Stability analysis of discrete time control structures

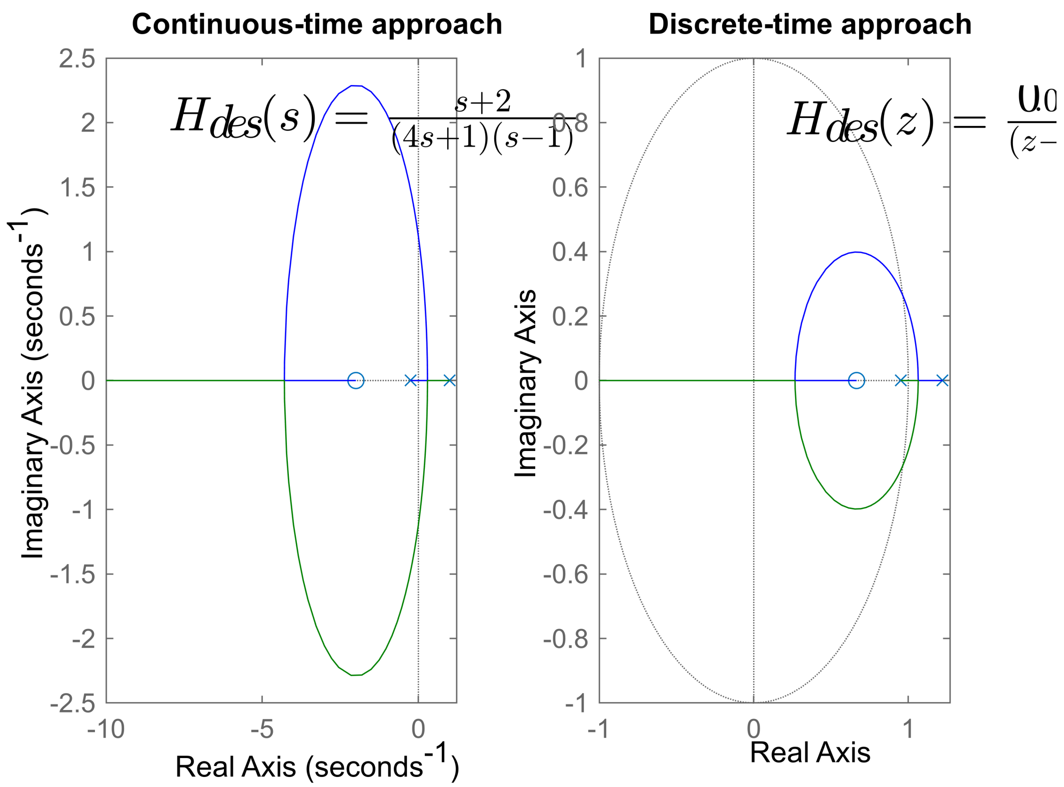
Theoretical aspects

- Stability analysis (Jury's stability criterion)
- Root Locus for discrete-time systems
- Grid of constant damping factors and natural frequencies
- Matlab functions: *c2d, rlocus, zgrid, feedback, pzmap*

Negative feedback structures with data sampling

Stability analysis depending on the proportional gain, k

```
%% S1: - Stability analysis depending on k
clear;% erase all the existing workspace variables
num=[1 2];den=[4 -3 -1];% the nominator and denominator of the process
Hp=tf(num,den);% the process transfer function
T=0.2;% the sampling period
Hdes=c2d(Hp,T,'zoh');% the open loop discrete time transfer function
subplot(121);rlocus(Hp);title('Continuous-time approach');
text(-8,2,'$H_{des}(s)=\frac{s+2}{(4s+1)(s-1)}$','Interpreter','Latex','FontSize',18)
subplot(122);rlocus(Hdes);title('Discrete-time approach')

text(0.2,0.8,'$H_{des}(z)=\frac{0.064563 (z-0.6655)}{(z-1.221) (z-0.9512)}$','Interpreter','Lat
```



