



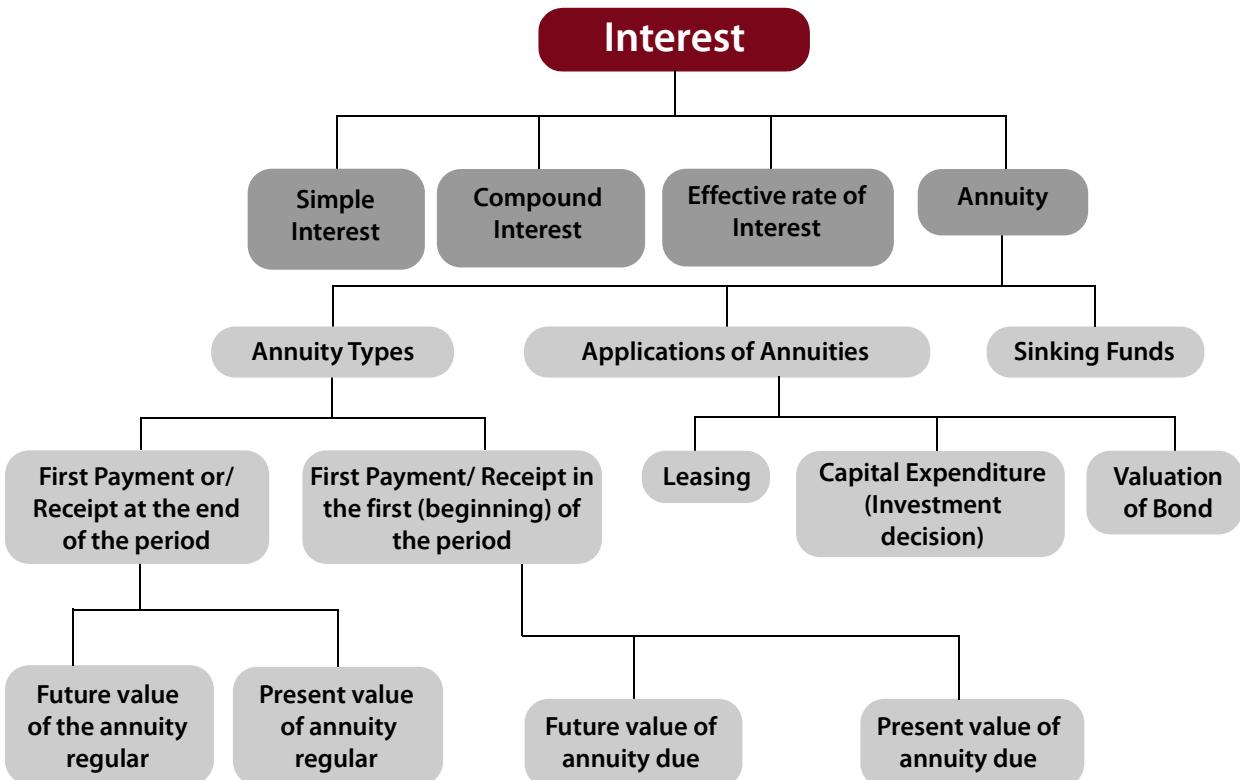
MATHEMATICS OF FINANCE

LEARNING OBJECTIVES

After studying this chapter students will be able to understand:

- ◆ The concept of interest, related terms and computation thereof;
- ◆ Difference between simple and compound interest;
- ◆ The concept of annuity;
- ◆ The concept of present value and future value;
- ◆ Use of present value concept in Leasing, Capital expenditure and Valuation of Bond.

CHAPTER OVERVIEW





4.1 INTRODUCTION

People earn money for spending it on housing, food, clothing, education, entertainment etc. Sometimes extra expenditures have also to be met with. For example there might be a marriage in the family; one may want to buy house, one may want to set up his or her business, one may want to buy a car and so on. Some people can manage to put aside some money for such expected and unexpected expenditures. But most people have to borrow money for such contingencies. From where they can borrow money?

Money can be borrowed from friends or money lenders or Banks. If you can arrange a loan from your friend it might be interest free but if you borrow money from lenders or Banks you will have to pay some charge periodically for using money of money lenders or Banks. This charge is called interest.

Let us take another view. People earn money for satisfying their various needs as discussed above. After satisfying those needs some people may have some savings. People may invest their savings in debentures or lend to other person or simply deposit it into bank. In this way they can earn interest on their investment.

Most of you are very much aware of the term interest. Interest can be defined as the price paid by a borrower for the use of a lender's money.

We will know more about interest and other related terms later.



4.2 WHY IS INTEREST PAID?

Now question arises why lenders charge interest for the use of their money. There are a variety of reasons. We will now discuss those reasons.

- Time value of money:** Time value of money means that the value of a unity of money is different in different time periods. The sum of money received in future is less valuable than it is today. In other words the present worth of money received after some time will be less than money received today. Since money received today has more value rational investors would prefer current receipts to future receipts. If they postpone their receipts they will certainly charge some money i.e. interest.
- Opportunity Cost:** The lender has a choice between using his money in different investments. If he chooses one he forgoes the return from all others. In other words lending incurs an opportunity cost due to the possible alternative uses of the lent money.
- Inflation:** Most economies generally exhibit inflation. Inflation is a fall in the purchasing power of money. Due to inflation a given amount of money buys fewer goods in the future than it will now. The borrower needs to compensate the lender for this.
- Liquidity Preference:** People prefer to have their resources available in a form that can immediately be converted into cash rather than a form that takes time or money to realize.
- Risk Factor:** There is always a risk that the borrower will go bankrupt or otherwise default on the loan. Risk is a determinable factor in fixing rate of interest.

A lender generally charges more interest rate (risk premium) for taking more risk.



4.3 DEFINITION OF INTEREST AND SOME OTHER RELATED TERMS

Now we can define interest and some other related terms.

4.3.1 Interest

Interest is the price paid by a borrower for the use of a lender's money. If you borrow (or lend) some money from (or to) a person for a particular period you would pay (or receive) more money than your initial borrowing (or lending). This excess money paid (or received) is called interest. Suppose you borrow (or lend) ₹ 50,000 for a year and you pay (or receive) ₹ 55,000 after one year the difference between initial borrowing (or lending) ₹ 50,000 and end payment (or receipts) ₹ 55,000 i.e. ₹ 5,000 is the amount of interest you paid (or earned).

4.3.2 Principal

Principal is initial value of lending (or borrowing). If you invest your money the value of initial investment is also called principal. Suppose you borrow (or lend) ₹ 50,000 from a person for one year. ₹ 50,000 in this example is the 'Principal.' Take another example suppose you deposit ₹ 20,000 in your bank account for one year. In this example ₹ 20,000 is the principal.

4.3.3 Rate of Interest

The rate at which the interest is charged for a defined length of time for use of principal generally on a yearly basis is known to be the rate of interest. Rate of interest is usually expressed as percentages. Suppose you invest ₹ 20,000 in your bank account for one year with the interest rate of 5% per annum. It means you would earn ₹ 5 as interest every ₹ 100 of principal amount in a year.

Per annum means for a year.

4.3.4 Accumulated amount (or Balance)

Accumulated amount is the final value of an investment. It is the sum total of principal and interest earned. Suppose you deposit ₹ 50,000 in your bank for one year with an interest rate of 5% p.a. you would earn interest of ₹ 2,500 after one year. (method of computing interest will be illustrated later). After one year you will get ₹ 52,500 (principal+ interest), ₹ 52,500 is accumulated amount here.

Amount is also known as the balance.



4.4 SIMPLE INTEREST AND COMPOUND INTEREST

Now we can discuss the method of computing interest. Interest accrues as either simple interest or compound interest. We will discuss simple interest and compound interest in the following paragraphs:

4.4.1 Simple Interest

Now we would know what is simple interest and the methodology of computing simple interest

and accumulated amount for an investment (principal) with a simple rate over a period of time. As you already know the money that you borrow is known as principal and the additional money that you pay for using somebody else's money is known as interest. The interest paid for keeping ₹ 100 for one year is known as the rate percent per annum. Thus if money is borrowed at the rate of 8% per annum then the interest paid for keeping ₹ 100 for one year is ₹ 8. The sum of principal and interest is known as the Amount.

Clearly the interest you pay is proportionate to the money that you borrow and also to the period of time for which you keep the money; the more the money and the time, the more the interest. Interest is also proportionate to the rate of interest agreed upon by the lending and the borrowing parties. Thus interest varies directly with principal, time and rate.

Simple interest is the interest computed on the principal for the entire period of borrowing. It is calculated on the outstanding principal balance and not on interest previously earned. It means no interest is paid on interest earned during the term of loan.

Simple interest can be computed by applying following formulas:

$$I = Pit$$

$$A = P + I$$

$$= P + Pit$$

$$= P(1 + it)$$

$$I = A - P$$

Here,

A = Accumulated amount (final value of an investment)

P = Principal (initial value of an investment)

i = Annual interest rate in decimal.

I = Amount of Interest

t = Time in years

Let us consider the following examples in order to see how exactly are these quantities related.

Example 1: How much interest will be earned on ₹ 2000 at 6% simple interest for 2 years?

Solution: Required interest amount is given by

$$I = P \times i \times t$$

$$\begin{aligned} &= 2,000 \times \frac{6}{100} \times 2 \\ &= ₹ 240 \end{aligned}$$

Example 2: Sania deposited ₹ 50,000 in a bank for two years with the interest rate of 5.5% p.a. How much interest would she earn?

Solution: Required interest amount is given by

$$I = P \times i \times t$$

$$= ₹ 50,000 \times \frac{5.5}{100} \times 2 \\ = ₹ 5,500$$

Example 3: In example 2 what will be the final value of investment?

Solution: Final value of investment is given by

$$A = P(1 + it) \\ = ₹ 50,000 \left(1 + \frac{5.5}{100} \times 2 \right) \\ = ₹ 50,000 \left(1 + \frac{11}{100} \right) \\ = ₹ \frac{50,000 \times 111}{100} \\ = ₹ 55,500$$

or

$$A = P + I \\ = ₹ (50,000 + 5,500) \\ = ₹ 55,500$$

Example 4: Sachin deposited ₹ 1,00,000 in his bank for 2 years at simple interest rate of 6%. How much interest would he earn? How much would be the final value of deposit?

