



## RESUMEN

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# OPERATION OF ELECTRIC POWER SYSTEMS

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### Autor

LEÓN PITA Pedro

Este documento contiene un resumen y problemas de apoyo para la prueba del 31 oct 2017. FELIZ HALLOWEEN. No se recomienda usarlo como único soporte para el estudio. Cualquier errata, comentario o añadido, enviar el PDF comentado a [pleonpita@gmail.com](mailto:pleonpita@gmail.com), gracias.

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## Parte I

# Power Systems Basics

## 1. Introduction and basic concepts

### 1.1. Correlation between GDP and electricity consumption

GDP (*PIB*) and electricity consumption are correlated but this correlation has started to disappear as our society creates wealth with other meanings, not energy.

### 1.2. Electricity, a complex commodity

- It cannot be stored.
- Cannot control the flow.
- Ensure supply/demand balance.

### 1.3. Need to transport electricity

- Demand: located at urban and industrial areas.
- Generation: conditioned by the availability of the resources.

### 1.4. General figures

$$1Mtoe = 11,6TWh$$

#### European countries

- Installed capacity  $\in [81, 190] \text{ GW}$
- Annual production  $\in [250, 650] \text{ TWh}$

#### The economic weight of each activity

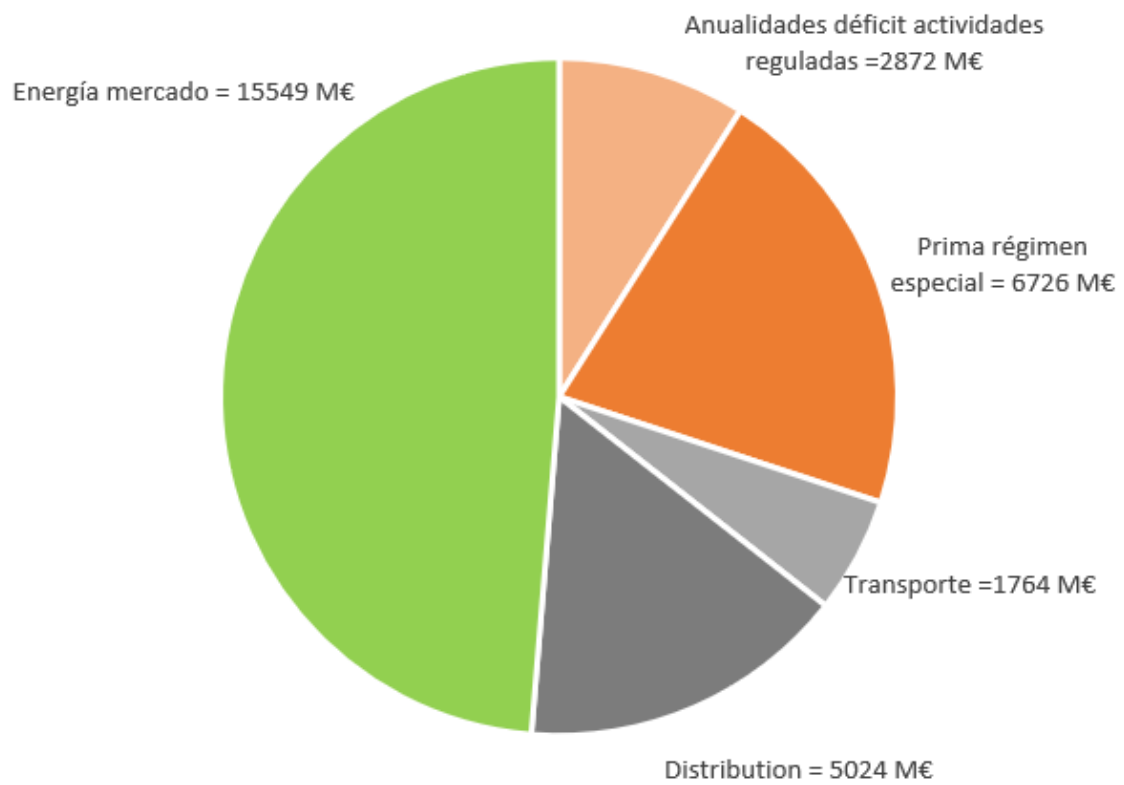


Figura 1: Omie regulation (green), generation (orange) and transport and distribution (grey)

#### Spain relevant data

- *Precio medio* - 45 €/MWh
- *Consumo anual en España* - 240 TWh
- *Consumo anual mundial* - 22.000 TWh

## 2. Technical and organizational historic evolution

### 2.1. Relevant dates

- In the **70's**, oil crisis forced in Spain the creation of Endesa to build an energy network to avoid relying only on oil to produce energy.
- **1982**, first electricity market in Chili.
- **1998**, creation of Spanish electricity market.

