CHAPTER 2

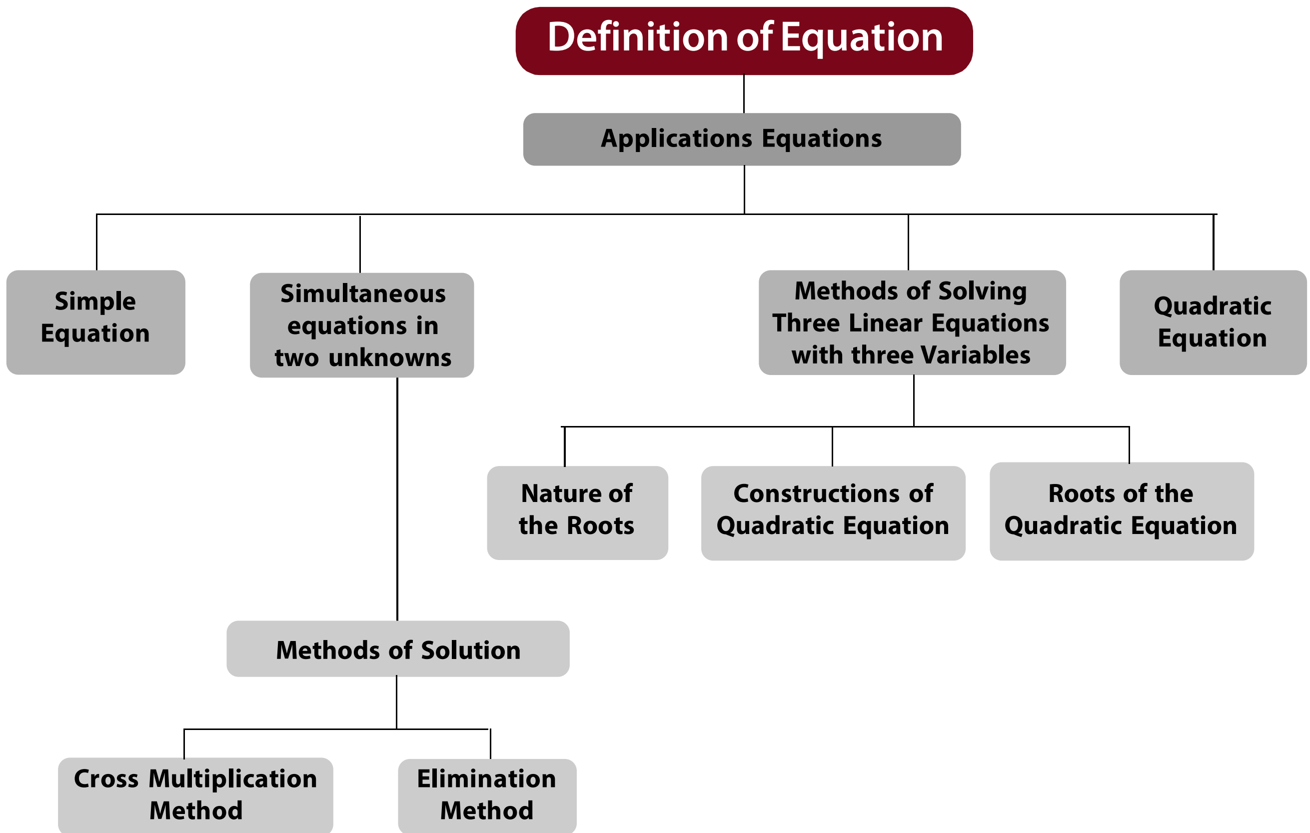
EQUATIONS

LEARNING OBJECTIVES

After studying this chapter, you will be able to:

- Understand the concept of equations and its various degrees – linear, simultaneous, quadratic and cubic equations;
- Know how to solve the different equations using different methods of solution; and

CHAPTER OVERVIEW



2.1 INTRODUCTION

Equation is defined to be a mathematical statement of equality. If the equality is true for certain value of the variable involved, the equation is often called a conditional equation and equality sign '=' is used; while if the equality is true for all values of the variable involved, the equation is called an identity.

For Example: $\frac{x+2}{3}+\frac{x+3}{2}=3$ holds true only for $x=1$.

So it is a conditional. On the other hand, $\frac{x+2}{3}+\frac{x+3}{2}=\frac{5x+13}{6}$

is an identity since it holds for all values of the variable x.

Determination of value of the variable which satisfy an equation is called solution of the equation or root of the equation. An equation in which highest power of the variable is 1 is called a Linear (or a simple) equation. This is also called the equation of degree 1. Two or more linear equations involving two or more variables are called *Simultaneous Linear Equations*. An equation of degree 2 (Highest Power of the variable is 2) is called *Quadratic equation* and the equation of degree 3 is called *Cubic Equation*.

For Example: $8x+17(x–3) = 4 (4x–9) + 12$ is a Linear equation.

$3x^2 + 5x + 6 = 0$ is a Quadratic equation.

$4x^3 + 3x^2 + x–7 = 1$ is a Cubic equation.

$x + 2y = 1$, $2x + 3y = 2$ are jointly called Simultaneous equations.

2.2 SIMPLE EQUATION

A simple equation in one unknown x is in the form $ax + b = 0$.

Where a, b are known constants and a # 0

Note: A simple equation has only one root.

Example: $\frac{4x}{3}-1=\frac{14}{15}x+\frac{19}{5}$.

Solution: By transposing the variables in one side and the constants in other side we have

$$\frac{4x}{3}-\frac{14x}{15}=\frac{19}{5}+1 \text{ or } \frac{(20-14)x}{15}=\frac{19+5}{5} \text{ or } \frac{6x}{15}=\frac{24}{5}.$$

$$x=\frac{24\text{x}15}{5\text{x}6}=12$$

EXERCISE (A)

Choose the most appropriate option (a) (b) (c) or (d).

1. The equation $-7x + 1 = 5 - 3x$ will be satisfied for x equal to:

a) 2 b) –1 c) 1 d) none of these

2. The root of the equation $\frac{x+4}{4} + \frac{x-5}{3} = 11$ is

a) 20 b) 10 c) 2 d) none of these

3. Pick up the correct value of x for $\frac{x}{30} = \frac{2}{45}$

a) $x = 5$ b) $x = 7$ c) $x = 1\frac{1}{3}$ d) none of these

4. The solution of the equation $\frac{x+24}{5} = 4 + \frac{x}{4}$

a) 6 b) 10 c) 16 d) none of these

5. 8 is the solution of the equation

a) $\frac{x+4}{4} + \frac{x-5}{3} = 11$ b) $\frac{x+4}{2} + \frac{x+10}{9} = 8$

c) $\frac{x+24}{5} = 4 + \frac{x}{4}$ d) $\frac{x-15}{10} + \frac{x+5}{5} = 4$

6. The value of y that satisfies the equation $\frac{y+11}{6} - \frac{y+1}{9} = \frac{y+7}{4}$ is

a) –1 b) 7 c) 1 d) $-\frac{1}{7}$

7. The solution of the equation (p+2) (p–3) + (p+3) (p–4) = p(2p–5) is

a) 6 b) 7 c) 5 d) none of these

8. The equation $\frac{12x+1}{4} = \frac{15x-1}{5} + \frac{2x-5}{3x-1}$ is true for

a) $x=1$ b) $x=2$ c) $x=5$ d) $x=7$

9. Pick up the correct value x for which $\frac{x}{0.5} - \frac{1}{0.05} + \frac{x}{0.005} - \frac{1}{0.0005} = 0$

a) $x=0$ b) $x = 1$ c) $x = 10$ d) none of these

ILLUSTRATIONS:

1. The denominator of a fraction exceeds the numerator by 5 and if 3 be added to both the fraction becomes $\frac{3}{4}$. Find the fraction.

Let x be the numerator and the fraction be $\frac{x}{x+5}$. By the question $\frac{x+3}{x+5+3}=\frac{3}{4}$ or $4x + 12 = 3x + 24$ or $x = 12$

The required fraction is $\frac{12}{17}$.

2. If thrice of A's age 6 years ago be subtracted from twice his present age, the result would be equal to his present age. Find A's present age.

Let x years be A's present age. By the question

$2x–3(x–6) = x$

or $2x–3x + 18 = x$

or $–x + 18 = x$

or $2x = 18$

or $x=9$

$\therefore$ A's present age is 9 years.

3. A number consists of two digits the digit in the ten's place is twice the digit in the unit's place. If 18 be subtracted from the number the digits are reversed. Find the number.

Let x be the digit in the unit's place. So the digit in the ten's place is $2x$. Thus the number becomes $10(2x) + x$. By the question

$20x + x– 18 = 10x + 2x$

or $21x – 18 = 12x$

or $9x = 18$

or $x = 2$

So the required number is 10 (2 × 2) + 2 = 42.

4. For a certain commodity the demand equation giving demand 'd' in kg, for a price 'p' in rupees per kg. is d = 100 (10 – p). The supply equation giving the supply s in kg. for a price p in rupees per kg. is s = 75(p – 3). The market price is such at which demand equals supply. Find the market price and quantity that will be bought and sold.

Given d = 100(10 – p) and s = 75(p – 3).

