Synergistic Information$^{3.0}$

Antoine Dubus and Patrick Legros*

December 9, 2020

Very preliminary, do not circulate.

Mergers fail, claimed efficiencies at the time of merger review are often not realized, or is there are realized, they aer not passed through to consumers. Is this because firms pretend the synergies, efficiency gains are present in the hope of convincing authorities to give them an instrument for more market power? Is it because the firms haven't done the homework and incorrectly evaluated the extent of synergies? Or is it because synergies exist "on average" only?

If firms collaborate before engaging into a full merger review, they may learn the extent of potential synergies. Joint ventures, sharing of information are typical cases of such collaborations. You know Wardwell algorithm and data are the source of value created to customers, the sharing of data among firms is another example of such collaboration.

Arrow famously pointed out the difficulty of inducing such collaborations among competitors, especially if what is shared is an idea that can be replicated at no cost. But even if there is a cost to replication, providing assets to competitors enhances their ablity to compete, hence is costly for the firm that shares these assets. Our point in this paper is to show that this competitive disadvantage can turn into more efficient merger decisions, which is at the benefit of the firms, and sometimes consumers.

1 Model

- There are two firms, indexed by $1, 2$, where 2 is a dominant firm (Google) and 1 is a firm that has developped a new product of platform and has a stock of data of mass 1 generated by this activity. Firm 1 is able to give a utility level of u to its customers and absent other competition can fix a price equal to u, making a profit of u (assume a mass one of

*We would like to thank.

customers interested by the product.)

- Firm 2 has developped other products and has its own stock of data. cOmbining data from 1 and 2 will enhance the value to customers to $u(\theta)$, where $u(0) = 0$ and $u(\theta)$ increasing in θ. (That $u(0) = u$ reflects the fact that absent synergies the value to customer is the same whether firm 1 offers the product of the merged firm [12] offers the product.)

- The value of θ is unknown to firms, but each know that it has a distribution $F(\theta)$, with continuous density and no atom.

- Generating synergies is costly, it requires for instance the development of new algorithms or code, further marketing efforts. Let C be this cost.

No sharing

- Firm 1 can share $s\%$ of its data with firm 2, possibly at an agreed upon price $T(s)$: at this time of sharing, firms $1, 2$ only known that θ has distribution $F(\theta)$. Upon receiving s, firm 2 can

  - either invest c, in which case the synergy θ is learned (to simplify by both firms, the case where firm 2 gets this information privately is for an extension), and it is known that the product provides value $u(\theta)s$ to customers if a share s of the data of firm 1 is used. Once θ is known, Firm 2 can then make a TIOLIT offer to firm 1 for creating a merger, or can choose to use the information to compete with firm 1. If there is a merger customers will have value $u(\theta)$ (since all data from firm 1 is part of the assets of the merged firm). If there is not a merger, firm 2 has a product competiting with that of firm 1 that provides value $u(\theta)s$ to customers.

  - Or not invest. In this case, firms $1, 2$ still do not know the extend of the synergies and decide for a merger under imperfect information.

- If firm 1 does not share data, merger happens under imperfect information. (Note that sharing $s = 0$ is equivalent to not sharing because even if firm 2 invests C the value to customers is $u(\theta)s \equiv 0$ independently of the true value of θ, hence firm 2 cannot compete with firm 1.)

2 Analysis

2.1 Competition

Suppose that firm 1 shares $s > 0$, and let us ignore for the moment the possibility of a merger. If firm 2 invests, it can provide its customes a value $u(\theta)s$ while firm 1 can provide a value u. Assuming Bertrand competition, it follows that the equilibrium price paid by the consumers and the profit per consumer are

$$\begin{cases} p = v(\theta)s - u, \ \pi_1(\theta, s) = 0, \ \pi_2(\theta, s) = v(\theta)s - u & \text{if } v(\theta)s - u \geq 0 \\ p = u - v(\theta)s \ \pi_1(\theta, s) = u - v(\theta)s, \ \pi_2(\theta, s) = 0 & \text{if } v(\theta)s - u \leq 0. \end{cases} \tag{1}$$

If firm 2 does not invest, its profit per consumer is equal to zero, that of firm 1 is equal to u and synergies are not learned.

If mergers are not possible, it should be clear that firm 1 has no incentive to share information. Note that absent a merger, firm 2 will invest in C only if

$$\int_{\theta: v(\theta)s - u \geq 0} (v(\theta)s - u) dF(\theta) \geq C.$$

The complication is that investing in C may provide useful information for negotiating the purchase of assets of firm 1 at the merger phase, hence firm 2 may decide to invest in c even if the previous inequality fails.

2.2 Merger

Suppose that no data is shared. At the time of the merger the expected value is equal to u and the merged firm has access to the full stock of data $s = 1$. Therefore, the expected value if there is no investment is equal to u and is equal to $\int v(\theta) dF(\theta) - C$ if there is investment. The value of the merger is therefore

$$W^M := \max \{u, \mathbb{E}[v(\theta)] - c.\}$$

Note that if there is investment, the merged firm over-invest when $v(\theta) - c < u$.

