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Promises and Communication

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Abstract

Keywords:

JEL codes:

Players have basic types C, D with payoffs when there is no communication

Own\ Opponent	H	L
H	a	10
L	24	20

Type C

Own\ Opponent	H	L
H	22	10
L	24	20

Type D

Table 1: Regime NC ($a \in \{30, 50\}$)

For each type, a proportion π of players keeps promises while a proportion $1 - \pi$ bears a small cost (equal to 1) from breaking promises. If during a communication phase a message is sent, the understanding is that the player promises to take action H in the continuation game. A type who does not feel bound to promises has same payoffs as in Table 1 for the action H, and the payoffs minus one for the action L; a type who is bound by her promise has a dominant strategy to play H in the

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continuation game (or faces a large cost-of-lying). This regime is call CT ("cheap talk").

The last regime F is when there is also an exogenous cost of sending messages and we fix this cost at 3. The difference with the previous regime CT is that a player incurs the cost of 3 independently of her future actions, while in the CT regime, she incurs the cost-of-lying, of not keeping promises, only if she plays L later on. Hence, the exogenous cost makes it uniformly less appealing to communicate. Note that once the cost of communication is paid, the marginal incentives for playing H versus L are identical to the CT regimes (but all payoffs are decreased by 3).

1 No-Communication (NC)

It is clear that types D players have a dominant strategy to play L, and it is sufficient to consider the strategy of types C players, that is the probability x that they play H. Because a player has a half-chance of being paired with a C player, playing H yields an expected payoff of $\frac{1}{2}(xa + (1-x)10) + \frac{1}{2}10 = 10 + (a-10)\frac{x}{2}$, while playing L gives an expected payoff of $20 + 2x$. Therefore, the best response of type C is

$$BR(x) = \begin{cases} 0 & \text{if } x < \frac{20}{a-14} \\ [0,1] & \text{if } x = \frac{20}{a-14} \\ 1 & \text{if } x > \frac{20}{a-14} \end{cases}$$

Clearly, as $a = 30$, $\frac{20}{a-14} > 1$, and therefore the only equilibrium is for both types to play L? However, if $a = 50$, there are three equilibria: the two pure strategy profiles $(L, L), (H, L)$ and the mixed strategy profile $(x = 5/9, L)$.

Proposition 1. *If communication is not possible:*

(i) If $a = 30$, the only equilibrium is for both types to play L.

(ii) If $a = 50$, there are two pure strategy equilibria (L, L) and (H, L), and another equilibrium where types C play H with probability $5/9$ and type D play L.

2 Communication with Cost-of-Lying (CT)

In the communication stage, actions are $\{0,1\}$, where 0 means not sending a message and 1 sending a message. A strategy for types C is the probability α of sending a message and a strategy for D is the probability β of sending a message.

After the communication stage, there can be four states $(i,j) \in \{0,1\}^2$. We use the convention that for a given player (i,j) denotes the action i that the player has taken during communication and the action j that the opponent has taken during communication. Therefore, a player who observes (i,j) knows that the opponent observes (j,i).

There are four possible types of players $C_1, C_\infty, D_1, D_\infty$ where the subscript 1 indexes a player who has a cost-of-lying of 1 and the subscript ∞ indexes a player who is bound by promises.

Without communication, type D has a dominant strategy to play L in the continuation game. Type D_∞ can, by communicating, commit to play H but the opportunity cost of doing so is to be able to have a higher payoff if he faces a player who plays H by not communicating. By contrast for a D_1 type, communication does not bring such a cost but may a-contrario induce the other player to play H. Communication is therefore subject to a basic adverse selection problem, as types who do not keep their promises are more likely to be willing to communicate.

Strategies in the communication game are $(\alpha_1, \alpha_\infty, \beta_1, \beta_\infty)$, where α_k is the strategy of C_k and β_k is the strategy of D_k.

Once a state (i,j) is realized, each player updates his belief about the type of the opponent. For instance, in state $(1,1)$ the posterior that the opponent is type C_∞ is

$$p(C_\infty|1) = \frac{\pi\alpha_\infty}{\pi(\alpha_\infty + \beta_\infty) + (1-\pi)(\alpha_1 + \beta_1)}.$$

Given these posterior beliefs, players play state-contingent strategies in the continuation game. For D_1 and for D_∞ who does not communicate the dominant strategy is L, while for D_∞ and C_∞ who communicate the dominant strategy is to play H. Finally, for C_1 or C_∞ who does not communicate, we assume that they use the same strategy in the continuation game (this is consistent with identifying a 'type' with the payoff structure), and we let $x(i,j)$ be the probability that a player facing state (i,j) plays H.

Table 2 summarizes this discussion. A no-communication outcome is always

	C_1	C_∞	D_1	D_∞
Message	$x(1,j)$	H	L	H
No Message	$x(0,j)$	$x(0,j)$	L	L

Table 2: Strategies ($j \in \{0,1\}$)

possible and we will be searching for equilibria in which communication yields more coordination on H than in regime NC.

2.1 All Types Communicate

Communication serves as a commitment device when there is a positive cost-of-lying. For instance types C_∞ and D_∞ are fully committed to play H after sending a message. This commitment by types C_∞ and D_∞ induces also type C_1 to play H more often than in the regime NC. However commitment entails a cost if the benefit from coordination on H is small.

Formally, if all types send a message, the posterior beliefs are the same as the prior. Let $x(1,1)$ be the equilibrium action of types C and assume that $x(i,0)=0$ for all i (this gives the most chances to this equilibrium).

By sending a message, a type D_∞ player faces types C_1 who will play H with probability $x(1,1)$, types C_∞, D_∞ who play H with probability 1 and a type D_1 player who will play L, therefore her expected payoff by sending a message is equal to $10+12\pi+6(1-\pi)x(1,1)$. Hence her maximum payoff from communication when $x(1,1)=1$ is equal to $16+6\pi$.

By not sending a message, type D_∞ faces types C_1 and D_1 who play L and types C_∞ and D_∞ who play H. Contrary to the previous case, the player can now play L. Therefore her expected payoff is $\pi 24+(1-\pi)20=20+4\pi$, which is clearly greater than $16+6\pi$ for any value of π.

Hence, there is no equilibrium in which all types communicate.

2.2 A Basic Adverse Selection Problem: Only Types C Communicate

Suppose that types C communicate, while types D do not communicate (it should be clear shortly that if D_1 does not communicate, then D_∞ does not communicate either: they both have the same expected payoff by not communicating but type D_∞ has a lower expected payoff than type D_1 by communicating since she is constrained in her best-responses).

For a type D_1, playing L in the continuation game is a dominant strategy (if she communicates, while her payoffs if playing L are decreased by 1, they still dominate the payoffs from playing H). Hence, by sending a message D_1 has an expected payoff of

$$\frac{1}{2}(\pi 24 + (1-\pi)(20 + 4x(1,1)) + \frac{1}{2}(20) - 1$$

while by not sending a message, she has an expected payoff of

$$\frac{1}{2}(\pi 24 + (1-\pi)(20 + 4x(1,0)) + \frac{1}{2}(20)$$

and therefore not sending a message is (weakly) optimal for D_1 when $x(1,1) - x(1,0) \leq \frac{1}{2(1-\pi)}$. In state $(1,0)$, the posterior beliefs of the communicating player is that she faces a type D with probability one. Therefore, a type C_1 who communicated but has an expected payoff of 10 if she plays H an an expected payoff of $20 - 1$ if she plays L. Therefore, she must use $x(1,0) = 0$. It follows that the condition for type D_1 not to communicate is

$$x(1,1) \leq \frac{1}{2(1-\pi)}. \tag{1}$$

This condition holds if $\pi \geq \frac{1}{2}$. If $\pi < \frac{1}{2}$, the condition holds only if $x(1,1) < 1$, implying that C_1 plays a mixed strategy in the continuation game or plays L.

Case $x(1,1) = 0$**.** After communicating type C_∞ faces an opponent who plays H only if that player is of type C_∞. Hence, her expected payoff is equal to $\frac{\pi}{2}a + (1 - \frac{1}{2}\pi)10 = 10 + \frac{\pi}{2}(a - 10)$. By not communicating, she faces opponents who play the same actions, that is only type C_∞ plays H. By not communicating and playing H when the state is $(0,1)$ and playing L when the state is $(0,0)$, C_∞ gets $\frac{1}{2}(\pi a + (1-\pi)) + \frac{1}{2}20 = 15 + \frac{\pi}{2}(a - 10)$ which is larger than C_∞'s expected payoff from communication. A contradiction.

Case $x(1,1) \in (0,1)$**.** There are two out-of-equilibrium states for player C (when she deviates by not communicating). If the state is $(0,0)$ the same logic as in state $(1,0)$ implies that $x(0,0) = 0$ is optimal. If the state is $(0,1)$ our player believes that she faces a type C_1 who is in state $(1,0)$ and plays L and a type C_∞ who plays H. Hence, playing H is weakly optimal for types C_1, C_∞ in state $(0,1)$ when $\pi a + (1-\pi)10 \geq \pi 24 + (1-\pi)20$, that is when

$$x(0,1) > 0 \Leftrightarrow \pi > \frac{10}{a - 14}.$$

We now turn to the other incentive compatibility conditions for players of type C. Because $x(1,1) \in (0,1)$, a player of type C_1 must be indifferent between playing H and L in state $(1,1)$. Because she sent a message, C_1 incurs a cost-of-lying of 1 if she plays L and she is willing to play the mixed strategy when

$$\begin{aligned}&\pi a + (1-\pi)(10 + (a-10)x(1,1)) \\ &\quad = \pi 24 + (1-\pi)(20 + 4x(1,1)) - 1\end{aligned}$$

or that

$$x(1,1) = \frac{9 - \pi(a - 14)}{(1-\pi)(a-14)}. \tag{2}$$

The bound is less than one since $a \geq 23$. It is positive when $\pi \leq \frac{9}{a-14}$, which is smaller than the bound $\frac{10}{a-14}$ for which $x(0,1)$ is positive. Hence, necessarily $x(0,1) = 0$.

