HW#7 Matrix Operations

SP1) $A = [\,7.2 \;\; -4.3 \;\; 0.6 \;\; 1.7\,]$

$B = [\,-11.0 \;\; 11.8 \;\; 2.4 \;\; -1.9\,]$

$C = \begin{bmatrix} 1.7 \\ 1.0 \\ -1.0 \\ 4.3 \end{bmatrix}$ $D = \begin{bmatrix} -2.4 \\ -0.7 \\ -6.8 \\ 3.0 \end{bmatrix}$

$A + B = [\,(7.2 - 11.0) \;\; (-4.3 + 11.8) \;\; (0.6 + 2.4) \;\; (1.7 - 1.9)\,]$

$$\boxed{\vec{A} + \vec{B} = [\,-3.8 \;\; 7.5 \;\; 3 \;\; -.2\,]}$$

$A - B = [\,(7.2 + 11.0) \;\; (-4.3 - 11.8) \;\; (0.6 - 2.4) \;\; (1.7 + 1.9)\,]$

$$\boxed{\vec{A} - \vec{B} = [\,18.2 \;\; -16.1 \;\; -1.8 \;\; 3.6\,]}$$

$$\vec{C}+\vec{D} = \begin{bmatrix} (1.7-2.4) \\ (1.0-0.7) \\ (-1.0-6.8) \\ (4.3+3.0) \end{bmatrix} = \begin{bmatrix} -.7 \\ .3 \\ -7.8 \\ 7.3 \end{bmatrix}$$

$$\vec{C}-\vec{D} = \begin{bmatrix} (1.7+2.4) \\ (1.0+0.7) \\ (-1.0+6.8) \\ (4.3-3.0) \end{bmatrix} = \begin{bmatrix} 4.1 \\ 1.7 \\ 5.8 \\ 1.3 \end{bmatrix}$$

SP2)

$$R_1 = 3A - 2B$$

$$R_2 = 5C + 7D$$

$$\vec{R}_1 = 3\left[7.2\ \ -4.3\ \ 0.6\ \ 1.7\right] - 2\left[-11.0\ \ 11.8\ \ 2.4\ \ -1.9\right]$$

$$\vec{R}_1 = \left[21.6\ \ -12.9\ \ 1.8\ \ 5.1\right] + \left[22\ \ -23.6\ \ -4.8\ \ 3.8\right]$$

$$\boxed{\vec{R}_1 = \left[43.6\ \ -36.5\ \ 3\ \ 8.9\right]}$$

$R_2 = 5c + 2d$

$$\vec{R}_2 = 5\begin{bmatrix} 1.7 \\ 1.0 \\ -1.0 \\ 4.3 \end{bmatrix} + 2\begin{bmatrix} -2.4 \\ -0.7 \\ -6.8 \\ 3.0 \end{bmatrix}$$

$$\vec{R}_2 = \begin{bmatrix} 8.5 \\ 5 \\ -5 \\ 21.5 \end{bmatrix} + \begin{bmatrix} -4.8 \\ -1.4 \\ -13.6 \\ 6 \end{bmatrix}$$

$$\boxed{\vec{R}_2 = \begin{bmatrix} 3.7 \\ 3.6 \\ -18.6 \\ 27.5 \end{bmatrix}}$$

SP3.) $E = [7\ -1\ 4\ 2\ -8]$

$F = [1\ 2\ 9\ 0\ -4]$

$2E - 3F + R_3 = 0$, $\boxed{\vec{R}_3 = [-11\ 8\ 19\ -4\ 4]}$

$R_3 = 3F - 2E$

$\vec{R}_3 = 3[1\ 2\ 9\ 0\ -4] - 2[7\ -1\ 4\ 2\ -8]$

$\vec{R}_3 = [3, 6, 27, 0\ -12] + [-14\ 2\ -8\ -4\ 16]$

SP4)

$$\vec{R} = (\vec{A}\cdot\vec{B})(2\vec{A}+\vec{B})$$

$$\vec{A}:\vec{B} = (7.2)(-11.0) + (-4.3)(11.8) + (0.6)(2.4) + (1.7)(-1.9)$$

$$\vec{A}\cdot\vec{B} = -131.73$$

$$2\vec{A} = [14.4 \ -8.6 \ 1.2 \ 3.4]$$

$$(2\vec{A}+\vec{B}) = [(14.4-11.0) \ (-8.6+11.8) \ (0.6+2.4) \ (1.7-1.9)]$$

$$(2\vec{A}+\vec{B}) = [3.4 \ \ 3.2 \ \ 3 \ \ -0.2]$$

$$\boxed{\vec{R} = -131.73 \begin{bmatrix} 3.4 \\ 3.2 \\ 3 \\ -0.2 \end{bmatrix} = \begin{bmatrix} -447.882 \\ -421.536 \\ -395.19 \\ 26.346 \end{bmatrix}}$$

SP5.)

