Hw 9 Gauss-Jordan matrix methods

sp1)

$$\begin{bmatrix} 2 & -2 & 1 \\ 3 & 2 & -5 \\ -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -16 \\ 8 \end{bmatrix}$$

ex)

```
import numpy as np
A= np.array ([[2,-2,1],[3,2,-5], [-1,2,3]])
R= np.array ([[10],[-16], [8] ])
# creating a augmented matrix
AR= np.append (A, R, axis=1)
```

append it into a column vector

appending R into A.

```
for i in range(3):
    for j in range(3):
        A[i,j] = A[i,j]/A[i,i]
```

- $i=0,\ j=0$

$$\begin{bmatrix} 2 & -2 & 1 \\ 3 & 2 & -5 \\ -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -16 \\ 8 \end{bmatrix}$$

$$A\vec{x} = \vec{b}$$

$$\vec{A}|\vec{b} = \left[\begin{array}{ccc|c} 2 & -2 & 1 & 10 \\ 3 & 2 & -5 & -16 \\ -1 & 2 & 3 & 8 \end{array}\right] \xrightarrow{(R_1 \leftrightarrow R_3)}$$

$$\sim \left[\begin{array}{ccc|c} -1 & 2 & 3 & 8 \\ 3 & 2 & -5 & -16 \\ 2 & -2 & 1 & 10 \end{array}\right] \xrightarrow{(3R_1 + R_2) \to R_2}$$

$$(3R_1) = \left[\begin{array}{ccc|c} -3 & 6 & 9 & 24 \end{array}\right]$$

$$(3R_1 + R_2) = \left[\begin{array}{ccc|c} 0 & 8 & 4 & 8 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} -1 & 2 & 3 & 8 \\ 0 & 8 & 4 & 8 \\ 2 & -2 & 1 & 10 \end{array}\right] \xrightarrow{(2R_1 + R_3) \to R_3}$$

$$2R_1 = \left[\begin{array}{ccc|c} -2 & 4 & 6 & 16 \end{array}\right]$$

$$(2R_1 + R_3) = \left[\begin{array}{ccc|c} 0 & 2 & 7 & 26 \end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c} -1 & 2 & 3 & 8 \\ 0 & 8 & 4 & 8 \\ 0 & 2 & 7 & 26 \end{array}\right] \xrightarrow{(R_2-4R_3)\to R_3}$$

$-4R_3 = [0\ -8\ -28 \mid -104]$

$(R_2-4R_3) = [0\ 0\ -24 \mid -96]$

$$\sim\left[\begin{array}{ccc|c} -1 & 2 & 3 & 8 \\ 0 & 8 & 4 & 8 \\ 0 & 0 & -24 & -96 \end{array}\right] \xrightarrow[(\frac{1}{8}R_2 \to R_2)\ (\frac{-1}{24}R_3 \to R_3)]{(-R_1 \to R_1)}$$

$$\sim\left[\begin{array}{ccc|c} 1 & -2 & -3 & -8 \\ 0 & 1 & 1/2 & 1 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$x_3 = 4$

$x_2 + \frac{1}{2}x_2 = 1; \quad x_2 = 1 - \frac{1}{2}(4) = -1$

$x_1 - 2x_2 - 3x_3 = -8; \quad x_1 = -8 + 3(4) + 2(-1) = 2$

SP2.)

$$\begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & -2 & 1 & -4 \\ 3 & -1 & -2 & -1 \\ -1 & 2 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -3 \\ 13 \end{bmatrix}$$

$$A\vec{x} = \vec{b}$$

$$A|\vec{b} = \left[\begin{array}{cccc|c} 2 & 1 & -1 & 2 & 0 \\ 1 & -2 & 1 & -4 & 3 \\ 3 & -1 & -2 & -1 & -3 \\ -1 & 2 & 1 & -2 & 13 \end{array}\right] \xrightarrow{(R_1 \longleftrightarrow R_2)}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & 1 & -4 & 3 \\ 2 & 1 & -1 & 2 & 0 \\ 3 & -1 & -2 & -1 & -3 \\ -1 & 2 & 1 & -2 & 13 \end{array}\right] \xrightarrow{(R_2 \longleftrightarrow R_4)}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & 1 & -4 & 3 \\ -1 & 2 & 1 & -2 & 13 \\ 3 & -1 & -2 & -1 & -3 \\ 2 & 1 & -1 & 2 & 0 \end{array}\right] \xrightarrow{(R_1 + R_2) \to R_2}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & 1 & -4 & 3 \\ 0 & 0 & 2 & -6 & 16 \\ 3 & -1 & -2 & -1 & -3 \\ 2 & 1 & -1 & 2 & 0 \end{array}\right] \xrightarrow{(-3R_1 + R_3) \to R_3}$$

$$-3R_1 = \left[\begin{array}{cccc|c} -3 & 6 & -3 & 12 & -9 \end{array}\right]$$

$$(-3R_1 + R_3) = \left[\begin{array}{cccc|c} 0 & 5 & -5 & 11 & -12 \end{array}\right]$$