Solution: (a) Required interest amount is given by

$$I = P \times it \\ = ₹ 1,00,000 \times \frac{6}{100} \times 2 \\ = ₹ 12,000$$

(b) Final value of deposit is given by

$$A = P + I \\ = ₹ (1,00,000 + 12,000) \\ = ₹ 1,12,000$$

Example 5: Find the rate of interest if the amount owed after 6 months is ₹ 1050, borrowed amount being ₹ 1000.

Solution: We know $A = P + Pit$

$$\text{i.e. } 1050 = 1000 + 1000 \times i \times (6/12)$$

- $50 = 500 i$
- $i = 1/10 = 10\%$

Example 6: Rahul invested ₹ 70,000 in a bank at the rate of 6.5% p.a. simple interest rate. He received ₹ 85,925 after the end of term. Find out the period for which sum was invested by Rahul.

Solution: We know $A = P(1+it)$

$$\text{i.e. } 85,925 = 70,000 \left(1 + \frac{6.5}{100} \times t \right)$$

$$\Rightarrow 85,925/70,000 = \frac{100+6.5t}{100}$$

$$\Rightarrow \frac{85,925 \times 100}{70,000} - 100 = 6.5t$$

$$\Rightarrow 22.75 = 6.5t$$

$$\Rightarrow t = 3.5$$

$$\therefore \text{time} = 3.5 \text{ years}$$

Example 7: Kapil deposited some amount in a bank for $7 \frac{1}{2}$ years at the rate of 6% p.a. simple interest. Kapil received ₹ 1,01,500 at the end of the term. Compute initial deposit of Kapil.

Solution: We know $A = P(1+it)$

$$\text{i.e. } 1,01,500 = P \left(1 + \frac{6}{100} \times \frac{15}{2} \right)$$

$$\Rightarrow 1,01,500 = P \left(1 + \frac{45}{100} \right)$$

$$\Rightarrow 1,01,500 = P \left(\frac{145}{100} \right)$$

$$\Rightarrow P = \frac{1,01,500 \times 100}{145}$$

$$= ₹ 70,000$$

$$\therefore \text{Initial deposit of Kapil} = ₹ 70,000$$

Example 8: A sum of ₹ 46,875 was lent out at simple interest and at the end of 1 year 8 months the total amount was ₹ 50,000. Find the rate of interest percent per annum.

Solution: We know $A = P(1+it)$

$$\text{i.e. } 50,000 = 46,875 \left(1 + i \times 1 \frac{8}{12} \right)$$

$$\Rightarrow 50,000 / 46,875 = 1 + \frac{5}{3} i$$

$$\Rightarrow (1.067 - 1) \times 3/5 = i$$

$$\Rightarrow i = 0.04$$

$$\Rightarrow \text{rate} = 4\%$$

Example 9: What sum of money will produce ₹ 28,600 as an interest in 3 years and 3 months at 2.5% p.a. simple interest?

Solution: We know $I = P \times it$

$$\text{i.e. } 28,600 = P \times \frac{2.5}{100} \times 3 \frac{3}{12}$$

$$\Rightarrow 28,600 = \frac{2.5}{100} P \times \frac{13}{4}$$

$$\Rightarrow 28,600 = \frac{32.5}{400} P$$

$$\begin{aligned}\Rightarrow P &= \frac{28,600 \times 400}{32.5} \\ &= ₹ 3,52,000\end{aligned}$$

∴ ₹ 3,52,000 will produce ₹ 28,600 interest in 3 years and 3 months at 2.5% p.a. simple interest

Example 10: In what time will ₹ 85,000 amount to ₹ 1,57,675 at 4.5 % p.a. ?

Solution: We know

$$A = P (1 + it)$$

$$\Rightarrow 1,57,675 = 85,000 \left(1 + \frac{4.5}{100} \times t \right)$$

$$\Rightarrow \frac{1,57,675}{85,000} = \frac{100 + 4.5t}{100}$$

$$\Rightarrow 4.5t = \left[\frac{1,57,675}{85,000} \times 100 \right] - 100$$

$$\Rightarrow t = \frac{85.5}{4.5} = 19$$

∴ In 19 years ₹ 85,000 will amount to ₹ 1,57,675 at 4.5% p.a. simple interest rate.



EXERCISE 4 (A)

Choose the most appropriate option (a) (b) (c) or (d).

1. S.I on ₹ 3,500 for 3 years at 12% p.a. is
 (a) ₹ 1,200 (b) ₹ 1,260 (c) ₹ 2,260 (d) none of these
2. $P = 5,000, R = 15, T = 4 \frac{1}{2}$ using $I = PRT/100$, I will be
 (a) ₹ 3,375 (b) ₹ 3,300 (c) ₹ 3,735 (d) none of these
3. If $P = 5,000, T = 1, I = ₹ 300$, R will be
 (a) 5% (b) 4% (c) 6% (d) none of these
4. If $P = ₹ 4,500, A = ₹ 7,200$, then Simple interest i.e. I will be
 (a) ₹ 2,000 (b) ₹ 3,000 (c) ₹ 2,500 (d) ₹ 2,700
5. $P = ₹ 12,000, A = ₹ 16,500, T = 2 \frac{1}{2}$ years. Rate percent per annum simple interest will be
 (a) 15% (b) 12% (c) 10% (d) none of these
6. $P = ₹ 10,000, I = ₹ 2,500, R = 12 \frac{1}{2}\%$ SI. The number of years T will be
 (a) $1 \frac{1}{2}$ years (b) 2 years (c) 3 years (d) none of these
7. $P = ₹ 8,500, A = ₹ 10,200, R = 12 \frac{1}{2}\%$ SI, t will be.
 (a) 1 yr. 7 mth. (b) 2 yrs. (c) $1 \frac{1}{2}$ yr. (d) none of these
8. The sum required to earn a monthly interest of ₹ 1,200 at 18% per annum SI is
 (a) ₹ 50,000 (b) ₹ 60,000 (c) ₹ 80,000 (d) none of these
9. A sum of money amount to ₹ 6,200 in 2 years and ₹ 7,400 in 3 years. The principal and rate of interest are
 (a) ₹ 3,800, 31.58% (b) ₹ 3,000, 20% (c) ₹ 3,500, 15% (d) none of these
10. A sum of money doubles itself in 10 years. The number of years it would triple itself is
 (a) 25 years. (b) 15 years. (c) 20 years (d) none of these

4.4.2 Compound Interest

We have learnt about the simple interest. We know that if the principal remains the same for the entire period or time then interest is called as simple interest. However in practice the method according to which banks, insurance corporations and other money lending and deposit taking companies calculate interest is different. To understand this method we consider an example :

Suppose you deposit ₹ 50,000 in ICICI bank for 2 years at 7% p.a. compounded annually. Interest will be calculated in the following way:

INTEREST FOR FIRST YEAR

$$I = Pit$$

$$= ₹ 50,000 \times \frac{7}{100} \times 1 = ₹ 3,500$$

INTEREST FOR SECOND YEAR

For calculating interest for second year principal would not be the initial deposit. Principal for calculating interest for second year will be the initial deposit plus interest for the first year. Therefore principal for calculating interest for second year would be

$$= ₹ 50,000 + ₹ 3,500$$

$$= ₹ 53,500$$

$$\begin{aligned}\text{Interest for the second year} &= ₹ 53,500 \times \frac{7}{100} \times 1 \\ &= ₹ 3,745\end{aligned}$$

Total interest = Interest for first year + Interest for second year

$$= ₹ (3,500 + 3,745)$$

$$= ₹ 7,245$$

This interest is ₹ 245 more than the simple interest on ₹ 50,000 for two years at 7% p.a. As you must have noticed this excess in interest is due to the fact that the principal for the second year was more than the principal for first year. The interest calculated in this manner is called compound interest.

Thus we can define the compound interest as the interest that accrues when earnings for each specified period of time added to the principal thus increasing the principal base on which subsequent interest is computed.

Example 11: Saina deposited ₹ 1,00,000 in a nationalized bank for three years. If the rate of interest is 7% p.a., calculate the interest that bank has to pay to Saina after three years if interest is compounded annually. Also calculate the amount at the end of third year.

Solution: Principal for first year ₹ 1,00,000

Interest for first year = Pit

$$\begin{aligned}&= 1,00,000 \times \frac{7}{100} \times 1 \\ &= ₹ 7,000\end{aligned}$$

Principal for the second year = Principal for first year + Interest for first year

$$\begin{aligned}&= ₹ 1,00,000 + ₹ 7,000 \\ &= ₹ 1,07,000\end{aligned}$$

$$\begin{aligned}\text{Interest for second year} &= 1,07,000 \times \frac{7}{100} \times 1 \\ &= ₹ 7,490\end{aligned}$$

$$\begin{aligned}\text{Principal for the third year} &= \text{Principal for second year} + \text{Interest for second year} \\ &= 1,07,000 + 7,490 \\ &= 1,14,490\end{aligned}$$

$$\begin{aligned}\text{Interest for the third year} &= ₹ 1,14,490 \times \frac{7}{100} \times 1 \\ &= ₹ 8,014.30\end{aligned}$$

$$\begin{aligned}\text{Compound interest at the end of third year} &= ₹ (7,000 + 7,490 + 8,014.30) \\ &= ₹ 22,504.30\end{aligned}$$

$$\begin{aligned}\text{Amount at the end of third year} &= \text{Principal (initial deposit)} + \text{compound interest} \\ &= ₹ (1,00,000 + 22,504.30) \\ &= ₹ 1,22,504.30\end{aligned}$$

Difference between simple interest and compound interest

Now we can summarize the main difference between simple interest and compound interest. The main difference between simple interest and compound interest is that in simple interest the principal remains constant throughout whereas in the case of compound interest principal goes on changing at the end of specified period. For a given principal, rate and time the compound interest is generally more than the simple interest.

