Contents

1 Leaky Waveguide Modes

1.1 TE

Why use this convention as opposed to in Yariv and Yeh? In Yariv and Yeh, outside the semiconductor, $k_{1x} = \left[\beta^2 - \left(\frac{n_1\omega}{c}\right)^2\right]^{1/2}$. Since $\beta > \frac{n_1\omega}{c}$, k_{1x} is real. In this derivation, k_{1x} must be imaginary to be confined. Yariv and Yeh focuses on guided modes, where as this is for leaky waveguide modes.

$k_{1x} = \left[\left(\frac{n_1\omega}{c}\right)^2 - \beta^2\right]^{1/2}$ and $k_{2x} = \left[\left(\frac{n_2\omega}{c}\right)^2 - \beta^2\right]^{1/2}$.

For TE modes, there are only components E_y, H_x, H_z. E_y is out of plane. For even mode, $\mathbf{H}(-\mathbf{r}) = \mathbf{H}(\mathbf{r})$ and $-\mathbf{E}(-\mathbf{r}) = \mathbf{E}(\mathbf{r})$.

$$\hat{O}_{M_x}\mathbf{E}(\mathbf{r}) = M_x\mathbf{E}(M_x\mathbf{r}) \tag{1.1}$$

$(E_x, E_y, E_z)(x, y, z) = (-E_x, E_y, E_z)(-x, y, z)$.

For even TE modes, E_y is the same above and below the midplane of the thin film or $x = 0$. $E_y(x, y, z) = E_y(-x, y, z)$. Thus, for even TE modes

$$ik_{2x}\tan(k_{2x}\frac{1}{2}d) = k_x \tag{1.2}$$

and

$$k_x^2 + k_{2x}^2 = V \tag{1.3}$$

or

$$\boxed{\tan(k_{2x}\frac{1}{2}d) = -i\frac{k_x}{k_{2x}}.} \tag{1.4}$$

For odd modes, $-\mathbf{H}(-\mathbf{r}) = \mathbf{H}(\mathbf{r})$ and $\mathbf{E}(-\mathbf{r}) = \mathbf{E}(\mathbf{r})$.

$$\hat{O}_{M_x}\mathbf{E}(\mathbf{r}) = -M_x\mathbf{E}(M_x\mathbf{r}) \tag{1.5}$$

$(E_x, E_y, E_z)(x, y, z) = (E_x, -E_y, -E_z)(-x, y, z)$.

For TE modes, E_y has a sign change about the midplane of the thin film or $x = 0$. $E_y(x, y, z) = -E_y(-x, y, z)$. For odd TE modes,

$$\boxed{k_{2x}\cot(k_{2x}\frac{1}{2}d) = ik_x} \tag{1.6}$$

The two equations can be combined

$$\tan(k_{2x}d) = -\frac{i2k_x k_{2x}}{(k_{2x}^2 + k_x^2)} \tag{1.7}$$

1.2 TM

For TM modes, there are only components H_y, E_x, E_z. H_y is out of plane. For even modes, $\mathbf{H}(-\mathbf{r}) = \mathbf{H}(\mathbf{r})$

$$\hat{O}_{M_x}\mathbf{H}(\mathbf{r}) = M_x\mathbf{H}(M_x\mathbf{r}) \tag{1.8}$$

$(H_x, H_y, H_z)(x, y, z) = -(-H_x, H_y, H_z)(-x, y, z) = (H_x, -H_y, -H_z)(-x, y, z)$ For TM modes, H_y is the flipped above and below the midplane of the thin film or $x = 0$. $H_y(x, y, z) = -H_y(-x, y, z)$.

For TM modes, E_y has a sign change about the midplane of the thin film or $x = 0$. The solution is

$$\boxed{k_{2x}\cot(k_{2x}\frac{1}{2}d) = i\frac{n_2^2}{n_1^2}k_x} \tag{1.9}$$

For odd modes, $\mathbf{H}(-\mathbf{r}) = -\mathbf{H}(\mathbf{r})$

$$\hat{O}_{M_x}\mathbf{H}(\mathbf{r}) = -M_x\mathbf{H}(M_x\mathbf{r}) \tag{1.10}$$

$(H_x, H_y, H_z)(x, y, z) = -(-H_x, H_y, H_z)(-x, y, z) = (H_x, -H_y, -H_z)(-x, y, z)$. For TM modes, H_y is the same above and below the midplane of the thin film or $x = 0$. $H_y(x, y, z) = H_y(-x, y, z)$.

$$\boxed{ik_{2x}\tan(k_{2x}\frac{1}{2}d) = \frac{n_2^2}{n_1^2}k_x} \tag{1.11}$$

The way it is described in some textbooks, TM even means H_y is even and TM odd means H_y is odd. However, since H is a pseudovector, these definitions should be flipped.

