Name:

Algebra II
Examination 8 (Project)

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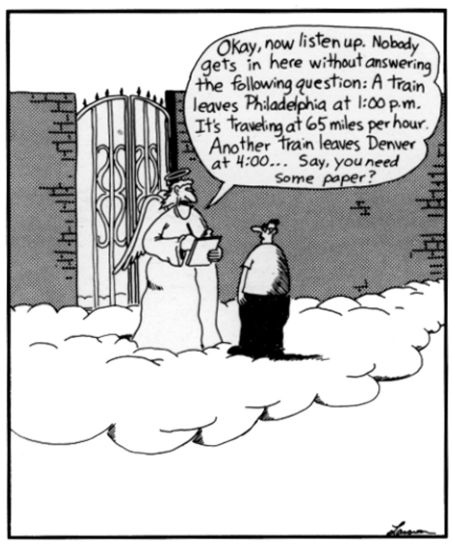
Thursday, January 27, 2022

The examination contains five problems which are worth 20 points each, and two bonus problems worth an additional 20 points each, for a maximum of 100 points. The problems have been selected from the first three exams of the first semester.



Math phobic's nightmare

Prob 1	Prob 2	Prob 3	Prob 4	Prob 5	Bonus1	Bonus 2	Total Score

Problem 1. (Sets of Numbers)
Recall our familiar sets of numbers:

$$\begin{aligned}
\text{Natural Numbers:} \quad & \mathbb{N} = \left\{1, 2, 3, \dots\right\} \\
\text{Integers:} \quad & \mathbb{Z} = \left\{\dots, -2, -1, 0, 1, 2, \dots\right\} \\
\text{Rational Numbers:} \quad & \mathbb{Q} = \left\{\frac{p}{q} \,\middle|\, p, q \in \mathbb{Z}, q \neq 0\right\} \\
\text{Real Numbers:} \quad & \mathbb{R} = \left\{ \text{ numbers given by decimal expansions } \right\} \\
\text{Complex Numbers:} \quad & \mathbb{C} = \left\{a + ib \mid a, b \in \mathbb{R} \text{ and } i^2 = -1\right\}
\end{aligned}$$

Note that

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}.$$

Of these sets of numbers, state the smallest set which contains all of the solutions to the given equation. Write the symbol of the appropriate set in the blank next to the equation.

(a) ____________ $2x - 8 = 0$

(b) ____________ $3x + 9 = 0$

(c) ____________ $6x - 15 = 0$

(d) ____________ $x^2 - 3 = 0$

(e) ____________ $x^2 + 4 = 0$

(f) ____________ $x^2 - 7x - 44 = 0$

(g) ____________ $|x - 5| = 3$

(h) ____________ $\sqrt{x - 5} = 3$

(i) ____________ $\sqrt{5} = x - 3$

(j) ____________ $x^2 - x - 1 = 0$

Problem 2. (True/False)
Circle the letter corresponding to the best answer.

$\mathbb{Z} \subset \mathbb{R}$

(T) True **(F)** False

$3 \in \{1, 2, 4\}$.

(T) True **(F)** False

$0.\overline{9} = 1.\overline{0}$.

(T) True **(F)** False

$2 \subset \{1, 2, 4\}$.

(T) True **(F)** False

Every real number is rational.

(T) True **(F)** False

The integers are closed under division.

(T) True **(F)** False

The rational numbers are closed under square roots.

(T) True **(F)** False

A linear equation with integer coefficient has a solution in $\mathbb{Z}$.

(T) True **(F)** False

A quadratic equation with real coefficients has a solution in $\mathbb{C}$.

(T) True **(F)** False

A rational number is real if and only if its decimal expansion terminates or repeats.

(T) True **(F)** False

Problem 3. (Equation of a Line and Circle)
Justify your answer by showing your work.
Let $A = (8, 2)$ and $B = (-1, 5)$.

(a) Find the slope of the line through A and B.

(b) Find the point-slope equation of the line through A and B.