Since the market price is such that demand (d) = supply (s) we have

100 (10 – p) = 75 (p – 3) or 1000 – 100p = 75p – 225

or – 175p = – 1225. $\therefore p = \frac{-1225}{-175} = 7$.

So market price of the commodity is ₹ 7 per kg.

$\therefore$ the required quantity bought = 100 (10 – 7) = 300 kg.

and the quantity sold = 75 (7 – 3) = 300 kg.

EXERCISE (B)

Choose the most appropriate option (a) (b) (c) or (d).

1. The sum of two numbers is 52 and their difference is 2. The numbers are

   a) 17 and 15 b) 12 and 10 c) 27 and 25 d) none of these

2. The diagonal of a rectangle is 5 cm and one of at sides is 4 cm. Its area is

   a) 20 sq.cm. b) 12 sq.cm. c) 10 sq.cm. d) none of these

3. Divide 56 into two parts such that three times the first part exceeds one third of the second by 48. The parts are.

   a) (20, 36) b) (25, 31) c) (24, 32) d) none of these

4. The sum of the digits of a two digit number is 10. If 18 be subtracted from it the digits in the resulting number will be equal. The number is

   a) 37 b) 73 c) 75 d) none of these numbers.

5. The fourth part of a number exceeds the sixth part by 4. The number is

   a) 84 b) 44 c) 48 d) none of these

6. Ten years ago the age of a father was four times of his son. Ten years hence the age of the father will be twice that of his son. The present ages of the father and the son are.

   a) (50, 20) b) (60, 20) c) (55, 25) d) none of these

7. The product of two numbers is 3200 and the quotient when the larger number is divided by the smaller is 2.The numbers are

   a) (16, 200) b) (160, 20) c) (60, 30) d) (80, 40)

8. The denominator of a fraction exceeds the numerator by 2. If 5 be added to the numerator the fraction increases by unity. The fraction is.

   a) $\frac{5}{7}$ b) $\frac{1}{3}$ c) $\frac{7}{9}$ d) $\frac{3}{5}$

9. Three persons Mr. Roy, Mr. Paul and Mr. Singh together have ₹ 51. Mr. Paul has ₹ 4 less than Mr. Roy and Mr. Singh has got ₹ 5 less than Mr. Roy. They have the money as.

   a) (₹ 20, ₹ 16, ₹ 15) b) (₹ 15, ₹ 20, ₹ 16)
   c) (₹ 25, ₹ 11, ₹ 15) d) none of these

10. A number consists of two digits. The digits in the ten's place is 3 times the digit in the unit's place. If 54 is subtracted from the number the digits are reversed. The number is

    a) 39 b) 92 c) 93 d) 94

11. One student is asked to divide a half of a number by 6 and other half by 4 and then to add the two quantities. Instead of doing so the student divides the given number by 5. If the answer is 4 short of the correct answer then the number was

    a) 320 b) 400 c) 480 d) none of these.

12. If a number of which the half is greater than $\frac{1}{5}$ th of the number by 15 then the number is

a) 50 b) 40 c) 80 d) none of these.

2.3 SIMULTANEOUS LINEAR EQUATIONS IN TWO UNKNOWNS

The general form of a linear equations in two unknowns x and y is $ax + by + c = 0$ where a, b are non-zero coefficients and c is a constant. Two such equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ form a pair of simultaneous equations in x and y. A value for each unknown which satisfies simultaneously both the equations will give the roots of the equations.

2.4 METHOD OF SOLUTION

1. **Elimination Method:** In this method two given linear equations are reduced to a linear equation in one unknown by eliminating one of the unknowns and then solving for the other unknown.

Example 1: Solve: $2x + 5y = 9$ and $3x - y = 5$.

Solution: $2x + 5y = 9$ …….. (i)

$3x - y = 5$ ………(ii)

By making (i) x 1, $2x + 5y = 9$

and by making (ii) x 5, $15x - 5y = 25$

Adding $17x = 34$ or $x = 2$. Substituting this values of x in (i) i.e. $5y = 9 - 2x$ we find;

$5y = 9 - 4 = 5 \therefore y = 1 \therefore x = 2, y = 1$.

2. **Cross Multiplication Method:** Let two equations be:

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

We write the coefficients of x, y and constant terms and two more columns by repeating the coefficients of x and y as follows:

$$\begin{array}{ccccccc} 1 & \underline{\quad x \quad} & 2 & \underline{\quad y \quad} & 3 & \underline{\quad 1 \quad} & 4 \\ b_1 & & c_1 & & a_1 & & b_1 \\ b_2 & & c_2 & & a_2 & & b_2 \end{array}$$

and the result is given by: $\dfrac{x}{b_1c_2 - b_2c_1} = \dfrac{y}{(c_1a_2 - c_2a_1)} = \dfrac{1}{(a_1b_2 - a_2b_1)}$

so the solution is : $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ $\quad y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$.

Example 2: Solve $3x + 2y + 17 = 0, 5x - 6y - 9 = 0$

Solution: $3x + 2y + 17 = 0$ .... (i)

$5x - 6y - 9 = 0$ ....(ii)

Method of elimination: By (i) x 3 we get $9x + 6y + 51 = 0$ ...... (iii)

Adding (ii) & (iii) we get $14x + 42 = 0$

$$\text{or } x = -\frac{42}{14} = -3$$

Putting $x = -3$ in (i) we get $3(-3) + 2y + 17 = 0$

$$\text{or, } 2y + 8 = 0 \text{ or, } y = -\frac{8}{2} = -4$$

So $x = -3$ and $y = -4$

Method of cross-multiplication: $3x + 2y + 17 = 0$

$5x - 6y - 9 = 0$

$$\frac{x}{2(-9) - 17(-6)} = \frac{y}{17 \times (5) - 3(-9)} = \frac{1}{3(-6) - 5 \times (2)}$$

$$\text{or, } \frac{x}{84} = \frac{y}{112} = \frac{1}{-28}$$

$$\text{or } \frac{x}{3} = \frac{y}{4} = \frac{1}{-1}$$

or $x = -3, y = -4$

2.5 METHOD OF SOLVING SIMULTANEOUS LINEAR EQUATION WITH THREE VARIABLES

Example 1: Solve for x, y and z:

$2x - y + z = 3, x + 3y - 2z = 11, 3x - 2y + 4z = 1$

Solution: (a) Method of elimination

$2x - y + z = 3$ ....(i)

$x + 3y - 2z = 11$ .... (ii)

$3x - 2y + 4z = 1$ .... (iii)

By (i) × 2 we get

$4x - 2y + 2z = 6$ .... (iv)

By (ii) + (iv), $5x + y = 17$ ....(v) [the variable z is thus eliminated]

By (ii) × 2, $2x + 6y - 4z = 22$ ....(vi)

By (iii) + (vi), $5x + 4y = 23$ ....(vii)

By (v) – (vii), $-3y = -6$ or $y = 2$

Putting $y = 2$ in (v) $5x + 2 = 17$, or $5x = 15$ or, $x = 3$

Putting $x = 3$ and $y = 2$ in (i)

$2 \times 3 - 2 + z = 3$

or $6 - 2 + z = 3$

or $4 + z = 3$

or $z = -1$

So $x = 3$, $y = 2$, $z = -1$ is the required solution.

(Any two of 3 equations can be chosen for elimination of one of the variables)

(b) Method of cross multiplication

We write the equations as follows:

$2x - y + (z - 3) = 0$

$x + 3y + (-2z - 11) = 0$

By cross multiplication

$$\frac{x}{-1(-2z-11)-3(z-3)} = \frac{y}{(z-3)-2(-2z-11)} = \frac{1}{(2\times3)-(1(-1))}$$

$$\frac{x}{20\text{-}z} = \frac{y}{5z+19} = \frac{1}{7}$$

$$x = \frac{20-z}{7}, \quad y = \frac{5z+19}{7}$$

Substituting above values for x and y in equation (iii) i.e. $3x - 2y + yz = 1$, we have

$$3\left(\frac{20-z}{7}\right) - 2\left(\frac{5z+19}{7}\right) + 4z = 1$$

or $60 - 3z - 10z - 38 + 28z = 7$

or $15z = 7 - 22$ or $15z = -15$ or $z = -1$

Now $x = \dfrac{20-(-1)}{7} = \dfrac{21}{7} = 3$, $\quad y = \dfrac{5(-1)+19}{7} = \dfrac{14}{7} = 2$

Thus $x = 3$, $y = 2$, $z = -1$

Example 2: Solve for x, y and z :

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=5, \quad \frac{2}{x}-\frac{3}{y}-\frac{4}{z}=-11, \quad \frac{3}{x}+\frac{2}{y}-\frac{1}{z}=-6$$

Solution: We put u = $\frac{1}{x}$; v = $\frac{1}{y}$; w = $\frac{1}{z}$ and get

$u + v + w = 5$ .... (i)

$2u - 3v - 4w = -11$........ (ii)

$3u + 2v - w = -6$........ (iii)

By (i) + (iii) $4u + 3v = -1$ .... (iv)

By (iii) x 4 $12u + 8v - 4w = -24$ .... (v)

By (ii) – (v) $-10u - 11v = 13$

or $10u + 11v = -13$ .... (vi)

By (iv) × 11 $44x + 33v = -11$ ....(vii)

By (vi) × 3 $30u + 33v = -39$ ....(viii)

By (vii) – (viii) $14u = 28$ or $u = 2$

Putting u = 2 in (iv) $4 \times 2 + 3v = -1$

or $8 + 3v = -1$

or $3v = -9$ or $v = -3$

Putting u = 2, v = –3 in (i) or $2-3 + w = 5$

or $-1 + w = 5$ or $w = 5 + 1$ or $w = 6$

Thus $x = \frac{1}{u} = \frac{1}{2}$, $y = -\frac{1}{v} = \frac{1}{-3}$, $z = \frac{1}{w} = \frac{1}{6}$ is the solution.

