# Mapping location on a bone to a corresponding location on a bone with different size

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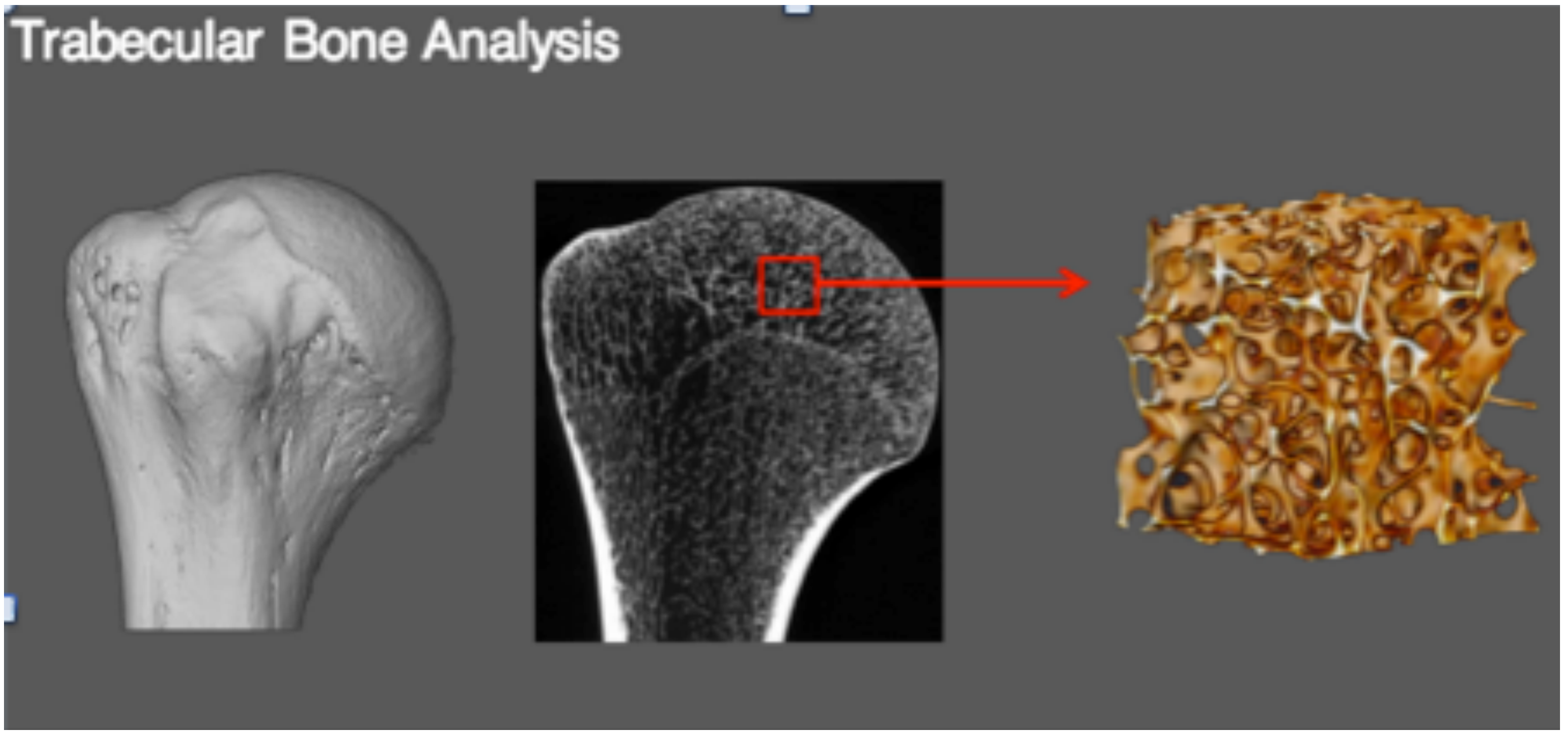
# Analysis of bone structure requires selecting the same small area across multiple specimens

Bones differ in size, orientation, and shape
(diagram courtesy of Timothy Ryan, Penn State)



