

# SPECTRAL MOMENTS AND TIME DOMAIN REPRESENTATION OF PHOTOACOUSTIC SIGNALS USED FOR DETECTION OF CRUDE OIL IN PRODUCED WATER

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## ABSTRACT

This paper reports the result from signal processing of the output from a photoacoustic(PA) instrument monitoring crude oil in water. The application involves the deployment of sensor heads directly into the produced water pipelines. It is highly sensitive and able to detect hydrocarbons in the range 0 - 1200 parts per million (ppm). The instrument delivers a signal where the usual method for representing the photoacoustic response is the peak-to-peak value. This value is found by taking the difference between the highest and lowest peaks in the signal. Signal processing illustrated the need for a linear phase filter, this led to the use of spectral moments for finding an effective representation for the photoacoustic response. The arithmetic mean of the signal was shown to be linked to the spectral moments through Parseval's theorem. This greatly simplified the computational procedure. By using the average of the signal instead of the peak-to-peak value, the computer and A/D card can be omitted when computing the photoacoustic response. This is advantageous in a downhole environment where high temperature and limited resources are found.

## 1. INTRODUCTION

In the production of oil and gas significant amounts of water are also recovered. This water is processed further before let out to sea, certain legislation [1] direct the allowed amount of crude oil that the produced water can contain. The method used to monitor produced water today is with infrared spectrometry [2] and the method requires that samples are retrieved from the pipeline. The oil industry has for some time evaluated various methods to find an in-line, continuous monitoring that can be used in field applications top-side, sub-sea and downhole [3]. A photoacoustic instrument [4] for detection of crude oil was developed at Heriot-Watt university in Edinburgh, Scotland. In cooperation with Kværner Oilfield Products a prototype was developed [5]. Further evaluation of the

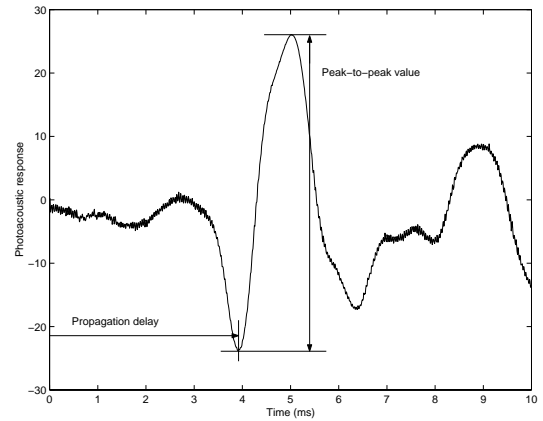


Figure 1: Typical photoacoustic signal

signal has been performed in our Department and is presented in this report [6].

## 2. PRINCIPLES OF PHOTOACOUSTIC DETECTION

The principle behind photoacoustic detection is governed by the fact that crude oil has a much higher response to wavelengths in the near infrared than water. A high energy laser is directed at the sample, the energy is absorbed and we get an area of heating resulting in an expanding region followed by a relaxation. The pressure wave produced can be picked up by a piezoelectric transducer. A typical photoacoustic signal over an average of 1500 pulses is shown in Fig. 1. The method that has been used to represent the oil content is the peak-to-peak value. Other physical parameters that are also known to affect the photoacoustic response [7] is the temperature, salinity and pressure. These parameters must also be included in an equation that models the oil content.

### 3. SPECTRAL MOMENTS

The spectral moments [8] retrieve information directly from the Fourier spectrum. They are advantageous when the overall shape is important and not the fine structure of the original spectrum itself. One of the advantages is the insensitivity to changes in the phase of the signal. Filters often distort the phase and may distort the amplitude spectrum. The spectral moments are often used in image processing, where the central moments are derived from the spectral moments and work as a transformation that can mirror image, rotate and resize an image. The spectral moments can also be viewed as a way to use statistics to analyse the power spectrum. The different moments have a statistical interpretation that will be described below. When working on a discrete-time signal the one dimensional spectral moments are defined in Eq. 1. The length of the power spectrum is  $N$ , and the sampling interval is  $\Delta t$ .

