Mapping a location on a bone to a corresponding location on a bone with a different size
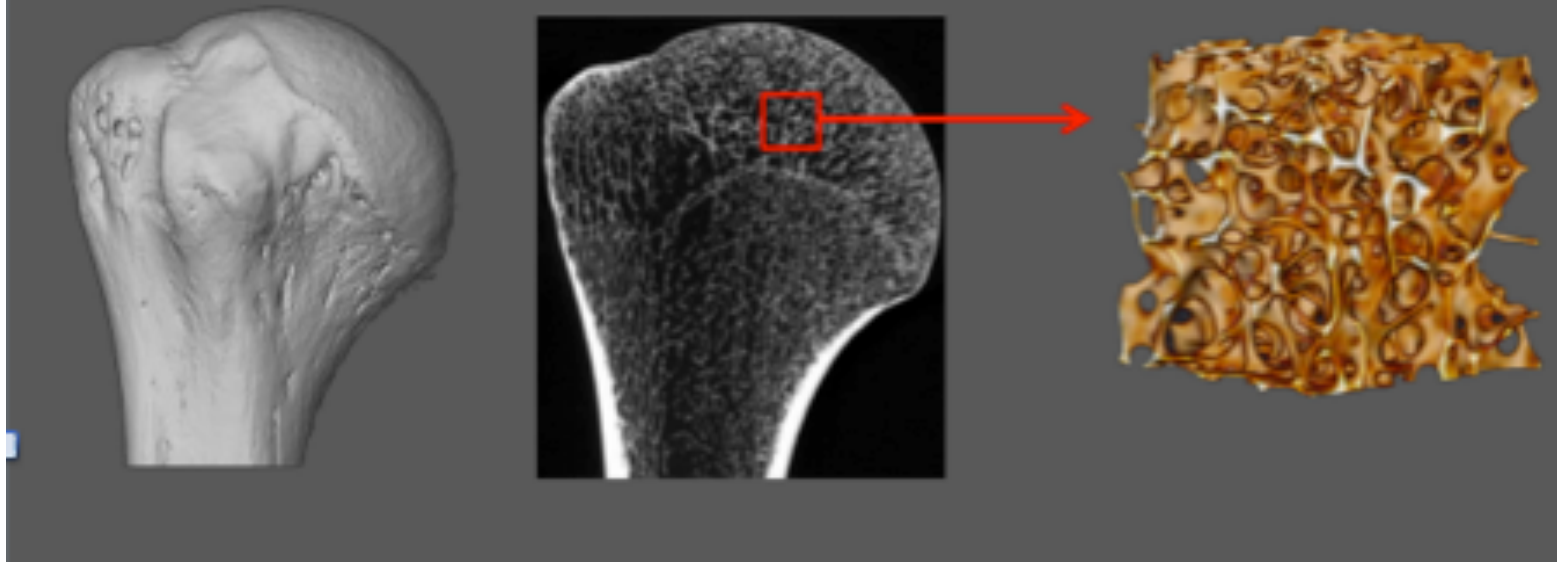
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Analysis of bone structure requires selecting the same small area across multiple specimens

Bones differ in size, orientation, and shape
(diagram courtesy of Timothy Ryan, Penn State)

Trabecular Bone Analysis

Idea: generate a unique coordinate system ρ(x, y) and θ(x, y)

* Discussion focuses on 2D case for simplicity
* Steps:
    * Choose two bones (reference and specimen) and a landmark point on the surface of each (e.g. look for max curvature). Also compute a middle point.
    * Solve a Laplace equation for ρ and θ. Solution ρ grows radially from 0 to 1 with 0/1 Dirichlet boundary conditions on the interior point and boundary, resp. Solution θ varies in the angle direction and requires 0/1 Dirichlet on the upper/lower side of the line connecting the interior point to the landmark and Neumann on the boundary
    * Choose point (x, y) on the reference bone and interpolate to get ρ and θ (interpolation)
    * Search for (x, y) on the specimen bone that give the above ρ and θ. This requires solving a nonlinear system iteratively (e.g. Newton's algorithm)