### 2.2. Omie

*Operador de Mercado Ibérico de Electricidad* is the entity in charge of making agreements between sales and generators. Final consumers will buy electricity from sales/retailers.

### 2.3. Vertical integration vs. deregulation

- Vertically integrated company: company that generates, transports, distributes and commercializes electricity. Strong economies of scale lead to monopolies.
- Deregulation: opening the market to new companies.

The change from regulated monopolies to competition was motivated by a reduction of economies of scale due to new generation technologies. First, generation and retail were opened to competition while transmission and distribution remained regulated.

### 3. Basic characteristics and technical aspects

#### 3.1. Consumption

There are issues related to consumption: the aggregated curves of offer and demand, factors affecting the demand, the load-duration curve and the quality of service.

##### Aggregated curves of offer and demand

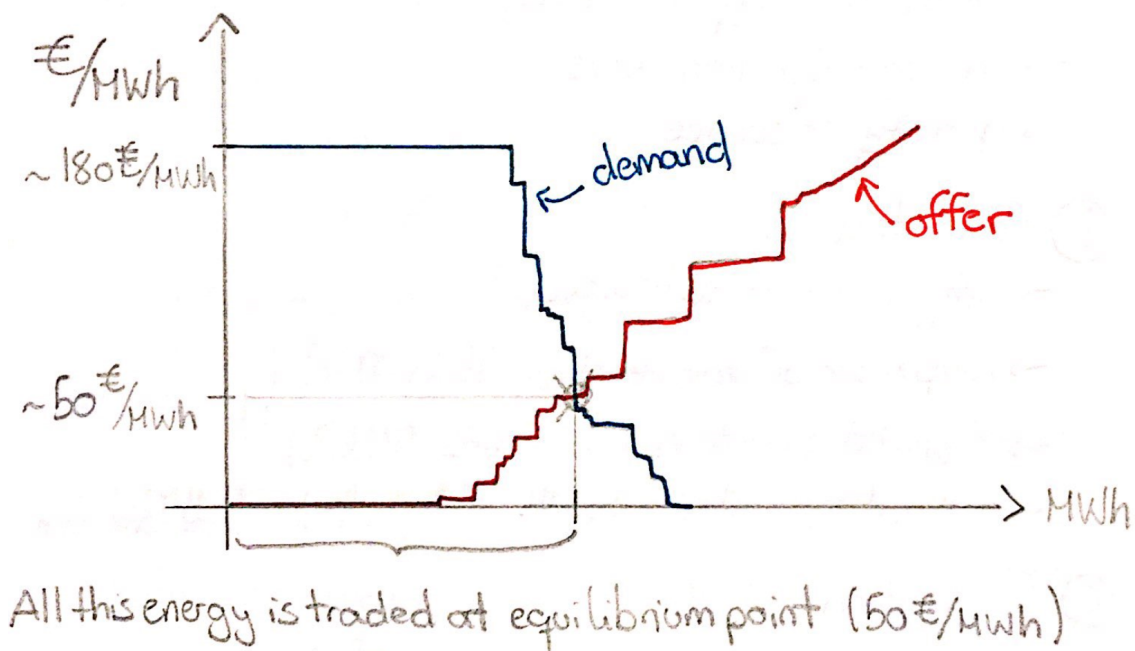


Figura 2: Example of an aggregated curve offer-demand

##### Factors affecting the demand

- Working patterns (*efecto calendario*, economic activity and temperature among others).
- Demand is almost not affected by electricity prices.

**The load-duration curve** (*curva agregada de demanda*) for a given period of time the temporary demand is re-ordered in a curve from the highest to the lowest demand in decreasing order. The area below this curve is the total year consumption.