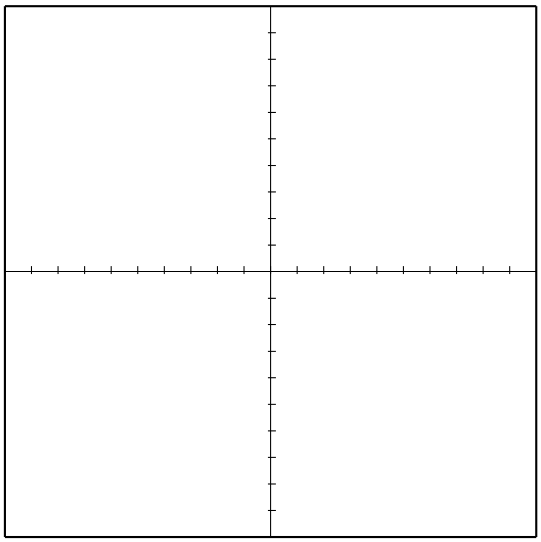
(c) Find the slope-intercept equation of the line through A and B.

(d) Find the distance from A to B.

(e) Find the equation of the circle centered at A and passing through B.

Problem 4. (Graphing) Fill out the charts, and sketch the graph.

(a) Consider the linear function $f(x) = -\frac{1}{2}(x-2)+3$. Find the slope-intercept form $f(x) = mx + b$ of the function, and identify the numbers m and b. Find the slope, the y-intercept, and the x-intercept (if any) of the line. Graph the line and label these points.



Linear Function: $f(x) = -\frac{1}{2}(x-2)+3$

Standard Form:

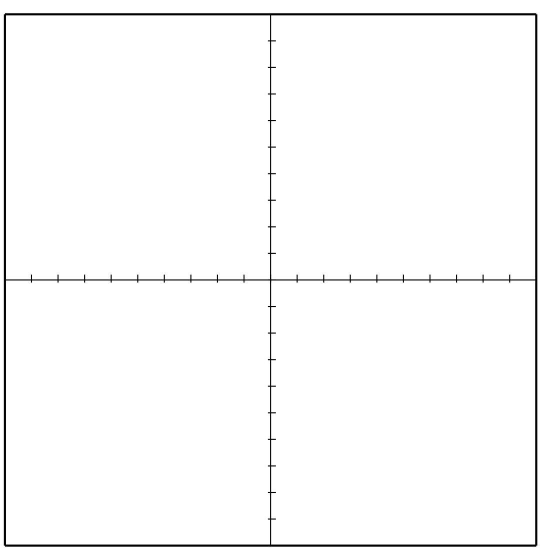
m: **b:**

Slope:

y-intercept:

x-intercept:

(b) Consider the quadratic function $f(x) = x^2 - 4x - 5$. Find the standard form $f(x) = ax^2 + bx + c$ and the shifted form $f(x) = a(x-h)^2 + k$. Identify the constants a, b, c, h, and k. Find the zeros, intercepts, and vertex. Graph the function and label these points.



Quadratic Function: $f(x) = x^2 - 4x - 5$

Standard Form:

Shifted Form:

a: **b:** **c:** **h:** **k:**

Discriminant:

Zeros:

y-intercept:

x-intercept(s):

Vertex:

Problem 5. (Polynomial Division)
Let $f(x) = x - 3$ and $g(x) = x^3 - 2x^2 - 23x + 60$.

(a) Divide f into g. Find the quotient and the remainder.

(b) Using part **(a)**, find the zeros of g. Write the solution set to the equation $x^3 - 2x^2 - 23x + 60 = 0$.

Problem 6. (Real Bonus)
Solve the problem.

(a) Find two numbers whose product is 7 and whose difference is 11.

(b) The equation $3x^2 - 8x + c = 0$ has exactly one solution. Find c.

Problem 7. (Complex Bonus)
Every complex number can be written in the form $a + bi$, where $a, b \in \mathbb{R}$ and $i^2 = -1$.

Suppose $a, b, c, d \in \mathbb{R}$. Then $a + bi = c + di$ if and only if $a = c$ and $b = d$.

(a) Let $a + bi = i^{123}$, where $a, b \in \mathbb{R}$. Find a and b. Justify your answer.

(b) Suppose that $(a + bi)(2 - 5i) = i$. Find a and b.