Patrick Legros

April 26, 2021

1 The Model

Let

$$\theta := \frac{\sigma_2(A) - \sigma_2(B)}{\sigma_1(B) - \sigma_1(A)} \tag{1}$$

be the learning index. The continuation value of an entrepreneur is

$$w(\alpha) := \alpha \mathbb{E}[\max(k, 1)] + (1 - \alpha)\mathbb{E}[k]$$

and can be rewritten as

$$w(\alpha) = \mathbb{E}[k] + \alpha\delta \tag{2}$$

where δ is the option value of being a worker instead of an entrepreneur:

$$\delta := \mathbb{E}[\max(k, 1)] - \mathbb{E}[k]. \tag{3}$$

The continuation value of a worker is equal to $w(1)$. We assume that doing task A is efficient from an expected total output perspective when an agent is hired in a firm: hence, $\sigma_1(A) + \sigma_2(A)w(1) > \sigma_1(B) + \sigma_2(B)w(1)$.

Assumption 1. *Task A in the first period maximizes the two-period expected output of a worker in a firm:*

$$\theta w(1) > 1.$$

We assume that $F(k)$ satisfies the following hazard rate condition.

Assumption 2. $\frac{F(k)}{f(k)}$ *is a non-decreasing function of* k*.*

Incentive of firms Firms capture a second period profit only if a worker does not get offers, hence gets $\sigma_1(\tau)(1-\beta)+(1-\alpha)\sigma_2(\tau)\mathbb{E}[\max(1-k,0)]$ since the residual profit in the second period of a firm is 1 minus the compensation that needs to be paid to workers to compensate for their second period outside option of k. [Btw, we need to assume that the k projects are short lived? Otherwise an entrepreneur in the second period has a project which yields in expection $\mathbb{E}[\max(k_1,k_2)]]$ since he can always continue his initial project...] Now, $\mathbb{E}[\max(1-k,0)] = 1-\mathbb{E}[\min(k,1)]$. A firm is therefore willing to implement task A in the first period of employment when

$$\alpha \leq \alpha^F(\beta) := 1-\theta(1-\beta)\delta. \tag{4}$$

Remark 1. A sufficient condition for α^F to be less than zero is that

$$\beta < 1-\frac{1}{\theta\mathbb{E}[\max(1-k,0)]}$$

under this conditions, firms are short-termist for any labor market condition. If the condition fails, firms are shot termist only if $\alpha \in (\alpha^F,1]$. If $\theta\mathbb{E}[\max(1-k,0)]$ is greater than one, then there exists β^* such that firms are shortermist if $\alpha > \alpha^F(\beta)$ and $\beta > \beta^*$.

Proposition 1. *Firms are short termist if* $\alpha > \alpha^F(\beta)$*.*

Incentives of agents Agents who contemplate entrepreneurship in the first period anticipate the optimal task they will use upon entrepreneurship as a function of their project. Choosing A is best if

$$\sigma_1(A)k+\sigma_2(A)w(\alpha) \geq \sigma_1(B)k+\sigma_2(B)w(\alpha),$$

or

$$k \leq k^A(\alpha) := \theta w(\alpha). \tag{5}$$

The optimal task $\tau^A(k)$ is then

$$\tau^A(k) := \begin{cases} A & \text{if } k \leq \theta w(\alpha) \\ B & \text{if } k \geq \theta w(\alpha). \end{cases}$$

Agents are *willing* to be entrepreneurs if employment yields a lower surplus. Let $\tau^F = B$ if, and only if, firms are shorttermists. Then an agent is willing to become an entrepreneur when

$$\sigma_1(\tau^A(k))k + \sigma_2(\tau^A(k))w(\alpha) \geq \sigma_1(\tau^F) + \sigma_2(\tau^F)w(1)$$

of

$$k \geq k^E(\alpha) := \frac{\sigma_1(\tau^F) + \sigma_2(\tau^F)w(1)}{\sigma_1(\tau^A(k))} - \frac{\sigma_2(\tau^A(k))}{\sigma_1(\tau^A(k))}w(\alpha). \tag{6}$$

For consistency, if $k^E(\alpha) < k^A(\alpha)$, it must be the case that $\tau^A(k^E(\alpha)) = A$, and otherwise, if $k^E(\alpha) > k^A(\alpha)$, that $\tau^A(k^E(\alpha)) = B$.

Discontinuity at $\alpha^F(\beta)$. There is a discountinuity at $\alpha^F(\beta)$ because the value of being an entrepreneur is continuous in α but the option of being a worker discontinually changes at $\alpha^F(\beta)$, hence the value of being an entrepreneur must increase by a first-order at α^F.