Example 3: Solve for x, y and z:

$$\frac{xy}{x+y} = 70, \quad \frac{xz}{x+z} = 84, \quad \frac{yz}{y+z} = 140$$

Solution: We can write as

$$\frac{x+y}{xy} = \frac{1}{70} \text{ or } \frac{1}{x}+\frac{1}{y}=\frac{1}{70} \quad \text{....... (i)}$$

$$\frac{x+z}{xz} = \frac{1}{84} \text{ or } \frac{1}{z}+\frac{1}{x}=\frac{1}{84} \quad \text{........ (ii)}$$

$$\frac{y+z}{yz}=\frac{1}{140} \text{ or } \frac{1}{y}+\frac{1}{z}=\frac{1}{140} \quad \text{....... (iii)}$$

By (i) + (ii) + (iii), we get $2\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right) = \frac{1}{70} + \frac{1}{84} + \frac{1}{140} = \frac{14}{420}$

or $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{7}{420}=\frac{1}{60}$ ……(iv)

By (iv)–(iii) $\frac{1}{x}=\frac{1}{60}-\frac{1}{140}=\frac{4}{420}$ or $x = 105$

By (iv)–(ii) $\frac{1}{y}=\frac{1}{60}-\frac{1}{84}=\frac{2}{420}$ or $y = 210$

By (iv)–(i) $\frac{1}{z}=\frac{1}{60}-\frac{1}{70}$ or $z = 420$

Required solution is $x = 105$, $y = 210$, $z = 420$

EXERCISE (C)

Choose the most appropriate option (a), (b), (c) or (d).

1. The solution of the set of equations $3x + 4y = 7$, $4x - y = 3$ is

   a) (1, –1) b) (1, 1) c) (2, 1) d) (1, –2)

2. The values of x and y satisfying the equations $\frac{x}{2}+\frac{y}{3}=2$, $x + 2y = 8$ are given by the pair.

   a) (3, 2) b) (–2, –3) c) (2, 3) d) none of these

3. $\frac{x}{p}+\frac{y}{q}=2$, $x + y = p + q$ are satisfied by the values given by the pair.

   a) (x=p, y=q) b) (x=q, y=p) c) (x=1, y=1) d) none of these

4. The solution for the pair of equations

   $\frac{1}{16x}+\frac{1}{15y}=\frac{9}{20}$, $\frac{1}{20x}-\frac{1}{27y}=\frac{4}{45}$ is given by

   (a) $\left(\frac{1}{4},\frac{1}{3}\right)$ (b) $\left(\frac{1}{3},\frac{1}{4}\right)$ (c) (3 4) (d) (4 3)

5. Solve for x and y: $\frac{4}{x}-\frac{5}{y}=\frac{x+y}{xy}+\frac{3}{10}$ and $3xy = 10(y-x)$.

   a) (5, 2) b) (–2, –5) c) (2, –5) d) (2, 5)

6. The pair satisfying the equations $x + 5y = 36$, $\frac{x+y}{x-y}=\frac{5}{3}$ is given by

   a) (16, 4) b) (4, 16) c) (4, 8) d) none of these.

7. Solve for x and y : $x-3y = 0$, $x+2y = 20$.

   a) $x = 4, y = 12$ b) $x = 12, y = 4$ c) $x = 5, y = 4$ d) none of these

8. The simultaneous equations $7x-3y = 31$, $9x-5y = 41$ have solutions given by

a) (–4, –1) b) (–1, 4) c) (4, –1) d) (3, 7)

9. $1.5x + 2.4y = 1.8$, $2.5(x+1) = 7y$ have solutions as

a) (0.5, 0.4) b) (0.4, 0.5) c) $(\frac{1}{2}, \frac{2}{5})$ d) (2, 5)

10. The values of x and y satisfying the equations

$\frac{3}{x+y}+\frac{2}{x-y}=3$, $\frac{2}{x+y}+\frac{3}{x-y}=3\frac{2}{3}$ are given by

a) (1, 2) b) (–1, –2) c) $(1, \frac{1}{2})$ d) (2, 1)

EXERCISE (D)

Choose the most appropriate option (a), (b), (c) or (d) as the solution to the given set of equations :

1. $1.5x + 3.6y = 2.1$, $2.5(x+1) = 6y$

a) (0.2, 0.5) b) (0.5, 0.2) c) (2, 5) d) (–2, –5)

2. $\frac{x}{5}+\frac{y}{6}+1=\frac{x}{6}+\frac{y}{5}=28$

a) (6, 9) b) (9, 6) c) (60, 90) d) (90, 60)

3. $\frac{x}{4}=\frac{y}{3}=\frac{z}{2}$; $7x + 8y + 5z = 62$

a) (4, 3, 2) b) (2, 3, 4) c) (3, 4, 2) d) (4, 2, 3)

4. $\frac{xy}{x+y}=20$, $\frac{yz}{y+z}=40$, $\frac{zx}{z+x}=24$

a) (120, 60, 30) b) (60, 30, 120) c) (30, 120, 60) d) (30, 60, 120)

5. $2x + 3y + 4z = 0$, $x + 2y - 5z = 0$, $10x + 16y - 6z = 0$

a) (0,0,0) b) (1, –1, 1) c) (3, 2, –1) d) (1, 0, 2)

6. $\frac{1}{3}(x+y) + 2z = 21$, $3x - \frac{1}{2}(y+z) = 65$, $x + \frac{1}{2}(x+y-z) = 38$

a) (4, 9, 5) b) (2, 9, 5) c) (24, 9, 5) d) (5, 24, 9)

7. $\frac{4}{x}-\frac{5}{y}=\frac{x+y}{xy}+\frac{3}{10}$ $3xy = 10(y-x)$

a) (2, 5) b) (5, 2) c) (2, 7) d) (3, 4)

8. $\frac{x}{0.01}+\frac{y+0.03}{0.05}=\frac{y}{0.02}+\frac{x+0.03}{0.04}=2$

a) (1, 2) b) (0.1, 0.2) c) (0.01, 0.02) d) (0.02, 0.01)

9. $\frac{xy}{y-x}=110,\ \frac{yz}{z-y}=132,\ \frac{zx}{z+x}=\frac{60}{11}$

a) (12, 11, 10) b) (10, 11, 12) c) (11, 10, 12) d) (12, 10, 11)

10. $3x-4y+70z = 0,\ 2x+3y-10z = 0,\ x+2y+3z = 13$

a) (1, 3, 7) b) (1, 7, 3) c) (2, 4, 3) d) (–10, 10, 1)

2.6 PROBLEMS LEADING TO SIMULTANEOUS EQUATIONS

ILLUSTRATIONS:

1. If the numerator of a fraction is increased by 2 and the denominator by 1 it becomes 1. Again if the numerator is decreased by 4 and the denominator by 2 it becomes 1/2 . Find the fraction.

SOLUTION: Let x/y be the required fraction.

By the question $\frac{x+2}{y+1}=1,\ \frac{x-4}{y-2}=\frac{1}{2}$

Thus $x + 2 = y + 1$ or $x - y = -1$ .... (i)

and $2x - 8 = y - 2$ or $2x - y = 6$ .... (ii)

By (i) – (ii) $-x = -7$ or $x = 7$

from (i) $7 - y = -1$ or $y = 8$

So the required fraction is 7/8.

2. The age of a man is three times the sum of the ages of his two sons and 5 years hence his age will be double the sum of their ages. Find the present age of the man?

SOLUTION: Let x years be the present age of the man and sum of the present ages of the two sons be y years.

By the condition $x = 3y$ .... (i)

and $x + 5 = 2\ (y + 5 + 5)$ ....(ii)

From (i) & (ii) $3y + 5 = 2\ (y + 10)$

or $3y + 5 = 2y + 20$

or $3y - 2y = 20 - 5$

or $y = 15$

$\therefore\ x = 3 \times y = 3 \times 15 = 45$

Hence the present age of the man is 45 years

3. A number consist of three digit of which the middle one is zero and the sum of the other digits is 9. The number formed by interchanging the first and third digits is more than the original number by 297 find the number.

SOLUTION: Let the number be $100x + y$.we have $x + y = 9$……(i)

Also $100y + x = 100x + y + 297$ ……………………………. (ii)

From (ii) $99(x - y) = -297$

or $x - y = -3$ ……………………………………………… (iii)

Adding (i) and (iii) $2x = 6$ or $x = 3$ $\therefore$ from (i) $y = 6$

$\therefore$ Hence the number is 306.

