Assignment #3

1. In a city with one hundred taxis, 1 is blue and 99 are green. A witness observes a hit-and-run by a taxi at night and recalls that the taxi was blue, so the police arrest the blue taxi driver who was on duty that night. The driver proclaims his innocence and hires you to defend him in court. You hire a scientist to test the witness' ability to distinguish blue and green taxis under conditions similar to the night of accident. The data suggests that the witness sees blue cars as blue 99% of the time and green cars as blue 2% of the time. Write a speech for the jury to give them reasonable doubt about your client's guilt. Your speech need not be longer than the statement of this question. Keep in mind that most jurors have not taken this course, so an illustrative table may be easier for them to understand than fancy formulas.

2. There are 8 cards in a hat: $1\heartsuit, 1\spadesuit, 1\diamondsuit, 1\clubsuit, 2\heartsuit, 2\spadesuit, 2\diamondsuit, 2\clubsuit$ You draw one card at random. If its rank is 1 you draw one more card; if its rank is two you draw two more cards. The cards are drawn without replacement. Hint: Use a tree diagram.
   (a) What is the probability that the sum of all the cards that you draw is 3?
   (b) What is the probability that the sum of all the cards that you draw is 6?

3. There are four dice in a drawer: one tetrahedron (4 sides), one hexahedron (i.e., cube, 6-sides), and two octahedra (8 sides). Your friend secretly grabs one of the four dice at random. Let s be the number of sides on the chosen die.

   Now your friend rolls the chosen die and without showing the die to you rolls it... Let R be the result of the roll.
   (a) Use Bayes rule to find $P(s = k|R = 3)$ for $k = 4, 6, 8$. Which die is most likely if R = 3?
   (b) Which die is most likely if R = 6? Hint: You can either repeat the computation in (b), or you can reason based on your result in (b).
   (c) Which die is most likely if R = 7? No computations are needed!

   Questions from textbook:

4. Police plan to enforce speed limits by using radar traps at four different locations within the city limits. The radar traps at each of the locations L1, L2, L3, and L4 will be operated 40%, 30%, 20%, and 30% of the time. If a person who is speeding on her way to work has probabilities of 0.2, 0.1, 0.5, and 0.2, respectively, of passing through these locations, what is the probability that she will receive a speeding ticket?

5. If the person in the previous question received a speeding ticket on her way to work, what is the probability that she passed through the radar trap located at L2?

6. A regional telephone company operates three identical relay stations at different locations. During a one-year period, the number of malfunctions reported by each station and the causes are shown below.

   Suppose that a malfunction was reported and it was found to be caused by other human errors. What is the probability that it came from station C?

7. Determine the value c so that each of the following functions can serve as a probability distribution of the discrete random variable X: (a) $f(x) = c(x^2 + 4)$, for $x = 0, 1, 2, 3$;

   (b) $f(x) = c\binom{2}{x}\binom{3}{3-x}$, for $x = 0, 1, 2$.

8. The shelf life, in days, for bottles of a certain prescribed medicine is a random variable having the density function

$$f(x) = \begin{cases} \frac{20{,}000}{(x+100)^3}, x > 0 \\ 0, \text{elsewhere} \end{cases} \tag{1}$$

   Find the probability that a bottle of this medicine will have a shell life of (a) at least 200 days;

   (b) anywhere from 80 to 120 days.

9. Let W be a random variable giving the number of heads minus the number of tails in three tosses of a coin.

   (a) List the elements of the sample space S for the three tosses of the coin and to each sample point assign a value w of W.

   (b) Find the probability distribution of the random variable W, assuming that the coin is biased so that a head is twice as likely to occur as a tail.

10. The waiting time, *in hours*, between successive speeders spotted by a radar unit is a continuous random variable with cumulative distribution function

$$F(x) = \begin{cases} 0, x < 0, \\ 1 - e^{-8x}, x \geq 0. \end{cases} \tag{2}$$

   Find the probability of waiting less than 12 minutes between successive speeders (a) using the cumulative distribution function of X;

   (b) using the probability density function of X.