The two equations can be combined to be

$$\tan(k_{2x}d) = -\frac{i2\bar{k_x}k_{2x}}{(k_{2x}^2 + \bar{k_x}^2)} \tag{1.12}$$

where

$$\bar{k_x} = \frac{n_2^2}{n_1^2}k_x \tag{1.13}$$

1.3 Normal Incidence; $\beta = 0$

At normal incidence, the even TE mode becomes

$$\begin{aligned} n_2 k \tan(n_2 k \frac{1}{2} d) &= -i n_1 k \\ \tan(n_2 k \frac{1}{2} d) &= -i \frac{n_1}{n_2} \end{aligned} \tag{1.14}$$

where $k = \frac{\omega}{c}$.

$$\boxed{\cot(n_2 k \frac{1}{2} d) = i \frac{n_2}{n_1}} \tag{1.15}$$

The odd TE mode becomes

$$\boxed{\tan(n_2 k \frac{1}{2} d) = -i \frac{n_2}{n_1}} \tag{1.16}$$

The TM modes result in the same equations for $\beta = 0$. These equations are the same as in Linyou's, except for the 1/2 [1].

1.4 Pala

Following [2], the solution to 50 nm Si waveguide modes with SiO_2 on both sides is. Use script `Pala.m`.

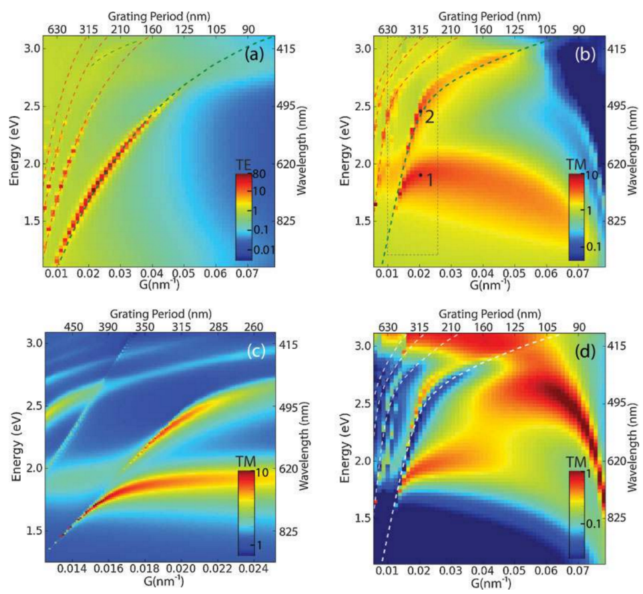


Figure 1: (a) TE illumination. (b) TM illumination.

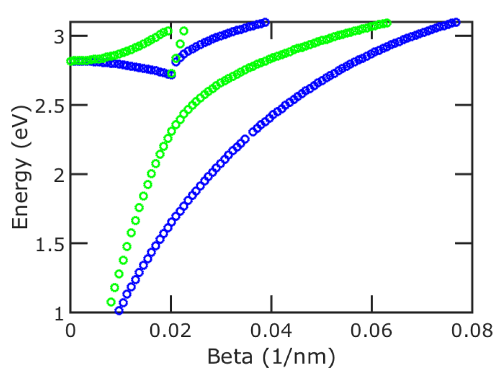


Figure 2: Leaky TE Modes in blue, TM modes in green

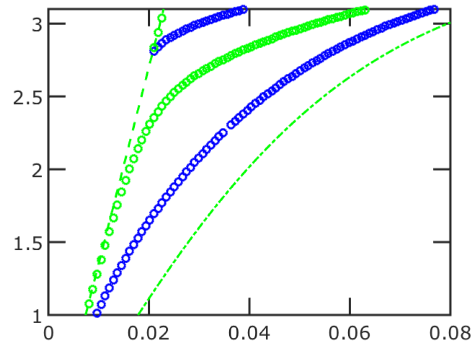


Figure 3: Guided TE Modes in blue, TM modes in green using Yariv's equations

1.5 Joannopoulos

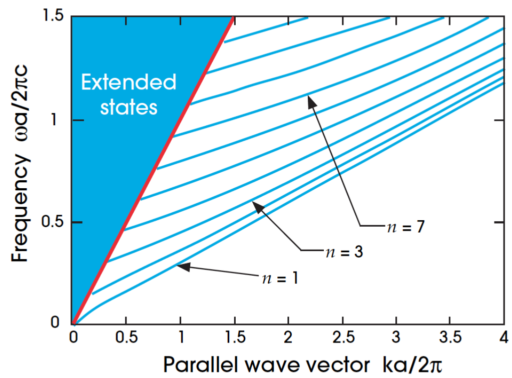


Figure 4: Figure from paper.