The bound must be non-negative and smaller than the bound $\frac{1}{2(1-\pi)}$ required for incentive compatibility of type D_1, that is we need

$$\frac{9-\pi(a-14)}{(1-\pi)(a-14)} \leq \frac{1}{2(1-\pi)}.$$

If $a = 30$, the condition holds if $\pi \geq 1/16$; if $a = 50$, the condition holds for any π. Hence a mixed strategy is possible when $a = 30$ if $\pi \in [1/16, 9/16]$ and when $a = 50$ if $\pi \in [0, 9/36]$.

What is left to verify is that types C_k prefer to communicate. It is enough to verify that type C_∞ prefers to communicate: a player of type C_∞ has a lower expected payoff from communication than type C_1 but the same expected payoff if she does not communicate as type C_1.

We consider the two values of a.

If $a = 30$, C_∞'s expected payoff from communication is:

$$\frac{1}{2}(30\pi + (1-\pi)(10 + 20x(1,1))) + \frac{1}{2}10,$$

or

$$10(1 + \pi + (1-\pi)x(1,1)).$$

If C_∞ does not communicate, she plays L in any state $(0, j)$. Hence her expected payoff is equal to $\frac{1}{2}(20 + 4\pi) + \frac{1}{2}(20) = 20 + 2\pi$, which is greater than what she obtains under communication for any π. Hence, it is not possible for only types C to communicate.

If $a = 50$, From above, communication requires $\pi \leq \frac{9}{36}$. Hence C_∞'s expected payoff from communication is:

$$\frac{1}{2}(50\pi + (1-\pi)(10 + 40x(1,1))) + \frac{1}{2}10,$$

or, using $x(1,1) = \frac{9-36\pi}{36(1-\pi)} = \frac{1-4\pi}{4(1-\pi)}$,

$$10(1 + 2\pi + 2(1-\pi)x(1,1)) = 15$$

which is less than the payoff of $20+2\pi$ and C_∞ does not communicate.

It is therefore not possible to have an equilibrium where only types C communicate and play a mixed strategy in the continuation game.

Case $x(1,1)=1$**.** This case is consistent with the incentive compatibility condition for D_1 only if $\pi \geq \frac{1}{2}$.

Player C_1 prefers to play H in state $(1,1)$: she gets a by doing so while gets only 23 by playing L.

Turning to the incentives to communicate, we borrow from the previous case, and note that if type C_∞ does not communicate she plays $x(0,1)>0$ only if $\pi \geq \frac{10}{a-14}$.

<u>If $a=30$</u>, and $\pi < \frac{5}{8}$, $x(0,1)=0$ and by not communicating, type C_∞ plays L in all states and therefore has an expected payoff of (only an opponent of type C_∞ plays H) $\frac{\pi}{2}24+\frac{1-\pi}{2}20+\frac{1}{2}20=20+2\pi$. By communicating, she faces a type C with probability $1/2$ and gets 30 and with probability $1/2$ she faces a type D and gets 10 (since she plays H against L), therefore an expected payoff of 20, smaller than when she does not communicate.

If $\pi > \frac{5}{8}$, $x(0,1)=1$, and not communicating yields $\frac{\pi}{2}30+\frac{1-\pi}{2}10+\frac{1}{2}20=15+10\pi$. Hence in the region $\pi \geq \frac{1}{2}$, communicating is not a best response for type C_∞ as $\pi \geq \frac{5}{8}$.

<u>If $a=50$</u>. Because $\pi \geq \frac{1}{2} > \frac{10}{a-14}$, $x(0,1)=1$. Under communication, player C_∞ gets 50 against types C_∞ and C_1 but only 10 against the other types, hence an expected payoff of 30. By not communicating, she gets 50 against types C_∞ only, 10 against type C_1 (who has state $(1,0)$ and plays L) and 20 against types D (since she faces state $(0,0)$ and plays L). Hence her expected payoff under no-communication is $15+20\pi$, which is smaller than 30 when $\pi \leq \frac{3}{4}$.

Proposition 2. *When there are cost of lying and communication is possible, an equilibrium in which types C communicate and types D do not communicate:*

(i) does not exist when $a=30$.

(ii) exists when $a=50$ if, and only if $\pi \in \left[\frac{1}{2},\frac{3}{4}\right]$ and $x(1,1)=x(0,1)=1$, $x(0,0)=x(1,0)=0$.

Remark 1. We have not considered the possibility of mixed strategy in communication. Intuitively, a mixed strategy equilibrium will induce more cooperation the more likely type C_∞ communicates because this type is more committed to play H following communication. However, a simple revealed preference argument suggests that whenever C_∞ is willing to communicate, type C_1 is also willing to communicate (remember that the strategic behaviors of types C_1, C_∞ are the same when they do not communicate, but that type C_∞ is more constrained than type C_1 in her best-response). Hence, mixed strategies are likely to decreases the level of cooperation conditional on one of the parties communicating.

2.3 Accepting Adverse Selection: All but D_∞ Communicate

Assume now that types C communicate and that type D_1 also communicates.

In the continuation game, D_1 plays L independently of the communication outcome. D_∞ plays L in the equilibrium states $(0, j)$, but plays H in the off equilibrium states $(1, j)$.

In the continuation game, C_∞ plays H in the states $(1, j)$. In state $(1, 0)$, C_1 believes that she faces a type D_∞ who will play L and therefore plays L, or $x(1, 0) = 0$ (she bears a cost of lying of 1 but prefers to play L and get 19 than playing H and get 10).

We will first analyze the optimal continuation actions $x(1, 1), x(0, 1)$ (after the communication phase) and then the incentives of different types to communicate.

Continuation Actions

If C_k does not communicate (off-equilibrium) and the state is $(0, 0)$, C_k believes that the other player is type D_∞. As D_∞ plays L, C_k will also play L in this case. In the other off-equilibrium state $(0, 1)$, C_k's posterior belief is $p(C_1|1) = p(D_1|1) = \frac{1-\pi}{2-\pi}$, $p(C_\infty|1) = \frac{\pi}{2-\pi}$. C_k's expected payoff from playing H is

$$\frac{1-\pi}{2-\pi}10 + \frac{\pi}{2-\pi}a + \frac{1-\pi}{2-\pi}10 = \frac{20 + (a-20)\pi}{2-\pi}.$$

C_k's expected payoff from playing L is

$$\frac{1-\pi}{2-\pi}20 + \frac{\pi}{2-\pi}24 + \frac{1-\pi}{2-\pi}20 = \frac{40-16\pi}{2-\pi}$$

Therefore, C_k will play L in state $(0,1)$ when $a = 30$ and $\pi \in (0, 10/13)$, plays H if $\pi > 10/13$ and is indifferent when $\pi = 10/13$. When $a = 50$, type C_k plays L if $\pi \in (0, 10/23)$, plays H if $\pi > 10/23$ and is indifferent between the two actions when $\pi = 10/23$.

Consider now the possible values of $x(1,1)$ in the continuation game for C_1. In the state $(1,1)$, the posterior belief is $p(C_1|1) = p(D_1|1) = \frac{1-\pi}{2-\pi}$, $p(C_\infty|1) = \frac{\pi}{2-\pi}$. The only strategy to determine is that of C_1 since, as discussed above, C_∞ plays H after sending a message and type D_1 plays L.

In state $(1,1)$, C_1 playing H dominates playing L when

$$\begin{aligned}\frac{1-\pi}{2-\pi}(10+(a-10)x(1,1)) + \frac{\pi}{2-\pi}(a) + \frac{1-\pi}{2-\pi}(10)&\\ \geq \frac{1-\pi}{2-\pi}(20+4x(1,1)) + \frac{\pi}{2-\pi}(24) + \frac{1-\pi}{2-\pi}(20) - 1&\end{aligned}$$

or

$$\pi \geq \frac{18-(a-14)x(1,1)}{a-5-(a-14)x(1,1)}.$$

When $a = 50$, this condition is equivalent to $x(1,1) \geq \frac{18-45\pi}{36(1-\pi)} = \frac{2-5\pi}{4(1-\pi)}$, and because the right hand side is inferior to one for any π, it can be optimal for C_1 to play H. Precisely, either C_1 plays the pure strategy H or if $\pi < \frac{2}{5}$ plays the mixed strategy $x(1,1) = \frac{2-5\pi}{4(1-\pi)}$.

By contrast, the condition when $a = 30$ can be satisfied only if π is large enough. Indeed, the condition reduces then to $x(1,1) \geq \frac{18-25\pi}{16(1-\pi)}$, with a right hand side less than one only if $\pi \geq \frac{2}{9}$.

To summarize, players of type C_1 play $x(1,1) > 0$ in the following cases:

- When $a = 30$ and $\pi \geq \frac{2}{9}$, in which case either $x(1,1) = 1$; or $x(1,1) = \frac{18-25\pi}{16(1-\pi)}$ if $\pi < \frac{18}{25}$.