It should be clear that $k^A(\alpha)$ is an increasing function of α – as α increases, the continuation value $w(\alpha)$ of an entrepreneur increases and therefore experimenting by doing A is less costly. By the same logic, $k^E(\alpha)$ is a decreasing function of α – as α increases, the risk of entreprenurial activity (losing the option to work in a firm tomorrow) is lower and therefore individuals are more willing to be entrepreneurs.

Entrepreneurship activity. The measure of entrepreneurs is equal to $1-\alpha+\alpha\Pr[k \geq k^E(\alpha)] = 1-\alpha F(k^E(\alpha))$. Therefore the variation with respect to α is $-(F(k^E(\alpha))+\alpha\frac{dk^E(\alpha)}{d\alpha}f(k^E(\alpha)))$.

Proposition 2. *As $\alpha \neq \alpha^F(\beta)$, the measure of entrepreneurs increases with α if and only if*

$$-\alpha\frac{dk^E(\alpha)}{d\alpha} \geq \frac{F(k^E(\alpha))}{f(k^E(\alpha))}.$$

Note that the value of β affects $k^E(\alpha)$ only for $\alpha > \alpha^F(\beta)$, that is when firms are short-termist for $\beta < 1$ and $\alpha \in (\alpha^F(\beta), 1]$ while firms are not short-termist in this range when $\beta = 1$.

1.1 No Contracting Friction: $\beta = 1$.

In this case no firm is short-termist. As we have seen, $k^A(\alpha)$ is increasing and $k^E(\alpha)$ is a decreasing function of α. It is always the case that $k^E(1) < k^A(1)$, but whether or not $k^E(0) > k^A(0)$ depends on the option δ.

Assume by way of contradiction that $k^E(1) > k^A(1)$. Hence the marginal entrepreneur will choose not to experiment and do task B. Therefore (**??**) implies that

$$\begin{aligned}
k^E(1) &= \frac{\sigma_1(A)+(\sigma_2(A)-\sigma_2(B))w(1)}{\sigma_1(B)} \\
&= \frac{\sigma_1(A)+(\sigma_1(B)-\sigma_1(A))\theta w(1)}{\sigma_1(B)} \\
&= \frac{\sigma_1(A)}{\sigma_1(B)} + \left(1-\frac{\sigma_1(A)}{\sigma_1(B)}\right)k^A(1) \\
&< k^A(1)
\end{aligned}$$

where the last inequality follows $k^A(1) > 1$ by our assumption **??** that task A is output efficient in firms. Therefore $k^E(1) < k^A(1)$. Note that when $k^E(\alpha) < k^A(\alpha)$, $k^E(\alpha) = 1 + \frac{\sigma_2(A)}{\sigma_1(A)}\delta(1-\alpha)$, hence that $k^E(1) = 1$.

At $\alpha = 0$, if $k^E(0) < k^A(0)$, the marginal entrepreneur does task A and therefore,

$$k^E(0) = 1 + \frac{\sigma_2(A)}{\sigma_1(A)}\delta$$

which is indeed smaller that $k^A(0) := \theta\mathbb{E}[k]$ only if θ is high enough

Lemma 1. *If $\beta = 1$,*

(i) (High Learning Benefit) When $1 + \frac{\sigma_2(A)}{\sigma_1(A)}\delta < \theta\mathbb{E}[k]$, the marginal entrepreneur $k^E(\alpha)$ does task A for any value of α.

(ii) (Low Learning Benefit) If $1 + \frac{\sigma_2(A)}{\sigma_1(A)}\delta > \theta\mathbb{E}[k]$, there exists α^* such that the marginal entrepreneur $k^E(\alpha)$ does A if $\alpha > \alpha^*$ and does B otherwise.

Proof. In a high learning environment, $k^E(0) < k^A(0)$ together with k^E decreasing and k^A increasing in α proves the result.

In a low learning environment, since $k^E(0) > k^A(0)$ while $k^E(1) < k^A(1)$ and the two functions have opposite monotonicity, there exists a unique value α^* solving $k^E(\alpha^*) = k^A(\alpha^*)$. □

We can now evaluate the change in the measure of entrepreneurs as α varies. In a high learning environment, the condition in Proposition **??** reduces to

$$\alpha\frac{\sigma_2(A)}{\sigma_1(A)}\delta \geq \frac{F(k^E(\alpha))}{f(k^E(\alpha))},$$

By assumption **??** and $k^E(\alpha)$ decreasing in α, and the displayed condition is more likely to be satisfied the larger α is. At $\alpha = 1$, $\frac{F(k^E(\alpha))}{f(k^E(\alpha))} = \frac{F(1)}{f(1)}$. At $\alpha = 0$, the condition is clearly violated.

