Synergistic Information$^{3.0}$

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Very preliminary, do not circulate.

At time 0, firm 2 offers the option to firm 1 to share amount of information s^* at price p, in which case firm 2 has full information about θ when merger happens at time 1. If firm 1 does not share data, firm 2 will offer a revelation mechanism $\{\alpha(\theta), T(\theta)\})$ to firm 1.

At time 0, let $\mathcal{N}$ be the set of types of firm 1 that firm 2 expects *not to share*. We claim that $\mathcal{N} = [\theta_0, \overline{\theta}]$.

Assumption 1 (Convenient reformulation)**.** *loss from sharing data if competition is equal to $L(\theta)$, and we assume that $L(0) = 0$, $L'(0) = 0$, $L'(\theta) > 0, L''(\theta) < 0$, with $X > L(\overline{\theta})$. (This last part is to simplify the derivation of the condition for having the mixed regime of sharing and no sharing. Also having ls^* is sort of window dressing since firm 1 cannot share other amounts.)*

Optimal mechanism at $t = 1$ given $\mathcal{N}$.

At this stage, firm 1 has for outside option not to accept the mechanism and obtain a profit level of $\max_e eX - \frac{e^2}{2} = \frac{X^2}{2}$, independent of θ. The problem is a standard screening problem for firm 2, with the caveat that firm 1's effort when choosing the element (α, T) solves $\max_e \alpha X(1+\theta)e + T - \frac{e^2}{2}$, or $e = \alpha X(1+\theta)$, increasing in θ.

If there is truth telling, type θ has payoff

$$U(\theta) = \alpha(\theta)^2 \frac{X^2(1+\theta)^2}{2} + T(\theta)$$

Hence, by choosing $(\alpha(\hat{\theta}), T(\hat{\theta})$, type θ has payoff

$$U(\hat{\theta}|\theta) = \alpha(\hat{\theta})^2 \frac{X^2(1+\theta)^2}{2} + T(\hat{\theta}).$$

*We would like to thank.

Noting that $U(\hat{\theta}|\theta) = U(\hat{\theta}) + \alpha(\hat{\theta})X^2(\theta - \hat{\theta})\left(1 + \frac{\theta+\hat{\theta}}{2}\right)$, the incentive condditions $U(\theta) \geq U(\hat{\theta}|\theta)$ and $U(\hat{\theta}) \geq U(\theta|\hat{\theta})$ yield

$$\alpha(\hat{\theta})X^2(\theta - \hat{\theta})\left(1 + \frac{\theta+\hat{\theta}}{2}\right) \leq U(\theta) - U(\hat{\theta}) \leq \alpha(\theta)X^2(\theta - \hat{\theta})\left(1 + \frac{\theta+\hat{\theta}}{2}\right)$$

Therefore, as $\theta > \hat{\theta}$, we must have $\alpha(\theta) \geq \alpha(\hat{\theta})$ and $U(\theta) \geq U(\hat{\theta})$. The standard results follow: firm 2 will want to bind the participation constraint of type $\theta_0 := \inf \mathcal{N}$ and $U(\theta)$ is an increasing function of θ.

Now, if a type shares information, firm 2 extracts all surplus at the merger stage, and therefore type θ has payoff from sharing equal to $\frac{(X-L(\theta))^2}{2} + p$. Her outside option is the best of not doing anything and get $\frac{X^2}{2}$ or plays the mechanism $\{\alpha(\theta), T(\theta)\}$.

This allows us to show that $\mathcal{N}$ is an interval $[\theta_0, \overline{\theta}]$. Indeed, suppose that types θ, θ' are in $\mathcal{N}$. Any type in (θ, θ') has the option not ot share data and play the mechanims and obtain at least $U(\theta|\hat{\theta})$, which is strictly greater than $\max\left\{\frac{X^2}{2}, \frac{(X-L(\theta))^2}{2} + p\right\}$: the first element is because $U(\theta|\hat{\theta}) \geq U(\theta_0|\hat{\theta}) \geq \frac{X^2}{2}$ and the second element is because type θ prefers not to share than to share.

