Prelab 4: Rutherford Scattering

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1.

This lab goals of this experiment is to measure the Rutherford cross-section, and understand how scattering is used in nuclear experiments. Before Rutherford's scattering can be addressed, one needs to define what is a scattering experiment and the main purposes of it. In a scattering experiment you have a beam that can hits target/targets that then either misses the target or hits it and scatters. In nuclear physics we care about using these experiments to get info on the target, projectile or it's interaction with each other. In many forms these experiments allow us scientists to probe internal structure of the atom and discover atomic properties. each target is defined by its specific cross-sectional area and what happens is that you have a distribution of scattered particles that hit multiple cross-sections that then allow you to probe internal structures and gain dynamical properties [1]. In most cases of scattering experiments we care about defining the scattering distribution by the probability that something scatter on a target per solid angle which leads to defining the differential cross-section and that is what Rutherford's cross section is [2]. Rutherford's gold foil experiment had a beam of alpha particles that hit or missed an arrangement of gold foil. The beam transverses and once it gets closed to the target the field created by the gold uses columbic repulsion to scatter the alpha particles. This is what led to the conclusion that an atoms mass is concentrated in the center (nucleus) [1]. [Count: 247]

2.

Definitions

- θ : Scattering angle
- T : Incident kinetic energy
- T' : Scattering kinetic energy
- M_0 : Alpha mass
- M_1 : Gold, copper, or silver mass

Expression for K

$$K = \left(\frac{M_0 \cos\theta + \sqrt{M_1^2 - M_0^2 \sin^2\theta}}{M_0 + M_1} \right)^2$$

Relationship between T' and T

$$T' = K\,T$$

Given Parameters

- $T = 2\,\mathrm{MeV},\ 6\,\mathrm{MeV}$
- $M_0 = 4\,\mathrm{u}$
- $M_1 = \begin{cases} 197\,\mathrm{u} & \text{(Gold)} \\ 108\,\mathrm{u} & \text{(Silver)} \\ 63.54\,\mathrm{u} & \text{(Copper)} \end{cases}$
- $\theta = 45^\circ,\ 135^\circ$ (converted to radians in calculation)

Notes

- u is the atomic mass unit.

Material	Initial Energy (MeV)	Scattering Angle (degrees)	Recoil Energy (MeV)
Gold	2	45	1.97635
Gold	2	135	1.86603
Gold	6	45	5.92905
Gold	6	135	5.59809
Silver	2	45	1.95707
Silver	2	135	1.76232
Silver	6	45	5.87120
Silver	6	135	5.28696
Copper	2	45	1.92751
Copper	2	135	1.61246
Copper	6	45	5.78252
Copper	6	135	4.83739

Table 1: Recoil energy of alpha particles after scattering off different materials at various initial energies and angles.

3.

Rutherford Differential Cross-Section

The Rutherford differential cross-section for scattering is given by:

$$\frac{d\sigma}{d\Omega} = 1.296 \left(\frac{\text{mb} \cdot \text{MeV}^2}{\text{sr}}\right) \left(\frac{Z_0 Z_1}{E_0}\right)^2 \left[\frac{1}{\sin^4\left(\frac{\theta}{2}\right)} - 2\left(\frac{M_0}{M_1}\right)^2\right]$$

where:

- $\frac{d\sigma}{d\Omega}$ is the differential cross-section measured in millibarns per steradian (mb/sr).
- Z_0 is the atomic number of the incident particle (for alpha particles, $Z_0 = 2$).
- Z_1 is the atomic number of the target nucleus (for Gold, $Z_1 = 79$).

- E_0 is the beam energy of the incident particle, measured in MeV.
- θ is the scattering angle in the laboratory frame, measured in degrees.
- M_0 is the mass of the incident particle in atomic mass units (for alpha particles, $M_0 = 4\,\mathrm{u}$).
- M_1 is the mass of the target nucleus in atomic mass units (for Gold, $M_1 = 197\,\mathrm{u}$).

Plot Description

The plot represents the variation of the Rutherford differential cross-section for a pure Gold target under two different conditions:

1. **Cross-section vs. Beam Energy**
   The first plot shows the differential cross-section as a function of beam energy (E_0), with a constant scattering angle $\theta = 60^\circ$. The beam energy varies between 2 MeV and 6 MeV.

2. **Cross-section vs. Scattering Angle**
   The second plot displays the differential cross-section as a function of scattering angle (θ), with a constant beam energy of $E_0 = 4.5\,\mathrm{MeV}$. The scattering angle varies between 25° and 155°.

