Contents

1 Nanowires

For nanowires, the leaky resonance modes are according to Fountaine [1]

$$\pm\left(\frac{1}{k_{cyl}^2-k_{air}^2}\right)^2\left(\frac{k_z m}{k_0 a}\right)^2=\left(\frac{\epsilon_{cyl}}{k_{cyl}}\frac{J'_m(k_{cyl}a)}{J_m(k_{cyl}a)}-\frac{1}{k_{air}}\frac{H'_m(k_{air}a)}{H_m(k_{air}a)}\right)\times\left(\frac{1}{k_{cyl}}\frac{J'_m(k_{cyl}a)}{J_m(k_{cyl}a)}-\frac{1}{k_{air}}\frac{H'_m(k_{air}a)}{H_m(k_{air}a)}\right) \tag{1.1}$$

k_{cyl} is the transverse component of the wavevector inside the cylinder.

$$k_{cyl}=\left[\left(\frac{n_{cyl}\omega}{c}\right)^2-\beta^2\right]^{1/2} \tag{1.2}$$

$k_{air}=\left[\left(\frac{\omega}{c}\right)^2-\beta^2\right]^{1/2}$

According to Linyou's dissertation,

$$\kappa^2=k_0^2n^2-\beta^2 \tag{1.3}$$

$$\gamma^2=k_0^2-\beta^2 \tag{1.4}$$

κ and γ are the wave vectors in the transverse direction inside and outside of the cylindrical structure. If $\gamma^2<0$ and $\kappa^2>0$, then it is a guided mode. If $\gamma^2>0$, then it is a leaky mode.

$$\left(\frac{1}{\kappa^2}-\frac{1}{\gamma^2}\right)^2\left(\frac{\beta m}{a}\right)^2=k_0^2\left(n^2\frac{J'_m(\kappa a)}{\kappa J_m(\kappa a)}-n_0^2\frac{H'_m(\gamma a)}{\gamma H_m(\gamma a)}\right)\left(\frac{J'_m(\kappa a)}{\kappa J_m(\kappa a)}-\frac{H'_m(\gamma a)}{\gamma H_m(\gamma a)}\right) \tag{1.5}$$

This is the same as above, where $\kappa=k_{cyl}$ and $\gamma=k_{air}$. $\beta=k_z$. $n^2=\epsilon_{cyl}$.

Inside a subwavelength hole, the propagating modes are [2]

$$\left(\frac{1}{k_{cyl}^2-k_1^2}\right)^2\left(\frac{k_z m}{k_0 a}\right)^2=\left(\frac{\epsilon_{cyl}}{k_{cyl}}\frac{J'_m(k_{cyl}a)}{J_m(k_{cyl}a)}-\frac{\epsilon_1}{k_1}\frac{H'_m(k_1a)}{H_m(k_1a)}\right)\times\left(\frac{1}{k_{cyl}}\frac{J'_m(k_{cyl}a)}{J_m(k_{cyl}a)}-\frac{1}{k_{air}}\frac{H'_m(k_1a)}{H_m(k_1a)}\right) \tag{1.6}$$

1.1 Modal Cutoff

The lower limit $\beta_j=kn_{cl}$ of permissble values of the bound mode propagation constant is called modal cutoff. Modal cutoff, or just cutoff is defined by

$$U_j=V;W_j=0. \tag{1.7}$$

Below cutoff, these modes propagate with loss and are the leaky modes. Leaky modes are also referred to as quasiguided modes.

According to [3], leaky modes are bound modes below cutoff. The modal parameters are now complex.

The leaky-mode propagation constant satisfies,

$$0 \leq \beta^R < kn_{cl} \tag{1.8}$$

and

$$U^r > V \tag{1.9}$$

.

The cladding parameter is $Q = \left[\beta^2 - \left(\frac{n_1 \omega}{c}\right)^2\right]^{1/2} = iW$.

Just below cutoff, W is almost pure imaginary. Q is almost real.

Imaginary part is indicative of the radiative loss of the mode. This transitions to zero as the mode transitions from leaky to guided.

References

[1] K. T. Fountaine, W. S. Whitney, and H. A. Atwater, "Resonant absorption in semiconductor nanowires and nanowire arrays: Relating leaky waveguide modes to bloch photonic crystal modes," *Journal of Applied Physics*, vol. 116, no. 15, 2014. [Online]. Available: http://scitation.aip.org/content/aip/journal/jap/116/15/10.1063/1.4898758

[2] P. B. Catrysse and S. Fan, "Propagating plasmonic mode in nanoscale apertures and its implications for extraordinary transmission," *Journal of Nanophotonics*, vol. 2, no. 1, pp. 021 790–021 790–20, 2008. [Online]. Available: http://dx.doi.org/10.1117/1.2890424

[3] A. W. Snyder and J. Love, *Optical Waveguide Theory*, 1st ed. Springer, Nov. 1983.