4.4.3 Conversion period

In the example discussed above the interest was calculated on yearly basis i.e. the interest was compounded annually. However in practice it is not necessary that the interest be compounded annually. For example in banks the interest is often compounded twice a year (half yearly or semi annually) i.e. interest is calculated and added to the principal after every six months. In some financial institutions interest is compounded quarterly i.e. four times a year. The period at the end of which the interest is compounded is called conversion period. When the interest is calculated and added to the principal every six months the conversion period is six months. In this case number of conversion periods per year would be two. If the loan or deposit was for five years then the number of conversion period would be ten.

Typical conversion periods are given below:

Conversion period	Description	Number of conversion period in a year
1 day	Compounded daily	365
1 month	Compounded monthly	12
3 months	Compounded quarterly	4
6 months	Compounded semi annually	2
12 months	Compounded annually	1

4.4.4 Formula for compound interest

Taking the principal as P , the rate of interest per conversion period as i (in decimal), the number of conversion period as n , the accrued amount after n payment periods as A_n we have accrued amount at the end of first payment period

$$A_1 = P + P i = P (1 + i);$$

at the end of second payment period

$$\begin{aligned} A_2 &= A_1 + A_1 i = A_1 (1 + i) \\ &= P (1 + i) (1 + i) \\ &= P (1 + i)^2; \end{aligned}$$

at the end of third payment period

$$\begin{aligned} A_3 &= A_2 + A_2 i \\ &= A_2 (1 + i) \\ &= P(1 + i)^2(1 + i) \\ &= P(1 + i)^3 \end{aligned}$$

$$\begin{aligned} A_n &= A_{n-1} + A_{n-1} i \\ &= A_{n-1} (1 + i) \\ &= P (1 + i)^{n-1} (1 + i) \\ &= P(1 + i)^n \end{aligned}$$

Thus the accrued amount A_n on a principal P after n conversion periods at i (in decimal) rate of interest per conversion period is given by

$$A_n = P (1 + i)^n$$

$$\text{where, } i = \frac{\text{Annual rate of interest}}{\text{Number of conversion periods per year}}$$

$$\begin{aligned} \text{Interest} &= A_n - P = P (1 + i)^n - P \\ &= P [(1+i)^n - 1] \end{aligned}$$

n is total conversions i.e. $t \times$ no. of conversions per year

Note : Computation of a shall be quite simple with a calculator. However compound interest table and tables for at various rates per annum with (a) annual compounding ; (b) monthly compounding and (c) daily compounding are available.

Example 12: ₹ 2,000 is invested at annual rate of interest of 10%. What is the amount after two years if compounding is done (a) Annually (b) Semi-annually (c) Quarterly (d) monthly.

Solution: (a) Compounding is done annually

Here principal $P = ₹ 2,000$; since the interest is compounded yearly the number of conversion periods n in 2 years are 2. Also the rate of interest per conversion period (1 year) i is 0.10

$$\begin{aligned} A_n &= P (1 + i)^n \\ A_2 &= ₹ 2,000 (1 + 0.1)^2 \\ &= ₹ 2,000 \times (1.1)^2 \\ &= ₹ 2,000 \times 1.21 \\ &= ₹ 2,420 \end{aligned}$$

(b) For semiannual compounding

$$n = 2 \times 2 = 4$$

$$i = \frac{0.1}{2} = 0.05$$

$$\begin{aligned} A_4 &= 2,000 (1+0.05)^4 \\ &= 2,000 \times 1.2155 \\ &= ₹ 2,431 \end{aligned}$$

(c) For quarterly compounding

$$n = 4 \times 2 = 8$$

$$i = \frac{0.1}{4} = 0.025$$

$$\begin{aligned} A_8 &= 2,000 (1+ 0.025)^8 \\ &= 2,000 \times 1.2184 \\ &= ₹ 2,436.80 \end{aligned}$$

(d) For monthly compounding

$$n = 12 \times 2 = 24, i = 0.1/12 = 0.00833$$

$$\begin{aligned} A_{24} &= 2,000 (1 + 0.00833)^{24} \\ &= 2,000 \times 1.22029 \\ &= ₹ 2,440.58 \end{aligned}$$

Example 13: Determine the compound amount and compound interest on ₹ 1000 at 6% compounded semi-annually for 6 years. Given that $(1 + i)^n = 1.42576$ for $i = 3\%$ and $n = 12$.

Solution: $i = \frac{0.06}{2} = 0.03; n = 6 \times 2 = 12$

$$P = 1,000$$

$$\begin{aligned}\text{Compound Amount } (A_{12}) &= P(1 + i)^n \\ &= ₹ 1,000(1 + 0.03)^{12} \\ &= 1,000 \times 1.42576 \\ &= ₹ 1,425.76 \\ \text{Compound Interest} &= ₹ (1,425.76 - 1,000) \\ &= ₹ 425.76\end{aligned}$$

Example 14: Compute the compound interest on ₹ 4,000 for 1½ years at 10% per annum compounded half-yearly.

Solution: Here principal $P = ₹ 4,000$. Since the interest is compounded half-yearly the number of conversion periods in 1½ years are 3. Also the rate of interest per conversion period (6 months) is $10\% \times 1/2 = 5\%$ (0.05 in decimal).

Thus the amount A_n (in ₹) is given by

$$\begin{aligned}A_n &= P(1 + i)^n \\ A_3 &= 4,000(1 + 0.05)^3 \\ &= 4,630.50\end{aligned}$$

The compound interest is therefore ₹ (4,630.50 - 4,000)

$$= ₹ 630.50$$

To find the Principal/Time/Rate

The Formula $A_n = P(1 + i)^n$ connects four variables A_n , P , i and n .

Similarly, $C.I. (\text{Compound Interest}) = P[(1+i)^n - 1]$ connects C.I., P , i and n . Whenever three out of these four variables are given the fourth can be found out by simple calculations.

Examples 15: On what sum will the compound interest at 5% per annum for two years compounded annually be ₹ 1,640?

Solution: Here the interest is compounded annually the number of conversion periods in two years are 2. Also the rate of interest per conversion period (1 year) is 5%.

$$n = 2 \quad i = 0.05$$

We know

$$C.I. = P[(1+i)^n - 1]$$

➤ $1,640 = P[(1+0.05)^2 - 1]$

➤ $1,640 = P(1.1025 - 1)$

➤ $P = \frac{1,640}{0.1025} = 16,000$

Hence the required sum is ₹ 16,000.

Example 16: What annual rate of interest compounded annually doubles an investment in 7

years? Given that $2^{\frac{1}{7}} = 1.104090$

Solution: If the principal be P then $A_n = 2P$.

Since $A_n = P(1+i)^n$

➤ $2P = P(1+i)^7$

➤ $2^{1/7} = (1+i)$

➤ $1.104090 = 1+i$

➤ $i = 0.10409$

∴ Required rate of interest = 10.41% per annum

Example 17: In what time will ₹ 8,000 amount to ₹ 8,820 at 10% per annum interest compounded half-yearly?

Solution: Here interest rate per conversion period

$$(i) = \frac{10}{2} \% \\ = 5\% (= 0.05 \text{ in decimal})$$

Principal (P) = ₹ 8,000

Amount (A_n) = ₹ 8,820

We know

$A_n = P(1+i)^n$

➤ $8,820 = 8,000(1+0.05)^n$

➤ $\frac{8,820}{8,000} = (1.05)^n$

➤ $1.1025 = (1.05)^n$

➤ $(1.05)^2 = (1.05)^n$

➤ $n = 2$

Hence number of conversion period is 2 and the required time = $n/2 = 2/2 = 1$ year

Example 18: Find the rate percent per annum if ₹ 2,00,000 amount to ₹ 2,31,525 in $1\frac{1}{2}$ year interest being compounded half-yearly.

Solution: Here $P = ₹ 2,00,000$

Number of conversion period (n) = $1\frac{1}{2} \times 2 = 3$

Amount (A_3) = ₹ 2,31,525

We know that

$$A_3 = P(1+i)^3$$

$$\Rightarrow 2,31,525 = 2,00,000(1+i)^3$$

$$\Rightarrow \frac{2,31,525}{2,00,000} = (1+i)^3$$

$$\Rightarrow 1.157625 = (1+i)^3$$

$$\Rightarrow (1.05)^3 = (1+i)^3$$

$$\Rightarrow i = 0.05$$

i is the Interest rate per conversion period (six months) = $0.05 = 5\%$ &

Interest rate per annum = $5\% \times 2 = 10\%$

Example 19: A certain sum invested at 4% per annum compounded semi-annually amounts to ₹78,030 at the end of one year. Find the sum.

Solution: Here $A_n = 78,030$

$$n = 2 \times 1 = 2$$

$$i = 4 \times 1/2 \% = 2\% = 0.02$$

$$P(\text{in ₹}) = ?$$

We have

$$A_n = P(1+i)^n$$

$$\Rightarrow A_2 = P(1+0.02)^2$$

$$\Rightarrow 78,030 = P(1.02)^2$$

$$\Rightarrow P = \frac{78,030}{(1.02)^2}$$

$$= 75,000$$

Thus the sum invested is ₹ 75,000 at the begining of 1 year.

Example 20: ₹ 16,000 invested at 10% p.a. compounded semi-annually amounts to ₹ 18,522. Find the time period of investment.

Solution: Here $P = ₹ 16,000$

$$A_n = ₹ 18,522$$

$$i = 10 \times 1/2 \% = 5\% = 0.05$$

$$n = ?$$

We have $A_n = P(1 + i)^n$

$$\Rightarrow 18,522 = 16,000(1+0.05)^n$$

$$\Rightarrow \frac{18,522}{16,000} = (1.05)^n$$

$$\Rightarrow (1.157625) = (1.05)^n$$

$$\Rightarrow (1.05)^3 = (1.05)^n$$

$$\Rightarrow n = 3$$

Therefore time period of investment is three half years i.e. $1\frac{1}{2}$ years.

Example 21: A person opened an account on April, 2011 with a deposit of ₹ 800. The account paid 6% interest compounded quarterly. On October 1 2011 he closed the account and added enough additional money to invest in a 6 month time-deposit for ₹ 1,000, earning 6% compounded monthly.

