Subband Transforms for Adaptive Direct Sequence Spread Spectrum Receivers

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Abstract

Adaptive DSSS receivers have advantages over their fixed-matched filter counterparts including interference cancellation capabilities and simplification of PN code acquisition. However, convergence using an LMS algorithm may be too slow in situations with relatively high SNR. The use of a RLS algorithm will improve convergence speed but at significantly increased computational cost; fast RLS algorithms cannot be used because the filter is updated at the symbol rate rather than at every sample. In this paper, we examine subband receivers which utilize multiple, shorter length adaptive filters with the goal of speeding up convergence and reducing computation while maintaining equivalent BERs.

1 Introduction

Adaptive, Direct Sequence Spread Spectrum (DSSS) digital receivers have several advantages over their fixed matched-filter counterparts [1]. These advantages include the ability to minimize the effects of multiple-access (MA) interference, narrow band interference, and intersymbol interference (ISI) without having information about the channel or interferers. Another advantage of this receiver is that it requires no information about the PN code (other than its length) and thus does not require a code acquisition phase.

In the fractionally-spaced (FS) adaptive DSSS receiver illustrated in Fig. 1, a received baseband signal, $x(n)$ which is the sum of a desired component and interference, is passed through the adaptive filter, $\mathbf{w}$. The adaptive filter length, N, is equal to the PN code length times the number of samples per chip. The filter output is sampled at the symbol rate, T_s and compared with a known training sequence, $d(n)$. The resulting difference or error, $e(n)$ is used to adjust the filter coefficients. After a training period, the coefficient vector is an approximation of the pulse-shaped PN sequence (if the interference is moderate) and may be fixed or updated in decision-directed mode.

The input correlation matrix (assuming uncorrelated symbols), $\mathbf{R}$ is the outer product of the spreading sequence, $[s_1, s_2 \ldots s_L]^T$ with itself plus a diagonal matrix, $\sigma^2\mathbf{I}$ representing the zero mean, σ^2 variance, uncorrelated, additive white Gaussian noise (AWGN). Mathematically, the correlation matrix is given by

$$\mathbf{R} = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_L \end{bmatrix} \begin{bmatrix} s_1 & s_2 & \ldots & s_L \end{bmatrix} + \sigma^2\mathbf{I}. \quad (1)$$

The eigenvalues of (1) are given by $\{s_1^2 + s_2^2 + \ldots + s_L^2 + \sigma^2, \sigma^2, \ldots, \sigma^2\}$ and the eigenvalue spread is

$$\frac{\lambda_{max}}{\lambda_{min}} = \frac{s_1^2 + s_2^2 + \ldots + s_L^2 + \sigma^2}{\sigma^2}. \quad (2)$$

The eigenvalue spread for such a matrix can be very large if the noise level is low; such ill-conditioning will lead to slow convergence of the LMS adaptive filter which can be a problem in a fast-changing environment. The use of a RLS algorithm will improve convergence speed but at significantly increased computational cost especially in the case of long PN codes; fast RLS algorithms cannot be used because the filter is updated at the symbol rate rather than at every sample [1].