5p2.)

a)

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 2 & 0 \\ 1 & -2 & 1 & -4 & 3 \\ 3 & -1 & -2 & -1 & -3 \\ -1 & 2 & 1 & -2 & 13 \end{array}\right] \xrightarrow{(R_1 \leftrightarrow R_2)}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -4 & 3 \\ 2 & 1 & -1 & 2 & 0 \\ 3 & -1 & -2 & -1 & -3 \\ -1 & 2 & 1 & -2 & 13 \end{array}\right] \xrightarrow{(-2R_1 + R_2) \to R_2}$$

$$-2R_1 = \left[\begin{array}{cccc|c} -2 & 4 & -2 & 8 & -6 \end{array}\right]$$

$$R_2 = \left[\begin{array}{cccc|c} 2 & 1 & -1 & 2 & 0 \end{array}\right]$$

$$-2R_1 + R_2 = \left[\begin{array}{cccc|c} 0 & 5 & -3 & 10 & -6 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -4 & 3 \\ 0 & 5 & -3 & 10 & -6 \\ 3 & -1 & -2 & -1 & -3 \\ -1 & 2 & 1 & -2 & 13 \end{array}\right] \xrightarrow{(-3R_1 + R_3) \to R_3}$$

$$-3R_1 = \left[\begin{array}{cccc|c} -3 & 6 & -3 & 12 & -9 \end{array}\right]$$

$$R_3 = \left[\begin{array}{cccc|c} 3 & -1 & -2 & -1 & -3 \end{array}\right]$$

$$-3R_1 + R_3 = \left[\begin{array}{cccc|c} 0 & 5 & -5 & 11 & -12 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -4 & 3 \\ 0 & 5 & -3 & 10 & -6 \\ 0 & 5 & -6 & 11 & -12 \\ -1 & 2 & 1 & -2 & 13 \end{array}\right] \xrightarrow{(R_1 + R_4) \to R_4}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -4 & 3 \\ 0 & 5 & -3 & 10 & -6 \\ 0 & 5 & -5 & 11 & -12 \\ 0 & 0 & 2 & -6 & 16 \end{array}\right] \xrightarrow{(R_2-R_3)\to R_3}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -4 & 3 \\ 0 & 5 & -3 & 10 & -6 \\ 0 & 0 & 2 & -1 & 6 \\ 0 & 0 & 2 & -6 & 16 \end{array}\right] \xrightarrow{(R_3-R_4)\to R_4}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -4 & 3 \\ 0 & 5 & -3 & 10 & -6 \\ 0 & 0 & 2 & -1 & 6 \\ 0 & 0 & 0 & 5 & -10 \end{array}\right]$$

$5x_4 = -10$; $x_4 = -2$

$2x_3 - x_4 = 6$; $x_3 = 2$

$5x_2 - 3x_3 + 10x_4 = -6$; $x_2 = 4$

$x_1 - 2x_2 + x_3 - 4x_4 = 3$; $x_1 = 1$

SP3)

$$\left[\begin{array}{cccc|c} 4 & 3 & 2 & 1 & 17 \\ 2 & -1 & 2 & -4 & 11 \\ 1 & 2 & -2 & -1 & 8 \\ -2 & 4 & 5 & -1 & 15 \end{array}\right] \xrightarrow{(R_1-2R_2)\to R_2} (R_1-4R_3)\to R_3 \quad (R_1+2R_4)\to R_4$$