$\vec{G} = [2\ -3\ 5]$, $\vec{H} = [1\ 4\ -2]$

$$\boxed{\vec{G} \cdot \vec{H} = (2)(1) + (-3)(4) + (5)(-2) = -20}$$

$$|\vec{G}| = \sqrt{(2)^2 + (-3)^2 + (5)^2} = \sqrt{38}$$

$$|\vec{H}| = \sqrt{(1)^2 + (4)^2 + (-2)^2} = \sqrt{21}$$

$$\vec{G} \cdot \vec{H} = |\vec{G}||\vec{H}| \cos\theta_{GH}$$

$$\frac{-20}{(\sqrt{38})(\sqrt{21})} = \cos\theta_{GH}$$

$$\boxed{\theta_{GH} = \cos^{-1}\left(\frac{-20}{(\sqrt{38})(\sqrt{21})}\right) = 135.07^\circ}$$

SP6.)

$$C = 3A - 2B$$

$$A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \\ -4 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -3 \\ 1 & 2 \\ -2 & -5 \end{bmatrix}$$

$$3A = \begin{bmatrix} 3(2) & 3(-1) \\ 3(0) & 3(3) \\ 3(-4) & 3(1) \end{bmatrix} = \begin{bmatrix} 6 & -3 \\ 0 & 9 \\ -12 & 3 \end{bmatrix}$$

$$2B = \begin{bmatrix} 2(4) & 2(-3) \\ 2(1) & 2(2) \\ 2(-2) & 2(-5) \end{bmatrix} = \begin{bmatrix} 8 & -6 \\ 2 & 4 \\ -4 & -10 \end{bmatrix}$$

$$C = 3A - 2B = \begin{bmatrix} 6 & -3 \\ 0 & 9 \\ -12 & 3 \end{bmatrix} - \begin{bmatrix} 8 & -6 \\ 2 & 4 \\ 4 & -10 \end{bmatrix}$$

$$\boxed{C = \begin{bmatrix} -2 & 3 \\ -2 & 5 \\ -16 & 13 \end{bmatrix}}$$

Sp7.)

$\vec{C}\vec{D}$

$$\vec{C} = \overbrace{\begin{bmatrix} 4 & 0 & -2 & 1 \\ 3 & -2 & 4 & 3 \end{bmatrix}}^{\text{"}2\times 4\text{"}}$$

$$\vec{D} = \underbrace{\begin{bmatrix} 3 \\ -2 \\ 1 \\ 4 \end{bmatrix}}_{\text{"}4\times 1\text{"}}$$

$2\times 4 \quad 4\times 1$

"2×1"

$$CD = \begin{bmatrix} 4(3) + (0)(-2) + (-2)(1) + (1)(4) \\ (3)(3) + (-2)(-2) + (4)(1) + (3)(4) \end{bmatrix}$$

$$\boxed{CD = \begin{bmatrix} 14 \\ 29 \end{bmatrix}}$$

Sp8.) CED

$$E = \begin{bmatrix} -2 & 1 & 9 & -2 \\ 3 & -1 & 2 & 7 \\ 0 & -2 & -3 & -9 \\ 5 & 7 & 1 & 6 \end{bmatrix}$$

$$CE = \begin{bmatrix} 4 & 0 & -2 & 1 \\ 3 & -2 & 4 & 3 \end{bmatrix} \begin{bmatrix} -2 & 1 & 9 & -2 \\ 3 & -1 & 2 & 7 \\ 0 & -2 & -3 & -9 \\ -5 & 7 & 1 & 6 \end{bmatrix}$$

$$CE = \begin{bmatrix} (-8-5) & (4+4+7) & (36+6+1) & (8+18+6) \\ (-6-6-15) & (3+2-8+21) & (27-4-12+3) & (-6-14-36+18) \end{bmatrix}$$

$$CE = \begin{bmatrix} -13 & 15 & 43 & 32 \\ -27 & 18 & 14 & -38 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 \\ -2 \\ 1 \\ 4 \end{bmatrix}$$

$2 \times 4 \quad 4 \times 1$

2×1

$$CED = \begin{bmatrix} (-13)(3) + (15)(-2) + (43)(1) + (32)(4) \\ (-27)(3) + (18)(-2) + (14)(1) + (-38)(4) \end{bmatrix}$$

$$\boxed{CED = \begin{bmatrix} 102 \\ -255 \end{bmatrix}}$$

SP9.)

$$F = \overbrace{\begin{bmatrix} -1 & 2 & 2 & 6 \\ 7 & -3 & -4 & 0 \end{bmatrix}}^{\text{"2 x 4"}}$$

$$G = \begin{bmatrix} 6 & 3 \\ -1 & 0 \\ 0 & -4 \\ 2 & 1 \end{bmatrix} \Bigg\} \text{"4x2"}$$

$$FG = \begin{bmatrix} (-6-2+12) & (-3-8+6) \\ (42+3) & (21+16) \end{bmatrix}$$

$$\boxed{FG = \begin{bmatrix} 4 & -5 \\ 45 & 37 \end{bmatrix}}$$

$$GF \begin{bmatrix} (-6+21) & (12-9) & (12-12) & (36) \\ (1) & (-2) & (-2) & (-6) \\ (-28) & (12) & (16) & 0 \\ (-2+7) & (4-3) & (4-4) & (12) \end{bmatrix}$$

$$\boxed{GF = \begin{bmatrix} 15 & 3 & 0 & 36 \\ 1 & -2 & -2 & -6 \\ -28 & 12 & 16 & 0 \\ 5 & 1 & 0 & 12 \end{bmatrix}}$$