- When $a = 30$ and $\pi \leq \frac{2}{9}$, then $x(1,1) = 0$.
- or when $a = 50$, in which case either $x(1,1) = 1$; or $x(1,1) = \frac{2-9\pi}{4(1-\pi)}$ if $\pi < \frac{2}{5}$.

For instance, if $\pi = 1/2$, C_1 plays H with probability one if $a = 50$ but can play also the mixed strategy 11/16 if $a = 30$.

Incentives to Communicate

Type D_∞. By not communicating, a player of type D_∞ has expected payoff of $20 + 2\pi$ since any opponent plays L in the equilibrium state $(1, 0)$ except C_∞ who plays H.

By deviating and communicating, D_∞ plays H in the continuation game. She will get $10 + 12x(1,1)$ if the opponent is C_1, 22 if the opponent is C_∞ and 10 if the opponent is D_1 or D_∞. Hence her expected payoff is equal to $10+6\pi+6x(1,1)(1-\pi)$ which is clearly smaller than $20 + 2\pi$. Hence D_∞ is incentive compatible.

Type D_1. By not communicating, D_1 has also a payoff of $20 + 2\pi$. In equilibrium D_1 communicates, plays L and bears a cost-of-lying. Therefore her expected payoff is equal to $19+2\pi+2x(1,1)(1-\pi)$, which is smaller than $20+2\pi$ when $x(1,1) < \frac{1}{2(1-\pi)}$ the same condition we had in the regime where only types C communicate. This means D_1 will communicate only if

$$x(1,1) \geq \frac{1}{2(1-\pi)}.$$

When $\pi > \frac{1}{2}$, then $\frac{1}{2(1-\pi)} > 1$ and D_1 will not communicate. When $\pi \leq 1/2$, then $\frac{1}{2(1-\pi)} \leq 1$ and D_1 can prefer communication if the condition for $x(1,1)$ holds.

When $a = 30$, then the mixed strategy $x(1,1) = \frac{18-25\pi}{16(1-\pi)}$ is not greater than $\frac{1}{2(1-\pi)}$ when $\pi \geq \frac{2}{5}$. In this case, D_1 will communicate if $x(1,1) = 1$. If $\pi \in (0, 2/5)$, then both mixed strategy $x(1,1) = \frac{18-25\pi}{16(1-\pi)}$ and $x(1,1) = 1$ satisfies the condition and D_1 will communicate.

When $a = 50$ and $\pi \leq \frac{2}{5}$, the mixed strategy is $x(1,1) = \frac{2-9\pi}{4(1-\pi)}$ which is indeed smaller than $\frac{1}{2(1-\pi)}$. Hence, D_1 cannot be incentive compatible with the mixed strategy $x(1,1) = \frac{2-9\pi}{4(1-\pi)}$. If $\pi \in (0, 1/2)$, then C_1 can also play a pure strategy $x(1,1) = 1$ which is also incentive compatible for D_1.

Type C_∞. (If this type is incentive compatible, type C_1 is also incentive compatible.) If a player of type C does not communicate, her opponent will believe that she is type D_∞. Types D_k and C_1 will play L, while type C_∞ will play H.

Case $a = 30$. As discussed in the analysis of the continuation games, if $\pi < 2/9$, C_1 plays L in state $(1,1)$ and types C_k also play L in state $(0, j)$. But then, communication is dominated by no communication for players of type C_∞ (they do not change the actions of their opponent by communicating, but are constrained in their best-response).

Hence, C_∞ can be incentive compatible only if $\pi > 2/9$. In this case, types C_1 play $x(1,1) > 0$, but $x(1,0) = 0$ and communication modifies the play of such opponents. By communicating a player C_∞ has payoff $\frac{1-\pi}{2}(10 + 20x(1,1)) + \frac{\pi}{2}30 + \frac{1}{2}10$ while by not communicating and playing L her payoff is $\frac{1-\pi}{2}(20 + 4x(0,1)) + \frac{\pi}{2}24 + \frac{1}{2}20 = 20 + 2\pi + 2x(0,1)(1-\pi)$. Hence, even if $x(0,1) = 0$, communication is best when $x(1,1) > \frac{5-4\pi}{5(1-\pi)}$, which is impossible since the bound is greater than one.

Case $a = 50$. As mixed strategy in state $(1,1)$ is not incentive compatible for type D_1, we will analyse only the case $x(1,1) = 1$. In the continuation game, $x(1,1) = 1$, and in state $(0,1)$, types C_k play H only if $\pi \geq \frac{10}{23}$.

(i) Suppose that $\pi > \frac{10}{23}$. If C_∞ communicates her expected payoff is $\frac{1-\pi}{2}(10 + 40x(1,1)) + \frac{\pi}{2}50 + \frac{1}{2}10 = 10 + 20\pi + 20(1-\pi)x(1,1) = 30$.

Suppose now that C_∞ does not communicate. She can now play L if she faces D_∞, a gain. If she faces an opponent who communicates, she plays H in state $(0,1)$, but faces a more aggressive opponent C_1 who plays L instead of $x(1,1)$, a loss. The expected payoff is $\frac{1-\pi}{2}(10) + \frac{\pi}{2}50 + \frac{1-\pi}{2}10 + \frac{\pi}{2}20 = 10 + 25\pi$. It is incentive compatible for player D_1 to communicate if $\pi \in (0, 1/2)$ and $x(1,1) = 1$. Therefore, C_∞'s expected payoff from communication is 30 and it is larger than her expected payoff from no communication $10 + 25\pi$ when $\pi \in [10/23, 1/2]$.

(ii) Suppose now that $\pi \leq \frac{10}{23}$. If the state is $(0,1)$, type C_k plays L. The payoff under no communication is then $20 + 2\pi$ while the payoff under communication is still $10 + 20\pi + 20(1-\pi)x(1,1) = 30$. It is incentive compatible for player D_1 to communicate since $\pi \leq \frac{10}{23} < \frac{1}{2}$ and $x(1,1) = 1$. The payoff of type C_∞ from communication is 30 when $x(1,1) = 1$, which is always larger than $20 + 2\pi$.

Proposition 3. *An equilibrium for which all types except D_∞ communicate:*

(i) does not exist when $a = 30$;

(ii) does not exist when $a = 50$ and $\pi > 1/2$;

(iii) exists when $a = 50$ and $\pi \in [0, 1/2]$; in this case the continuation equilibrium entails pure strategies, in particular $x(1,1) = 1$ and (out-of-the equilibrium) $x(0,1) = 0$ if $\pi \in [0, 10/23]$ and $x(0,1) = 1$ and (out-of-the equilibrium) $x(0,1) = 1$ if $\pi \in (10/23, 1/2]$.

2.4 Accepting Adverse Selection: Only Type C_1 Communicates

As discussed in the previous two sections, when $a = 30$, there is no equilibrium in which both types $C_1; C_\infty$ communicate. Because types C_1 have a lower opportunity cost of communicating (they can play L at cost 1 if they choose to while C_∞ are committed to playing H), we consider now equilibria in which C_1 communicates but C_∞ does not communicate.

For such a profile, posterior beliefs in states $(i,1)$ are $p(C_1|1) = 1$, $p(t|1) = 0$ for $t \neq C_1$, and in states $(i,0)$, $p(C_1|0) = 0$, $p(C_\infty|0) = p(D_\infty|0) = \frac{\pi}{1+\pi}$, $p(D_1|0) = \frac{1-\pi}{1+\pi}$.

In the continuation game, type D_1 players play L independent of the communication outcome, type D_∞ players play L if the communication outcome is $(0,j)$, while play H in the off equilibrium states $(1,j)$. In the off equilibrium states $(1,j)$, type C_∞ players also play H.

In state $(0,0)$, types C_k may be induced to play H if it is likely that they are facing types C_∞. Intuitively, if $x(0,0)$ is positive, it increases the opportunity cost of type C_1 to communicate, and we will turn to this later.

$$\frac{\pi}{1+\pi}(10 + (a-10)x(0,0)) + \frac{1}{1+\pi}10 \geq \frac{\pi}{1+\pi}(20 + 4x(0,0)) + \frac{1}{1+\pi}20,$$

hence, $x(0,0) \geq \frac{10}{a-14}\frac{1+\pi}{\pi}$. This is possible only if the bound is less than one, hence if $\pi \geq \frac{10}{a-24}$.

$$x(0,0) \geq 0 \Leftrightarrow \pi \geq \frac{10}{a-24} \tag{3}$$

Consider the case $a = 30$. Continuation strategies: From (3), when $a = 30$ the lower bound is greater than one and therefore $x(0,0) = 0$.

In state $(0,1)$, the best response of C_k is to play H when $10 + 20x(1,0) \geq 20 + 4x(1,0)$, or when $x(1,0) \geq \frac{5}{8}$.

Consider the continuation best responses of type C_1.

In state $(1,1)$, she plays H as long as $10 + 20x(1,1) \geq 20 + 4x(1,1) - 1$ or $x(1,1) \geq \frac{9}{16}$. Hence there are two possible Nash equilibria in state $(1,1)$

$$x(1,1) \in \left\{1, \frac{9}{16}\right\}. \tag{4}$$

In state $(1,0)$, playing H is a best response if (type D plays L while type C_∞ plays $x(0,1)$):

$$\frac{\pi}{1+\pi}(10 + 20x(0,1)) + \frac{1}{1+\pi}10 \geq \frac{\pi}{1+\pi}(20 + 4x(0,1)) + \frac{1}{1+\pi}20 - 1,$$

and the condition will hold in the best case scenario $x(0,1) = 1$ when $\pi \geq \frac{9}{7}$ which is clearly not possible. Therefore, $x(1,0) = 0$.

