Weibull

$$E[x] = \frac{1}{\beta}\Gamma\left(1+\frac{1}{\tau}\right)$$

1- Définition de l'espérance

$$E[x] = \int_0^\infty x \cdot f_x(x)\, dx$$

$$= \int_0^\infty x \cdot \beta\tau\,(\beta x)^{\tau-1} e^{-(\beta x)^\tau}$$

2- changement variable

$$u = (\beta x)^\tau$$

$$du = \tau(\beta x)^{\tau-1} \cdot \beta\, dx$$

$$= \int_0^\infty \frac{u^{1/\tau}}{\beta} e^{-u}\, du$$

3- on veut retrouver une gamma

☆forme de base $\dfrac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}$

Donc on a $\lambda = 1$

$\alpha = (1/\tau + 1)$

$$\frac{\Gamma(\frac{1}{\tau}+1)}{\beta} \underbrace{\int_0^\infty \frac{u^{(1/\tau+1)-1} \cdot 1^{(1/\tau+1)} \cdot e^{-u}}{\Gamma(\frac{1}{\tau}+1)}\, du}_{\int_0^\infty f_x(x) = 1}$$

Donc

$$E[x] = \frac{\Gamma(1/\tau + 1)}{\beta}$$

$$var(x) = E[x^2] - E[x]^2$$

on l'a déjà prouvé

$$E[x^2] = \int_0^\infty x^2 \cdot f_x(x)\,dx$$

$$= \int_0^\infty x \cdot \beta\tau (\beta x)^{\tau-1} e^{-(\beta x)^\tau}$$

2- changement variable

$u = (\beta x)^\tau$

$du = \tau(\beta x)^{\tau-1} \cdot \beta\,dx$

$$= \int_0^\infty \frac{u^{2/\tau}}{\beta^2} e^{-u}\,du$$

3- on veut retrouver une gamma

☆forme de base $\quad \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}$

Donc on a $\lambda = 1$

$\alpha = (2/\tau + 1)$

$$\frac{\Gamma(\frac{2}{\tau}+1)}{\beta^2} \underbrace{\int_0^\infty \frac{u^{(2/\tau+1)-1} \cdot 1^{(2/\tau+1)} \cdot e^{-u}}{\Gamma(\frac{2}{\tau}+1)}\,du}_{\int_0^\infty f_x(x) = 1}$$

Donc

$$E[x^2] = \frac{\Gamma(2/\tau+1)}{\beta^2}$$

$$var(x) = \frac{\Gamma(\frac{2}{\tau}+1)}{\beta^2} - \left(\frac{1}{\beta}\Gamma(1/\tau+1)\right)^2$$

$$M_X(t) = \underbrace{\sum_{k=0}^{\infty}}_{} \frac{t^k}{\beta^k k!} \Gamma\left(1 + \frac{k}{\tau}\right)$$

la somme sortira pas par magie

donc on se rappelle

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$E\left[e^{tx}\right] = E\left[\sum_0^{\infty} \frac{(tx)^k}{k!}\right] = \sum_0^{\infty} \frac{t^k}{k!} \underbrace{E\left[X^k\right]}_{}$$

mêmes étapes que $E[X]$ et $E[X^2]$

$$\int_0^{\infty} \frac{u^{k/\tau}}{\beta^k} e^{-u} du$$

$$\vdots$$

$$\frac{1}{\beta^k} \Gamma\left(\frac{k}{\tau} + 1\right)$$

$$M_X(t) = \sum_0^{\infty} \frac{t^k}{k!} \cdot \frac{1}{\beta^k} \Gamma(k/\tau + 1)$$