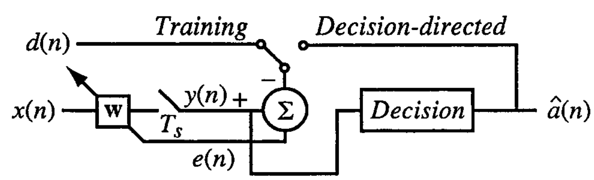

Figure 1: *Fractionally-spaced adaptive DSSS receiver.*

Subband and wavelet transforms have been previously applied to spread spectrum communications. A notable overview of the applications are in [2]. In previous work, we proposed the subband, adaptive DSSS receiver in order to introduce parallelism into the architecture. Such parallelism could allow implementation of high-speed receivers using relatively low-speed digital hardware [3, 4]. As illustrated in Fig. 2, the received digitized baseband signal is decomposed by a linear transformation, $\mathbf{T}$ into M lower rate signals and adaptive filtering is performed in the subbands. The shorter adaptive filters have the potential to speed up convergence and reduce computation while maintaining equivalent bit error rates (BERs). Sampling at the symbol rate can be done in the subbands due to the assumption that the number of samples belonging to one symbol is a multiple of M. The subband signals are added after weighting by the gain factors $\alpha_1, \ldots, \alpha_M$ which can also be made adaptive in order to speed up convergence. Standard transforms such as DCT and Hadamard were used and performance (BER and convergence rates) was reported using simple LMS adaptive filters [4].

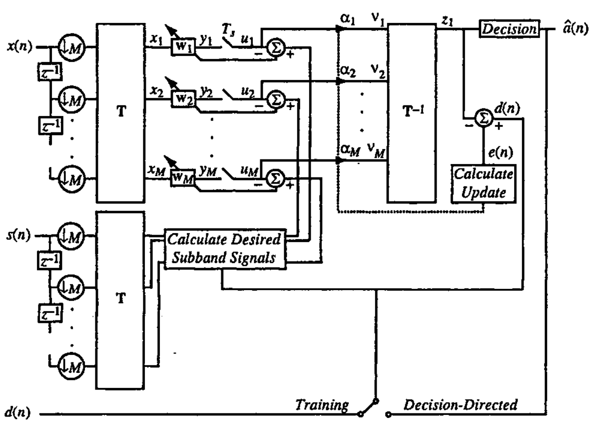

Figure 2: *Subband, adaptive DSSS receiver.*

In this paper, we further examine the subband, adaptive DSSS receiver with the focus being on the transform itself and how it effects the BER and convergence rate of the receiver.

2 Effect of the Subband Transform on Bit Error Rate

A common assumption about the error signal of an adaptive filter is that it is white gaussian noise after convergence of the filter [1]. Therefore the mean-squared error (MSE) after convergence is directly related to the BER. In order to understand the effect of the subband transform on BER, we could calculate the minimum MSE (MMSE) at the receiver output using a general transform and arbitrary input autocorrelations. As it turns out, the MMSE derived in this way is not too useful for purposes of optimizing the transform for minimum error, because it is a complicated rational function involving the elements of the transformation matrix [5]. Even for the simplest case (two subbands, length two subband filters), it is still difficult to work with.

As an alternative, we consider calculating a lower bound on the output signal-to-noise ratio (SNR) and designing $\mathbf{T}$ to maximize this bound. (Later in the paper, we will discuss the relation between maximizing the lower bound and the actual SNR.) Here, we assume additive, white gaussian noise (AWGN) with variance σ^2 but no interference. The derivation is presented for the case of a two subband receiver with length two subband filters—other cases can be easily generalized for arbitrary parameters. We refer the reader to Fig. 2 for notation used in this section.

In order to simplify the analysis, we assume the gain factors are fixed. We denote L as the number of samples per symbol, and assume $N = L/M$ is an integer. Next, we define

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 & \ldots & \mathbf{x}_M \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1M} \\ x_{21} & x_{22} & \cdots & x_{2M} \\ \vdots & \vdots & \vdots & \vdots \\ x_{N1} & x_{N2} & \cdots & x_{NM} \end{bmatrix} \quad (3)$$

as the $N \times M$ matrix whose columns are vectors of subband (transformed) samples with

$$x_{ij} = \sum_{m=1}^{M} t_{jm} x\left(m + (i-1)M\right) \quad (4)$$

where x is the received input signal and t_{ij} is the i,jth element of $\mathbf{T}$. We define

$$\mathbf{W} = \begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 & \ldots & \mathbf{w}_M \end{bmatrix} \quad (5)$$

as the matrix whose columns are the coefficient vectors of the adaptive subband filters. The oversampled and pulse shaped spreading sequence, s is partitioned into vectors of length M which form the columns of

$$\mathbf{S} = \begin{bmatrix} s_1 & s_{M+1} & \cdots & s_{(N-1)M+1} \\ s_2 & s_{M+2} & \cdots & s_{(N-1)M+2} \\ \vdots & \vdots & \vdots & \vdots \\ s_M & s_{2M} & \cdots & s_{NM} \end{bmatrix}. \quad (6)$$