Incentive compatibility requires that for almost all $\theta \in [\theta_0, \overline{\theta}]$,

$$\dot{U}(\theta) = \alpha^2(\theta)X^2(1+\theta),$$

hence in 2's problem we can replace $T(\theta)$ by $-U(\theta_0) - \int_{\theta_0}^{\theta} \alpha(t)^2 X^2(1+t)dt$, and after integration by parts, and binding θ_0's participation constraint obtain the maximization problem

$$\frac{X^2}{1-F(\theta_0)} \int_{\theta_0}^{\overline{\theta}} \alpha(\theta)(1+\theta)^2 - \alpha^2(\theta)(1+\theta)\left(\frac{(1+\theta)}{2} + \frac{1-F(\theta)}{f(\theta)}\right) dF(\theta) \tag{1}$$

This is maximized for

$$\alpha^*(\theta) = \frac{(1+\theta)}{(1+\theta) + 2\frac{1-F(\theta)}{f(\theta)}}. \tag{2}$$

Remark 1. *The rent $\dot{U}(\theta)$ is then equal to $X^2 \frac{(1+\theta)^3}{(1+\theta+2\ell(\theta))^2}$*

Hence, conditional on firm 1 not sharing, firm 2's expected payoff is

$$\frac{X^2}{1-F(\theta_0)} \int_{\theta_0}^{\overline{\theta}} \frac{(1+\theta)^3}{2((1+\theta)+2\ell(\theta))} f(\theta)d\theta$$

Is it obvious that firm 2 will always desire to merge? I think yes, but this should be established. Akin to showing that firm 2 does not want to exclude some types.

A: if the outside option of firm 2 is zero (no profits from firm 1 sharing), desire to merge is straightforward.

Equilibrium Data Sharing

There are three possible cases: when $\theta_0 = 0$ and there is no sharing of data, when $\theta_0 \in (0, \overline{\theta})$, and there is a mixture of data sharing by firms with low values of θ, and when $\theta_0 = \overline{\theta}$ and all firms share data.

Because type θ_0 has no rent if firm 1 does not share data (that is $U(\theta_0) = \frac{X^2}{2}$) it must be the case that it has also zero rent if firm 1 shares data. Hence, we must have $\frac{(X-L(\theta_0))^2}{2} + p = \frac{X^2}{2}$, or

$$p = \frac{L(\theta_0)}{2}(2X - L(\theta_0)). \tag{3}$$

For each $\theta < \theta_0$, at the merger stage, firm 2 has perfect information and offers a merger contract $(\alpha(\theta), T(\theta))$ that binds the participation constraint of firm 1. Because the effort level is $\alpha(\theta)X(1+\theta)$, the binding participation constraint is $T(\theta) = \frac{\alpha^2 X^2 (1+\theta)^2}{2} - \frac{X^2}{2}$

because the profit of firms that have shared information is $\frac{(X-L(\theta))^2}{2}$, and the participation constraint is $T(\theta) + \frac{\alpha^2 X^2(1+\theta)^2}{2} \geq \frac{(X-L(\theta))^2}{2}$, the binding transfer is $T(\theta) = -\frac{\alpha^2 X^2(1+\theta)^2}{2} + \frac{(X-L(\theta))^2}{2}$

, and firm 2 solves

$$\max_{\alpha} (1-\alpha)\frac{\alpha X^2(1+\theta)^2}{2} - \frac{\alpha^2 X^2(1+\theta)^2}{2} + \frac{X^2}{2},$$

$\max_{\alpha} (1-\alpha)\alpha X^2(1+\theta)^2 + \frac{\alpha^2 X^2(1+\theta)^2}{2} - \frac{(X-L(\theta))^2}{2}$

that is

$$\max_{\alpha} \alpha(1-2\alpha)\frac{X^2(1+\theta)^2}{2},$$

$\max_{\alpha} \alpha(1-\frac{\alpha}{2})\frac{X^2(1+\theta)^2}{2}$,

and offers $\alpha(t) = \frac{1}{4}$ and $T(\theta) = \frac{X^2(1+\theta^2)}{32} - \frac{X^2}{2}$.[1]

[1] This assumes that firm 1 can make transfer payments at the time of the merger, in exchange for a share in the merged firm profits. If firm 1 cannot make transfers, that is if T must be non-negative, then firm 1 will make rents at the merger phase.

and offers $\alpha(t) = 1$ and $T(\theta) = -\frac{X^2(1+\theta)^2}{2} + \frac{(X-L(\theta))^2}{2}$. Ce resultat est naturel: en donnant tout la part à la firm 1, firm 2 maximise son incitation à faire un effort, puis elle extrait tout le surplus à travers le transfert

Hence, the expected payoff of firm 2 if firm 1 shares data is (using (3))

$$\frac{X^2(1+\theta)^2}{16} - p = \frac{X^2(1+\theta)^2}{16} - \frac{L(\theta_0)}{2}(2X - L(\theta_0)).$$

$\frac{X^2(1+\theta)^2}{2} - p - \frac{(X-L(\theta))^2}{2} = \frac{X^2(1+\theta)^2}{2} - \frac{(X-L(\theta))^2}{2} - \frac{L(\theta_0)}{2}(2X - L(\theta_0))$

The higher p is, the more likely that firm 1 shares data, that is that $\theta_0 > 0$. If the rents $\dot{U}(\theta)$ that have to be given to firm 1 ofr $\theta > \theta_0$ are large on average, firm 2 may prefer to induce data sharing for all types, that is choose $\theta_0 = \overline{\theta}$.