It follows ,that type C_∞ does not play H in the state $(0,1)$. It is also true for off equilibrium state $(0,1)$ for type C_1, $x(0,1) = 0$. Note that, it is also true that $x(0,0) = 0$ for both C_k types.

Communication incentives: Given these continuation strategies, a player of type C_1 can get the full coordination payoff $a = 30$ only if she meets another type C_1 and will get 20 minus the cost-of-lying if she meets another type. Therefore, sending a message is optimal for C_1 when $\frac{1-\pi}{2}(10 + 20x(1,1)) + \frac{1+\pi}{2}19 \geq 20$, that is

$$C_1 \text{ communicates } \Leftrightarrow x(1,1) \geq \frac{11 - 9\pi}{20(1-\pi)}, \tag{5}$$

where the bound is inferior to one when $\pi \in [0, 9/11]$ and has value in $\left[\frac{11}{20}, 1\right]$. Comparing with (4), C_1 prefers communication if $x(1,1) = 1$ and $\pi \in [0, 9/11]$ or $x(1,1) = 9/16$ and $\pi \in [0, 1/9]$.

Type C_∞ would be committed to play H if she sends a message and therefore can get 30 by meeting a C_1 type, but 10 by meeting any other type (including the other players of type C_∞ who are supposed not to send a message in equilibrium). Hence, her expected payoff is $\frac{1-\pi}{2}(10 + 20x(1,1)) + \frac{1+\pi}{2}10 = 10 + 10x(1,1)(1-\pi)$ which is strictly less than her expected payoff of 20 by not sending a message, unless $\pi = 0$ and $x(1,1) = 1$. Hence C_∞ prefers not to send a message.

Consider now the incentive of D_1 to send a message. By not sending a message she has an expected payoff of 20 because $x(1,0) = x(0,1) = x(0,0) = 0$. By sending a message she gets $24x(1,1) + 20(1 - x(1,1)) - 1 = 19 + 4x(1,1)$ when meeting a C_1 player and $20 - 1 = 19$ when meeting other types, so her expected payoff is $\frac{1-\pi}{2}(19 + 4x(1,1)) + \frac{1+\pi}{2}19 = 19 + 2x(1,1)(1-\pi)$. Hence,

$$D_1 \text{ does not communicate } \Leftrightarrow x(1,1) \leq \frac{1}{2(1-\pi)}. \tag{6}$$

If $x(1,1) = 1$, (6) is possible only if $\pi \geq \frac{1}{2}$; for the mixed strategy $x(1,1) = \frac{9}{16}$ (6) requires that $\pi \geq \frac{1}{9}$. Note that, C_1 communicates and plays $x(1,1) = 1$ when $\pi \in [0, 9/11]$ or $x(1,1) = 9/16$ when $\pi \in [0, 1/9]$

Proposition 4. *When $a = 30$, an equilibrium where only C_1 communicates exists under the following conditions on the continuation strategies and prior belief π.*

(i) $x(1,0) = x(0,0) = x(0,1) = 0$

(ii) $x(1,1) = \frac{9}{16}$ *if* $\pi = \frac{1}{9}$.

(iii) $x(1,1) = 1$ *if* $\pi \in \left[\frac{1}{2}, \frac{9}{11}\right]$

Consider now the case $a = 50$. From (3), when $a = 50$, it is possible to have $x(0,0) = 0$, or $x(0,0)$ positive if $\pi \geq \frac{5}{13}$, in which case either $x(0,0) = 1$ or $x(0,0) = \frac{5}{18}\frac{1+\pi}{\pi}$.

We consider the cases $x(0,0)=0$ and $x(0,0)>0$ in turn.

Case $x(0,0)=0$. In state $(0,1)$, the best response of C_∞ (also for C_1 in off equilibrium) is to play H when $10+40x(1,0)\geq 20+4x(1,0)$, or when $x(1,0)\geq \frac{5}{18}$.

$$x(0,1)>0 \Leftrightarrow x(1,0)\geq \frac{5}{18}. \tag{7}$$

Consider the continuation best responses of type C_1. In state $(1,1)$, she plays H as long as $10+40x(1,1)\geq 20+4x(1,1)-1$ or $x(1,1)\geq \frac{1}{4}$. Hence there are two possible (symmetric) continuation equilibria, $x(1,1)=\frac{1}{4}$ and $x(1,1)=1$.

In state $(1,0)$, playing H is a best response if (type D plays L while type C_∞ plays $x(0,1)$):

$$\frac{\pi}{1+\pi}(10+40x(0,1))+\frac{1}{1+\pi}10\geq \frac{\pi}{1+\pi}(20+4x(0,1))+\frac{1}{1+\pi}20-1,$$

that is

$$x(1,0)>0 \Leftrightarrow x(0,1)\geq \frac{1+\pi}{4\pi} \tag{8}$$

Note that (7) implies that $x(1,0)>0$ and therefore from (8) that $x(0,1)\geq \frac{1+\pi}{4\pi}$, which can happen only if π is not smaller than $\frac{1}{3}$. Note also that if $x(0,1)<1$, it must be that $x(1,0)=\frac{5}{18}$ and by (8) that $x(0,1)=\frac{1+\pi}{4\pi}$.

Lemma 1. *The unique continuation strategies when $a=50$ and $x(0,0)=0$ are the following.*

(i) $x(1,1)=1$ *or* $x(1,1)=\frac{1}{4}$.

(ii) $x(0,1)=x(1,0)=0$.

(iii) *If* $\pi\geq\frac{1}{3}$,

- $x(1,0)=x(0,1)=1$
- $x(0,1)=\frac{1}{4}\frac{1+\pi}{\pi}$ *and* $x(1,0)=\frac{5}{18}$.

We consider the incentive of type D_1 to deviate and communicate (which implies that type D_∞ cannot gain either). By not communicating, in state $(0,0)$ the player expects to face opponents who play L and in state $(0,1)$ an opponent of type C_1 who plays $x(1,0)$. Hence her expected payoff is

$$U(D_1) := \frac{1-\pi}{2}(20+4x(1,0)) + \frac{1+\pi}{2}20 = 20 + 2(1-\pi)x(1,0).$$

By communicating, type D_1 loses 1 in cost of lying in the continuation game, but the benefit is to "fool" C_1 and induce her to play H with probability $x(1,1)$, but also type C_∞ who will play H with probability $x(0,1)$. Type D_1 player's expected payoff from communicating is then $\frac{1-\pi}{2}(20+4x(1,1)) + \frac{\pi}{2}(20+4x(0,1)) + \frac{1}{2}20 - 1$, or

$$\hat{U}(D_1) := 19 + 2(1-\pi)x(1,1) + 2\pi x(0,1).$$

Hence, incentive compatibility $U(D_1) \geq \hat{U}(D_1)$ requires that

$$(1-\pi)(x(1,1) - x(1,0)) + \pi x(0,1) \leq \frac{1}{2} \tag{9}$$

We consider the three cases of Lemma 1.

- If $x(0,1) = x(1,0) = 0$ the condition reduces to $x(1,1) \leq \frac{1}{2(1-\pi)}$. If $x(1,1) = 1$, it must be that $\pi \geq \frac{1}{2}$. The mixed strategy $x(1,1) = \frac{1}{4}$ is possible for any value of π.

  C_1 must prefer to communicate. C_1's expected payoff from no communication is 20, as her opponent will play L in this case. C_1's expected payoff from communication and playing H if the state is (1,1) and L otherwise is $\frac{1-\pi}{2}(10+40x(1,1)) + \frac{\pi}{2}(20-1) + \frac{1}{2}(20-1) = \frac{29}{2} + \frac{9}{2}\pi + 20x(1,1)(1-\pi)$. When $x(1,1) = 1/4$ then C_1 will not communicate, but when $x(1,1) = 1$ then C_1 will communicate if $\pi \leq \frac{29}{31}$. So, in this case, the equilibrium can exist when $\pi \in \left[\frac{1}{2}, \frac{29}{31}\right]$ and $x(1,1) = 1$.

  Finally, C_∞'s must prefer not to communicate. C_∞'s expected payoff from no communication is the same as C_1 and equals to 20. C_∞'s expected payoff from communication is $\frac{1-\pi}{2}(10+40x(1,1)) + \frac{\pi}{2}10 + \frac{1}{2}(10) = 10 + 20x(1,1)(1-\pi)$. When $\pi \geq 1/2$ then C_∞ prefers no communication.

- $\pi \geq \frac{1}{3}$ and $x(1,0) = x(0,1) = 1$. Because C_1 and C_∞ use the same strategy after communication (they both weakly prefer to play H in all states) and also when they do not communicate, they must have the same expected payoffs. This means, both types have to be indifferent between communication and no communication, as they cannot have opposite incentives. Type C_k's expected payoff from communication and playing H in states $(1,1)$ and $(1,0)$ is $\frac{\pi}{2}50 + \frac{\pi}{2}(10 + 40x(1,1)) + \frac{1}{2}10 = 10 + 20\pi + 20x(1,1)(1-\pi)$. Type C_k's payoff from no communication and playing H in state $(0,1)$ and L in state $(0,0)$ is $\frac{1-\pi}{2}50 + \frac{1+\pi}{2}20 = 35 - 15\pi$. If $x(1,1) = 1$, then type C_k is indifferent between communication and no communication if $x(1,1) = 1$ and $\pi = 1/3$ or $x(1,1) = 1/4$ and $\pi = 2/3$. Note that, both cases are consistent with D_1's incentive for not communicating. If $x(1,1) = 1$, D_1 does not want communication if $\pi \leq 1/2$ and if $x(1,1) = 1/4$, D_1 does not want communication if $\pi \leq 5/7$.

