

# Synergistic Information<sup>3.0</sup>

Antoine Dubus and Patrick Legros\*

November 30, 2020

**Very preliminary, do not circulate.**

At time 0, firm 2 offers the option to firm 1 to share amount of information  $s^*$  at price  $p$ , in which case firm 2 has full information about  $\theta$  when merger happens at time 1. If firm 1 does not share data, firm 2 will offer a revelation mechanism  $\{\alpha(\theta), T(\theta)\}$  to firm 1.

At time 0, let  $\mathcal{N}$  be the set of types of firm 1 that firm 2 expects *not to share*. We claim that  $\mathcal{N} = [\theta_0, \bar{\theta}]$ .

**Assumption 1** (Convenient reformulation). *loss from sharing data if competition is equal to  $L(\theta)$ , and we assume that  $L(0) = 0$ ,  $L'(0) = 0$ ,  $L'(\theta) > 0$ ,  $L''(\theta) < 0$ , with  $X > L(\bar{\theta})$ . (This last part is to simplify the derivation of the condition for having the mixed regime of sharing and no sharing. Also having  $ls^*$  is sort of window dressing since firm 1 cannot share other amounts.)*

**Optimal mechanism at  $t = 1$  given  $\mathcal{N}$ .**

At this stage, firm 1 has for outside option not to accept the mechanism and obtain a profit level of  $\max_e eX - \frac{e^2}{2} = \frac{X^2}{2}$ , independent of  $\theta$ . The problem is a standard screening problem for firm 2, with the caveat that firm 1's effort when choosing the element  $(\alpha, T)$  solves  $\max_e \alpha X(1 + \theta) + T - \frac{e^2}{2}$ , or  $e = X \frac{1+\theta}{2}$ , increasing in  $\theta$ .

If there is truth telling, type  $\theta$  has payoff

$$U(\theta) = \alpha(\theta)^2 \frac{X^2(1 + \theta)^2}{2} + T(\theta)$$

Hence, by choosing  $(\alpha(\hat{\theta}), T(\hat{\theta}))$ , type  $\theta$  has payoff

$$U(\hat{\theta}|\theta) = \alpha(\hat{\theta})^2 \frac{X^2(1 + \theta)^2}{2} + T(\hat{\theta}).$$

---

\*We would like to thank.

Noting that  $U(\hat{\theta}|\theta) = U(\hat{\theta}) + \alpha(\hat{\theta})X^2(\theta - \hat{\theta})\left(1 + \frac{\theta + \hat{\theta}}{2}\right)$ , the incentive conditions  $U(\theta) \geq U(\hat{\theta}|\theta)$  and  $U(\hat{\theta}) \geq U(\theta|\hat{\theta})$  yield

$$\alpha(\theta)X^2(\theta - \hat{\theta})\left(1 + \frac{\theta + \hat{\theta}}{2}\right) \leq U(\theta) - U(\hat{\theta}) \leq \alpha(\hat{\theta})X^2(\theta - \hat{\theta})\left(1 + \frac{\theta + \hat{\theta}}{2}\right)$$

Therefore, as  $\theta > \hat{\theta}$ , we must have  $\alpha(\theta) \geq \alpha(\hat{\theta})$  and  $U(\theta) \geq U(\hat{\theta})$ . The standard results follow: firm 2 will want to bind the participation constraint of type  $\theta_0 := \inf \mathcal{N}$  and  $U(\theta)$  is an increasing function of  $\theta$ .

Now, if a type shares information, firm 2 extracts all surplus at the merger stage, and therefore type  $\theta$  has payoff from sharing equal to  $\frac{(X - \theta \ell s^*)^2}{2} + p$ . Her outside option is the best of not doing anything and get  $\frac{X^2}{2}$  or plays the mechanism  $\{\alpha(\theta), T(\theta)\}$ .

This allows us to show that  $\mathcal{N}$  is an interval  $[\theta_0, \bar{\theta}]$ . Indeed, suppose that types  $\theta, \theta'$  are in  $\mathcal{N}$ . Any type in  $(\theta, \theta')$  has the option not to share data and play the mechanism and obtain at least  $U(\theta|\hat{\theta})$ , which is strictly greater than  $\max\left\{\frac{X^2}{2}, \frac{(X - L(\theta))^2}{2} + p\right\}$ : the first element is because  $U(\theta|\hat{\theta}) \geq U(\theta_0|th) \geq \frac{X^2}{2}$  and the second element is because type  $t$  prefers not to share than to share.

