Calculus I Worksheet 4 - Rational Functions

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February 5, 2009

A *rational function* is a function of the form

$$f(x) = \frac{g(x)}{h(x)},$$

where $g(x)$ and $h(x)$ are polynomials.

A rational function is in *lowest form* if the numerator and the denominator have no common complex zeros. Assume that $f(x) = g(x)/h(x)$ is a rational function in lowest form.

The *degree* of $f(x)$ is $\max\{\deg(g), \deg(h)\}$.

The *zeros* of $f(x)$ are the zeros of $g(x)$; that is, they are the solutions to $g(x) = 0$.

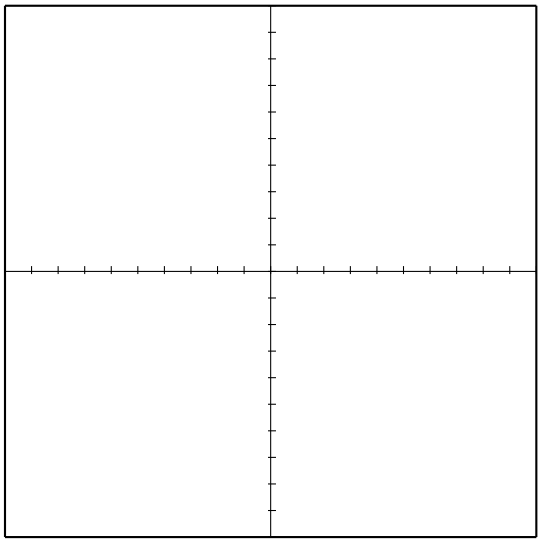
The *poles* of $f(x)$ the zeros of $h(x)$; that is, they are the solutions to $h(x) = 0$.

The *y-intercept* of $f(x)$ is the point $(0, f(0))$.

The *x-intercepts* of $f(x)$ are the points $(z, 0)$, where z is a *real* zero of $f(x)$.

The *vertical asymptotes* of $f(x)$ are the lines $x = p$, where p is a *real* pole of $f(x)$.

The *polynomial asymptote* of $f(x)$ is the polynomial equation $y = q(x)$, where $q(x)$ is the quotient when $g(x)$ is divided by $h(x)$ using polynomial division.



Equation: $y = \dfrac{6}{x-2}$

Degree:

Zeros:

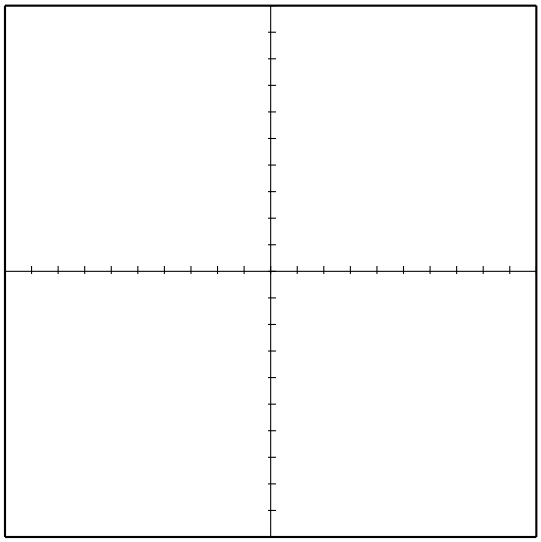
Poles:

y-intercept:

x-intercepts:

Vertical Asymptotes:

Polynomial Asymptote:



Equation: $y = \dfrac{4x+2}{3x-6}$

Degree:

Zeros:

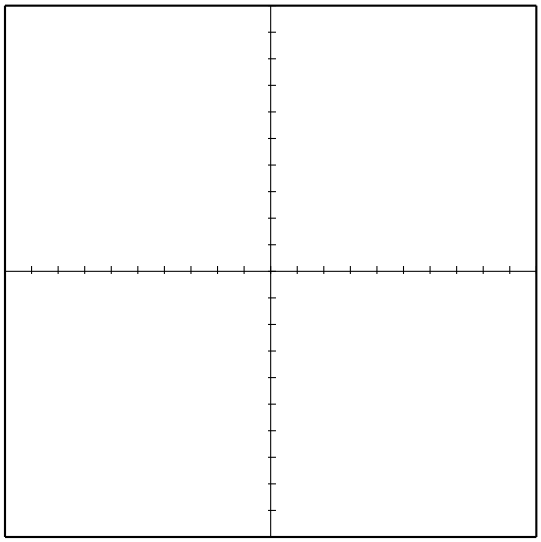
Poles:

y-intercept:

x-intercepts:

Vertical Asymptotes:

Polynomial Asymptote:



Equation: $y = \dfrac{x-5}{x^2+x-6}$

Degree:

Zeros:

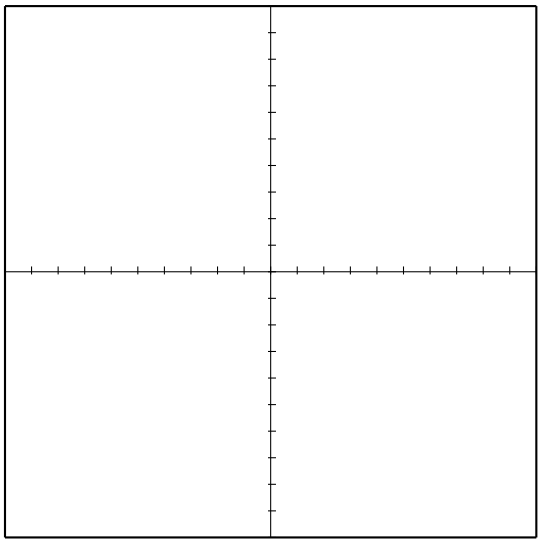
Poles:

y-intercept:

x-intercepts:

