Assignment #5

1. A large company has an inspection system for the batches of small compressors purchased from vendors. A batch typically contains 15 compressors. In the inspection system, a random sample of 5 is selected and all are tested. Suppose there are 2 faulty compressors in the batch of 15.
   (a) What is the probability that for a given sample there will be 1 faulty compressor?
   (b) What is the probability that inspection will discover both faulty compressors?

2. On average, 3 traffic accidents per month occur at a certain intersection. What is the probability that in any given month at this intersection
   (a) exactly 5 accidents will occur?
   (b) fewer than 3 accidents will occur?
   (c) at least 2 accidents will occur?

3. A production process outputs items in lots of 50. Sampling plans exist in which lots are pulled aside periodically and exposed to a certain type of inspection. It is usually assumed that the proportion defective is very small. It is important to the company that lots containing defectives be a rare event. The current inspection plan is to periodically sample randomly 10 out of the 50 items in a lot and, if none are defective, to perform no intervention.
   (a) Suppose in a lot chosen at random, 2 out of 50 are defective. What is the probability that at least 1 in the sample of 10 from the lot is defective?
   (b) From your answer to part (a), comment on the quality of this sampling plan.
   (c) What is the mean number of defects found out of 10 items sampled?

4. Buses arrive at a specified stop at 15-minute intervals starting at 7 A.M. That is, they arrive at 7, 7:15, 7:30, 7:45, and so on. If a passenger arrives at the stop at a time that is uniformly distributed between 7 and 7:30, find the probability that he waits (a) less than 5 minutes for a bus;
   (b) at least 12 minutes for a bus.

5. If X is a normal random variable with mean 3 and variance 16, find
   (a) $P(X < 11)$

   (b) $P(X > -1)$

6. Data from the National Oceanic and Atmospheric Administration indicate that the yearly precipitation in Los Angeles is a normal random variable with a mean of 12.08 inches and a standard deviation of 3.1 inches. Find the probability that the total precipitation during the next 2 years will exceed 25 inches.

7. The average grade for an exam is 74, and the standard deviation is 7.
   (a) What is the 25% quartile?
   (b) What is the 50% quartile?
   (c) What is the 75% quartile?
   (d) If we want to curve the exam so that 20% of the class is given As, and the grades are curved to follow a normal distribution, what is the lowest possible A and the highest possible B?

8. There are four independent random variables that each have a uniform distribution on the interval $[0, 4]$.
   (a) What is the mean of the sum of the four variables?
   (b) What is the variance of the sum of the four variables?
   (c) What is the lower limit of the range?
   (d) What is the upper limit of the range?

9. The life, in years, of a certain type of electrical switch has an exponential distribution with an average life $\beta = 2$. If 1 of this type of switch is installed, what is the probability that it will fail during the first year?

10. A certain type of device has an advertised failure rate of 0.01 per hour. The failure rate is constant and the exponential distribution applies.
    (a) What is the mean time to failure?

    (b) What is the probability that 200 hours will pass before a failure is observed?