

A *quadratic function* is a polynomial of degree two.  
The *standard form* of a quadratic function is

$$f(x) = ax^2 + bx + c,$$

where  $a$ ,  $b$ , and  $c$  are real numbers. The graph is a *parabola* which opens upward if  $a > 0$  and opens downward if  $a < 0$ . The  $y$ -intercept is the point  $(0, f(0))$ , and we see that  $f(0) = c$ . The zeros of the function are the values of  $x$  such that  $f(x) = 0$ . The *quadratic formula* says that  $f(x) = 0$  if and only if

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The *discriminant* of the quadratic function is

$$\Delta = b^2 - 4ac;$$

this determines the number of real zeros. There are three cases:

- (a) if  $b^2 - 4ac > 0$ , there are two real zeros;
- (b) if  $b^2 - 4ac = 0$ , there is one real zero;
- (c) if  $b^2 - 4ac < 0$ , there are no real zeros.

The  $x$ -intercepts (if any) are the points  $(x, 0)$ , where  $x$  is a real zero.

The *shifted form* of a quadratic function is

$$f(x) = a(x - h)^2 + k,$$

where  $a$ ,  $h$ , and  $k$  are real numbers. The shifted form tells how the graph of  $f(x)$  is obtained from the graph of  $x^2$ , as follows:

- (a) shift horizontally by  $h$ ;
- (b) stretch vertically by  $|a|$ ;
- (c) reflect across the  $x$ -axis if  $a$  is negative;
- (d) shift vertically by  $k$ .

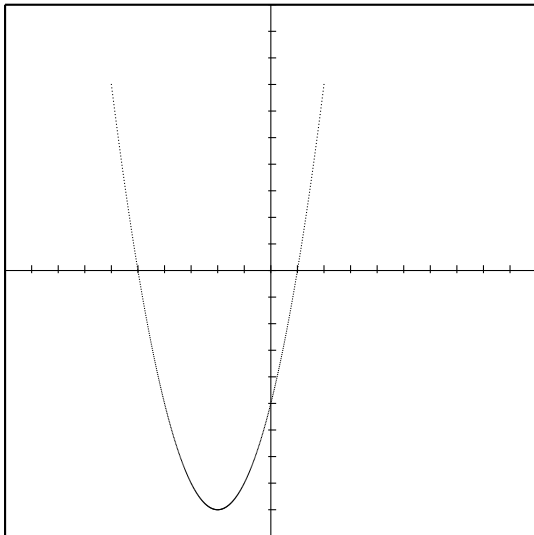
The point  $(h, k)$  where the graph turns around is called the *vertex*. Thus  $k$  is the *minimum value* of the function if  $a > 0$ , and is the *maximum value* of the function if  $a < 0$ .

We can convert from standard form to shifted form by completing the square, which leads to:

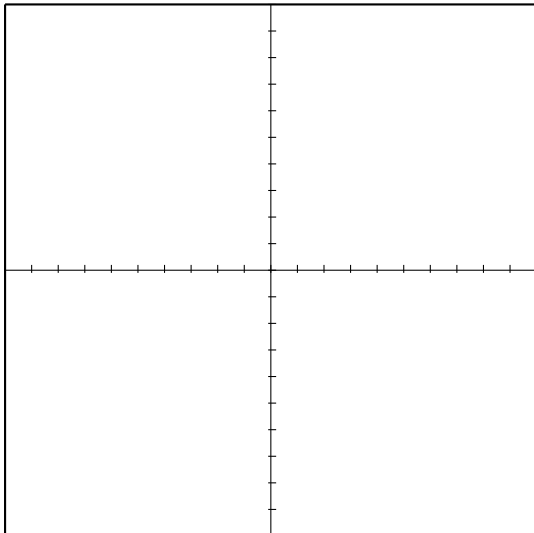
$$h = -\frac{b}{2a} \quad \text{and} \quad k = c - \frac{b^2}{4a}.$$

We can convert from shifted form to normal form by squaring and simplifying, which leads to:

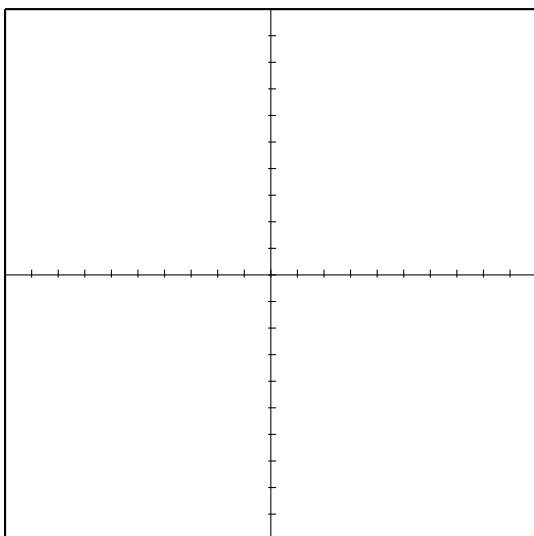
$$b = -2ah \quad \text{and} \quad c = ah^2 + k.$$