- (a) How much additional amount did the person invest on October 1?
- (b) What was the maturity value of his time deposit on April 1 2012?
- (c) How much total interest was earned?

Given that $(1 + i)^n$ is 1.03022500 for $i=1\frac{1}{2}\% = 0.015$ and $(1 + i)^n$ is 1.03037751 for $i = \frac{1}{2}\% = 0.005$ and $n = 6$.

Solution: (a) The initial investment earned interest for April-June and July- September quarter i.e. for two quarters. In this case $i = 6/4 = 1\frac{1}{2}\% = 0.015$, $n = \frac{6}{12} \times 4 = 2$

$$\begin{aligned} \text{and the compounded amount} &= 800(1 + 0.015)^2 \\ &= 800 \times 1.03022500 \\ &= ₹ 824.18 \end{aligned}$$

$$\begin{aligned} \text{The additional amount invested} &= ₹ (1,000 - 824.18) \\ &= ₹ 175.82 \end{aligned}$$

- (b) In this case the time-deposit earned interest compounded monthly for six months.

$$\begin{aligned} \text{Here } i &= \frac{6}{12} = 1/2\% = (0.005) \quad n = 6 \quad \text{and} \quad P = ₹ 1,000 \\ &= \frac{6}{12} \times 12 \end{aligned}$$

$$\text{Maturity value} = 1,000(1+0.005)^6$$

$$= 1,000 \times 1.03037751 \\ = ₹ 1,030.38$$

(c) Total interest earned = ₹ (24.18 + 30.38) = ₹ 54.56



4.5 EFFECTIVE RATE OF INTEREST

If interest is compounded more than once a year the effective interest rate for a year exceeds the per annum interest rate. Suppose you invest ₹ 10,000 for a year at the rate of 6% per annum compounded semi annually. Effective interest rate for a year will be more than 6% per annum since interest is being compounded more than once in a year. For computing effective rate of interest first we have to compute the interest. Let us compute the interest.

$$\text{Interest for first six months} = ₹ 10,000 \times 6/100 \times 6/12 \\ = ₹ 300$$

Principal for calculation of interest for next six months

$$= \text{Principal for first period one} + \text{Interest for first period} \\ = ₹ (10,000 + 300) \\ = ₹ 10,300$$

$$\text{Interest for next six months} = ₹ 10,300 \times 6/100 \times 6/12 = ₹ 309$$

Total interest earned during the current year

$$= \text{Interest for first six months} + \text{Interest for next six months} \\ = ₹ (300 + 309) = ₹ 609$$

Interest of ₹ 609 can also be computed directly from the formula of compound interest.

We can compute effective rate of interest by following formula

$$I = PEt$$

Where I = Amount of interest

E = Effective rate of interest in decimal

t = Time period

P = Principal amount

Putting the values we have

$$609 = 10,000 \times E \times 1$$

$$\Rightarrow E = \frac{609}{10,000}$$

$$= 0.0609 \text{ or}$$

$$= 6.09\%$$

Thus if we compound the interest more than once a year effective interest rate for the year will be more than actual interest rate per annum. But if interest is compounded annually effective interest rate for the year will be equal to actual interest rate per annum.

So effective interest rate can be defined as the equivalent annual rate of interest compounded annually if interest is compounded more than once in a year.

The effective interest rate can be computed directly by following formula:

$$E = (1 + i)^n - 1$$

Where E is the effective interest rate

i = actual interest rate in decimal

n = number of conversion period

Example 22: ₹ 5,000 is invested in a Term Deposit Scheme that fetches interest 6% per annum compounded quarterly. What will be the interest after one year? What is effective rate of interest?

Solution: We know that

$$I = P [(1+i)^n - 1]$$

Here P = ₹ 5,000

i = 6% p.a. = 0.06 p.a. or 0.015 per quarter

n = 4

and I = amount of compound interest

putting the values we have

$$\begin{aligned} I &= ₹ 5,000 [(1+0.015)^4 - 1] \\ &= ₹ 5,000 \times 0.06136355 \\ &= ₹ 306.82 \end{aligned}$$

For effective rate of interest using $I = PEt$ we find

$$306.82 = 5,000 \times E \times 1.$$

$$\begin{aligned} \Rightarrow E &= \frac{306.82}{5000} \\ &= 0.0613 \text{ or } 6.13\% \end{aligned}$$

Note: We may arrive at the same result by using

$$\begin{aligned} E &= (1+i)^n - 1 \\ \Rightarrow E &= (1 + 0.015)^4 - 1 \\ &= 1.0613 - 1 \\ &= .0613 \text{ or } 6.13\% \end{aligned}$$

We may also note that effective rate of interest is not related to the amount of principal. It is related to the interest rate and frequency of compounding the interest.

Example 23: Find the amount of compound interest and effective rate of interest if an amount of ₹ 20,000 is deposited in a bank for one year at the rate of 8% per annum compounded semi annually.

Solution: We know that

$$I = P [(1+i)^n - 1]$$

here $P = ₹ 20,000$

$$i = 8\% \text{ p.a.} = 8/2 \% \text{ semi annually} = 0.04$$

$$n = 2$$

$$I = ₹ 20,000 [(1+0.04)^2 - 1]$$

$$= ₹ 20,000 \times 0.0816$$

$$= ₹ 1,632$$

Effective rate of interest:

We know that

$$I = PEt$$

$$\triangleright 1,632 = 20,000 \times E \times 1$$

$$\begin{aligned} \triangleright E &= \frac{1632}{20000} = 0.0816 \\ &= 8.16\% \end{aligned}$$

Effective rate of interest can also be computed by following formula

$$\begin{aligned} E &= (1 + i)^n - 1 \\ &= (1 + 0.04)^2 - 1 \\ &= 0.0816 \quad \text{or } 8.16\% \end{aligned}$$

Example 24: Which is a better investment 3% per year compounded monthly or 3.2% per year simple interest? Given that $(1+0.0025)^{12} = 1.0304$.

Solution: $i = 3/12 = 0.25\% = 0.0025$

$$\begin{aligned} n &= 12 \\ E &= (1 + i)^n - 1 \\ &= (1 + 0.0025)^{12} - 1 \\ &= 1.0304 - 1 = 0.0304 \\ &= 3.04\% \end{aligned}$$

Effective rate of interest (E) being less than 3.2%, the simple interest 3.2% per year is the better investment.



EXERCISE 4 (B)

Choose the most appropriate option (a) (b) (c) or (d).

1. If $P = ₹ 1,000$, $R = 5\%$ p.a, $n = 4$; What is Amount and C.I. is

(a) ₹ 1,215.50, ₹ 215.50	(b) ₹ 1,125, ₹ 125
(c) ₹ 2,115, ₹ 115	(d) none of these
2. ₹ 100 will become after 20 years at 5% p.a compound interest of

(a) ₹ 250	(b) ₹ 205	(c) ₹ 165.33	(d) none of these
-----------	-----------	--------------	-------------------
3. The effective rate of interest corresponding to a nominal rate 3% p.a payable half yearly is

(a) 3.2% p.a	(b) 3.25% p.a	(c) 3.0225% p.a	(d) none of these
--------------	---------------	-----------------	-------------------
4. A machine is depreciated at the rate of 20% on reducing balance. The original cost of the machine was ₹ 1,00,000 and its ultimate scrap value was ₹ 30,000. The effective life of the machine is

(a) 4.5 years (appx.)	(b) 5.4 years (appx.)
(c) 5 years (appx.)	(d) none of these
5. If $A = ₹ 1,000$, $n = 2$ years, $R = 6\%$ p.a compound interest payable half-yearly, then principal (P) is

(a) ₹ 888.50	(b) ₹ 885	(c) 800	(d) none of these
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6. The population of a town increases every year by 2% of the population at the beginning of that year. The number of years by which the total increase of population be 40% is

(a) 7 years	(b) 10 years	(c) 17 years (app)	(d) none of these
-------------	--------------	--------------------	-------------------
7. The difference between C.I and S.I on a certain sum of money invested for 3 years at 6% p.a is ₹ 110.16. The principle is

(a) ₹ 3,000	(b) ₹ 3,700	(c) ₹ 12,000	(d) ₹ 10,000
-------------	-------------	--------------	--------------
8. The useful life of a machine is estimated to be 10 years and cost ₹ 10,000. Rate of depreciation is 10% p.a. The scrap value at the end of its life is

(a) ₹ 3,486.78	(b) ₹ 4,383	(c) ₹ 3,400	(d) none of these
----------------	-------------	-------------	-------------------
9. The effective rate of interest corresponding a nominal rate of 7% p.a convertible quarterly is

(a) 7%	(b) 7.5%	(c) 5%	(d) 7.18%
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10. The C.I on ₹ 16000 for $1 \frac{1}{2}$ years at 10% p.a payable half -yearly is

(a) ₹ 2,222	(b) ₹ 2,522	(c) ₹ 2,500	(d) none of these
-------------	-------------	-------------	-------------------
11. The C.I on ₹ 40000 at 10% p.a for 1 year when the interest is payable quarterly is

(a) ₹ 4,000	(b) ₹ 4,100	(c) ₹ 4,152.51	(d) none of these
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4.6 ANNUITY

In many cases you must have noted that your parents have to pay an equal amount of money regularly like every month or every year. For example payment of life insurance premium, rent of your house (if you stay in a rented house), payment of housing loan, vehicle loan etc. In all these cases they pay a constant amount of money regularly. Time period between two consecutive payments may be one month, one quarter or one year.

Sometimes some people received a fixed amount of money regularly like pension, rent of house etc. In all these cases annuity comes into the picture. When we pay (or receive) a fixed amount of money periodically over a specified time period we create an annuity.

Thus annuity can be defined as a sequence of periodic payments (or receipts) regularly over a specified period of time.

There is a special kind of annuity also, that is called Perpetuity. It is one where the receipt or payment takes place forever. Since the payment is forever we cannot compute a future value of perpetuity. However we can compute the present value of the perpetuity. We will discuss later about future value and present value of annuity.

To be called annuity a series of payments (or receipts) must have following features:

- (1) Amount paid (or received) must be constant over the period of annuity and
 - (2) Time interval between two consecutive payments (or receipts) must be the same.

Consider following tables. Can payments/receipts shown in the table for five years be called annuity?

Table - 4.1

Year end	Payments/Receipts (₹)	Year end	Payments/Receipts (₹)
I	5,000	I	5,000
II	6,000	II	5,000
III	4,000	III	–
IV	5,000	IV	5,000
V	7,000	V	5,000

Table - 4.2

Table - 4.3

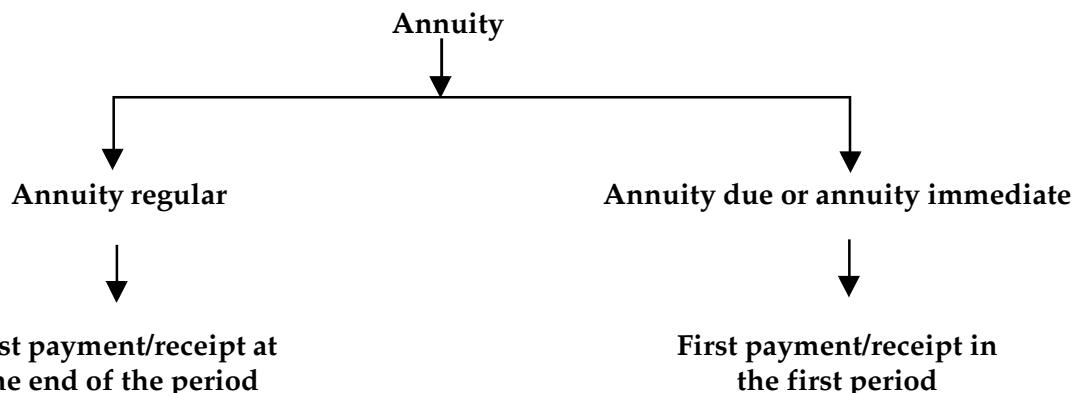
Year end	Payments/Receipts (₹)
I	5,000
II	5,000
III	5,000
IV	5,000
V	5,000

Payments/Receipts shown in table 4.1 cannot be called annuity. Payments/Receipts though have been made at regular intervals but amount paid are not constant over the period of five years.

Payments/receipts shown in table 4.2 cannot also be called annuity. Though amounts paid/received are same in every year but time interval between different payments/receipts is not equal. You may note that time interval between second and third payment/receipt is two year and time interval between other consecutive payments/receipts (first and second third and fourth and fourth and fifth) is only one year. You may also note that for first two years the payments/receipts can be called annuity.

Now consider table 4.3. You may note that all payments/receipts over the period of 5 years are constant and time interval between two consecutive payments/receipts is also same i.e. one year. Therefore payments/receipts as shown in table-4.3 can be called annuity.

4.6.1 Annuity regular and Annuity due/immediate



Annuity may be of two types:

- (1) **Annuity regular:** In annuity regular first payment/receipt takes place at the end of first period. Consider following table:

Table - 4.4

Year end	Payments/Receipts (₹)
I	5,000
II	5,000
III	5,000
IV	5,000
V	5,000

We can see that first payment/receipts takes place at the end of first year therefore it is an annuity regular.

- (2) **Annuity Due or Annuity Immediate:** When the first receipt or payment is made today (at the beginning of the annuity) it is called annuity due or annuity immediate. Consider following table:

Table - 4.5

In the beginning of	Payment/Receipt (₹)
I year	5,000
II year	5,000
III year	5,000
IV year	5,000
V year	5,000

We can see that first receipt or payment is made in the beginning of the first year. This type of annuity is called annuity due or annuity immediate.

4.7 FUTURE VALUE

Future value is the cash value of an investment at some time in the future. It is tomorrow's value of today's money compounded at the rate of interest. Suppose you invest ₹ 1,000 in a fixed deposit that pays you 7% per annum as interest. At the end of first year you will have ₹ 1,070. This consist of the original principal of ₹ 1,000 and the interest earned of ₹ 70. ₹ 1,070 is the future value of ₹ 1,000 invested for one year at 7%. We can say that ₹ 1000 today is worth ₹ 1070 in one year's time if the interest rate is 7%.

Now suppose you invested ₹ 1,000 for two years. How much would you have at the end of the second year. You had ₹ 1,070 at the end of the first year. If you reinvest it you end up having ₹ $1,070(1+0.07) = ₹ 1144.90$ at the end of the second year. Thus ₹ 1,144.90 is the future value of ₹ 1,000 invested for two years at 7%. We can compute the future value of a single cash flow by applying the formula of compound interest.

We know that

$$A_n = P(1+i)^n$$

Where A = Accumulated amount

n = number of conversion period

i = rate of interest per conversion period in decimal

P = principal

Future value of a single cash flow can be computed by above formula. Replace A by future value (F) and P by single cash flow (C.F.) therefore

$$F = C.F. (1 + i)^n$$

Example 25: You invest ₹ 3000 in a two year investment that pays you 12% per annum. Calculate the future value of the investment.

Solution: We know

$$F = C.F. (1 + i)^n$$

where F = Future value

C.F. = Cash flow = ₹ 3,000

i = rate of interest = 0.12

n = time period = 2

$$F = ₹ 3,000(1+0.12)^2$$

$$= ₹ 3,000 \times 1.2544$$

$$= ₹ 3,763.20$$

4.7.1 Future value of an annuity regular

Now we can discuss how do we calculate future value of an annuity.

Suppose a constant sum of ₹ 1 is deposited in a savings account at the end of each year for four years at 6% interest. This implies that ₹ 1 deposited at the end of the first year will grow for three years, ₹ 1 at the end of second year for 2 years, ₹ 1 at the end of the third year for one year and ₹ 1 at the end of the fourth year will not yield any interest. Using the concept of compound interest we can compute the future value of annuity. The compound value (compound amount) of ₹ 1 deposited in the first year will be

$$\begin{aligned} A_3 &= ₹ 1 (1 + 0.06)^3 \\ &= ₹ 1.191 \end{aligned}$$

The compound value of ₹ 1 deposited in the second year will be

$$\begin{aligned} A_2 &= ₹ 1 (1 + 0.06)^2 \\ &= ₹ 1.124 \end{aligned}$$

The compound value of ₹ 1 deposited in the third year will be

$$\begin{aligned} A_1 &= ₹ 1 (1 + 0.06)^1 \\ &= ₹ 1.06 \end{aligned}$$

and the compound value of ₹ 1 deposited at the end of fourth year will remain ₹ 1.

The aggregate compound value of ₹ 1 deposited at the end of each year for four years would be:

$$₹ (1.191 + 1.124 + 1.060 + 1.00) = ₹ 4.375$$

This is the compound value of an annuity of ₹ 1 for four years at 6% rate of interest.

The above computation is summarized in the following table:

Table 4.6

End of year	Amount Deposit (₹)	Future value at the end of fourth year (₹)
0	-	-
1	1	1 (1 + 0.06) ³ = 1.191
2	1	1 (1 + 0.06) ² = 1.124
3	1	1 (1 + 0.06) ¹ = 1.060
4	1	1 (1 + 0.06) ⁰ = 1
	Future Value	4.375

The computation shown in the table can be expressed as follows:

$$A(4, i) = A(1+i)^0 + A(1+i) + A(1+i)^2 + A(1+i)^3$$

$$\text{i.e. } A(4, i) = A [1+(1+i) + (1+i)^2 + (1+i)^3]$$

In above equation A is annuity, A(4, i) is future value at the end of year four, i is the rate of interest shown in decimal.

We can extend above equation for n periods and rewrite as follows:

$$A(n, i) = A(1+i)^0 + A(1+i)^1 + \dots + A(1+i)^{n-2} + A(1+i)^{n-1}$$

$$\text{Here } A = \text{Re.1}$$

Therefore

$$\begin{aligned} A(n, i) &= 1(1+i)^0 + 1(1+i)^1 + \dots + 1(1+i)^{n-2} + 1(1+i)^{n-1} \\ &= 1 + (1+i)^1 + \dots + (1+i)^{n-2} + (1+i)^{n-1} \end{aligned}$$

[a geometric series with first term 1 and common ratio (1+i)]

$$= \frac{1 \cdot [1 - (1+i)^n]}{1 - (1+i)}$$

$$= \frac{1-(1+i)^n}{-i}$$

$$= \frac{(1+i)^n - 1}{i}$$

If A be the periodic payments, the future value $A(n, i)$ of the annuity is given by

$$A(n, i) = A \left[\frac{(1+i)^n - 1}{i} \right]$$

Example 26: Find the future value of an annuity of ₹ 500 made annually for 7 years at interest rate of 14% compounded annually. Given that $(1.14)^7 = 2.5023$.

Solution: Here annual payment $A = ₹ 500$

$$n = 7$$

$$i = 14\% = 0.14$$

Future value of the annuity

$$\begin{aligned} A(7, 0.14) &= 500 \left[\frac{(1+0.14)^7 - 1}{(0.14)} \right] \\ &= \frac{500 \times (2.5023 - 1)}{0.14} \\ &= ₹ 5,365.35 \end{aligned}$$

Example 27: ₹ 200 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the future value of this annuity after 10th payment? Given that $(1.005)^{10} = 1.0511$

Solution: Here $A = ₹ 200$

$$n = 10$$

$$i = 6\% \text{ per annum} = 6/12\% \text{ per month} = 0.005$$

Future value of annuity after 10 months is given by

$$\begin{aligned} A(n, i) &= A \left[\frac{(1+i)^n - 1}{i} \right] \\ A(10, 0.005) &= 200 \left[\frac{(1+0.005)^{10} - 1}{0.005} \right] \\ &= 200 \left[\frac{1.0511 - 1}{0.005} \right] \end{aligned}$$

$$\begin{aligned}
 &= 200 \times 10.22 \\
 &= ₹ 2,044
 \end{aligned}$$

4.7.2 Future value of Annuity due or Annuity Immediate

As we know that in Annuity due or Annuity immediate first receipt or payment is made today. Annuity regular assumes that the first receipt or the first payment is made at the end of first period. The relationship between the value of an annuity due and an ordinary annuity in case of future value is:

Future value of an Annuity due/Annuity immediate = Future value of annuity regular $\times (1+i)$
where i is the interest rate in decimal.