We try to duplicate this with $a = 50$ nm. I The minimum $\omega = 0$. The maximum $\omega =$

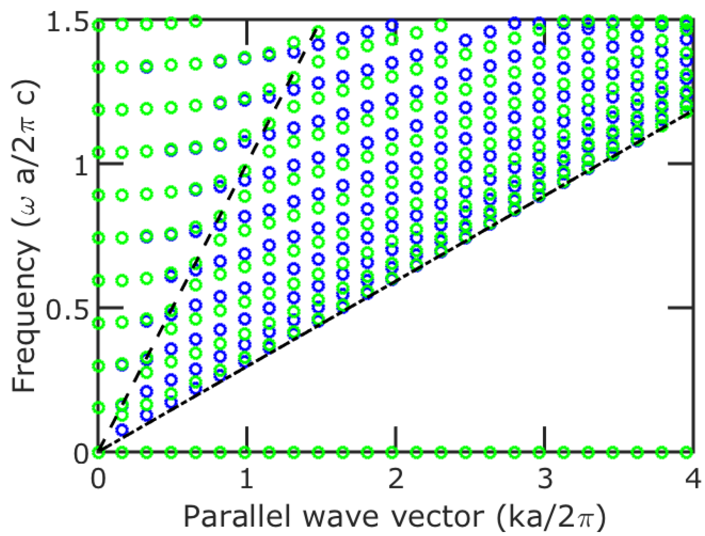


Figure 5: Figure from paper.

2 With back metal

For TE modes,

$$\boxed{k_{2x} \cot(k_{2x} d) = i k_x} \tag{2.1}$$

The equation is just like the equation for an odd TE mode except that the thickness is now d instead of $d/2$ in the equation. This intuitively makes sense since the $E_y = 0$ at $x = 0$. Thus, only the odd TE solutions from before are possible.

For TM modes,

$$\boxed{ik_{2x}\tan(k_{2x}d) = \frac{n_2^2}{n_1^2}k_x} \tag{2.2}$$

The equation is just like the equation for an odd TE mode except that the thickness is now d instead of $d/2$ in the equation. This intuitively makes sense since the $E_y = 0$ at $x = 0$. Thus, only the odd TE solutions from before are possible.

2.1 Normal Incidence

In the limit $\beta \to 0$, $k_x = n_1 k$ and $k_{2x} = n_2 k$.

$$\boxed{\tan(n_2 kd) = -i\frac{n_2}{n_1}} \tag{2.3}$$

This is just the odd mode before with thickness d instead of $d/2$.

2.2 Our Hemisphere Paper

This calculates the waveguide modes for 100 nm thick Si on metal. Use script `Gao_Hemisphere.m`.

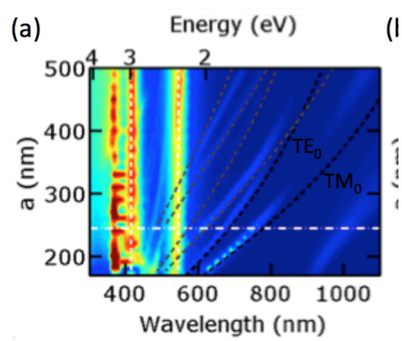


Figure 6: Figure from paper.

