Assignment #2

1. (a) In how many ways can 6 people be lined up to get on a bus?

   $6! = 720$.

   (b) If 3 specific persons, among 6, insist on following each other (but not in any specific order), how many ways are possible?

   Treat the three specific people as a single block. Along with the remaining three individuals, this gives a total of 4 objects to arrange. The 4 objects can be arranged in 4! ways. Within the block, the three people can be arranged among themselves in 3! ways. Thus, the total number of possible lineups is

   $$4! \times 3! = 144.$$

   (c) If 2 specific persons, among 6, refuse to follow each other, how many ways are possible?

   First count the total number of possible lineups:

   $$6! = 720.$$

   Now count the number of lineups in which the two specific people do stand next to each other. Treat the two people as a single block. Together with the remaining four individuals, this gives 5 objects to arrange, which can be done in 5! ways. The two people within the block can be arranged in 2! ways.

   $$6! - 5! \times 2! = 720 - 240 = 480.$$

2. Four married couples have bought 8 seats in the same row for a concert. In how many different ways can they be seated

   (a) with no restrictions?

   $8! = 40320$

   (b) if each couple is to sit together?

   Treat each couple as a single block. Since there are 4 couples, the blocks can be arranged in 4! ways. Within each couple, the two individuals can switch seats, giving 2 possible arrangements per couple. Since there are 4 couples, this contributes a factor of 2^4. Thus, the total number of seating arrangements is

   $$4! \times 2^4 = 384.$$

   (c) if all the men sit together to the right of all the women? All the men sit in the right four seats and all the women sit in the left four seats.

All women occupy the left four seats, and all men occupy the right four seats. The four women can be arranged among themselves in 4! ways, and the four men can be arranged among themselves in 4! ways. Therefore, the total number of seating arrangements is

$$4! \times 4! = 576.$$

3. A poker hand consists of 5 cards out of 52 cards.

   (a) How many total poker hands are there?

   $\binom{52}{5} = 2598960$.

   (b) A one-pair hand consists of two cards having one rank (2, 3,... , 9, 10, J, Q, K, A) and the other three cards having three other rank. How many 5 poker card hands have exactly one two of a kind?

   First choose the rank of the pair. There are 13 possible ranks:

   $$\binom{13}{1}.$$

   Next, choose 2 of the 4 suits for the pair:

   $$\binom{4}{2}.$$

   Now choose the ranks of the remaining three cards from the remaining 12 ranks:

   $$\binom{12}{3}.$$

   Finally, for each of these three ranks, choose one of the 4 possible suits:

   $$\binom{4}{1}^3.$$

   Multiplying these choices together gives the total number of one-pair hands:

   $$\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}^3 = 1098240.$$

   (c) What is the probability of getting a two of a kind in a poker hand.

   1098240/2598960 = 0.42257
   42.2%

4. Before the distribution of certain statistical software, every fourth compact disk (CD) is tested for accuracy. The testing process consists of running four inde-

pendent programs and checking the results. The failure rates for the four testing programs are, respectively, 0.01, 0.03, 0.02, and 0.01.

(a) What is the probability that a tested CD failed at least one test?

$1 - 0.99 \times 0.97 \times 0.98 \times 0.99 = 0.068$
6.8%

(b) Given that a CD was tested, what is the probability that it failed program 2 or 3?

$1 - 0.97 \times 0.98 = 0.049$
4.9%

(c) In a sample of 100 tests, how many CDs would you expect would fail at least one test?

100 CDs $100 \times 6.8\% = 6.8$.

(d) Given that a CD was defective, what is the probability that it was tested?

The testing is independent of whether the CD is defective or not. $P(tested) = 0.25$

5. Pollution of the rivers in the United States has been a problem for many years. Consider the following events:
A: the river is polluted,
B: a sample of water tested detects pollution,
C : fishing is permitted.
Assume $P(A) = 0.3$, $P(B|A) = 0.75$, $P(B|A') = 0.20$, $P(C|A \cap B) = 0.20$, $P(C|A' \cap B) = 0.15$, $P(C|A \cap B') = 0.80$, and $P(C|A' \cap B') = 0.90$.

(a) Find $P(A \cap B \cap C)$.

