1

04/01/2023 Mathematical Modelling 1-2.

$$\begin{pmatrix} \frac{\dot{R}}{R} \\ \frac{\dot{S}}{S} \\ \frac{\dot{F}}{F} \end{pmatrix} = \begin{pmatrix} r_R \\ r_S \\ r_F \end{pmatrix} + \begin{pmatrix} a_{RR} & a_{RS} & a_{RF} \\ a_{SR} & a_{SS} & a_{SF} \\ a_{FR} & a_{FS} & a_{FF} \end{pmatrix} \begin{pmatrix} R \\ S \\ F \end{pmatrix}$$

r_R: independent of S, F,
dependent on R
should ~~be positive~~ have a positive affect on $\dot{R}$

r_S: independent of R, F
dependent on S
should have a positive affect on $\dot{S}$
Probably lower than r_R

r_F: independent of R, S
Should have a negative affect on $\dot{F}$, as foxes should be dependent on R, S populations, as they must hunt to reproduce + survive.
Die out if no food (Rabbits or Sheep).

04/01/2023 Mathematical Modelling 1-2.

Rabbits:

~~a_{RR}~~

a_{RS}: How is the rabbit population affected by sheep?

- negatively, through food competition!

a_{RF}: How is the rabbit population affected by foxes?

- negatively, more significantly than a_{RS}

Sheep:

- Similar to rabbits.

a_{SR}: How sheep are affected by rabbits

- negative food competition.

Smaller than a_{RS}?

- ~~mean~~ ~~rabbits~~ more rabbits, an individual rabbit isn't gonna affect as much as a sheep.

~~Foxes~~ a_{SF}: How sheeps are affected by Foxes.

- negatively, Smaller than a_{RF}?

③

04/01/23

Mathematical Modelling 1-2.

foxes:

a_{FR} Rabbits + Sheep should have
a_{FS} a positive affect on the number of foxes.

a_{FR} should be smaller than a_{FS},
as there are more rabbits, but

$a_{FR} \cdot R > a_{FS} \cdot S$, for typical values of R, S; as rabbits should be easier to hunt.

Self dependent:

a_{RR}, a_{SS}, a_{FF}

- This is how the species competes with itself.
- How do the animals rely on 'Strength in numbers?'
- How do we punish overpopulation?
- How do these species function in a vacuum?
  - $F, S = 0$, $R \neq 0$
  - finite area, cannot support an infinite number of rabbits.
  - so it must be negative

④

04/01/23

Math Modelling 1-2.

for rabbits

$$\frac{\dot{R}}{R} = r_r + a_{rr} \cdot R$$

$$\dot{R} = R(r_r + a_{rr} R)$$

when $\dot{R} = 0$:

$R = 0 \Rightarrow$ species cannot become negative, or return from extinction.

$$0 = r_r + a_{rr} R$$

$$r_r = -\underbrace{a_{rr}}_{\text{negative}} R$$

$$\frac{r_r}{-a_{rr}} = R_{max} \Rightarrow \text{max achievable population of Rabbits.}$$

what about other species?

$$\dot{R} = R\Big(r_r + \underbrace{a_{rr} R}_{\text{negative}} + \underbrace{a_{rs} S}_{\text{negative}} + \underbrace{a_{rf} F}_{\text{negative}}$$

$$+ \underbrace{O(S,F)}_{\text{'some' negative}}\Big)$$

$$\Rightarrow 0 = r_r + a_{rr} R + O(S,F)$$

$$\frac{r_r}{-a_{rr} - O(S,F)} = R < R_{max}$$

05/01/23

Mathematical Modellin 1-2.

$$\frac{r_r}{-a_{rr}} = R_{max} =$$ Carrying capacity of Rabbits.

as per breif,

$$R_{max} > S_{max}$$

$$\boxed{\frac{r_r}{-a_{rr}} > \frac{r_s}{-a_{ss}}}$$ Must be maintained.

What about foxes?

we could give them a carrying capacity:

$$\frac{r_F}{-a_{ff}} = F_{max}$$

but r_f is negative, so for F_{max} to be positive, a_{ff} would also have to be positive.

But this doesnt seem right! (Foxes becoming more effective in larger quantities).

In isolation $(R, S = 0)$

$$\frac{\dot{F}}{F} = r_F + a_{FF} \cdot F$$

$$\dot{F} = r_F \cdot F + a_{FF} \cdot F^2$$

~~But~~ In this scenario we would want foxes to die out due to lack of food, but for $F > \frac{-r_f}{a_{ff}}$, $\dot{F} > 0$!

no page 6,

7

05/01/23

Mathematical Modelling 1-2

However, a_{ff} affects $\dot{F}$ much more significantly than r_f, so while any $a_{ff} \leq 0$ is valid, it should probably be close to zero.

So what do we know now?

- $r_R > r_S > 0 > r_F$

- $a_{RF} < a_{RS} < 0$

- $a_{SF} < a_{SR} < 0$

- $a_{FR} > a_{FS} > 0$

- a_{RR}, a_{SS} controls the carrying capacity of sheep and Rabbits
    - $R_{max} = \frac{r_R}{-a_{rr}}$
    - $S_{max} = \frac{r_S}{-a_{SS}}$
    - Values of a_{rr}, a_{SS} must maintain $R_{max} > S_{max}$

- $a_{FF} \leq 0$; $\cancel{F_{max} = \begin{cases} \frac{-r_F - \sqrt{r_F^2 - 4a_{FF}(a_{FR}R + a_{FS}S)}}{2a_{FF}}, & a_{FF} < 0 \\ \frac{a_{FR}R + a_{FS}S}{-r_F}, & a_{FF} = 0 \end{cases}}$