ENGR 0021 Recitation 2

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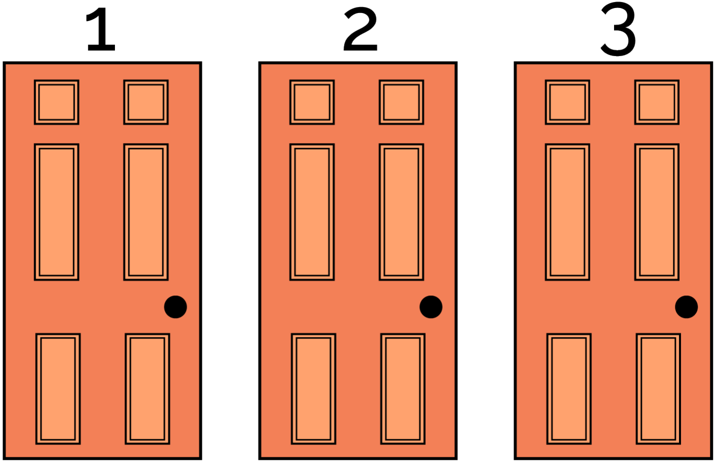
Let's Play a Game



The Monty Hall Problem: The Math

Let C_i be the event that the car is behind Door i ($P(C_i) = 1/3,\ \forall i$). Let M_j be the event that Monty opens Door j ($i \neq j$). Suppose you pick Door 1, after which Monty opens Door 3. We want to calculate $P(C_2|M_3)$.

Likelihoods $P(M_3|C_i)$:

If C_1, Monty can open 2 or 3 $\rightarrow$ $P(M_3|C_1) = 1/2$
If C_2, Monty can only open 3 $\rightarrow$ $P(M_3|C_2) = 1$
If C_3, Monty cannot open 3 $\rightarrow$ $P(M_3|C_3) = 0$

We can find $P(C_2|M_3)$ using Bayes' Rule:

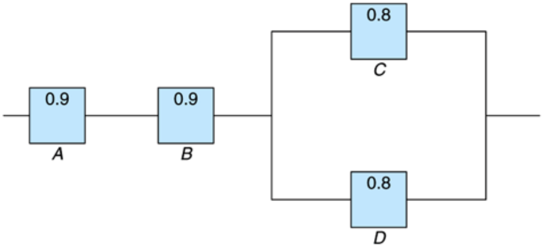
$$P(C_2|M_3) = \frac{P(M_3|C_2)P(C_2)}{P(M_3|C_1)P(C_1) + P(M_3|C_2)P(C_2) + P(M_3|C_3)P(C_3)}$$
$$= \frac{1 \cdot \frac{1}{3}}{\frac{1}{2} \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 0 \cdot \frac{1}{3}} = \frac{2}{3} \rightarrow \text{Switching doubles chances!}$$

Chapter 2, Slide 80 (Example 2.39)

An electrical system consists of four components (illustrated below). The system works is components A and B work and if either of components C or D work. The probability each component works (the reliability) is pictured in the figure.

(a) Find the probability that the entire system works.

(b) Find the probability that component C does not work given the entire system works. Assume components function independently.



Chapter 2, Slide 81 (Example 2.39)

Part 1:

We are given $P(A) = 0.9$, $P(B) = 0.9$, $P(C) = 0.8$, and $P(D) = 0.8$. Let $P(Sys)$ be the probability of the system working (this is the probability we want to find).

$$P(Sys) = P(A) \cdot P(B) \cdot P(C \cup D)$$

The probability of either of components C or D working is:

$$P(C \cup D) = 1 - P(C^c \cap D^c) = 1 - (0.2 \cdot 0.2) = 0.96$$

Chapter 2, Slide 81 (Example 2.39)

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Therefore,

$$\begin{aligned} P(Sys) &= P(A) \cdot P(B) \cdot P(C \cup D) \\ &= 0.9 \cdot 0.9 \cdot 0.96 = 0.7776 \end{aligned}$$

Chapter 2, Slide 82 (Example 2.39)

Part 2:

Now, we want to find the probability that C fails given the system works. If $P(Sys)$ is the probability the system fails, how can we represent this probability in terms of our events?

$$P(C^c|Sys)$$

We can calculate this using the conditional probability formula:

$$P(C^c|Sys) = \frac{P(C^c \cap Sys)}{P(Sys)}$$

How can we calculate $P(C^c \cap Sys)$?

$$\begin{aligned} P(C^c \cap Sys) &= P(A) \cdot P(B) \cdot P(C^c) \cdot P(D) \\ &= 0.9 \cdot 0.9 \cdot 0.2 \cdot 0.8 = 0.1296 \end{aligned}$$

Chapter 2, Slide 82 (Example 2.39) Continued

Part 2 (Continued):

Plugging all of this in, we have:

$$\begin{aligned} P(C^c|Sys) &= \frac{P(C^c \cap Sys)}{P(Sys)} \\ &= \frac{P(A) \cdot P(B) \cdot P(C^c) \cdot P(D)}{P(Sys)} \\ &= \frac{0.9 \cdot 0.9 \cdot 0.2 \cdot 0.8}{0.7776} \\ &= \frac{0.1296}{0.76} = \frac{1}{6} \end{aligned}$$

Homework 2, Question 2: The Problem

Four married couples have bought 8 seats in the same row for a concert.

(a) How many different ways can they be seated with no restrictions?

(b) How many different ways can they be seated if each couple is to sit together?

(c) How many different ways can they be seated if all the men sit in the right four seats and all the women sit in the left four seats?