$P(A \cap B \cap C) = P(A)P(B|A)P(C|A \cap B) = 0.3 \times 0.75 \times 0.2 = 0.045$
4.5%

(b) Find $P(B'|A')$

$P(B'|A') = 1 - P(B|A') = 1 - 0.2 = 0.8$

(c) Find $P(B'|A)$

$P(B'|A) = 1 - P(B|A) = 1 - 0.75 = 0.25$

(d) Find $P(B' \cap C)$. Hint: Use a Venn Diagram or the Law of Total Probability.

$$\begin{aligned}
P(B' \cap C) &= P(A \cap B' \cap C) + P(A' \cap B' \cap C) && (1)\\
&= P(C|A \cap B')P(A \cap B') + P(C|A' \cap B')P(A' \cap B') && (2)\\
&= P(C|A \cap B')P(B'|A)P(A) + P(C|A' \cap B')P(B'|A')P(A') && (3)\\
&= 0.8 \times 0.25 \times 0.3 + 0.9 \times 0.8 \times (1 - 0.3) && (4)\\
&= 0.564 && (5)
\end{aligned}$$

(e) Find the probability that the river is polluted, given that fishing is permitted and the sample tested did not detect pollution.

This is

$$\begin{aligned}
P(A|B' \cap C) &= P(A \cap B' \cap C)/P(B' \cap C) && (6)\\
&= P(C|A \cap B')P(A \cap B')/P(B' \cap C) && (7)\\
&= P(C|A \cap B')P(B'|A)P(A)/P(B' \cap C) && (8)\\
&= 0.8 \times 0.25 \times 0.3/0.564 && (9)\\
&= .1064 && (10)
\end{aligned}$$

6. How many distinct permutations can be made from the letters of the word INFINITY?

$\frac{8!}{3!2!1!1!1!} = 3360$

7. An experiment involves tossing a pair of dice, one green and one red, and recording the numbers that come up. If x equals the outcome on the green die and y the outcome on the red die find

(a) the probability of event A that the sum is greater than 8

For greater than 8, $(4 + 3 + 2 + 1)/36 = 5/18$

(b) the probability of event C that a number greater than 4 comes up on the green die;

Roll a 5 or 6 1/3

(c) the probability of event $A \cap C$.

$1/6 \times (1/2) + 1/6 \times (2/3) = 7/36.$

8. Suppose that in a senior college class of 500 students it is found that 210 smoke, 258 drink alcoholic beverages, 216 eat between meals, 122 smoke and drink alcoholic beverages, 83 eat between meals and drink alcoholic beverages, 97 smoke

and eat between meals, and 52 engage in all three of these bad health practices. If a member of this senior class is selected at random, find the probability that the student

(a) smokes but does not drink alcoholic beverages;

$(210 - 122)/500 = 22/125$

(b) eats between meals and drinks alcoholic beverages but does not smoke;

$(83 - 52)/500 = 31/500$

(c) neither smokes nor eats between meals.

$(500 - 210 - 216 + 97)/500 = 171/500$

9. If 3 books are picked at random from a shelf containing 5 novels, 3 books of poems, and a dictionary, what is the probability that

(a) the dictionary is selected?

$1 - 8/9 \times 7/8 \times 6/7 = 0.33$

(b) 2 novels and 1 book of poems are selected?

Total possibilities are $\binom{9}{3} = 84$. Possibilities where 2 novels and 1 book of poems are selected are $\binom{5}{2} \times \binom{3}{1} = 30$. Probability is 30/84 or 35.7%

10. For married couples living in a certain suburb, the probability that the husband will vote on a bond referendum is 0.21, the probability that the wife will vote on the referendum is 0.28, and the probability that both the husband and the wife will vote is 0.15. What is the probability that

(a) at least one member of a married couple will vote?

$0.21 + 0.28 - 0.15 = 0.34$

(b) a wife will vote, given that her husband will vote?

$P(W|H) = P(H \cap W)/P(H) = 0.15/0.21 = 5/7$

(c) a husband will vote, given that his wife will not vote?

$P(H|W') = P(H \cap W')/P(W') = (0.21 - 0.15/)/(1 - 0.28) = 1/12$