EXERCISE (E)

Choose the most appropriate option (a), (b), (c) or (d).

1. Monthly incomes of two persons are in the ratio 4 : 5 and their monthly expenses are in the ratio 7 : 9. If each saves ₹ 50 per month find their monthly incomes.

   a) (500, 400) b) (400, 500) c) (300, 600) d) (350, 550)

2. Find the fraction which is equal to 1/2 when both its numerator and denominator are increased by 2. It is equal to 3/4 when both are increased by 12.

   a) 3/8 b) 5/8 c) 2/8 d) 2/3

3. The age of a person is twice the sum of the ages of his two sons and five years ago his age was thrice the sum of their ages. Find his present age.

   a) 60 years b) 52 years c) 51 years d) 50 years

4. A number between 10 and 100 is five times the sum of its digits. If 9 be added to it the digits are reversed find the number.

   a) 54 b) 53 c) 45 d) 55

5. The wages of 8 men and 6 boys amount to ₹ 33. If 4 men earn ₹ 4.50 more than 5 boys determine the wages of each man and boy.

   a) (₹ 1.50, ₹ 3) b) (₹ 3, ₹ 1.50)

   c) (₹ 2.50, ₹ 2) d) (₹ 2, ₹ 2.50)

6. A number consisting of two digits is four times the sum of its digits and if 27 be added to it the digits are reversed. The number is :

   a) 63 b) 35 c) 36 d) 60

7. Of two numbers, 1/5th of the greater is equal to 1/3rd of the smaller and their sum is 16. The numbers are:

   a) (6, 10) b) (9, 7) c) (12, 4) d) (11, 5)

8. y is older than x by 7 years 15 years back x's age was 3/4 of y's age. Their present ages are:

a) (x=36, y=43) b) (x=50, y=43)
c) (x=43, y=50) d) (x=40, y=47)

9. The sum of the digits in a three digit number is 12. If the digits are reversed the number is increased by 495 but reversing only of the ten's and unit digits increases the number by 36. The number is

a) 327 b) 372 c) 237 d) 273

10. Two numbers are such that twice the greater number exceeds twice the smaller one by 18 and 1/3rd of the smaller and 1/5th of the greater number are together 21. The numbers are:

a) (36, 45) b) (45, 36) c) (50, 41) d) (55, 46)

11. The demand and supply equations for a certain commodity are 4q + 7p = 17 and $p = \frac{q}{3} + \frac{7}{4}$. respectively where p is the market price and q is the quantity then the equilibrium price and quantity are:

(a) $2, \frac{3}{4}$ (b) $3, \frac{1}{2}$ (c) $5, \frac{3}{5}$ (d) None of these.

2.7 QUADRATIC EQUATION

An equation of the form $ax^2 + bx + c = 0$ where x is a variable and a, b, c are constants with $a \neq 0$ is called a quadratic equation or equation of the second degree.

When b=0 the equation is called a pure quadratic equation; when $b \neq 0$ the equation is called an affected quadratic.

Examples: i) $2x^2 + 3x + 5 = 0$

ii) $x^2 - x = 0$

iii) $5x^2 - 6x - 3 = 0$

The value of the variable say x is called the root of the equation. A quadratic equation has got two roots.

How to find out the roots of a quadratic equation:

$ax^2 + bx + c = 0 \quad (a \neq 0)$

or $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

or $x^2 + 2\frac{b}{2a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$

or $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$

$$\text{or } x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$\text{or } x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Sum and Product of the Roots:

Let one root be α and the other root be β

$$\text{Now } \alpha + \beta = \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{-b+\sqrt{b^2-4ac}-b-\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-2b}{2a} = \frac{-b}{a}$$

$$\text{Thus sum of roots} = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coeffient of } x^2}$$

$$\text{Next } \alpha\beta = \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right) = \frac{c}{a}$$

$$\text{So the product of the roots} = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

2.8 HOW TO CONSTRUCT A QUADRATIC EQUATION

For the equation $ax^2 + bx + c = 0$ we have

$$\text{or } x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\text{or } x^2 - \left(-\frac{b}{a}\right)x + \frac{c}{a} = 0$$

or x^2 – (Sum of the roots) x + Product of the roots = 0

2.9 NATURE OF THE ROOTS

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

i) If $b^2-4ac = 0$ the roots are real and equal;

ii) If $b^2-4ac > 0$ then the roots are real and unequal (or distinct);

iii) If $b^2-4ac < 0$ then the roots are imaginary;

iv) If $b^2–4ac$ is a perfect square ($\neq$ 0) the roots are real, rational and unequal (distinct);

v) If $b^2–4ac$ >0 but not a perfect square the roots are real, irrational and unequal.

Since $b^2 – 4ac$ discriminates the roots $b^2 – 4ac$ is called the discriminant in the equation $ax^2 + bx + c = 0$ as it actually discriminates between the roots.

Note: (a) Irrational roots occur in conjugate pairs that is if $(m + \sqrt{n})$ is a root then $(m - \sqrt{n})$ is the other root of the same equation.

(b) If one root is reciprocal to the other root then their product is 1 and so $\frac{c}{a} = 1$ i.e. c = a

(c) If one root is equal to other root but opposite in sign then.

their sum = 0 and so $\frac{b}{a} = 0$. i.e. b = 0.

Example 1: Solve $x^2 – 5x + 6 = 0$

Solution: 1st method : $x^2 – 5x + 6 = 0$

or $x^2 –2x –3x +6 = 0$

or $x(x–2) – 3(x–2) = 0$

or $(x–2)(x–3) = 0$

or $x = 2$ or 3

2nd method (By formula) $x^2 – 5x + 6 = 0$

Here a = 1, b = –5 , c = 6 (comparing the equation with $ax^2 + bx + c = 0$)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{25-24}}{2}$$

$$= \frac{5 \pm 1}{2} = \frac{6}{2} \text{ and } \frac{4}{2}; \quad \therefore x = 3 \text{ and } 2$$

Example 2: Examine the nature of the roots of the following equations.

i) $x^2 – 8x + 16 = 0$ ii) $3x^2 – 8x + 4 = 0$

iii) $5x^2 – 4x + 2 = 0$ iv) $2x^2 – 6x – 3 = 0$

Solution: (i) a = 1, b = –8, c = 16

$b^2 – 4ac = (–8)^2 – 4 \times 1 \times 16 = 64 – 64 = 0$

The roots are real and equal.

(ii) $3x^2 – 8x + 4 = 0$

a = 3, b = –8, c = 4

$b^2 - 4ac = (-8)^2 - 4 \times 3 \times 4 = 64 - 48 = 16 > 0$ and a perfect square

The roots are real, rational and unequal

(iii) $5x^2 - 4x + 2 = 0$

$b^2 - 4ac = (-4)^2 - 4 \times 5 \times 2 = 16 - 40 = -24 < 0$

The roots are imaginary and unequal

(iv) $2x^2 - 6x - 3 = 0$

$b^2 - 4ac = (-6)^2 - 4 \times 2\ (-3)$

$= 36 + 24 = 60 > 0$

The roots are real and unequal. Since $b^2 - 4ac$ is not a perfect square the roots are real irrational and unequal.

ILLUSTRATIONS:

1. If α and ß be the roots of $x^2 + 7x + 12 = 0$ find the equation whose roots are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$.

SOLUTION: Now sum of the roots of the required equation

$= (\alpha+\beta)^2 + (\alpha-\beta)^2 = (-7)^2 + (\alpha+\beta)^2 - 4\alpha\beta$

$= 49 + (-7)^2 - 4x12$

$= 49 + 49 - 48 = 50$

Product of the roots of the required equation $= (\alpha+\beta)^2(\alpha-\beta)^2$

$= 49\ (49–48) = 49$

Hence the required equation is

x^2 – (sum of the roots) x + product of the roots = 0

or $x^2 - 50x + 49 = 0$

2. If α, β be the roots of $2x^2 - 4x - 1 = 0$ find the value of $\dfrac{\alpha^2}{\beta} + \dfrac{\beta^2}{\alpha}$

SOLUTION: $\alpha + \beta = \dfrac{-(-4)}{2} = 2,\ \alpha\beta = \dfrac{-1}{2}$

$$\therefore \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$\frac{2^3 - 3\left(-\frac{1}{2}\right).2}{\left(-\frac{1}{2}\right)} = -22$$

3. Solve x : $4^x - 3.2^{x+2} + 2^5 = 0$

SOLUTION: $4^x - 3.2^{x+2} + 2^5 = 0$

or $(2^x)^2 - 3.2^x.\ 2^2 + 32 = 0$

or $(2^x)^2 - 12.\ 2^x + 32 = 0$

or $y^2 - 12y + 32 = 0$ (taking $y = 2^x$)

or $y^2 - 8y - 4y + 32 = 0$

or $y(y - 8) - 4(y - 8) = 0$ $\quad\therefore (y - 8)(y - 4) = 0$

either $y - 8 = 0$ or $y - 4 = 0$ $\quad\therefore y = 8$ or $y = 4$.

$\Rightarrow 2^x = 8 = 2^3$ or $2^x = 4 = 2^2$ Therefore $x = 3$ or $x = 2$.

4. Solve $\left(x - \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right) = 7\frac{1}{4}$.

SOLUTION: $\left(x - \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right) = 7\frac{1}{4}$.

$$\left(x - \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right) = \frac{29}{4}.$$

or $$\left(x + \frac{1}{x}\right)^2 - 4 + 2\left(x + \frac{1}{x}\right) = \frac{29}{4}$$

[as $(a - b)^2 = (a + b)^2 - 4ab$]

or $p^2 + 2p - \frac{45}{4} = 0$ $\qquad$ Taking $p = x + \frac{1}{x}$

or $4p^2 + 8p - 45 = 0$

or $4p^2 + 18p - 10p - 45 = 0$

or $2p(2p + 9) - 5(2p + 9) = 0$

or $(2p - 5)(2p + 9) = 0$.

$\therefore$ Either $2p + 9 = 0$ or $2p - 5 = 0$ $\quad\Rightarrow p = -\frac{9}{2}$ or $p = \frac{5}{2}$

$\therefore$ Either $x+\dfrac{1}{x} = -\dfrac{9}{2}$ or $x+\dfrac{1}{x} = \dfrac{5}{2}$

i.e. Either $2x^2 + 9x + 2 = 0$ or $2x^2 - 5x + 2 = 0$

i.e. Either $x = \dfrac{-9\pm\sqrt{81-16}}{4}$ or, $x = \dfrac{5\pm\sqrt{25-16}}{4}$

i.e. Either $x = \dfrac{-9\pm\sqrt{65}}{4}$ or $x = 2$ or $\dfrac{1}{2}$.

5. Solve $2^{x-2} + 2^{3-x} = 3$

SOLUTION: $2^{x-2} + 2^{3-x} = 3$

or $2^x.\ 2^{-2} + 2^3.\ 2^{-x} = 3$

or $\dfrac{2^x}{2^2}+\dfrac{2^3}{2^x}=3$

or $\dfrac{t}{4}+\dfrac{8}{t}=3$ when $t = 2^x$

or $t^2 + 32 = 12t$

or $t^2 - 12t + 32 = 0$

or $t^2 - 8t - 4t + 32 = 0$

or $t(t-8) - 4(t-8) = 0$

or $(t-4)(t-8) = 0$

$\therefore$ $t = 4, 8$

For $t = 4$, $2^x = 4 = 2^2$ i.e. $x = 2$

For $t = 8$, $2^x = 8 = 2^3$ i.e. $x = 3$

6. If one root of the equation is $2-\sqrt{3}$ form the equation given that the roots are irrational

SOLUTION: Other root is $2+\sqrt{3}$ $\therefore$ sum of two roots $= 2-\sqrt{3} + 2 + \sqrt{3} = 4$

Product of roots $= (2-\sqrt{3})(2+\sqrt{3}) = 4 - 3 = 1$

$\therefore$ Required equation is : $x^2 -$ (sum of roots)x + (product of roots) = 0

or $x^2 - 4x + 1 = 0$.

7. If $\alpha\ \beta$ are the two roots of the equation $x^2 - px + q = 0$ form the equation

whose roots are $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$.

SOLUTION: As α, β are the roots of the equation $x^2 - px + q = 0$

$\alpha + \beta = -(-p) = p$ and $\alpha\beta = q$.

Now $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} = \frac{p^2-2q}{q}$; and $\frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha} = 1$

∴ Required equation is $x^2 - \left(\frac{p^2-2q}{q}\right)x + 1 = 0$

or $q\,x^2 - (p^2 - 2q)\,x + q = 0$

8. If the roots of the equation $p(q - r)x^2 + q(r - p)x + r(p - q) = 0$ are equal show that $\frac{2}{q} = \frac{1}{p} + \frac{1}{r}$.

SOLUTION: Since the roots of the given equation are equal the discriminant must be zero ie. $q^2(r - p)^2 - 4.\ p(q - r)\ r(p - q) = 0$

or $q^2 r^2 + q^2 p^2 - 2q^2 rp - 4pr\,(pq - pr - q^2 + qr) = 0$

or $p^2q^2 + q^2r^2 + 4p^2r^2 + 2q^2pr - 4p^2qr - 4pqr^2 = 0$

or $(pq + qr - 2rp)^2 = 0$

∴ $pq + qr = 2pr$

or $\frac{pq+qr}{2pr} = 1$ or, $\frac{q}{2}\cdot\frac{(p+r)}{pr} = 1$ or, $\frac{1}{r}+\frac{1}{p} = \frac{2}{q}$

EXERCISE (F)

Choose the most appropriate option (a) (b) (c) or (d).

1. If the roots of the equation $2x^2 + 8x - m^3 = 0$ are equal then value of m is

   (a) – 3 (b) – 1 (c) 1 (d) – 2

2. If $2^{2x+3} - 3^2.\ 2^x + 1 = 0$ then values of x are

   (a) 0, 1 (b) 1, 2 (c) 0, 3 (d) 0, – 3

3. The values of $4+\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+.....\infty}}}$

   (a) $1\pm\sqrt{2}$ (b) $2+\sqrt{5}$ (c) $2\pm\sqrt{5}$ (d) none of these

4. If α, β be the roots of the equation $2x^2 - 4x - 3 = 0$
the value of $\alpha^2 + \beta^2$ is

a) 5 b) 7 c) 3 d) – 4

5. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals then $\frac{b^2}{ac} + \frac{bc}{a^2}$ is equal to

a) 2 b) –2 c) 1 d) –1

6. The equation $x^2 - (p+4)x + 2p + 5 = 0$ has equal roots the values of p will be.

a) ± 1 b) 2 c) ± 2 d) –2

7. The roots of the equation $x^2 + (2p-1)x + p^2 = 0$ are real if.

a) $p \geq 1$ b) $p \leq 4$ c) $p \geq 1/4$ d) $p \leq 1/4$

8. If $x = m$ is one of the solutions of the equation $2x^2 + 5x - m = 0$ the possible values of m are

a) (0, 2) b) (0, –2) c) (0, 1) d) (1, –1)

9. If p and q are the roots of $x^2 + 2x + 1 = 0$ then the values of $p^3 + q^3$ becomes

a) 2 b) –2 c) 4 d) – 4

10. If L + M + N = 0 and L, M, N are rationals the roots of the equation $(M+N-L)x^2+(N+L-M)x+(L+M-N) = 0$ are

a) real and irrational b) real and rational
c) imaginary and equal d) real and equal

11. If α and β are the roots of $x^2 = x + 1$ then value of $\frac{\alpha^2}{\beta} - \frac{\beta^2}{\alpha}$ is

a) $2\sqrt{5}$ b) $\sqrt{5}$ c) $3\sqrt{5}$ d) $-2\sqrt{5}$

12. If $p \neq q$ and $p^2 = 5p - 3$ and $q^2 = 5q - 3$ the equation having roots as $\frac{p}{q}$ and $\frac{q}{p}$ is

a) $x^2 - 19x + 3 = 0$ b) $3x^2 - 19x - 3 = 0$
c) $3x^2 - 19x + 3 = 0$ d) $3x^2 + 19x + 3 = 0$

13. If one root of $5x^2 + 13x + p = 0$ be reciprocal of the other then the value of p is

a) –5 b) 5 c) 1/5 d) –1/5

EXERCISE (G)

Choose the most appropriate option (a) (b) (c) or (d).

1. A solution of the quadratic equation $(a+b-2c)x^2 + (2a-b-c)x + (c+a-2b) = 0$ is

a) $x = 1$ b) $x = -1$ c) $x = 2$ d) $x = -2$

2. If the root of the equation $x^2-8x+m = 0$ exceeds the other by 4 then the value of m is

a) m = 10 b) m = 11 c) m = 9 d) m = 12

3. The values of x in the equation

$7(x+2p)^2 + 5p^2 = 35xp + 117p^2$ are

a) (4p, –3p) b) (4p, 3p) c) (–4p, 3p) d) (–4p, –3p)

4. The solutions of the equation $\frac{6x}{x+1}+\frac{6(x+1)}{x}=13$ are

a) (2, 3) b) (3, –2) c) (–2, –3) d) (2, –3)

5. The satisfying values of x for the equation

$\frac{1}{x+p+q}=\frac{1}{x}+\frac{1}{p}+\frac{1}{q}$ are

a) (p, q) b) (–p, –q) c) (p, –p) d) (–p, q)

6. The values of x for the equation $x^2 + 9x + 18 = 6 - 4x$ are

a) (1, 12) b) (–1, –12) c) (1, –12) d) (–1, 12)

7. The values of x satisfying the equation

$\sqrt{(2x^2+5x-2)}-\sqrt{(2x^2+5x-9)} = 1$ are

a) (2, –9/2) b) (4, –9) c) (2, 9/2) d) (–2, 9/2)

8. The solution of the equation $3x^2-17x + 24 = 0$ are

a) (2, 3) b) $(2, 3\frac{2}{3})$ c) $(3, 2\frac{2}{3})$ d) $(3, \frac{2}{3})$

9. The equation $\frac{3(3x^2+15)}{6} + 2x^2 + 9 = \frac{2x^2+96}{7} + 6$

has got the solution as

a) (1, 1) b) (1/2, –1) c) (1, –1) d) (2, –1)

10. The equation $\left(\frac{l-m}{2}\right)x^2-\left(\frac{l+m}{2}\right)x + m = C$ has got two values of x to satisfy the equation given as

a) $\left(1,\frac{2m}{l-m}\right)$ b) $\left(1,\frac{m}{l-m}\right)$ c) $\left(1,\frac{2l}{l-m}\right)$ d) $\left(1,\frac{1}{l-m}\right)$

2.10 PROBLEMS ON QUADRATIC EQUATION

1. Difference between a number and its positive square root is 12; find the numbers?

Solution: Let the number be x.

Then $x - \sqrt{x} = 12$ …………… (i)

$(\sqrt{x})^2 - \sqrt{x} - 12 = 0$. Taking y = $\sqrt{x}$, $y^2 - y - 12 = 0$

or $(y-4)(y+3) = 0$ ∴ Either y = 4 or y = – 3 i.e. Either $\sqrt{x} = 4$ or $\sqrt{x} = -3$

If $\sqrt{x} = -3$ $x = 9$ if does not satisfy equation (i) so $\sqrt{x}$ =4 or x=16.

2. A piece of iron rod costs ₹ 60. If the rod was 2 metre shorter and each metre costs ₹ 1.00 more, the cost would remain unchanged. What is the length of the rod?

Solution: Let the length of the rod be x metres. The rate per meter is ₹ $\frac{60}{x}$.

New Length = $(x - 2)$; as the cost remain the same the new rate per meter is $\frac{60}{x-2}$

As given $\frac{60}{x-2} = \frac{60}{x} + 1$

or $\frac{60}{x-2} - \frac{60}{x} = 1$

or $\frac{120}{x(x-2)} = 1$

or $x^2 - 2x = 120$

or $x^2 - 2x - 120 = 0$ or $(x - 12)(x + 10) = 0$.

Either $x = 12$ or $x = -10$ (not possible)

∴ Hence the required length = 12m.

3. Divide 25 into two parts so that sum of their reciprocals is 1/6.

Solution: let the parts be x and $25 - x$

By the question $\frac{1}{x} + \frac{1}{25-x} = \frac{1}{6}$

or $\frac{25-x+x}{x(25-x)} = \frac{1}{6}$

or $150 = 25x - x^2$

or $x^2 - 25x + 150 = 0$

or $x^2 - 15x - 10x + 150 = 0$

or $x(x-15) - 10(x-15) = 0$

or $(x-15)(x-10) = 0$

or $x = 10, 15$

So the parts of 25 are 10 and 15.

EXERCISE (H)

Choose the most appropriate option (a) (b) (c) or (d).

1. The sum of two numbers is 8 and the sum of their squares is 34. Taking one number as x form an equation in x and hence find the numbers. The numbers are

   a) (7, 10) b) (4, 4) c) (3, 5) d) (2, 6)

2. The difference of two positive integers is 3 and the sum of their squares is 89. Taking the smaller integer as x form a quadratic equation and solve it to find the integers. The integers are.

   a) (7, 4) b) (5, 8) c) (3, 6) d) (2, 5)

3. Five times of a positive whole number is 3 less than twice the square of the number. The number is

   a) 3 b) 4 c) –3 d) 2

4. The area of a rectangular field is 2000 sq.m and its perimeter is 180m. Form a quadratic equation by taking the length of the field as x and solve it to find the length and breadth of the field. The length and breadth are

   a) (205m, 80m) b) (50m, 40m) c) (60m, 50m) d) none

5. Two squares have sides p cm and (p + 5) cms. The sum of their squares is 625 sq. cm. The sides of the squares are

   a) (10 cm, 30 cm) b) (12 cm, 25 cm)
   c) 15 cm, 20 cm) d) none of these

6. Divide 50 into two parts such that the sum of their reciprocals is 1/12. The numbers are

   a) (24, 26) b) (28, 22) c) (27, 23) d) (20, 30)

7. There are two consecutive numbers such that the difference of their reciprocals is 1/240. The numbers are

   a) (15, 16) b) (17, 18) c) (13, 14) d) (12, 13)

8. The hypotenuse of a right–angled triangle is 20cm. The difference between its other two sides be 4cm. The sides are

   a) (11cm, 15cm) b) (12cm, 16cm) c) (20cm, 24cm) d) none of these

9. The sum of two numbers is 45 and the mean proportional between them is 18. The numbers are

a) (15, 30) b) (32, 13) c) (36, 9) d) (25, 20)

10. The sides of an equilateral triangle are shortened by 12 units 13 units and 14 units respectively and a right angle triangle is formed. The side of the equilateral triangle is

a) 17 units b) 16 units c) 15 units d) 18 units

11. A distributor of apple Juice has 5000 bottle in the store that it wishes to distribute in a month. From experience it is known that demand D (in number of bottles) is given by $D = -2000p^2 + 2000p + 17000$. The price per bottle that will result zero inventory is

a) ₹ 3 b) ₹ 5 c) ₹ 2 d) none of these.

12. The sum of two irrational numbers multiplied by the larger one is 70 and their difference is multiplied by the smaller one is 12; the two numbers are

a) $3\sqrt{2}, 2\sqrt{3}$ (b) $5\sqrt{2}, 3\sqrt{5}$ (c) $2\sqrt{2}, 5\sqrt{2}$ d) none of these.

2.11 SOLUTION OF CUBIC EQUATION

On trial basis putting if some value of x stratifies the equation then we get a factor. This is a trial and error method. With this factor to factorise the LHS and then other get values of x.

ILLUSTRATIONS:

1. Solve $x^3 - 7x + 6 = 0$

Putting $x = 1$ L.H.S is Zero. So $(x-1)$ is a factor of $x^3 - 7x + 6$

We write $x^3 - 7x + 6 = 0$ in such a way that $(x-1)$ becomes its factor. This can be achieved by writing the equation in the following form.

or $x^3-x^2+x^2-x-6x+6 = 0$

or $x^2(x-1) + x(x-1) - 6(x-1) = 0$

or $(x-1)(x^2+x-6) = 0$

or $(x-1)(x^2+3x-2x-6) = 0$

or $(x-1)\{ x(x+3) - 2(x+3) \} = 0$

or $(x-1)(x-2)(x+3) = 0$

$\therefore$ or $x = 1, 2, -3$

2. Solve for real ***x*:** $x^3 + x + 2 = 0$

SOLUTION: By trial we find that $x = -1$ makes the LHS zero. So $(x + 1)$ is a factor of $x^3 + x + 2$

We write $x^3 + x + 2 = 0$ as $x^3 + x^2 - x^2 - x + 2x + 2 = 0$

or $x^2(x + 1) - x(x + 1) + 2(x + 1) = 0$

or $(x + 1)(x^2 - x + 2) = 0$.

Either $x + 1 = 0$; $x = -1$

or $x^2 - x + 2 = 0$ i.e. $x = -1$

i.e. $x = \frac{1 \pm \sqrt{1-8}}{2} = \frac{1 \pm \sqrt{-7}}{2}$

As $x = \frac{1 \pm \sqrt{-7}}{2}$ is not real, $x = -1$ is the required solution.

EXERCISE (I)

Choose the most appropriate option (a), (b), (c) or (d)

1. The solution of the cubic equation $x^3-6x^2+11x-6 = 0$ is given by the triplet :

   a) (–1, 1 –2) b) (1, 2, 3) c) (–2, 2, 3) d) (0, 4, –5)

2. The cubic equation $x^3 + 2x^2 - x - 2 = 0$ has 3 roots namely.

   a) (1, –1, 2) b) (–1, 1, –2) c) (–1, 2, –2) d) (1, 2, 2)

3. x, $x - 4$, $x + 5$ are the factors of the left–hand side of the equation.

   a) $x^3 + 2x^2 - x - 2 = 0$ b) $x^3 + x^2 - 20x = 0$

   c) $x^3 - 3x^2 - 4x + 12 = 0$ d) $x^3 - 6x^2 + 11x - 6 = 0$

4. The equation $3x^3 + 5x^2 = 3x + 5$ has got 3 roots and hence the factors of the left–hand side of the equation $3x^3 + 5x^2 - 3x - 5 = 0$ are

   a) $x - 1$, $x - 2$, $x - 5/3$ b) $x - 1$, $x + 1$, $3x + 5$ c) $x + 1$, $x - 1$, $3x - 5$ d) $x - 1$, $x + 1$, $x - 2$

5. The roots of the equation $x^3 + 7x^2 - 21x - 27 = 0$ are

   a) (– 3, – 9, – 1) b) (3, – 9, – 1) c) (3, 9, 1) d) (– 3, 1, 9)

6. The roots of $x^3 + x^2 - x - 1 = 0$ are

   a) (– 1, – 1, 1) b) (1, 1, – 1) c) (– 1, – 1, – 1) d) (1, 1, 1)

7. The satisfying value of $x^3 + x^2 - 20x = 0$ are

   a) (1, 4, – 5) b) (2, 4, – 5) c) (0, – 4, 5) d) (0, 4, – 5)

8. The roots of the cubic equation $x^3 - 6x^2 + 9x - 4 = 0$ are

   a) (4, 1, –1) b) (–4, –1, –1) c) (–4, –1, 1) d) (1, 1, 4)

9. If $4x^3 + 8x^2 - x - 2 = 0$ then value of $(2x+3)$ is given by

   a) 4, –1, 2 b) –4, 2, 1 c) 2, –4, –1 d) none of these.

