Comp Methods Hw 10

Spl.)

$$A = \begin{bmatrix} 6 & 12 & 24 \\ 2 & 11 & 29 \\ 4 & 10 & 24 \end{bmatrix}$$

$$A = \begin{bmatrix} L_{00} & 0 & 0 \\ L_{10} & L_{11} & 0 \\ L_{20} & L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} 1 & U_{01} & U_{02} \\ 0 & 1 & U_{12} \\ 0 & 0 & 1 \end{bmatrix}$$

$a_{00} = L_{00}$; $U_{01} = \frac{a_{01}}{L_{00}} = \frac{12}{6} = 2$

$L_{00} = 6$ $U_{02} = \frac{a_{02}}{L_{00}} = \frac{24}{6} = 4$

$a_{10} = L_{10} = 2$;

$a_{11} = L_{10}U_{01} + L_{11}$; $L_{11} = a_{11} - L_{10}U_{01}$

$L_{11} = 11 - (2)(2) = 7$

$U_{12} = (a_{12} - L_{10}U_{02}) / L_{11}$;

$U_{12} = (29 - (2)(4.)) / 7 = 21/7 = 3$

$L_{20} = 4$

$L_{21} = 2$ $L_{21} = (a_{21} - L_{20}U_{01}) = 10 - (4)(2)$

$$L_{22} = a_{22} - L_{20}U_{02} - L_{21}U_{12}$$

$$L_{22} = 24 - (4)(4) - (2)(3)$$

$$L_{22} = 2$$

$$U = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} 6 & 0 & 0 \\ 2 & 7 & 0 \\ 4 & 2 & 2 \end{bmatrix}$$

Sp2) $A\vec{x} = \vec{b}$

$$\begin{bmatrix} 2 & -6 & 6 \\ 3 & -7 & 13 \\ -2 & 2 & 11 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -13 \\ 21 \end{bmatrix}$$

- $U_{0j} = \frac{a_{0j}}{L_{00}}$, $L_{i0} = a_{i0}$
- $L_{ij} = a_{ij} - \sum_{k=0}^{j-1} L_{ik} U_{kj}$
- $U_{ij} = \frac{a_{ij} - \sum_{k=0}^{j-1} L_{ik} U_{kj}}{L_{ii}}$

$$U = \begin{bmatrix} 1 & U_{01} & U_{02} \\ 0 & 1 & U_{12} \\ 0 & 0 & 1 \end{bmatrix}, \quad L = \begin{bmatrix} L_{00} & 0 & 0 \\ L_{10} & L_{11} & 0 \\ L_{20} & L_{21} & L_{22} \end{bmatrix}$$

- $L_{00} = 2$, $U_{01} = \frac{-6}{2} = -3$

  $U_{02} = \frac{6}{2} = 3$
- $L_{10} = 3$,
- $U_{12} = a_{12} - (L_{10})(U_{02})/2$

  $U_{12} = 13 - (3)(3) = 4$
- $L_{20} = -2$

$\cdot L_{11} = a_{11} - (L_{10})(U_{01})$

$L_{11} = -7 - (3)(-3) = 2$

$\cdot L_{21} = a_{21} - (L_{20})(U_{01})$

$\cdot L_{21} = 2 - (-2)(-3) = -4$

$\cdot L_{22} = a_{22} - (L_{20})(U_{02}) - (L_{21})(U_{12})$

$\cdot L_{22} = 11 - (-2)(3) - (-4)(2) = 25$

$$L = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 2 & 0 \\ -2 & -4 & 25 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 7 \\ -13 \\ 21 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & -3 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \qquad \vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$L\vec{y} = \vec{b}$

$U\vec{x} = \vec{y}$

$$L|b = \left[\begin{array}{ccc|c} 2 & 0 & 0 & 7 \\ 3 & 2 & 0 & -13 \\ -2 & -4 & 25 & 21 \end{array}\right]$$

$$\vec{y} = \begin{bmatrix} 1 \\ -8 \\ -0.36 \end{bmatrix}$$

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$$U|\vec{y} = \left[\begin{array}{ccc|c} 1 & -3 & 3 & 1 \\ 0 & 1 & 2 & -8 \\ 0 & 0 & 1 & -0.36 \end{array}\right]$$

$z = -0.36$

$y + 2z = -8$, $y = -7.28$

$x - 3y + 3z = 1$, $x = -19.76$

$$\vec{x} = \begin{bmatrix} -19.76 \\ -7.28 \\ -0.36 \end{bmatrix}$$

3.)

$A^{-}A = I$

$$A = \begin{bmatrix} -1 & 2 & 2 \\ 0 & 2 & -0.5 \\ 0.5 & 1 & -2 \end{bmatrix}$$

$A|I \sim I|A^{-1}$

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 2 & 0 \\ 0.5 & 1 & 0.5 \end{bmatrix}$$

$$AI = \left[\begin{array}{ccc|ccc} -1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0.5 & 1 & 0.5 & 0 & 0 & 1 \end{array}\right] \xrightarrow{(.5R_1 + R_3) \to R_3}$$

$$\sim \left[\begin{array}{ccc|ccc} -1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0.5 & 0.5 & 0 & 1 \end{array}\right] \xrightarrow{(R_2 - R_3) \to R_3}$$

$$\sim \left[\begin{array}{ccc|ccc} -1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0.5 & 0.5 & -1 & 1 \end{array}\right] \xrightarrow{(R_3 + R_2) \to R_2}$$