The subband version of the spreading sequence is given by

$$\mathbf{S}_{\text{sub}} = (\mathbf{TS})^T$$

$$= \begin{bmatrix} \mathbf{S}_{s1} & \mathbf{S}_{s2} & \ldots & \mathbf{S}_{sM} \end{bmatrix}$$

$$= \begin{bmatrix} S_{s11} & S_{s12} & \ldots & S_{s1M} \\ S_{s21} & S_{s22} & \ldots & S_{s2M} \\ \vdots & \vdots & \vdots & \vdots \\ S_{sN1} & S_{sN2} & \ldots & S_{sNM} \end{bmatrix}. \quad (7)$$

The signal power at the receiver output (after the rightmost summation in Fig. 2) is given by

$$P_s = \left[\left(\mathbf{S}_{s1}^T \mathbf{S}_{s1} \right) \alpha_1 a + \left(\mathbf{S}_{s2}^T \mathbf{S}_{s2} \right) \alpha_2 a \right]^2 \quad (8)$$

where $\mathbf{S}_{si}$ is the subband version of the oversampled pulse shaped spreading sequence defined in (7) and a is the data bit (either +1 or −1). The noise power, assuming that the noise outputs of the subbands are independent (true if the rows of $\mathbf{T}$ are orthogonal), is given by

$$P_n = \left(\mathbf{S}_{s1}^T \mathbf{S}_{s1} \right) \sigma^2 \left(t_{11}^2 + t_{12}^2 \right) \alpha_1^2 + \left(\mathbf{S}_{s2}^T \mathbf{S}_{s2} \right) \sigma^2 \left(t_{21}^2 + t_{22}^2 \right) \alpha_2^2. \quad (9)$$

The output SNR is then the ratio of (8) to (9)

$$\text{SNR} = \frac{P_s}{P_n} \quad (10)$$

$$= \frac{\left[\left(\mathbf{S}_{s1}^T \mathbf{S}_{s1} \right) \alpha_1 a + \left(\mathbf{S}_{s2}^T \mathbf{S}_{s2} \right) \alpha_2 a \right]^2}{\sigma^2 \left[\left(\mathbf{S}_{s1}^T \mathbf{S}_{s1} \right) \left(t_{11}^2 + t_{12}^2 \right) \alpha_1^2 + \left(\mathbf{S}_{s2}^T \mathbf{S}_{s2} \right) \left(t_{21}^2 + t_{22}^2 \right) \alpha_2^2 \right]}.$$

Calculating the gradient of (10) with respect to the entries of $\mathbf{T}$, t_{ij} leads to a system of nonlinear equations which is difficult to solve. Therefore some simplification of the problem is necessary.

We assume that the absolute value of all elements of the oversampled pulse shaped spreading sequence and all elements of $\mathbf{T}$ are smaller or equal 1. Then $\mathbf{S}_{si}^T \mathbf{S}_{si} \leq 8$ and

$$\text{SNR} \geq \frac{\left[\left(\mathbf{S}_{s1}^T \mathbf{S}_{s1} \right) \alpha_1 a + \left(\mathbf{S}_{s2}^T \mathbf{S}_{s2} \right) \alpha_2 a \right]^2}{8\sigma^2 \left[\left(t_{11}^2 + t_{12}^2 \right) \alpha_1^2 + \left(t_{21}^2 + t_{22}^2 \right) \alpha_2^2 \right]}. \quad (11)$$

The problem is now to maximize this lower bound.