Somewhat unusual, if $\theta_0 \in (0, \overline{\theta})$, the equilibrium payoff of firm 1 is not monotonic in θ: it is decreasing in $\theta < \theta_0$ because firms with high θ lose more than lower types if there is data sharing and competition, hence are at a disadvantage when the merger happens.

Hence, for a given θ_0, the expected payoff of firm 2 is

$$V(\theta_0) := \int_0^{\theta_0} \left[\frac{X^2(1+\theta)^2}{16} - \frac{L(\theta_0)}{2}(2X - L(\theta_0)) \right] f(\theta)d\theta + \int_{\theta_0}^{\overline{\theta}} X^2 \frac{(1+\theta)^3}{2(1+\theta) + \frac{1-F(\theta)}{f(\theta_0)}} f(\theta)d\theta.$$

$V(\theta_0) := \int_0^{\theta_0} \left[\frac{X^2(1+\theta)^2}{2} - \frac{(X-L(\theta))^2}{2} - \frac{L(\theta_0)}{2}(2X - L(\theta)) \right] f(\theta)d\theta + \int_{\theta_0}^{\overline{\theta}} X^2 \frac{(1+\theta)^3}{2(1+\theta)+\frac{1-F(\theta)}{f(\theta_0)}} f(\theta)d\theta.$

Direct computation shows that

$$\begin{aligned} V'(\theta_0) := &\left[\frac{X^2}{16}(1+\theta_0)^2 - \frac{L(\theta_0)}{2}(2X - L(\theta_0)) \right] f(\theta_0) - F(\theta_0)L'(\theta_0)(X - L(\theta_0)) \\ &- X^2 \frac{(1+\theta_0)^3}{2(1+\theta_0) + \ell(\theta)} f(\theta_0) \end{aligned}$$

$V'(\theta_0) := \left[\frac{X^2}{2}(1+\theta_0)^2 - \frac{X^2}{2} \right] f(\theta_0) - F(\theta_0)L'(\theta_0)(X - L(\theta_0)) - X^2 \frac{(1+\theta_0)^3}{2(1+\theta_0)+\ell(\theta)} f(\theta_0)$

hence,

$$V'(0) = f(0)X^2 \left[\frac{1}{16} - \frac{1}{2 + \ell(0)} \right]$$

$V'(0) = -f(0)X^2 \frac{1}{2+\ell(0)}$

noting that $\ell(0) = \frac{1}{f(0)}$, we will have data sharing if

$$0 < f(0) < \frac{1}{8}.$$

we have data sharing if $0 < f(0)$ (always the case)

Could we have *only* data sharing? A necessary condition is that $V'(\bar{\theta}) \geq 0$. However,

$$V'(\bar{\theta}) = -f(\bar{\theta})\frac{7X^2(1+\bar{\theta})^2}{16} - \frac{L(\theta_0)}{2}(2X - L(\theta_0))f(\bar{\theta}) - L'(\bar{\theta})(X - L(\bar{\theta}))$$

is negative by Assumption 1.

Proposition 1. *Suppose that $0 < f(0) < \frac{1}{8}$, then in equilibrium firm 2 offers a price for data sharing that induces a positive measure of types to share data and the other types not to share data.*

1 Antoine's stuff

If merger occurs:

- Industry profits: $X(1+\theta)e - \frac{e^2}{2}$, where e is an effort exerted by firm 1, inducing quadratic costs
- Firm 2 gets the whole profits, and gives a share α to firm 1 to exert positive efforts
- the profits of the firms are thus

$$\Pi_2(\alpha(\theta)) = (1 - \alpha(\theta))X(1+\theta)e - T(\theta)$$

$$\Pi_1(\alpha(\theta)) = \alpha(\theta)X(1+\theta)e - \frac{e^2}{2} + T(\theta).$$