Incentive compatibility requires that for almost all  $\theta \in [\theta_0, \bar{\theta}]$ ,

$$\dot{U}(\theta) = \alpha^2(\theta)X^2(1 + \theta),$$

hence in 2's problem we can replace  $T(\theta)$  by  $-U(\theta_0) - \int_{\theta_0}^{\theta} \alpha^2(t)2X^2(1 + t)dt$ , and after integration by parts, and binding  $\theta_0$ 's participation constraint obtain the maximization problem

$$\frac{X^2}{1 - F(\theta_0)} \int_{\theta_0}^{\bar{\theta}} \alpha(\theta)(1 + \theta)^2 - \alpha^2(\theta)(1 + \theta) \left( \frac{(1 + \theta)}{2} + \frac{1 - F(\theta)}{f(\theta)} \right) dF(\theta) \quad (1)$$

This is maximized for

$$\alpha^*(\theta) = \frac{(1 + \theta)}{(1 + \theta) + 2\frac{1 - F(\theta)}{f(\theta)}}. \quad (2)$$

**Remark 1.** The rent  $\dot{U}(\theta)$  is then equal to  $X^2 \frac{(1 + \theta)^3}{(1 + \theta + 2\ell(\theta))^2}$

Hence, conditional on firm 1 not sharing, firm 2's expected payoff is

$$\frac{X^2}{1 - F(\theta_0)} \int_{\theta_0}^{\bar{\theta}} \frac{(1 + \theta)^3}{2(1 + \theta) + \ell(\theta)} f(\theta) d\theta$$

Is it obvious that firm 2 will always desire to merge? I think yes, but this should be established. Akin to showing that firm 2 does not want to exclude some types.

## Equilibrium Data Sharing

There are three possible cases: when  $\theta_0 = 0$  and there is no sharing of data, when  $\theta_0 \in (0, \bar{\theta})$ , and there is a mixture of data sharing by firms with low values of  $\theta$ , and when  $\theta_0 = \bar{\theta}$  and all firms share data.

Because type  $\theta_0$  has no rent if firm 1 does not share data (that is  $U(\theta_0) = \frac{X^2}{2}$ ) it must be the case that it has also zero rent if firm 1 shares data. Hence, we must have  $\frac{(X-L(\theta_0))^2}{2} + p = \frac{X^2}{2}$ , or

$$p = \frac{L(\theta_0)}{2}(2X - L(\theta_0)). \quad (3)$$

For each  $\theta < \theta_0$ , at the merger stage, firm 2 has perfect information and offers a merger contract  $(\alpha(\theta), T(\theta))$  that binds the participation constraint of firm 1. Because the effort level is  $\frac{\alpha(\theta)X(1+\theta)}{2}$ , the binding participation constraint if  $T(\theta) = \frac{\alpha^2 X^2 (1+\theta)^2}{2} - \frac{X^2}{2}$ , and firm 2 solves

$$\max_{\alpha} (1 - \alpha) \frac{\alpha X^2 (1 + \theta)^2}{2} - \frac{\alpha^2 X^2 (1 + \theta)^2}{2} + \frac{X^2}{2},$$

that is

$$\max_{\alpha} \alpha(1 - 2\alpha) \frac{X^2 (1 + \theta)^2}{2},$$

and offers  $\alpha(t) = \frac{1}{4}$  and  $T(\theta) = \frac{X^2(1+\theta^2)}{32} - \frac{X^2}{2}$ .<sup>1</sup> Hence, the expected payoff of firm 2 if firm 1 shares data is (using (3))

$$\frac{X^2(1 + \theta)^2}{16} - p = \frac{X^2(1 + \theta)^2}{16} - \frac{L(\theta_0)}{2}(2X - L(\theta_0)).$$

The higher  $p$  is, the more likely that firm 1 shares data, that is that  $\theta_0 > 0$ . If the rents  $\dot{U}(\theta)$  that have to be given to firm 1 for  $\theta > \theta_0$  are large on average, firm 2 may prefer to induce data sharing for all types, that is choose  $\theta_0 = \bar{\theta}$ .