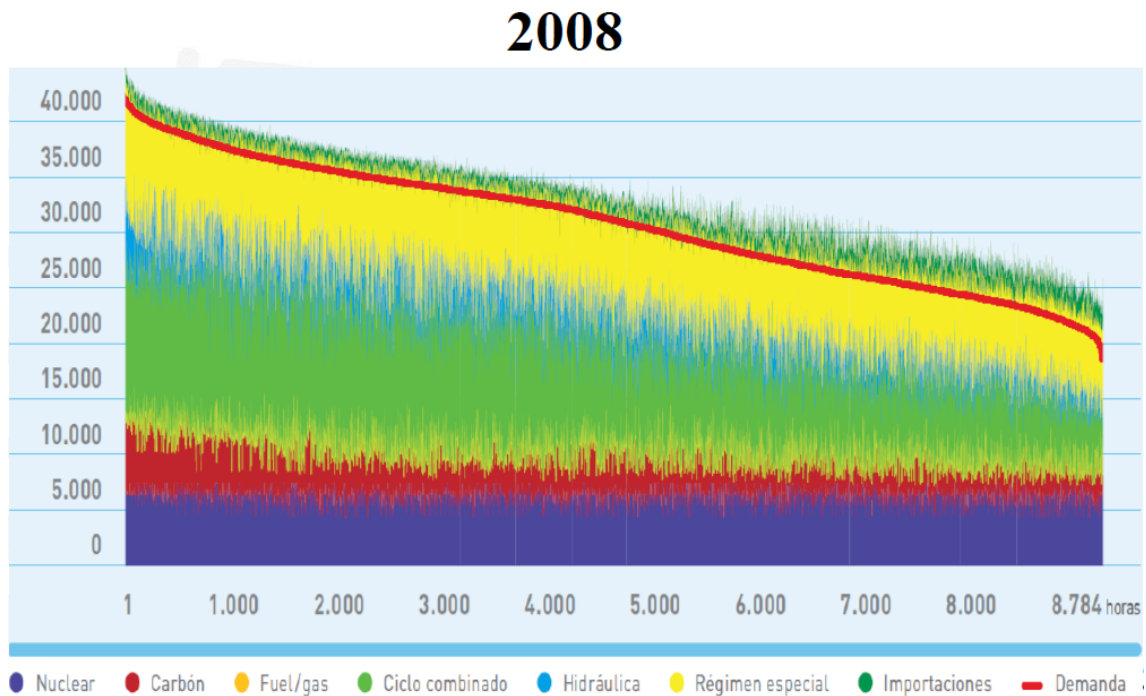


Figura 3: Distribution of technologies in a load-duration curve

#### Quality of service:

- **Commercial quality:** installation time.
- **Supply quality:** continuity of supply, voltage quality (steady frequency).

### 3.2. Generation

The need for different generation technologies come from a changing demand so the generation must be **flexible**. For this, long-run technologies are installed to assure a constant production and flexible technologies are turned on and off to answer the real-time demand. There are also political and environmental considerations such as using carbon free technologies and the adverse social reaction to nuclear power plants after Fukushima disaster.

Comparison of non-renewable energies:

- Efficiency: [49,58] % for CCGT <sup>1</sup> and [37,44] % for coal.
- Investment cost: 1000€/kW CCGT, 2 times for coal and 4 times for nuclear.

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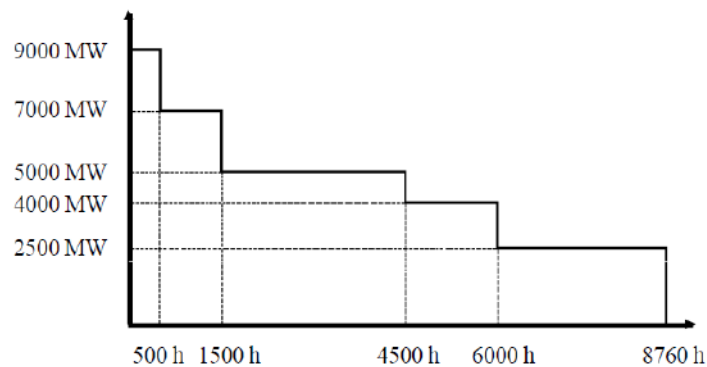
<sup>1</sup>Combined Cycle Gas Turbine



- Construction period (years): 2-3 CCGT, 3-5 coal and 5 nuclear.
- $CO_2$  emissions: 0,45 kg/kWh for CCGT and 1 kg/kWh for coal.