$$m'_n = \frac{2}{N} \sum_{l=0}^{(N/2)-1} S(l)(l/N\Delta t)^n \quad (1)$$

The computations are often limited to a band of frequencies due to prior knowledge of the physics of the generation of the spectrum. This corresponds to a filtering process where the frequencies of interest are extracted. In the discrete case the spectral estimate is done in the frequency range  $c_1$  to  $c_2$  of length  $I$ , where  $I = c_2 - c_1 + 1$ . The result is Eq. 2.

$$m'_n = \frac{1}{I} \sum_{l=c_1}^{c_2} S(l)(l/N\Delta t)^n \quad (2)$$

The zero order moment will in the discrete case look like Eq. 3.

$$m_0 = m'_0 = \frac{1}{I} \sum_{l=c_1}^{c_2} S(l) \quad (3)$$

The zero-order spectral moment is basically the average of the values in the interval  $c_1$  to  $c_2$ , and is proportional to the mean energy in that interval. If we use Eq. 1 to take the zero order spectral moment over all frequencies in the signal, it can be shown that it can also be estimated directly from the variance of the time series itself. When only working on parts of the spectrum it is necessary to normalize the spectrum with  $m_0$  before taking the higher spectral moments. This will make the moments depend only on the shape of the spectrum and not the level. The discrete first order moment is therefore as given in Eq. 4

$$m_1 = \frac{1}{Im_0} \sum_{l=c_1}^{c_2} S(l)(l/N\Delta t)^n \quad (4)$$

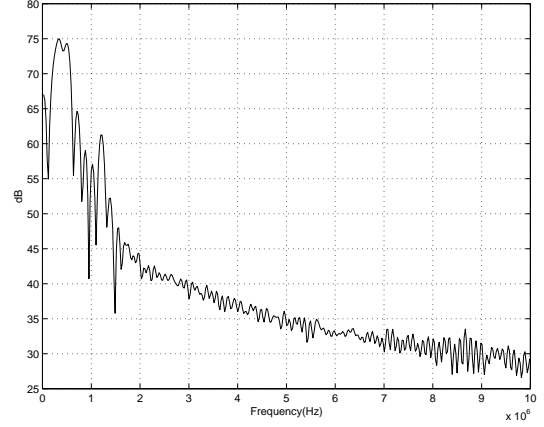


Figure 2: Plot of the FFT of a photoacoustic signal.

The first order spectral moment is interpreted as the mean frequency,  $\bar{f}$ , of the signal in the interval  $I$ . To get higher order shape information one needs to define the central moments, Eq. 5.

$$M_n = \frac{1}{m_0} \int_{c_1}^{c_2} (f - \bar{f})^n S(f) df \quad (5)$$

The discrete version of this formula is given by equation 6

$$M_n = \frac{1}{Im_0} \sum_{l=c_1}^{c_2} S(l) \left( \frac{l}{N\Delta t} - \bar{l} \right)^n \quad (6)$$

Where the mean frequency  $\bar{l}$  is the normalized value found from the discrete 1. order moment. The second order central moment can be interpreted as the variance of the power spectrum, or the squared bandwidth. The third order is the skewness, and the fourth order is the kurtosis. The central moments can be easily found from the moments.

### 4. A COMPUTATIONAL METHOD USING SPECTRAL MOMENTS

In the photoacoustic signal of our data a lot of high frequency background noise exists. Low frequency noise from pumps and valves can appear in the production environment, and filtering will greatly reduce such disturbance. A typical signal is illustrated in figure 1. Taking the Fast Fourier Transform (FFT) of this signal result in figure 2. The figure shows one tenth of the frequency range of the spectrum, the magnitude is in dB. Little frequency information exists beyond  $2MHz$ , with the strongest components in the signal ranging from  $0,1 - 0,6MHz$ . Filtering of the low frequencies will also remove possible noise from pumps and valves. Filters have a tendency to distort the shape of the time signal and in this case the