Calculating the future value of the annuity due involves two steps.

- Step-1 Calculate the future value as though it is an ordinary annuity.
- Step-2 Multiply the result by $(1+i)$

Example 28: Z invests ₹ 10,000 every year starting from today for next 10 years. Suppose interest rate is 8% per annum compounded annually. Calculate future value of the annuity. Given that $(1 + 0.08)^{10} = 2.15892500$.

Solution: Step-1: Calculate future value as though it is an ordinary annuity.

Future value of the annuity as if it is an ordinary annuity

$$= ₹ 10,000 \left[\frac{(1+0.08)^{10}-1}{0.08} \right]$$

$$= ₹ 10,000 \times 14.4865625$$

$$= ₹ 1,44,865.625$$

Step-2: Multiply the result by $(1+i)$

$$= ₹ 1,44,865.625 \times (1+0.08)$$

$$= ₹ 1,56,454.875$$



4.8 PRESENT VALUE

We have read that future value is tomorrow's value of today's money compounded at some interest rate. We can say present value is today's value of tomorrow's money discounted at the interest rate. Future value and present value are related to each other in fact they are the reciprocal of each other. Let's go back to our fixed deposit example. You invested ₹ 1000 at 7% and get ₹ 1,070 at the end of the year. If ₹ 1,070 is the future value of today's ₹ 1000 at 7% then ₹ 1,000 is present value of tomorrow's ₹ 1,070 at 7%. We have also seen that if we invest ₹ 1,000 for two years at 7% per annum we will get ₹ 1,144.90 after two years. It means ₹ 1,144.90 is the future value of today's ₹ 1,000 at 7% and ₹ 1,000 is the present value of ₹ 1,144.90 where time period is two years and rate of interest is 7% per annum. We can get the present value of a cash flow (inflow or outflow) by applying compound interest formula.

The present value P of the amount A_n due at the end of n period at the rate of i per interest period may be obtained by solving for P the below given equation

$$A_n = P(1 + i)^n$$

$$\text{i.e. } P = \frac{A_n}{(1+i)^n}$$

- Computation of P may be simple if we make use of either the calculator or the present value

table showing values of $\frac{1}{(1+i)^n}$ for various time periods/per annum interest rates.

- For positive i the factor $\frac{1}{(1+i)^n}$ is always less than 1 indicating thereby future amount has smaller present value.

Example 29: What is the present value of ₹ 1 to be received after two years compounded annually at 10% interest rate?

Solution: Here $A_n = ₹1$

$$i = 10\% = 0.1$$

$$n = 2$$

$$\begin{aligned} \text{Required present value} &= \frac{A_n}{(1+i)^n} \\ &= \frac{1}{(1+0.1)^2} \\ &= \frac{1}{1.21} = 0.8264 \\ &= ₹ 0.83 \end{aligned}$$

Thus ₹ 0.83 shall grow to ₹ 1 after 2 years at 10% interest rate compounded annually.

Example 30: Find the present value of ₹ 10,000 to be required after 5 years if the interest rate be 9%. Given that $(1.09)^5 = 1.5386$.

Solution: Here $i = 0.09 = 9\%$

$$n = 5$$

$$A_n = 10,000$$

$$\begin{aligned}
 \text{Required present value} &= \frac{A_n}{(1+i)^n} \\
 &= \frac{10,000}{(1+0.09)^5} \\
 &= \frac{10,000}{1.5386} = ₹ 6,499.42
 \end{aligned}$$

4.8.1 Present value of an Annuity regular: We have seen how compound interest technique can be used for computing the future value of an Annuity. We will now see how we compute present value of an annuity. We take an example, Suppose your mom promise you to give you ₹ 1,000 on every 31st December for the next five years. Suppose today is 1st January. How much money will you have after five years from now if you invest this gift of the next five years at 10%? For getting answer we will have to compute future value of this annuity.

But you don't want ₹ 1,000 to be given to you each year. You instead want a lump sum figure today. Will you get ₹ 5,000. The answer is no. The amount that she will give you today will be less than ₹ 5,000. For getting the answer we will have to compute the present value of this annuity. For getting present value of this annuity we will compute the present value of these amounts and then aggregate them. Consider following table:

Table 4.7

Year End	Gift Amount (₹)	Present Value [A _n / (1 + i) ⁿ]
I	1,000	1,000/(1 + 0.1) = 909.091
II	1,000	1,000/(1 + 0.1) ² = 826.446
III	1,000	1,000/(1 + 0.1) ³ = 751.315
IV	1,000	1,000/(1 + 0.1) ⁴ = 683.013
V	1,000	1,000/(1 + 0.1) ⁵ = 620.921
Present Value		= 3790.786

Thus the present value of annuity of ₹ 1,000 for 5 years at 10% is ₹ 3,790.79

It means if you want lump sum payment today instead of ₹ 1,000 every year you will get ₹ 3,790.79.

The above computation can be written in formula form as below.

The present value (V) of an annuity (A) is the sum of the present values of the payments.

$$\therefore V = \frac{A}{(1+i)^1} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \frac{A}{(1+i)^4} + \frac{A}{(1+i)^5}$$

We can extend above equation for n periods and rewrite as follows:

$$V = \frac{A}{(1+i)^1} + \frac{A}{(1+i)^2} + \dots + \frac{A}{(1+i)^{n-1}} + \frac{A}{(1+i)^n} \dots \dots \dots \quad (1)$$

multiplying throughout by $\frac{1}{(1+i)}$ we get

$$\frac{V}{(1+i)} = \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \dots + \frac{A}{(1+i)^n} + \frac{A}{(1+i)^{n+1}} \dots \dots \dots \quad (2)$$

subtracting (2) from (1) we get

$$V - \frac{V}{(1+i)} = \frac{A}{(1+i)^1} - \frac{A}{(1+i)^{n+1}}$$

$$\text{Or} \quad V(1+i) - V = A - \frac{A}{(1+i)^n}$$

$$\text{Or} \quad V_i = A \left[1 - \frac{1}{(1+i)^n} \right]$$

$$\therefore V = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right] = A.P(n, i)$$

$$\text{Where, } P(n, i) = \frac{(1+i)^n - 1}{i(1+i)^n}$$

Consequently $A = \frac{V}{P(n, i)}$ which is useful in problems of amortization.

A loan with fixed rate of interest is said to be amortized if entire principal and interest are paid over equal periods of time by way of sequence of equal payment.

$A = \frac{V}{P(n,i)}$ can be used to compute the amount of annuity if we have present value (V), n the number of time period and/ the rate of interest in decimal.

Suppose your dad purchases a car for ₹ 5,50,000. He gets a loan of ₹ 5,00,000 at 15% p.a. from a Bank and balance 50,000 he pays at the time of purchase. Your dad has to pay whole amount of loan in 12 equal monthly instalments with interest starting from the end of first month.

Now we have to calculate how much money has to be paid at the end of every month. We can compute equal instalment by following formula

$$A = \frac{V}{P(n,j)}$$

Here $V = ₹ 5,00,000$

n = 12

$$i = \frac{0.15}{12} = 0.0125$$

$$P(n, i) = \frac{(1+i)^n - 1}{i(1+i)^n}$$

$$\begin{aligned} P(12, 0.0125) &= \frac{(1+0.0125)^{12} - 1}{0.0125(1+0.0125)^{12}} \\ &= \frac{1.16075452 - 1}{0.0125 \times 1.16075452} \\ &= \frac{0.16075452}{0.01450943} = 11.079 \\ \therefore A &= \frac{5,00,000}{11.079} = ₹ 45130.43 \end{aligned}$$

Therefore your dad will have to pay 12 monthly instalments of ₹ 45,130.43.

Example 31: S borrows ₹ 5,00,000 to buy a house. If he pays equal instalments for 20 years and 10% interest on outstanding balance what will be the equal annual instalment?

Solution: We know

$$A = \frac{V}{P(n, i)}$$

$$\text{Here } V = ₹ 5,00,000$$

$$n = 20$$

$$i = 10\% \text{ p.a.} = 0.10$$

$$\begin{aligned} \therefore A &= \frac{V}{P(n, i)} = ₹ \frac{5,00,000}{P(20, 0.10)} \\ &= ₹ \frac{5,00,000}{8.51356} [\text{P}(20, 0.10) = 8.51356 \text{ from table 2(a)}] \\ &= ₹ 58,729.84 \end{aligned}$$

Example 32: ₹ 5,000 is paid every year for ten years to pay off a loan. What is the loan amount if interest rate be 14% per annum compounded annually?

Solution: $V = A \times P(n, i)$

$$\text{Here } A = ₹ 5,000$$

$$n = 10$$

$$\begin{aligned}
 i &= 0.14 \\
 V &= 5000 \times P(10, 0.14) \\
 &= 5000 \times 5.21611 = ₹ 26,080.55
 \end{aligned}$$

Therefore the loan amount is ₹ 26,080.55

Note: Value of $P(10, 0.14)$ can be seen from table 2(a) or it can be computed by formula derived in preceding paragraph.

Example 33: Y bought a TV costing ₹ 13,000 by making a down payment of ₹ 3000 and agreeing to make equal annual payment for four years. How much would be each payment if the interest on unpaid amount be 14% compounded annually?

Solution: In the present case we have present value of the annuity i.e. ₹ 10,000 (13,000-3,000) and we have to calculate equal annual payment over the period of four years.

We know that

$$V = A.P(n, i)$$

Here $n = 4$ and $i = 0.14$

$$\begin{aligned}
 A &= \frac{V}{P(n, i)} \\
 &= \frac{10,000}{P(4, 0.14)} \\
 &= \frac{10,000}{2.91371} \text{ [from table 2(a)]} \\
 &= ₹ 3,432.05
 \end{aligned}$$

Therefore each payment would be ₹ 3,432.05

4.8.2 Present value of annuity due or annuity immediate

Present value of annuity due/immediate for n years is the same as an annuity regular for $(n-1)$ years plus an initial receipt or payment in beginning of the period. Calculating the present value of annuity due involves two steps.