- Similar to the previous case, $\pi \geq \frac{1}{3}$ and $x(0,1) = \frac{1+\pi}{4\pi}, x(1,0) = \frac{5}{18}$ requires that both C types have to be indifferent between communication and no communication. Because C_1 and C_∞ use the same strategy after communication (they both weakly prefer to play H in all states) and also when they do not communicate, they must have the same expected payoffs. Type C_k's expected payoff from communication and playing H in states[1] $(1,1)$ and $(1,0)$ is $\frac{1-\pi}{2}(10 + 40x(0,1)) + \frac{\pi}{2}(10 + 40x(1,1)) + \frac{1}{2}10 = 15 + 5\pi + 20x(1,1)(1-\pi)$. Type C_k's payoff from no communication and playing H in state $(0,1)$[2] and L in state $(0,0)$ is $\frac{1-\pi}{2}(10 + 40x(1,0)) + \frac{1+\pi}{2}20 = \frac{5}{9}(37 - \pi)$. If $x(1,1) = 1$, then type C_k is indifferent between communication and no communication if $x(1,1) = 1$ and $\pi = 1$ or $x(1,1) = 1/4$ and $\pi = 1$. Note that, **none** of these cases are consistent with D_1's incentive for not communicating. If $x(1,1) = 1$, D_1 does not want communication if $\pi \leq 1/2$ and if $x(1,1) = 1/4$, D_1 does not want communication if $\pi \leq 5/7$.

Case $x(0,0) > 0$ (and $\pi \geq 5/13$). Note that type C_k players' incentives in $(1,0)$,

[1] As C_1 is indifferent to play H or L in the state $(1,0)$, we can assume that she plays H.
[2] As C_1 is indifferent to play H or L in the state $(1,0)$, we can assume that she plays H.

$(0,1)$ and $(1,1)$ states are the same as in the case $x(0,0) = 0$ described above and we have to check the incentives of the players to communicate. We analyse the three different cases of Lemma 1.

- If $x(0,1) = x(1,0) = 0$. If D_1 communicates and plays L she gets $\frac{1-\pi}{2}(20 + 4x(1,1)) + \frac{\pi}{2}20 + \frac{1}{2}20 - 1 = 19 + 2x(1,1)(1-\pi)$. If D_1 does not communicate and plays L she gets $\frac{1-\pi}{2}(20) + \frac{\pi}{2}(20 + 4x(0,0)) + \frac{1}{2}20 = 20 + 2x(0,0)\pi$. Not communicating is best when $x(1,1) = 1/4$. When $x(1,1) = 1$, not communicating is best only if $x(0,0) \geq \frac{1-2\pi}{2\pi}$, which is possible because by assumption $\pi \geq \frac{5}{13}$ implies that $\frac{1-2\pi}{2\pi} \leq 1$. Note that both the pure strategy $x(0,0) = 1$ and the mixed strategy $x(0,0) = \frac{5(1+\pi)}{18\pi}$ satisfy this condition.

  C_1's expected payoff from communication and playing H in the state $(1,1)$ and L in the state $(1,0)$ is $\frac{1-\pi}{2}(10 + 40x(1,1)) + \frac{\pi}{2}19 + \frac{1}{2}19$. If C_1 does not communicate, her expected payoff from playing H in the state $(0,0)$[3] and L in the state $(0,1)$ is $\frac{1-\pi}{2}20 + \frac{\pi}{2}(10 + 40x(0,0)) + \frac{1}{2}10$. If $x(1,1) = \frac{1}{4}$, then C_1 communicates if $x(0,0) \leq \frac{9(1+\pi)}{40\pi}$, but as $x(0,0) \in \left\{1, \frac{5(1+\pi)}{18\pi}\right\}$ and $\pi \geq 5/13$ this condition never holds. If $x(1,1) = 1$, then C_1 communicates if $x(0,0) \leq \frac{39-21\pi}{40\pi}$. This condition is satisfied for $x(0,0) = 1$ when $\pi \in \left[\frac{5}{13}, \frac{39}{61}\right]$, and is satisfied for $x(0,0) = \frac{5(1+\pi)}{18\pi}$ when $\pi \in \left[\frac{5}{13}, \frac{251}{289}\right]$.

  Finally, we have to check C_∞'s does not have incentives to communicate. C_∞'s expected payoff from no communication is the same as C_1's payoff from no communication and is given as $\frac{1-\pi}{2}20 + \frac{\pi}{2}(10 + 40x(0,0)) + \frac{1}{2}10 = 15 - 5\pi + 20x(0,0)\pi$. C_∞'s expected payoff from communication is $\frac{1-\pi}{2}(10 + 40x(1,1)) + \frac{\pi}{2}10 + \frac{1}{2}10 = 10 + 20(1-\pi)x(1,1)$. Hence C_∞ does not communicate when

$$x(0,0) \geq \left(x(1,1) - \frac{1}{4}\right)\frac{1-\pi}{\pi}$$

  Clearly, the condition holds when $x(1,1) = \frac{1}{4}$ but as we have seen, the mixed strategy is incompatible with C_1 communicating.

[3] As $x(0,0) > 0$ then type C_1 at least weekly prefers H over L.

When $x(1,1)=1$, the condition is $x(0,0)\geq\frac{3}{4}\frac{1-\pi}{\pi}$. The pure strategy $x(0,0)=1$ satisfies the condition only if $\pi\geq\frac{3}{7}$. The mixed strategy $x(0,0)=\frac{5(1+\pi)}{18\pi}$ satisfies the condition if $\pi\geq\frac{17}{37}$.

Therefore, incentive compatibility is satisfied only if $x(1,1)=1$ and either

- $x(0,0)=1$ and $\pi\in\left[\frac{3}{7},\frac{39}{61}\right]$
- or $x(0,0)=\frac{5(1+\pi)}{18\pi}$ and $\pi\in\left[\frac{17}{37},\frac{251}{289}\right]$.

The last two cases correspond to $x(1,0)x(0,1)>0$, which is possible when $\pi\geq\frac{5}{13}$ by Lemma 1.

- Case $x(1,0)=x(0,1)=1$. Types C_1 plays H in all states after communication, hence has the same expected payoff as a type C_∞ who deviates and communicates. When not communicating, types C_1,C_∞ have the same best responses and expected payoffs. therefore, they must both be indifferent between communicating and not communicating (and using H as a continuation strategy)

$$\frac{1-\pi}{2}(10+40x(1,1))+\frac{\pi}{2}(50)+\frac{1}{2}10=\frac{1-\pi}{2}(50)+\frac{\pi}{2}(10+40x(0,0))+\frac{1}{2}10,$$

and

$$x(0,0)=x(1,1)\frac{1-\pi}{\pi}+\frac{2\pi-1}{\pi}.$$

If $x(1,1)=1$, $x(0,0)=1$ for all π. If $x(1,1)=\frac{1}{4}$, $x(0,0)=\frac{7\pi-3}{4\pi}$ which is consistent with $x(0,0)\in\left\{1,\frac{5(1+\pi)}{18\pi}\right\}$, either when $\pi=1$ (for the pure strategy) or $\pi=\frac{37}{53}$ (for the mixed strategy), in which case $x(0,0)=\frac{25}{37}$.

Hence we can have $x(1,0)=x(0,1)=x(1,1)=1$ in the following cases

- $\pi\geq\frac{5}{13}$, and $x(0,0)=x(1,1)=1$,
- $\pi=1$, $x(1,1)=\frac{1}{4}$, $x(0,0)=1$
- $\pi=\frac{37}{53}$, $x(1,1)=\frac{1}{4}$ $x(0,0)=\frac{25}{37}$.

Now we have to check D_1's incentive to communicate.

$$\frac{1-\pi}{2}(24)+\frac{\pi}{2}(20+4x(0,0))+\frac{1}{2}20 \geq \frac{1-\pi}{2}(20+4x(1,1))+\frac{\pi}{2}(24)+\frac{1}{2}20-1,$$

and

$$x(0,0) \geq \frac{1}{2\pi}(2x(1,1)(1-\pi)-3+4\pi).$$

This condition holds for all the three equilibrium candidates above.

- Case $x(0,1)=\frac{1+\pi}{4\pi}, x(1,0)=\frac{5}{18}$. Using the same argument as in the previous case, C_1, C_∞ must be indifferent between communication and no communication. The condition is now,

$$\frac{1-\pi}{2}(10+40x(1,1))+\frac{\pi}{2}(10+40x(0,1))+\frac{1}{2}10=\frac{1-\pi}{2}(10+40x(1,0))+\frac{\pi}{2}(10+40x(0,0))+\frac{1}{2}10$$

  or, using the values for $x(0,1), x(1,0)$

$$x(0,0)=x(1,1)\frac{1-\pi}{\pi}+\frac{19\pi-1}{36\pi}.$$

  If $x(1,1)=1$, $x(0,0)=\frac{35-17\pi}{36\pi}$, which is consistent with $x(0,0)=1$ only if $\pi=\frac{35}{53}$. It is consistent with $x(0,0)=\frac{5(1+\pi)}{18\pi}$ only if $\pi=\frac{25}{27}$.

  If $x(1,1)=\frac{1}{4}$, $x(0,0)=\frac{4+5\pi}{18\pi}$. This is consistent with $x(0,0)=1$ when $\pi=\frac{4}{13}$, a contradiction to the condition that $\pi \geq \frac{5}{13}$. It is clearly inconsistent with $x(0,0)=\frac{5(1+\pi)}{18\pi}$.