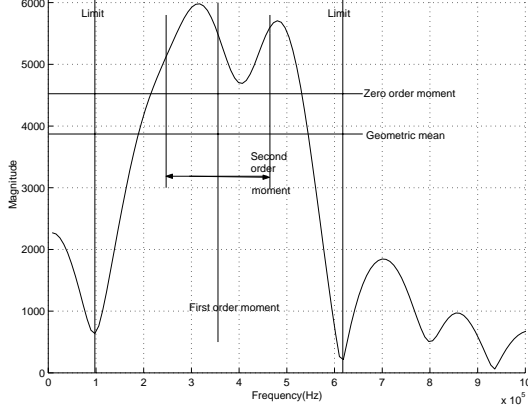


Figure 3: Visualisation of the three first spectral moments.

peak-to-peak value of the signal, as is the case of non-linear phase filters. Because little was gained in accuracy when filtering, alternative methods to represent the photoacoustic response were sought. One such method is to use the spectral moments. Spectral moments are usually equated over the whole spectrum. In this case we wanted to remove the influence of frequencies that can contribute to unwanted noise in the signal. These limits are the lines marked with limit in Fig. 3. The figure is a zoomed in FFT of the photoacoustic signal. For illustrative purposes the magnitude of the spectrum is displayed here. The first three moments and their interpretation is illustrated in Fig. 3. Each moment is found as described below. The zero order moment is defined as the mean power of the signal, and is found with Eq. 7.

$$\overline{m}_a = \frac{1}{I} \sum_{l=c_1}^{c_2} S(l) \quad (7)$$

Another value that gives a similar representation to the zero order moment is the geometric mean. This value has the same unit as arithmetic mean, and it is interesting simply because it is easy to compute.

$$\overline{m}_g = \left( \prod_{l=c_1}^{c_2} S(l) \right)^{1/I} \quad (8)$$

The two methods have similar results, but the geometric mean favors the high values in the spectrum. Both values are marked off in the figure describing their relation to the spectrum. The geometric mean came to the attention when the Fourier transform was displayed in dB. This resulted in the use of the logarithm on each element before computation of the mean magnitude of the spectral moments. The result was interesting so the case was further researched. The relation

between the geometric mean and the arithmetic mean of the logarithmic spectrum is shown in Eq. 9.

$$\begin{aligned} \frac{1}{N} \sum_{n=1}^N 20 \log x_n &= 10 \sum_{n=1}^N \log(x_n^2)^{1/N} \\ &= 10 (\log(x_1^2)^{1/N} + \log(x_2^2)^{1/N} + \dots + \log(x_N^2)^{1/N}) \\ &= 10 \log(\prod_{n=1}^N x_n^2)^{1/N} \end{aligned} \quad (9)$$

The mean used on the logarithm of the power spectrum, gives a scaled version of the geometric mean, scaled by 10 times the logarithm. When estimating the oil concentration we are after a proportional number so the change in scale will not disturb the result. Let us return to Fig. 3 and look at the first order moment. This is interpreted as the mean frequency. The second order moment is the standard deviation of an equivalent Gaussian spectrum. It is a measure of the width of the spectrum about the mean. The square root of the second order moment is also marked off in the figure and equals the distance from the mean to either one of the two lines. Higher order moments are impossible to display graphically in two dimensions. The third order moment is a measure of the skewness in the spectrum, this value will be zero if the spectrum is totally symmetrical about the zero order moment. Instead of using the peak-to-peak value we can use the output from the spectral moments. Each moment has its own interpretation as described above, and might distill information that can improve the result when finding the oil concentration. The zero order moment has the largest correlation with the peak-to-peak value. With statistical analyses we have shown that for our data, the zero order moment is a better representation of the oil concentration than the peak-to-peak value. The second order moment also has strong influence when representing the photoacoustic response, but little or no information was found in the higher order moments.

## 5. FINDING THE PHOTOACOUSTIC RESPONSE DIRECTLY FROM THE TIME DOMAIN

The filtering process is split in two stages. First the signal is filtered with a lowpass filter, this attenuates the high frequency components in the signal and helps to avoid aliasing when sampling the signal at a lower frequency. Next step is to create a highpass filter that removes the low frequency components below 0.1 MHz. This is hard to achieve with the high sampling frequency of 100 MHz. Thus the signal was first decimated before the highpass filter was applied. The decimation process corresponds to sampling at a lower frequency, and shows that it is not necessary to use the high sampling frequency of 100 MHz used in the instrument today. Even with a lower sampling frequency it will be fairly easy to reconstruct the low-

pass filtered signal by zero padding and interpolation. From the filtered signal it is important to find a good representation for the photoacoustic response. There are three ways to find this value. The first method is the peak-to-peak value of the signal. The second method is to use the arithmetic mean power. The third method is to make a summation over the magnitude of the signal. The arithmetic mean power of the signal is related to the frequency domain by Parseval's theorem [9]. Parseval's theorem is valid for the power spectrum of Fourier transformed signals. In the discrete case the relation is expressed in Eq. 10