If we constrain the denominator in (11) to be

$$\left(t_{11}^2 + t_{12}^2 \right) \alpha_1^2 + \left(t_{21}^2 + t_{22}^2 \right) \alpha_2^2 = k \quad (12)$$

where k is a constant, then we need only maximize the numerator. Because the data is assumed to be binary, the square can be dropped in order to find the extrema. Using the method of Lagrange multipliers, leads to the following eigenvalue problem

$$\begin{bmatrix} 2\alpha_1^2 \mathbf{A}_1 & 0 \\ 0 & 2\alpha_1^2 \mathbf{A}_2 \end{bmatrix} \begin{bmatrix} t_{11} \\ t_{12} \\ t_{21} \\ t_{22} \end{bmatrix} = \mathbf{0} \quad (13)$$

where

$$\mathbf{A}_i = \begin{bmatrix} \frac{s_1^2 + s_3^2}{\alpha_i} - \lambda & \frac{s_1 s_2 + s_3 s_4}{\alpha_i} \\ \frac{s_1 s_2 + s_3 s_4}{\alpha_i} & \frac{s_2^2 + s_4^2}{\alpha_i} - \lambda \end{bmatrix}. \quad (14)$$

Since the submatrices, $\mathbf{A}_1$ and $\mathbf{A}_2$ in (13) are identical except for the factors α_1, α_2, the eigenvectors of

$$\mathbf{B} = \begin{bmatrix} s_1^2 + s_3^2 & s_1 s_2 + s_3 s_4 \\ s_1 s_2 + s_3 s_4 & s_2^2 + s_4^2 \end{bmatrix} \quad (15)$$

solve the optimization problem and yield $\mathbf{T}$. For the general case of M subbands with subband filter length N, the entries of $\mathbf{B}$ are given by

$$b_{mn} = \sum_{i=1}^{M} s_{(i-1)M+m} s_{(i-1)M+n}. \quad (16)$$

The algorithm for finding the optimal transformation matrix can be summarized as follows. Build the matrix $\mathbf{B}$ with entries as in (16) using the spreading sequence which is assumed to be given. Find the eigenvectors of $\mathbf{B}$ and use them as the rows of $\mathbf{T}$ (in any order).

One open question is the relation between the lower bound of the output SNR, which was maximized, and the true SNR. A numerical evaluation of the exact SNR equation (10) for different matrices reveals that the matrix $\mathbf{T}$, optimized with the method above leads to the maximum SNR value. It should be noted that all orthonormal matrices also lead to the same maximum SNR value, but the theory for why this happens is beyond the scope of this paper. For more details see [5].

While in theory all orthonormal matrices lead to the maximum SNR, simulation results (shown below) demonstrate that the optimized transform performs slightly better than other standard transforms (DCT, Hadamard, Identity) which are also (scaled) orthonormal matrices. The main reason is the fact that the SNR maximization was done assuming a matched filter, however an adaptive filter converges to the Wiener solution. Another reason is the additional noise each adaptive filter creates because the tap vector is wandering around the optimal solution. Therefore it is better, regarding the output SNR, to concentrate the signal energy into as few subbands as possible (which is what is ultimately happening with the optimal transform) while the subband filters containing no signal energy adapt to the zero vector. This feature may be exploited to reduce computation in subbands with little or no signal energy.

The BER performance of the proposed subband, adaptive DSSS receiver was simulated and compared to theory, the matched filter (MF) receiver, and the fullband adaptive DSSS receiver. These results are shown in Fig. 3 and illustrate that the optimal transform is slightly better than the standard transforms. System parameters include a length 31 PN sequence, chip pulses shaped with a square-root raised cosine (SRRC) filter (50% excess bandwidth), and a NLMS adaptive algorithm; we assume perfect carrier and chip synchronization. For each simulation point, 2×10^5 symbols are used, the MSE curves are averaged over 100 simulation runs. In addition, we assume four other users (MA interference) and three narrowband interferences (sinusoids), each 6dB stronger than the desired signal; zero mean, white Gaussian noise is also added to achieve a desired SNR which during training is 6dB. In the fullband receiver, the adaptive filter length is 128 (length 31 PN sequence, four samples per chip which yields 124 samples, resampled to get 128 samples). In the subband receiver, we use $M = 4$ subbands with a subband adaptive filter length of 32.

