SETS, RELATIONS AND FUNCTIONS

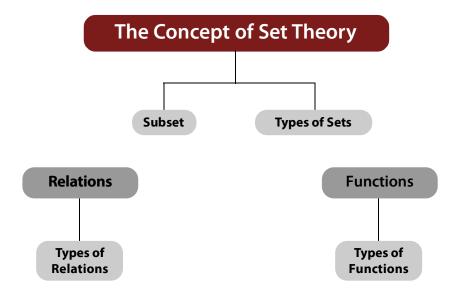


LEARNING OBJECTIVES

After reading this chapter, students will be able to understand:

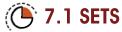
- Understand the concept of set theory.
- Appreciate the basics of functions and relations.
- Understand the types of functions and relations.
- Solve problems relating to sets, functions and relations.

CHAPTER OVERVIEW []



In our mathematical language, everything in this universe, whether living or non-living, is called an object.

If we consider a collection of objects given in such a way that it is possible to tell beyond doubt whether a given object is in the collection under consideration or not, then such a collection of objects is called a *well-defined collection of objects*.



A set is defined to be a collection of well-defined distinct objects. This collection may be listed or described. Each object is called an element of the set. We usually denote sets by capital letters and their elements by small letters.

Example: A = {a, e, i, o, u}

B = {2, 4, 6, 8, 10}

C = {pqr, prq, qrp, rqp, qpr, rpq}

D = {1, 3, 5, 7, 9}

E = {1,2}

etc.

This form is called Roster or Braces form. In this form we make a list of the elements of the set and put it within braces { }.

Instead of listing we could describe them as follows:

A = the set of vowels in the alphabet

B = The set of even numbers between 2 and 10 both inclusive.

C = The set of all possible arrangements of the letters p, q and r

D = The set of odd digits between 1 and 9 both inclusive.

E = The set of roots of the equation $x^2 - 3x + 2 = 0$

Set B, D and E can also be described respectively as

B = $\{x : x = 2m \text{ and } m \text{ being an integer lying in the interval } 0 < m < 6\}$

D = $\{2x - 1 : 0 < x < 5 \text{ and } x \text{ is an integer}\}$

 $E = \{x : x^2 - 3x + 2 = 0\}$

This form is called set-Builder or Algebraic form or Rule Method. This method of writing the set is called Property method. The symbol : or/reads 'such that'. In this method, we list the property or properties satisfied by the elements of the set.

We write, $\{x:x \text{ satisfies properties } P\}$. This means, "the set of all those x such that x satisfies the properties P".

A set may contain either a finite or an infinite number of members or elements. When the number of members is very large or infinite it is obviously impractical or impossible to list them all. In such case.

we may write as:

N = The set of natural numbers = $\{1, 2, 3....\}$

W = The set of whole numbers = $\{0, 1, 2, 3, \ldots\}$

etc.

- I. The members of a set are usually called elements. In $A = \{a, e, i, o, u\}$, a is an element and we write $a \in A$ i.e. a belongs to A. But 3 is not an element of $B = \{2, 4, 6, 8, 10\}$ and we write $3 \notin B$. i.e. 3 does not belong to B.
- II. If every element of a set P is also an element of set Q we say that P is a subset of Q. We write $P \subset Q$. Q is said to be a superset of P. For example $\{a,b\} \subset \{a,b,c\}$, $\{2,4,6,8,10\} \subset N$. If there exists even a single element in A, which is not in B then A is not a subset of B.
- III. If P is a subset of Q but P is not equal to Q then P is called a proper subset of Q.
- IV. Φ has no proper subset.

Illustration: {3} is a proper subset of {2, 3, 5}. But {1, 2} is not a subset of {2, 3, 5}.

Thus if $P = \{1, 2\}$ and $Q = \{1, 2, 3\}$ then P is a subset of Q but P is not equal to Q. So, P is a proper subset of Q.

To give completeness to the idea of a subset, we include the set itself and the empty set. The empty set is one which contains no element. The empty set is also known as **null or void** set usually denoted by $\{\}$ or Greek letter Φ , to be read as phi. For example the set of prime numbers between 32 and 36 is a null set. The subsets of $\{1, 2, 3\}$ include $\{1, 2, 3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$, $\{1\}$, $\{2\}$, $\{3\}$ and $\{\}$.

A set containing n elements has 2^n subsets. Thus a set containing 3 elements has 2^3 (=8) subsets. A set containing n elements has 2^n –1 proper subsets. Thus a set containing 3 elements has 2^n –1 =7 subsets. The proper subsets of { 1,2,3} include {1, 2}, {1, 3}, {2, 3}, {1}, {2}, {3}, { } }.

Suppose we have two sets A and B. The intersection of these sets, written as $A \cap B$ contains those elements which are in A and are also in B.

For example $A = \{2, 3, 6, 10, 15\}$, $B = \{3, 6, 15, 18, 21, 24\}$ and $C = \{2, 5, 7\}$, we have $A \cap B = \{3, 6, 15\}$, $A \cap C = \{2\}$, $B \cap C = \Phi$, where the intersection of B and C is empty set. So, we say B and C are disjoint sets since they have no common element. Otherwise sets are called overlapping or intersecting sets. The union of two sets, A and B, written as $A \cup B$ contain all these elements which are in either A or B or both.

So
$$A \cup B = \{2, 3, 6, 10, 15, 18, 21, 24\}$$

$$A \cup C = \{2, 3, 5, 6, 7, 10, 15\}$$

A set which has at least one element is called non-empty set . Thus the set $\{\,0\,\}$ is non-empty set. It has one element say 0.

Singleton Set: A set containing one element is called Singleton Set.

For example {1} is a singleton set, whose only element is 1.

Equal Set: Two sets A & B are said to be equal, written as A = B if every element of A is in B and every element of B is in A.

Illustration: If $A = \{2, 4, 6\}$ and $B = \{2, 4, 6\}$ then A = B.

Remarks: (I) The elements of the two sets may be listed in any order.

Thus, $\{1, 2, 3\} = \{2, 1, 3\} = \{3, 2, 1\}$ etc.

(II) The repetition of elements in a set is meaningless.

Example: $\{x : x \text{ is a letter in the word "follow"}\} = \{f, o, l, w\}$

Example: Show that Φ , $\{0\}$ and 0 are all different.

Solution: Φ is a set containing no element at all; $\{0\}$ is a set containing one element, namely 0. And 0 is a number, not a set.

Hence Φ , {0} and 0 are all different.

The set which contains all the elements under consideration in a particular problem is called the universal set denoted by S. Suppose that P is a subset of S. Then the complement of P, written as P^c (or P') contains all the elements in S but not in P. This can also be written as S - Por $S \sim P$. $S - P = \{x : x \in S, x \notin P\}$.

For example let $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

 $P = \{0, 2, 4, 6, 8\}$

 $Q = \{1, 2, 3, 4, 5\}$

Then $P' = \{1, 3, 5, 7, 9\}$ and $Q' = \{0, 6, 7, 8, 9\}$

Also $P \cup Q = \{0, 1, 2, 3, 4, 5, 6, 8\}, (P \cup Q)' = \{7, 9\}$

 $P \cap Q = \{2, 4\}$

 $P \cup Q' = \{0, 2, 4, 6, 7, 8, 9\}, (P \cap Q)' = \{0, 1, 3, 5, 6, 7, 8, 9\}$

 $P' \cup Q' = \{0, 1, 3, 5, 6, 7, 8, 9\}$

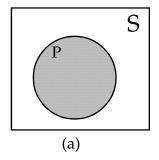
 $P' \cap Q' = \{7, 9\}$

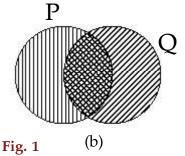
So it can be noted that $(P \cup Q)' = P' \cap Q'$ and $(P \cap Q)' = P' \cup Q'$. These are known as De Morgan's laws.



7.2 VENN DIAGRAMS

We may represent the above operations on sets by means of Euler - Venn diagrams.



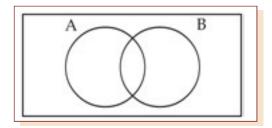


Thus Fig. 1(a) shows a universal set S represented by a rectangular region and one of its subsets P represented by a circular shaded region.

The un-shaded region inside the rectangle represents P'.

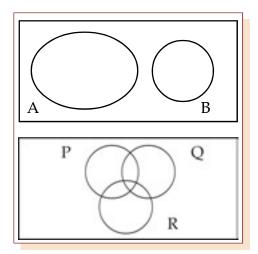
Figure 1(b) shows two sets P and Q represented by two intersecting circular regions. The total shaded area represents PUQ, the cross-hatched "intersection" represents $P \cap Q$.

The number of distinct elements contained in a finite set A is called its **cardinal number**. It is denoted by n(A). For example, the number of elements in the set $R = \{2, 3, 5, 7\}$ is denoted by n(R). This number is called the cardinal number of the set R.



Thus $n(AUB) = n(A) + n(B) - n(A \cap B)$

If A and B are disjoint sets, then n(AUB) = n(A) + n(B) as $n(A \cap B) = 0$



For three sets P, Q and R

$$n(PUQUR) = n(P) + n(Q) + n(R) - n(P \cap Q) - n(Q \cap R) - n(P \cap R) + n(P \cap Q \cap R)$$

When P, Q and R are disjoint sets

$$= n(P) + n(Q) + n(R)$$

Illustration: If $A = \{2, 3, 5, 7\}$, then n(A) = 4

Equivalent Set: Two finite sets A & B are said to be equivalent if n(A) = n(B).

Clearly, equal sets are equivalent but equivalent sets need not be equal.

Illustration: The sets $A = \{1, 3, 5\}$ and $B = \{2, 4, 6\}$ are equivalent but not equal.

Here n(A) = 3 = n(B) so they are equivalent sets. But the elements of A are not in B. Hence they are not equal though they are equivalent.

Power Set: The collection of all possible subsets of a given set A is called the power set of A, to be denoted by P(A).

Illustration: (i) If $A = \{1, 2, 3\}$ then

$$P(A) = \{ \{1, 2, 3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1\}, \{2\}, \{3\}, \Phi \} \}$$

(ii) If
$$A = \{1, \{2\}\}\)$$
, we may write $A = \{1, B\}$ when $B = \{2\}$ then

$$P(A) = \{ \Phi, \{1\}, \{B\}, \{1, B\} \} = \{ \Phi, \{1\}, \{\{2\}\}, \{1, \{2\}\} \}$$

EXERCISE 7 (A)

Choose the most appropriate option or options (a) (b) (c) or (d).