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2 \quad (10)$$

where  $x[n]$  is an  $N$ -point sequence, and  $X[k]$  is the  $N$  point FFT of the signal. This equation should be compared to the equation for  $m_0$  (Eq. 3), where  $S(l)$  is the discrete spectrum of length  $I = c_2 - c_1$ . The term  $S(l)$  equals the term for the power spectrum  $|X[k]|^2$  from Eq. 10. Eq. 11 therefore gives the following relation in the time domain for  $m_0$ .

$$m_0 = \sum_{n=0}^{N-1} |x[n]|^2 \quad (11)$$

This shows that it is possible to compute the zero order spectral moment directly from the time domain itself. If filtering is done first, we get the same effect as using the zero order moment over a range of frequencies in the spectrum. This method performed very well on our data. Running the dataset through this method produced a similar results as the zero order spectral moment. This shows that the total energy in the signal can be used to find the oil concentration. The decimation process is important to make the computation of the energy fast and efficient. Instead of depending only on the two values used by the the peak-to-peak method, this method use the energy in the whole signal. At the same time the method is easy to implement both in hardware and software. Another, and by far, the simplest method for finding the photoacoustic response is by averaging the magnitude of the filtered signal as in Eq. 12.

$$m_0 = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]| \quad (12)$$

In a hardware implementation a magnitude operation followed by a standard averaging filter can be used to find the photoacoustic response value at certain time intervals. This value is not related directly to the spectral moments in the same way as the signal energy in Eq. 11. However, we wanted to take a closer look at it because of the ease of implementation and

the close similarity to the signal energy. The average of the filtered signal and the statistical performance is compared to the other techniques in the next section. This can be a possible implementation when running the instrument in a downhole environment, it removes the need of a computer or a A/D card to equate the photoacoustic response.

### 5.1. A statistical view of the data

It is interesting to take a look at the statistical similarity between the parameters of the zero order moment, the signal energy and the average magnitude of the signal. A mathematical connection exists through Parseval's theorem between the zero order moment and signal energy, statistics shows that the average magnitude gives similar performance. One can, however, not expect that the three methods give exactly the same result. First of all the average magnitude method is not mathematically equivalent to the other two methods. Second, there will also be small differences in the way the filtering is done in the time domain and the frequency domain for the energy and the zero order moment method respectively. We have used linear regression to show the similarity between the methods. The models used are presented in Eqs. 13, 14, and 15

$$oc_i = \alpha_0 + \alpha_1 m0_i + \alpha_2 temp_i + \alpha_3 sal_i + \epsilon_i \quad (13)$$

$$oc_i = \alpha_0 + \alpha_1 filter_i + \alpha_2 temp_i + \alpha_3 sal_i + \epsilon_i \quad (14)$$

$$oc_i = \alpha_0 + \alpha_1 average_i + \alpha_2 temp_i + \alpha_3 sal_i + \epsilon_i \quad (15)$$

Here  $oc_i$  is the oil concentration using the values from the oil dispenser, where the subscript  $i$  represents observation number  $i$ . The term  $m0_i$  in equation 13 is the zero order moment. The  $filter_i$  uses the mean energy of the filtered signal, and  $average_i$  use the average magnitude of the signal. The  $temp_i$  and  $sal_i$  represent the temperature and salinity respectively. All the variables were normalized before doing regression, this makes the output parameters unusable for calibration of the instrument, but suitable for comparison of equations. The output parameters are displayed in table 1. The table displays values that are

	Eq. 13	Eq. 14	Eq. 15
$\alpha_0$	$-5.0 \times 10^{-16}$	$-2.6 \times 10^{-16}$	$3.6 \times 10^{-16}$
$\alpha_1$	0.7753	0.7736	0.8313
$\alpha_2$	-0.2418	-0.2428	-0.2092
$\alpha_3$	-0.1821	-0.1816	-0.1329

Table 1: The table display the value of the four parameters in the three equations 13, 14 and 15 after running linear regression.

close to each other. They are not equal, but indicate

that they are highly correlated. The residual sum of squares (RSS) is 601 for Eq. 13, 606 for Eq. 14 and 445 for Eq. 15. Surprisingly the average magnitude gives the best fit. Also we see that the performance is almost identical for Eqs. 13 and 14 as expected. A bonus is that the average magnitude is easiest to implement and at the same time gives best result.