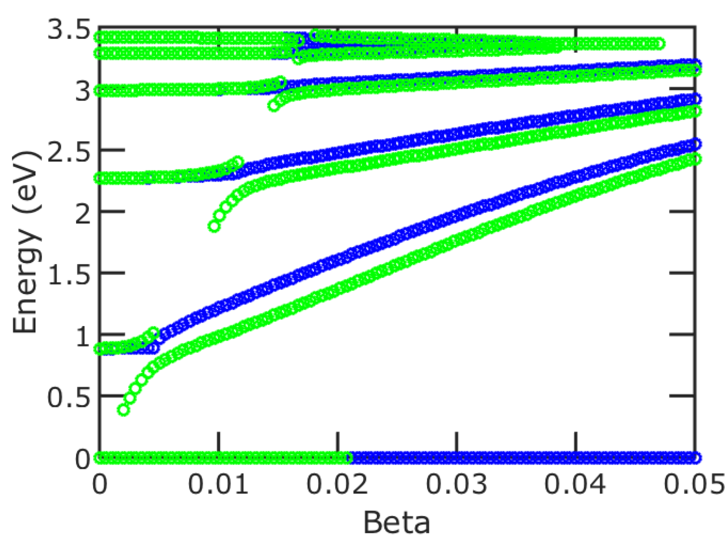


Figure 7: Leaky TE Modes in blue. TM modes in green.

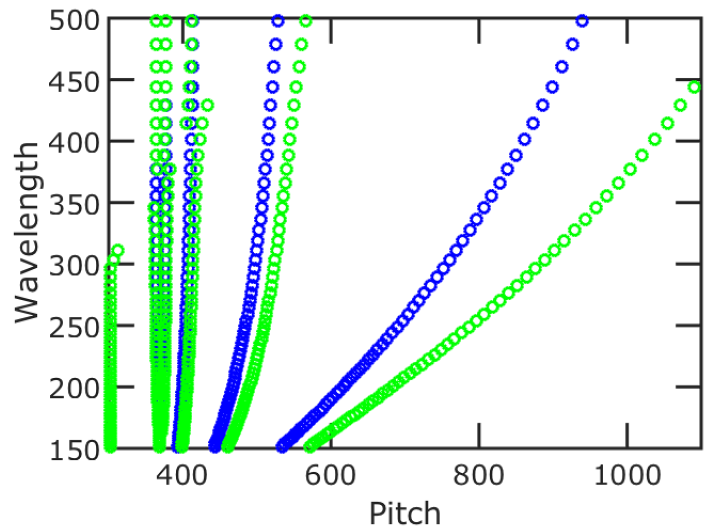


Figure 8: Leaky TE modes in blue. Leaky TM modes in green.

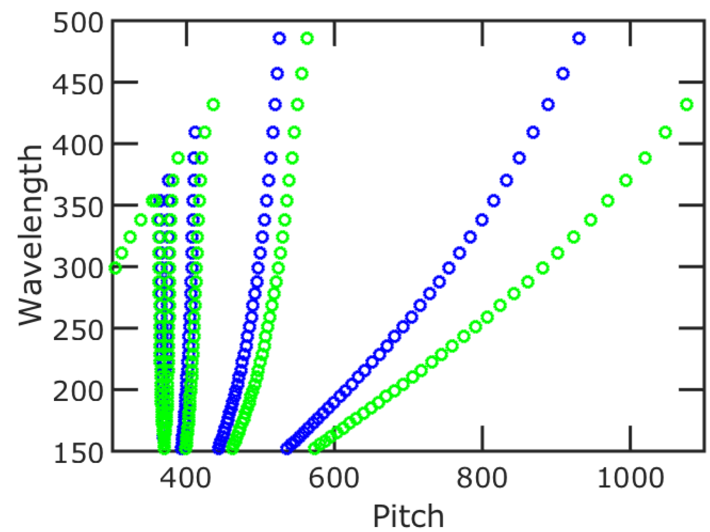


Figure 9: Guided Waveguide modes.