Somewhat unusual, if  $\theta_0 \in (0, \bar{\theta})$ , the equilibrium payoff of firm 1 is not monotonic in  $\theta$ : it is decreasing in  $\theta < \theta_0$  because firms with high  $\theta$  lose more than lower types if there is data sharing and competition, hence are at a disadvantage when the merger happens.

Hence, for a given  $\theta_0$ , the expected payoff of firm 2 is

$$V(\theta_0) := \int_0^{\theta_0} \left[ \frac{X^2(1 + \theta)^2}{16} - \frac{L(\theta_0)}{2}(2X - L(\theta_0)) \right] f(\theta) d\theta + \int_{\theta_0}^{\bar{\theta}} X^2 \frac{(1 + \theta)^3}{2(1 + \theta) + \frac{1-F(\theta)}{f(\theta_0)}} f(\theta) d\theta.$$

<sup>1</sup>This assumes that firm 1 can make transfer payments at the time of the merger, in exchange for a share in the merged firm profits. If firm 1 cannot make transfers, that is if  $T$  must be non-negative, then firm 1 will make rents at the merger phase.

Direct computation shows that

$$V'(\theta_0) := \left[ \frac{X^2}{16}(1 + \theta_0)^2 - \frac{L(\theta_0)}{2}(2X - L(\theta_0)) \right] f(\theta_0) - F(\theta_0)L'(\theta_0)(X - L(\theta_0)) \\ - X^2 \frac{(1 + \theta_0)^3}{2(1 + \theta_0) + \ell(\theta)} f(\theta_0)$$

hence,

$$V'(0) = f(0)X^2 \left[ \frac{1}{16} - \frac{1}{2 + \ell(0)} \right]$$

noting that  $\ell(0) = \frac{1}{f(0)}$ , we will have data sharing if

$$0 < f(0) < \frac{1}{8}.$$

Could we have *only* data sharing? A necessary condition is that  $V'(\bar{\theta}) \geq 0$ . However,

$$V'(\bar{\theta}) = -f(\bar{\theta}) \frac{7X^2(1 + \bar{\theta})^2}{16} - \frac{L(\bar{\theta})}{2}(2X - L(\bar{\theta}))f(\bar{\theta}) - L'(\bar{\theta})(X - L(\bar{\theta}))$$

is negative by Assumption 1.

**Proposition 1.** *Suppose that  $0 < f(0) < \frac{1}{8}$ , then in equilibrium firm 2 offers a price for data sharing that induces a positive measure of types to share data and the other types not to share data.*

## 1 Antoine's stuff

If merger occurs:

- Industry profits:  $X(1 + \theta)e - \frac{e^2}{2}$ , where  $e$  is an effort exerted by firm 1, inducing quadratic costs
- Firm 2 gets the whole profits, and gives a share  $\alpha$  to firm 1 to exert positive efforts
- the profits of the firms are thus

$$\Pi_2(\alpha(\theta)) = (1 - \alpha(\theta))X(1 + \theta)e - T(\theta)$$

$$\Pi_1(\alpha(\theta)) = \alpha(\theta)X(1 + \theta)e - \frac{e^2}{2} + T(\theta).$$

- firm 2 does not know the type of firm 1, and thus ignores whether the share  $\alpha$  that maximizes its profits is enough to ensure participation.
- the optimal effort for firm 1 is

$$e^* = \alpha(\theta)X(1 + \theta)$$

and

$$\Pi_1^* = \frac{\alpha(\theta)^2 X^2 (1 + \theta)^2}{2} + T(\theta)$$

which is larger than  $X$  for

$$\theta \geq \frac{\sqrt{2(X - T(\theta))}}{\alpha(\theta)X} - 1$$

- With a fixed  $\alpha$  and a fixed  $T$ , the profits of firm 2 thus are

$$\int_{\frac{\sqrt{2(X-T)}}{\alpha X} - 1}^{\bar{\theta}} \alpha(1 - \alpha)X^2 \frac{(1 + \theta)^2}{2} - T dF(\theta)$$