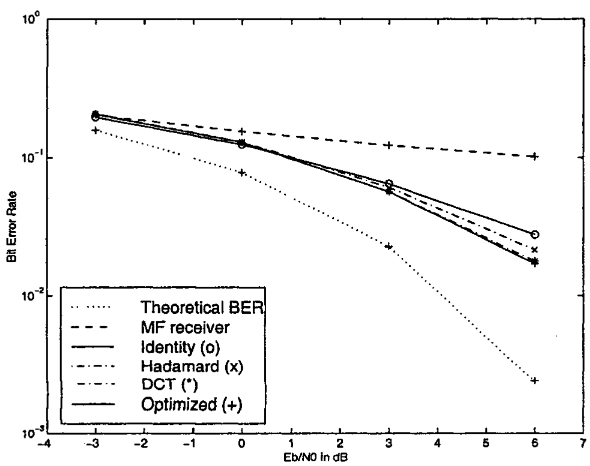

Figure 3: *Bit error rate for the subband receiver using various transforms.*

3 Eigenvalue and Convergence Analysis with LMS/NLMS Filters

In this section, we investigate how $\mathbf{T}$ effects the eigenvalue spread of the autocorrelation matrices of the subbands. As is known, this determines the convergence speed of the LMS/NLMS subband adaptive filters [6].

3.1 Desired User in AWGN

As described earlier, the autocorrelation matrix of the received signal is given by (1). The subband autocorrelation matrices are also outer products plus AWGN. For the case of two subbands and length two subband filters, the autocorrelation matrix of the first subband is

$$\mathbf{R}_{11} = \begin{bmatrix} \rho_1^2 & \rho_1\rho_3 \\ \rho_1\rho_3 & \rho_1^2 \end{bmatrix} + \sigma^2 \left(t_{11}^2 + t_{12}^2\right) \mathbf{I} \quad (17)$$

where

$$\begin{aligned} \rho_1 &= (t_{11}s_1 + t_{12}s_2) \\ \rho_3 &= (t_{11}s_3 + t_{12}s_4). \end{aligned} \quad (18)$$

The eigenvalue spread of subband 1 is then

$$\frac{\lambda_{\max}}{\lambda_{\min}} = \frac{\rho_1^2 + \rho_3^2 + \sigma^2 \left(t_{11}^2 + t_{12}^2\right)}{\sigma^2 \left(t_{11}^2 + t_{12}^2\right)}. \quad (19)$$

This expression shows that designing $\mathbf{T}$ to minimize the eigenvalue spread in subband 1 (and thus speed up convergence in LMS/NLMS case) is in conflict with maximizing the output SNR in subband 1 since

$$\rho_1^2 + \rho_3^2 = \mathbf{s}_1^T \mathbf{s}_1, \quad (20)$$

appears also in the numerator of the SNR as in (10). This is, of course, a measure of the signal energy in subband 1. To see this conflict another way, for maximum SNR in subband 1, the first row of $\mathbf{T}$, $[t_{11}\ t_{12}]$, should be correlated with the oversampled, pulse-shaped spreading sequence, $s_1, \ldots, s_4$, as much as possible. For minimum eigenvalue spread in subband 1, $[t_{11}\ t_{12}]$ needs to be orthogonal to the spreading sequence and thus in direct contrast to maximizing the SNR of subband 1.

