1 With Series Resistance

The ideal diode equation is

$$I = I_0(e^{qV_d/kT} - 1) \tag{1.1}$$

I_0 is the reverse saturation current, q is the fundamental charge, V_d is the voltage across the diode, k is the Boltzmann constant, and T is the temperature in Kelvin.
As $V_d \to -\infty, I = -I_0$,

The current of a diode with a series resistance is

$$\begin{aligned} I &= I_0(e^{q(V_a - IR_s)/nkT} - 1) &(1.2)\\ &= I_0(e^{(V_a - IR_s)/nV_T} - 1) &(1.3)\end{aligned}$$

where

$$V_a - IR_s = V_d \tag{1.4}$$

and $V_T = kT/q$.

Introduce w

$$w = \frac{I_0 R_s}{V_T}\left(\frac{I}{I_0} + 1\right) \tag{1.5}$$

Substitute in the ideal diode equation

$$w = \frac{I_0 R_s}{nV_T}\left[(e^{q(V_a - IR_s)/nkT}\right] \tag{1.6}$$

Now we need to get the equation in the form we^w

$$\begin{aligned} we^w &= \frac{I_0 R_s}{nV_T}\left[(e^{(V_a - IR_s)/nV_T}\right] e^{\frac{I_0 R_s}{nV_T}\left(\frac{I}{I_0}+1\right)} &(1.7)\\ &= \frac{I_0 R_s}{nV_T}\left[(e^{\frac{V_a}{nV_T} - \frac{IR_s}{nV_T} + \frac{I_0 R_s I/I_0}{nV_T} + \frac{I_0 R_s}{V_T}})\right] &(1.8)\\ &= \frac{I_0 R_s}{nV_T}\left[(e^{(V_a + I_0 R_s)/V_T}\right] &(1.9)\end{aligned}$$

Using the Lambert $W-$function,

$$\begin{aligned} w &= W\left(we^w\right) &(1.10)\\ w &= W\left(\frac{I_0 R_s}{nV_T}\left[(e^{(V_a + I_0 R_s)/nV_T}\right]\right) &(1.11)\\ \frac{I_0 R_s}{nV_T}\left(\frac{I}{I_0} + 1\right) &= W\left(\frac{I_0 R_s}{nV_T}\left[(e^{(V_a + I_0 R_s)/nV_T}\right]\right) &(1.12)\\ I &= \frac{nV_T}{R_s} W\left(\frac{I_0 R_s}{nV_T}\left[(e^{(V_a + I_0 R_s)/nV_T}\right]\right) - I_0 &(1.13)\end{aligned}$$

2 With Series Resistance and Shunt Resistance

If the shunt resistance across the junction is included into the diode equation while the shunt losses at the periphery of the diode are considered negligible, then the diode equation becomes

$$I = I_0(e^{(V_a - IR_s)/nV_T} - 1) + \frac{V_a - IR_s}{R_{sh}} \tag{2.1}$$

$$= \frac{R_{sh}}{R_s + R_{sh}}\left[I_0(e^{(V_a - IR_s)/nV_T} - 1) + \frac{V_a}{R_{sh}}\right] \tag{2.2}$$

We again introduce w [1].

$$w = \left(I - \frac{V_a - I_0 R_{sh}}{R_{sh} + R_s}\right)\frac{R_s}{nV_T} \tag{2.3}$$

$$\tag{2.4}$$

Substituting the equation for I (Equation 2.2) into the expression for w,

$$w = \frac{I(R_{sh} + R_s) - V_a + I_0 R_{sh}}{R_{sh} + R_s}\frac{R_s}{nV_T} \tag{2.5}$$

$$= \frac{I_0 R_s R_{sh}}{nV_T(R_s + R_{sh})} e^{\frac{V_a - IRs}{nV_T}} \tag{2.6}$$

Thus,

$$we^w = \frac{I_0 R_s R_{sh}}{nV_T(R_s + R_{sh})} \exp\left[\frac{V_a - IRs}{nV_t}\right] \exp\left[\left(I - \frac{V_a - I_0 R_{sh}}{R_{sh} + R_s}\right)\frac{R_s}{nV_T}\right] \tag{2.7}$$

$$= \frac{I_0 R_s R_{sh}}{nV_T(R_s + R_{sh})} \exp\left[\frac{R_{sh}(V_a + I_0 R_s)}{nV_T(R_{sh} + R_s)}\right] \tag{2.8}$$

Then, applying the Lambert W function like before, we are left with an explicit equation for current.

$$w = W[we^w] \tag{2.9}$$

$$\frac{R_s}{nV_T}\left[I - \frac{V_a - I_0 R_{sh}}{R_{sh} + R_s}\right] = W\left[\frac{I_0 R_s R_{sh}}{nV_T(R_{sh} + R_s)} e^{\frac{R_{sh}(V_a + I_0 R_s)}{nV_T(R_{sh} + R_s)}}\right] \tag{2.10}$$

$$I = \frac{nV_T}{R_s} W\left[\frac{I_0 R_s R_{sh}}{nV_T(R_{sh} + R_s)} e^{\frac{R_{sh}(V_a + I_0 R_s)}{nV_T(R_{sh} + R_s)}}\right] + \left(\frac{V_a - I_0 R_{sh}}{R_{sh} + R_s}\right) \tag{2.11}$$

