Chapter 7: Functions of Random Variables

- Linear Combinations of Random Variables
  - Mean
  - Variance

Linear Combinations of Independent Normals

If $X_1, \ldots, X_n$ are independent random variables having normal distributions with means $\mu_1, \mu_2, \ldots, \mu_n$ and variances $\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2$, then the random variable

$$Y = a_1 X_1 + a_2 X_2 + \cdots + a_n X_n$$

has a normal distribution with mean

$$\mu_Y = a_1 \mu_1 + a_2 \mu_2 + \cdots + a_n \mu_n$$

and variance

$$\sigma_Y^2 = a_1^2 \sigma_1^2 + a_2^2 \sigma_2^2 + \cdots + a_n^2 \sigma_n^2.$$

Example: Sum of Normal Random Variables

The time that it takes a randomly selected rat of a certain subspecies to find its way through a maze is a normally distributed random variable with $\mu = 1.5$ min and $\sigma = 0.35$ min. Suppose five rats are selected.

Let $X_1, \ldots, X_5$ denote their times in the maze. Assuming the X_i's to be a random sample from this normal distribution, what is the probability that the total time

$$Y = X_1 + \cdots + X_5$$

for the five rats is between 6 and 8 min?

Example: Solution