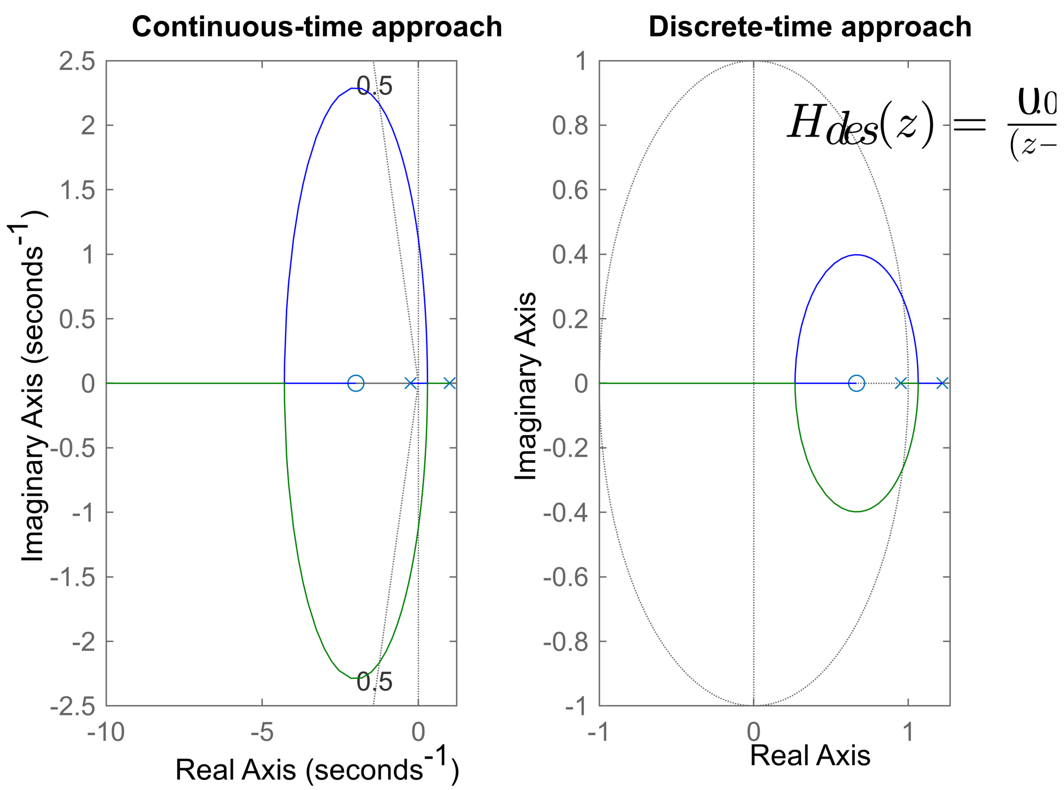
Dynamic behaviors depending on the proportional gain k

Evaluating the pole location depending on the gain k, varied inside the stable interval $(3.76, 40.3)$, the next types can be obtained:

