Synergistic Information[3.0]

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Abstract

Very preliminary, do not circulate.

1 Literature

- Optimal merger policy when firms have private information Besanko and Spulber (1993)
- Information sharing in oligopolies

  Vives (1984); Gal-Or (1986) sharing information can have pro or anti competitive effects depending on the nature of competition (cournot vs bertrand)

  They justify that we look at l positive and negative
- Disclosure to consumers
- Data as assets

  Stucke and Grunes (2016) discuss how mergers are now motivated by the acquisition of a firm data set
- Computer science and data synergies:

  Bertschinger et al. (2014); Griffith and Koch (2014); Olbrich et al. (2015) discuss how information synergies can arise when merging two data sets,

  justify why we focus on information synergies
- Joint ventures before merger
- incomplete contracts and cooperative investments (Che and Hausch, 1999)

2 Concrete example

- Profit sharing mechanisms between two firms: Politics, Science and the Remarkable Race for a Coronavirus Vaccine.

*We would like to thank.

3 Introduction

Idea: firm 1 with dataset may have a merger opportunity in the future with another firm 2 using also data. If there are synergies, the merger is beneficial, but without prior infomration, firm 2 may be reluctant to merge. Revelation of information about synergies can be done in one of two ways:

- By prior sharing of some of the data from firm 1. If the amount of data that is shared is sufficiently large, this will enable firm 2 to learn about the synergy level; otherwise there is no learning. For the example below, the assumption is that sharing below s^* does not bring information, but sharing about s^* brings information. A more 'continuous' model will have a change in the precision of information continuous as a function of s.

- Or by waiting and bargaining under incomplete information on the part of firm 2 about the level of the synergy. To induce revelation of information, the firms may have to walk away from the merger with some probability.

Hence, waiting to merge generates ex-post inefficiencies while not waiting insures ex-post efficiency (since there is symmetric information at the time of the merger). From firm 1's point of view, sharing of data brings a competitive disadvantage if the merger fails and firms compete (the case of sharing for collusion to be analyzed next). Hence, to induce firm 1 to share, firm 2 has to offer a high price for the shared data, and give rents to firms when the synergy is low. By contrast, waiting and bargaining under asymmetric information gives rents to firms 1 when the synergy is high.

4 Model

- If firm 1 shares s and the firms compete, the profits are $X - \theta ls$ for 1 and θs for 2.

- Firm 2 has full negotiation power. (Needed: firm 2 cannot commit to a mechanism at the merging stage? Result below seems fine even if firm 2 can commit. TO BE VERIFIED.)

- There are two relevant levels of sharing: $s = 0$ and s^*. If 0, firm 2 does not know θ at the time of merger; if sharing is s^*, firm 2 knows θ.

- θ has ex-ante distribution $F(\theta)$, with continuous density and support $[0, \overline{\theta}]$ ($\overline{\theta} = \infty$ is allowed), MLRP satisfied.

Steps to follow:

(1) Let T the price that firm 2 offers to pay for data s^*.

(2) If firm 1 accepts, s^* is shared and firm 2 learns θ. When merger opportunity arises, firm 2 extracts all the surplus from merger. Let $U_1(\theta|s = s^*)$ be the expected utility of type θ of doing data sharing.

(3) Let $\mathcal{N}$ be the subset of types that accept T.

(4) Given $\mathcal{N}$, there is an optimal revelation mechanism that maximizes firm 2's expected payoff subject to the IR and IC conditions for firm 1. Let $U(\theta|s = 0)$ be the expected payoff of firm 1 of type θ when playing this mechanism (corresponding to the 'sunk' belief of firm 2 of facing firms 1 with types in $\mathcal{N}$. Note that we can compute this value for any θ, which is obviously necessary in order to verify the incentives of different types to do data sharing or not.)

(5) Go back and verify that for each $\theta \in \mathcal{N}$, $U_1(\theta|s = 0) \geq U_1(\theta|s = s^*)$ and for each $\theta \notin \mathcal{N}$, $U_1(\theta|s = 0) \leq U_1(\theta|s = s^*)$

Note that we assume that firm 2 does not commit to the mechanism used at stage (4). Probably not necessary, but facilitates derivations; also quite relevant in the Anton and Yao (2002) type of environment.

