VECTOR CALCULUS
DR. PAUL L. BAILEY

Responses 0330
Sunday, March 29, 2020

Solutions coming soon to a theater near you.

Problem 1. When evaluating a path integral, you are always asked to make sure the parametrization for the path is smooth. What does that mean and why is that a necessary condition before integrating?

Problem 2. Consider the vector fields in $\mathbb{R}^2$ given by

$$\vec{F}(x,y) = \langle x, y\rangle \quad \text{and} \quad \vec{G}(x,y) = \langle -y, x\rangle.$$

Let C denote the unit circle, oriented counterclockwise.

(a) Draw a small sketch of each of these vector fields.

(b) Compare the flow of $\vec{F}$ versus $\vec{G}$ along C.

(c) Compare the flux of $\vec{F}$ versus $\vec{G}$ along C.

Write clearly, in complete sentences. Explain your reasoning.

Problem 3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a continuous real valued function defined on $\mathbb{R}^2$.
Let C be a smooth curve in $\mathbb{R}^2$ parameterized by $\vec{r} : [0,1] \to \mathbb{R}^2$ with parameter by t. Compare and contrast the integrals below numerically (what "answer" do you get?), geometrically, and in words.

$$\int_C f(x,y)\,ds \quad \text{versus} \quad \int_0^1 f(\vec{r}(t))\,dt$$

Problem 4. Let $\vec{F} = \langle -y, x\rangle$. Let C_1 denote the semicircle of radius 1 in the upper half plane from $(1,0)$ to $(-1,0)$ Let C_2 denote the line segment along the x-axis from $(1,0)$ to $(-1,0)$.

(a) Compute the flow of $\vec{F}$ along C_1.

(b) Compute the flow of $\vec{F}$ along C_2.

(c) Compare these values. Are they the same? Is $\vec{F}$ conservative?