  Hence, we can have the mixed strategies in states $(1,0),(0,1)$ only in the following cases.

  - $\pi=\frac{35}{53}$, $x(1,1)=x(0,0)=1$
  - $\pi=\frac{25}{27}$, $x(1,1)=1$, $x(0,0)=\frac{26}{45}$.

Now we have to check D_1's incentive to communicate.

$$\frac{1-\pi}{2}(20+4x(1,0))+\frac{\pi}{2}(20+4x(0,0))+\frac{1}{2}20 \geq \frac{1-\pi}{2}(20+4x(1,1))+\frac{\pi}{2}(20+4x(0,1))+\frac{1}{2}20-1.$$

This condition is satisfied for the two equilibrium candidates above.

Proposition 5. *When $a = 50$, an equilibrium where only C_1 communicates exists under the following conditions on the continuation strategies and prior belief π.*

(a) If $\pi \in \left[\frac{1}{2}, \frac{29}{31}\right]$, $x(0,0) = x(0,1) = x(1,0) = 0$ *and* $x(1,1) = 1$.

(b) If $\pi \in \left[\frac{3}{7}, \frac{39}{61}\right]$, $x(0,1) = x(1,0) = 0$ *and* $x(1,1) = x(0,0) = 1$.

(c) If $\pi \in \left[\frac{17}{37}, \frac{251}{289}\right]$, $x(0,1) = x(1,0) = 0$, $x(0,0) = \frac{5(1+\pi)}{18\pi}$ *and* $x(1,1) = 1$.

(d) If $\pi \in \left[\frac{5}{13}, 1\right]$, $x(0,1) = x(1,0) = x(1,1) = x(0,0) = 1$.

(e) $\pi = \frac{35}{53}$, $x(1,0) = \frac{22}{35}$, $x(0,1) = \frac{5}{18}$, $x(1,1) = x(0,0) = 1$

(f) $\pi = \frac{25}{27}$, $x(1,0) = \frac{13}{25}$, $x(0,1) = \frac{5}{18}$, $x(1,1) = 1$, $x(0,0) = \frac{26}{45}$.

(g) $\pi = 1$, $x(0,1) = x(1,0) = x(0,0) = 1$, $x(1,1) = \frac{1}{4}$.

(h) $\pi = \frac{37}{53}$, $x(0,1) = x(1,0) = 1$, $x(1,1) = \frac{1}{4}$, $x(0,0) = \frac{25}{37}$.

The table below contrasts the different "generic" equilibrium outcomes (that is when the equilibrium behavior exists for a non-empty interval of π) in under the three regimes of communication under cheap talk.

3 Exogenous Cost of Communication (F)

We introduce now the exogenous cost (equal to 3) of sending messages. The exogenous cost is sunk once players choose their actions and therefore the set of continuation equilibria is the same as in regime CT.

There are two opposite forces generated by an exogenous cost of communication. First, as in Spence (1973) credible signalling is facilitated. Indeed, types D pay 3 to communicate but can gain at most 2 when facing an opponent of type C who plays H, while types C can gain more than 3 when a is large by communicating and inducing cooperation. But because communication is costly, types C players may be reluctant to engage in communication.

	Who Communicates		
	C_1, C_∞, D_1	C_1, C_∞	C_1
$a = 30$	n.a.	n.a.	$x(i,j) = 0, (i,j) \neq (1,1)$ (a) $x(1,1) = \frac{9}{16}$ if $\pi = \frac{1}{9}$ (b) $x(1,1) = 1$ if $\pi \in \left[\frac{1}{2}, \frac{9}{11}\right]$
$a = 50$	(a) $x(i,j) = 0, (i,j) \neq (1,1)$ $x(1,1) = 1$ if $\pi \in \left[0, \frac{10}{23}\right]$ (b) $x(0,0) = x(1,0) = 0$ $x(0,1) = x(1,1) = 1$ if $\pi \in \left[\frac{10}{23}, \frac{1}{2}\right]$	$x(0,0) = x(1,0) = 0$ $x(1,1) = x(0,1) = 1$ $\pi \in \left[\frac{1}{2}, \frac{3}{4}\right]$	(a) $x(i,j) = 0, (i,j) \neq (1,1)$ $x(1,1) = 1,\ \pi \in \left[\frac{1}{2}, \frac{29}{31}\right]$ (b) $x(1,0) = x(0,1) = 0$ $x(1,1) = x(0,0) = 1,\ \pi \in \left[\frac{3}{7}, \frac{39}{61}\right]$ (c) $x(0,0) = \frac{5(1+\pi)}{18\pi}$, $x(1,0) =$ $x(0,1) = 0, x(1,1) = 1,\ \pi \in \left[\frac{17}{37}, \frac{251}{289}\right]$ $(d)\pi \in \left[\frac{5}{13}, 1\right]$, $x(i,j) = 1$

Table 3: Cheap Talk Equilibria

Hence, there can be only two types of equilibria: when both C_1, C_∞ communicate and when only C_1 communicates. Compared to our analysis in section 2 we can ignore the incentive compatibility constraint for communication by D_1, that $x(1,1) \leq \frac{1}{2(1-\pi)}$. Because the cost of communication is sunk, the *equilibrium* continuation equilibria are identical to what we derived in the previous section. However the $off-equilibrium$ continuation best response of type C_∞ is more binding because of the lower payoff under communication due to the exogenous cost of communication.

3.1 Only Types C Communicate

Proposition 2 implies that this profile cannot be part of an equilibrium if $a = 30$. Continuation strategies are the same in this case as in the cheap talk, but we have to check the incentives of type C players to communicate. Analyses for $x(1,1) = 0$ and $0 < x(1,1) < 1$ hold also in this case, as C_∞ type does not prefer communication for any values of π. If $x(1,1)$ and $\pi \leq \frac{5}{8}$, then $x(0,1) = 0$. C_∞'s expected payoff from communication is 17, while her expected payoff from no communication is $20 + 2\pi$. So, C_∞ will not communicate. If $x(1,1)$ and $\pi \geq \frac{5}{8}$, then $x(0,1) = 1$. C_∞'s expected payoff from communication is 17, while her expected payoff from no communication is $15 + 10\pi$. So, C_∞ will not communicate as $\pi \geq \frac{5}{8}$. When $a = 30$, there is no equilibrium where only C types communicate.

We can therefore restrict attention to the case $a = 50$. Equilibrium strategies are $x(0,0) = x(1,0) = 0$ and $x(1,1) = 1$.[4] Because there is no longer a binding incentive compatibility condition for types D, the restriction on π that we had in Proposition 2 is no longer present, but the incentive for C_∞ to deviate and not communicate is more binding. Precisely, consider incentives for type C_∞ to deviate and not communicate. In equilibrium, type C_∞ has an expected payoff of $\frac{1}{2}50 + \frac{1}{2}10 - 3 = 27$. If the player does not communicate she can play either L or H in the state $(0,1)$, but will play L in the state $(0,0)$. C_∞'s payoff from playing L in the state $(0,1)$ is $20 + 2\pi$. C_∞'s payoff from playing H in the state $(0,1)$ is $15 + 20\pi$. So, if $\pi \leq \frac{5}{18}$ then C_∞ (also type C_1) will play L in $(0,1)$, if $\pi \geq \frac{5}{18}$ will play H in state $(0,1)$.

If $\pi \leq \frac{5}{18}$, then $20 + 2\pi < 27$ and C_∞ will communicate. If $\pi \geq \frac{5}{18}$, then $15 + 20\pi \leq 27$ if $\pi \leq \frac{3}{5}$.

Hence, if $\pi \leq \frac{5}{18}$ then $x(0,0) = x(1,0) = x(0,1) = 0$ and $x(1,1) = 1$ is an equilibrium. If $\pi \in \left[\frac{5}{18}, \frac{3}{5}\right]$ then $x(0,0) = x(1,0) = 0$ and $x(0,1) = x(1,1) = 1$ is an equilibrium

3.2 Only C_1 Communicates

Because types D are incentive compatible, condition (6) is no longer present. However, the incentive compatibility condition for type C_1 may be more difficult to satisfy than in the cheap talk situation.

[4] Analyses for $x(1,1) = 0$ and $0 < x(1,1) < 1$ hold also in this case, as C_∞ type does not prefer communication for any values of π.

Case $a = 30$

In equilibrium, a player of type C_1 is in state $(1,1)$ and if communication is beneficial, it must coordinate on H with high enough probability. As we have seen in the cheap talk section, there are two possible continuation equilibria in state $(1,1)$: either the pure strategy $x(1,1) = 1$ when $\pi \in \left[0, \frac{9}{11}\right]$ or the mixed strategy $x(1,1) = \frac{9}{16}$ if $\pi \in \left[0, \frac{1}{9}\right]$. In the other states $(i,j) \neq (1,1)$, $x(i,j) = 0$. It is sufficient to verify that C_1 wants to communicate and C_∞ prefers not to communicate.

No communication yields C_k an expected payoff of 20. By communicating, C_∞ playing H in all states has an expected payoff of $\frac{1-\pi}{2}(10 + 20x(1,1)) + \frac{1+\pi}{2}10 - 3$. Therefore

$$C_\infty \text{ does not communicate } \Leftrightarrow x(1,1) \leq \frac{13}{10(1-\pi)}.$$

Because the bound is greater than 1, C_∞ is incentive compatible.

