Homework 5

This homework is due by the beginning of class Tuesday December 5. It will be accepted for full credit if uploaded to blackboard by 5pm December 5. It will be accepted with a 20% deduction if uploaded by 9am Wednesday Dec. 6 after which it will not be accepted.

1. Reduce INDEPENDENT SET to CLIQUE. (This means, prove that you can solve INDEPENDENT SET if you can solve CLIQUE. More specifically, suppose you can answer the question "Does graph G have a clique of size j" in polynomial time. Prove that you can solve "Does graph G have an independent set of size k in polynomial time"

2. The KNAPSACK problem is NP-Complete. It can be phrased as the following: "Given $s_1, s_2, s_3, \ldots s_n$, and a target value T, does some subset of the numbers exactly sum to T? Another way to phrase the same question is: does there exist a set $X \subseteq \{1, 2, \ldots n\}$ such that $\sum_{i \in X} s_i = T$? In class we created a dynamic programming algorithm for this problem, with runtime that is $O(nT)$. That looks like a polynomial time algorithm; but KNAPSACK is NP-Complete. Explain.

3. Suppose that someone gives you an oracle (a magic algorithm whos running time you can consider to be constant) to answer the Independent-Set decision problem: "Does graph G have an indepdent set of size k?". Show how to use this to solve for the actual elements of the independent set. Give the run-time of your algorithm (assuming the the oracle runs in constant time), and argue that your algorithm is correct.