Name:

Algebra II
Examination 17 (Project)

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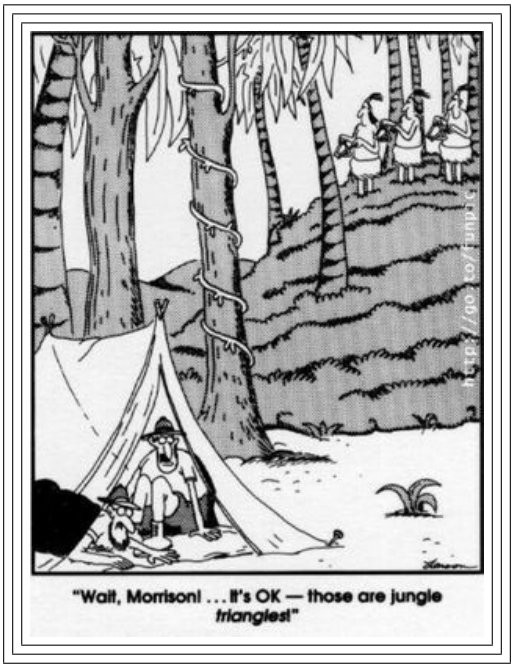
Tuesday, April 26, 2022

The examination contains 20 multiple choice questions worth 5 points each, and two bonus problems worth an additional 10 points each, for a maximum of 100 points.

- Compound Interest: $A(t) = A_0\left(1 + \frac{r}{f}\right)^{ft}$
- Exponential Growth: $A(t) = A_0 e^{rt}$
- Change of Base: $\log_b(x) = \frac{\log_a(x)}{\log_a(b)} = \frac{\ln(x)}{\ln(a)}$
- Law of Sines: $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$
- Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos(C)$



MC #	MC Points	Bonus 1	Bonus 2	Raw Score	Adj Score

Question 1. Consider the equation

$$x = 1 + 2.$$

Find x.

(A) 1

(B) 2

(C) 3

(D) 6

(E) Cannot be determined from the information given.

Question 2. Consider the equation

$$5x - 2 = 2x + 11.$$

Find x.

(A) 1.857

(B) 3.000

(C) 3.333

(D) 4.333

(E) 5.222

Question 3. Which of the following functions is NOT injective (one-to-one)?

(A) $f(x) = e^x$

(B) $f(x) = \ln(x)$

(C) $f(x) = 2x - 7$

(D) $f(x) = x^3 - x$

(E) $f(x) = x^3 + x$

Question 4. Consider the points

$$A = (-23, 50) \text{ and } B = (31, 71).$$

The slope of the line through A and B is

(A) 0.381

(B) 0.389

(C) 2.571

(D) 2.625

(E) 15.125

Question 5. Consider the equation

$$x = \log_5(1234).$$

Find x.

(A) 0.226

(B) 1.234

(C) 3.091

(D) 4.423

(E) 7.118

Question 6. Consider the equation

$$\log_x(43) = 17.$$

Find x.

(A) 1.068

(B) 1.248

(C) 2.529

(D) 60.000

(E) 731.000

Question 7. Consider the equation

$$e^{2x} - 2e^x = 15.$$

Find x.

(A) 0.213

(B) 0.631

(C) 1.098

(D) 1.609

(E) 2.718

Question 8. Consider the equation

$$\sin x = \frac{1}{4}.$$

Find x.

(A) 12.500°

(B) 13.862°

(C) 14.478°

(D) 15.000°

(E) 25.000°

Question 9. A circular arc of length $s = 2357$ has a central angle of $\theta = 47°$. What is the approximate radius of the circle?

(A) 1178

(B) 1933

(C) 2357

(D) 2835

(E) 2873

Question 10. A circular sector of area $A = 50$ is cut from a circle of radius 10. What is the approximate angle of the sector?

(A) 23°

(B) 28°

(C) 47°

(D) 57°

(E) 62°

Question 11. A triangle has vertices A, B, and C, with opposite sides of length a, b, and c. Let $a = 8$, $b = 15$, and $c = 17$. Find the approximate angle at A.