# Idea: generate a unique coordinate system $\rho(x, y)$ and $\theta(x, y)$

* Discussion focuses on 2D case for simplicity
* Steps:
  * Choose two bones (reference and specimen) and a landmark point on the surface of each (e.g. look for max curvature). Also compute a middle point.
  * Solve a Laplace equation for $\rho$ and $\theta$. Solution $\rho$ grows radially from 0 to 1 with 0/1 Dirichlet boundary conditions on the interior point and boundary, resp. Solution $\theta$ varies in the angle direction and requires 0/1 Dirichlet on the upper/lower side of the line connecting the interior point to the landmark and Neumann on the boundary
  * Choose point (x, y) on the reference bone and interpolate to get $\rho$ and $\theta$ (interpolation)
  * Search for (x, y) on the specimen bone that gives the above $\rho$ and $\theta$. This requires solving a small nonlinear system (e.g. Newton's algorithm)

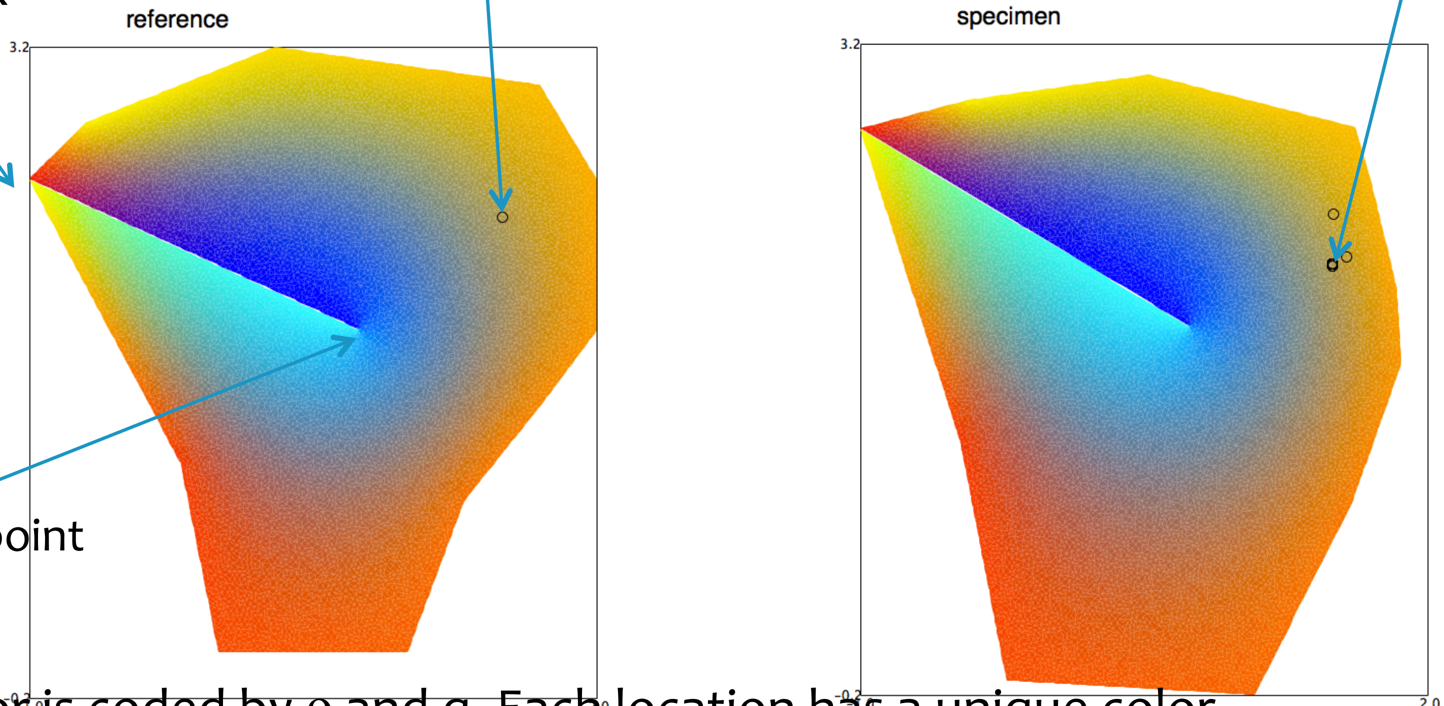
# Toy problem involving significant morphological differences

Location on reference bone

Corresponding location on specimen bone

landmark

Interior point




Color is coded by ρ and q. Each location has a unique color.
Goal is to find the specimen point that has the same color as the color on the reference point

# 3D requires solving for 3 Laplace fields

* Boundary becomes a surface
* Tetrahedral mesh in the interior (different for each bone)
* I think this requires two landmark points
* Radial and two angle-like fields
* Perhaps, may not need to solve for the two angle-like fields (can cook up angular fields)