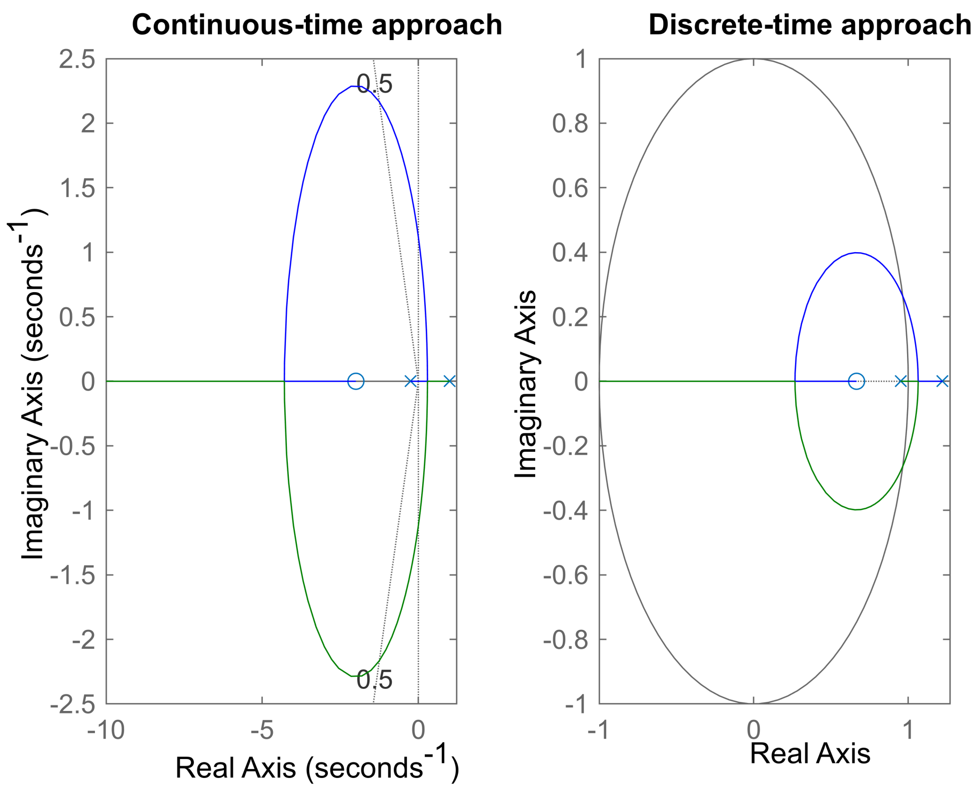
- $k = 3.76, \zeta = 0$, UNDAMPED system, both poles are located exactly on the unity circle
- $k \in (3.76, 25.4), \zeta \in (0, 1)$, UNDERDAMPED system, pair of complex conjugated poles, inside the unity circle
- $k = 25.4. \zeta = 1$, CRITICALLY DAMPED system, real pole with multiplicity order 2
- $k \in (25.4, 40.3), \zeta > 1$, OVERDAMPED system, two real and distinctive poles, inside the unity circle
- $k = 40.3, \zeta = 0$, UNDAMPED system, one pole located exactly on the unity circle (in -1)

Damping ratio (ζ) constant locus

```
%% S2: - damping ratio constant locus ζ = 0.5 (σ ≈ 16% )
subplot(121);rlocus(Hp);title('Continuous-time approach');sgrid(0.5,[]);
```




```
subplot(122);rlocus(Hdes);title('Discrete-time approach');zgrid(0.5,[])
```



Problems

3.1 For the next time control structure with the sampling period T=0.05sec

```
clear variables;

k=1;
s=tf('s');
T = 0.05;
Hc = k/s;
Hp = 2400/(s+20)/(s+40);
Hd = Hc * Hp;
```

1. Discretize the controller ($H_c(s) = \frac{k}{s}$) using tustin (bilinear) transformation

```
Hc = c2d(Hc,T,'tustin')
```

```
Hc =
 
  0.025 z + 0.025
  ---------------
       z - 1
 
Sample time: 0.05 seconds
Discrete-time transfer function.
```

2. Discretize the process ($H_p(s) = \dfrac{2400}{(s+20)(s+40)}$) using zero order hold method

```
Hp = c2d(Hp,T,'zoh')
```

```
Hp =

      1.199 z + 0.441
  ------------------------
  z^2 - 0.5032 z + 0.04979

Sample time: 0.05 seconds
Discrete-time transfer function.
```

