Synergistic information*

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Abstract

Very preliminary, do not circulate.

1 Model

- Two firms, 1 and 2.
- Firm 1 is endowed with a data set X. Firms make profits using data/
- Firm 2 want to merge with Firm 1. The merged entity makes profits $\Pi_m = X(1+\theta)$ where θ is the synergistic value of information owned by Firm 1 when combined with the data owned by firm 2.
- Firm 2 and a regulator believe that the type of Firm 1 is uniformly distributed over $[\underline{\theta}, \overline{\theta}]$.
- Firm 1 can share s information from the total data set X, which is verifiable by Firm 2 and the regulator.
- The regulator will choose whether to allow the merger or not. The regulator tradeoffs the cost from increased market power (assumed to simplify to be proportional to the industry profit) and the gain from synergies. There is some uncertainty on the weight w the regulator will put on synergies, and this uncertainty is resolved only at the time the regulator evaluates the merger proposal, that is after firms $1,2$ have played their mechanism, and s has been shared.
- If the merger fails, the information that has been shared by Firm 1 can be used by Firm 2 during the competition, and information sharing changes the profit of Firm 1: $\Pi_{c1} = X - \theta l(s)$; and of Firm 2: $\Pi_{c2} = \theta s$. We assume Inada conditions $l(0) = 0, l'(0) = 0$.
- There are two cases to consider:
  - $l(s) \geq 0$: sharing information increases competition and incurs a loss for a firm
  - $l(s) < 0$: sharing information facilitates coordination and increases the profits of a firm (à la **?** for instance)
- Firm 2 offers a menu $(s, T(s))$ and firm 1 chooses – as a function of her type θ – how much to share. Conditional on obverving s, both firms decide to approach the regulator to authorize the merger. We assume that there is no renegotiation on $T(s)$ at this stage.
- If the merger occurs, Firm 2 agrees to make a transfer $T(s)$ to Firm 1 and be residual claimant on the profit.[1]

*We would like to thank.

[1]This payment scheme is without loss of generality. What matters is that the profit of firm 1 in case of merger depends on its true type, for otherwise revelation of information is impossible. Also, for incentive compatibility, it is clear that if two θs share the same s, they will claim to be of the type that minimizes $T(\theta, s)$.

The timing of the game is the following.

($U[a, b]$ denotes the uniform distribution on $[a, b]$; we could use more general distributions...)

1. Firm 2 offers a rmenu $(s, T(s)$, s being the sharing of data that firm 1 should do and $T(s)$ is the price firm 2 agrees to pay to firm 1 if the merger is agreed and the regulator authorizes it.
2. Firm 1 privately learns its type $\theta \sim U[\underline{\theta}, \overline{\theta}]$ and makes an announcement in the mechanism and plays s.
3. If both firms anticipate to be better off via a merger, they approach the regulator and ask for the merger to be approved. At this point, the regulator sees $s, T(s)$.
4. The regulator observes a draw w from a distribution F on $\mathbb{R}_+$ with a continuous density. The regulator decides to allow or to prevent the merger. The market structure, profits and welfare, are realized.

1.1 The Regulator's Problem

The regulator maximizes a social welfare function that weights the social cost of high industry profits and the social benefit of synergies. Contrary to the usual view that synergies are created only during the merger, synergies endogenously happen without a merger if firm 1 shares some of its data with firm 2. Hence, when evaluating a merger proposal, the regulator will compare the *relative synergy gain* to the *relative industry profit gain*.

At the time the regulator has to evaluate a merger, firm 1 has already shared s with firm 2, following a strategy $\sigma(\theta)$ in the sharing game. Let $\sigma^{-1} = \{\theta | \sigma(\theta) = s\}$ be the set of types of firm 1 consistent with a sharing of s given the equilibrium strategy σ. Hence, conditional on observing s, the regulator believes that the expected synergy is the conditional expectation

$$\theta_s := \mathbb{E}[\theta | \theta \in \sigma^{-1}(s)].$$

If the merger is allowed and firm 2 takes control of firm 1, the loss from industry profit is $-[X(1+\theta)]$ while the synergy gain is $w[X(1+\theta)]$, hence welfare is