- firm 2 does not know the type of firm 1, and thus ignores whether the share α that maximizes its profits is enough to ensure participation.
- the optimal effort for firm 1 is

$$e^* = \alpha(\theta)X(1+\theta)$$

  and

$$\Pi_1^* = \frac{\alpha(\theta)^2 X^2(1+\theta)^2}{2} + T(\theta)$$

which is larger than X for

$$\theta \geq \frac{\sqrt{2(X - T(\theta))}}{\alpha(\theta)X} - 1$$

- With a fixed α and a fixed T, the profits of firm 2 thus are

$$\int_{\frac{\sqrt{2(X-T)}}{\alpha X}-1}^{\overline{\theta}} \alpha(1-\alpha)X^2\frac{(1+\theta)^2}{2} - TdF(\theta)$$

- firm 2 can offer a screening contract composed of a couple $(T(\theta), \alpha(\theta))$, such that the profit of firm 1 of type θ selecting couple $(\alpha(\hat{\theta}), T(\hat{\theta}))$ is

$$U(\hat{\theta}|\theta) = \alpha(\hat{\theta})X(1+\theta)e + T(\hat{\theta}) - \frac{e^2}{2}$$

and with the equilibrium value of e

$$U(\hat{\theta}|\theta) = \frac{\alpha^2(\hat{\theta})X^2(1+\theta)^2}{2} + T(\hat{\theta})$$

- incentive compatibility constraint implies

$$\frac{\alpha^2(\hat{\theta})X^2[(2+\theta+\hat{\theta})(\theta-\hat{\theta})]}{2} \leq U(\theta) - U(\hat{\theta}) \leq \frac{\alpha^2(\theta)X^2[(2+\theta+\hat{\theta})(\theta-\hat{\theta})]}{2}$$

- This implies that $\alpha(\theta)$ is non decreasing almost everywhere, and that

$$\dot{U}(\theta) = \alpha^(\theta)2X^2(1+\theta)$$

- The profit maximizing function of firm 2 is

$$\begin{aligned}
&\max_{\alpha(.)} \int_{\underline{\theta}}^{\overline{\theta}} (1-\alpha(\theta))\alpha(\theta)X^2(1+\theta)^2 - T(\theta)dF(\theta) \\
\iff &\max_{\alpha(.)} \int_{\underline{\theta}}^{\overline{\theta}} (1-\alpha(\theta))\alpha(\theta)X^2(1+\theta)^2 - U(\theta) + \frac{\alpha^2(\theta)X^2(1+\theta)^2}{2} dF(\theta) \\
&\qquad U(\theta) \geq \frac{X^2}{2} \qquad IR \\
&\qquad U(\theta) \geq U(\hat{\theta}|\theta) \qquad IC
\end{aligned} \tag{4}$$

- This is equivalent to maximizing

$$
\begin{aligned}
&\int_{\underline{\theta}}^{\overline{\theta}} (1-\alpha(\theta))\alpha(\theta)X^2(1+\theta)^2 - U(\theta) + \frac{\alpha^2(\theta)X^2(1+\theta)^2}{2} dF(\theta) \\
=&\int_{\underline{\theta}}^{\overline{\theta}} \alpha(\theta)X^2(1+\theta)^2 - U(\theta) - \frac{\alpha^2(\theta)X^2(1+\theta)^2}{2} dF(\theta) \\
=&X^2 \int_{\underline{\theta}}^{\overline{\theta}} \alpha(\theta)(1+\theta)^2 - \int_{\underline{\theta}}^{\theta} \alpha^2(\theta)(1+\theta)d\theta - \frac{\alpha^2(\theta)(1+\theta)^2}{2} dF(\theta) \\
=&X^2 \int_{\underline{\theta}}^{\overline{\theta}} \alpha(\theta)(1+\theta)^2 - \alpha^2(\theta)(1+\theta)\left(\frac{(1+\theta)}{2} + \frac{1-F(\theta)}{f(\theta)}\right) dF(\theta)
\end{aligned}
\tag{5}
$$

- This is maximized for

$$
\alpha^*(\theta) = \frac{(1+\theta)f(\theta)}{(1+\theta)f(\theta)+2-2F(\theta)}
$$

Question: how to model partial information sharing with moral hazard?

- Without mergers, firm 1 makes profits $Xe - \frac{e^2}{2}$
- with information sharing, firm 1 makes profits $(X - ls^*)e - \frac{e^2}{2}$ and firm 2 makes profits $e\theta s$
- sharing information thus decreases the incentives to effort of firm 1
- in turn, this changes the outside option of firm 1 when merging with prior information sharing:

$$
\alpha^2(\theta)X^2(1+\theta) \geq (X - ls^*)^2
$$

- The optimal share of information is identical to above, however the transfer changes as the bargaining power of Firm 1 decreases with prior information sharing

References