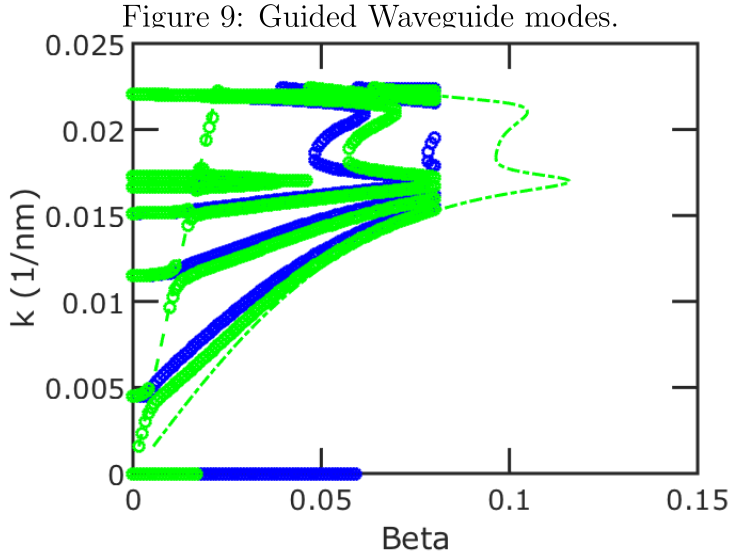


Figure 10

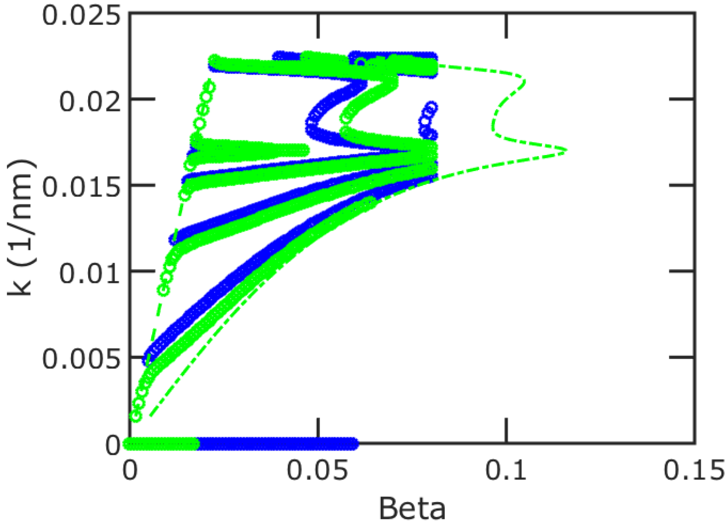
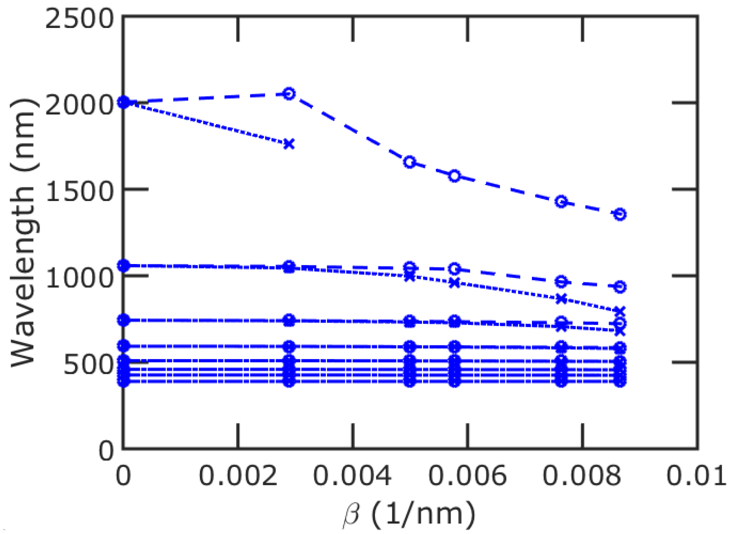


Figure 11: Using Yariv's equations

2.3 Our Double Sided Metal Nanomesh Paper



3 Both sides metal

4 Yariv

4.1 Symmetric Slab Waveguides

4.1.1 Guided TE Modes

$$h = \left[\left(\frac{n_2 \omega}{c} \right)^2 - \beta^2 \right]^{1/2} \tag{4.1}$$

$$q = \left[\beta^2 - \left(\frac{n_1 \omega}{c} \right)^2 \right]^{1/2} \tag{4.2}$$

The symmetric modes are

$$h \tan(\frac{1}{2} hd) = q \tag{4.3}$$

The antisymmetric modes are

$$h \cot(\frac{1}{2} hd) = -q. \tag{4.4}$$

The two equations can be combined

$$\tan(hd) = \frac{2hq}{h^2 - q^2} \tag{4.5}$$

For $\beta = 0$, the symmetric modes are

$$\tan(\frac{1}{2}n_2kd) = i\frac{n_1}{n_2} \tag{4.6}$$

or

$$\cot(\frac{1}{2}n_2kd) = -i\frac{n_2}{n_1} \tag{4.7}$$

and the antisymmetric modes are

$$\cot(\frac{1}{2}n_2kd) = -i\frac{n_1}{n_2} \tag{4.8}$$

or

$$\tan(\frac{1}{2}n_2kd) = i\frac{n_2}{n_1} \tag{4.9}$$

Note that these equations are slightly different from Linyou's. Linyou's equations are for the even mode, $\cot(\frac{1}{2}n_2kd) = i\frac{n_2}{n_1}$ and the odd mode, $\tan(\frac{1}{2}n_2kd) = -i\frac{n_2}{n_1}$.