Vertical Asymptotes:

Polynomial Asymptote:



Equation: $y = \dfrac{x^2-x-2}{x+2}$

Degree:

Zeros:

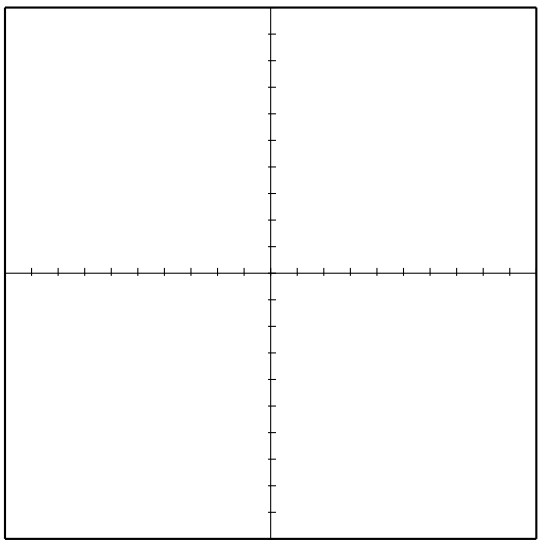
Poles:

y-intercept:

x-intercepts:

Vertical Asymptotes:

Polynomial Asymptote:



Equation: $y = \dfrac{x^2-49}{x^2-25}$

Degree:

Zeros:

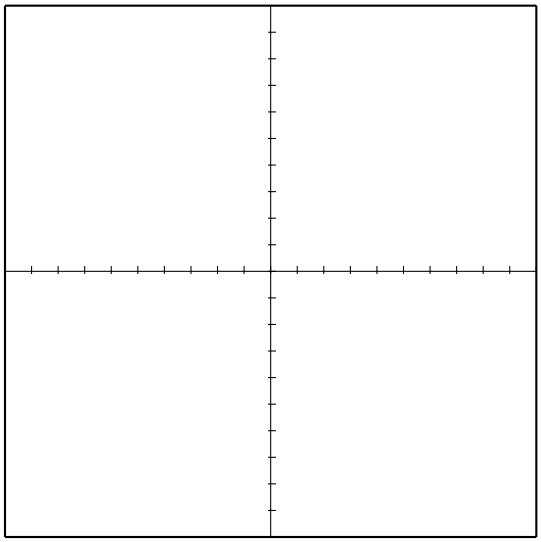
Poles:

y-intercept:

x-intercepts:

Vertical Asymptotes:

Polynomial Asymptote:



Equation: $y = \dfrac{x^3 - x}{x^2 - 9}$

Degree:

Zeros:

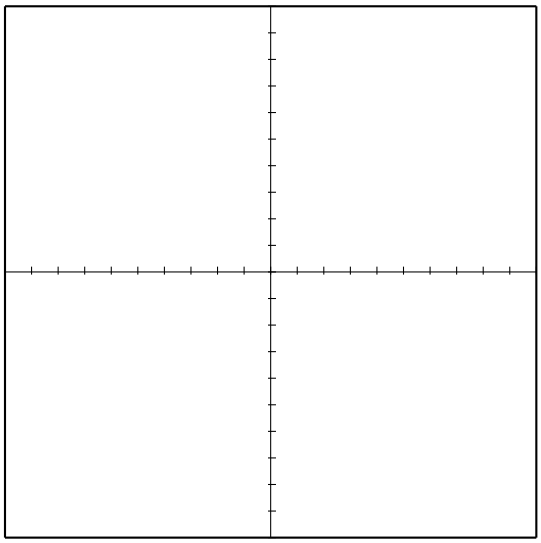
Poles:

y-intercept:

x-intercepts:

Vertical Asymptotes:

Polynomial Asymptote:



Equation: $y = \dfrac{x^2 - 25}{x^3 - 3x^2 - 4x + 12}$

Degree:

Zeros:

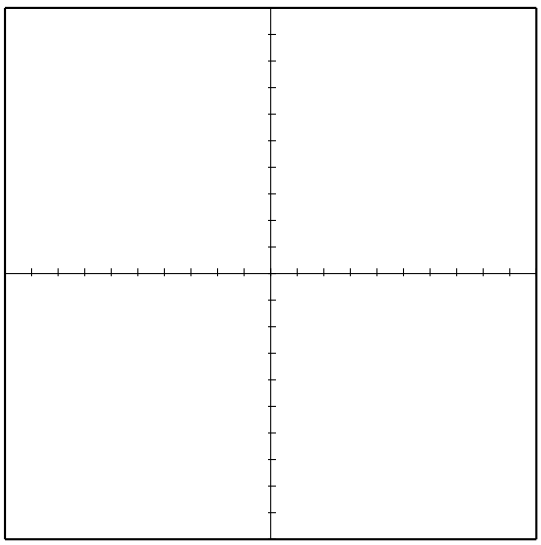
Poles:

y-intercept:

x-intercepts:

Vertical Asymptotes:

Polynomial Asymptote:



Equation: $y = \dfrac{x^3 - 6x + 7}{x + 1}$

Degree:

Zeros:

Poles:

y-intercept:

x-intercepts:

Vertical Asymptotes:

Polynomial Asymptote: