Optimisation projects
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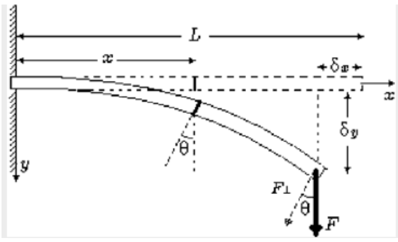
Exercise 1 : Application to the cantilever beam cross section optimisation

The objective of the lab work is to make a global analysis and synthesis in order to find the best section for the problem the can support a concentrated force of F = 20kN applied to the free end of the cantilever beam (see the figure).



Beam parameter :

Young modulus	E= 2. 1.e11	Pa
Density (mass/volume)	ρ = Ro = 7800	Kg / m3
Admissible stress yield (max)	σ_a =sigma= 2 1.e8	Pa
Length of the beam	L = 2	m
Concentrated Force	F = 20 000	N
Admissible deflection (max)	da = Δ_a = 0.01	m
Admissible mass (max)	ma = μ_a = 300	kg

General equations of the flexion of the cantilever beam

stress :	$\sigma_{max} <= \sigma_a$	$\sigma_{max} = (FL/\mathbf{I}) * v$ (v=h/2)
Deflection	$\Delta <= \Delta_a$	$\Delta = FL^3 / (3 E \mathbf{I})$
Masse	m <= ma	$m = \rho$ L S (S = section)

Where $\mathbf{I}$ represent the Second moment of the cross section and S is the section.
For the bounds it is accepted that the h>L/5.

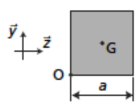
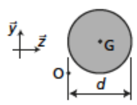
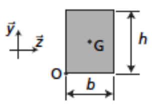
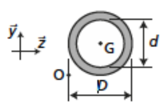
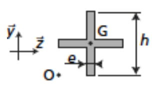
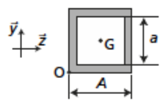
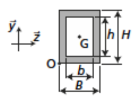
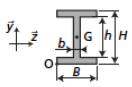
For the beam of square section : $\mathbf{I} = a^4 / 12$ and $S = a^2$

so that : $\sigma_{max} = 6FL / a^3$ $\Delta = 4 FL^3 / (E a^4)$ and Mass = ρ L a^2

For the beam of disc section : $\mathbf{I} = \pi D^4 / 64$ and $S = \pi D^2 / 4$

so that : $\sigma_{max} = 32$ F L / πD^3 , $\Delta = 64 FL^3 / (3 E \pi D^4)$ and Mass = ρ L $\pi D^2 / 4$

The objective of this optimization problem is to select the best section among the following section

One variable				
Two variables				
four variables				

The formulas of the section and the second moment and the section are given in Annex 1.

1/ Problems with One variable (Square and disc)

1.1./ P1: Mass minimization subject to the constraints (σ_{max} <= σ_a , Δ < = Δ_a)

- Formulate the problem of minimizing the mass subject to the constraint of maximum deflection and stress yield.
- Without using the optimization tool, find by hand the value of corresponding value of "a" (the square side length) and "D" (the diameter if the disc section) that minimize the mass (s.t. the constraints).
- Use MATLAB to find the same solution (select the appropriate lower and upper bounds),
- Plot the mass curve and show on the curve the limit corresponding the stress yield constraint, deflection and bounds.

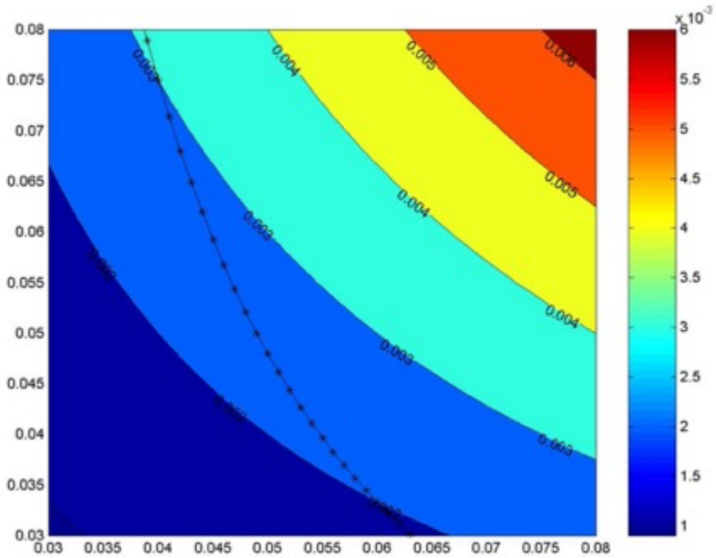
1.2./ P2: Deflection minimization subject to the constraints (σ_{max} <= σ_a , mass < ma)

- Apply the same steps as 1.1/ for the minimizing the Deflection subject to the constraint of maximum mass and stress yield.

2/ Problem with two variables (Hollow disc, Hollow square, Rectangle and plus section)

- For each section write the equations of the mass, deflection stress yield
- Formulate the two optimization problems (Mass and deflection : as in 1.1)
- Use MATLAB to find the optimum (select the appropriate lower and upper bounds),
- Plot the contour of the objective function and the line constraint of the constraints

This figure is an example for the plot of the contour of mass including the stress yield constraint line for the problem of the beam



2/ Problem with four variables (Hollow square, and IPN)

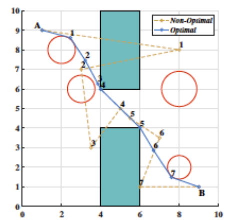
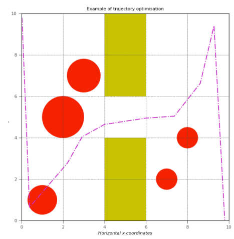
Use MATLAB to find the optimum (select the appropriate lower and upper bounds), for the two minimization problem (mass and deflection)

Exercise 2 : Trajectory optimization

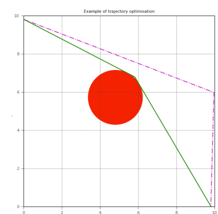
Consider a room of size 10x10. The room has obstacles in the form of "c" circles of given radii and centers. Two rectangular walls, each of size 4x2 with their origins at (4, 0) and (4, 6) separates the test region into two rooms. A trajectory AB has to travel from the left side of the room (preferably from the top) to the right side of the room (preferably to the bottom). The trajectory has to pass through p intermediate points before it has to reach its destination B. The objective of the project is to optimise this trajectory containing the intermediate points (guess points) in such a way that they pass either tangentially to the circles or away from the circles as well as pass in between the walls as it tries to go from the left to the right side of the room.

The discretization has to be done for each individual segments of the trajectory with n number of points. For example, if there is one guess point, the discretization has to be done between A and the guess point followed by discretization between the guess point and final point B. If multiple guess points are initialized, successful discretization on the segments between the guess points has to be done.

The number of circles , the number of intermediates point, the starting point A and the goal B are given by the user.

The objective is to find the best trajectory between point A and B avoiding obstacle like circles and two rectangles representing the walls separating two rooms.	 

3.1 For a given point A(1, 5) and B(9,5) and a circle centered at (5,5) with r= 2 , find the best trajectory from A to B avoiding the circle (no wall) :

a/ The trajectory can be represented by two segments, b/ the trajectory can be represented by 3 segments c/ and n segments	

3.2. For a given point A and B and a 2 circles, find the best trajectory from A to B avoiding the circles (no wall)

3.3. For a given point A and B and a n circles, find the best trajectory from A to B avoiding the circles (no wall)

3.4. For a given point A and B and a n circles, find the best trajectory from A to B avoiding the circles and the wall.

You can use or update the following algorithm

```
function D=distance_segment_circle(A,B,C, n)
% A et B two point of the segement
% C center of the circle R the radius
% n number of discretization
for i=0:n
    M=A+(i/n)*(B-A);     % M is a given point between A and B
    d(i+1)=norm(M-C);
end
D=min(d);
end
```

Suppose you have the list of points of the trajectory [P1 ; P2 ; … ; Pn].

In order to calculate the objective function value and the constraints, it is recommended to create a new list like : [A; P1 ; P2 ; … ; Pn; B]

For each step 3.1 to 3.4, plot the initial trajectory, the final trajectory and the environment (circles and wall).

Exercise 3:

3.1. Implement the algorithm of the dichotomous optimization method for a single variable problem.

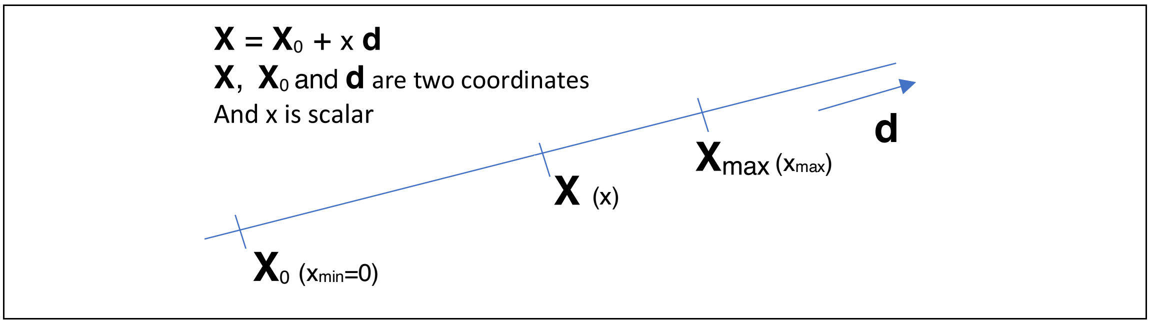
Apply the algorithm to find the minim of : $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 1$

Test the algorithm with different initial intervals. It is strongly recommended to define a Matlab function for the objective function.

3.2. Extension of dichotomous method to multivariable optimization problem

For a given point $\mathbf{X}_0$ and a given direction $\mathbf{d}$, one can find all the point of the line Δ defined by ($\mathbf{X}_0$, and $\mathbf{d}$) $\mathbf{X} = \mathbf{X}_0 + x\,\mathbf{d}$. $f(\mathbf{X}_0 + x\,\mathbf{d})$ is only function of the single variable x. For the extended algorithm, it is recommended to estimate the extreme point on the line Δ (x_{min} correspond to $\mathbf{X}_0$ and x_{max} correspond to the extreme point).

Estimate



Apply the extended algorithm to the 2 variables optimization problem :

$$f(\mathbf{X}) = (3X_1 + 2X_2 - 1)^2 + (X_1 - X_2 + 1)^2 \quad , \mathbf{X}_0 = (0,1) \text{ and } \mathbf{d} = (0.5\ ,\ 1)$$

X_1 and X_2 are the two components of $\mathbf{X} = (X_1, X_2)$ (X_1 and X_2 in the interval [-10, 10])

Plot the corresponding contour, and the single variable line $f(\mathbf{X}_0 + x\,\mathbf{d})$.

Use different tests and comment the results.

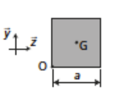
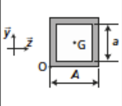
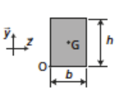
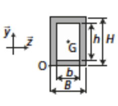
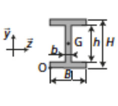
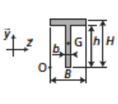
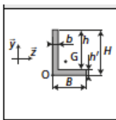
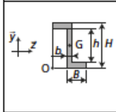
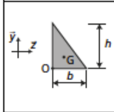
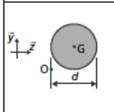
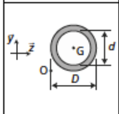
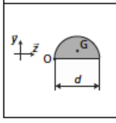
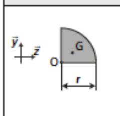
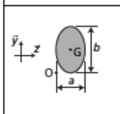
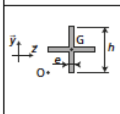
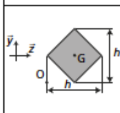
Exercise 4 :

4.1. Implement the algorithm of the Simulated Annealing method for a single variable problem and for 2 variables optimization problem.

4.2. Test the method to the same previous function of exercise 1 :

$$f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 1 \quad \text{and } f(\mathbf{X}) = (3X_1 + 2X_2 - 1)^2 + (X_1 - X_2 + 1)^2$$

Annex 1 : proprieties of sections

	$S = a^2$	$z_G = \frac{a}{2}$ $y_G = \frac{a}{2}$	$\mathcal{H}_{Oz} = \frac{a^3}{2}$ $\mathcal{H}_{Oy} = \frac{a^3}{2}$	$I_{Gz} = \frac{a^4}{12}$ $I_{Gy} = \frac{a^4}{12}$
	$S = A^2 - a^2$	$z_G = \frac{A}{2}$ $y_G = \frac{A}{2}$	$\mathcal{H}_{Oz} = \frac{A^3 - a^2 A}{2}$ $\mathcal{H}_{Oy} = \frac{A^3 - a^2 A}{12}$	$I_{Gz} = \frac{A^4 - a^4}{12}$ $I_{Gy} = \frac{A^4 - a^4}{12}$
	$S = bh$	$z_G = \frac{b}{2}$ $y_G = \frac{h}{2}$	$\mathcal{H}_{Oz} = \frac{bh^2}{2}$ $\mathcal{H}_{Oy} = \frac{b^2 h}{2}$	$I_{Gz} = \frac{bh^3}{12}$ $I_{Gy} = \frac{hb^3}{12}$
	$S = BH - bh$	$z_G = \frac{B}{2}$ $y_G = \frac{H}{2}$	$\mathcal{H}_{Oz} = \frac{BH^2 - bhH}{2}$ $\mathcal{H}_{Oy} = \frac{B^2 H - bhB}{2}$	$I_{Gz} = \frac{BH^3 - bh^3}{12}$ $I_{Gy} = \frac{HB^3 - hb^3}{12}$
	$S = BH - h(B-b)$	$z_G = \frac{B}{2}$ $y_G = \frac{H}{2}$	$\mathcal{H}_{Oz} = \frac{BH^2 - hH(B-b)}{2}$ $\mathcal{H}_{Oy} = \frac{B^2 H - hB(B-b)}{2}$	$I_{Gz} = \frac{BH^3 - h^3(B-b)}{12}$ $I_{Gy} = \frac{B^3 H - h(B^3 - b^3)}{12}$
	$S = BH - h(B-b)$	$z_G = \frac{B}{2}$ $y_G = \frac{H_{Oz}}{S}$	$\mathcal{H}_{Oz} = \frac{BH^2 - h^2(B-b)}{2}$ $\mathcal{H}_{Oy} = \frac{B^2 H - hB(B-b)}{2}$	$I_{Gz} = \frac{BH^3 - h^3(B-b)}{3} - y_G^2 S$ $I_{Gy} = \frac{B^3 H - h(B^3 - b^3)}{12}$
	$S = BH - h(B-b)$	$z_G = \frac{H_{Oy}}{S}$ $y_G = \frac{H_{Oz}}{S}$	$\mathcal{H}_{Oz} = \frac{(B-b)h'^2 + bH^2}{2}$ $\mathcal{H}_{Oy} = \frac{B^2 h' + hb^2}{2}$	$I_{Gz} = \frac{(B-b)h'^3 + bH^3}{3} - y_G^2 S$ $I_{Gy} = \frac{h' B^3 + hb^3}{3} - z_G^2 S$
	$S = BH - h(B-b)$	$z_G = B - \frac{b}{2}$ $y_G = \frac{H}{2}$	$\mathcal{H}_{Oz} = \frac{BH^2 - hH(B-b)}{2}$ $\mathcal{H}_{Oy} = z_G S$	$I_{Gz} = \frac{BH^3 - h^3(B-b)}{12}$ $I_{Gy} = \frac{2hb^3 + (H-h)(2B-b)^3}{24}$
	$S = \frac{1}{2} bh$	$z_G = \frac{b}{3}$ $y_G = \frac{h}{3}$	$\mathcal{H}_{Oz} = \frac{bh^2}{6}$ $\mathcal{H}_{Oy} = \frac{b^2 h}{6}$	$I_{Gz} = \frac{bh^3}{36}$ $I_{Gy} = \frac{hb^3}{36}$
	$S = \frac{\pi d^2}{4}$	$z_G = \frac{d}{2}$ $y_G = \frac{d}{2}$	$\mathcal{H}_{Oz} = \frac{\pi d^3}{8}$ $\mathcal{H}_{Oy} = \frac{\pi d^3}{8}$	$I_{Gz} = \frac{\pi D^4}{64}$ $I_{Gy} = \frac{\pi D^4}{64}$
	$S = \frac{\pi(D^2 - d^2)}{4}$	$z_G = \frac{D}{2}$ $y_G = \frac{D}{2}$	$\mathcal{H}_{Oz} = \frac{\pi(D^3 - d^2 D)}{8}$ $\mathcal{H}_{Oy} = \frac{\pi(D^3 - d^2 D)}{8}$	$I_{Gz} = \frac{\pi(D^4 - d^4)}{64}$ $I_{Gy} = \frac{\pi(D^4 - d^4)}{64}$
	$S = \frac{\pi d^2}{8}$	$z_G = \frac{d}{2}$ $y_G = \frac{2d}{3\pi}$	$\mathcal{H}_{Oz} = \frac{d^3}{12}$ $\mathcal{H}_{Oy} = \frac{\pi d^3}{16}$	$I_{Gz} = \frac{d^4}{16}\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$ $I_{Gy} = \frac{\pi d^4}{128}$
	$S = \frac{\pi r^2}{4}$	$z_G = \frac{4r}{3\pi}$ $y_G = \frac{4r}{3\pi}$	$\mathcal{H}_{Oz} = \frac{r^3}{3}$ $\mathcal{H}_{Oy} = \frac{r^3}{3}$	$I_{Gz} = \frac{r^4}{2}\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$ $I_{Gy} = \frac{r^4}{2}\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$
	$S = \frac{\pi ab}{4}$	$z_G = \frac{a}{2}$ $y_G = \frac{b}{2}$	$\mathcal{H}_{Oz} = \frac{\pi ab^2}{8}$ $\mathcal{H}_{Oy} = \frac{\pi a^2 b}{8}$	$I_{Gz} = \frac{\pi ab^3}{64}$ $I_{Gy} = \frac{\pi a^3 b}{64}$
	$S = 2eh - e^2$	$z_G = \frac{h}{2}$ $y_G = \frac{h}{2}$	$\mathcal{H}_{Oz} = \frac{2eh^2 - e^2 h}{2}$ $\mathcal{H}_{Oy} = \frac{2eh^2 - e^2 h}{2}$	$I_{Gz} = \frac{eh^3 + e^3 h - e^4}{12}$ $I_{Gy} = \frac{eh^3 + e^3 h - e^4}{12}$
	$S = \frac{h^2}{2}$	$z_G = \frac{h}{2}$ $y_G = \frac{h}{2}$	$\mathcal{H}_{Oz} = \frac{h^3}{4}$ $\mathcal{H}_{Oy} = \frac{h^3}{4}$	$I_{Gz} = \frac{h^4}{48}$ $I_{Gy} = \frac{h^4}{48}$