Step 1: Compute the present value of annuity as if it were a annuity regular for one period short.

Step 2: Add initial cash payment/receipt to the step 1 value.

Example 34: Suppose your mom decides to gift you ₹ 10,000 every year starting from today for the next five years. You deposit this amount in a bank as and when you receive and get 10% per annum interest rate compounded annually. What is the present value of this annuity?

Solution: It is an annuity immediate. For calculating value of the annuity immediate following steps will be followed:

Step 1: Present value of the annuity as if it were a regular annuity for one year less i.e. for four years

$$= ₹ 10,000 \times P(4, 0.10)$$

$$= ₹ 10,000 \times 3.16987$$

$$= ₹ 31,698.70$$

Step 2: Add initial cash deposit to the step 1 value

$$₹ (31,698.70 + 10,000) = ₹ 41,698.70$$



4.9 SINKING FUND

It is the fund credited for a specified purpose by way of sequence of periodic payments over a time period at a specified interest rate. Interest is compounded at the end of every period. Size of the sinking fund deposit is computed from $A = P \cdot A(n, i)$ where A is the amount to be saved, P the periodic payment, n the payment period.

Example 35: How much amount is required to be invested every year so as to accumulate ₹ 300000 at the end of 10 years if interest is compounded annually at 10%?

Solution: Here $A = 3,00,000$

$$n = 10$$

$$i = 0.1$$

Since $A = P \times A(n, i)$

$$300000 = P \cdot A(10, 0.1)$$

$$= P \times 15.9374248$$

$$\therefore P = \frac{3,00,000}{15.9374248} = ₹ 18,823.62$$

This value can also be calculated by the formula of future value of annuity regular.

We know that

$$A(n, i) = A \left[\frac{(1+i)^n - 1}{i} \right]$$

$$300000 = A \left[\frac{(1+0.1)^{10} - 1}{0.1} \right]$$

$$300000 = A \times 15.9374248$$

$$A = \frac{3,00,000}{15.9374248}$$

$$= ₹ 18,823.62$$



4.10 APPLICATIONS

4.10.1 Leasing

Leasing is a financial arrangement under which the owner of the asset (lessor) allows the user of the asset (lessee) to use the asset for a defined period of time (lease period) for a consideration (lease rental) payable over a given period of time. This is a kind of taking an asset on rent. How can we decide whether a lease agreement is favourable to lessor or lessee, it can be seen by following example.

Example 36: ABC Ltd. wants to lease out an asset costing ₹ 3,60,000 for a five year period. It has fixed a rental of ₹ 1,05,000 per annum payable annually starting from the end of first year. Suppose rate of interest is 14% per annum compounded annually on which money can be invested by the company. Is this agreement favourable to the company?

Solution: First we have to compute the present value of the annuity of ₹ 1,05,000 for five years at the interest rate of 14% p.a. compounded annually.

The present value V of the annuity is given by

$$\begin{aligned} V &= A.P(n, i) \\ &= 1,05,000 \times P(5, 0.14) \\ &= 1,05,000 \times 3.43308 = ₹ 3,60,473.40 \end{aligned}$$

which is greater than the initial cost of the asset and consequently leasing is favourable to the lessor.

Example 37: A company is considering proposal of purchasing a machine either by making full payment of ₹ 4,000 or by leasing it for four years at an annual rate of ₹ 1,250. Which course of action is preferable if the company can borrow money at 14% compounded annually?

Solution: The present value V of annuity is given by

$$\begin{aligned} V &= A.P(n, i) \\ &= 1,250 \times P(4, 0.14) \\ &= 1,250 \times 2.91371 = ₹ 3,642.11 \end{aligned}$$

which is less than the purchase price and consequently leasing is preferable.

4.10.2 Capital Expenditure (investment decision)

Capital expenditure means purchasing an asset (which results in outflows of money) today in anticipation of benefits (cash inflow) which would flow across the life of the investment. For taking investment decision we compare the present value of cash outflow and present value of cash inflows. If present value of cash inflows is greater than present value of cash outflows decision should be in the favour of investment. Let us see how do we take capital expenditure (investment) decision.

Example 38: A machine can be purchased for ₹ 50000. Machine will contribute ₹ 12000 per year for the next five years. Assume borrowing cost is 10% per annum compounded annually. Determine whether machine should be purchased or not.

Solution: The present value of annual contribution

$$\begin{aligned} V &= A.P(n, i) \\ &= 12,000 \times P(5, 0.10) \\ &= 12,000 \times 3.79079 \\ &= ₹ 45,489.48 \end{aligned}$$

which is less than the initial cost of the machine. Therefore machine must not be purchased.

Example 39: A machine with useful life of seven years costs ₹ 10,000 while another machine with useful life of five years costs ₹ 8,000. The first machine saves labour expenses of ₹ 1,900 annually and the second one saves labour expenses of ₹ 2,200 annually. Determine the preferred course of action. Assume cost of borrowing as 10% compounded per annum.

Solution: The present value of annual cost savings for the first machine

$$\begin{aligned} &= ₹ 1,900 \times P(7, 0.10) \\ &= ₹ 1,900 \times 4.86842 \\ &= ₹ 9,249.99 \\ &= ₹ 9,250 \end{aligned}$$

Cost of machine being ₹ 10,000 it costs more by ₹ 750 than it saves in terms of labour cost.

The present value of annual cost savings of the second machine

$$\begin{aligned} &= ₹ 2,200 \times P(5, 0.10) \\ &= ₹ 2,200 \times 3.79079 \\ &= ₹ 8,339.74 \end{aligned}$$

Cost of the second machine being ₹ 8,000 effective savings in labour cost is ₹ 339.74. Hence the second machine is preferable.

4.10.3 Valuation of Bond

A bond is a debt security in which the issuer owes the holder a debt and is obliged to repay the principal and interest. Bonds are generally issued for a fixed term longer than one year.

The bond issuer enters into contract with bondholder to pay interest.

Example 40: An investor intends purchasing a three year ₹ 1,000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and the investor requires a rate of return of 14%?

Solution: Present value of the bond

$$\begin{aligned} &= \frac{100}{(1+0.14)^1} + \frac{100}{(1+0.14)^2} + \frac{100}{(1+0.14)^3} + \frac{1,000}{(1+0.14)^3} \\ &= 100 \times 0.87719 + 100 \times 0.769467 + 100 \times 0.674972 + 1,000 \times 0.674972 \\ &= 87.719 + 76.947 + 67.497 + 674.972 \\ &= 907.125 \end{aligned}$$

Thus the purchase value of the bond is ₹ 907.125



4.11 PERPETUITY

Perpetuity is an annuity in which the periodic payments or receipts begin on a fixed date and continue indefinitely or perpetually. Fixed coupon payments on permanently invested (irredeemable) sums of money are prime examples of perpetuities.

The formula for evaluating perpetuity is relatively straight forward. Two points which are important to understand in this regard are:-

- (a) The value of the perpetuity is finite because receipts that are anticipated far in the future have extremely low present value (today's value of the future cash flows).
- (b) Additionally, because the principal is never repaid, there is no present value for the principal.

Therefore, the price of perpetuity is simply the coupon amount over the appropriate discount rate or yield.

4.11.1 Calculation of multi period perpetuity:

The formula for determining the present value of multi-period perpetuity is as follows:

$$PVA_{\infty} = \frac{R}{(1+i)^1} + \frac{R}{(1+i)^2} + \frac{R}{(1+i)^3} + \dots + \frac{R}{(1+i)^n} = \sum_{n=1}^{\infty} \frac{R}{(1+i)^n} = \frac{R}{i}$$

Where:

R = the payment or receipt each period

i = the interest rate per payment or receipt period

Example 41: Ramesh wants to retire and receive ₹ 3,000 a month. He wants to pass this monthly payment to future generations after his death. He can earn an interest of 8% compounded annually. How much will he need to set aside to achieve his perpetuity goal?

Solution:

$$R = ₹ 3,000$$

$$i = 0.08/12 \text{ or } 0.00667$$

Substituting these values in the above formula, we get

$$\begin{aligned} PVA &= \frac{₹ 3,000}{0.00667} \\ &= ₹ 4,49,775 \end{aligned}$$

If he wanted the payments to start today, he must increase the size of the funds to handle the first payment. This is achieved by depositing ₹ 4,52,775 (PV of normal perpetuity + perpetuity received in the beginning = 4,49,775 + 3,000) which provides the immediate payment of

₹ 3,000 and leaves ₹ 4,49,775 in the fund to provide the future ₹ 3,000 payments.

4.11.2 Calculation of Growing Perpetuity:

A stream of cash flows that grows at a constant rate forever is known as growing perpetuity.

The formula for determining the present value of growing perpetuity is as follows:

$$PVA = \frac{R}{(1+i)^1} + \frac{R(1+g)}{(1+i)^2} + \frac{R(1+g)^2}{(1+i)^3} + \dots + \frac{R(1+g)^\infty}{(1+i)^\infty}$$

$$\sum_{n=1}^{\infty} \frac{R(1+g)^{n-1}}{(1+i)^n} = \frac{R}{i-g}$$

Example 42: Assuming that the discount rate is 7% per annum, how much would you pay to receive ₹ 50, growing at 5%, annually, forever?

Solution:

$$PVA = \frac{R}{i-g} = \frac{50}{0.07 - 0.05} = 2,500$$

Where

R = Cash flow stream, i = interest rate or discount rate, g = growth rate in interest

Calculating Rate of Return:

- 1) Calculating the rate of return provides important information that can be used for future investments. For example, if you invested in a stock that showed a substantial gain after several months of performance, you may decide to purchase more of that stock. If the stock showed a continual loss, it may be wise to conduct research to find a better-performing stock.
- 2) Calculating the rate of return is that it allows you to gauge your investment and decision-making skills. Investments that create a gain or profit are great. However, if you continually make investments at a loss, then you may want to change your investment strategies. A great attribute of successful business people is knowing how and when to make investments, as is knowing when to change strategies. With a firm grasp of calculating the rate of return, you can manage and monitor your investments at various stages to determine the outcome of your investments. This leads to a higher level of confidence and the skills necessary to be a savvy investor.