Suppose that the firms agree that firm 1 will share s with firm 2, and that firm 2 agrees to pay $T(s)$ to firm 1 for this amount of data.

Upon receiving s, firm 2 can decide to invest c in order to learn θ. In this case, the two firms anticipate payoffs $\pi_i(\theta, s)$ as given by (1) if there is no merger. Because, $W^M \geq \pi_1(\theta, s) + \pi_2(\theta, s)$, a merger is always beneficial. Firm 2 can make at TIOLI offer to buy firm 1's asset at a price $p(\theta, s)$ that will make firm 1 indifferent between merging and not merging, that is

$$p(\theta, s) := \pi_1(\theta, s). \tag{2}$$

It will be useful to use the notation

$$\sigma := v^{-1}$$

clearly, $\sigma(s)$ is an increasing function of s and $v(\sigma(s)) = s$. Using (1)-(2), if $\theta \geq \sigma(s)$, firm 1 has a zero profit if there is competition, hence firm 2 can merge with firm 1 by offering a zero price and get the full surplus $v(\theta)$. If $\theta < \sigma(s)$, firm 1 makes a profit if there is competition and will merge only if the price is at least equal to $u - v(\theta)s$; hence firm 2 can make a profit of at most $(1 + s)v(\theta) - u$ from the merger, which is greater than her payoff under competition only if $\theta \geq \sigma(1 + s)$. It follows that the expected payoff of firm 2 of paying $T(s)$ to get s and investing c following sharing of data is

$$\int_{\sigma(u/(1+s))}^{\sigma(u/s)} ((1 + s)v(\theta) - u)dF(\theta) + \int_{\sigma(u/s)}^{\infty} v(\theta)dF(\theta) + -C - T(s) \tag{3}$$

By contrast firm 1 has an expected payoff of

$$\int_{0}^{\sigma(u/(1+s))} (u - v(\theta)s)dF(\theta) + T(s). \tag{4}$$

Therefore, it must be the case that firm 2 offers a price

$$T(s) := u - \int_{0}^{\sigma(u/(1+s))} (u - v(\theta)s)dF(\theta),$$

for sharing s and has an expected payoff (substituting $T(s)$ in (3)) equal to

$$w(s) = \int_{0}^{\sigma(u/(1+s))} (u-v(\theta)s)dF(\theta)+\int_{\sigma(u/(1+s))}^{\sigma(u/s)} ((1+s)v(\theta)-u)dF(\theta)+\int_{\sigma(u/s)}^{\infty} v(\theta)dF(\theta)-u-c.$$

The variation of this value is

$$w'(s) = -\frac{u^2}{(1+s)^3}\sigma'\left(\frac{u}{1+s}\right) f\left(\frac{u}{1+s}\right) - \int_{0}^{\sigma(u/(1+s))} v(\theta)dF(\theta) + \int_{\sigma(u/(1+s)}^{\sigma(u/s)} v(\theta)dF(\theta).$$

Example 1. *For instance, if $s \in [0, 1]$ and θ is uniformly distributed on $[0, 1]$ and $v(\theta) = \theta$,*

$$w'(s) = u^2 \left(\frac{1}{2s^2} - \frac{1}{(1+s)^2} - \frac{1}{(1+s)^3} gfdgt\right)$$

which is positive for all $s \in [0, 1]$. Hence, in this example firm 2 will ask firm 1 to share all its data.

The alternative is not to share data. In this case, firm 2 makes a TIOLI offer to buy firm 1 at price u and firm 2 makes profit $w(0) := W^M - u$.

Note that $\lim_{s\downarrow 0} w(s) = \int_{\sigma(u)}^{\infty}(v(\theta) - 2u)dF(\theta) - c$ and *is not equal* to what happens if there is no sharing: firm 2 gets then $W^M - u = \max\{0, \mathbb{E}[v(\theta)] - u - c\}$. It remains therefore to show that the optimal sharing dominates no sharing for firm 2. We continue here with the specification in the example. Then $\mathbb{E}[v(\theta)] = \int_0^1 \theta d\theta = \frac{1}{2}$, and firm 2' payoff if there is no sharing is

$$w_0 := \max(0, \frac{1}{2} - u - c).$$

while under sharing it is

$$w(1) = \frac{1}{2} + \frac{u^2}{8} - u - c.$$

Suppose $u + c \geq \frac{1}{2}$**,**

that is $w_0 = 0$. It is not beneficial to invest in learning the synergies if there is no sharing and a merge. In this case, the merger is neutral for consumers and the firms, and sharing is optimal whenever and

- $c < \frac{1}{2}$ and $u \in \left[\frac{1}{2}(1-2c), 4 - 2\sqrt{2c+3}\right] \cup [2\sqrt{2c+3} + 4, \infty]$.
- $c = \frac{1}{2}$ and $u = 0$ or $u \geq 8$
- $c > \frac{1}{2}$ and $u \geq 2\sqrt{2c+3} + 4$

Suppose now $u + c < \frac{1}{2}$**,**

and $w_0 = \frac{1}{2} - (u + c)$. Then sharing is optimal when (necesarily $c < \frac{1}{2}$)

$$0 \leq u < \frac{1}{2}(1 - 2c)$$

References