As an illustration of the mechanics of the model, suppose firm 2 has full bargaining power. Firms that accept to share s^* for a price of T anticipate that firm 2 will make a TIOLI offer if a merger possibily arises and will extract all the surplus. Hence, the payoff of firms that accept the offer to share is

$$U_1(\theta|s = s^*) = X + T - \theta l s^*$$

which is a decreasing function of θ. By contrast, firms that do not share data anticipate that firm 2 will make an offer that gives them an informational rent, that is $U_1(\theta|s = 0)$ is increasing in θ. As we will see, this is a standard screening problem and, because of a lack of commitment of firm 2, firm 1 anticipates that the participation constraint of the lowest type θ_0 who does not share data will be binding and that all types greater than θ_0 get a rent. It follows that $\mathcal{N}$ is an interval $[\theta_0, \overline{\theta}]$. Furthermore, because θ_0 must be indifferent between sharing and not sharing data, we need $T + X - \theta_0 ls^* = X$, or

$$T = \theta_0 ls^*. \tag{1}$$

Following sharing by type θ, if a merger opportunity arises, firm 1 will accept the merger if she receives a payoff of $X - \theta ls^*$. Therefore, the expected payoff to firm 2 of inducing sharing by type θ is equal to $X(1+\theta) - (X - \theta ls^*) - T = \theta(X + ls^*) - T$. Firm 1 has expected payoff $X - \theta ls^* + T$.

Mechanism if firm 2 believes that $\theta \in \mathcal{N}$ at the merging phase.

At the merging stage, firm 2 believes that types have a distribution with support on $\mathcal{N} = [\theta_0, \overline{\theta}]$. Note that all types in $\mathcal{N}$ do not share data and have the same outside option of X. The value to the merger is $X(1+\theta)$. There is no loss of generality in assuming that firm 1 gets all the surplus in exchange for paying a price to firm 2.

A mechanism is then a menu $\{(p(\theta), z(\theta)); t \in \mathcal{N}\}$, where $p(\theta)$ is the price paid by firm 1 to firm 2 and $z(\theta)$ is the probability that firm 2 agrees to the merger. It should be clear that if θ_0 does not get a rent in the mechanism, types $\theta < \theta_0$ get a negative rent if they do not share data; by contrast they get a positive rent equal to $ls^*(\theta_0 - \theta)$ if they share data. The participation constraint of firm 1 is

$$U(\theta) := -p(\theta) + z(\theta)X(1+\theta) + (1 - z(\theta))X \geq X \tag{2}$$

or

$$p(\theta) \leq z(\theta)X\theta$$

while the truth-telling constraint is

$$\theta \in \arg\max_{\hat{\theta}} U(\hat{\theta}|\theta) := -p(\hat{\theta}) + X + z(\hat{\theta})X\theta$$

Usual manipulations yield

$$Xz(\hat{\theta})(\theta - \hat{\theta}) \leq U(\theta) - U(\hat{\theta}) \leq Xz(\theta)(\theta - \hat{\theta})$$

hence that $U(\theta)$ and $z(\theta)$ are almost everywhere non-decreasing function. Moreover, by the envelop theorem, $\dot{U}(\theta) = z(\theta)X$.

Hence, firm 2 offers a mechanism (p, z) to solve $\max_{\{p(\cdot),s(\cdot)\}} \int_{\theta_0}^{\bar{t}} p(\theta)f(\theta)dt$ subject to the IR and IC constraints. By using $p(\theta) = X + z(\theta)X\theta - U(\theta)$, the problem can be rewritten as

$$\max_{(p(\cdot);z(\dot))} \int_{\theta_0}^{\overline{\theta}} (-U(\theta) + X + Xz(\theta)\theta)\frac{f(\theta)}{1 - F(\theta_0)}d\theta$$

$$U(\theta) \geq X \qquad \text{(IR)}$$

$$\dot{U}(\theta) = z(\theta)X \qquad \text{(IC)}$$

Standard derivations yield to the equivalent problem

$$\max_{\{z(\theta)\}} \int_{\theta \in \mathcal{N}} z(\theta)\left(\theta - \frac{1 - F(\theta)}{f(\theta)}\right) f(\theta)d\theta.$$