$$W_m = (w-1)X(1+\theta_s)$$

If the merger is prevented or the firms decide not to go ahead with it, firm 2 benefits from the information given by firm 1 (by a factor of θs, where θ is the true state of the world), and firm 1 has a loss of $l(s)$, implying that the total industry profit increases by $\theta s - l(s)$. Therefore, the regulator believes that the realized industry profit will be $X_1 - l(s) + X_2 + \theta_s s$ if he does not authorize the merger, while the synergy benefit would be $w[X + \theta_s s]$.[2] Hence the expected welfare under competition is

$$W_c = -X + l(s) - \theta_s s + w(X + \theta_s s)$$

The regulator allows the merger when $W_m \geq W_c$, or

$$w \geq w^*(s) := 1 + \frac{l(s)}{(X-s)} \tag{1}$$

[2] As it should be, the regulator ignores the market loss $l(s)$ to firm 1.

Our specification implies therefore that the probability that the merger is authorized depdns only on the share s but not on the beliefs of the regulator about θ.

At the ex-ante stage, when firms play their mechanism to share information, the probability that a merger will be approved when firm 1 shares s with firm 2 is then equal to

$$a(s) := 1 - F(w^*(s)).$$

TBC....

2 Playing the Regulatory Game

At the time firms $1, 2$ play the mechanism, the probabilities $a(s)$ are taken as given because if called upon to act, the regulator has sunk beliefs (he believes that types in $\sigma^{-1}(s)$ of firm 1 have shared s.) In the mechanism, firm 2 will ask for a merger only if when firm 1 claims to be of type θ, firm 1 agrees to share a quantity $s = s(\theta)$ of data. As is standard, there are two types of incentive conditions to be satisfied: one related to using the requested share, and the other related to telling the truth. If these conditions are satisfied, it must also be the case that firm 1 prefers to play the mechanism than not playing it, and foregoing the possibility of a merger.

The interim individual rationality constraint of firm 1 is then

$$a(s(\theta))T(\theta, s(\theta)) + (1 - a(s(\theta)))(X - l(s(\theta))) \geq X. \tag{2}$$

The Incentives to Share.

Suppose first that firm 1 of type θ does not lie and announces θ. WIth some abuse of notation we write $T(\theta) := T(\theta, s(\theta))$. If firm 1 shares $s(\theta)$ as recommended by the mechanism, there will be a merger with probability $a(s(\theta))$, hence firm 1's expected profit is

$$U(\theta) := a(s(\theta))(X(1 + \theta) - T(\theta)) + (1 - a(s(\theta)))(X_1 - l(s(\theta))),$$

while by deviating to $s \neq s(\theta)$ there is competition for sure and firm 1's payoff is $X - l(s)$. Clearly, the optimal deviation is $s = 0$. Hence, following the recommended share while telling the truth is optimal when the IR constraint (**??**) is satisfied.

If type θ does not tell the truth and claims $\hat{\theta} \neq \theta$, he will choose to follow the recommendation $s(\hat{\theta})$ when the IR condition for type $\hat{\theta}$ is satisfied since by deviating to $\hat{\theta}$ and doing $s(\hat{\theta})$, type θ gets

$$U(\hat{\theta}|\theta) := a(s(\hat{\theta}))(X(1 + \theta) - T(\hat{\theta})) + (1 - a(s(\hat{\theta})))(X - l(s(\hat{\theta})))$$

which is greater than X when the IR constraint of $\hat{\theta}$ is satisfied.

It remains therefore to verify that type θ does not want to deviate to $\hat{\theta}$ and follow the recommended shares, that is that $U(\theta) \geq U(\hat{\theta}|\theta)$. Now,

$$U(\hat{\theta}|\theta) = U(\hat{\theta}) + a(s(\hat{\theta}))X(\theta - \hat{\theta})$$

and therefore the two incentive constraints $U(\hat{\theta}|\theta) \leq U(\theta)$ and $U(\theta|\hat{\theta}) \geq U(\hat{\theta})$ yield

$$a(s(\hat{\theta}))X(\theta - \hat{\theta}) \leq U(\theta) - U(\hat{\theta}) \leq a(s(\theta))X(\theta - \hat{\theta})$$

Therefore, $a(\theta)$ and $U(\theta)$ are monotonic non-decreasing functions.

