Sample Midterm: CSCI 6212 Algorithms

Policies: Test will be closed book, closed notes. You may bring in one sheet of paper. Tests will be taken individually, with not conversation or discussions with other classmates.

1. Short Answer

   (a) Suppose that in the Gale-Shapley algorithm, a mans proposal has just been accepted. True or false: He is guaranteed to remain engaged (to this person or someone else) for the remainder of the algorithms execution.

   (b) Suppose that in the Gale-Shapley algorithm, a woman has just accepted a proposal. True or false: She is guaranteed to remain engaged (to this person or someone else) for the remainder of the algorithms execution.

   (c) As a function of n, what is the asymptotic running time of the following function? (Express your running time using Θ notation.)

```
void scareMe(int n) {
i = n;
    while (i > 0) {
    for (j = 1 to i) print("boo!\n");
        i = floor(i/2);
    }
}
```

2. Consider using a simple linked list to store items, and assume you will never get duplicate elements. You use a simple linked list as a dictionary, insert any new elements at the beginning of the list. Suppose in your problem domain, there may be any number of insert operations, but at most one lookup operation.

   (a) What is the worst-case running-time of the operations performed on this data structure under the assumptions above? Briefly justify your answer.

   (b) What is the worst-case amortized running-time of the operations performed on this data structure under the assumptions above? Briefly justify your answer.

3. Consider the recurrence $T(n) = 4T(\frac{n}{2}) + n$, Using Θ-notation, give a tight asymptotic bound on $T(n)$. (E.g., $T(n)$ is $\Theta(n)$ or $\Theta(nlogn)$, etc.)

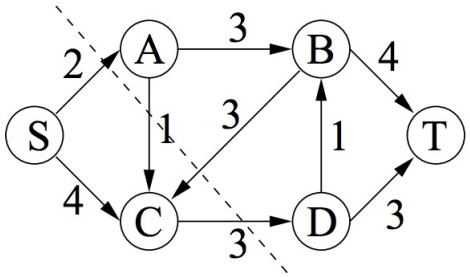
4. Let G be a connected undirected graph $G = (V, E)$ in which each edges weight is either 1 or 2. Present an $O(n + m)$ time algorithm to compute a minimum spanning tree for G, where $n = |V|$ and $m = |E|$. Explain your algorithms correctness and derive its running time. (Hint: This can be done by a variant of DFS or BFS.)

5. Give an example of a directed graph with negative-weight edges for which Dijkstras algorithm produces incorrect answers. Show why Dijkstras algorithm fails.

6. Edge-disjoint paths Given a directed graph $G = (V, E)$ with vertices $s, t \in V$, give an algorithm that finds the maximum number of edge-disjoint paths from s to t.

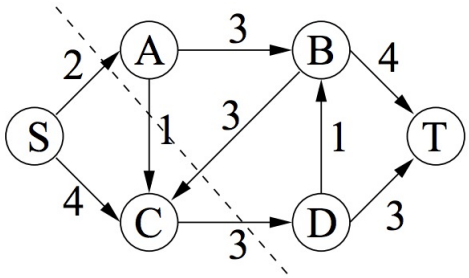
7. Alice wants to send Bob a file over a wired network. Their computers are connected to each other by multiple routers and multiple different routes. Given the bandwidth of each connection (edge weights), Alice needs to calculate the route that can send the file the fastest. How do you modify Dijkstras algorithm to find the path with the highest (minimum) bandwidth?

8. Problem 1. (12 points) Network Flows In the flow network illustrated below, each directed edge is labeled with its capacity.



(a) The dotted line represents a cut. Is this a minimum cut separating S from T?

(b) If you run the Ford Fulkerson Algorithm, what is a possible result in terms of flow after 1 iteration? Show the path chosen and the flow along that path?



(c) Show a possible solution at the conclusion fo the Ford Fulkerson algorithm with maximum flow