Standard Signalling (Without Cost of Lying)

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Players have equiprobable types C, D with payoff structures

Own\ Opponent	H	L
H	50	10
L	24	20

Type C

Own\ Opponent	H	L
H	22	10
L	24	20

Type D

Table 1: Regime NC

1 No Communication (NC)

Types D play L. Let x be the probability that type C plays H. Then $x > 0$ when

$$\frac{1}{2}(10 + (50 - 10)x) + \frac{1}{2}(10) \geq \frac{1}{2}(20 + 4x) + \frac{1}{2}(20)$$

that is when

$$x \geq \frac{5}{9}$$

Hence, either $x = 0$, or $x = 1$ or $x = \frac{5}{9}$.

2 Communication without Cost (CWC)

It is clear that if C communicates with probability one, D will communicate. Indeed, if D does not communicate, in state $(1,0)$ types C plays L with probability one and D has a payoff of 20 by not communicating. By communicating, D has a payoff of $\frac{1}{2}(20+4x(1,1))+\frac{1}{2}(20)$, larger than 20 if $x(1,1)>0$. Hence if types C communicate with probability one, either they do not play H or if they do types D also communicate. This situation is observationally equivalent to the previous regime of no communication.

Another possibility is when types C and D play a mixed strategy in communication. Let α the strategy of C and β the strategy of D in communication. If the opponent has communicated, an agent believes that she faces a type C with probability $\frac{\alpha}{\alpha+\beta}$. If the opponent has not communicated, an agent believes that she is facing a type C with probability $\frac{1-\alpha}{2-\alpha-\beta}$.

We will be looking at equilibria consistent with types C playing H with positive probability, and both types communicating, that is $\alpha\beta>0$.

2.1 Continuation Strategies

State $(1,1)$**.** It is optimal to set $x(1,1)>0$ if $\frac{\alpha}{\alpha+\beta}(10+40x(1,1))+\frac{\beta}{\alpha+\beta}10$ is greater than $\frac{\alpha}{\alpha+\beta}(20+4x(1,1))+\frac{\beta}{\alpha+\beta}20$, that is

$$x(1,1)>0 \Leftrightarrow x(1,1)\geq\frac{5}{18}\frac{\alpha+\beta}{\alpha}. \tag{1}$$

A necessary condition is that $\alpha\geq\frac{5}{18}\beta$.

State $(0,1)$**.** A non-communicating type C facing a communicating opponent plays H when (the belief structure is the same as in state $(1,1)$ but the players expects a communicating type C to play $x(1,0)$),

$$x(1,0)\geq\frac{5}{18}\frac{\alpha+\beta}{\alpha} \tag{2}$$

and the necessary condition is the same as for having $x(1,1) > 0$.

State $(1,0)$**.** An opponent of type C plays $x(0,1)$, and therefore playing H is optimal for a type C who communicates when

$$(1-\alpha)(10+40x(0,1)) + (1-\beta)10 \geq (1-\alpha)(20+4x(0,1)) + (1-\beta)20,$$

$$x(0,1) \geq \frac{5}{18}\frac{2-\alpha-\beta}{1-\alpha} \tag{3}$$

and a necessary condition is that

$$1-\alpha \geq \frac{5}{18}(1-\beta)$$

State $(0,0)$**.** The belief structure is the same as that of a communicating type C in state $(1,0)$, and therefore $x(0,0) > 0$ when

$$x(0,0) \geq \frac{5}{18}\frac{2-\alpha-\beta}{1-\alpha} \tag{4}$$

with the same necessary condtion as in state $(1,0)$.

It is possible to have all $x(i,j) > 0$ when the bounds in (**??**) and (**??**) are not greater than one, that is when

$$\frac{5}{13}\beta \leq \alpha \leq \frac{5}{13}\beta + \frac{8}{13}. \tag{5}$$

2.2 Incentives to Communicate

It is possible that $x(i,j) > 0$ for each state (i,j). The necessary condition is (**??**). In this case, types C weakly prefer to communicate and when

$$\begin{aligned} &\frac{\alpha}{2}(10+40x(1,1)) + \frac{1-\alpha}{2}(10+40x(0,1)) + \frac{1}{2}10 \\ &\quad \geq \frac{\alpha}{2}(10+40x(1,0)) + \frac{1-\alpha}{2}(10+40x(0,0)) + \frac{1}{2}10 \end{aligned} \tag{6}$$

Note that the condition reduces to

$$\alpha(x(1,1) - x(1,0)) + (1-\alpha)(x(0,1) - x(0,0)) \geq 0. \tag{7}$$

If this condition binds, it also implies that type D is indifferent between communicating and not communicating. If there is a strict inequality, types D communicate with probability one.

"Calibration" In the experiment, we have $\alpha = 82\%$ and $\beta = 43\%$: these values satisfy the necessary condition (**??**). Because $\frac{\alpha+\beta}{\alpha} = \frac{125}{82}$ and $\frac{2-\alpha-\beta}{1-\alpha} = \frac{85}{18}$. Therefore, $x(0,0) > 0$ requires $x(0,0) > \frac{425}{324}$, clearly absurd. Hence $x(0,0) = 0$. Similarly, $x(1,0) > 0$ requires that $x(0,1) \geq \frac{425}{324}$, which is also absurd. Hence, $x(i,j) = 0$ for all $(i,j) \neq (1,1)$, in contradiction with the experimental results.

3 Costly Communication

When a fee of 3 has to be paid in order to communicate, types D cannot benefit from communicating. Indeed, the gain from communication is equal to $2\alpha(x(1,1) - x(1,0)) + 2(1-\alpha)(x(0,1) - x(0,0))$ which is less than 2 and cannot compensate for the exogenous cost of 3 of sending a message.

Therefore $\beta = 0$. The continuation strategies of type C are as in the previous case. However the incentive of type C to communicate is different.

If $x(i,j) = 0$ for $(i,j) \neq (1,1)$, we need

$$20\alpha x(1,1) \geq 3,$$

hence a minimum value of α of 15% when $x(1,1) = 1$. If $x(1,1) = \frac{5}{18}$), the condition is that $\alpha \geq 54\%$.

If now $x(i,j) > 0$ for all $(i,j) \neq (1,1)$, the incentive condition for types

C to communicate is

$$20(\alpha(x(1,1)-x(1,0)+(1-\alpha)(x(0,1)-x(0,0)))) \geq 3. \tag{8}$$

There are many solutions satisfying this condition as well as (**??**)-(**??**).

"Calibration". In the experiment, $\alpha = 0.50$, $\beta \approx 0$, $x(1,1) = 1$, $x(0,1) = 80\%, x(1,0) = 70\%, x(0,0) = 31\%$. The only theoretical prediction that roughly matches the order of magnitudes of these values are $x(1,1) = x(1,0) = x(0,1) = 1$ and $x(0,0) = \frac{5}{18}\frac{2-\alpha}{1-\alpha}$. For these values, the left hand side of (**??**) is $\frac{5}{18}(2-\alpha)$, which can be equal to $\frac{3}{20}$ when $\alpha = \frac{41}{25}$, clearly impossible. (Alternatively, if $\alpha = \frac{1}{2}$, the left hand side has value $\frac{15}{36}$, that is three times smaller than $\frac{3}{20}$.)

We conclude that a standard model of signalling without cost-of-lying cannot explain the experimental data.