Clearly, by MLRP, there exists a unique value θ^* solving

$$\theta^* = \frac{1 - F(\theta^*)}{f(\theta^*)}.$$

Hence, the optimal solution is to set $z(\theta) = 0$ for $\theta < \theta^*$ and $z(\theta) = 1$ for $\theta > \theta^*$. It follows that the expected payoff to firm 2 is (using $T = \theta_0 ls^*$)[1]

$$V(\theta_0) := \begin{cases} \int_0^{\theta_0}(\theta X - (\theta_0 - \theta)ls^*)f(\theta)d\theta + X\int_{\theta^*}^{\overline{\theta}}\left(\theta - \frac{1-F(\theta)}{f(\theta)}\right)f(\theta)d\theta & \text{if } \theta_0 \leq \theta^* \\ \int_0^{\theta_0}(\theta X - (\theta_0 - \theta)ls^*)f(\theta)d\theta + X\int_{\theta_0}^{\overline{\theta}}\left(\theta - \frac{1-F(\theta)}{f(\theta)}\right)f(\theta)d\theta & \text{if } \theta_0 \geq \theta^* \end{cases} \tag{3}$$

The solution to $\max_{\theta_0} V(\theta_0)$ necessarily involves $\theta_0 > 0$ under the assumption that $f(0)$ is positive, that is in equilibrium, data sharing happens.

Proposition 1. *(i) If $f(0)$ is positive, firm 2 chooses $\theta_0 > 0$, that is there is data sharing in equilibrium.*

(ii) If the competitive loss from data sharing ls^* is greater than $X\theta^* f(\overline{\theta})$, in equilibrium $\theta_0 \in (0, 1)$ and some firms choose data sharing while others, those with high type, choose not to share data.

(iii) If $ls^* \geq X\frac{1-F(\theta^*)}{F(\theta^*)}$, then in equilibrium $\theta < \theta_0$, that is firms will not merge for $\theta \in (\theta_0, \theta^*)$.

Proof. To show (i), it is enough to show that $V(\theta_0)$ is increasing at 0. If $\theta_0 = 0$, all firms are expected not to share data and as θ_0 increases beyond 0, the value of $V(\theta_0)$ is given by the

[1] Indeed, if $\theta_0 < \theta^*$, firm 2 optimally commits not to merge when $\theta \in (\theta_0, \theta^*)$, hence gets a surplus only if $t < \theta_0$ or $t \geq \theta^*$.

first function in (3). Therefore, $V'(\theta_0) = \theta_0 X f(\theta_0) - F(\theta_0) ls^*$ which is equal to zero at $\theta_0 = 0$. However,

$$V''(\theta_0) = f(\theta_0)(X - ls^*) + \theta_0 X f'(\theta_0)$$

and therefore if $f(0) > 0$ and $f'(0) < \infty$, $v''(0)$) is positive, implying the result. To show (ii)), note that at $\theta_0 = \overline{\theta}$, $\overline{\theta} > \theta^*$, and therefore in a neighborhood of $\overline{\theta}$, $V(\theta_0)$ is the function in the second part of (3), where all types choose to share data. Then,

$$\lim_{\theta_0 \uparrow \overline{\theta}} V'(\theta_0) = X f(\overline{\theta})\theta^* - ls^*$$

and under the condition in the proposition, $\lim_{\theta_0 \uparrow \overline{\theta}} V'(\theta_0) < 0$, implying that $\theta_0 < \overline{\theta}$.

To show (iii), assume that $\theta_0 > \theta^*$. Then using the second function in (3), we have

$$V'(\theta_0) = X f(\theta_0)\theta^* - F(\theta_0) ls^*$$

A sufficient condition for having $\theta_0 < \theta^*$ is that for all $\theta_0 \geq \theta^*$, $V'(\theta_0)$ is negative. This requires that ls^* be larger than $X\frac{f(\theta_0)}{F(\theta_0)}$. By MLRP, it must then be that $X\frac{f(\theta^*)}{F(\theta^*)}$, and using the definition of θ^* that $ls^* \geq X\frac{1-F(\theta^*)}{F(\theta^*)}$, as claimed. □

An example is for the exponential distribution with positive parameter λ. For this distribution, $\theta^* = \frac{1}{\lambda}$, $f(0) = \lambda$ and $f(\infty) = 0$. The condition in (i) is satisfied; because $f(\infty) = 0$, the condition in (ii) holds for all values of ls^*. Finally, the right hand side in condition in (iii) is equal to $\frac{(1+\lambda)e^{-\lambda\theta_0}}{1+(\lambda\theta_0-1)e^{-\lambda\theta_0}}$, a decreasing function of $\theta_0 \geq \frac{1}{\lambda}$, hence with a maximum value of $\frac{\lambda+1}{e}$. Hence, it is sufficient for (iii) that (in fact could derive a necessary and sufficient condition by directly looking at $V'(\theta)$ for the exponential distribution).

$$ls^* > \frac{\lambda+1}{e}.$$

Under condition (iii), there is no merger for types in the interval (θ_0, θ^*), which is an inefficient outcome from the point of view of the industry. A conjecture is that when the bargaining power is more evenly split among the two firms, the likelihood of such an outcome decreases.

