Assignment #4

1. A coin is tossed twice. Let Z denote the number of heads on the first toss and W the total number of heads on the 2 tosses. If the coin is unbalanced and a head has a 40% chance of occurring, find

   (a) the joint probability distribution of W and Z;

   (b) the marginal distribution of W;

   (c) the marginal distribution of Z;

   (d) the probability that at least 1 head occurs.

2. If a dealer's profit, in units of $5000, on a new automobile can be looked upon as a random variable X having the density function

$$f(x) = \begin{cases} 2(1-x), 0 < x < 1 \\ 0, \text{ elsewhere.} \end{cases} \tag{1}$$

   find the average profit per automobile.

3. The distribution of the number of imperfections per 10 meters of synthetic fabric is given by

x	0	1	2	3	4
f(x)	0.41	0.37	0.16	0.05	0.01

   (a) Plot the probability function.

   (b) Find the expected number of imperfections, $E(X) = \mu$.

   (c) Find $E(X^2)$.

4. Suppose X and Y have joint pdf $f(x, y) = c(x^2 + xy)$ on $[0, 1] \times [0, 1]$.

   (a) Find c and the joint cdf $F(x, y)$.

   (b) Find the marginal cumulative distribution functions $F(x)$ and $F(y)$ and the marginal pdf $f(x)$ and $f(y)$.

   (c) Find E(X).

   (d) Find Var(X).

   (e) Find the covariance of X and Y.

   (f) Find the correlation of X and Y.

5. Suppose X and Y are random variables with the following joint pmf. Are X and Y independent?

$X\backslash Y$	1	2	3
1	1/18	1/9	1/6
2	1/9	1/6	1/18
3	1/6	1/18	1/9

6. Alice and Bob are trying to meet for lunch and both will arrive, independently of each other, uniformly at random between noon and 1 pm. Let A and B be the number of minutes after noon at which Alice and Bob arrive, respectively. Then A and B are independent $\text{Unif}(0, 60)$. Hint: For parts (c–e) you might find it easiest to find the fraction of the square $[0, 60] \times [0, 60]$ filled by the event.

   (a) Find the joint pdf $f(a, b)$ and joint cdf $F(a, b)$.

   (b) Find the probability that Alice arrives before 12:30.

   (c) Find the probability that Alice arrives before 12:15 and Bob arrives between 12:30 and 12:45 in two ways: (i) using independence; (ii) using area in $[0, 60] \times [0, 60]$.

   (d) Find the probability that Alice arrives less than five minutes after Bob.

   (e) Now suppose Alice and Bob will leave if they have to wait more than 15 minutes for the other to arrive. What is the probability that they will have lunch together?

7. A nationwide survey of college seniors by the University of Michigan revealed that almost 70% disapprove of daily pot smoking, according to a report in Parade. If 12 seniors are selected at random and asked their opinion, find the probability that the number who disapprove of smoking pot daily (you can assume exactly 70% disapprove) is

   (a) anywhere from 7 to 9;

   (b) at most 5;

   (c) not less than 8.

8. A manufacturer knows that on average 20% of the electric toasters produced require repairs within 1 year after they are sold. When 20 toasters are randomly selected, find appropriate numbers x and y such that
   (a) the probability that at least x of them will require repairs is less than 0.5;

   (b) the probability that at least y of them will not require repairs is greater than 0.8.

9. A large company has an inspection system for the batches of small compressors purchased from vendors. A batch typically contains 15 compressors. In the inspection system, a random sample of 5 is selected and all are tested. Suppose there are 2 faulty compressors in the batch of 15.

    (a) What is the probability that for a given sample there will be 1 faulty compressor?

    (b) What is the probability that inspection will discover both faulty compressors?

10. On average, 3 traffic accidents per month occur at a certain intersection. What is the probability that in any given month at this intersection

    (a) exactly 5 accidents will occur?

    (b) fewer than 3 accidents will occur?

    (c) at least 2 accidents will occur?