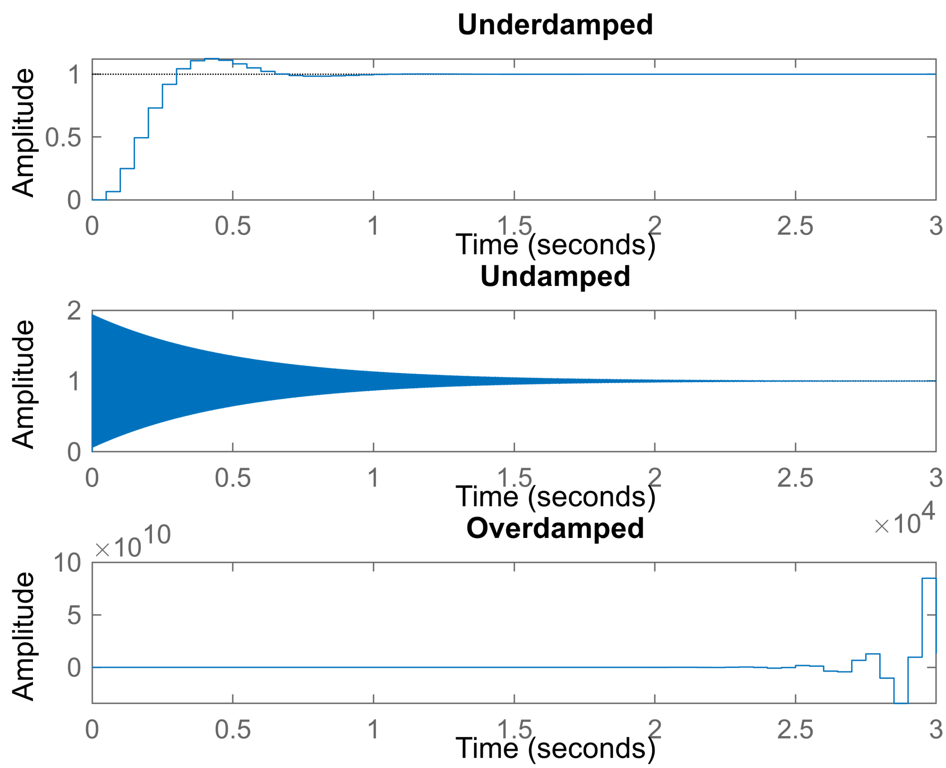
3. Use *zpk* function to write on your notebook the open loop transfer function

```
Hd = Hc*Hp;
Hd = zpk(Hd)
```

```
Hd =

   0.029968 (z+1) (z+0.3679)
  --------------------------
  (z-1) (z-0.3679) (z-0.1353)

Sample time: 0.05 seconds
Discrete-time zero/pole/gain model.
```

4. Analyze the stability of the closed loop system depending on $k \in (0, \infty)$; draw on your notebook the root locus and mention the obtained values of k directly on the graphics

```
Hc = k/s;
Hc = c2d(Hc,T,'tustin');
Hd = Hc*Hp;

figure,
subplot(3,1,1); step(feedback(2.18*Hd,1),3); title("Underdamped")
subplot(3,1,2); step(feedback(9.27*Hd,1)); title("Undamped")
subplot(3,1,3); step(feedback(42*Hd,1),3); title("Overdamped")
```

