HW5:

Monday, April 21, 2025 1:08 PM

5.1 Show that EQ_{CFG} is undecidable.

Assume R decides EQ_{CFG}, and construct TM S deciding ALL_{CFG}

Let S = "On input ⟨G⟩,

1) Construct CFG G_{all} such that $L(G_{all}) = \Sigma^*$
2) Use R to determine if $L(G) = L(G_{all})$
3) Accept if yes, otherwise reject."

5.2 Show that EQ_{CFG} is co-Turing-recognizable.

Assume R recognizes $\overline{EQ_{CFG}}$ and construct TM S that recognizes it.

Let S = "On input $\langle G_1, G_2 \rangle$,

1) Enemurate all strings $w \in \Sigma^*$
2) For each string w, simulate a decider for whether $w \in L(G_1)$ and whether $w \in L(G_2)$
3) If $w \in L(G_1)$ and $w \notin L(G_2)$ or $w \notin L(G_1)$ and $L \in (G_2)$
4) S accepts, otherwise reject

5.3 Find a match in the following instance of the Post Correspondence Problem.

$$\left\{ \left[\frac{\text{ab}}{\text{abab}}\right], \left[\frac{\text{b}}{\text{a}}\right], \left[\frac{\text{aba}}{\text{b}}\right], \left[\frac{\text{aa}}{\text{a}}\right] \right\}$$

The sequence is, $\frac{ab}{abab}, \frac{ab}{abab}, \frac{aba}{b}, \frac{b}{a}, \frac{b}{a}, \frac{aa}{a}, \frac{aa}{a}$

abababa bb aaaa

ababababbaaaa

⇒[1,1,3,2,2,4,4]

5.9: Let T = {⟨M⟩| M is a TM that accepts w^R whenever it accepts w}. Show that T is undecidable. (Some Editions have a different 5.9 Exercise)

Construct M' such that

is undecidable. (Some Editions have a different 5.9 Exercise)

Construct M' such that

M' = "On input x:

1) If $x = w$, run M on w

2) If M accepts w, accepts

3) If $x = w^R$, rejects

4) Otherwise, reject."

S = "On input <M, w>:

1) construct M'

2) Run R on <M'>

3) If R accepts, reject

4) If R rejects, accept."