points each, and two bonus problems worth ten points each.



Prob1	Prob2	Prob3	Prob4	Prob5	Prob6	Prob7	Prob8	Prob9	Prob10	Bonus1	Bonus2	Total

Problem 1. (Cube Roots)
Solve the equation

$$\sqrt[3]{x^2 - 8x - 6} = 3.$$

Correctly write the solution set.

Problem 2. (Exponents)
Simplify

$$8^{2/3} + 9^{3/2}.$$

The answer should be an integer.

Problem 3. (Equation of a Circle)
Write the equation of the circle with center $(2, 5)$ which passes through $(5, 9)$.

Problem 4. (Factoring Cubics)
Let $f(x) = x^3 - 11x + 10$. Notice that $f(1) = 0$. Divide $f(x)$ by $(x - 1)$ to find the other two zeros of f. Write the solution set for the equation $f(x) = 0$.

Problem 5. (Solving rational equations)
Let $f(x) = \dfrac{x^2 - x - 2}{x + 4}$. Solve the equation $f(x) = 2$. Correctly write the solution set.

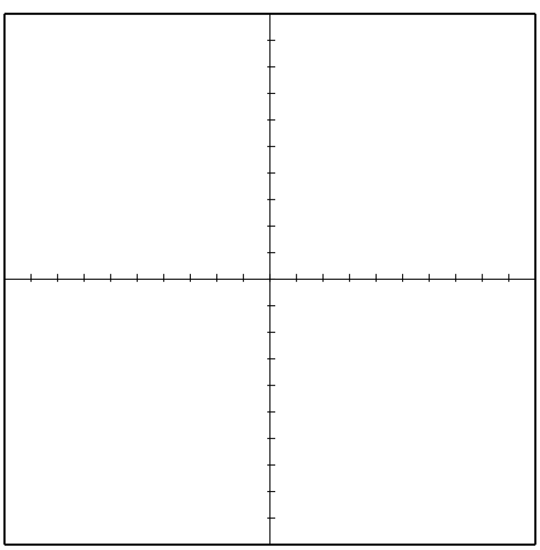
Problem 6. (Remainder Theorem)
Let $g(x) = 2x^5 - 20x^4 + 19x^3 - 12x^2 + 30x + 33$ and $f(x) = x - 9$. Find the quotient and remainder when g is divided by f.

Problem 7. (Domain)

Let $f(x) = \dfrac{\sqrt{x+10}}{x^2-4}$. Find the domain of f. Write your answer as the union of disjoint intervals.

Problem 8. (Graphing)

Consider the polynomial function $f(x) = x^3 - x^2 - 9x + 9$. Find its degree, leading coefficient, constant coefficient, zeros, and end behavior. Find the y-intercept and x-intercepts. Graph the function and label these points.



Polynomial: $f(x) = x^3 - x^2 - 9x + 9$

Degree:

Leading Coefficient:

Constant Coefficient:

Zeros:

y-intercept:

x-intercepts:

End Behavior:

Problem 9. (Standard Sets)
Of the sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, and $\mathbb{C}$, state the smallest set which contains all solutions to the given equation.

(a) $2x - 5 = 5x - 2$

(b) $x^2 - 2x + 2 = 0$

(c) $x^2 - 3x + 2 = 0$

(d) $x^3 - 3x + 2 = 0$

(e) $x^3 - 3x + 2 = 2$

Problem 10. (Set Operations)
Compute the following sets. Write your answer using correct set notation.

Let $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8\}$, and $C = [4, 7]$.

(a) $A \cup B$

(b) $A \cap B$

(c) $A \smallsetminus B$

(d) $B \smallsetminus A$

(e) $C \smallsetminus B$

Problem 11. (Bonus - Matching)
Match the function $f(x)$ on the left with its inverse $g(x)$ on the right. Write the number of the matching inverse in the blank next to each function $f(x)$. Use each $g(x)$ exactly once.

(a) ______ $f(x) = x^2$

(b) ______ $f(x) = 2x + 8$

(c) ______ $f(x) = \dfrac{x}{x+1}$

(d) ______ $f(x) = \sqrt{x+1}$

(e) ______ $f(x) = \sqrt[3]{x}$

(f) ______ $f(x) = \dfrac{1}{x^2+1}$

(g) ______ $f(x) = x$

(h) ______ $f(x) = \sqrt{x} - 1$

(i) ______ $f(x) = \dfrac{1}{x}$

(j) ______ $f(x) = \dfrac{1}{2}x + 4$

(1) $g(x) = x^3$

(2) $g(x) = \dfrac{1}{2}x - 4$

(3) $g(x) = \dfrac{x}{1-x}$

(4) $g(x) = x$

(5) $g(x) = \sqrt{\dfrac{1-x}{x}}$

(6) $g(x) = x^2 - 1$

(7) $g(x) = \sqrt{x}$

(8) $g(x) = (x+1)^2$

(9) $g(x) = 2x - 8$

(10) $g(x) = \dfrac{1}{x}$

Definition 1. Let $f : A \to B$. We say that f is *injective* (or *one-to-one*) on A if, for every $a_1, a_2 \in A$, we have

$$f(a_1) = f(a_2) \quad \Rightarrow \quad a_1 = a_2.$$

Problem 12. (Bonus - Injectivity)
Find an interval A such that the given function is injective on A.

(a) $f(x) = 7x - 81$

(b) $f(x) = x^2 - 6x + 9$

(c) $f(x) = \sqrt{x}$

(d) $f(x) = |x - 2| + 5$

(e) $f(x) = x^4 - 10x^2 + 9$