- 1. The number of subsets of the set $\{2, 3, 5\}$ is
 - (a) 3
- (b) 8

(c) 6

- (d) none of these
- 2. The number of subsets of a set containing n elements is
 - (a) 2^n
- (b) 2^{-n}

(c) n

(d) none of these

- 3. The null set is represented by
 - $(a)\{\Phi\}$
- (b) { 0 }

(c) **Φ**

- (d) none of these
- 4. $A = \{2, 3, 5, 7\}$, $B = \{4, 6, 8, 10\}$ then $A \cap B$ can be written as
 - (a) { }
- (b) {Φ}

- (c) (AUB)'
- (d) None of these
- 5. The set $\{x \mid 0 < x < 5\}$ represents the set when x may take integral values only
 - (a) {0, 1, 2, 3, 4, 5} (b) {1, 2, 3, 4}
- (c) {1, 2, 3, 4, 5}
- (d) none of these

- 6. The set {0, 2, 4, 6, 8, 10} can be written as
 - (a) $\{2x \mid 0 < x < 5\}$ (b) $\{x : 0 < x < 5\}$
- (c) $\{2x : 0 \leq x \leq 5\}$
- (d) none of these

From Q.7 to Q.10 The data to be used If $P = \{1, 2, 3, 5, 7\}$, $Q = \{1, 3, 6, 10, 15\}$,

Universal Set $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

- 7. The cardinal number of $P \cap Q$ is
 - (a) 3
- (b) 2

(c) 0

(d) none of these

- 8. The cardinal number of $P \bigcup Q$ is
 - (a) 10
- (b) 9

(c) 8

(d) none of these

- 9. n (P') is
 - (a) 10
- (b) 5

(c) 6

(d) none of these

- 10. n(Q') is
 - (a) 4
- (b) 10

(c) 4

(d) none of these

- 11. The set of cubes of the natural number is
 - (a) a finite set
- (b) an infinite set
- (c) a null set
- (d) none of these

12.	The set $\{2^x \mid x \text{ is any positive rational number}\}$ is							
	(a) an infinite set	(b) a null set	(c) a finite set	(d) none of these				
13.	$\{1-(-1)^x\}$ for all integer x is the set							
	(a) {0}	(b) {2}	(c) {0, 2}	(d) none of these				
14.	E is a set of positive	ve even numbers and O	is a set of positive odd	numbers, then $E \cup O$ is a				
	(a) set of whole no	umbers (b) N	(c) a set of rational nur	mber (d) none of these				
15.	If R is the set of p	ositive rational numbers	s and E is the set of real	numbers then				
	(a) R⊆E	(b) R ⊂ E	(c) $E \subset R$	(d) none of these				
16.	If N is the set of n	atural numbers and I is	the set of positive integ	gers, then				
	(a) N = I	(b) N⊂I	(c) N⊆I	(d) none of these				
17.	If I is the set of iso	osceles triangles and E is	s the set of equilateral tr	riangles, then				
	(a) I⊂E	(b) E⊂I	(c) $E = I$	(d) none of these				
18.	If R is the set of isosceles right angled triangles and I is set of isosceles triangles, then							
	(a) $R = I$	(b) R⊃I	(c) R⊂I	(d) none of these				
19.	${n(n+1)/2 : n \text{ is a}}$	positive integer} is						
	(a) a finite set	(b) an infinite set	(c) is an empty set	(d) none of these				
20.	If $A = \{1, 2, 3, 5, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7,$	7}, and B = $\{x^2 : x \in A\}$						
	(a) n(b) = n(A)	(b) $n(B) > n(A)$	(c) $n(A) = n(B)$	$(d) \ n(A) < n(B)$				
21.	$A \cup A$ is equal to							
	(a) A	(b) E	(c) 	(d) none of these				
22.	$A \cap A$ is equal to							
	(a) 	(b) A	(c) E	(d) none of these				
23.	$(A \cup B)'$ is equal to)						
	(a) $(A \cap B)'$	(b) A∪B'	(c) A' ∩ B'	(d) none of these				
24.	$(A \cap B)'$ is equal to)						
	(a) (A'∪B)'	(b) A'∪ B'	(c) $A' \cap B'$	(d) none of these				
25.	$A \cup E$ is equal to (E is a superset of A)						
	(a) A	(b) E	(c) 	(d) none of these				
26.	$A \cap E$ is equal to ((E is a superset of A)						
	(a) A	(b) E	(c) 	(d) none of these				

- 27. $E \cup E$ is equal to E is a superset of A
 - (a) E
- (b) **b**

(c) 2E

(d) none of these

- 28. $A \cap E'$ is equal to E is a superset of A
 - (a) E
- (b) ϕ

(c) A

(d) none of these

- 29. $A \cap \phi$ is equal to E is a superset of A
 - (a) A
- (b) E

(c) **b**

(d) none of these

- 30. AUA' is equal to E is a superset of A
 - (a) E
- (b) ϕ

(c) A

- (d) none of these
- 31. If $E = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, the subset of E satisfying 5 + x > 10 is
 - (a) {5, 6, 7, 8, 9} (b) {6, 7, 8, 9}
- (c) {7, 8, 9}
- (d) none of these
- 32. If $A\Delta B = (A-B) \cup (B-A)$ and $A = \{1, 2, 3, 4\}$, $B = \{3,5,7\}$ than $A\Delta B$ is
 - (a) {1, 2, 4, 5, 7} (b) {3}

- (c) {1, 2, 3, 4, 5, 7}
- (d) none of these

[Hint : If A and B are any two sets, then

 $A - B = \{ x : x \in A, x \notin B \}.$

[i.e. A - B Contains all elements of A but not in B].



7.3 PRODUCT OF SETS

Ordered Pair: Two elements a and b, listed in a specific order, form an ordered pair, denoted by (a, b).

Cartesian Product of sets : If A and B are two non-empty sets, then the set of all ordered pairs (a, b) such that a belongs to A and b belongs to B, is called the Cartesian product of A and B, to be denoted by $A \times B$.

Thus, $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

If $A = \Phi$ or $B = \Phi$, we define $A \times B = \Phi$

Illustration: Let $A = \{1, 2, 3\}, B = \{4, 5\}$

Then $A \times B = \{ (1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5) \}$

Example: If $A \times B = \{ (3, 2), (3, 4), (5, 2), (5, 4) \}$, find A and B.

Solution: Clearly A is the set of all first co-ordinates of $A \times B$, while B is the set of all second co-ordinates of elements of $A \times B$.

Therefore $A = \{3, 5\}$ and $B = \{2, 4\}$

Example: Let $P = \{1, 3, 6\}$ and $Q \{3, 5\}$

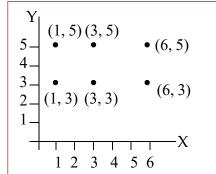
The product set $P \times Q = \{(1, 3), (1, 5), (3, 3), (3, 5), (6, 3), (6, 5)\}$.

Notice that $n(P \times Q) = n(P) \times n(Q)$ and ordered pairs (3, 5) and (5, 3) are not equal. and $Q \times P = \{(3, 1), (3, 3), (3, 6), (5, 1), (5, 3), (5, 6)\}$

So $P \times Q \neq Q \times P$; but $n(P \times Q) = n(Q \times P)$.

Illustration: Here n(P) = 3 and n(Q) = 2, $n(P \times Q) = 6$. Hence $n(P \times Q) = n(P) \times n(Q)$. and $n(P \times Q) = n(Q \times P) = 6.$

We can represent the product set of ordered pairs by plotting points in the XY plane.



If X=Y= the set of all natural numbers, the product set XY represents an infinite equal lattice of points in the first quadrant of the XY plane.

7.4 RELATIONS AND FUNCTIONS

Any subset of the product set X.Y is said to define a **relation** from X to Y and any relation from X to Y in which no two different ordered pairs have the same first element is called a **function**. Let A and B be two non-empty sets. Then, a rule or a correspondence f which associates to each element x of A, a unique element, denoted by f(x) of B, is called a function or **mapping** from A to B and we write $f : A \rightarrow B$

The element f(x) of B is called the image of x, while x is called the pre-image of f(x).



7.5 DOMAIN & RANGE OF A FUNCTION

Let $f: A \rightarrow B$, then A is called the domain of f, while B is called the co-domain of f.

The set $f(A) = \{f(x) : x \in A\}$ is called the range of f.

Illustration: Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 4, 9, 16, 25\}$

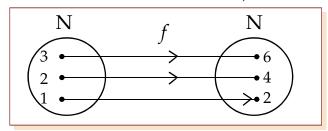
We consider the rule $f(x) = x^2$. Then f(1) = 1; f(2) = 4; f(3) = 9 & f(4) = 16.

Then clearly each element in A has a unique image in B.

So, $f: A \rightarrow B: f(x) = x^2$ is a function from A to B.

Here domain of $f = \{1, 2, 3, 4\}$ and range of $f = \{1, 4, 9, 16\}$

Example: Let N be the set of all natural numbers. Then, the rule



$$f: N \rightarrow N: f(x) = 2x$$
, for all $x \in N$

is a function from N to N, since twice a natural number is unique.

Now,
$$f(1) = 2$$
; $f(2) = 4$; $f(3) = 6$ and so on.

Here domain of
$$f = N = \{1, 2, 3, 4, \dots \}$$
; range of $f = \{2, 4, 6, \dots \}$

This may be represented by the mapping diagram or arrow graph.



(7.6 VARIOUS TYPES OF FUNCTIONS

One-One Function: Let $f: A \rightarrow B$. If different elements in A have different images in B, then f is said to be a one-one or an injective function or mapping.

Illustration: (i) Let $A = \{1, 2, 3\}$ and $B = \{2, 4, 6\}$

Let us consider $f : A \rightarrow B : f(x) = 2x$.

Then
$$f(1) = 2$$
; $f(2) = 4$; $f(3) = 6$

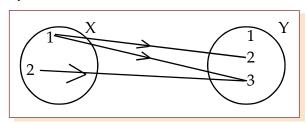
Clearly, f is a function from A to B such that different elements in A have different images in B. Hence f is one-one.

Remark : Let $f : A \rightarrow B$ and let $x_1, x_2 \in A$.

Then $x_1 = x_2$ implies $f(x_1) = f(x_2)$ is always true.

But $f(x_1) = f(x_2)$ implies $x_1 = x_2$ is true only when f is one-one.

(ii) let $x=\{1, 2, 3, 4\}$ and $y=\{1, 2, 3\}$, then the subset $\{(1, 2), (1, 3), (2, 3)\}$ defines a relation on X.Y.



Notice that this particular subset contains all the ordered pair in X.Y for which the X element (x) is less than the Y element (y). So in this subset we have X<Y and the relation between the set, is "less than". This relation is not a function as it includes two different ordered pairs (1, 2), (1, 3) have same first element.

$$X.Y=\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\}$$

(3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3)}

The subset $\{(1, 1), (2, 2), (3, 3)\}$ defines the function y = x as different ordered pairs of this subset have different first element.

Onto or Surjective Functions : Let $f : A \rightarrow B$. If every element in B has at least one pre-image in A, then f is said to be an onto function.

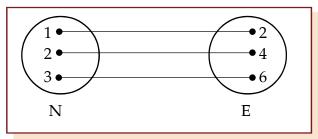
If f is onto, then corresponding to each $y \in B$, we must be able to find at least one element $x \in A$ such that y = f(x)

Clearly, f is onto if and only if range of f = B

Illustration: Let N be the set of all natural numbers and E be the set of all even natural numbers. Then, the function

$$f: N \rightarrow E: f(x) = 2x$$
, for all $x \in N$

is onto, since each element of E is of the form 2x, where $x \in N$ and the same is the f-image of $x \in N$.



Represented on a mapping diagram it is a on to mapping of X onto Y.

Bijection Function : A one-one and onto function is said to be bijective.

A bijective function is also known as a one-to-one correspondence.

Identity Function : Let A be a non-empty set. Then, the function I defined by

$$I: A \rightarrow A: I(x) = x$$
 for all $x \in A$ is called an identity function on A.

It is a one-to-one and onto function with domain A and range A.

Into Functions: Let $f: A \to B$. There exists even a single element in B having no pre-image in A, then f is said to be an into function.

Illustration: Let $A = \{2, 3, 5, 7\}$ and $B = \{0, 1, 3, 5, 7\}$. Let us consider $f : A \rightarrow B$;

$$f(x) = x - 2$$
. Then $f(2) = 0$; $f(3) = 1$; $f(5) = 3 & f(7) = 5$.

It is clear that f is a function from A to B.

Here there exists an element 7 in B, having no pre-mage in A.

So, f is an into function.

Constant Function: Let $f: A \to B$, defined in such a way that all the elements in A have the same image in B, then f is said to be a constant function.

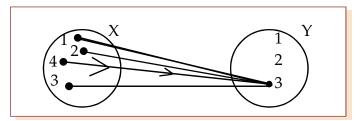
Illustration: Let A = $\{1, 2, 3\}$ and B = $\{5, 5, 5\}$ or B= $\{5\}$. Let f : A \rightarrow B : f (x) = 5 for all x \in A.

Then, all the elements in A have the same image namely 5 in B.

So, f is a constant function.

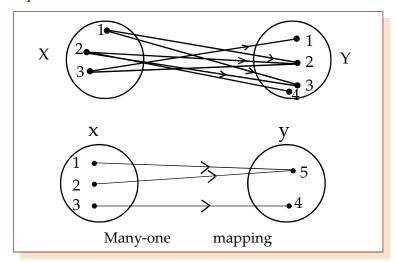
Remark: The range set of a constant function is a singleton set.

Example: Another subset of X.Y is {(1, 3), (2, 3), (3, 3), (4, 3)}



This relation is a function (a constant function). It is represented on a mapping diagram and is a many-one mapping of X into Y.

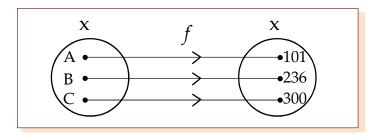
Let us take another subset $\{(4, 1), (4, 2), (4, 3)\}$ of X.Y. This is a relation but not a function. Here different ordered pairs have same first element so it is not a function.



There are two example of many-one mapping.

Equal Functions: Two functions f and g are said to be equal, written as f = g if they have the same domain and they satisfy the condition f(x) = g(x), for all x.

A function may simply pair people and the corresponding seat numbers in a theatre. It may simply associate weights of parcels with portal delivery charge or it may be the operation of squaring, adding to doubling, taking the log of etc.



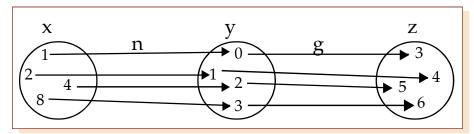
The function f here assigning a locker number to each of the persons A, B and C. Names are associated with or mapped onto, locker numbers under the function f.

We can write

$$f: X \to Y$$

$$f(x) = y$$

$$f(B) = 236$$



This diagram shows the effect of two functions n and g on the sets X, Y and Z

 $n: X \rightarrow Y$ and $g: Y \rightarrow Z$

If x, y, z are corresponding elements of X, Y and Z, we write n(x) = y and g(y) = z

Thus g n(1) = 0 and g(0) = 3, so that g(n(1)) = g(0) = 3 we can write it as

g(n(1)) or g on (1) = 3 But g(1) = 4 and n(g(1)) = n(4) = 2

So $g(n) \neq n(g)$ [(or, g (o n) \neq n (o g))]

The function g(n) or n(g) is called a composite function. As n(8) = 3, we say that 3 is the image of 8 under the mapping (or function) n.

Inverse Function: Let f be a one-one onto function from A to B. Let y be an arbitrary element of B. Then f being onto, there exists an element x in A such that f(x) = y.

As f is one-one this x is unique.

Thus for each $y \in B$, there exists a unique element $x \in A$ such that f(x) = y.

So, we may define a function, denoted by f^{-1} as:

 $f^{-1}: B \rightarrow A: f^{-1}(y) = x \text{ if and only if } f(x) = y.$

The above function f⁻¹ is called the inverse of f.

A function is invertible if and only if f is one-one onto.

Remarks: If f is one-one onto then f^{-1} is also one-one onto.

Example: net A = $\{1, 3, 5, 7\}$ and B = $\{3, 4, 9\}$ So, f; A \rightarrow B given by $\{(1, 2), (3, 3), (5, 4), (7, 9)\}$ and it is one-one on to mapping. Now if then is there of f.

If $f: A \rightarrow B$ then $f^{-1}: B \rightarrow A$.

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7.14

EXERCISE 7 (B)

Choose the most appropriate option/options (a) (b) (c) or (d).

If $A = \{x, y, z\}$, $B = \{p, q, r, s\}$ which of the relations on A to B are functions.

(a)
$$\{(x, p), (x, q), (y, r), (z, s)\}, (b) \{(x, s), (y, s), (z, s)\}$$

(c)
$$\{(y, p), (y, q), (y, r), (z, s), (d) \{(x, p), (y, r), (z, s)\}$$

2. $\{(x, y) | x+y = 5\}$ where $x, y \in R$ is a

(a) not a function (b) a composite function (c) one-one mapping (d) none of these

 $\{(x, y) | x = 4\}$ where $x, y \in R$ is a

(a) not a function (b) function

(c) one-one mapping (d) none of these

4. $\{(x, y), y=x^2\}$ where $x, y \in R$ is

(a) not a function (b) a function

(c) inverse mapping (d) none of these

5. $\{(x, y) \mid x < y\}$ where $x, y \in R$ is

(a) not a function (b) a function

(c) one-one mapping

(d) none of these

The domain of $\{(1, 7), (2, 6)\}$ is 6.

(a) {1, 6}

(b) {7, 6}

(c) {1, 2}

(d) {6, 7}

The range of $\{(3, 0), (2, 0), (1, 0), (0, 0)\}$ is

(a) {0, 0}

(b) {0}

 $(c) \{0, 0, 0, 0\}$

(d) none of these

The domain and range of $\{(x,y) : y = x^2\}$ where $x, y \in R$ is

(a) (reals, natural numbers)

(b) (reals, non-negative reals)

(c) (reals, reals)

(d) none of these

9. Let the domain of x be the set {1}. Which of the following functions gives values equal to 1

(a)
$$f(x) = x^2$$
, $g(x) = x$

(b) f(x) = x, g(x) = 1-x

(c)
$$f(x) = x^2 + x + 2$$
, $g(x) = (x+1)^2$

(d) none of these

10. If f(x) = 1/1-x, f(-1) is

(a) 0

(b) $\frac{1}{2}$

(c) 0

(d) none of these

11. If g(x) = (x-1)/x, $g(-\frac{1}{2})$ is

(a) 1

(b) 2

(c) 3/2

(d) 3

12. If f(x) = 1/1-x and g(x) = (x-1)/x, then f(x) = 1/1-x is

(a) x

(b) 1/x

(c) -x

(d) none of these

13. If f(x) = 1/1-x and g(x) = (x-1)/x, then gof(x) is

(a) x-1

(b) x

(c) 1/x

(d) none of these

- 14. The function $f(x) = 2^x$ is
 - (a) one-one mapping

(b) one-many

(c) many-one

- (d) none of these
- 15. The range of the function $f(x) = \log_{10}(1 + x)$ for the domain of real values of x when $0 \le x < 9$ is
 - (a) [0, 1]
- (b) [0, 1, 2]
- (c) {0, 1}
- (d) none of these

- 16. The Inverse function f^{-1} of f(x) = 2x is
 - (a) 1/2x
- (b) $\frac{x}{2}$

(c) 1/x

(d) none of these

- 17. If f(x) = x+3, $g(x) = x^2$, then fog(x) is
 - (a) $x^2 + 3$
- (b) $x^2 + x + 3$
- (c) $(x + 3)^2$
- (d) none of these

- 18. If f(x) = x+3, $g(x) = x^2$, then
- f(x).g(x) is

- (a) $(x + 3)^2$
- (b) $x^2 + 3$
- (c) $x^3 + 3x^2$
- (d) none of these

- 19. The Inverse h^{-1} when $h(x) = \log_{10} x$ is
 - (a) $\log_{10} x$
- (b) 10^{x}

- (c) $\log_{10}(1/x)$
- (d) none of these
- 20. For the function $h(x) = 10^{1+x}$ the domain of real values of x where $0 \le x \le 9$, the range is
 - (a) $10 \le h(x) \le 10^{10}$

- (b) $0 \le h(x) \le 10^{10}$
- (c) 0 < h(x) < 10

(d) none of these

Different types of relations

Let $S = \{a, b, c,\}$ be any set then the relation R is a subset of the product set $S \times S$

i) If R contains all ordered pairs of the form (a, a) in S×S, then R is called reflexive. In a *reflexive* relation 'a' is related to itself .

For example, 'Is equal to' is a reflexive relation for a = a is true.

ii) If $(a, b) \in R \Rightarrow (b, a) \in R$ for every $a, b \in S$ then R is called symmetric

For example $a=b \Rightarrow b=a$. Hence the relation 'is equal to' is a symmetric relation.

iii) If $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \Rightarrow R$ for every $a, b, c, \in S$ then R is called *transistive*.

For example a = b, $b = c \Rightarrow a = c$. Hence the relation 'is equal to' is a transitive relation.

A relation which is reflexive, symmetric and transitive is called an *equivalence relation* or simply an *equivalence*. 'is equal to' is an equivalence relation.

Similarly, the relation " is parallel to " on the set S of all straight lines in a plane is an equivalence relation.

Illustration: The relation "is parallel to" on the set S is

(1) reflexive, since a || a for a \in S

- (2) symmetric, since a $|| b \Rightarrow b || a$
- (3) transitive, since a \parallel b , b \parallel c \Rightarrow a \parallel c

Hence it is an equivalence relation.

Domain & Range of a relation : If R is a relation from A to B, then the set of all first coordinates of elements of R is called the domain of R, while the set of all second co-ordinates of elements of R is called the range of R.

So, Dom (R) =
$$\{a : (a, b) \in R\}$$
 & Range (R) = $\{b : (a, b) \in R\}$

Illustration: Let $A = \{1, 2, 3\}$ and $B = \{2, 4, 6\}$

Then
$$A \times B = \{(1, 2), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 2), (3, 4), (3, 6)\}$$

By definition every subset of $A \times B$ is a relation from A to B.

Thus, if we consider the relation

$$R = \{ (1, 2), (1, 4), (3, 2), (3, 4) \}$$
 then Dom $(R) = \{1, 3\}$ and Range $(R) = \{2, 4\}$

From the product set X. $Y = \{(1, 3), (2, 3), (3, 3), (4, 3), (2, 2), (3, 2), (4, 2), (1, 1), (2, 1), (3, 1), (4, 1)\}$, the subset $\{(1, 1), (2, 2), (3, 3)\}$ defines the relation 'Is equal to', the subset $\{(1, 3), (2, 3), (1, 2)\}$ defines 'Is less than', the subset $\{(4, 3), (3, 2), (4, 2), (2, 1), (3, 1), (4, 1)\}$ defines 'Is greater than' and the subset $\{(4, 3), (3, 2), (4, 2), (2, 1), (3, 1), (4, 1), (1, 1), (2, 2), (3, 3)\}$ defines to greater 'In greater than or equal'.

Illustration: Let $A = \{1, 2, 3\}$ and $b = \{2, 4, 6\}$

Then
$$A \times B = \{(1, 2), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 2), (3, 4), (3, 6)\}$$

If we consider the relation = $\{(1, 2), (1, 4), (3, 4)\}$ then Dom (R) = $\{1, 3\}$ and Range = $\{2, 4\}$. Here the relation "Is less than".

Identity Relation: The relation $I = \{(a, a) : a \in A\}$ is called the identity relation on A.

Illustration: Let $A = \{1, 2, 3\}$ then $I = \{(1, 1), (2, 2), (3, 3)\}$

Inverse Relation: If R be a relation on A, then the relation R⁻¹ on A, defined by

 $R^{-1} = \{(b, a) : (a, b) \in R\}$ is called an inverse relation on A.

Clearly , Dom (R^{-1}) = Range (R) & Range (R^{-1}) = Dom (R).

Illustration: Let $A = \{1, 2, 3\}$ and $R = \{(1, 2), (2, 2), (3, 1), (3, 2)\}$

Then R being a subset of a \times a, it is a relation on A. Dom (R) = $\{1, 2, 3\}$ and Range (R) = $\{2, 1\}$

Now, $R^{-1} = \{(2, 1), (2, 2), (1, 3), (2, 3)\}$. Here, Dom $(R^{-1}) = \{2, 1\} = Range(R)$ and

Range $(R^{-1}) = \{1, 2, 3\} = Dom(R)$.

Illustration: Let $A = \{1, 2, 3\}$, then

(i)
$$R1 = \{(1, 1), (2, 2), (3, 3), (1, 2)\}$$

Is reflexive and transitive but not symmetric, since $(1, 2) \in R$, but (2, 1) does not belong to R.

(ii)
$$R2 = \{(1, 1), (2, 2), (1, 2), (2, 1)\}$$

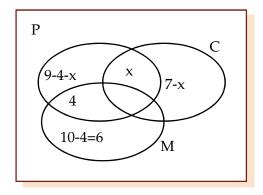
is symmetric and transitive but not reflexive, since (3, 3) does not belong to R_a .

(iii)
$$R3 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$$

is reflexive and symmetric but not transitive , since $(1, 2) \in R3 \& (2, 3) \in R3$ but (1, 3) does not belong to R3.

Problems and solution using Venn Diagram

1. Out of a group of 20 teachers in a school, 10 teach Mathematics, 9 teach Physics and 7 teach Chemistry. 4 teach Mathematics and Physics but none teach both Mathematics and Chemistry. How many teach Chemistry and Physics? How many teach only Physics?



Let x be the no. of teachers who teach both Physics & Chemistry.

$$9 - 4 - x + 6 + 7 - x + 4 + x = 20$$

or
$$22 - x = 20$$

or
$$x = 2$$

Hence, 2 teachers teach both Physics and Chemistry and 9 - 4 - 2 = 3 teachers teach only Physics.

2. A survey shows that 74% of the Indians like grapes, whereas 68% like bananas.

What percentage of the Indians like both grapes and bananas?

Solution: Let P & Q denote the sets of Indians who like grapes and bananas respectively. Then n(P) = 74, n(Q) = 68 and $n(P \cup Q) = 100$.

We know that
$$n(P \cap Q) = n(P) + n(Q) - n(P \cup Q) = 74 + 68 - 100 = 42$$
.

Hence, 42% of the Indians like both grapes and bananas.

- 3. In a class of 60 students, 40 students like Maths, 36 like Science, and 24 like both the subjects. Find the number of students who like
 - (i) Maths only
- (ii) Science only
- (iii) either Maths or Science

(iv) neither Maths nor Science

Solution: Let M = students who like Maths and S = students who like Science

Then n(M) = 40, n(S) = 36 and n (M \cap S) = 24

Hence,

- (i) $n(M) n(M \cap S) = 40 24 = 16 = number of students like Maths only.$
- (ii) $n(S) n(M \cap S) = 36 24 = 12 = number of students like Science only.$
- (iii) $n(M \cup S) = n(M) + n(S) n(M \cap S) = 40 + 36 24 = 52 =$ number of students who like either Maths or Science.
- (iv) $n(M \cup S)^c = 60 n(M \cup S) = 60 52 = 8 = number of students who like neither Maths nor Science.$



- A set is defined to be a collection of well-defined distinct objects. This collection may be listed or described. Each object is called an element of the set. We usually denote sets by capital letters and their elements by small letters.
- Singleton Set: A set containing one element is called Singleton.
- ◆ Equal Set: Two sets A & B are said to be equal, written as A = B if every element of A is in B and every element of B is in A.
- ♦ Universal Set: The set which contains all the elements under consideration in a particular problem is called *the universal set* denoted by S. Suppose that P is a subset of S. Then the complement of P, written as P^c (or P') contains all the elements in S but not in P. This can also be written as S P or $S \sim P$. $S P = \{x : x \in S, x \notin P\}$.
- If A and B are two sets then

$$n(AUB) = n(A) + n(B) - n(A \cap B)$$

- If A and B are disjoint sets, then n(AUB) = n(A) + n(B) as $n(A \cap B) = 0$
- For three sets P, Q and R

$$n(PUQUR) = n(P) + n(Q) + n(R) - n(P \cap Q) - n(Q \cap R) - n(P \cap R) + n(P \cap Q \cap R)$$

When P, Q and R are disjoint sets

$$= n(P) + n(Q) + n(R)$$

- Equivalent Set: Two finite sets A & B are said to be equivalent if n(A) = n(B).
- **Power Set:** The collection of all possible subsets of a given set A is called the power set of A, to be denoted by P(A).
 - 1. A set containing n elements has 2ⁿ subsets.
 - 2. A set containing n elements has 2ⁿ-1 proper subsets

- Ordered Pair: Two elements a and b, listed in a specific order, form an ordered pair, denoted by (a, b).
- ◆ Cartesian Product of sets: If A and B are two non-empty sets, then the set of all ordered pairs (a, b) such that a belongs to A and b belongs to B, is called the Cartesian product of A and B, to be denoted by A × B.

Thus, $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

If $A = \Phi$ or $B = \Phi$, we define $A \times B = \Phi$

• **Relation and Function:** Any subset of the product set X.Y is said to define a **relation** from X to Y and any relation from X to Y in which no two different ordered pairs have the same first element is called a **function**.

Let A and B be two non-empty sets. Then, a rule or a correspondence f which associates to each element x of A, a unique element, denoted by f(x) of B, is called a function or **mapping** from A to B and we write $f: A \rightarrow B$

The element f(x) of B is called the image of x, while x is called the pre-image of f(x).

Let $f: A \rightarrow B$, then A is called the domain of f, while B is called the co-domain of f.

The set $f(A) = \{ f(x) : x \in A \}$ is called the range of f.

- \bullet One-one Function: Let $f: A \rightarrow B$. If different elements in A have different images in B, then f is said to be a one-one or an injective function or mapping.
- ◆ Onto or Surjective Functions: Let f : A→B. If every element in B has at least one preimage in A, then f is said to be an onto function.

If f is onto, then corresponding to each $y \in B$, we must be able to find at least one element $x \in A$ such that y = f(x)

Clearly, f is onto if and only if range of f = B

• **Bijection Function:** A one-one and onto function is said to be bijective.

A bijective function is also known as a one-to-one correspondence.

• Identity Function: Let A be a non-empty set . Then, the function I defined by

 $I: A \rightarrow A: I(x) = x$ for all $x \in A$ is called an identity function on A.

• It is a one-to-one onto function with domain A and range A.

Into Functions: Let $f : A \rightarrow B$. There exists even a single element in B having no pre-image in A, then f is said to be an into function.

- Constant Function: Let $f: A \to B$, defined in such a way that all the elements in A have the same image in B, then f is said to be a constant function.
- Equal Functions: Two functions f and g are said to be equal, written as f = g if they have the same domain and they satisfy the condition f(x) = g(x), for all x.
- Inverse Function: Let f be a one-one onto function from A to B. Let y be an arbitrary element of B. Then f being onto, there exists an element x in A such that f(x) = y.

A function is invertible if and only if f is one-one onto.

Different types of relations:

Let $S = \{a, b, c,\}$ be any set then the relation R is a subset of the product set $S \times S$

- i) If R contains all ordered pairs of the form (a, a) in S×S, then R is called reflexive. In a *reflexive* relation 'a' is related to itself.
 - For example, 'Is equal to' is a reflexive relation for a = a is true.
- ii) If $(a, b) \in R \Rightarrow (b, a) \in R$ for every $a, b \in S$ then R is called symmetric For Example $a = b \Rightarrow b = a$. Hence the relation 'is equal to' is a symmetric relation.
- iii) If $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \Rightarrow R$ for every $a, b, c, \in S$ then R is called *transistive*.

For Example a = b, $b = c \Rightarrow a = c$. Hence the relation 'is equal to' is a transitive relation.

A relation which is reflexive, symmetric and transitive is called an *equivalence relation* or simply an *equivalence*. 'is equal to' is an equivalence relation.

Similarly, the relation "is parallel to" on the set S of all straight lines in a plane is an equivalence relation.

◆ **Domain & Range of a relation:** If R is a relation from A to B, then the set of all first coordinates of elements of R is called the domain of R, while the set of all second co-ordinates of elements of R is called the range of R.

So, Dom (R) = $\{a : (a, b) \in R\}$ & Range (R) = $\{b : (a, b) \in R\}$

EXERCISE 7 (C)

Choose the most appropriate option/options (a) (b) (c) or (d).

1. "Is smaller than" over the set of eggs in a box i	1.	"Is smaller than'	over the set of eggs in a	box is
--	----	-------------------	---------------------------	--------

- (a) Transitive (T) (b) Symmetric (S)
- (c) Reflexive (R)
- (d) Equivalence (E)
- 2. "Is equal to" over the set of all rational numbers is
 - (a) (T)
- (b) (S)

(c) (R)

(d) E

[By using using R = Reflexive; T = Transitive, S = Symmetric and E = Equivalence from Q.No. 2 to 8]

3. "has the same father as" over the set of children

- (a) R
- (b) S

(c) T

(d) E

4. "is perpendicular to" over the set of straight lines in a given plane is

- (a) R
- (b) S

(c) T

(d) E

5. "is the reciprocal of" over the set of non-zero real numbers is

- (a) S
- (b) R

(c) T

(d) none of these

6. $\{(x,y)/x \in x, y \in y, y = x\}$ is

	(a) R	(b) S	(c) T	(d) E
7.	$\{(x,y) / x + y = 2x$	where x and y are posi-	tive integers}, is	
	(a) R	(b) S	(c) T	(d) E
8.	"Is the square of"	over n set of real numbe	ers is	
	(a) R	(b) S	(c) T	(d) none of these
9.	If A has 32 element in $A \cap B$ is	its, B has 42 elements and	d A \cup B has 62 elements	, the number of elements
	(a) 12	(b) 74	(c) 10	(d) none of these
10.	In a group of 20 ch drinking coffee bu		ot coffee and 13 like tea	. The number of children
	(a) 6	(b) 7	(c) 1	(d) none of these
11.	The number of su	bsets of the sets $\{6, 8, 11\}$	} is	
	(a) 9	(b) 6	(c) 8	(d) none of these
12.	The sets $V = \{x / another if x is equals = 0\}$		0) and $S = \{x : x^2 + x - x^2 \}$	$-2 = 0$ } are equal to one
	(a) – 2	(b) 2	(c) ½	(d) none of these
13.	If the universal set	$t E = \{x \mid x \text{ is a positive i} \}$	nteger <25}, A = {2, 6, 8	8, 14, 22}, B = {4, 8, 10, 14}
	(a) $(A \cap B)' = A' \cup B$	B' (b) $(A \cap B)' = A' \cap B'$	(c) $(A' \cap B)' = \phi$	(d) none of these
14.	If the set P has 3 e	elements, Q four and R t	two then the set $P \times Q$	× R contains
	(a) 9 elements	(b) 20 elements	(c) 24 elements	(d) none of these
15.	Given $A = \{2, 3\}, 1$	$B = \{4, 5\}, C = \{5, 6\} \text{ then}$	$A \times (B \cap C)$ is	
	(a) {(2, 5), (3, 5)}	(b) {(5, 2), (5, 3)}	(c) {(2, 3), (5, 5)}	(d) none of these
16.				d the newspaper X and ersons not reading X and
	(a) 2,000	(b) 3,000	(c) 2,500	(d) none of these
17.	If $A = \{ 1, 2, 3, 5, $	7} and $B = \{1, 3, 6, 10, 1\}$	5}. Cardinal number of	A-B is
	(a) 3	(b) 4	(c) 6	(d) none of these
18.	Which of the diag	ram is graph of a functi	on	
	Y •	Y	Y	Y
	(a) • • •	(b) \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	(c)	(d)

19.	9. At a certain conference of 100 people there are 29 Indian women and 23 Indian men. Out of these Indian people 4 are doctors and 24 are either men or doctors. There are no foreign doctors. The number of women doctors attending the conference is															
	(a) 2			(b)	4			(c)	1			(d)	none	of the	ese	
20.	Let A	$= \{a,$	b}. Set	t of sı	ıbsets (of A is	calle	d pow	er set	of A	denote	ed by	P(A).	Now	n(P(A) is	
	(a) 2			(b)	4			(c)	3			(d)	none	of the	ese	
21.	21. Out of 2000 employees in an office 48% preferred Coffee (c), 54% liked (T), 64% used to smoke (S). Out of the total 28% used C and T, 32% used T and S and 30% preferred C and S, only 6% did none of these. The number having all the three is															
	(a) 36	0		(b)	300			(c)	380			(d)	none	of the	se	
22.	Refer	red to	the da	ata of	Q. 21	the nu	mber	of em	ploye	es hav	ing T	and S	but n	ot C is	S	
	(a) 20	0		(b)	280			(c)	300			(d)	none	of the	se	
23.	Refer	red to	the d	ata o	f Q. 21	the n	umbe	r of e	nploy	ees p	referri	ng on	ly cof	fee is		
	(a) 10	00		(b)	260			(c)	160			(d)	none	of the	ese	
24.	If f(x)	= x +	- 3, g(x) = x	² , then	g of (x) is									
	(a) (x	+ 3)2		(b)	$x^2 + 3$			(c)	$x^{2}(x +$	- 3),		(d)	none	of the	ese	
25.	If f(x)	= 1/	1–x, tl	nen f	f(x) is											
	(a) 1-	-x		(b)	(x-1)/	x		(c)	x/(x-	1)		(d)	none	of the	ese	
ΔN	SWE	DÇ														
	ercise															
1.	b	2.	a	3.	С	4.	a	5.	b	6.	С	7.	b	8.	С	
9.	a	10.	b	11.	b	12.	a	13.	С	14.	b	15.	b	16.	a	
17.	. b	18.	С	19.	b	20.	С	21.	a	22.	b	23.	С	24.	b	
25.	. b	26.	a	27.	a	28.	b	29.	c	30.	a	31.	b	32.	a	
Exc	ercise	7(B)														
1.	b, d	2.	c	3.	a	4.	b	5.	a	6.	c	7.	b	8.	b	
9.	a	10.	b	11.	d	12.	a	13.	b	14.	a	15.	a	16.	b	
17.	. a	18.	С	19.	b	20.	a									
Ev	arcica	7(C)														

3. d

11. c

19. c

1. a

9. a

17. a25. b

2.

18. b

10. b

d

4. b

12. a

20. b

5.

a

21. a

6. d

13. a **14.** c **15.** a

22. b

7.

d

23. c

8.

16. b

24.

d



7.7 CONCEPT OF LIMIT

I) We consider a function f(x) = 2x. If x is a number approaching to the number 2 then f(x) is a number approaching to the value 2x2=4

The following table shows f(x) for different values of x approaching 2

X	f(x)
1.90	3.8
1.99	3.98
1.999	3.998
1.9999	3.9998
2	4

Here x approaches 2 from values of x<2 and for x being very close to 2 f(x) is very close 4. This situation is defined as left-hand limit of f(x) as x approaches 2 and is written as $\lim f(x) = 4$, $x \rightarrow 2$ -

X	f(x)
2.0001	4.0002
2.001	4.002
2.01	1.02
2.0	4

Here x approaches 2 from values of x greater than 2 and for x being very close 2f(x) is very close to 4. This situation is defined as right-hand limit of f(x) as x approaches 2 and is written as $\lim_{x \to 2} f(x) = 4$ as $x \to 2$

So we write

$$\lim_{x \to 2^{-}} f(x) = \lim_{x \to 2^{-+}} f(x) = 4$$

Thus $\lim_{x\to a} f(x)$ is said to exist when both left-hand and right-hand limits exists and they are equal. We write as

$$\lim_{x \to a^{-}} f(x) = \lim_{x \to a^{-+}} f(x) = \lim_{x \to a} f(x)$$

Thus $\lim_{x\to a} f(x)$ is said to exist when both left-hand and right-hand limits exists and they are equal. We write as

$$\lim_{x \to a} f(x) = \lim_{x \to a+} f(x) = \lim_{x \to a} f(x)$$

Thus, if
$$\lim_{h\to 0} f(a+h) = \lim_{h\to 0} f(a-h)$$
, $(h > 0)$

then
$$\lim_{x\to a} f(x)$$
 exists

We now consider a function defined by

$$fx = \begin{cases} 2x-2 & \text{for } x < 0 \\ 1 & \text{for } x = 0 \\ 2x+2 & \text{for } x > 0 \end{cases}$$

We calculate limit of f(x) as x tend to zero. At x = 0, f(x) = 1 (given). If x tends to zero from left-hand side for the value of x < 0, f(x) is approaching (2x0) - 2 + -2 which is defined as left-hand limit of f(x) as $x \to 0$. We can write this as

$$\lim_{x\to 0-}=-2$$

Similarly if x approaches zero from right-hand side for values of x>0, f(x) is approaching $2 \times 0 + 2 = 2$. We can write this as $\lim_{x \to 0+} f(x) = -2$.

In this case both left-hand and right-hand exist, but they are not equal. So we may conclude that $\lim_{x\to 0} f(x)$ does not exist.



7.8. USEFUL RULES (THEOREMS) ON LIMITS

Let
$$\lim_{x\to a} f(x) = l$$
 and $\lim_{x\to a} g(x) = m$

where I and m are finite quantities

i)
$$\lim_{x \to a} f \{f(x) + g(x)\} = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) = l + m$$

That is limit of the sum of the difference of two functions is equal to the sum of their limits

ii)
$$\lim_{x \to a} \{ f(x) - g(x) \} = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) = l - m$$

That is limit of the difference of two functions is equal to difference of their limits.

iii)
$$\lim_{x \to a} \{f(x) \cdot g(x)\} = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) = lm$$

That is limit of the product of two functions is equal to the product of their limits

$$\lim_{x \to a} \left\{ f(x) / g(x) \right\} = \left\{ \lim_{x \to a} f(x) \right\} / \left\{ \lim_{x \to a} g(x) \right\} = l / m \text{ if } m \neq 0$$

That is limit of the quotient of two functions is equal to the quotient of their limits.

v) $\lim_{x\to a} c = c$ where c is constant

That is limit of a constant is the constant

$$Vi) \quad \lim_{x \to a} cf(x) = c \lim_{x \to a} f(x)$$

vii)
$$\lim_{x \to a} F\{f(x)\} = F\{\lim_{x \to a} F(x)\} = F(l)$$

viii)
$$\lim_{x\to 0+} \frac{1}{x} = \lim_{h\to 0} \frac{1}{h} \to +\infty$$
 (h>0)

$$\lim_{x\to 0+} \frac{1}{x} = \lim_{h\to 0} \frac{1}{-h} \to -\infty \quad \text{(h>0)}$$

 ∞ is a very-every large number called infinity

Thus $\lim_{x\to 0+} \frac{1}{x}$ does not exist.

Example 1: Evaluate (i) $\lim_{x\to 2} (3x+9)$;

(ii)
$$\lim_{x\to 5} \frac{1}{x-1}$$

(ii)
$$\lim_{x\to 5} \frac{1}{x-1}$$
 (iii) $\lim_{x\to a} \frac{1}{x-a}$

Solution:

(i)
$$\lim_{x\to 2} (3x+9) = 3.2+9 = (6+9) = 15$$

(ii)
$$\lim_{x\to 5} \frac{1}{x-1} = \frac{1}{5-1} = \frac{1}{4}$$

(iii)
$$\lim_{x\to a} \frac{1}{x-a}$$
 does not exist, since $\lim_{x\to a^+} \frac{1}{x-a} \to +\infty$ and $\lim_{x\to a^-} \frac{1}{x-a} \to -\infty$

Example-2 Evaluate
$$\lim_{x\to 2} \frac{x^2 - 5x + 6}{x - 2}$$

Solution: At x = 2 the function becomes undefined as 2-2 =0 and division by zero is not mathematically defined.

So
$$\lim_{x\to 2} \left\{ x^2 - 5x + 6 / (x - 2) \right\} = \lim_{x\to 2} \left\{ (x - 3) / (x - 2) \right\} = \lim_{x\to 2} (x - 3) (\because x - 2 \neq 0)$$

$$= 2-3 = -1$$

Example-2 Evaluate
$$\lim_{x\to 2} \frac{x^2 + 2x - 1}{\sqrt{x^2 + 2}}$$

$$\lim_{x \to 2} \frac{x^2 + 2x - 1}{\sqrt{x^2 + 2}} = \frac{\lim_{x \to 2} (x^2 + 2x - 1)}{\lim_{x \to 2} \sqrt{x^2 + 2}} = \frac{\lim_{x \to 2} (x^2 + \lim_{x \to 2} 2x - 1)}{\sqrt{\lim_{x \to 2} x^2 + 2}}$$

$$\frac{(2)^2 + 2x \ 2 - 1}{\sqrt{(2)^2 + 2}} = \frac{7}{\sqrt{6}}$$



SOME IMPORTANT LIMITS

We now state some important limits

a)
$$\lim_{x\to 0} \frac{(e^x - 1)}{x} = 1$$

b)
$$\lim_{x\to 0} \frac{a^x - 1}{x} = \log_e \ a(a > 0)$$

$$c) \qquad \lim_{x \to 0} \frac{\log(1+x)}{x} = 1$$

d)
$$\lim_{x \to x} \left(1 + \frac{1}{x} \right)^x = e$$
 or $\lim_{x \to 0} \frac{(1+x)^{\frac{1}{x}}}{x} = e$

e)
$$\lim_{x\to 0} \frac{x^n - a^n}{x} = na^{n-1}$$

f)
$$\lim_{x \to 0} \frac{(1+x)^n - 1}{x} = n$$

- (A) The number e called exponential number is given by e = 2.718281828_____=2.7183. This number e is one of the useful constants in mathematics.
- (B) In calculus all logarithms are taken with respect to base 'that is $\log x = \log_e x$.

ILLUSTRATIVE EXAMPLES

Example 1: Evaluate:
$$\lim_{x\to 3} \frac{x^2 - 6x + 9}{x - 3}$$
, where $f(x) = \frac{x^2 - 6x + 9}{x - 3}$. Aso find f (3)

Solution: At x = 3 the function is undefined as division by zero is meaningless. While taking the limit as $x \to 3$, x cannot be exactly equal to 3 i.e. $x-3 \ne 0$ and consequently division by x-3 is permissible.

Now
$$\lim_{x \to 3} \frac{x^2 - 6x + 9}{x - 3} = \lim_{x \to 3} \frac{(x - 3)^2}{x - 3} = \lim_{x \to 3} (x - 3) = 3 - 3 = 0$$
 $f(3) = \frac{0}{0}$ is undefined

The reader may compute the left-hand and the right-hand limits as an exercise.

Example 2: A function is defined as follows:

$$fx = \begin{cases} -3x & \text{when } x < 0 \\ 2x & \text{when } x > 0 \end{cases}$$

Test the existence of $\lim_{x\to 0} f(x)$.

Solution: For x approaching 0 from the left x < 0.

Left-hand limit =
$$\lim_{x\to 0^{-}} f(x) = \lim_{x\to 0^{-}} (-3x) = 0$$

When x approaches 0 from the right x > 0

Right-hand limit =
$$\lim_{x\to 0+} f(x) = \lim_{x\to 0+} 2x = 0$$

Since L.H. limit = R.H. Limit, the limit exists. Thus, $\lim_{x\to 0} f(x) = 0$.

Example 3:
$$\lim_{x\to 3} \frac{x^2 + 4x + 3}{x^2 + 6x + 9}$$
.

Solution:
$$\frac{x^2 + 4x + 3}{x^2 + 6x + 9} = \frac{x^2 + 3x + x + 3}{(x+3)^2} = \frac{x(x+3) + 1(x+3)}{(x+3)^2} = \frac{(x+3)(x+1)}{(x+3)^2} = \frac{x+1}{x+3}$$

$$\lim_{x \to 3} \frac{x^2 + 4x + 3}{x^2 + 6x + 9} = \lim_{x \to 3} \frac{x + 1}{x + 3} = \frac{4}{3} = \frac{2}{3}.$$

Example 4: Find the following limits:

(i)
$$\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$$
 (ii)

(ii)
$$\lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \text{ if } h > 0.$$

Solution:

(i)
$$\frac{\sqrt{x}-3}{x-9} = \frac{\sqrt{x}-3}{(\sqrt{x}+3)(\sqrt{x}-3)} = \frac{1}{\sqrt{x}+3}$$
 $\therefore \lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{6}$.

(ii)
$$\frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}} :: \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$
$$= \frac{1}{\lim_{h \to 0} \sqrt{x+h} + \lim_{h \to 0} \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}.$$

Example 5: Find $\lim_{x\to 0} \frac{3x+|x|}{7x-5|x|}$.

Solution: Right-hand limit =
$$\lim_{x \to 0+} \frac{3x + |x|}{7x - 5|x|} = \lim_{x \to 0+} \frac{3x + x}{7x - 5x} = \lim_{x \to 0+} 2 = 2$$

Left-hand limit =
$$\lim_{x\to 0} \frac{3x+|x|}{7x-5|x|} = \lim_{x\to 0} \frac{3x-(x)}{7x-5(-x)} = \lim_{x\to 0} \frac{1}{6} = \frac{1}{6}$$
.

Since Right-hand limit \neq Left-hand limit the limit does not exist.

Example 6: Evaluate = $\lim_{x\to 0} \frac{e^x - e^{-x}}{x}$

Solution:
$$= \lim_{x \to 0} \frac{e^x - e^{-x}}{x} == \lim_{x \to 0} \frac{(e^x - 1) - (e^{-x} - 1)}{x} = \lim_{x \to 0} \frac{e^x - 1}{x} = \lim_{x \to 0} \frac{e^x - 1}{x} = 1 - 1 = 0$$

Example 7: Find $\lim_{x \to a} \left(1 + \frac{9}{x}\right)^x$. (Form 1)

Solution: It may be noted that $\frac{x}{9}$ approaches α as x approaches ∞ . i.e. $\lim_{x\to\infty}\frac{x}{9}\to\infty$

$$\lim_{x \to \infty} \left(1 + \frac{9}{x} \right)^x = \lim_{x/9 \to \alpha} \left\{ \left(1 + \frac{1}{\frac{x}{9}} \right)^{x/9} \right\}^9$$

Substitution x / 9 = z the above expression takes the form $\lim_{z \to a} \left\{ \left(1 + \frac{1}{z}\right)^z \right\}^9$

$$= \left\{ \lim_{z \to \infty} \left(1 + \frac{1}{z} \right)^z \right\}^9 = e^9.$$
 Example 8: Evaluate: $\lim_{z \to \infty} \frac{2x+1}{x^3+1}$.

Solution: As x approaches ∞ , 2x + 1 and $x^3 + 1$ both approach ∞ and therefore the given function takes the form $\frac{\infty}{\infty}$ which is determinate. Therefore, instead of evaluating directly let us try for suitable algebraic transformation so that the indeterminate for is avoided.

$$\lim_{x \to \infty} \frac{x^{\frac{2}{2} + \frac{1}{3}}}{1 + x^{\frac{1}{3}}} = \frac{\lim_{x \to \infty} \left(x^{\frac{2}{2} + \frac{1}{3}}\right)}{\lim_{x \to \infty} \left(1 + x^{\frac{1}{3}}\right)} = \frac{\lim_{x \to \infty} x^{\frac{2}{2} + \lim_{x \to \infty} x^{\frac{1}{3}}}}{\lim_{x \to \infty} 1 + \lim_{x \to \infty} x^{\frac{1}{3}}} = \frac{0 + 0}{1 + 0} = \frac{0}{1} = 0$$

Example 9: Find
$$\lim_{x\to\infty} \frac{1^2 + 2^2 + 3^2 + \dots + x^2}{x^3}$$

$$\lim_{x \to \infty} \frac{\left[x(x+)(2x+1)\right]}{6x^3} = \frac{1}{6} \lim_{x \to \infty} \left\{ \left(1 + \frac{1}{x}\right) \left(2 + \frac{1}{x}\right) \right\}$$

$$=\frac{1}{6} \times 1 \times 2 = \frac{1}{3}$$
.



7.9 CONTINUITY:

By the term continuous we mean some thing which goes on without interruption and without abrupt changes. Here in mathematics the term continuous carries the same meaning. Thus, we define continuity of a function in the following way.

A function f(x) is said to be continuous at x=a if and only if

- f(x) is defined x=a
- $\lim_{x \mapsto a^{-}} f(x) = \lim_{x \mapsto a^{+}} f(x)$
- (iii) $\lim_{x \mapsto a} f(x) = f(a)$

In the second condition both left-hand and right-hand limits are equal. In the third condition limiting value of the function must be equal to its function value at x=a

Useful Information:

- The sum, difference and product of two continuous functions is a continuous function. This property holds good for any finite number of functions.
- (ii) The quotient of two continuous functions is continuous function provided the denominator is not equal to zero.

Example:1 f(x) = [

$$= \frac{3}{2} - x \qquad \text{when } \frac{1}{2} < x < 1$$

$$=\frac{1}{2}$$
 when $x=\frac{1}{2}$

Discuss the continuity of f(x) at $x = \frac{1}{2}$

$$\lim_{x \to \frac{1}{2^{-}}} f(x) = \lim_{x \to \frac{1}{2^{+}}} \left(\frac{1}{2} - x \right) = \frac{1}{2} - \frac{1}{2} = 0$$

$$\lim_{x \to \frac{1}{2}^{+}} f(x) = \lim_{x \to \frac{1}{2}^{+}} \left(\frac{3}{2} - x \right) = \frac{3}{2} - \frac{1}{2} = 1$$

Solution: Since LHL ≠ RHL

 $\lim_{x \to 0} f(x)$ does not exist

Moreover f(1/2)

Hence f(x) is not continuous at x=1/2, ie f(x) is discontinuous at x=1/2

Example 2: Find points of discontinuity of the function $f(x) = \frac{x^2 + 2x + 5}{x^2 - 3x + 2}$

Solution:
$$f(x) = \frac{x^2 + 2x + 5}{x^2 - 3x + 2}$$

For x=1 and x=2 the denominator becomes zero and the function f(x) is undefined at x=1 and x=2. Hence the points of discontinuity are at x=1 and x=2

Example 3: A function g(x) is defined as follows

$$g(x) = x \text{ when } 0 < x < 1$$

=2-x when
$$x \ge 1$$

is g(x) is continuous at x=1

Solution:

$$\lim_{x \to 1^-} g(x) = \lim_{x \to 1^+} g(x)$$

$$\lim_{x \to 1+} g(x) = \lim_{x \to 1+} (2 - x) = 2 - 1 = 1$$

$$\therefore \lim_{x \mapsto 1^{-}} g(x) = \lim_{x \mapsto 1^{-}} g(x) = 1$$

Also
$$g(1) = 2-1=1$$

and
$$\lim_{x \to 1} g(x) = g(1) = 1$$

Hence f(x) is coninuous at x=1

Example 4: The function $f(x) = \frac{x^2 - 9}{x - 3}$ is undefined at x=3. What value must be assigned to f (3), if f(x) is to be continuous at x=3

Solution: When x approaches 3, $x \ne 3$ ie $x-3\ne 0$

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{(x-3)(x+3)}{(x-3)}$$
$$= \lim_{x \to 3} (x+3) = 3+3=6$$

Therefore if f(x) is to be continuous at $x=3=f(3)=\lim_{x\to 3}f(x)=6$

Example 5: Is the function f(x) = |x| continuous at x = 0

Solution: We know that |x| = x when x > 0

$$= 0$$
 when $x = 0$

$$= -x$$
 when $x < 0$

$$\lim_{x \to 0^{-}} f(x) = \lim_{x \to 0^{-}} (-x) = 0 \text{ and } \lim_{x \to 0^{+}} f(x) = \lim_{x \to 0^{+}} x = 0$$

Hence
$$\lim_{x \to 0} f(x) = 0 = f(0)$$

So f(x) is continuous at x = 0

ADDITIONAL QUESTION BANK

- 1. Following set notations represent: $A \subset B$; $x \notin A$; $A \supset B$; $\{0\}$; $A \not\subset B$
 - (a) A is a proper subset of B; x is not an element of A; A contains B; singleton with an only element zero; A is not contained in B.
 - (b) A is a proper subset of B; x is an element of A; A contains B; singleton with an only element zero; A is contained in B.
 - (c) A is a proper subset of B; x is not an element of A; A does not contains B; contains elements other than zero; A is not contained in B.
 - (d) None
- 2. Represent the following sets in set notation: Set of all alphabets in English language, set of all odd integers less than 25, set of all odd integers, set of positive integers x satisfying the equation $x^2+5x+7=0$:
 - (a) A = {x:x is an alphabet in English}, I = {x:x is an odd integer>25}, I = {2, 4, 6, 8} I = {x: $x^2+5x+7=0$ }
 - (b) A={x:x is an alphabet in English}, I = {x:x is an odd integer<25}, I = {1, 3, 5, 7} I = {x: $x^2+5x+7=0$ }
 - (c) A = {x:x is an alphabet in English}, I = {x:x is an odd integer £ 25}, I = {1, 3, 5, 7} $I = {x: x^2 + 5x + 7 = 0}$
 - (d) None
- 3. Rewrite the following sets in a set builder form: $A=\{a, e, i, o, u\}$ $B=\{1, 2, 3, 4\}$ C is a set of integers between -15 and 15.
 - (a) $A=\{x:x \text{ is a consonant}\}$, $B=\{x:x \text{ is an irrational number}\}$, $C=\{x:-15 < x < 15 \land x \text{ is a fraction}\}$
 - (b) A={x:x is a vowel}, B={x:x is a natural number}, C={x: $-15^3x^315 \land x$ is a whole number}
 - (c) $A=\{x:x \text{ is a vowel}\}$, $B=\{x:x \text{ is a natural number}\}$, $C=\{x:-15 < x < 15 \land x \text{ is a whole number}\}$
 - (d) None
- 4. If $V=\{0, 1, 2, ...9\}$, $X=\{0, 2, 4, 6, 8\}$, $Y=\{3, 5, 7\}$ and $Z=\{3, 7\}$ then $Y \cup Z$, $(V \cup Y) \cap X$, $(X \cup Z) \cup V$ are respectively:
 - (a) $\{3, 5, 7\}, \{0, 2, 4, 6, 8\}, \{0, 1, 2, ...9\}$ (b) $\{2, 4, 6\}, \{0, 2, 4, 6, 8\}, \{0, 1, 2, ...9\}$
 - (c) {2, 4, 6}, {0, 1, 2, ...9}, {0, 2, 4, 6, 8} (d) None
- 5. In question No. (4) $(X \cup Y) \cap Z$ and $(\phi \cup V) \cap \phi$ are respectively:
 - (a) $\{0, 2, 4, 6, 8\}$, ϕ (b) $\{3, 7\}$, ϕ (c) $\{3, 5, 7\}$, ϕ (d) None

6.		${x:x + 2 = 0}$ R= ${x \in of x \text{ equal to } }$		nd S={x:x²+4x+4	=0} then V, R, S are equal for the	
	(a) 0		(b) -1	(c) – 2	(d) None	
7.	$B=\{x:$				{x:x is a letter in the word <i>flower</i> } word <i>wolf</i> } D={x:x is a letter in the	
	(a) I	B=C=D and all the	ese are subsets of t	he set A		
	(b) I	B=C≠D	(c) B≠C≠D	(d) None		
8.			ctness or otherwise, c, d} (iii) $\{b\} \in \{\{b\}\}$,	g statements: – (i) $\{a, b, c\} = \{c, b, a\}$ and $\phi \subset \{\{b\}\}$.	
	(a) Oı	nly (iv) is incorred	ct	(b) (i) (ii) are in	correct	
	(c) (ii)) (iii) are incorrec	t	(d) All are incom	rrect	
9.	are co	orrect: $-$ (i) $B \subset A$		$E \text{ (iv) } D \subset E \text{ (v) } D$	which of the following statements $D \subset B$ (vi) $D = A$ (vii) $B \not\subset C$ (viii) $E \cap A$	
	(a) (i)	(ii) (iii) (ix) (x) (x	iii) only are correc	t (b) (ii) (iii) (iv) ((x) (xii) (xiii) only are correct	
	(c) (i)	(ii) (iv) (ix) (xi) (x	kiii) only are correc	ct(d) None		
10.	10. Let $A = \{0\}$, $B = \{0\ 1\}$, $C = \emptyset$, $D = \{\emptyset\}$, $E = \{x \mid x \text{ is a human being } 300 \text{ years old}\}$, $F = \{x \mid x \in A \text{ ar } x \in B\}$ state which of the following statements are true: $-(i) A \subset B(ii) B = F(iii) C \subset D(iv) = E(v) A = F(vi) F = 1 \text{ and } (vii) E = C = D$					
	(a) (i)	(iii) (iv) and (v) o	only are true	(b) (i) (ii) (iii) ar	nd (iv) are true	
	(c) (i)	(ii) (iii) and (vi) o	only are true	(d) None		
11.			of the following solution $\{0\} \in A \text{ (vii) } \emptyset$		$ae: -(i) \{1\} \subset A (ii) \{1\} \in A (iii) \phi \in$	
	(a) (i)	(iv) and (vii) onl	y are true	(b) (i) (iv) and (vi) only are true	
	(c) (ii)) (iii) and (vi) onl	y are true	(d) None		
12.	{ <i>y</i> : <i>y</i> =	$=a^2$; a is an integ		a positive integer	ty: $-(i) X = \{1, 2, 3, \dots 500\}$ (ii) $Y = \{x \in A : x \in A \}$ or multiple of 2 (iv) $A : X \in A : x \in A $	
	(a) f	finite infinite infir	nite empty	(b) infinite infir	nite finite empty	
	(c) i	nfinite finite infir	nite empty	(d) None		
13.	If A =	$= \{1, 2, 3, 4\} B = \{2,$	$(3,7,9)$ and $C = \{1$, 4, 7, 9} then		
	(a) A	$A \cap B \neq \phi \ B \cap C \neq$	$\phi A \cap C \neq \phi \text{ but } A$	$A \cap B \cap C = \emptyset$		
		•	$\phi A \cap C = \phi A \cap I$	·		
	` ,	·	$\phi A \cap C \neq \phi A \cap B$	$B \cap C \neq \emptyset$		
	(d) I	None				

14.	If the universal set is $X = \{x: x \in \mathbb{N}, 1 \le x \le 12\}$ and $A = \{1, 9, 10\}$, $B = \{3, 4, 6, 11, 12\}$ and $C = \{2, 5, 6\}$ are subsets of X then set $A \cup (B \cap C)$ is								
	(a) {3, 4, 6, 12}	(b) {1, 6, 9, 10}	(c) {2, 5, 6, 11}	(d) None					
15.	As per question No.(1	.4) the set (A \cup B) \wedge	\cap (A \cup C) is						
	(a) {3, 4, 6, 12}	(b) {1, 6, 9, 10}	(c) {2, 5, 6, 11}	(d) None					
16.	groups < ₹6000/-, 600	00/- to ₹10999/-, ₹1 60, 20, 50 families on	11000/-, to ₹15999/-, ₹ le set is available to 152,	noticed that for income 16000 and above No. TV , 308, 114, 46 families and					
	$C = \{x \mid x \text{ is a family } \}$	with income less that, $E = \{x \mid x \text{ is a family}\}$	an ₹ 6000/-}, D = $\{x \mid x x $ ly with income ₹ 11000	s a family with one set, is a family with income /- to ₹ 15999/-}, find the					
	(ii) $A \cup E$								
	(a) 152, 580	(b) 152, 20	(c) 152, 50	(d) 152, 496					
17.	As per question No.(16) find the number of families in each of the following sets: –								
	(i) $(A \cup B)' \cap E$ (ii) $(C \cup D \cup E) \cap (A \cup B)'$								
	(a) 20, 50	(b) 152, 20	(c) 152, 50	(d) 20, 140					
18.	As per question No.(1	6) express the follo	wing sets in set notatio	n: –					
	(i) {x x is a family with one set and income of less than ₹ 11000/-}								
	(ii) {x x is a family with no set and income over ₹ 16000/-}								
	(a) $(C \cup D) \cap B$		(b) $(A \cup B)' \cap (C \cup D \cup E)'$						
	(c) Both		(d) None						
19.	As per question No.(16) express the following sets in set notation: –								
	(i) $\{x \mid x \text{ is a family with two or more sets or income of } ₹ 11000/- to ₹ 15999/-\}$								
	(ii) $\{x \mid x \text{ is a family with no set}\}$								
	(a) $(A \cup E)$	(b) $(A \cup B)'$	(c) Both	(d) None					
20.	If $A = \{a, b, c, d\}$ list to	he element of powe	r set P (a)						
	(a) {\phi {a} {b}({c} {d} {a}	b} {a, c} {a, d} {b, c	} {b, d} {c, d}						
	(b) {a, b, c} {a, b, d} {a	(b) {a, b, c} {a, b, d} {a, c, d} {b, c, d}							
	(c) {a, b, c, d}								
	(d) All the above elem	nents are in P (a)							

21.	. If four members a, b, c, d of a decision making body are in a meeting to pass a resolution where rule of majority prevails list the wining coalitions. Given that a, b, c, d own 50%, 20%, 15%, 15% shares each.						
	(c) {b, c} {b, d} {c, d} {a,	c, d} {b, c, d} {a} {	{b} {	c} {d} \(\phi \) (d) None			
22.	As per question No.(21) with same order of options (a) (b) (c) and (d) list the blocking conditions.						
23.	As per question No.(22 conditions.	1) with same ord	der o	of options (a) (b) (c) a	nd (d) list the losing		
24.	If $A = \{a, b, c, d, e, f\} B =$	= {a, e, i, o, u} and	1 C =	= {m, n, o, p, q, r, s, t, u	$\}$ then $A \cup B$ is		
	(a) {a, b, c, d, e, f, i, o, u	}	(b)	{a, b, c, i, o, u}			
	(c) {d, e, f, i, o, u}		(d)	None			
25.	As per question No.(24)	$A \cup C$ is					
	(a) {a, b, c, d, e, f, m, n,	o, p, q, r, s, t, u}	(b)	{a, b, c, s, t, u}			
	(c) {d, e, f, p, q, r}		(d)	None			
26.	As per question No.(24)	$B \cup C$ is					
	(a) {a, e, i, o, u, m, n, p,	q, r, s, t}	(b)	{a, e, i, r, s, t}			
	(c) {i, o, u, p, q, r}		(d)	None			
27.	As per question No.(24)	A - B is					
	(a) {b, c, d, f}	(b) {a, e, i, o}		(c) {m, n, p, q}	(d) None		
28.	As per question No.(24)	$A \cap B$ is					
	(a) {a, e}	(b) {i, o}		(c) {o, u}	(d) None		
29.	As per question No.(24)	$B \cap C$ is					
	(a) {a, e}	(b) {i, o}		(c) {o, u}	(d) None		
30.	As per question No.(24)	$A \cup (B - C)$ is					
	(a) {a, b, c, d, e, f, i}	(b) {a, b, c, d, e, f	f, o}	(c) {a, b, c, d, e, f, u}	(d) None		
31.	As per question No.(24)	$A \cup B \cup C$ is					
	(a) {a, b, c, d, e, f, i, o, u		t}	(b) {a, b, c, r, s, t}			
	(c) { d, e, f, n, p, q}			(d) None			
32.	As per question No.(24)	$A \cap B \cap C$ is		. ,			
	-	(b) {a, e}		(c) { m, n}	(d) None		
33.	If A = {3, 4, 5, 6} B = {3, set U = {3, 4,, 11, 12		6, 8,	10, 12, 7} then A' is (gi	ven that the universal		
	(a) {7, 8,12, 13}		(b)	{4, 6, 8, 10,13}			
	(c) { 3, 4, 5, 9, 11, 13}		(d)	None			

- 34. As per question No.(33) with the same order of options (a), (b), (c) and (d) the set B' is 35. As per question No.(33) with the same order of options (a), (b), (c) and (d) the set C' is 36. As per question No.(33) the set (A')' is (a) {3, 4, 5, 6} (b) {3, 7, 9, 5} (c) {8, 10, 11, 12, 13} (d) None 37. As per question No.(33) the set (B')' is (a) {3, 4, 5, 6} (b) {3, 7, 9, 5} (c) {8, 10, 11, 12, 13} (d) None 38. As per question No.(33) the set $(A \cup B)'$ is (b) {3, 7, 9, 5} (a) {3, 4, 5, 6} (c) {8, 10, 11, 12, 13} (d) None 39. As per question No.(33) the set $(A \cap B)'$ is (a) {8, 10, 11, 12, 13} (b) {4, 6, 7,13} (c) {3, 4, 5, 7, 8,....13} (d) None 40. As per question No.(33) the set $A' \cup C'$ is (a) {8, 10, 11, 12, 13} (b) {4, 6, 7,13} (c) {3, 4, 5, 7, 8,....13} (d) None 41. If $A = \{1, 2, ...9\}$, $B = \{2, 4, 6, 8\}$ $C = \{1, 3, 5, 7, 9\}$, $D = \{3, 4, 5\}$ and $E = \{3, 5\}$ what is set S if it is also given that $S \subset D$ and $S \not\subset B$ (a) {3, 5} (b) {2, 4} (c) {7, 9} (d) None 42 As per question No.(41) what is set S if it is also given that $S \subset B$ and $S \not\subset C$ (c) {7, 9} (d) None (a) {3, 5} (b) {2, 4} 43. If $U = \{1, 2, ...9\}$ be the universal set $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6, 8\}$ then the set $A \cup B$ is (a) {1, 2, 3, 4, 6, 8} (b) {2, 4} (c) {5, 6, 7, 8, 9} (d) {5, 7, 9} 44. As per question No.(43) with the same order of options (a) (b) (c) and (d) the set $A \cap B$ is 45. As per question No.(43) with the same order of options (a) (b) (c) and (d) the set A' is 46. As per question No.(43) with the same order of options (a) (b) (c) and (d) the set $(A \cup B)'$ is 47. As per question No.(43) the set $(A \cap B)'$ is (a) {1, 2, 3, 4, 6, 8} (b) {2, 4} (c) {5, 6, 7, 8, 9} (d) {1, 3, 5, 6, 7, 8, 9} 48. Let P = (1, 2, x), Q = (a x y), R = (x, y, z) then $P \times Q$ is (a) {(1, a) (1, x) (1, y); (2, a) (2, x) (2, y); (x, a) (x, x) (x, y)} (b) {(1, x); (1, y); (1, z); (2, x); (2, y); (2, z); (x, x) (x, y) (x, z)} (c) $\{(a, x) (a, y) (a, z); (x, x) (x, y) (x, z); (y, x) (y, y) (y, z)\}$
- 49. As per question No.(48) with the same order of options (a) (b) (c) and (d) then the set $P \times R$ is
- 50. As per question No.(48) with the same order of options (a) (b) (c) and (d) then the set $Q \times R$ is

(d) $\{(1, x) (1, y) (2, x) (2, y) (x, x) (x, y)\}$

51.	As per question No.(48) with the same order of options (a) (b) (c) and (d) then the set $(P \times Q) \cap (P \times R)$ is							
52.	As per question No.(48) the set $(R \times Q) \cap (R \times P)$ is							
	(a) {(a, x), (a, y), (a, z)	, (x, x), (x, y), (x, z),	(y, x), (y, y), (y, z)					
	(b) {(1, x), (1, y), (2, z)	x), (2, y), (x, x), (x, y)	}					
	(c) $\{(x, x), (y, x), (z, x)\}$	x)}						
	(d) {(1, a), (1, x), (1, y), (1, y), (y, x), (z, 1), (z, 2)), (x, a), (x, x), (x, y), (x,	1), (x, 2), (y, 1), (y, 2),				
53.	As per question No. (No.(52) the set $(P \times Q)$		der of options (a) (b) (c)	and (d) as in question				
54.	If P has three elements $P \times Q \times R$ will have	Q four and R two how	w many elements does the	e Cartesian product set				
	(a) 24	(b) 9	(c) 29	(d) None				
55.	Identify the elements (5, 3), (6, 1), (6, 2), (6, 3)		3} and $P \times Q = \{(4, 1), (4, 4), (4, $	2), (4, 3), (5, 1), (5, 2),				
	(a) {3, 4, 5}	(b) {4, 5, 6}	(c) {5, 6, 7}	(d) None				
56.	If $A = \{2, 3\}$, $B = \{4, 5\}$, $C = \{5, 6\}$ then $A \times (B \cup C)$ is							
	(a) {(2, 4), (2, 5), (2, 6)), (3, 4), (3, 5), (3, 6)}						
	(b) $\{(2,5),(3,5)\}$							
	(c) {(2, 4), (2, 5), (3, 4)), (3, 5), (4, 5), (4, 6), (5	5, 5), (5, 6)}					
	(d) None							
57.	As per question No. $A \times (B \cap C)$ is	.(56) with the same	order of options (a) (b	o) (c) and (d) the set				
58.	As per question No.(5 $(B \times C)$ is	6) with the same orde	er of options (a) (b) (c) an	d (d) the set (A \times B) \cup				
59.	If A has 32 elements elements in $A \cap B$	B has 42 elements ar	nd $A \cup B$ has 62 elemen	ts find the number of				
	(a) 74	(b) 62	(c) 12	(d) None				
60.	1 1	_	3,000 read Telegraph and do not read any paper?	23,000 read Times of				
	(a) 3,000	(b) 2,000	(c) 4,000	(d) None				
61.	-		and 64% cocoa. Of the tot coa. Only 6% did none of					
	(a) 360	(b) 280	(c) 160	(d) None				

- 62. As per question No.(61) with the same order of options (a), (b), (c) and (d) find the number having tea and cocoa but not coffee.
- 63. As per question No.(61) with the same order of options (a), (b), (c) and (d) find the number having only coffee.
- 64. Complaints about works canteen had been about Mess (M) Food (F) and Service (S). Total complaints 173 were received as follows: –

$$n(M)=110, n(F)=55, n(S)=67, n(M\cap F\cap S')=20, n(M\cap S\cap F')=11$$
 and $n(F\cap S\cap M')=16.$ Determine the complaints about all the three.

(a) 6 (b) 53

(c)35

(d) None

- 65. As per question No. (64) with the same order of options (a), (b), (c) and (d) determine the complaints about two or more than two.
- 66. Out of total 150 students 45 passed in Accounts 50 in Maths. 30 in Costing 30 in both Accounts and Maths. 32 in both Maths and Costing 35 in both Accounts and Costing. 25 students passed in all the three subjects. Find the number who passed at least in any one of the subjects.

(a) 63

(b) 53

(c)73

(d) None

67. After qualifying out of 400 professionals, 112 joined industry, 120 started practice and 160 joined as paid assistants. There were 32, who were in both practice and service 40 in both practice and assistantship and 20 in both industry and assistantship. There were 12 who did all the three. Find how many could not get any of these.

(a) 88

(b) 244

(c) 122

(d) None

- 68. As per question No. (67) with the same order of options (a) (b) (c) and (d) find how many of them did only one of these.
- 69. A marketing research team interviews 50 people about their drinking habits of tea coffee or milk or A B C respectively. Following data is obtained but the Manager is not sure whether these are consistent.

Category	No.	Category	No.
ABC	3	A	42
AB	7	В	17
BC	13	C	27
AC	18		

- (a) Inconsistent since $42 + 17 + 27 7 13 18 + 3 \neq 50$
- (b) Consistent
- (c) Cannot determine due to data insufficiency
- (d) None

70.	On a survey of 100 boys it was found that 50 used white shirt 40 red and 30 blue. 20 we habituated in using both white and red shirts 15 both red and blue shirts and 10 blue a white shirts. Find the number of boys using all the colours.							
	(a) 20	(b) 25		(c) 30		(6	d) None	
71.	As per question No.(70 boys used all the color				of the white re	te red or blue colours and 20		
	(a) Inconsistent since is used two colours	no. of boys u	sed all t	he thre	e colours can	't more	e then no. of boys	
	(b) Consistent							
	(c) Cannot determine	due to data i	nsufficie	ncy				
	(d) None							
72.	Out of 60 students 25 failed in paper (1), 24 in paper (2), 32 in paper (3), 9 in paper alone, 6 in paper (2) alone, 5 in papers (2) and (3) only and 3 in papers (1) and (2) or Find how many failed in all the three papers.							
	(a) 10	(b) 60		(c) 50		(c	d) None	
73.	As per question No. (7	2) how many	y passed	in all t	he three pape	rs?		
	(a) 10	(b) 60		(c) 50		(c	d) None	
74.	Asked if you will cast	your vote for	a party	the foll	lowing feedba	ck is o	btained: –	
			Yes		No	Don'	't know	
	Adult Male		10		20	5		
	Adult Female		20		15	5		
	Youth over 18 years		10		5	10		
	If A = set of Adult C = set of negative opinion			nen and	Youth $Y = se$	t of po	sitive opinion N =	
	(a) 25	(b) 40		(c) 20		(c	d) None	
75.	As per question No. (74) with the san	ne order	of optio	ns (a), (b), (c) a	and (d)	the set $n(A \cap C)$ is	
76.	As per question No. $n(Y \cup N)'$ is	(74) with the	e same o	order o	of options (a),	(b), (c) and (d) the set	
77.	As per question No. $n[A \cap (Y \cap N)']$ is	(74) with the	e same o	order o	of options (a),	(b), (c) and (d) the set	
78.	In a market survey you its correctness:	have obtained	d the follo	owing d	ata which you	like to	examine regarding	

April

59

May

62

June

62

April &

May

35

May &

June

33

April &

June

31

April May

June

22

Did not use the brand

Percentage answering 'Yes'

- (a) Inconsistent since $59 + 62 + 62 35 33 31 + 22 \neq 100$
- (b) Consistent
- (c) cannot determine due to data insufficiency
- (d) None
- 79. In his report an Inspector of an assembly line showed in respect of 100 units the following which you are require to examine.

Defect	Strength (S)	Flexibility (F)	Radius (R)	S and F	S and R	F and R	SFR
No. of pieces	35	40	18	7	11	12	3

- (a) No. of pieces with radius defect alone was -2 which was impossible
- (b) Report may be accepted
- (c) Cannot be determined due to data insufficiency (d) None
- 80. A survey of 1000 customers revealed the following in respect of their buying habits of different grades:

A grade only	A and C grades		A grade but not B grade	A grade	C and B grades	None
180	80	480	230	360	80	140

How many buy B grade?

(a) 280

- (b) 400
- (c) 50

- (d) None
- 81. As per question No. (80) with the same order of options (a) (b) (c) and (d) how many buy C grade if and only if they do not buy B grade?
- 82. As per question No. (80) with the same order of options (a) (b) (c) and (d) how many buy C and B grades but not the A grade?
- 83. Consider the following data: -

	Skilled & Direct Worker	Unskilled & Direct Worker	Skilled & Indirect Worker	Unskilled & Indirect Worker
Short Term	6	8	10	20
Medium Term	7	10	16	9
Long Term	3	2	8	0

If S M L T I denote short medium long terms skilled and indirect workers respectively find the number of workers in set M.

(a) 42

(b) 8

(c) 10

(d) 43

84. Consider the problem No. (83) and find the number of workers in set L \cap I.

(a) 42

(b) 8

(c) 10

(d) 43

85.	Consider the problem l	No. (83) and find t	he number of workers in se	number of workers in set $S \cap T \cap I$.					
	(a) 42	(b) 8	(c) 10	(d) 43					
86.	Consider the problem	Consider the problem No. (83) and find the number of workers in set							
	$(M \cup L) \cap (T \cup I).$								
	(a) 42	(b) 8	(c) 10	(d) 43					
87.	Consider the problem	Consider the problem No. (83) and find the number of workers in set							
	$S' \cup (S' \cap I)'.$								
	(a) 42	(b) 44	(c) 43	(d) 99					
88.	Consider the problem members. Pair is (S \cup N		t which set of the pair has	more workers as its					
	(a) $(S \cup M)' > L$	(b) $(S \cup M)' < L$	(c) $(S \cup M)' = L$	(d) None					
89.	Consider the problem members. Pair is (I \cap T		t which set of the pair has	more workers as its					
	(a) $(I \cap T)' > [S - (I \cap$	S')]	(b) $(I \cap T)' < [S - (I \cap S')]$						
	(c) $(I \cap T)' = [S - (I \cap$	S')]	(d) None						
90.		oup-II, 372 in grou	gregate, 166 in the aggregate. p-I, 590 in group-II and 12 e.						
	(a) 106	(b) 224	(c) 206	(d) 464					
91.	As per question No.(90) how many failed in the aggregate but not in group-II?								
	(a) 106	(b) 224	(c) 206	(d) 464					
92.	As per question No.(90) how many failed in group-I but not in the aggregate?								
	(a) 106	(b) 224	(c) 206	(d) 464					
93.	As per question No.(90) how many failed in group-II but not in group-I?								
	(a) 106	(b) 224	(c) 206	(d) 464					
94.	As per question No.(90) how many failed	l in aggregate or group-II b	out not in group-I?					
	(a) 206	(b) 464	(c) 628	(d) 164					
95.	As per question No.(90) how many failed	l in aggregate but not in gr	oup-I and group-II?					
	(a) 206	(b) 464	(c) 628	(d) 164					

ANSWERS

1.	(a)	2.	(b)	3.	(c)	4.	(a)	5.	(b)	6.	(c)
7.	(a)	8.	(a)	9.	(a)	10.	(a)	11.	(a)	12.	(a)
13.	(a)	14.	(b)	15.	(b)	16.	(d)	17.	(d)	18.	(c)
19.	(c)	20.	(d)	21.	(a)	22.	(b)	23.	(c)	24.	(a)
25.	(a)	26.	(a)	27.	(a)	28.	(a)	29.	(c)	30.	(a)
31.	(a)	32.	(a)	33.	(a)	34.	(b)	35.	(c)	36.	(a)
37.	(b)	38.	(c)	39.	(b)	40.	(c)	41.	(a)	42.	(b)
43.	(a)	44.	(b)	45.	(c)	46.	(d)	47.	(d)	48.	(a)
49.	(b)	50.	(c)	51.	(d)	52.	(c)	53.	(d)	54.	(a)
55.	(b)	56.	(a)	57.	(b)	58.	(c)	59.	(c)	60.	(a)
61.	(a)	62.	(b)	63.	(c)	64.	(a)	65.	(b)	66.	(b)
67.	(a)	68.	(b)	69.	(a)	70.	(b)	71.	(a)	72.	(a)
73.	(a)	74.	(a)	75.	(b)	76.	(c)	77.	(c)	78.	(a)
79.	(a)	80.	(a)	81.	(b)	82.	(c)	83.	(a)	84.	(b)
85.	(c)	86.	(d)	87.	(d)	88.	(c)	89.	(a)	90.	(a)
91.	(b)	92.	(c)	93.	(d)	94.	(c)	95.	(d)		

NOTES

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