Homework 4

Due Thursday, Nov. 2 at the beginning of class. Accepted with no penalty until Thursday, Nov. 2 at 5pm.

1. Given a connected weighted undirected graph $G = (V, E)$ where every edge e has a positive weight $w(e)$, find the weight of the shortest cycle in G. Argue that your algorithm is correct and derive its running time. For a graph with n nodes, a runtime of $O(n^3)$ is acceptable.

2. Let $G = (V, E)$ be a connected, weighted undirected graph and suppose you have already computed T, the shortest-path tree from source s. If all weights in G are increased by the same amount c (so that each edge e has a new weight $w'(e) = w(e) + c$. Either prove that the tree T is still the shortest-path tree (from source s) for the graph with the new weights, or give a counter example.

3. Let $G = (V, E)$ be a connected weighted undirected graph, and let T be a minimum spanning tree of G. Graph G and tree T are given. Assume the cost of one edge e in G is decreased from $w(e)$ to $w'(e)$. Design an algorithm to find a minimum spanning tree in the modified graph. The running time of your algorithm should be O(n). Describe all details and write a pseudo-code for your algorithm.