By communicating, C_1 plays H only in state $(1,1)$ and gets 19, the payoff net of the cost of lying, in other states. Hence, her expected payoff of $\frac{1-\pi}{2}(10+20x(1,1)) + \frac{1+\pi}{2}19 - 3$ and she prefers to communicate when

$$C_1 \text{ communicates } \Leftrightarrow x(1,1) \geq \frac{17 - 9\pi}{20(1-\pi)}.$$

If $x(1,1) = 1$, the condition requires $\pi \leq \frac{3}{11}$. If $x(1,1) = \frac{9}{16}$, we need $\pi \leq -\frac{23}{9}$, which cannot be true.

Hence, when $a = 30$, an equilibrium in which only C_1 communicates exists only if $x(i,j) = 0$ for $(i,j) \neq (1,1)$ and $x(1,1) = 1$ while $\pi \leq \frac{3}{11}$.

Case $a = 50$

We know from the cheap-talk case that either $x(1,1) \in \{1, 1/4\}$ or $x(1,1) = 1$. Furthermore, $x(0,0) = 0$ or, if $\pi \geq \frac{5}{13}$, $x(0,0) \in \left\{0, \frac{5}{18}\frac{1+\pi}{\pi}\right\}$.

Because the cases $x(0,1) > 0$ and $x(1,0) > 0$ are non-generic (see our discussion around Proposition 5), we focus on situations where $x(0,1) = x(1,0) = 0$.

Case $x(0,0) = 0$. C_∞ prefers not to communicate when

$$\frac{1-\pi}{2}(10 + 40x(1,1) + \frac{1+\pi}{2}10 - 3 \leq 20$$

that is

$$x(1,1) \leq \frac{13}{20(1-\pi)}.$$

The condition for C_1 to be willing to communicate is

$$\frac{1-\pi}{2}(10 + 40x(1,1) + \frac{1+\pi}{2}19 - 3 \leq 20,$$

that is

$$x(1,1) \geq \frac{17-9\pi}{40(1-\pi)}.$$

The condition cannot be satisfied if $x(1,1) = \frac{1}{4}$, but it can be satisfied when $x(1,1) = 1$ whenever $\pi \in \left[\frac{7}{20}, \frac{23}{31}\right]$.

In this case, $x(0,1) = x(1,0) = 1$, $x(1,1) = 1$ and $\pi = 8/15$ and $x(0,1) = x(1,0) = 1$, $x(1,1) = 1/4$ and $\pi = 23/30$ are also equilibria. If $\pi = \frac{103}{130}$, then $x(0,1) = \frac{1+\pi}{4\pi}$, $x(1,0) = \frac{5}{18}$ and $x(1,1) = 1$ is also an equilibrium. In these cases, C_k types are indifferent between communication and no communication.

Case $x(0,0) > 0$. As discussed, this case can happen when $\pi \geq \frac{5}{13}$. By not communicating, a type C player has an expected payoff of

$$\frac{1-\pi}{2}20 + \frac{\pi}{2}(10 + 40x(0,0)) + \frac{1}{2}10 = 15 - 5\pi + 20\pi x(0,0).$$

By communicating, type C_∞ has expected payoff of

$$\frac{1-\pi}{2}(10 + 40x(1,1)) + \frac{1+\pi}{2}10 - 3 = 7 + 20(1-\pi)x(1,1)$$

while type C_1 has expected payoff of

$$\frac{1-\pi}{2}(10 + 40x(1,1)) + \frac{1+\pi}{2}19 - 3 = \frac{23}{2} + \frac{9}{2}\pi + 20(1-\pi)x(1,1)$$

Therefore, C_∞ does not communicate and C_1 communicate when

$$\frac{7-19\pi}{2}\underbrace{\leq}_{IC_1} 20\left[(1-\pi)x(1,1)-\pi x(0,0)\right]\underbrace{\leq}_{IC_\infty} 8-5\pi$$

The condition cannot hold if $x(1,1) = 1/4$. If $x(1,1) = 1$, the condition is

$$12-15\pi \leq 20\pi x(0,0) \leq \frac{33}{2}-\frac{21}{2}\pi,$$

hence $x(0,0) = 1$ requires $\pi \in \left[\frac{12}{35}, \frac{33}{61}\right]$. If $\pi \geq \frac{5}{13}$, then a mixed strategy $x(0,0) = \frac{5}{18}\frac{1+\pi}{\pi}$ can also be possible; it satisfies the IC condition when $\pi \in \left[\frac{58}{185}, \frac{197}{289}\right]$, but because we need $\pi \geq \frac{5}{13}$, the condition for having $x(0,0) = \frac{5}{18}\frac{1+\pi}{\pi}$ is $\pi \in \left[\frac{5}{13}, \frac{197}{289}\right]$.

There are also equilibria where C types are indifferent between communication and no communication. If $\pi = 4/5$, then $x(1,1) = 1/4$, $x(0,0) = 5/8$ and $x(0,1) = x(1,0) = 1$. If $\pi = 77/130$, then $x(1,1) = 1$, $x(0,0) = 64/77$ and $x(0,1) = x(1,0) = 1$. If $\pi = 148/265$, then $x(1,1) = 1$, $x(0,0) = 1$, $x(0,1) = 413/592$ and $x(1,0) = 5/18$. If $\pi = 98/135$, then $x(1,1) = 1$, $x(0,0) = 1165/1764$, $x(0,1) = x(1,0) = 233/392$.

Proposition 6. *When there is an exogenous cost of communicating of* 3*, the only possible equilibrium with communication is when type C_1 communicates and the other types do not communicate. The "generic" profiles involve $x(0,1) = x(1,0) = 0$.*

(i) If $a = 30$, $\pi \leq \frac{3}{11}$ and $x(1,1) = 1$ and $x(i,j) = 0$ for $(i,j) \neq (1,1)$.

(ii) If $a = 50$,

- *Either $x(0,0) = 0$, $x(1,1) = 1$ and $\pi \in \left[\frac{7}{20}, \frac{23}{31}\right]$.*
- *Or, $x(0,0) = 1$, $x(1,1) = 1$ and $\pi \in \left[\frac{5}{13}, \frac{33}{61}\right]$.*
- *Or, $x(0,0) = \frac{5}{18}\frac{1+\pi}{\pi}$, $x(1,1) = 1$ and $\pi \in \left[\frac{5}{13}, \frac{197}{289}\right]$.*

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4 Comparing Equilibrium Outcomes

We can summarize our results by indicating whether an equilibrium can exist and what are the conditions on $x(0,0)$ and π consistent with existence. We restrict attention to "generic" outcomes (those equilibrium which exist for a set of priors with positive measure). To simplify, we indicate the continuation strategies only if they assign a positive probability to playing H; otherwise the implicit assumption is that $x(i,j)=0$. In the case where players cannot communicate,x is the probability that types C play H. No confusion should arise.

The only situation where $x(0,1)$ is nonzero generically is for CT50 where the state $(0,1)$ is an out-of-equilibrium state. See Table 4.

	Who Communicates			
	$\emptyset$	C_1, C_∞, D_1	C_1, C_∞	C_1
NC30	0	n.a.	n.a	n.a.
CT30	0	✗	✗	(a)$x(1,1)=\frac{9}{16}$, $\pi=\frac{1}{9}$ (b) $x(1,1)=1$, $\pi\left[\frac{1}{2},\frac{9}{11}\right]$
F30	0	✗	✗	$x(1,1)=1$, $\pi\leq\frac{3}{11}$
NC50	$\left\{0,1,\frac{5}{9}\right\}$	n.a.	n.a.	n.a.
CT50	$\left\{0,1,\frac{5}{9}\right\}$	$(a)x(1,1)=1$, $\pi\in\left[0,\frac{10}{23}\right]$ (b) $x(1,1)=1, x(0,1)=1$ $\pi\in\left(\frac{10}{23},\frac{1}{2}\right]$	$x(1,1)=x(0,0)=1$ $\pi\in\left[\frac{1}{2},\frac{3}{4}\right]$	(a) $x(1,1)=1$, $\pi\in\left[\frac{1}{2},\frac{29}{31}\right]$ (b) $x(1,1)=x(0,0)=1$, $\pi\in\left[\frac{3}{7},\frac{39}{61}\right]$ (c) $x(1,1)=1$, $x(0,0)=\frac{5(1+\pi)}{18\pi}$ $\pi\in\left[\frac{17}{37},\frac{251}{289}\right]$
F50	$\left\{0,1,\frac{5}{9}\right\}$	✗	✗	(a) $x(1,1)=1$, $\pi\in\left[\frac{7}{20},\frac{23}{31}\right]$ (b) $x(1,1)=x(0,0)=1$, $\pi\in\left[\frac{4}{5},\frac{11}{13}\right]$

Table 4: Equilibria

Comparing CT30 to CT50

- There are more modes of equilibria with CT40, in particular when all but D_∞ communicate and when only types C communicate.
- For the equilibrium in which only type C_1 communicates, there is communication for a larger set of priors. (When $\pi \leq \frac{1}{9}$, the only equilibrium with $a = 30$ is when C_1 only communicates; with $a = 50$, it is possible to have an equilibrium with C_1, C_∞, D_1 communicate and types C cooperate.
- Not only is there more often communication but it is also possible that types who do not communicate play H.

See figure 1. Unless otherwise indicates, $x(1,1) = 1$ and $x(1,0) = x(0,0) = x(0,1) = 0$.