Toy problem involving significant morphological differences

Corresponding location on specimen bone

Location on reference bone

landmark

Interior point
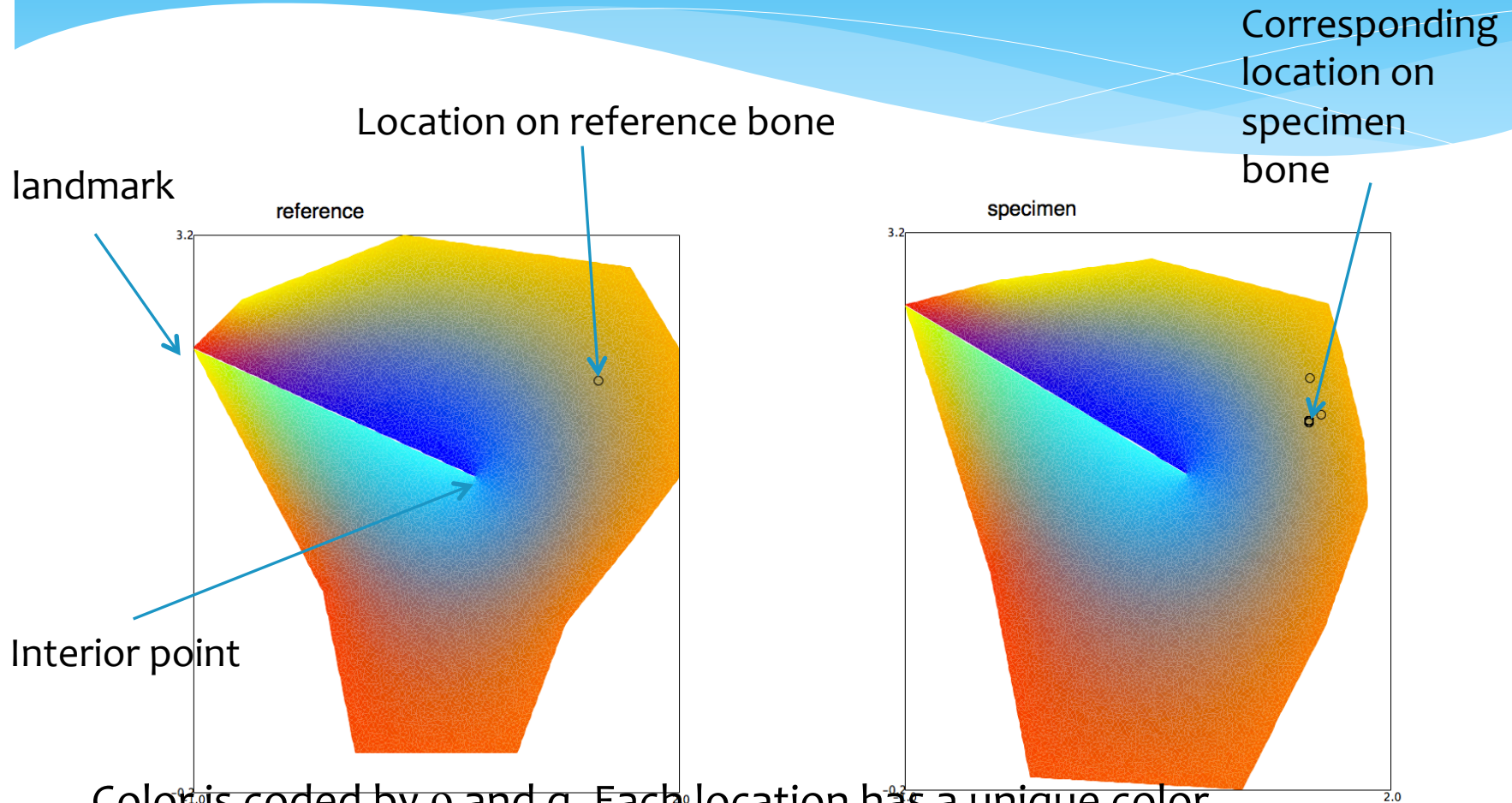
Color is coded by ρ and q. Each location has a unique color.
Goal is to find the specimen point that has the same color as
the color on the reference point

3D requires solving for 3 Laplace fields

* Boundary becomes a surface
* Tetrahedral mesh in the interior (different for each bone)
* I think this requires two landmark points
* Radial and two angle-like fields
* Perhaps, may not need to solve for the two angle-like fields (can cook up angular fields)