Remark 1. *The size of the competitive loss ls^* is key.. For instance, if $l = 0$, both conditions (ii)-(ii) do not hold. In this case, firm 2 may as well offer any price for data sharing (because firm 1 does not bear a competitive penalty). If l is very large, conditions (i)-(iii) hold at θ_0 close* *to zero, and the inefficiency is maximum as there is no merger for all $\theta \in [0, \theta^*)$. Interpret this in light of potential data sharing between Biotechs and Big pharmas.*

We need to provide interpretations, real life examples that could fit with the theoretical results.

5 Collusive data sharing

We now consider the case where there is a competitive *gain* of data sharing, that is $l < 0$.[2] For clarity we normalize the notations to $l := -l > 0$.

Firm 1 is willing to share data in case the merger does not occur, as profits without and with sharing can be written:

$$U_1(\theta|s = s^*) = X + \theta l s^* > U_1(\theta|s = 0) = X$$

Thus the payoff of a firm that accepts to share s^* for a price T anticipates that firm 2 will make a TIOLI offer if a merger possibly arises, and will extract all the surplus. Hence, the payoff of a firm that accepts to share is:

$$U_1(\theta|s = s^*) = X + T + tls^*$$

which is an increasing function of θ. On the other hand, firms that do not share data anticipate that firm 2 will make an offer that gives them an informational rent, that is $U_1(\theta|s = 0)$ is increasing in θ.

6 EXTENSIONS TO BE DONE

(order not necessarily sequential)

TBD 1. Look at the case where there is a competitive *gain* of data sharing, that is $l < 0$. Firm 1 is less reluctant to share with firm 2, so...?

TBD 2. Look at case where sharing s allows firm 2 to learn the true value of θ with probability $\alpha(s)$, an increasing function of s. (The case above coincides with $\alpha(s) = 0$ is $s < s^*$ and $\alpha(s) = 1$ if $s \geq s^*$.)

[2] This is in line with the theoretical literature on information sharing in oligopolies such as Vives (1984) and Gal-Or (1986).

TBD 3. The general case where firm 2 has bargaining power $\beta < 1$. Hence it is as if the two firms agree on a price T and a mechanism that satisfies IR and IC for firm 1 in order to maximize the weighted sum $(1_\beta)U_1 + \beta U_2$, that is $U_1 + U_2 - \beta U_1$: as β decreases. Is it less likely that the firms will *not* merge?

TBD 4. Uncertain merger opportunities. For instance a biotech may share data with a pharma who decides later on to merge with another firm or not to pursue the relationship. Or the regulator's decision is somewhat random.

TBD 5. Endogenous merger choice by the regulator. Seems complicated but may be worth thinking about it as the paper should probably say something about guidelines for regulating data sharing.

References

Anton, J. J. and Yao, D. A. (2002), 'The sale of ideas: Strategic disclosure, property rights, and contracting', *The Review of Economic Studies* **69**(3), 513–531.

Bertschinger, N., Rauh, J., Olbrich, E., Jost, J. and Ay, N. (2014), 'Quantifying unique information', *Entropy* **16**(4), 2161–2183.

Besanko, D. and Spulber, D. F. (1993), 'Contested mergers and equilibrium antitrust policy', *The Journal of Law, Economics, and Organization* **9**(1), 1–29.

Che, Y. and Hausch, D. (1999), 'Cooperative investments and the value of contracting', *American Economic Review* **89**(1), 125–147.

Gal-Or, E. (1986), 'Information transmission—cournot and bertrand equilibria', *The Review of Economic Studies* **53**(1), 85–92.

Griffith, V. and Koch, C. (2014), Quantifying synergistic mutual information, *in* 'Guided Self-Organization: Inception', Springer, pp. 159–190.

Olbrich, E., Bertschinger, N. and Rauh, J. (2015), 'Information decomposition and synergy', *Entropy* **17**(5), 3501–3517.

Stucke, M. E. and Grunes, A. P. (2016), 'Introduction: Big data and competition policy', *Big Data and Competition Policy, Oxford University Press (2016)* .

Vives, X. (1984), 'Duopoly information equilibrium: Cournot and bertrand', *Journal of economic theory* **34**(1), 71–94.