### Problem 1: Optimal Thermal Mix

Choose the total amount of power produced by each technology (Nuclear, Carbon, Fuel) given their fixed and variable costs and the total demand in the next load-duration curve.



Tecnología	N	C	F
Coste Fijo	210 €/kW	114 €/kW	72 €/kW
Coste Variable	6 €/MWh	18 €/MWh	30 €/MWh

Figura 4: Optimal thermal mix problem formulation

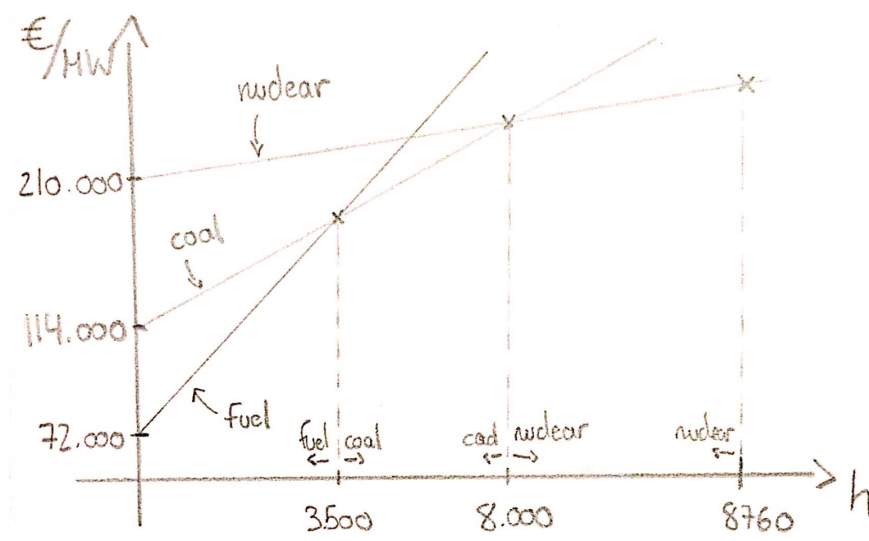


Figura 5: Optimal thermal mix problem solution

Threshold hours:

$$72 + 0,03 \cdot t = 114 + 0,018 \cdot t; t_{fuel-coal} = 3500$$

$$114 + 0,018 \cdot t = 210 + 0,006 \cdot t; t_{coal-nuclear} = 8000$$

Once the threshold hours are known the amount of power produced by each technology is calculated using the load-duration curve, to supply all the demand.

$$TotalCost = InstalledCapacity * FixedCost + SuppliedDemand * VariableCost * Time$$

**Fuel:** supplies the demand of the peak hours up to 1500h as the next demand block total time is 4500h > 3500h (threshold hour).

$$FuelTotalCost = 4000 \cdot 72 \cdot 10^3 + (4000 \cdot 30 \cdot 500 + 2000 \cdot 30 \cdot 1000) = 408m.eur$$

**Coal:** supplies the demand up to 6000h as the next demand block total time is 8760 > 8000h (threshold hour).

$$CoalTotalCost = 2500 \cdot 114 \cdot 10^3 + (2500 \cdot 18 \cdot 4500 + 1500 \cdot 18 \cdot 1500) = 528m.eur$$

**Nuclear:** supplies the demand up to 8760h.

$$NuclearTotalCost = 2500 \cdot 210 \cdot 10^3 + (2500 \cdot 6 \cdot 8760) = 656,4m.eur$$

$$TotalCost = 1592,4m.eur$$

## Problem 2: bids in an open market

In this case, the investment cost, variable cost and total production are available.

There are other 11 power plant in the bidding for a total demand of 1.800 MW.

Given plant:

- Annualized fixed cost = 70k€/MW
- Variable cost = 55 €/MWh
- Total production = 200 MW

In class, our team included the investment cost to compute the selling price:

$$SellingPrice = VariableCost + \frac{fixedCost}{8760h} = 55 + 8 = 63eur/MWh$$

Once the bid was close, the equilibrium price was revealed at 50€/MWh. Our generation unit was out of the bid and wouldn't produce any energy during the year, so there was no return of investment. Two conclusions:

- Investment cost shouldn't be internalized in the selling price because the opportunity of being cleared (produce) might be lost (selling price < equilibrium) and in that case the investment won't be paid either ways.
- Selling price must never be lower than the variable cost because in the case of being cleared (produce), the generation unit would lose money for each kW produced.