SP10)

$$H = \begin{bmatrix} 1 & 3 & 0 \\ 2 & -1 & 0 \\ 3 & 2 & 0 \end{bmatrix}$$

$$J = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 5 & 7 \end{bmatrix}$$

$$HJ = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{HJ = \vec{0}}$$

$$JH = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ (2+10+21) & (6-5+14) & (0) \end{bmatrix}$$

$$\boxed{JH = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 33 & 15 & 0 \end{bmatrix}}$$

Sp11.)

$$[B_1 B_2 B_3]^T = B_3^T B_2^T B_1^T$$

$$B_1 = \begin{bmatrix} -4 & 1 \\ 2 & 3 \end{bmatrix}, \quad B_2 \begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix}, \quad B_3 \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$$

$$B_1^T = \begin{bmatrix} -4 & 2 \\ 1 & 3 \end{bmatrix}, \quad B_2^T = \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix}, \quad B_3^T = \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix}$$

$$B_1 B_2 = \begin{bmatrix} (-8+0) & (-4-3) \\ (4+0) & (2-9) \end{bmatrix}$$

$$= \begin{bmatrix} -8 & -7 \\ 4 & -7 \end{bmatrix}$$

$$B_1 B_2 B_3 = \begin{bmatrix} (-24+7) & (16-28) \\ (12+7) & (-8-28) \end{bmatrix} = \begin{bmatrix} -17 & -12 \\ 19 & -36 \end{bmatrix}$$

$$(B_1 B_2 B_3)^T = \begin{bmatrix} -17 & 19 \\ -12 & -36 \end{bmatrix}$$

$$B_3^T B_2^T = \begin{bmatrix} (6-1) & (3) \\ (-4+4) & (-12) \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 0 & -12 \end{bmatrix}$$

$$\left[(B_3^T B_2^T) B_1^T \right] = \begin{bmatrix} (-20+3) & (10+9) \\ (-12) & (-36) \end{bmatrix}$$

$$\boxed{\begin{bmatrix} -17 & 19 \\ -12 & -36 \end{bmatrix} = \begin{bmatrix} -17 & 19 \\ -12 & -36 \end{bmatrix}}$$

SP 12.

$x + 2y + 3z = -5$

$3x + y - 3z = 4$

$-3x + 4y + 7z = -7$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & -3 \\ -3 & 4 & 7 \end{bmatrix}$

$\vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$\vec{b} = \begin{bmatrix} -5 \\ 4 \\ 7 \end{bmatrix}$

$\det(A) = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & -3 \\ -3 & 4 & 7 \end{bmatrix} \begin{matrix} 1 & 2 \\ 3 & 1 \\ -3 & 4 \end{matrix}$

$\det(A) = (7 + 18 + 36) - (-9 - 12 + 42)$

$\det(A) = 40$

$A_x = \begin{bmatrix} -5 & 2 & 3 \\ 4 & 1 & -3 \\ -7 & 4 & 7 \end{bmatrix} \begin{matrix} -5 & 2 \\ 4 & 1 \\ -7 & 4 \end{matrix}$

$\det(A_x) =$

$(-35 + 42 + 48) - (-21 + 60 + 56)$

$\det(A_x) = -40$

$A_y = \begin{bmatrix} 1 & -5 & 3 \\ 3 & 4 & -3 \\ -3 & -7 & 7 \end{bmatrix} \begin{matrix} 1 & -5 \\ 3 & 4 \\ -3 & -7 \end{matrix}$

$\det(A_y) = (28 - 45 - 63) - (-36 + 21 - 105)$

$\det(A_y) = 39$

$$\det(A_z) = \left[\begin{array}{ccc} 1 & 2 & -5 \\ 3 & 1 & 4 \\ -3 & 4 & -7 \end{array}\right] \begin{array}{cc} 1 & 2 \\ 3 & 1 \\ -3 & 4 \end{array}$$

$$\det(A_z) = (-7 - 24 - 60) - (15 + 16 - 42)$$

$$\det(A_z) = -80$$

$$\boxed{\begin{aligned} x &= \frac{\det(A_x)}{\det(A)} = \frac{-40}{-40} = 1 \\ y &= \frac{\det(A_y)}{\det(A)} = \frac{-39}{-40} = \frac{39}{40} \\ z &= \frac{\det(A_z)}{\det(A)} = \frac{-80}{-40} = 2 \end{aligned}}$$