Homework 2, Question 2: The Solution

(a) How many different ways can they be seated with no restrictions?

$$8! = 40,320 \text{ ways}$$

(b) How many different ways can they be seated if each couple is to sit together? If we view each couple as one unit, this is the same asking how many ways we can arrange four people (units). For each couple, however, we need to double the number of permutations, as they can create a new permutation by swapping seats. Thus,

$$4! \cdot 2^4 = 384 \text{ ways}$$

(c) How many different ways can they be seated if all the men sit in the right four seats and all the women sit in the left four seats? This is equivalent to independently arranging two groups of 4 and combining their arrangements afterwards. Thus,

$$4! \cdot 4! = 576 \text{ ways}$$

Question 1

DNA is made of sequences of the following nucleotides: A, C, G, T. How many sequences of length 3 are there? How many sequences of length 3 with no repeats are there? How many unique sets of length 3 are there?

Answer:
Sequences of Length 3: $4 \cdot 4 \cdot 4 = 64$
Sequences of Length 3 with No Repeats: $4 \cdot 3 \cdot 2 = 24$
Unique Sets of Length 3:
Sets imply that order does not matter $\rightarrow$ Combination

$$\binom{4}{3} = \frac{n!}{r!(n-r)!} = \frac{4!}{3! \cdot 1!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 1} = 4$$

Question 2

Suppose we flip a coin 5 times. How many ways can we get exactly 3 heads in 5 coin flips? What is the probability of getting exactly 3 heads in 5 coin flips?

Answer:

Ways to Get 3 Heads in 5 Flips:

$$\binom{5}{3} = \frac{n!}{r!(n-r)!} = \frac{5!}{3! \cdot 2!} = 10$$

Probability of Getting 3 Heads in 5 Flips:

$$P(\text{3 heads in 5 flips}) = \frac{\#\text{ of outcomes with 3 heads}}{\text{total }\#\text{ of outcomes}}$$

Total # of Outcomes in 5 Flips: $2^5 = 32$

$$P(\text{3 heads in 5 flips}) = \frac{10}{32} = \frac{5}{16}$$

Question 3

Suppose we have three mutually exclusive events A, B, and C. The probabilities of events A and B are 0.2 and 0.3 respectively ($P(A) = 0.2, P(B) = 0.3$). Can the probability of event C be 0.1? Can it be 0.7? What must the probability of event C be if events A, B, and C partition their sample space?

Answer:

Can $P(C) = 0.1$? $\rightarrow$ Yes!

Since these events are mutually exclusive,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

As $P(\cdot) \leq 1$ for any probability, $P(A \cup B \cup C) \leq 1$. This gives us:

$$0.2 + 0.3 + P(C) \leq 1 \rightarrow P(C) \leq 0.5$$

and $0.1 \leq 0.5$.

Question 3 (Continued)

Suppose we have three mutually exclusive events A, B, and C. The probabilities of events A and B are 0.2 and 0.3 respectively ($P(A) = 0.2, P(B) = 0.3$). Can the probability of event C be 0.1? Can it be 0.7? What must the probability of event C be if events A, B, and C partition their sample space?

Answer:

Can $P(C) = 0.7$? $\rightarrow$ No!

Since these events are mutually exclusive,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) = 0.2 + 0.3 + 0.7 = 1.2 \geq 1$$

Question 3 (Continued)

Suppose we have three mutually exclusive events A, B, and C. The probabilities of events A and B are 0.2 and 0.3 respectively ($P(A) = 0.2, P(B) = 0.3$). Can the probability of event C be 0.1? Can it be 0.7? What must the probability of event C be if events A, B, and C partition their sample space?

Answer:

Can $P(C) = 0.7$? $\rightarrow$ No!

Since these events are mutually exclusive,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) = 0.2 + 0.3 + 0.7 = 1.2 \geq 1$$

What is $P(C)$ if the events partition S? If A, B, and C partition S, $P(A) + P(B) + P(C) = 1$. Therefore,

$$P(C) = 1 - P(A) - P(B) = 1 - 0.2 - 0.3 = 0.5$$

Question 4

Ashley is a soccer player, and she has a lucky pair of socks. When Ashley wears the socks, her team wins 85% of their matches. Ashley is quite forgetful, however, and only remembers to bring the socks to 40% of matches. What is the probability Ashley remembers her socks and her team wins the match?

Answer:

$$P(\text{win} \cap \text{socks}) = P(\text{win}|\text{socks}) \cdot P(\text{socks}) = 0.85 \cdot 0.4 = 0.34$$

Question 5

In response to a zombie outbreak, your town installs a zombie detector. If someone is a zombie, it beeps 90% of the time; if they aren't, it stays silent 95% of the time). In your town, only 1 in 10 people are actually zombies. The detector beeps at your friend Matt. What is the probability that Matt is actually a zombie?

Answer:

Let Z be the event that someone is a zombie and B be the event that the detector beeps. The questions gives us these probabilities: $P(Z) = 0.1, P(B|Z) = 0.9, P(B^c|Z^c) = 0.95$

$$\text{Bayes' Rule: } P(Z|B) = \frac{P(B|Z)P(Z)}{P(B|Z)P(Z) + P(B|Z^c)P(Z^c)}$$

$$P(B|Z^C) = 1 - P(B^C|Z^C) = 1 - 0.95 = 0.05, P(Z^c) = 1 - P(Z) = 1 - 0.1 = 0.9$$

$$P(Z|B) = \frac{0.9 \cdot 0.1}{0.9 \cdot 0.1 + 0.05 \cdot 0.9} = \frac{2}{3}$$