**Problem 3: Allocation of hydro energy** (or 0 variable cost energy):

For a hydro plant, rain water is free, and their operational cost is low. We consider a maximum power output of 500 MW and an annual total production of 1.500 GWh.

- First approach: one allocation would be to supply a constant power during the 8.760h of the year. But then, peak hours would be supplied by the same amount of expensive source (last served production).
- Second approach: a more efficient way of allocating the hydro resources would be at peak hours, reducing the total cost and hydro plant will sell their energy at its highest price.

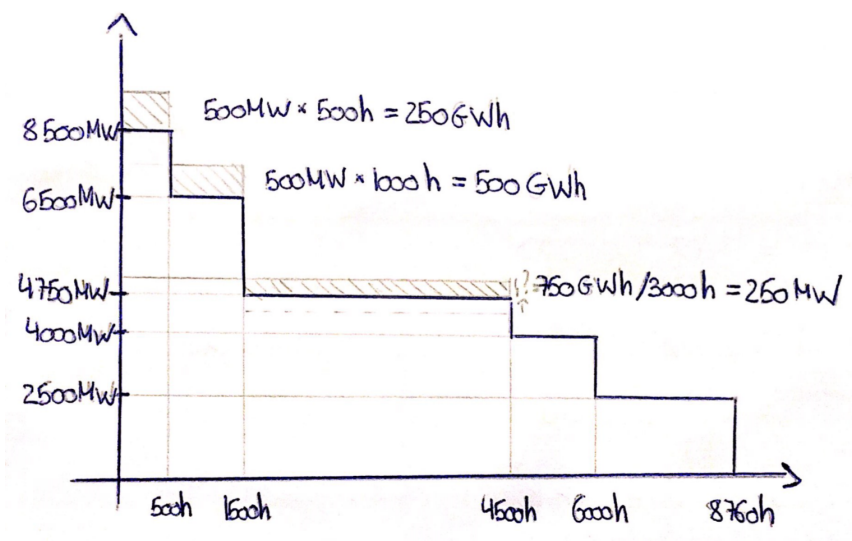


Figura 6: Optimal Thermal Mix Hydro new load-duration curve

Threshold hours remain the same as fixed and variable costs haven't changed: 3500, 8000 and 8760. Once the threshold hours are known, the amount of power installed for each technology is calculated using the new load-duration curve, to supply all the demand non-supplied by the hydro resources.

$$TotalCost = InstalledCapacity * FixedCost + SuppliedDemand * VariableCost * Time$$

**Fuel:** supplies the demand of the peak hours up to 1500h as the next demand block total time is 4500h > 3500h (threshold hour).

$$FuelTotalCost = 3750 \cdot 72 \cdot 10^3 + (3750 \cdot 30 \cdot 500 + 1750 \cdot 30 \cdot 1000) = 378,75m.eur$$

**Coal:** supplies the demand up to 6000h as the next demand block total time is 8760 > 8000h (threshold hour).

$$CoalTotalCost = 2250 \cdot 114 \cdot 10^3 + (2250 \cdot 18 \cdot 4500 + 1500 \cdot 18 \cdot 1500) = 479,25m.eur$$

**Nuclear:** supplies the demand up to 8760h.

$$NuclearTotalCost = 2500 \cdot 210 \cdot 10^3 + (2500 \cdot 6 \cdot 8760) = 656,4m.eur$$

$$TotalCost = 1514,4m.eur$$

$$TotalSavings = 1592,4 - 1514,4 = 78m.eur$$

**Mathematical formulation:** optimal mix problem

- Variables:
  - For each technology, the amount of energy supplied for each period,  $P_i$ .
  - Total amount of power installed (if investment is considered).
  - Amount of non-supplied demand for each period (if non-served power cost is considered).
- Constraints:
  - Minimize total cost,  $F_i(P)$ .
  - Supply total demand,  $D$ .
  - For each period and each technology, energy supplied must be lower or equal to the capacity installed  $P_i \leq \bar{P}_i$ .