Lemma 2. *In the high learning environment,* *(i) If* $\frac{\sigma_2(A)}{\sigma_1(A)} < \frac{F(1)}{f(1)}$*, the measure of entrepreneurs is a decreasing function of* α*.*

(ii) If $\frac{\sigma_2(A)}{\sigma_1(A)} > \frac{F(1)}{f(1)}$*, there exists* $\underline{\alpha}$*, such that the measure of entrepreneurs is decreasing on* $\alpha \in [0, \underline{\alpha}]$ *and is increasing on* $\alpha \in [\underline{\alpha}, 1]$*.*

Example 1 (Uniform Distribution)**.** *Suppose that* $k \sim U[0, \lambda]$*. The hazard rate is equal to* $k^E(\alpha)$*, independent of the parameter* λ*. Because* $k^E(1) = 1$*, the condition in the Lemma for having a non-monotonic relationship is that*

$$\frac{\sigma_2(A)}{\sigma_1(A)} < 1,$$

which is not possible. Therefore, for the uniform distribution, the measure of entrepreneurs is a decreasing function of α*.*

Example 2 (Exponential Distribution)**.** *Suppose that* k *is exponentially distributed with parameter* $\frac{1}{\lambda}$*:* $F(k) = 1 - e^{-k/\lambda}$*. the hazard rate is*

$$\frac{F(k)}{f(k)} = e^{\frac{k}{\lambda}-1}$$

and the condition for a local increase of entrepreneurs (remembering that $k^E(1) = 1$*)*

$$\frac{\sigma_2(A)}{\sigma_1(A)}\delta < e^{\frac{1}{\lambda}-1}$$

and there exists λ *small enough for which the inequality holds.*[1]

Bottom line: we should not make a big deal of non-monotonicity due to $\beta < 1$.... At the same time, there are no learning entrepreneurs, so entrepreneurs who go back to employment have learn less ($\sigma_2(\tau_1) \leq \sigma_2(A)$) than workers and therefore will fetch a lower wage on the market. By contrast, when $\beta < 1$ and $\alpha > \alpha^F(\beta)$, learning entrepreneurs fetch more than previous workers when they go back to employment.

[1] A small value of λ means that the average project of individuals has low value.

1.2 Contracting Friction: $\beta < 1$

((random notes, to be cleaned))

As there are contracting frictions, firms are short-termists when $\alpha > \alpha^F(\beta)$. For $\alpha < \alpha^F(\beta)$, firms behave as if there were no contracting frictions and experiment. It follows that in this range the occupational choice of individuals is unchanged by contracting frictions. When firms are short-termist, occupational choices are changed because the payoff under employment is now lower than in the case $\beta = 1$.

While $k^A(\alpha)$ is unchanged – conditional on entrepreneurship, the individuals chooses task A only if his project value is less than $k^A(\alpha)$ – the value of $k^E(\alpha)$ will decrease, but the variation of $k^E(\alpha)$ will be affected. It is convenient at this point to explicitly refer to β in addition to α in the value of the marginal project. Hence, let $k^E(\alpha, \beta)$ denote the value of the marginal project for which a first period entrepreneur is indifferent between the two tasks.

The difference $k^E(\alpha, 1) - k^E(\alpha, \beta)$ is strictly positive for $\beta < 1$ and $\alpha > \alpha^F(\beta)$, hence there are more entrepreneurs as contracting frictions increase (β decreases).

If contracting frictions are low ($\alpha^F(\beta) > \alpha^*$ (Lemma **??**), for $\alpha > \alpha^F(\beta)$, the new entrepreneurs are those who draw projects in $k^E(\alpha, \beta), k^E(\alpha, 1))$ and those choose task A, as would the marginal $k^E(\alpha, 1)$ would in the absence of contracting frictions.Hence the slope of $k^E(\alpha, \beta)$ is the same as the slope of $k^E(\alpha, \beta)$. However, the ratio $\frac{F(k^E(\alpha,\beta)}{f(k^E(\alpha,\beta)}$ decreases with β, and therefore the measure of entrepreneurs is more responsive to changes in α as β decreases.

If contracting frictions are high ($\alpha^F(\beta) < \alpha^*$, among the new entrepreneurs, the marginal entrepreneur may do B for α close to α^*, but for higher values of α, the cross effect is more complex. If $\alpha > \alpha^*$, we still have a higher variation in the measure of entrepreneurs with than without contracting imperfections. If α is closer to $\alpha^F(\beta)$ however, it is possible that the marginal entrepreneur does task A while the marginal entrepreneur does B when $\beta = 1$. Then, the absolute variation of $k^E(\alpha, \beta)$ is equal to $\frac{\sigma_2(A)}{\sigma_1(A)} w(\alpha)$ when $\beta < 1$, and is greater than absolute variation $\frac{\sigma_2(B)}{\sigma_1(B)} w(\alpha)$ of the marginal entrepreneur when $\beta = 1$.