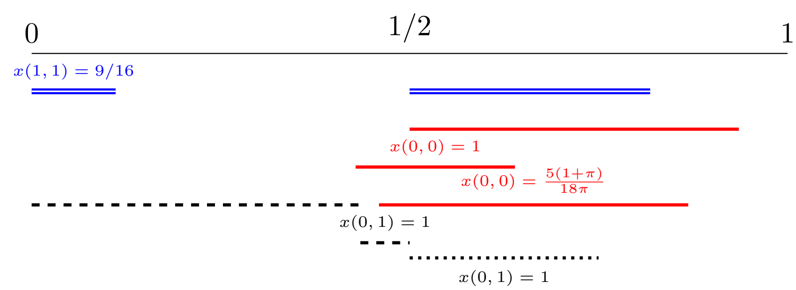

Figure 1: Compare CT30 (double,blue) - CT50 (red,black,orange)
red: only C_1, dashed: C_1, C_∞, D_1, dotted: C_1, C_∞

Comparing CT30 and F30

- In CT30 and F30, there is communication only by type C_1.
- If $\pi \leq \frac{3}{11}$, it is more likely that there is communication in F30 than C30 (communication happens there only if $\pi \leq \frac{1}{9} < \frac{3}{11}$.
- If $\pi \geq \frac{1}{2}$, there is no communication in F30 but communication in CT30.

See Figure 2. Unless otherwise indicates, $x(1,1) = 1$ and $x(1,0) = x(0,0) = x(0,1) = 0$.

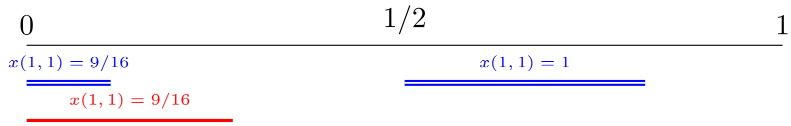

Figure 2: Compare CT30 (double, blue) - F30 (red)

Comparing CT50 and FT50

- There is communication equilibria for all $\pi \in \left[0, \frac{29}{31}\right]$ in CT50 but only in the set $\left[\frac{7}{20}, \frac{23}{31}\right] \cup \left[\frac{4}{5}, \frac{11}{20}\right]$ in F50.
- Moreover, in CT50 it is possible for different types to communicate while only type C_1 communicates in F50.
- In the region $\left[\frac{4}{5}, \frac{11}{20}\right]$, type C_∞ cooperates (in the equilibrium state $(0,0)$) with probability one in F50 but with probability less than one in CT50.

See figure **??**. Unless otherwise indicates, $x(1,1) = 1$ and $x(1,0) = x(0,0) = x(0,1) = 0$.

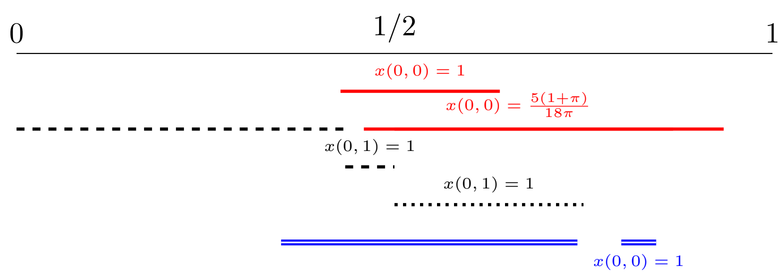

Figure 3: Compare CT50 (red,dashed,dotted) - F50 (double line, blue)
red: only C_1, dashed: C_1, C_∞, D_1, dotted: C_1, C_∞

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5 Theory vs Experimental Results

Some basic statistics:

- $\Pr(Comm|R)$ the probability of sending messages if regime $R \in \{CT, F\}$
- $\Pr(C|R)$, $\Pr(D|R)$, the probability that a C or D player communicates in regime R.
- $x(C|t,R), x(D|t,R)$, the probabilities that a C or D player plays H conditional on state t in regime R.

In the experiment, $\Pr(Comm|CT30) = 75\%$, $\Pr(Comm|CT50) = 75\%$, $\Pr(Comm|F30) = 17\%$, $\Pr(Comm|F50) = 25\%$. By Propositions **??** and 6, when $\pi = 1/2$ the theoretical probabilities are $\Pr(Comm|CT30) = 75\%$, $\Pr(Comm|CT50) = 75\%$, $\mathrm{PR}(Comm|F30) = 0$, $\Pr(Comm|F50) = 25\%$. Hence except for regime $F30$ there is a good fit – both quantitatively and 'qualitatively' – of the theory with the experimental values. (By Proposition 6, in regime $F30$ there is communication by C_1 if $\pi \leq 3/11$, in which case the probability is equal to $\frac{1-\pi}{2} = \frac{8}{22} = 36\%$ but the other probabilities will not match.)

TBC....

6 Cheap talk - Mixed communication strategies

6.1 Only C types communicates

In this section, we consider mixed communication strategies. We assume that C_∞ types communicate with a probability α, while C_1 always communicates. Posterior beliefs after communication are $p(C_1|1) = \frac{1-\pi}{1-\pi+\alpha\pi}$, $p(C_\infty|1) = \frac{\alpha\pi}{1-\pi+\alpha\pi}$ and $p(D_k|1) = 0$. Posterior beliefs after no communications are $p(C_1|0) = 0$, $p(C_\infty|0) = \frac{(1-\alpha)\pi}{1+(1-\alpha)\pi}$, $p(D_1|0) = \frac{1-\pi}{1+(1-\alpha)\pi}$ and $p(D_\infty|0) = \frac{\pi}{1+(1-\alpha)\pi}$.

D_1 plays L after all communication outcomes. D_∞ plays L if she does not communicate, but plays H after she communicates. C_∞ type plays H after she communicates. We will analyse the behavior of C_∞ type for the remaining states and behavior of C_1 type.

When the state is $(0,0)$, then C_k plays H if

$$\frac{(1-\alpha)\pi}{1+(1-\alpha)\pi}(10+(a-10)x(0,0))+\frac{1}{1+(1-\alpha)\pi}10 \geq \frac{(1-\alpha)\pi}{1+(1-\alpha)\pi}(20+4x(0,0))+\frac{1}{1+(1-\alpha)\pi}20,$$

or

$$x(0,0) \geq \frac{10(1+\pi(1-\alpha))}{\pi(1-\alpha)(a-14)}.$$

$x(0,0) > 0$ if $\pi \geq \frac{10}{(1-\alpha)(a-24)}$. When $a = 30$, then $x(0,0)$ cannot be positive, so $x(0,0) = 0$. When $a = 50$, then $x(0,0) > 0$ if $\pi \geq \frac{5}{13}\frac{1}{1-\alpha}$ and $\alpha \leq \frac{8}{13}$.

Case $a = 30$. In this case, $x(0,0) = 0$.

In the state $(0,1)$, C_k type plays H if

$$\frac{1-\pi}{1-\pi+\alpha\pi}(10+20x(1,0))+\frac{\alpha\pi}{1-\pi+\alpha\pi}30 \geq \frac{1-\pi}{1-\pi+\alpha\pi}(20+4x(1,0))+\frac{\alpha\pi}{1-\pi+\alpha\pi}24,$$

or

$$x(1,0) \geq \frac{10-10\pi-6\pi\alpha}{16(1-\pi)} = \frac{5-5\pi-3\pi\alpha}{8(1-\pi)}.$$

In the state $(1,1)$, C_1 type plays H if

$$\frac{1-\pi}{1-\pi+\alpha\pi}(10+20x(1,1))+\frac{\alpha\pi}{1-\pi+\alpha\pi}30 \geq \frac{1-\pi}{1-\pi+\alpha\pi}(20+4x(1,1))+\frac{\alpha\pi}{1-\pi+\alpha\pi}24-1,$$

or

$$x(1,1) \geq \frac{9-9\pi-7\pi\alpha}{16(1-\pi)}.$$

In the state $(1,0)$, C_1 type plays H if

$$\frac{(1-\alpha)\pi}{1+(1-\alpha)\pi}(10+20x(0,1))+\frac{1}{1+(1-\alpha)\pi}10 \geq \frac{(1-\alpha)\pi}{1+(1-\alpha)\pi}(20+4x(0,1))+\frac{1}{1+(1-\alpha)\pi}20-1,$$

or

$$x(0,1) \geq \frac{9(1+\pi(1-\alpha))}{16\pi(1-\alpha)}.$$

If $\alpha \leq \frac{2}{7}$ then $x(0,1)$ can be positive. If $\alpha \geq \frac{2}{7}$ then $x(0,1) = 0$ and $x(0,1) = 0$.

If $\alpha \geq \frac{2}{7}$, then $x(0,0) = x(0,1) = x(1,0) = 0$. C_∞ type has to be indifferent between communication and no communication.

$$\frac{\pi}{2}(10+20\alpha)+\frac{1-\pi}{2}(10+20x(1,1))+\frac{1}{2}10=20+2\pi\alpha,$$

or

$$x(1,1)=\frac{5+5\pi-8\pi\alpha}{10(1-\pi)}.$$

This can be true if $x(1,1)=1$, $\alpha=\frac{15\pi-5}{8\pi}$ and $\pi\leq\frac{5}{7}$ or $x(1,1)=\frac{9-9\pi-7\pi\alpha}{16(1-\pi)}$, $\alpha=\frac{90\pi-5}{29\pi}$ and $\pi\leq\frac{5}{61}$.

C_1's incentive to communicate...

D_1's incentive to not communicate...

References

Spence, M. (1973), 'Job market signaling', *The Quarterly Journal of Economics* **87**(3), 355–374.