### 3.3. Distribution

Transmission lines are usually **meshed** because:

- Increases reliability and reduces local reserve margins.
- Allows a more efficient short-term dispatch.
- Increases the short-circuit power (drawback).

Purposes of **substations**:

- Line interconnection buses.
- Transformation nodes that feed the distribution grids that reach consumers.
- Centres where system measurement, protection, interruption and dispatch equipment is sited.

### 3.4. Control and protection

- Primary control, **Frequency Containment Reserve**: local and automatic (done in seconds). Restores the generation demand balance but at a frequency different to the nominal value.
- Secondary control, **Frequency Restoration Reserves**: automatic or manual (done in minutes). Restores frequency, cross-border exchanges, and primary reserve.
- Tertiary control, **Replacement Reserves**: manual. Replaces secondary reserves.

## 4. Two management paradigms

As explained in subsection 2.3 there are two philosophies for operation of power systems: centralized (vertical integrated monopolies) and liberalized context (deregulation).

### 4.1. Centralized context

Management of power systems: one entity that controls the generation and retail. The expansion and operation are centralized in this entity that guarantee to recover investment and variable costs.

- Long term expansion: network and generation expansion under general environmental and strategic policies.
- Medium and short-term operation involves: hydro resources allocation and unit commitment (start up and shut down) among others.

### 4.2. Liberalized context

Management of power systems: market open to multiple entities for generation and retail. Competition encourages innovation, improving technologies and services. Decisions are freely taken by individuals, but investment recovery is not guaranteed.

- Long term expansion: generation seeks to maximize profit. Agent in charge of the network expansion have imperfect information about generation expansion.
- Medium and short-term operation involves: bid in the different markets (based on competitors' strategies).

## Parte II

# Economic dispatch (ED)

## 5. Characteristics power generation units

### 5.1. Input-output curves of thermal units

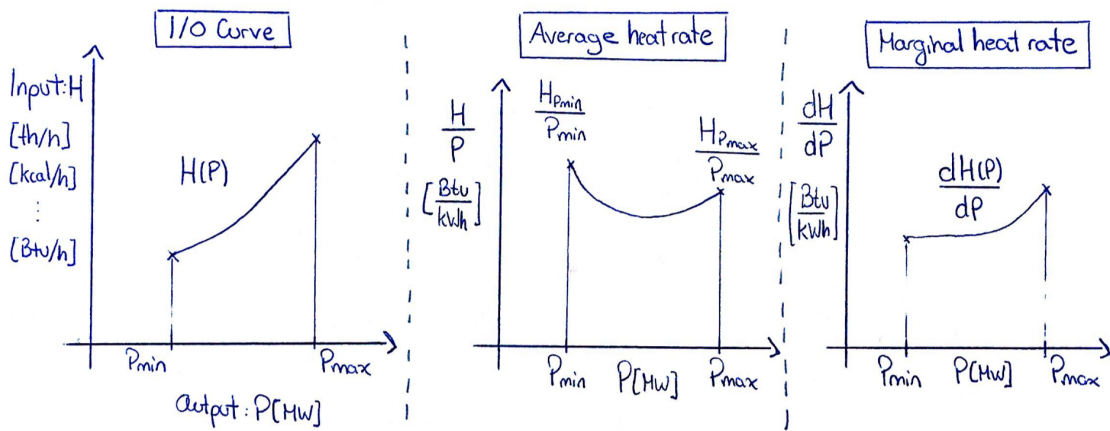


Figura 7: Input-output curve, average heat rate and incremental (marginal) heat rate (from left to right)

Input (heat rate) units are:

$$[th] = Thermie$$

$$[Btu] = BritishThermalUnit$$

$$1th = 1000kcal$$

$$1th = 3968,32Btu$$

$$1Btu \approx 0,252kcal$$

$$1kWh = 3412,14Btu = 859,85kcal$$

## 5.2. Input-out in term of cost

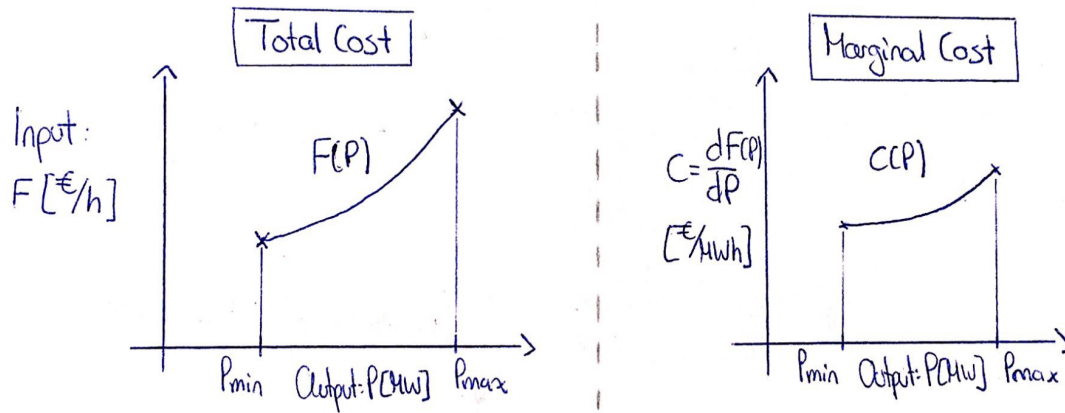


Figura 8: Total cost and marginal cost (from left to right)

The marginal cost curve will be the one used in the optimal dispatch problems, choosing the minimum marginal cost.

- If total cost ( $F$ ) is linear and pass by the origin  $(0,0)$  marginal cost = average cost.
- If  $\frac{d^2 F}{dP^2} > 0$  (la pendiente crece) then average cost < marginal cost.
- If  $\frac{d^2 F}{dP^2} < 0$  (la pendiente decrece) then marginal cost < average cost.

## 5.3. Hydro units characteristics

$$P(W) = \nu \cdot \rho \cdot g \cdot H \cdot q$$

Where  $\nu$  is the efficiency,  $(\rho \cdot g \cdot H)$  potential energy and  $q$  flow.

For hydroplants there is a minimum flow before which the turbine is not stable, so, there is a minimum generation ( $P_{min}$ ).



## 6. ED of thermal units

### 6.1. ED problem formulation

In an Economic Dispatch problem, the commitment of generation units has already been established. Therefore, investment cost is already done and doesn't depend on the amount of power produced. The optimum solution will be the one matching all generation units marginal cost.

However, generation units usually have a minimum ( $P_{min}$ ) and maximum ( $P_{max}$ ) load. In the case a generation unit has a very cheap cost then it will produce its total installed capacity and the other units will produce as much to match each other's marginal cost. It is proved then that sometimes matching all generation units marginal cost is sometimes impossible.

The same may happen to a very expensive generation unit that will always produce at its minimum load ( $P_{min}$ ). So, its marginal cost won't be equal to the others. Its production will only increase in peak hours.

The **System Marginal Cost** (SMC) is the cost of producing an extra W/kW/MW. The SMC is constrained by the generation limits and even if there is a generation unit with lower marginal cost, if this one is producing its maximum load, then it can't produce the extra MW.

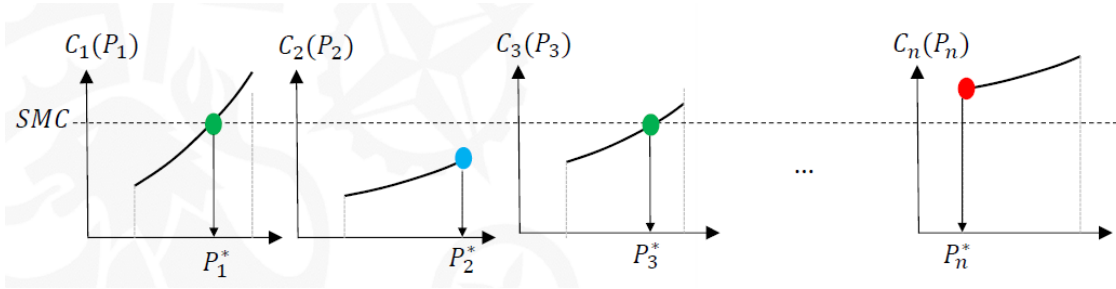


Figura 9: System Marginal Cost of 4 generation units

The generation unit in blue is the cheap one and it's producing its maximum load. The generation unit in red is the expensive one and it's producing its minimum load. Generation units in green are producing at the SMC and the extra MW will be distributed between both, reaching a new SMC.