10. The rational root of the equation $2x^3 - x^2 - 4x + 2 = 0$ is

    a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 2 d) – 2.

SUMMARY

- A simple equation in one unknown x is in the form $ax + b = 0$.

  Where a, b are known constants and $a \neq 0$
- The general form of a linear equations in two unknowns x and y is $ax + by + c = 0$ where a, b are non-zero coefficients and c is a constant. Two such equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ form a pair of simultaneous equations in x and y. A value for each unknown which satisfies simultaneously both the equations will give the roots of the equations.
- **Elimination Method:** In this method two given linear equations are reduced to a linear equation in one unknown by eliminating one of the unknowns and then solving for the other unknown.
- **Cross Multiplication Method:** Let two equations be:

  $a_1x + b_1y + c_1 = 0$

  $a_2x + b_2y + c_2 = 0$

  $$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \qquad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}.$$
- An equation of the form $ax^2 + bx + c = 0$ where x is a variable and a, b, c are constants with a ^{1} 0 is called a quadratic equation or equation of the second degree.

  When b=0 the equation is called a pure quadratic equation; when b= 0 the equation is called an affected quadratic.
- The roots of a quadratic equation:

  $$= \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
- The Sum and Product of the Roots of quadratic equation

  $$\text{sum of roots} = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coeffient of } x^2}$$

  $$\text{product of the roots} = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$
- To construct a quadratic equation for the equation $ax^2 + bx + c = 0$ we have

  x^2 – (Sum of the roots) x + Product of the roots = 0
- Nature of the roots

  $$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

i) If $b^2–4ac = 0$ the roots are real and equal;

ii) If $b^2–4ac > 0$ then the roots are real and unequal (or distinct);

iii) If $b^2–4ac < 0$ then the roots are imaginary;

iv) If $b^2–4ac$ is a perfect square ($\neq 0$) the roots are real, rational and unequal (distinct);

v) If $b^2–4ac > 0$ but not a perfect square the rots are real, irrational and unequal.

Since $b^2 – 4ac$ discriminates the roots $b^2 – 4ac$ is called the discriminant in the equation $ax^2 + bx + c = 0$ as it actually discriminates between the roots.

ANSWERS

Exercise (A)

1. (b) **2.** (a) **3.** (c) **4.** (c) **5.** (b) **6.** (d) **7.** (a) **8.** (d)
9. (c)

Exercise (B)

1. (c) **2.** (b) **3.** (a) **4.** (b) **5.** (c) **6.** (a) **7.** (d) **8.** (d)
9. (a) **10.** (c) **11.** (c) **12.** (a)

Exercise (C)

1. (b) **2.** (c) **3.** (a) **4.** (a) **5.** (d) **6.** (a) **7.** (b) **8.** (c)
9. (b) **10.** (d)

Exercise (D)

1. (a) **2.** (c) **3.** (a) **4.** (d) **5.** (a) **6.** (c) **7.** (a) **8.** (c)
9. (b) **10.** (d)

Exercise (E)

1. (b) **2.** (a) **3.** (d) **4.** (c) **5.** (b) **6.** (c) **7.** (a) **8.** (a)
9. (c) **10.** (b) **11.** (a)

Exercise (F)

1. (d) **2.** (d) **3.** (b) **4.** (b) **5.** (a) **6.** (c) **7.** (d) **8.** (b)
9. (b) **10.** (b) **11.** (d) **12.** (c) **13.** (b)

Exercise (G)

1. (b) **2.** (d) **3.** (a) **4.** (d) **5.** (b) **6.** (b) **7.** (a) **8.** (c)
9. (c) **10.** (a)

Exercise (H)

1.	(c)	**2.**	(b)	**3.**	(a)	**4.**	(b)	**5.**	(c)	**6.**	(d)	**7.**	(a)	**8.**	(b)
9.	(c)	**10.**	(a)	**11.**	(a)	**12.**	(c)								

Exercise (I)

1.	(b)	**2.**	(b)	**3.**	(b)	**4.**	(b)	**5.**	(b)	**6.**	(a)	**7.**	(d)	**8.**	(d)
9.	(a)	**10.**	(a)												

ADDITIONAL QUESTION BANK

1. Solving equation $x^2 - (a+b)\,x + ab = 0$ are, value(s) of x

   (a) a, b (b) a (c) b (d) None

2. Solving equation $x^2 - 24x + 135 = 0$ are, value(s) of x

   (a) 9, 6 (b) 9, 15 (c) 15, 6 (d) None

3. If $\frac{x}{b} + \frac{b}{x} = \frac{a}{b} + \frac{b}{a}$ the roots of the equation are

   (a) $a, b^2/a$ (b) $a^2, b/a^2$ (c) $a^2, b^2/a$ (d) a, b^2

4. Solving equation $\frac{6x+2}{4} + \frac{2x^2-1}{2x^2+2} = \frac{10x-1}{4x}$ we get roots as

   (a) ±1 (b) +1 (c) -1 (d) 0

5. Solving equation $3x^2 - 14x + 16 = 0$ we get roots as

   (a) ±1 (b) 2 and $\frac{8}{3}$ (c) 0 (d) None

6. Solving equation $3x^2 - 14x + 8 = 0$ we get roots as

   (a) ±4 (b) ±2 (c) $4, \frac{2}{3}$ (d) None

7. Solving equation $(b-c)\,x^2 + (c-a)\,x + (a-b) = 0$ following roots are obtained

   (a) $\frac{b-a}{b-c}, 1$ (b) $(a-b)(a-c), 1$ (c) $\frac{b-c}{a-b}, 1$ (d) None

8. Solving equation $7\sqrt{\frac{x}{1-x}} + 8\sqrt{\frac{1-x}{x}} = 15$ following roots are obtained

   (a) $\frac{64}{113}, \frac{1}{2}$ (b) $\frac{1}{50}, \frac{1}{65}$ (c) $\frac{49}{50}, \frac{1}{65}$ (d) $\frac{1}{50}, \frac{64}{65}$

9. Solving equation $6\left[\sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}}\right]=13$ following roots are obtained

(a) $\frac{4}{13},\frac{9}{13}$ (b) $\frac{-4}{13},\frac{-9}{13}$ (c) $\frac{4}{13},\frac{5}{13}$ (d) $\frac{6}{13},\frac{7}{13}$

10. Solving equation $z^2 - 6z + 9 = 4\sqrt{z^2 - 6z + 6}$ following roots are obtained

(a) $3+2\sqrt{3}, 3-2\sqrt{3}$ (b) 5, 1 (c) all the above (d) None

11. Solving equation $\frac{x+\sqrt{12p-x}}{x-\sqrt{12p-x}}=\frac{\sqrt{p}+1}{\sqrt{p}-1}$ following roots are obtained

(a) $3p$ (b) both $3p$ and $-4p$ (c) only $-4p$ (d) $-3p$, $4p$

12. Solving equation $(1+x)^{2/3}+(1-x)^{2/3}=4\left(1-x^2\right)^{1/3}$ are, values of x

(a) $\frac{5}{\sqrt{3}}$ (b) $-\frac{5}{\sqrt{3}}$ (c) $\pm\frac{5}{3\sqrt{3}}$ (d) $\pm\frac{15}{\sqrt{3}}$

13. Solving equation $(2x+1)(2x+3)(x-1)(x-2)=150$ the roots available are

(a) $\frac{1\pm\sqrt{129}}{4}$ (b) $\frac{7}{2}, -3$ (c) $-\frac{7}{2}, 3$ (d) None

14. Solving equation $(2x+3)(2x+5)(x-1)(x-2) = 30$ the roots available are

(a) $0,\frac{1}{2},-\frac{11}{4},\frac{9}{4}$ (b) $0,-\frac{1}{2},\frac{-1\pm\sqrt{105}}{4}$ (c) $0,-\frac{1}{2},-\frac{11}{4},-\frac{9}{4}$ (d) None

15. Solving equation $z+\sqrt{z}=\frac{6}{25}$ the value of z works out to

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{1}{25}$ (d) $\frac{2}{25}$

16. Solving equation $z^{10}-33z^5+32=0$ the following values of z are obtained

(a) 1, 2 (b) 2, 3 (c) 2, 4 (d) 1, 2, 3

17. When $\sqrt{2z+1}+\sqrt{3z+4}=7$ the value of z is given by

(a) 1 (b) 2 (c) 3 (d) 4

18. Solving equation $\sqrt{x^2-9x+18}+\sqrt{x^2+2x-15}=\sqrt{x^2-4x+3}$ following roots are obtained

(a) 3, $\frac{2\pm\sqrt{91}}{3}$ (b) $\frac{2\pm\sqrt{94}}{3}$ (c) 4, $-\frac{8}{3}$ (d) 3, $4-\frac{8}{3}$