Compare this with the previous equations. $k_{2x} = h$. However, $k_{1x} = iq$. When $\beta = 0$, $q = in_1k$ and $k_x = n_1k$. The symmetric mode is

$$k_{2x}\tan(\frac{1}{2}k_{2x}d) = -ik_x. \tag{4.10}$$

and the antisymmetric mode is

$$k_{2x}\cot(\frac{1}{2}k_{2x}d) = ik_x \tag{4.11}$$

For bound or guided modes, use q. For radiation modes, use k_{1x}.

Using previous notation,

$$k_{Si} = \left[\left(\frac{n_2\omega}{c}\right)^2 - \beta^2\right]^{1/2} \tag{4.12}$$

$$k_x = \left[\beta^2 - \left(\frac{n_1\omega}{c}\right)^2\right]^{1/2} \tag{4.13}$$

4.1.2 Guided TM Modes

The even modes are really odd, since the magnetic field is a pseudovector. The even modes are

$$h\tan(\frac{1}{2}hd) = \frac{n_2^2}{n_1^2}q \tag{4.14}$$

The odd modes are

$$h\cot(\frac{1}{2}hd) = \frac{n_2^2}{n_1^2}q \tag{4.15}$$

The two equations can be combined

$$\tan(hd) = \frac{2h\bar{q}}{h^2 - \bar{q}^2} \tag{4.16}$$

where

$$\bar{q} = \frac{n_2^2}{n_1^2}q \tag{4.17}$$

4.2 With Back Metal

On metal, the modes are simply the odd modes with the 1/2 removed For TE modes,

$$h \cot(hd) = -q \tag{4.18}$$

For TM modes,

$$h \tan(hd) = \frac{n_2^2}{n_1^2} q \tag{4.19}$$

4.3 Assymmetric Waveguides

$$h = \left[\left(\frac{n_2 \omega}{c} \right)^2 - \beta^2 \right]^{1/2} \tag{4.20}$$

$$q = \left[\beta^2 - \left(\frac{n_1 \omega}{c} \right)^2 \right]^{1/2} \tag{4.21}$$

$$p = \left[\left(\frac{n_3 \omega}{c} \right)^2 - \beta^2 \right]^{1/2} \tag{4.22}$$

The TE modes are given by

$$\tan(ht) = \frac{p + q}{h(1 - pq/h^2)} \tag{4.23}$$

$$\tan(ht) = \frac{h(p + q)}{h^2 - pq} \tag{4.24}$$

This is the same formula given in [3] where $\bar{\beta} = h$, $\beta_s = p$, and $\beta = q$.

For TM modes,

$$\tan(ht) = \frac{h(\bar{p} + \bar{q})}{(h^2 - \bar{p}\bar{q})} \tag{4.25}$$

$$\tag{4.26}$$

$\bar{p} = \frac{n_2^2}{n_3^2} p$ and $\bar{q} = \frac{n_2^2}{n_1^2} q$

Rearranging, we get

$$\tan(ht) = \frac{\bar{\epsilon} h(p + \epsilon_s q)}{\epsilon_s h^2 - \bar{\epsilon}^2 \bar{q}} \tag{4.27}$$

$$\tag{4.28}$$

which is the same as in [3].

References

[1] Y. Yu and L. Cao, "Coupled leaky mode theory for light absorption in 2D, 1D, and 0D semiconductor nanostructures," *Optics Express*, vol. 20, no. 13, pp. 13 847–13 856, Jun. 2012. [Online]. Available: http://www.opticsexpress.org/abstract.cfm?URI=oe-20-13-13847

[2] R. A. Pala, J. White, E. Barnard, J. Liu, and M. L. Brongersma, "Design of plasmonic thin-film solar cells with broadband absorption enhancements," *Advanced Materials*, vol. 21, no. 34, pp. 3504–3509, Sep. 2009. [Online]. Available: http://onlinelibrary.wiley.com/doi/10.1002/adma.200900331/abstract

[3] S. G. Tikhodeev, A. L. Yablonskii, E. A. Muljarov, N. A. Gippius, and T. Ishihara, "Quasiguided modes and optical properties of photonic crystal slabs," *Physical Review B*, vol. 66, no. 4, p. 045102, Jul. 2002. [Online]. Available: http://link.aps.org/doi/10.1103/PhysRevB.66.045102