TOPICS IN GEOMETRIC REPRESENTATION THEORY

LECTURES BY XIN JIN
NOTES BY PATRICK LEI

Abstract. The goal of this course is to introduce many geometric objects and tools that are in the heart of GRT. Below is the tentative plan of the course:

- Part I. I will talk about the geometry of complex semisimple Lie algebras and groups. Specific topics include
  (1) The geometry of flag variety, nilpotent cone, Grothendieck-Springer resolution, Steinberg variety and an introduction to Springer theory
  (2) The wonderful compactification of semisimple groups and more general toroidal compactifications of reductive groups.

  We will follow some standard text/notes (and some additional references):
  - Chriss-Ginzburg, *Representation theory and complex geometry*, mostly chapter 3 and part of chapter 2
  - Evens-Jones, *On the wonderful compactification*
- Part 2. We will focus on a class of completely integrable systems, called the universal centralizers J_G (or under other names: Toda systems, regular centralizer group schemes, or bi-Whittaker reductions), associated with a semisimple (or reductive) group G. This class of completely integrable systems lies at the crossroads of many different subjects in differential geometry, GRT (including geometric Langlands), (homological and enumerative) mirror symmetry, and mathematical physics (e.g. the class of Coulomb branches). Selected topics include:
  (1) A partial log-compactification of J_G, by Balibanu (as an application of (2) in part 1)
  (2) Symplectic geometry of J_G and sketch of its homological mirror symmetry (based on some of my recent work)
  (3) Depending on interest, I will sketch the proof of a hyperKahler metric on it constructed from gauge theory (following Donaldson, Kronheimer and Bielawski).
- Prerequisite: Basic knowledge of differential topology and complex geometry is assumed. Familiarity with semisimple Lie algebras and algebraic groups, as well as their representations, will be helpful but is not strictly required. We will follow the approach of Chriss–Ginzburg in recalling and supplementing the necessary background as needed.

Contents

Date: Spring 2026.