$R_1 = [4\ 3\ 2\ 1 \mid 17]$

$-2R_2 \to [-4\ 2\ -4\ 8 \mid -22]$

* $(R_1-2R_2) = [0\ 5\ -2\ 9 \mid -5]$

$R_1 = [4\ 3\ 2\ 1 \mid 17]$

$-4R_3 = [-4\ -8\ 8\ 4 \mid -32]$

* $R_1-4R_3 = [0\ -5\ 10\ 5 \mid -15]$

$R_1 = [4\ 3\ 2\ 1 \mid 17]$

$2R_4 = [-4\ 8\ 10\ -2 \mid 30]$

* $R_1+2R_4 = [0\ 11\ 12\ -1 \mid 37]$

$$\left[\begin{array}{cccc|c} 4 & 3 & 2 & 1 & 17 \\ 0 & 5 & -2 & 9 & -5 \\ 0 & -5 & 10 & 5 & -15 \\ 0 & 11 & 12 & -1 & 37 \end{array}\right] \xrightarrow{(R_2+R_3)\to R_3} (11R_2-5R_4)\to R_4$$

* $R_2+R_3 = [0\ 0\ 8\ 14 \mid -20]$

* $11R_2-5R_4 = [0\ 0\ 50\ 60 \mid -350]$

5P3)

$$\left[\begin{array}{cccc|c} 4 & 3 & 2 & 1 & 17 \\ 2 & -1 & 2 & -4 & 11 \\ 1 & 2 & -2 & -1 & 8 \\ -2 & 4 & 5 & -1 & 15 \end{array}\right] \xrightarrow{(R_1 \leftrightarrow R_3)}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & -1 & 8 \\ 2 & -1 & 2 & -4 & 11 \\ 4 & 3 & 2 & 1 & 17 \\ -2 & 4 & 5 & -1 & 15 \end{array}\right] \xrightarrow{(-2R_1 + R_2) \to R_2}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & -1 & 8 \\ 0 & -5 & 6 & -2 & -5 \\ 4 & 3 & 2 & 1 & 17 \\ -2 & 4 & 5 & -1 & 15 \end{array}\right] \xrightarrow{(-4R_1 + R_3) \to R_3}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & -1 & 8 \\ 0 & -5 & 6 & -2 & -5 \\ 0 & -5 & 10 & 5 & -15 \\ -2 & 4 & 5 & -1 & 15 \end{array}\right] \xrightarrow{(2R_1 + R_4) \to R_4}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & -1 & 8 \\ 0 & -5 & 6 & -2 & -5 \\ 0 & -5 & 10 & 5 & -15 \\ 0 & 8 & 1 & -3 & 31 \end{array}\right] \xrightarrow{(-R_2 + R_3) \to R_3}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & -1 & 8 \\ 0 & -5 & 6 & -2 & -5 \\ 0 & 0 & 4 & 7 & -10 \\ 0 & 8 & 1 & -3 & 31 \end{array}\right] \xrightarrow{(\frac{8}{5}R_2 + R_4) \to R_4}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & -1 & 8 \\ 0 & -3 & 6 & -2 & -5 \\ 0 & 0 & 4 & 7 & -10 \\ 0 & 0 & 9/5 & -7/15 & 33 \end{array}\right] \xrightarrow{\left(-\frac{9}{20}R_3 + R_4\right) \to R_4}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & -1 & 8 \\ 0 & -3 & 6 & -2 & -5 \\ 0 & 0 & 4 & 7 & -10 \\ 0 & 0 & 0 & -49/4 & 49/2 \end{array}\right] \xrightarrow{\left(-\frac{4}{49}R_4 \to R_4\right)}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & -1 & 8 \\ 0 & -3 & 6 & -2 & -5 \\ 0 & 0 & 4 & 7 & -10 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{\left(-7R_4 + R_3\right) \to R_3}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & -1 & 8 \\ 0 & -3 & 6 & -2 & -5 \\ 0 & 0 & 4 & 0 & 4 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{\left(2R_4 + R_3\right) \to R_3}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & -1 & 8 \\ 0 & -3 & 6 & 0 & -9 \\ 0 & 0 & 4 & 0 & 4 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{\left(R_4 + R_1\right) \to R_1}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & 0 & 6 \\ 0 & -3 & 6 & 0 & -9 \\ 0 & 0 & 4 & 0 & 4 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{\left(\frac{1}{4}R_3 \to R_3\right)}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & 0 & 6 \\ 0 & -3 & 6 & 0 & -9 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{\left(-6R_3 + R_2\right) \to R_2}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -2 & 0 & 6 \\ 0 & -5 & 0 & 0 & -15 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{(2R_3 + R_1) \to R_1}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 8 \\ 0 & -5 & 0 & 0 & -15 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{(-\frac{1}{5}R_2 \to R_2)}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 8 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{(-2R_2 + R_1) \to R_1}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

$$\boxed{\begin{aligned} x_1 &= 2 \\ x_2 &= 3 \\ x_3 &= 1 \\ x_4 &= -2 \end{aligned}}$$