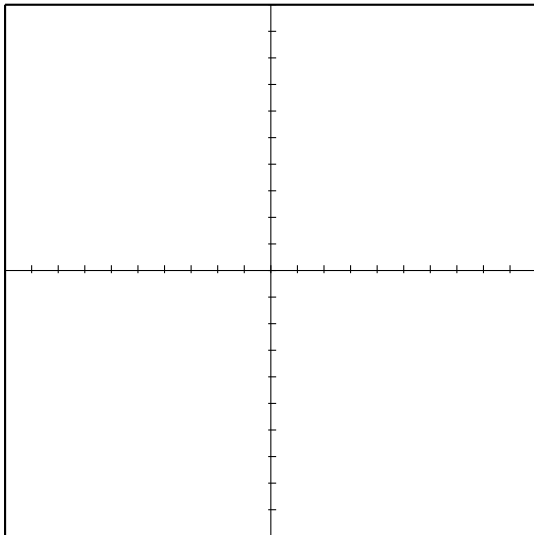
**Example:**  $f(x) = 4x - 5 + x^2$   
**Normal Form:**  $f(x) = x^2 + 4x - 5$   
**Shifted Form:**  $f(x) = (x + 2)^2 - 9$   
**a:** 1    **b:** 4    **c:** -5    **h:** -2    **k:** -9  
**Discriminant:** 36  
**Zeros:**  $x = -5$  and  $x = 1$   
**y-intercept:**  $(0, -5)$   
**x-intercept(s):**  $(-5, 0)$  and  $(1, 0)$   
**Vertex:**  $(-2, -9)$



**Quadratic Function:**  $f(x) = x^2 - 6x + 8$   
**Normal Form:**  
**Shifted Form:**  
**a:**    **b:**    **c:**    **h:**    **k:**  
**Discriminant:**  
**Zeros:**  
**y-intercept:**  
**x-intercept(s):**  
**Vertex:**



**Quadratic Function:**  $f(x) = (x + 2)^2 - 5$   
**Normal Form:**  
**Shifted Form:**  
**a:**    **b:**    **c:**    **h:**    **k:**  
**Discriminant:**  
**Zeros:**  
**y-intercept:**  
**x-intercept(s):**  
**Vertex:**



**Quadratic Function:**  $f(x) = x^2 - 6x + 9$

**Normal Form:**

**Shifted Form:**

**a:**      **b:**      **c:**      **h:**      **k:**

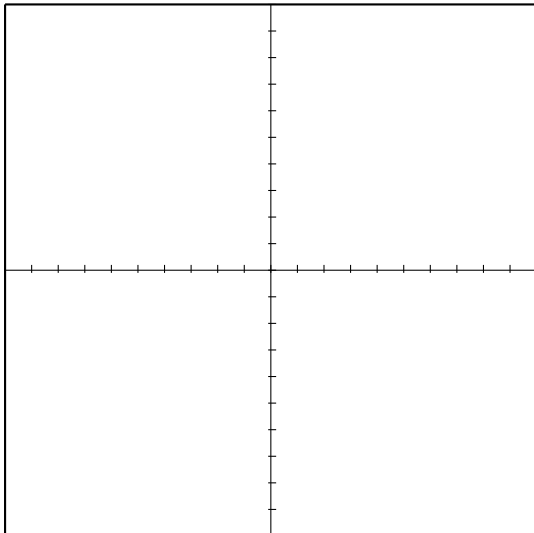
**Discriminant:**

**Zeros:**

***y*-intercept:**

***x*-intercept(s):**

**Vertex:**



**Quadratic Function:**  $f(x) = 9 - x^2$

**Normal Form:**

**Shifted Form:**

**a:**      **b:**      **c:**      **h:**      **k:**

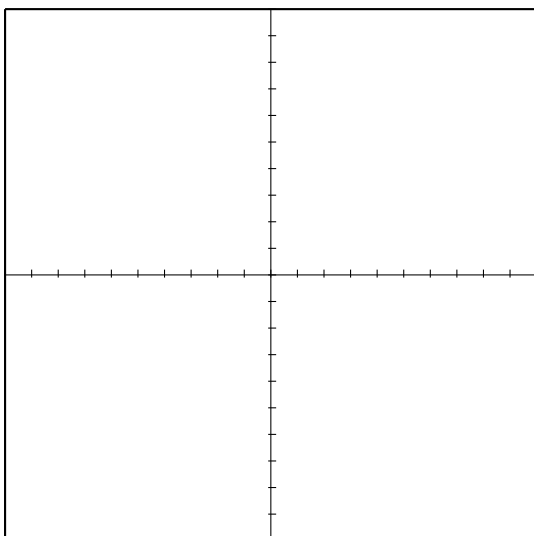
**Discriminant:**

**Zeros:**

***y*-intercept:**

***x*-intercept(s):**

**Vertex:**



**Quadratic Function:**  $f(x) = 6x - x^2$

**Normal Form:**

**Shifted Form:**

**a:**      **b:**      **c:**      **h:**      **k:**

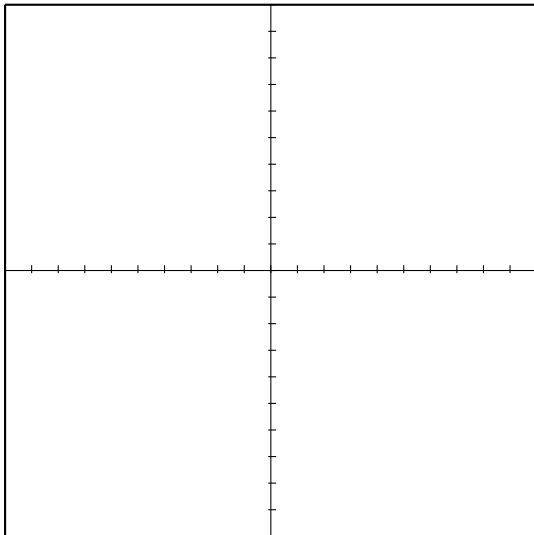
**Discriminant:**

**Zeros:**

***y*-intercept:**

***x*-intercept(s):**

**Vertex:**



**Quadratic Function:**  $f(x) = x^2 - 5x + 2$

**Normal Form:**

**Shifted Form:**

**a:**      **b:**      **c:**      **h:**      **k:**

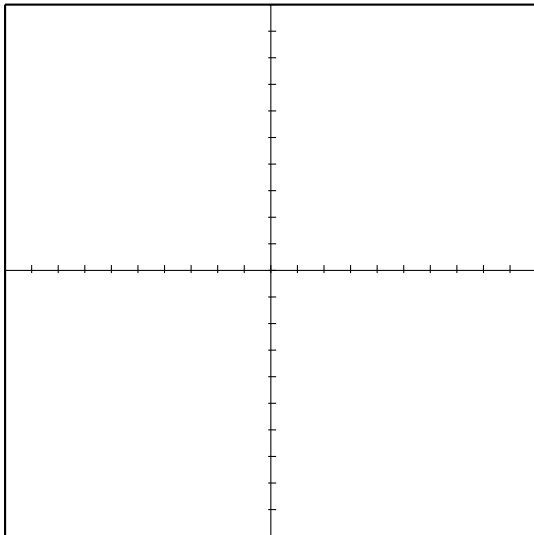
**Discriminant:**

**Zeros:**

***y*-intercept:**

***x*-intercept(s):**

**Vertex:**



**Quadratic Function:**  $f(x) = (3x - 7)(-x + 1)$

**Normal Form:**

**Shifted Form:**

**a:**      **b:**      **c:**      **h:**      **k:**

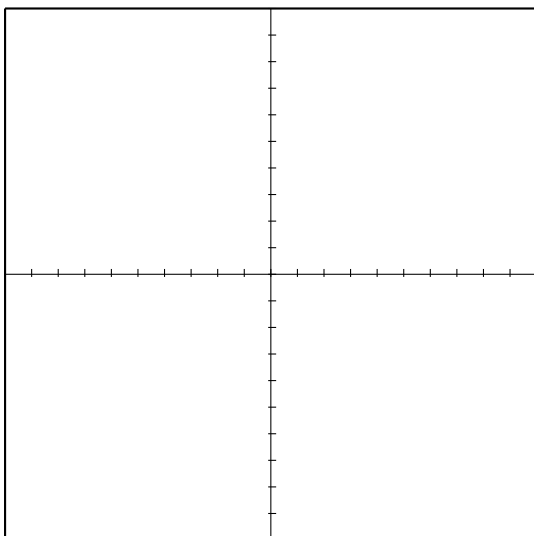
**Discriminant:**

**Zeros:**

***y*-intercept:**

***x*-intercept(s):**

**Vertex:**



**Quadratic Function:**  $f(x) = 6 + x^2 - 4x$

**Normal Form:**

**Shifted Form:**

**a:**      **b:**      **c:**      **h:**      **k:**

**Discriminant:**

**Zeros:**

***y*-intercept:**

***x*-intercept(s):**

**Vertex:**