## 6.2. Lambda iteration method

- Start with  $\lambda = \lambda_0$  (and set the value for  $\lambda_1$ )
- Calculate  $P_g \forall g$
- Calculate the error  $\epsilon = D - \sum_g P_g$
- Update  $\lambda$  and repeat 1-4 until  $|\epsilon(\lambda)| \leq \text{tolerance}$

## 6.3. Lagrangian and Karush-Kuhn-Tucker method

Example of 2 units:

Optimisation function:

$$\min F_1(P_1) + F_2(P_2)$$

Constraints:

s.t.

$$P_1 + P_2 = D : \lambda$$

$$P_1 - \bar{P}_1 \leq 0 : \bar{\mu}_1$$

$$\underline{P}_1 - P_1 \leq 0 : \underline{\mu}_1$$

$$P_2 - \bar{P}_2 \leq 0 : \bar{\mu}_2$$

$$\underline{P}_2 - P_2 \leq 0 : \underline{\mu}_2$$

Lagrange function:

$$L(P_1, P_2, \lambda, \bar{\mu}_1, \underline{\mu}_1, \bar{\mu}_2, \underline{\mu}_2) = F_1(P_1) + F_2(P_2)$$

$$+ \lambda \cdot (D - P_1 - P_2)$$

$$+ \bar{\mu}_1 \cdot (P_1 - \bar{P}_1) + \underline{\mu}_1 \cdot (\underline{P}_1 - P_1)$$

$$+ \bar{\mu}_2 \cdot (P_2 - \bar{P}_2) + \underline{\mu}_2 \cdot (\underline{P}_2 - P_2)$$

Derivatives of the Lagrange function:

$$\frac{\delta L}{\delta P_1} = 0; C_1(P_1) - \lambda + \bar{\mu}_1 - \underline{\mu}_1 = 0$$

$$\frac{\delta L}{\delta P_2} = 0; C_2(P_2) - \lambda + \bar{\mu}_2 - \underline{\mu}_2 = 0$$

$$\frac{\delta L}{\delta \lambda} = 0; D - P_1 - P_2 = 0$$

To solve the problem, an **initial hypothesis** is needed. For example: both generation units are marginal.

$$\bar{\mu}_1 = \underline{\mu}_1 = \bar{\mu}_2 = \underline{\mu}_2 = 0$$

$$C_1(P_1) = \lambda$$

$$C_2(P_2) = \lambda$$

$$P_1 + P_2 = D$$

Once  $P_1$  and  $P_2$  are calculated, the hypothesis needs to be checked. Imagine that, for example,  $P_1 > \bar{P}_1$  then the **new hypothesis** would be that unit 1 is at its maximum power:

$$\bar{\mu}_1 > 0; \underline{\mu}_1 = \bar{\mu}_2 = \underline{\mu}_2 = 0$$

$$C_1(P_1) = \lambda - \bar{\mu}_1$$

$$C_2(P_2) = \lambda$$

$$P_1 + P_2 = D$$

If unit 1 is producing at its maximum capacity then  $P_1$  is already known and equal to  $\bar{P}_1$ . Mathematically speaking, the unknown variable  $P_1$  is replaced by the new unknown  $\bar{\mu}_1$ . But still, there are 3 unknowns and 3 equations.

On the other hand, in the case  $P_1 < \underline{P}_1$  then the **new hypothesis** would be that unit 1 is at its minimum power:

$$\underline{\mu}_1 > 0; \underline{\mu}_1 = \bar{\mu}_2 = \underline{\mu}_2 = 0$$

$$C_1(P_1) = \lambda - \underline{\mu}_1$$

$$C_2(P_2) = \lambda$$

$$P_1 + P_2 = D$$

If unit 1 is producing at its minimum capacity then  $P_1$  is already known and equal to  $\underline{P}_1$ . Mathematically speaking, the unknown variable  $P_1$  is replaced by the new unknown  $\underline{\mu}_1$ . But still, there are 3 unknowns and 3 equations. Again, the hypothesis needs to be checked  $\underline{\mu}_1 > 0$  and  $\underline{P}_2 < P_2 < \bar{P}_2$ .