Units

- The units of the differential cross-section are given in millibarns per steradian (**mb/sr**), where:

$$1\,\mathrm{mb} = 10^{-31}\,\mathrm{m}^2.$$

- The beam energy E_0 is measured in MeV (Mega-electron Volts).
- The scattering angle θ is measured in degrees.
- The atomic masses M_0 and M_1 are measured in atomic mass units (**u**), where:

$$1\,\mathrm{u} = 931.494\,\mathrm{MeV}/c^2.$$

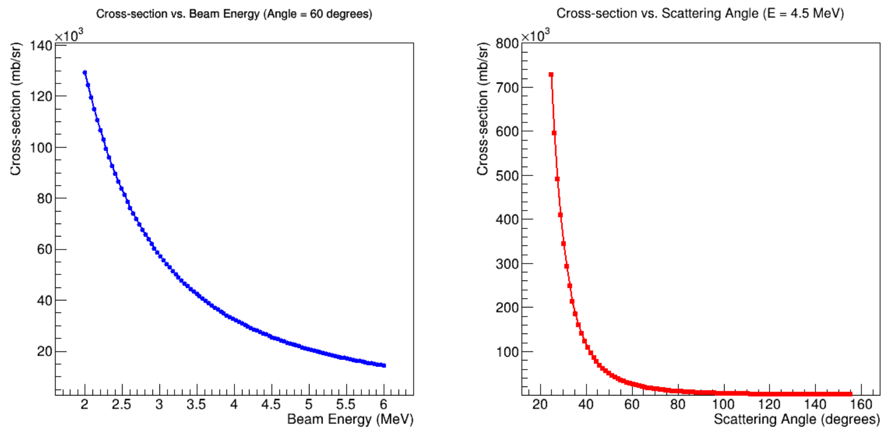

Figure 1: Variation of the Rutherford differential cross-section for a pure Gold target. The left plot shows the cross-section as a function of beam energy for a constant scattering angle of 60 degrees. The right plot shows the cross-section as a function of scattering angle for a constant beam energy of 4.5 MeV. The cross-section is measured in mb/sr (millibarns per steradian).

4.

Problem Description

The objective is to calculate the number of alpha particles detected by a silicon detector after scattering from a pure gold target. The calculation takes into account the Rutherford differential cross-section, beam intensity, target number density, and the geometric setup of the experiment.

Initial Variables and Known Constants

- **Beam Current,** I:

$$I = 1\,\mathrm{nA} = 1 \times 10^{-9}\,\mathrm{A}$$

- **Charge of an Alpha Particle,** q_α:

$$q_\alpha = 2e = 2 \times 1.6 \times 10^{-19}\,\mathrm{C}$$

- **Beam Duration,** t:

$$t = 1\,\text{minute} = 60\,\text{seconds}$$

- **Beam Energy,** E_0:

$$E_0 = 4.5\,\text{MeV}$$

- **Scattering Angle,** θ:

$$\theta = 60^\circ$$

- **Atomic Numbers:**
  - $Z_0 = 2$ (for alpha particles)
  - $Z_1 = 79$ (for gold)
- **Masses:**
  - $M_0 = 4$ (alpha mass in atomic mass units)
  - $M_1 = 197$ (gold mass in atomic mass units)
- **Number Density of Target,** n_t:

$$n_t = 8 \times 10^{17}\,\text{atoms/cm}^2$$

- **Detector Area,** A:

$$A = 1\,\text{mm}^2 = 1 \times 10^{-2}\,\text{cm}^2$$

- **Distance from Target to Detector,** r:

$$r = 10\,\text{cm}$$

Formulas Used in the Calculation

1. Total Number of Incident Alphas

The total number of alpha particles per second in the beam is calculated as:

$$\text{Number of alphas/second} = \frac{I}{q_\alpha}$$

where:

- I is the beam current,
- $q_\alpha = 2e$ is the charge of an alpha particle.

The total number of incident alphas over the time t is:

$$N_{\text{incident}} = \left(\frac{I}{q_\alpha}\right) \times t$$

2. Rutherford Differential Cross-Section

The Rutherford differential cross-section for scattering is given by:

$$\frac{d\sigma}{d\Omega} = 1.296 \left(\frac{Z_0 Z_1}{E_0}\right)^2 \left[\frac{1}{\sin^4\left(\frac{\theta}{2}\right)} - 2\left(\frac{M_0}{M_1}\right)^2\right]$$

where:

- Z_0 and Z_1 are the atomic numbers of the incident alpha and target nucleus, respectively,
- E_0 is the initial beam energy,
- θ is the scattering angle,
- M_0 and M_1 are the masses of the incident alpha and target nucleus, respectively.

3. Solid Angle Covered by the Detector

The solid angle covered by the detector is calculated as:

$$d\Omega = \frac{A}{r^2}$$

where:

- A is the detector area,
- r is the distance from the target to the detector.

4. Number of Alphas Detected

The total number of alpha particles detected by the silicon detector is given by:

$$N_{\text{detected}} = N_{\text{incident}} \times n_t \times \frac{d\sigma}{d\Omega} \times d\Omega$$

where:

- N_{incident} is the total number of incident alphas,
- n_t is the target number density,
- $\frac{d\sigma}{d\Omega}$ is the Rutherford differential cross-section,
- $d\Omega$ is the solid angle covered by the detector.

Parameter	**Value**
Total incident alphas	1.875×10^{11}
Differential cross-section at 60 degrees	25561.8 mb/sr
Number of alphas detected	383.427

Table 2: Results from the alpha detection calculation.