$$\sim \left[\begin{array}{ccc|ccc} -1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & -0.5 & 2 & -1 \\ 0 & 0 & 0.5 & -0.5 & 1 & -1 \end{array}\right] \xrightarrow{(-4R_3 + R_1) \to R_1}$$

$$\sim \left[\begin{array}{ccc|ccc} -1 & 2 & 0 & -3 & -4 & 4 \\ 0 & 2 & 0 & -0.5 & 2 & -1 \\ 0 & 0 & 0.5 & -0.5 & 1 & -1 \end{array}\right] \xrightarrow{(R_2 - R_1) \to R_1}$$

$$\sim \left[\begin{array}{ccc|ccc} -1 & 0 & 0 & -3.5 & 6 & -5 \\ 0 & 2 & 0 & -0.5 & 2 & -1 \\ 0 & 0 & 0.5 & -0.5 & 1 & -1 \end{array}\right] \xrightarrow{\substack{(-R_1 + R_2) \to R_2 \\ (\frac{1}{2}R_2) \to R_2 \\ (2R_3) \to R_3}}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3.5 & 6 & -5 \\ 0 & 1 & 0 & -0.25 & 1 & -0.5 \\ 0 & 0 & 1 & -1 & 2 & -2 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -3.5 & 6 & -5 \\ -0.25 & 1 & -0.5 \\ -1 & 2 & -2 \end{bmatrix}$$

sp4)
$$\begin{bmatrix} 4 & 0 & 1 & 0 & 1 \\ 2 & 5 & -1 & 1 & 0 \\ 1 & 0 & 3 & -1 & 0 \\ 0 & 1 & 0 & 4 & -2 \\ 1 & 0 & -1 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 32 \\ 19 \\ 14 \\ -2 \\ 4 \end{bmatrix}$$

Gauss - Seidel method

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=0}^{j=i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{j=n-1} a_{ij} x_j^{(k)}}{a_{ii}}$$

$$\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

1st iteration:

$$x_1^1 = \frac{32}{4} = 8$$

$$x_2^1 = (19/5) = 3.8$$

$$x_3^1 = (14/3) = 4.6667$$

$$x_4^1 = (-2/4) = -0.5$$

$$x_5^1 = (4/5) = 8.2$$

$$\vec{X}^1 = \begin{bmatrix} 8 \\ 3.8 \\ 4.6667 \\ -0.5 \\ 8.2 \end{bmatrix}$$

2nd iteration:

$$X_1^2 = \frac{32 - (1)(8.2) - (1)(4.6667)}{4}$$

$$*(X_1^2 = 4.783325)$$

$$X_2^2 = \frac{19 - (1)(-0.5) - (4.6667)(-1) - (8)(2)}{3.8}$$

$$*(X_2^2 = 2.14913\,1579)$$

$$X_3^2 = \frac{14 - (8.2)(0) - (-0.5)(-1) - (3.8)(0) - (8)(1)}{(-3)}$$

$$*(X_3^2 = 1.8333)$$

$$X_4^2 = \frac{-2 - (-2)(8.2) - (1)(3.8)}{(4)}$$

$$*(X_4^2 = 2.65)$$

$$X_5^2 = \frac{44 - (-1)(4.6667) - (1)(8)}{5}$$

$$*(X_5^2 = 7.53334)$$

3rd iteration :

$$\vec{X}^2 = \begin{bmatrix} 4.783325 \\ 2.149131579 \\ 1.8333 \\ 2.65 \\ 7.53334 \end{bmatrix}$$

$$X_1^3 = \frac{32 - (1)(7.53334) - (1)(1.8333)}{4}$$

$$*(X_1^3 = 5.65834)$$

$$X_2^3 = \frac{19 - (1)(2.65) - (-1)(1.8333) - (2)(7.53334)}{(5)}$$

$$*(X_2^3 = 1.72333)$$

$$X_3^3 = \frac{14 - (-1)(2.65) - (1)(4.783325)}{3}$$

$$*(X_3^3 = 3.95555)$$

$$X_4^3 = \frac{-2 - (-1)(7.53334) - (1)(2.149131579)}{4}$$

$$*(X_4^3 = 2.72939)$$

$$X_5^3 = \frac{41 - (-1)(1.8333) - (1)(4.783325)}{5}$$

$$*(X_5^3 = 7.609995)$$

$$\vec{X}^3 = \begin{bmatrix} 5.658334 \\ 1.72333 \\ 3.95556 \\ 2.72939 \\ 7.609995 \end{bmatrix}$$

Sp5.) $\text{Cond}([a]) = \|[a]\| \|[a]^{-1}\|$

$$a = \begin{bmatrix} -1 & 2 & 2 \\ 0 & 2 & -0.5 \\ 0.5 & 1 & -2 \end{bmatrix}$$

$$a^{-1} = \begin{bmatrix} -3.5 & 6 & -5 \\ -0.25 & 1 & -0.5 \\ -1 & 2 & -2 \end{bmatrix}$$

$\|a\|_\infty = 1 + 2 + 2 = 5$

$\|a\|_\infty^{-1} = 3.5 + 6 + 5 = 14.5$

$\text{Cond}([a]) = (5)(14.5) = 72.5$

Sp6.)

$\|a\|_1 = 1 + 2 + 2 = 5$

$\|a\|_1^{-1} = 3.5 + 6 + 5 = 14.5$

$\text{Cond}([a]) = (5)(14.5) = 72.5$