# ENGR 0021 Recitation 5

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# The Poisson Distribution

**When to use:**

- Events occur at an average rate $\lambda$ per unit time and are independent of one another.
- We are counting the number of events in an interval of length $t$.

**Probability Mass Function (PMF):**

If $X \sim Poisson(\lambda t)$, then:

$$P(X = x) = e^{-\lambda t} \frac{(\lambda t)^x}{x!}, \quad x = 0, 1, 2, \ldots$$

**Mean and Variance:**

$$E[X] = Var(X) = \lambda t$$

# Example 1: Creatures in Traps

**Problem:** Let $X$ be the number of creatures captured in a week. Suppose that $X$ has a Poisson distribution with $\lambda = 4.5$, so on average traps contain 4.5 creatures. Suppose:

$$X \sim Poisson(\lambda t = 4.5)$$

(a) Find $P(X = 5)$.

(b) Find $P(X \leq 5)$.

(c) Find $P(X \geq 5)$.

**Solutions:**

(a) $P(X = 5) = \frac{e^{-4.5}(4.5)^5}{5!} \approx 0.1708$

(b) $P(X \leq 5) = \sum_{x=0}^{5} \frac{e^{-4.5}(4.5)^x}{x!} \approx 0.7029$

(c) $P(X \geq 5) = 1 - P(X \leq 4) = 1 - \sum_{x=0}^{4} \frac{e^{-4.5}(4.5)^x}{x!} \approx 0.4679$

# Example 2: Arrivals at a Counter

**Problem:** Suppose pulses arrive at a counter at an average rate of 6 per minute, so $\lambda = 6$.

**(a)** Find the probability that in a 0.5-minute interval at least one pulse is received.

**Step 1: Adjust Rate.** New mean $\mu = \lambda t = 6 \times 0.5 = 3$ pulses.
**Step 2: Solve.** "At least one" is the complement of "None".

$$P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{e^{-3}(3^0)}{0!} = 1 - e^{-3} \approx 0.9502$$

# Example 2: Arrivals (Continued)

**(b)** Find the mean number of pulses received in the 30-second interval.

**Solution:**

- The rate is given as $\lambda = 6$ pulses per minute.
- The interval is 30 seconds, which is $t = 0.5$ minutes.
- For a Poisson distribution, the mean (expected value) is $E[X] = \lambda t$.

$$E[X] = 6 \times 0.5 = 3 \text{ pulses}$$

# Example 3: Poisson Approximation

**Problem:** A factory produces screws, which have 0.1% defect rate. A box contains 2000 screws.

**Question:** What is the probability the box has exactly 3 defective screws?

**Solution:**

- This is technically Binomial ($P(X = 2)$ where $n = 2000, p = 0.001$).
- Since $n$ is large ($n \geq 20$) and $p$ is small ($p \leq 0.05$), we approximate with Poisson.
- $\lambda = np = 2000(0.001) = 2$.

$$P(X = 3) = \frac{e^{-2}(2^3)}{3!} = \frac{e^{-2}(8)}{6} \approx 0.1804$$

# Is Poisson Appropriate?

**Scenario:** A store receives customers in bursts: when one arrives, their friends often arrive immediately after.

**Question:** Is Poisson a good model for the number of arrivals per minute? Why or why not?

**Answer: No.**
The Poisson distribution assumes that events occur **independently** of one another. If customers arrive in groups, the independence assumption is violated.

# Exercise 1 (HW 4 Problem 4)

**Problem:** Recall that an exponential random variable $X \sim \exp(\lambda)$ has mean $1/\lambda$ and PDF given by $f(x) = \lambda e^{-\lambda x}$ on $x \geq 0$.

**(a) Compute $P(X \geq x)$.**

# Exercise 1 (HW 4 Problem 4)

**(b)** Suppose that $X_1$ and $X_2$ are independent exponential random variables with mean $1/\lambda$. Let $T = \min(X_1, X_2)$. Find the CDF of $T$.
*(Hint: what is $P(T \geq t)$?)*

# Exercise 1 (HW 4 Problem 4)

**(c)** Suppose we test 3 different brands of light bulbs $B_1, B_2$, and $B_3$ whose lifetimes are independent exponential random variables with **means** $1/2, 1/3$, and $1/5$ years, respectively.

What is the expected time before **one** of the bulbs fails?
*(Hint: part (b) was a warmup for this problem.)*

# Exercise 2 (HW 4 Problem 6)

**Problem:** Suppose $X$ and $Y$ are random variables with the following joint PMF. Are $X$ and $Y$ independent?

| $X \setminus Y$ | 1 | 2 | 3 |
|---|---|---|---|
| 1 | $1/18$ | $1/9$ | $1/6$ |
| 2 | $1/9$ | $1/6$ | $1/18$ |
| 3 | $1/6$ | $1/18$ | $1/9$ |

# Exercise 2 (HW 4 Problem 6)