ENGR 0021 Recitation 4 (Exam 1 Review)

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Descriptive Statistics

Measures of Central Tendency

Key Concept: Robustness

- **Mean:** Highly sensitive to outliers.
- **Median:** Robust (resistant) to outliers.
- **Mode:** Most frequent value.

Example: Consider the dataset $\{1, 2, 3, 4, 100\}$.

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Example: Consider the dataset $\{1, 2, 3, 4, 100\}$.

- The mean is $\frac{110}{5} = 22$. The median is 3.
- If 100 changes to 1000, the mean jumps massively. The median stays 3.

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→ It stays the **same**. Shifting data does not change the "spread" or width of the distribution.

Probability Basics

Conditional Probability Practice (Coin Toss)

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- Event B (First is Heads): $\{$HHH, HHT, HTH, HTT$\}$.
  $\rightarrow |B| = 4$, so $P(B) = 4/8 = 1/2$

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Step 2: Apply Formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/8}{4/8} = \frac{2}{4} = \frac{1}{2}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{2/8}{3/8} = \frac{2}{3}$$

Unions and Independence

Union Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Question: Let $P(A) = 0.4$ and $P(B) = 0.5$.

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2. If independent, $P(A \cap B) = P(A)P(B) = 0.4 \times 0.5 = 0.2$.
$P(A \cup B) = 0.4 + 0.5 - 0.2 = 0.7$.

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Practical Reasoning:

- **Correlation (ρ)** is a linearity score. It only tests if a *straight line* fits the data.
- **Independence** means there is *no relationship whatsoever*.

What about a Parabola? Consider $Y = X^2$ (a perfect U-shape).

- Are they related? **Yes**, perfectly.
- Is the correlation zero? **Yes**, because a straight line fits a U-shape poorly.

Combinatorics

Counting Rules

Permutations (Order Matters): $P_k^n = \frac{n!}{(n-k)!}$
Combinations (Order Doesn't Matter): $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

Practice:

1. How many ways can you arrange 4 distinct books on a shelf?
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1. $4! = 4 \times 3 \times 2 \times 1 = 24$.

2. Order doesn't matter (committee members are equal). Use Combinations.
$\binom{10}{3} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120$.

The Urn Problem

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$$P(2R, 1G) = \frac{\binom{6}{2} \cdot \binom{4}{1}}{\binom{10}{3}}$$

$$= \frac{15 \cdot 4}{120} = \frac{60}{120} = 0.5$$

Random Variables

Expectation and Variance Rules

For any random variable X and constants a, b:

$$E[aX + b] = aE[X] + b$$
$$Var(aX + b) = a^2 Var(X) \quad \text{(Shift } b \text{ does nothing to spread!)}$$

Practice: Suppose $E[X] = 2$ and $Var(X) = 3$. Find $E[X^2]$ and $Var(3X + 5)$.

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1. Find $Var(3X + 5)$:
$Var(3X + 5) = 3^2 Var(X) = 9(3) = 27.$

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1. Find $Var(3X + 5)$:
$Var(3X + 5) = 3^2 Var(X) = 9(3) = 27.$

2. Find $E[X^2]$:
Recall $Var(X) = E[X^2] - (E[X])^2$.
$3 = E[X^2] - (2)^2 \implies E[X^2] = 3 + 4 = 7.$

Discrete Random Variables

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Part B: What is the CDF value $F(1)$?
$F(1) = P(X \leq 1) = P(X = 0) + P(X = 1) = 0.2 + 0.5 = 0.7$.

Joint Probabilities & Bayes

Joint Distributions

Marginal Distributions: To find the distribution of X from a joint table, sum across the rows (or columns) of Y.

Independence Check: X and Y are independent **only if** $P(X = x, Y = y) = P(X = x)P(Y = y)$ for **every** cell in the table.

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Conditional Probabilities: If you are asked to find $P(X = 1|Y = 2)$, for example, read the intersection from the table and divide by the marginal of Y.

$$P(X = 1|Y = 2) = \frac{P(X = 1 \cap Y = 2)}{P(Y = 2)}$$

Bayes' Theorem Practice

Scenario: A company has two machines.

- Machine 1 produces 60% of items, with a 5% defect rate.
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You pick an item at random and it is **defective**. What is the probability it came from Machine 1?

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- Let M_1 be Machine 1, D be Defective.
- We want $P(M_1|D)$.

$$P(M_1|D) = \frac{P(D|M_1)P(M_1)}{P(D|M_1)P(M_1) + P(D|M_2)P(M_2)}$$

$$= \frac{0.05(0.60)}{0.05(0.60) + 0.02(0.40)} = \frac{0.030}{0.030 + 0.008} = \frac{0.030}{0.038} \approx 0.789$$

Reminders

Tips:

1. Write the general formula (e.g., Bayes, Total Probability) before plugging in numbers. You cannot get partial credit without showing your work.
2. If you are stuck on a word problem with probabilities, try:
   - Defining the events which are occurring
   - Writing given probabilities in terms of events
   - Checking equations to determine what you might be able to do with what you are given
3. For continuous variables, $\int_{-\infty}^{\infty} f(x)dx = 1$.
4. If you get a negative probability or variance, you made a mistake.

Other:

1. You are allowed one $9.5'' \times 11''$ note sheet (front and back)

Good Luck on Tuesday!