3 PREVIOUS STUFF

We apply the revelation principle and focus our analysis on equilibrium with truthtelling. Firm 2 maximizes its profits by proposing an incentive compatible mechanism where Firm 1 reveals its type θ and sharing information $s(\theta)$. Sharing information is used to prevent mimicking between different types of θ and ensures that the equilibrium is separating. Firm 2 then proceeds to the transfer $T(.)$ that it proposed to Firm 1. Thus the objective function of Firm 2 is the following, under incentive compatibility constraints for Firm 1 in order to guarantee truthtelling:

$$max_{T(.),s}\{(1-F(\theta,s)[\frac{X_1(1+\theta)+X_2}{2}-T(\theta,s)]+F(\theta,s)(X_2+\theta s)\}$$

$$s.t.PC:(1-F(\theta,s))[\frac{X_1(1+\theta)+X_2}{2}+T(\theta,s)]+F(\theta,s)(X_1-l(s))\geq X_1 \tag{3}$$

$$s.t.\ \ IC:\ \ \forall\ \ \hat{\theta}_1\neq\theta,\ \ \Pi(\theta,s)\geq\Pi(\theta,\hat{\theta}_1,\hat{s}_1)$$

Patrick dans le pdf que tu m'as envoyé il y aussi une incentive compatibility constraint sur s: $IC_s:\forall\theta s(\theta)$ is maximized. Je ne suis pas sur que ca soit indispensable comme s est verifiable

The PC constraint is binding for the lowest type, and naturally satisfied for all other types as the outside option is constant.

Consider $\theta<\hat{\theta}_1$. IC constraints can be written:

$$\Pi(\theta,s)\geq\Pi(\theta,\hat{\theta}_1,\hat{s}_1)\ \ and\ \ \Pi(\hat{\theta}_1,\hat{s}_1)\geq\Pi(\hat{\theta}_1,\theta,s).$$

Rearranging the inequalities we can derive the following expression:

$$\begin{aligned}&(1-F(\hat{\theta}_1,\hat{s}_1))\frac{X_1}{2}(\hat{\theta}-\theta)\geq\Pi(\hat{\theta}_1,\hat{s}_1)-\Pi(\theta,s)\\&\geq(1-F(\theta,s))\frac{X_1}{2}(\hat{\theta}-\theta)\end{aligned} \tag{4}$$

We now show that $\psi(\theta)=s$ is increasing in θ. Consider $\hat{\theta}_1>\theta$:

$$(1-F(\hat{\theta}_1,\hat{s}_1))\frac{X_1}{2}(\hat{\theta}-\theta)$$

$$\geq(1-F(\theta,s))\frac{X_1}{2}(\hat{\theta}-\theta)$$

we can write

$$F(\theta,s)\geq F(\hat{\theta}_1,\hat{s}_1)$$

If we have single crossing conditions on $-F(.)$, we have $\hat{s}_1\geq s$.

Is it the case? let's look at the derivative of F with respect to θ and s:
$\frac{\partial^2F(\theta,s)}{\partial\theta\partial s}=\frac{1}{\overline{w}-\underline{w}}\frac{l'(s)(X_1-s)+l(s)}{\theta^2(X_1-s)^2}$
Whose sign is identical to that of $l'(s)(X_1-s)+l(s)$

- When $l'(s)(X_1-s)+l(s)\leq0$: $s(\theta)$ is increasing

- When $l'(s)(X_1 - s) + l(s) \geq 0$: $s(\theta)$ is decreasing

We now derive the expression of the first degree derivative of $\Pi(\theta, s)$ with respect with θ.

$$\begin{aligned} &(1 - F(\hat{\theta}_1, \hat{s}_1))\frac{X_1}{2} \geq \frac{\Pi(\hat{\theta}_1, \hat{s}_1) - \Pi(\theta, s)}{\hat{\theta} - \theta} \\ &\geq (1 - F(\theta, s))\frac{X_1}{2} \end{aligned} \tag{5}$$

Considering the limit case where $\hat{\theta}_1 \to \theta$ proves the differentiability of $\Pi(\theta, s)$, moreover by the sandwich theorem:

$$\frac{\partial \Pi(\theta, s)}{\partial \theta} = (1 - F(\theta, s))\frac{X_1}{2}$$