Based on the results of Section 2, however, a reasonable tradeoff between receiver convergence time and SNR performance is possible by distributing the signal energy as uniformly as possible between the subbands. As a side note, we can show in the case of Multiuser Interference that simultaneous maximization of the subband SNR and minimization of the subband eigenvalue spread is also not possible. However, as above, a reasonable tradeoff can be made between receiver convergence time and BER performance with appropriate choice of transform.

3.2 RLS Subband Adaptive Filters

The results in the previous section demonstrate that a subband transform for the adaptive DSSS receiver cannot be designed to improve the convergence speed of the LMS/NLMS filters while maintaining good BER (which is the ultimate goal). The situation is different if the RLS algorithm is used because the convergence speed of this algorithm depends on the filter length unlike that of LMS/NLMS algorithms which depend on the eigenvalue spread of the input correlation matrix. Of course the price paid for RLS's fast convergence is increased computation (proportional to the square of the filter length). Fig. 4 shows the BER of the fullband receiver as well as for the subband receiver (using a DCT) with four and eight subbands using RLS filters, under the same interference conditions as in the previous section. The scaling factor of the initial correlation matrix of the RLS algorithm provides some possibility to trade final MSE value for convergence (found during experiments), however it is difficult to adjust the final MSE to the same value as the fullband receiver. Therefore the receivers are compared with final MSE values and BERs not exactly equal. Comparison of convergence times of the fullband and subband adaptive receivers is based on the similar BER performance, e.g. the BER of the fullband receiver was adjusted by varying parameters in the RLS algorithm until it was approximately the same as the BER for the subband receiver. Then convergence time was then measured by how long it takes the MSE to come within 1dB of the steady-state value. Table 1 summarizes the simulation results along with the matched filter receiver and theoretical results. As seen from Table 1, the proposed subband receiver has a BER performance comparable to the fullband receiver. With $M = 8$ subbands, there is a slight degradation in BER (but still better than the matched filter receiver) due to losses associated with the higher-dimension decomposition. However, as expected when using the RLS algorithm, we achieve faster convergence with the shorter length filters.

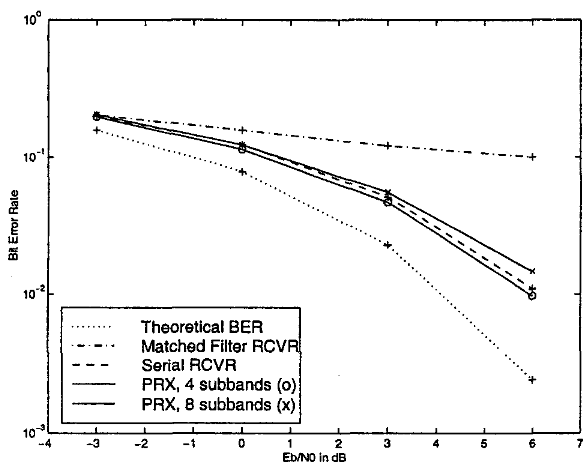

Figure 4: *Bit error rate for the subband receiver with RLS adaptive filters.*

4 Conclusions

In this paper, we have carefully analyzed the transform used in a proposed subband, adaptive DSSS receiver with the goal of improving BERs, convergence speed, and computational complexity. Our work shows that while an optimal transform can be designed to improve (slightly) BERs compared to standard transforms, it cannot improve convergence when using subband LMS/NLMS adaptive filters. However, when used with subband RLS adaptive filters, both convergence speed and computational complexity can be improved.

Table 1: Performance of Subband, Adaptive RLS DSSS Receiver.

Number of Subbands	*BER for Various SNRs*			*Conv. Time*
	0dB	3dB	6dB	
Theoretical	0.0880	0.027	0.0030	N/A
MF Receiver	0.1558	0.1210	0.0999	N/A
1 (Fullband)	0.1237	0.0515	0.0109	150 Sym
4	0.1140	0.0471	0.0097	100 Sym
8	0.1232	0.0556	0.0146	50 Sym

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