19. Solving equation $\sqrt{y^2+4y-21}+\sqrt{y^2-y-6}=\sqrt{6y^2-5y-39}$ following roots are obtained

(a) 2, 3, 5/3 (b) 2, 3, -5/3 (c) -2, -3, 5/3 (d) -2, -3, -5/3

20. Solving equation $6x^4+11x^3-9x^2-11x+6=0$ following roots are obtained

(a) $\frac{1}{2},-2,\frac{-1\pm\sqrt{37}}{6}$ (b) $-\frac{1}{2},2,\frac{-1\pm\sqrt{37}}{6}$ (c) $\frac{1}{2},-2,\frac{5}{6},\frac{-7}{6}$ (d) None

21. If $\frac{x-bc}{d+c}+\frac{x-ca}{c+a}+\frac{x-ab}{a+b}=a+b+c$ the value of x is

(a) $a^2+b^2+c^2$ (b) $a(a+b+c)$ (c) $(a+b)(b+c)$ (d) $ab+bc+ca$

22. If $\frac{x+2}{x-2}-\frac{x-2}{x+2}=\frac{x-1}{x+3}-\frac{x+3}{x-3}$ then the values of x are

(a) $0,\pm\sqrt{6}$ (b) $0,\pm\sqrt{3}$ (c) $0,\pm2\sqrt{3}$ (d) None

23. If $\frac{x-a}{b}+\frac{x-b}{a}=\frac{b}{x-a}+\frac{a}{x-b}$ then the values of x are

(a) $0,(a+b),(a-b)$ (b) $0,(a+b),\frac{a^2+b^2}{a+b}$ (c) $0,(a-b),\frac{a^2+b^2}{a+b}$ (d) $\frac{a^2+b^2}{a+b}$

24. If $\frac{x-a^2-b^2}{c^2}+\frac{c^2}{x-a^2-b^2}=2$ the value of is

(a) $a^2+b^2+c^2$ (b) $-a^2-b^2-c^2$ (c) $\frac{1}{a^2+b^2+c^2}$ (d) $-\frac{1}{a^2+b^2+c^2}$

25. Solving equation $\left(x-\frac{1}{x}\right)^2-6\left(x+\frac{1}{x}\right)+12=0$ we get roots as follows

(a) 0 (b) 1 (c) -1 (d) None

26. Solving equation $\left(x-\frac{1}{x}\right)^2-10\left(x-\frac{1}{x}\right)+24=0$ we get roots as follows

(a) 0 (b) 1 (c) -1 (d) $(2\pm\sqrt{5}),(3\pm\sqrt{10})$

27. Solving equation $2\left(x-\frac{1}{x}\right)^2-5\left(x+\frac{1}{x}+2\right)+18=0$ we get roots as under

(a) 0 (b) 1 (c) -1 (d) 2,1/2

28. If α, β are the roots of equation x^2-5x+6=0 and α > β thenthe equation with roots (α + β) and (α - β) is

(a) x^2-6x+5=0 (b) 2x^2-6x+5=0 (c) 2x^2-5x+6=0 (d) x^2-5x+6=0

29. If α, β are the roots of equation x^2-5x+6=0 and α > β then the equation with roots (α^2 + β) and (α + β^2) is

(a) x^2-9x+99=0 (b) x^2-18x+90=0 (c) x^2-18x+77=0 (d) None

30. If α, β are the roots of equation x^2-5x+6=0 and α > β then the equation with roots (αβ+α+β) and (αβ-α-β) is

(a) x^2-12x+11=0 (b) 2x^2-6x+12=0 (c) x^2-12x+12=0 (d) None

31. The condition that one of ax^2+bx+c=0 the roots of is twice the other is

(a) b^2=4ca (b) 2b^2=9(c+a) (c) 2b^2=9ca (d) 2b^2=9(c-a)

32. The condition that one of ax^2+bx+c=0 the roots of is thrice the other is

(a) 3b^2=16ca (b) b^2=9ca (c) 3b^2=-16ca (d) b^2=-9ca

33. If the roots of ax^2+bx+c=0 are in the ratio $\frac{p}{q}$ then the value of $\frac{b^2}{(ca)}$ is

(a) $\frac{(p+q)^2}{(pq)}$ (b) $\frac{(p+q)}{(pq)}$ (c) $\frac{(p-q)^2}{(pq)}$ (d) $\frac{(p-q)}{(pq)}$

34. Solving 6x+5y-16=0 and 3x-y-1=0 we get values of x and y as

(a) 1, 1 (b) 1, 2 (c) -1, 2 (d) 0, 2

35. Solving x^2+y^2-25=0 and x-y-1=0 we get the roots as under

(a) ±3 ±4 (b) ±2 ±3 (c) 0, 3, 4 (d) 0, -3, -4

36. Solving $\sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}-\frac{5}{2}=0$ and x+y-5=0 we get the roots as under

(a) 1, 4 (b) 1, 2 (c) 1, 3 (d) 1, 5

37. Solving $\frac{1}{x^2}+\frac{1}{y^2}-13=0$ and $\frac{1}{x}+\frac{1}{y}-5=0$ we get the roots as under

(a) $\frac{1}{8},\frac{1}{5}$ (b) $\frac{1}{2},\frac{1}{3}$ (c) $\frac{1}{13},\frac{1}{5}$ (d) $\frac{1}{4},\frac{1}{5}$

38. Solving $x^2+xy-21=0$ and $xy-2y^2+20=0$ we get the roots as under

(a) ±1, ±2 (b) ±2, ±3 (c) ±3, ±4 (d) None

39. Solving $x^2+xy+y^2=37$ and $3xy+2y^2=68$ we get the following roots

(a) ±3, ±4 (b) ±4, ±5 (c) ±2, ±3 (d) None

40. Solving $4^x.2^y=128$ and $3^{3x+2y}=9^{xy}$ we get the following roots

(a) $\frac{7}{4},\frac{7}{2}$ (b) 2, 3 (c) 1, 2 (d) 1, 3

41. Solving $9^x=3^y$ and $5^{x+y+1}=25^{xy}$ we get the following roots

(a) 1, 2 (b) 0, 1 (c) 0, 3 (d) 1, 3

42. Solving $9x+3y-4z=3$, $x+y-z=0$ and $2x-5y-4z=-20$ following roots are obtained

(a) 2, 3, 4 (b) 1, 3, 4 (c) 1, 2, 3 (d) None

43. Solving $x+2y+2z=0$, $3x-4y+z=0$ and $x^2+3y^2+z^2=11$ following roots are obtained

(a) 2, 1, -2 and -2, -1, 2 (b) 2, 1, 2 and -2, -1, -2

(c) only 2, 1, -2 (d) only -2, -1, 2

44. Solving $x^3-6x^2+11x-6=0$ we get the following roots

(a) -1, -2, 3 (b) 1, 2, -3 (c) 1, 2, 3 (d) -1, -2, -3

45. Solving $x^3+9x^2-x-9=0$ we get the following roots

(a) ±1, -9 (b) ±1, ±9 (c) ±1, 9 (d) None

46. It is being given that one of the roots is half the sum of the other two solving $x^3-12x^2+47x-60=0$ we get the following roots:

(a) 1, 2, 3 (b) 3, 4, 5 (c) 2, 3, 4 (d) -3, -4, -5

47. Solve $x^3+3x^2-x-3=0$ given that the roots are in arithmetical progression

(a) -1, 1, 3 (b) 1, 2, 3 (c) -3, -1, 1 (d) -3, -2, -1

48. Solve $x^3-7x^2+14x-8=0$ given that the roots are in geometrical progression

(a) ½, 1, 2 (b) 1, 2, 4 (c) ½ , -1, 2 (d) -1, 2, -4

49. Solve $x^3 - 6x^2 + 5x + 12 = 0$ given that the product of the two roots is 12

(a) 1, 3, 4 (b) -1, 3, 4 (c) 1, 6, 2 (d) 1, -6, -2

50. Solve $x^3 - 5x^2 - 2x + 24 = 0$ given that two of its roots being in the ratio of 3:4

(a) -2, 4, 3 (b) -1, 4, 3 (c) 2, 4, 3 (d) -2, -4, -3

ANSWERS

1.	(a)	**18.**	(a)	**35.**	(a)
2.	(b)	**19.**	(b)	**36.**	(a)
3.	(a)	**20.**	(a)	**37.**	(b)
4.	(b)	**21.**	(d)	**38.**	(c)
5.	(b)	**22.**	(d)	**39.**	(a)
6.	(c)	**23.**	(b)	**40.**	(a), (b)
7.	(a)	**24.**	(a)	**41.**	(a)
8.	(a)	**25.**	(b)	**42.**	(c)
9.	(a)	**26.**	(d)	**43.**	(a)
10.	(c)	**27.**	(d)	**44.**	(c)
11.	(a)	**28.**	(a)	**45.**	(a)
12.	(c)	**29.**	(c)	**46.**	(b)
13.	(a)	**30.**	(a)	**47.**	(c)
14.	(b)	**31.**	(c)	**48.**	(b)
15.	(c)	**32.**	(a)	**49.**	(b)
16.	(a)	**33.**	(a)	**50.**	(a)
17.	(d)	**34.**	(b)		

NOTES