- firm 2 can offer a screening contract composed of a couple  $(T(\theta), \alpha(\theta))$ , such that the profit of firm 1 of type  $\theta$  selecting couple  $(\alpha(\hat{\theta}), T(\hat{\theta}))$  is

$$U(\hat{\theta}|\theta) = \alpha(\hat{\theta})X(1 + \theta)e + T(\hat{\theta}) - \frac{e^2}{2}$$

and with the equilibrium value of  $e$

$$U(\hat{\theta}|\theta) = \frac{\alpha^2(\hat{\theta})X^2(1 + \theta)^2}{2} + T(\hat{\theta})$$

- incentive compatibility constraint implies

$$\frac{\alpha^2(\hat{\theta})X^2[(2 + \theta + \hat{\theta})(\theta - \hat{\theta})]}{2} \leq U(\theta) - U(\hat{\theta}) \leq \frac{\alpha^2(\theta)X^2[(2 + \theta + \hat{\theta})(\theta - \hat{\theta})]}{2}$$

- This implies that  $\alpha(\theta)$  is non decreasing almost everywhere, and that

$$\dot{U}(\theta) = \alpha(\theta)2X^2(1 + \theta)$$

- The profit maximizing function of firm 2 is

$$\begin{aligned}
& \max_{\alpha(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} (1 - \alpha(\theta)) \alpha(\theta) X^2 (1 + \theta)^2 - T(\theta) dF(\theta) \\
\iff & \max_{\alpha(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} (1 - \alpha(\theta)) \alpha(\theta) X^2 (1 + \theta)^2 - U(\theta) + \frac{\alpha^2(\theta) X^2 (1 + \theta)^2}{2} dF(\theta) \quad (4) \\
& U(\theta) \geq \frac{X^2}{2} \quad IR \\
& U(\theta) \geq U(\hat{\theta}|\theta) \quad IC
\end{aligned}$$

- This is equivalent to maximizing

$$\begin{aligned}
& \int_{\underline{\theta}}^{\bar{\theta}} (1 - \alpha(\theta)) \alpha(\theta) X^2 (1 + \theta)^2 - U(\theta) + \frac{\alpha^2(\theta) X^2 (1 + \theta)^2}{2} dF(\theta) \\
= & \int_{\underline{\theta}}^{\bar{\theta}} \alpha(\theta) X^2 (1 + \theta)^2 - U(\theta) - \frac{\alpha^2(\theta) X^2 (1 + \theta)^2}{2} dF(\theta) \\
= & X^2 \int_{\underline{\theta}}^{\bar{\theta}} \alpha(\theta) (1 + \theta)^2 - \int_{\underline{\theta}}^{\bar{\theta}} \alpha^2(\theta) (1 + \theta) d\theta - \frac{\alpha^2(\theta) (1 + \theta)^2}{2} dF(\theta) \\
= & X^2 \int_{\underline{\theta}}^{\bar{\theta}} \alpha(\theta) (1 + \theta)^2 - \alpha^2(\theta) (1 + \theta) \left( \frac{(1 + \theta)}{2} + \frac{1 - F(\theta)}{f(\theta)} \right) dF(\theta) \quad (5)
\end{aligned}$$

- This is maximized for

$$\alpha^*(\theta) = \frac{(1 + \theta) f(\theta)}{(1 + \theta) f(\theta) + 2 - 2F(\theta)}$$

Question: how to model partial information sharing with moral hazard?

- Without mergers, firm 1 makes profits  $Xe - \frac{e^2}{2}$
- with information sharing, firm 1 makes profits  $(X - ls^*)e - \frac{e^2}{2}$  and firm 2 makes profits  $e\theta s$
- sharing information thus decreases the incentives to effort of firm 1
- in turn, this changes the outside option of firm 1 when merging with prior information sharing:

$$\alpha^2(\theta) X^2 (1 + \theta) \geq (X - ls^*)^2$$

- The optimal share of information is identical to above, however the transfer changes as the bargaining power of Firm 1 decreases with prior information sharing

## References