(A) 23°

(B) 28°

(C) 47°

(D) 57°

(E) 62°

Question 12. A triangle has vertices A, B, and C, with opposite sides of length a, b, and c. Let $a = 8$, $b = 15$, and $B = 47°$. Find the approximate angle at A.

(A) 23°

(B) 28°

(C) 47°

(D) 57°

(E) 62°

Question 13. Consider the function

$$f(x) = \frac{\sqrt{x^2 - 4}}{x^3 - 9x}.$$

What is the fewest number of disjoint intervals required to write the domain of f?

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

Question 14. Consider the function

$$f(x) = 3 - \ln(x - 5).$$

Find the x-intercept of f.

(A) $(8.000, 0)$

(B) $(15.000, 0)$

(C) $(25.086, 0)$

(D) $(25.608, 0)$

(E) $(25.860, 0)$

Question 15. Consider the function

$$f(x) = x^2 - 7.$$

The positive solution to the equation $f(x) = 0$ is

(A) 2.500

(B) 2.568

(C) 2.625

(D) 2.646

(E) 2.750

Question 16. Consider the function

$$f(x) = x^2 - 7x - 11.$$

The positive solution to the equation $f(x) = 0$ is

(A) 3.019

(B) 4.618

(C) 6.038

(D) 8.322

(E) 10.019

Question 17. Consider the function

$$f(x) = x^2(5 - x) + 1.$$

How many x intercepts does this function have?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

Question 18. Billy Jo invested \$623.25 with 1.7% annual interest, compounded monthly. How much money will Billy Jo have after 20 years?

(A) \$623.25

(B) \$875.42

(C) \$6232.50

(D) \$31162.50

(E) \$35619.76

Question 19. Twenty tribbles were smuggled onto the Starship Enterprise. Tribble populations grow exponentially, and increase by 57% per day. How many tribbles were aboard after ten days?

(A) 20

(B) 77

(C) 1140

(D) 5977

(E) 11400

Question 20. Let A and B be intervals. Which of the following is NEVER true?

(A) $A \cap B$ is the empty set.

(B) $A \cap B$ is an single point.

(C) $A \setminus B$ is a single point.

(D) $A \cap B$ is the disjoint union of two intervals.

(E) $A \setminus B$ is the disjoint union of two intervals.

Problem 1. (Bonus - True/False)
Circle the letter corresponding to the best answer.

Every real number is rational.

(T) True **(F)** False

The integers are closed under division.

(T) True **(F)** False

The natural numbers are closed under addition.

(T) True **(F)** False

Every polynomial of odd degree has an x-intercept.

(T) True **(F)** False

Every quadratic function has exactly two x-intercepts.

(T) True **(F)** False

There exists a quadratic function whose range is $(0, \infty)$.

(T) True **(F)** False

Every nonconstant linear function has exactly one x-intercept.

(T) True **(F)** False

There exists a logarithmic function whose domain is $(0, \infty)$.

(T) True **(F)** False

A linear equation with integer coefficients has a solution in $\mathbb{Z}$.

(T) True **(F)** False

A quadratic equation with real coefficients has a solution in $\mathbb{C}$.

(T) True **(F)** False

Definition 1. Let $f : A \to B$, and let $D \subset B$. The *preimage* of D under f is

$$f^{-1}(D) = \{a \in A \mid f(a) \in D\}.$$

That is, the preimage of D is the set of all points in the domain which are mapped by f into D.

Problem 2. (Bonus - Preimage)
Let

$$A = \mathbb{R} \setminus \left\{ x \in \mathbb{R} \mid x = \frac{\pi}{2} + k\pi \text{ for some } k \in \mathbb{Z} \right\}.$$

Consider the function

$$f : A \to \mathbb{R} \quad \text{given by} \quad f(x) = \tan(x).$$

Let $D = [0, 1]$. Find $f^{-1}(D)$.