## 6. CONCLUSION

This paper proposes some alternative methods to the peak-to-peak value for finding the response from a photoacoustic signal. First the filtering process is important to reduce both high and low frequency noise. The components bearing information in the signal are found to be in the range  $0,1 - 0,6\text{MHz}$ , so passing these components should be an important characteristics of the filter.

The spectral moments are a new way to analyse the signal, they work directly on the spectrum and can be limited within these ranges. The zero order moment is the best candidate to replace the peak-to-peak value. The second order moment also shows ability to contribute new information when modelling the photoacoustic response. Little is known of the influence of the higher order moments, but there might be important information that can be used to get a better representation of the oil concentration.

The analysis also shows that the sampling frequency can be greatly reduced. Here it is important to have a lowpass filter that smooths the signal, and removes possible aliasing when sampling. The sampling can be safely reduced down to 1/25th of that of the original signal without loss of information. It should be possible to sample the signal at even lower frequencies. When using low frequency sampling an alternative method building on the zero order spectral moment is proposed. Instead of using the peak-to-peak value it is possible to use the arithmetic mean power over the signal to represent the oil concentration. When using this method it is necessary to perform some filtering of the signal, but it is probably sufficient to use a lowpass filter before sampling. In experiments with this method the signal was decimated down to 1/100th of the original signal, this represent a sampling frequency of 1MHz. This process still shows good ability to represent the photoacoustic response. The parameters from linear regression illustrate that both the arithmetic mean power and the average magnitude of the filtered signal produce similar results to the zero order spectral moment. This makes it possible to make a fast and stable method for monitoring. The photoacoustic response can then be estimated by means of a bandpass filter, a rectifier and an averaging filter. If the hardware filters are constructed with care, the A/D card and computer can be removed in

future applications.

## 7. ACKNOWLEDGEMENT

Want to thank Tone Schanke at Kværner Oilfield Products, Gunder Homstvedt at Hydro and Dr. Hugh MacKenzie at Heriot-Watt University for their involvement and contribution in the project.

## 8. REFERENCES

- [1] United Kingdom , "Prevention of Oil Pollution Act 1971", HMSO.
- [2] "Water analysis: Determination of oil in water Infrared spectrophotometric method", Norsk Standard NS9803, juni 1993.
- [3] T.Schanke, S.A.Kjølberg, F.Vogel, "Oil in Water Monitoring for Subsea and Downhole Separators", SPE 66538 ,Society of Petroleum Engineers, February 2001
- [4] Scott Stuart Freeborn, "Pulsed Laser Photoacoustic instrumentation for the monitoring of crude oil in produced water", Department of Physics, Herriot-Watt University, Edinburgh, September 1997.
- [5] Tone Schanke, Scott S. Freeborn, "Oil In Water - Downhole Monitoring: Norsk Hydro Tests", Kværner Oilfield Products, Heriot-Watt University, Document no. 407954, dec. 1998.
- [6] Fredrik Vogel, "Spectral moments and linear models used for photoacoustic detection of crude oil in produced water", University of Oslo, department of Informatics, April 2001.
- [7] P. Terzoudi, T. Whitaker, H.A. Mackenzie, "Use of Photo-acoustic measurement technology to measure hydrocarbon concentration levels in reinjection lines", Kværner Oilfield Products Ltd., Heriot-Watt University, COSWASS JIP, 2000.
- [8] Sverre Holm, IfI Memo, 13 august 1999
- [9] A.V. Oppenheim, R.W. Schaffer, "Discrete-Time Signal Processing", prentice-hall international inc., ISBN 0-13-216771-9, 1989.