To get the equation for voltage, we introduce

$$w = \frac{I(R_{sh} + R_s) + I_0 R_{sh} - V_a}{nV_T} \tag{2.12}$$

Substituting I into this expression, we get

$$w = \frac{I_0 R_{sh} e^{\frac{V_a - IR_s}{nV_T}}}{nV_T} \tag{2.13}$$

$$we^w = I_0 R_{sh} e^{\frac{R_{sh}(I_0+I)}{nV_T}} \tag{2.14}$$

Thus,

$$w = W[we^w] \tag{2.15}$$

$$\frac{I(R_{sh}+R_s)+I_0R_{sh}-V_a}{nV_T} = W\left[I_0R_{sh}e^{\frac{R_{sh}(I_0+I)}{nV_T}}\right] \tag{2.16}$$

$$V_a = I(R_{sh}+R_s)+I_0R_{sh}-V_a-nV_TW\left[I_0R_{sh}e^{\frac{R_{sh}(I_0+I)}{nV_T}}\right] \tag{2.17}$$

$$\tag{2.18}$$

3 Matlab Code

Use `iv.fit('Rs')` to do the fit with just a series resistance. Use `iv.fit('Rs_Rsh')` to do the fit with both series and shunt resistance.

This code uses `lsqcurvefit()` to fit a curve to the diode equation with series and shunt resistance. `lsqcurvefit()` is a nonlinear least-squares fit, and the parameters by which it fits can be set in `options`.
In order to fit the data using `lsqcurvefit()` it is necessary to have what are called guess parameters. These guess parameters are used as a starting point for each of the fit parameters, and are found using approximations.

3.1 Guessing the Series Resistance

In order to obtain an approximate value for the series resistance we assume that the shunt resistance is infinite thus causing the diode equation to take the form

$$I = I_0(e^{(V_a-IR_s)/nV_T}-1) \tag{3.1}$$

$$\tag{3.2}$$

Now taking the derivative with respect to V_a

$$\frac{dI}{dV_a} = I_0e^{(V_a-IR_s)/nV_T}(1-\frac{dI}{dV_a}R_s)/(nV_T) \tag{3.3}$$

Rearranging terms,

$$\frac{dI}{dV_a}\left[1+\frac{R_s}{nV_T}e^{(V_a-IR_s)/(nV_T)}\right] = \frac{I_0}{nV_T}e^{(V_a-IR_s)/(nV_T))} \tag{3.4}$$

$$\frac{dI}{dV_a} = \frac{\frac{I_0}{V_T}e^{(V_a-IR_s)/(nV_T)}}{1+\frac{I_0R_s}{V_T}e^{(V_a-IR_s)/(nV_T)}} \tag{3.5}$$

Recalling that

$$V_d = V_a - IR_s \tag{3.6}$$

We have

$$\frac{dI}{dV_a} = \frac{\frac{I_0}{V_T}e^{(V_d)/(nV_T)}}{1 + \frac{I_0 R_s}{V_T}e^{(V_d)/(nV_T)}} \tag{3.7}$$

Finally, we take the limit as V_d goes infinity, which corresponds to the large positive voltage regime. The 1 in the denominator becomes insignificant allowing the exponential terms to cancel and we are left with

$$\frac{dI}{dV_a} \approx \frac{1}{R_s} \tag{3.8}$$

So we can roughly approximate the series resistance as being the inverse of the slope of the diode curve at large positive voltages.

3.2 Guessing the Shunt Resistance

Using a similar method as in 3.1, we can find an approximate value for the shunt resistance. Starting with the full diode equation with R_{sh} and R_s

$$I = I_0(e^{(V_a - IR_s)/nV_T} - 1) + \frac{V_a - IR_s}{R_{sh}} \tag{3.9}$$

$$\tag{3.10}$$

If we take the limit as V_a goes to minus infinity then exponential term goes to zero, while the IR_s becomes insignificant relative to V_a. At large negative V_a,

$$I \approx -I_0 + \frac{V_a}{R_{sh}} \tag{3.11}$$

$$\tag{3.12}$$

Now taking the derivative with respect to V_a

$$\frac{dI}{dV_a} \approx \frac{1}{R_{sh}} \tag{3.13}$$

Thus, the shunt resistance is approximately the inverse of the slope of the diode curve at large negative voltages.

3.3 Guessing the Saturation Current

From the basic diode equation (Eq. 1.1), the saturation current, I_0, can be approximated as the minimum current. If one were to set a very large negative voltage, then the current would just be equal to the saturation current.

3.4 Guessing the Ideality Factor

The ideality factor, n, represents the deviation from an ideal diode, and takes into account all of the second order effects. We use an `n_guess` value of 2.

3.5 Bounds on Fitted Values

In order to constrain the fitted values to the real plane and help the program produce feasible results, bounds must be placed on the `lsqcurvefit()` guess values. The bounds for the Saturation current are between 0 and 1 Amps, because the saturation current will be very small in a decent diode. Bounds for the ideality factor are between 0 and 50, so reasonable values are used to fit the ideality factor. The bounds for the series resistance are between 0 and 10000 Ohms since we expect a relatively low series resistance. Whereas the bounds for the shunt resistance are between 0 and 10000000 Ohms since we expect a high shunt resistance.

References

[1] A. Ortiz-Conde, F. J. Garca Sánchez, and J. Muci, "Exact analytical solutions of the forward non-ideal diode equation with series and shunt parasitic resistances," *Solid-State Electronics*, vol. 44, no. 10, pp. 1861–1864, Oct. 2000. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0038110100001325