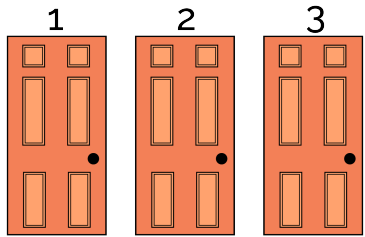
# More Probability

## Chapter 2

# Trees

- Help us
  - organize computations
  - compute total probabilities
  - compute Bayes' formula

# Let's Make a Deal (Monty Hall Problem)



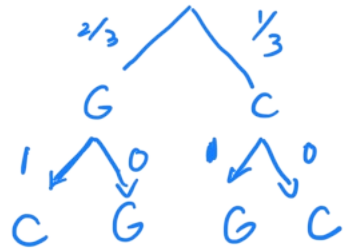

1. One door hides a car; two hide goats.
2. The contestant chooses a door.
3. Monty (who knows where the car is) opens a *different* door revealing a goat.
4. The contestant may switch doors or keep the original choice.

What is the best strategy for winning a car?
(a) Switch (b) Don't switch (c) It doesn't matter

# Monty Hall Problem Solution

O: original choice

S: switch




$P(C|S) =$

$2/3 \times 1 + 1/3 \times 0$

$= 2/3$

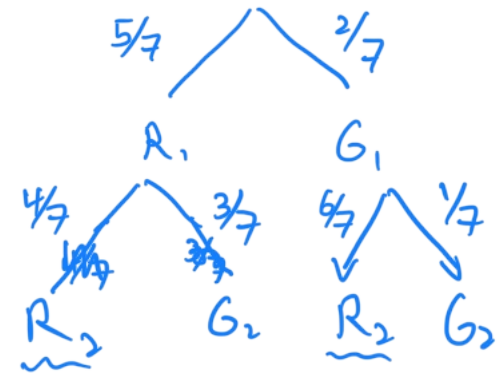
If you switch, you will win when your initial pick was a goat.

$P(C|S) = P(G|O)$

# Ball Example

- Example:
  - An urn contains 5 red balls and 2 green balls
  - A ball is drawn at random
  - The ball is replaced by a ball of the *other* color
- Questions:
  1. What is the probability the second ball is red?
  2. What is the probability the first ball was red given the second ball was red?

# Solution




$$P(R_2) = P(R_1 \cap R_2) + P(G_1 \cap R_2)$$
$$= \frac{5}{7} \times \frac{4}{7} + \frac{2}{7} \times \frac{6}{7}$$
$$= \frac{32}{49}$$

$$P(R_1 | R_2) = \frac{P(R_1 \cap R_2)}{P(R_2)} = \frac{\frac{20}{49}}{\frac{32}{49}} = \frac{20}{32} = \frac{5}{8}$$

# In-Class Exercise: Dice Game

$P(6\text{-sided} \mid \#)$

1. The Randomizer holds a 6-sided die in one fist and an 8-sided die in the other.
2. The Roller selects one of the Randomizer's fists and covertly takes the die.
3. The Roller rolls the die in secret and reports the result to the table.

**Question:** Given the reported number, what is the probability that the 6-sided die was chosen?
What is the probability that the 6-side died was chosen if the number rolled was 1 to 6?
What is the probability that the 6-side died was chosen if the number rolled was 7 to 8?

# Dice Game