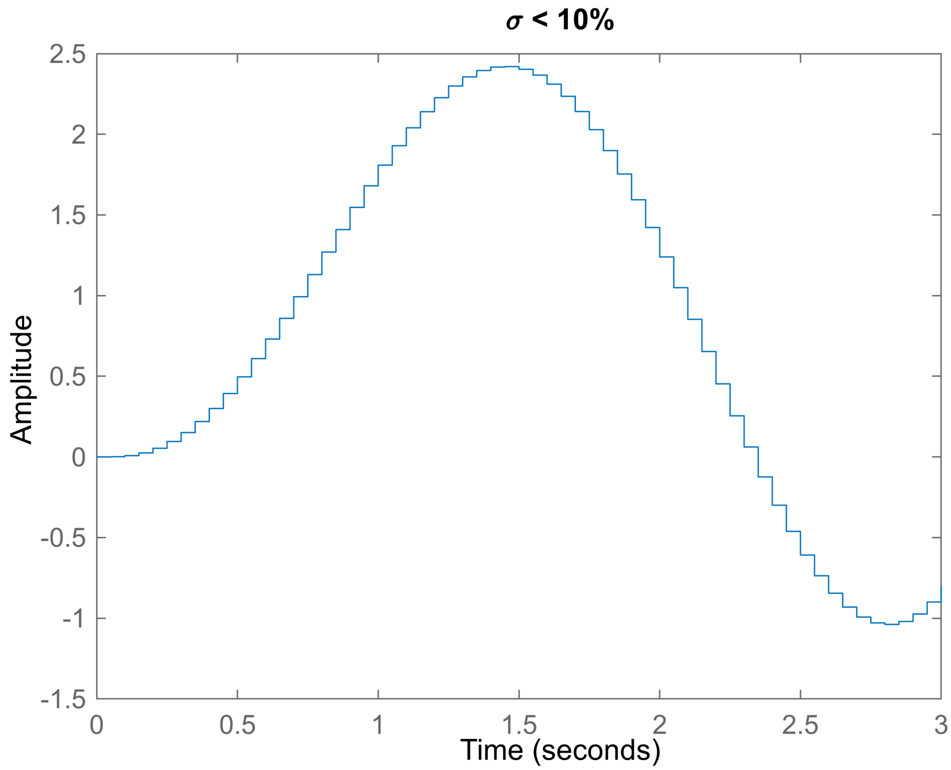


5. Analyze closed loop behavior depending on $k \in (0, \infty)$; use Matlab to generate the closed loop step response for all different behaviors that result depending on k; give suggestive titles to the generated plots

```
H0 = feedback(Hd,1);
```

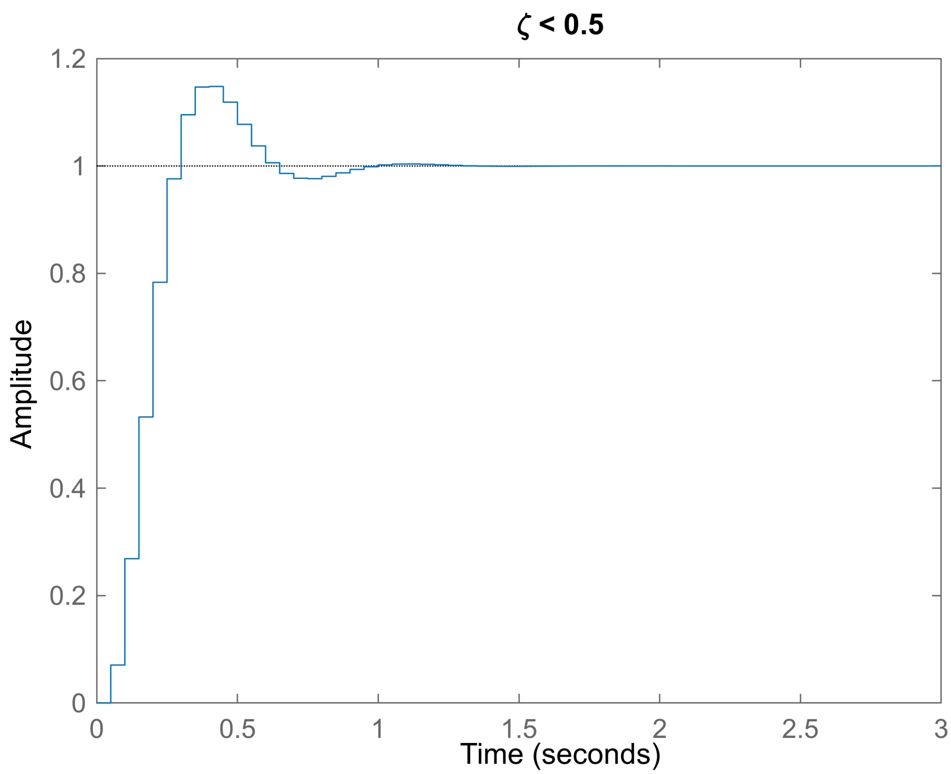
6. Use the zgrid function to obtain the value of k for which the overshoot of the closed loop is below 10%

```
figure,
step(feedback(1.87*zpk(Hc*Hd),1),3)
title('\sigma < 10%')
```



7. Generate the step response of the closed loop when the damping ratio is 5.5

```
figure,
step(feedback(2.36*zpk(Hc*Hp),1),3)
title('\zeta < 0.5')
```


ζ < 0.5
Amplitude
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
0.5
1.5
2
2.5
3