5.

Problem Description

We need to determine the number of alpha particles detected by a silicon detector when scattering occurs from a target foil made up of 80% Gold, 15% Silver, and 5% Copper. This calculation is based on the Rutherford differential cross-section and takes into account the different contributions from each element in the target mixture.

Given Information

- Target composition:

- 80% Gold
- 15% Silver
- 5% Copper

- **Beam current,** I:

$$I = 1\,\text{nA} = 1 \times 10^{-9}\,\text{A}$$

- **Charge of an alpha particle,** q_α:

$$q_\alpha = 2e = 2 \times 1.6 \times 10^{-19}\,\text{C}$$

- **Beam duration,** t:

$$t = 1\,\text{minute} = 60\,\text{seconds}$$

- **Beam energy,** E_0:

$$E_0 = 4.5\,\text{MeV}$$

- **Scattering angle,** θ:

$$\theta = 60^\circ$$

- **Atomic numbers:**
  - $Z_{\text{Au}} = 79$ (Gold)
  - $Z_{\text{Ag}} = 47$ (Silver)
  - $Z_{\text{Cu}} = 29$ (Copper)

- **Atomic masses:**
  - $M_{\text{Au}} = 197$ (Gold)
  - $M_{\text{Ag}} = 108$ (Silver)
  - $M_{\text{Cu}} = 63.5$ (Copper)

- **Number density of the target,** n_t:

$$n_t = 8 \times 10^{17}\,\text{atoms/cm}^2$$

- **Detector area,** A:

$$A = 1\,\text{mm}^2 = 1 \times 10^{-2}\,\text{cm}^2$$

- **Distance from target to detector,** r:

$$r = 10\,\text{cm}$$

Formulas to Use

1. Total Number of Incident Alphas

Calculate the total number of alpha particles that hit the target during the given time:

$$N_0 = \left(\frac{I}{q_\alpha}\right) \times t$$

where:

- I: Beam current,
- q_α: Charge of an alpha particle,
- t: Beam duration.

2. Rutherford Differential Cross-Section for Each Element

The Rutherford differential cross-section for element i is given by:

$$\frac{d\sigma_i}{d\Omega} = 1.296 \left(\frac{Z_0 Z_i}{E_0}\right)^2 \left[\frac{1}{\sin^4\left(\frac{\theta}{2}\right)} - 2\left(\frac{M_0}{M_i}\right)^2\right]$$

where:

- $Z_0 = 2$: Atomic number of alpha particles,
- Z_i: Atomic number of element i (Gold, Silver, or Copper),
- E_0: Beam energy,
- θ: Scattering angle,
- $M_0 = 4$: Mass of alpha particles,
- M_i: Mass of element i.

3. Solid Angle Covered by the Detector

The solid angle covered by the detector is:

$$\Delta\Omega = \frac{A}{r^2}$$

where:

- A: Area of the detector,
- r: Distance from the target to the detector.

4. Total Number of Alphas Detected

The total number of detected alphas, N_s, is calculated by considering the contributions from each element in the target mixture:

$$N_s = N_0 \cdot \Delta\Omega \cdot \left(\frac{d\sigma_{\mathrm{Au}}}{d\Omega} \cdot n_{\mathrm{Au}} + \frac{d\sigma_{\mathrm{Ag}}}{d\Omega} \cdot n_{\mathrm{Ag}} + \frac{d\sigma_{\mathrm{Cu}}}{d\Omega} \cdot n_{\mathrm{Cu}} \right)$$

where:

- N_0: Total incident alphas,
- $\Delta\Omega$: Solid angle,
- $\frac{d\sigma_i}{d\Omega}$: Differential cross-section for element i,
- n_i: Fraction of element i in the target (0.8 for Gold, 0.15 for Silver, 0.05 for Copper).

Steps to Calculate the Number of Alphas Detected

1. Calculate the total number of incident alphas, N_0.
2. Calculate the differential cross-section for each element (Gold, Silver, and Copper) at the given scattering angle using the Rutherford formula.
3. Calculate the solid angle, $\Delta\Omega$, covered by the detector.

4. Use the contributions from each element's differential cross-section and fraction in the target to calculate the total number of alphas detected, N_s, using the formula above.

Parameter	**Value**
Differential cross-section at 60 degrees (Gold)	25561.8 mb/sr
Differential cross-section at 60 degrees (Silver)	9046.51 mb/sr
Differential cross-section at 60 degrees (Copper)	3443.03 mb/sr
Total number of alphas detected	330.558

Table 3: Results from the alpha detection calculation for a mixed target foil of 80% Gold, 15% Silver, and 5% Copper.

Works Cited

[1] John R. Taylor. *Classical Mechanics*. Collisions. Sausalito, CA: University Science Books, 2005. Chap. 14.

[2] Dr. Paul King. *Physics 6751: Nuclear Lab Manual*. Version 24.5. Athens, Ohio, 2024.