Net Present Value Technique (NPV): The net present value technique is a discounted cash flow method that considers the time value of money in evaluating capital investments. An investment has cash flows throughout its life, and it is assumed that a rupee of cash flow in the early years of an investment is worth more than a rupee of cash flow in a later year.

The net present value method uses a specified discount rate to bring all subsequent net cash inflows after the initial investment to their present values (the time of the initial investment is year 0).

Determining Discount Rate

Theoretically, the discount rate or desired rate of return on an investment is the rate of return the firm would have earned by investing the same funds in the best available alternative investment that has the same risk. Determining the best alternative opportunity available is difficult in practical terms so rather than using the true opportunity cost, organizations often use an alternative measure for the desired rate of return. An organization may establish a minimum rate of return that all capital projects must meet; this minimum could be based on an industry average or the cost of other investment opportunities. Many organizations choose to use the overall cost of capital or Weighted Average Cost of Capital (WACC) that an organization has incurred in raising funds or expects to incur in raising the funds needed for an investment.

The net present value of a project is the amount, in current value of rupees, the investment earns after paying cost of capital in each period.



4.12 NET PRESENT VALUE

Net present value = Present value of net cash inflow – Total net initial investment

Since it might be possible that some additional investment may also be required during the life time of the project then appropriate formula shall be:

Net present value = Present value of cash inflow – Present value of cash outflow

The steps to calculate net present value are:-

1. Determine the net cash inflow in each year of the investment.
2. Select the desired rate of return or discounting rate or Weighted Average Cost of Capital.
3. Find the discount factor for each year based on the desired rate of return selected.
4. Determine the present values of the net cash flows by multiplying the cash flows by respective discount factors of respective period called Present Value (PV) of Cash flows
5. Total the amounts of all PVs of Cash Flows

Decision Rule:

If $NPV > 0$ Accept the Proposal

If $NPV < 0$ Reject the Proposal

Example 43: Compute the net present value for a project with a net investment of ₹ 1,00,000 and net cash flows for year one is ₹ 55,000; for year two is ₹ 80,000 and for year three is ₹ 15,000. Further, the company's cost of capital is 10%?

[PVIF @ 10% for three years are 0.909, 0.826 and 0.751]

Solution:

Year	Net Cash Flows	PVIF @ 10%	Discounted Cash Flows
0	(1,00,000)	1.000	(1,00,000)
1	55,000	0.909	49,995
2	80,000	0.826	66,080
3	15,000	0.751	11,265
Net Present Value			27,340

Recommendation: Since the net present value of the project is positive, the company should accept the project.

4.13 NOMINAL RATE OF RETURN

The nominal rate is the stated interest rate. If a bank pays 5% annually on a savings account, then 5% is the nominal interest rate. So if you deposit ₹ 100 for 1 year, you will receive ₹ 5 in interest. However, that Rs. 5 will probably be worth less at the end of the year than it would have been at the beginning. This is because inflation lowers the value of money. As goods, services, and assets, such as real estate, rise in price.

The nominal interest rate is conceptually the simplest type of interest rate. It is quite simply the stated interest rate of a given bond or loan. It is also defined as a stated interest rate. This interest works according to the simple interest and does not take into account the compounding periods.

Real Rate of Return: The real interest rate is so named because it states the “real” rate that the lender or investor receives after inflation is factored in; that is, the interest rate that exceeds the inflation rate.

A comparison of real and nominal interest rates can therefore be summed up in this equation:

$$\text{Nominal Rate of Return} - \text{Inflation} = \text{Real Rate of Return}$$

$$\text{Nominal Interest Rate} = \text{Real Interest Rate} + \text{Inflation}$$

Effective Rate:

It is the actual equivalent annual rate of interest at which an investment grows in value when interest is credited more often than once a year. If interest is paid m times in a year it can be found by calculating:

$$\left(1 + \frac{i}{m}\right)^m - 1$$

The chief advantage to knowing the difference between nominal, real and effective rates is that it allows consumers to make better decisions about their loans and investments. A loan with frequent compounding periods will be more expensive than one that compounds annually. A

bond that only pays a 1% real interest rate may not be worth investors' time if they seek to grow their assets over time. These rates effectively reveal the true return that will be posted by a fixed-income investment and the true cost of borrowing for an individual or business.

Effective and nominal interest rates allow banks to use the number that looks most advantageous to the consumer. When banks are charging interest, they advertise the nominal rate, which is lower and does not reflect how much interest the consumer would owe on the balance after a full year of compounding. On the other hand, with deposit accounts where banks are paying interest, they generally advertise the effective rate because it is higher than the nominal rate.



4.14 COMPOUND ANNUAL GROWTH RATE (CAGR)

Compound Annual Growth Rate (CAGR) is a business and investing specific term for the smoothed annualized gain of an investment over a given time period. It is not an accounting term, but remains widely used, particularly in growth industries or to compare the growth rates of two investments because CAGR dampens the effect of volatility of periodic returns that can render arithmetic means irrelevant. CAGR is often used to describe the growth over a period of time of some element of the business, for example revenue, units delivered, registered users, etc.

$$\text{CAGR} (t_0, t_n) = \left(\frac{V(t_n)}{V(t_0)} \right)^{\frac{1}{t_n - t_0}} - 1$$

Where $V(t_0)$ = Beginning Period ; $V(t_n)$ = End Period

Example: Suppose the revenues of a company for four years, $V(t)$ in the above formula, have been

Year	2013	2014	2015	2016
Revenues	100	120	160	210

Calculate Compound annual Growth Rate.

Solution:

$$t_n - t_0 = 2016 - 2013 = 3$$

The CAGR revenues over the three-year period from the end of 2013 to the end of 2016 is

$$\text{CAGR} (0, 3) = \left(\frac{210}{100} \right)^{\frac{1}{3}} - 1 = 1.2774 - 1 = 27.74\%$$

Applications: These are some of the common CAGR applications:

- Calculating average returns of investment funds.
- Demonstrating and comparing the performance of investment advisors.
- Comparing the historical returns of stocks with bonds or with a savings account.
- Forecasting future values based on the CAGR of a data series.
- Analyzing and communicating the behavior, over a series of years, of different business measures such as sales, market share, costs, customer satisfaction, and performance.


EXERCISE 4 (C)

Choose the most appropriate option (a) (b) (c) or (d).

1. The present value of an annuity of ₹ 3000 for 15 years at 4.5% p.a CI is
 (a) ₹ 23,809.41 (b) ₹ 32,218.76 (c) ₹ 32,908.41 (d) none of these
2. The amount of an annuity certain of ₹ 150 for 12 years at 3.5% p.a C.I is
 (a) ₹ 2,190.28 (b) ₹ 1,290.28 (c) ₹ 2,180.28 (d) none of these
3. A loan of ₹ 10,000 is to be paid back in 30 equal instalments. The amount of each installment to cover the principal and at 4% p.a CI is
 (a) ₹ 587.87 (b) ₹ 587 (c) ₹ 578.30 (d) none of these
4. $A = ₹ 1,200$ $n = 12$ years $i = 0.08$, $V = ?$

Using the formula $V = \frac{A}{i} \left[1 - \frac{1}{(1+i)^n} \right]$ value of v will be

- (a) ₹ 3,039 (b) ₹ 3,990 (c) ₹ 9,930 (d) 9,043.30
5. $a = ₹ 100$ $n = 10$, $i = 5\%$ find the FV of annuity
 Using the formula $FV = a / \{1 + i\}^n - 1\}$, FV is equal to
 (a) ₹ 1,258 (b) ₹ 2,581 (c) ₹ 1,528 (d) none of these
6. If the amount of an annuity after 25 years at 5% p.a C.I is ₹ 50,000 the annuity will be
 (a) ₹ 1,406.90 (b) ₹ 1,047.62 (c) ₹ 1,146.90 (d) none of these
7. Given annuity of ₹ 100 amounts to ₹ 3137.12 at 4.5% p.a C. I. The number of years will be
 (a) 25 years (appx.) (b) 20 years (appx.) (c) 22 years (d) none of these
8. A company borrows ₹ 10,000 on condition to repay it with compound interest at 5% p.a by annual installments of ₹ 1000 each. The number of years by which the debt will be clear is
 (a) 14.2 years (b) 10 years (c) 12 years (d) none of these
9. Mr. X borrowed ₹ 5,120 at $12 \frac{1}{2}\%$ p.a C.I. At the end of 3 yrs, the money was repaid along with the interest accrued. The amount of interest paid by him is
 (a) ₹ 2,100 (b) ₹ 2,170 (c) ₹ 2,000 (d) none of these
10. Mr. Paul borrows ₹ 20,000 on condition to repay it with C.I. at 5% p.a in annual installments of ₹ 2000 each. The number of years for the debt to be paid off is
 (a) 10 years (b) 12 years (c) 11 years (d) 14.2 years
11. A person invests ₹ 500 at the end of each year with a bank which pays interest at 10% p. a C.I. annually. The amount standing to his credit one year after he has made his yearly investment for the 12th time is.

- (a) ₹ 11,761.36 (b) ₹ 10,000 (c) ₹ 12,000 (d) none of these
12. The present value of annuity of ₹ 5,000 per annum for 12 years at 4% p.a C.I. annually is
 (a) ₹ 46,000 (b) ₹ 46,850 (c) ₹ 15,000 (d) ₹ 46,925.40
13. A person desires to create a fund to be invested at 10% CI per annum to provide for a prize of ₹ 300 every year. Using $V = a/I$ find V and V will be
 (a) ₹ 2,000 (b) ₹ 2,500 (c) ₹ 3,000 (d) none of these



SUMMARY

- ◆ **Time value of money:** Time value of money means that the value of a unity of money is different in different time periods. The sum of money received in future is less valuable than it is today. In other words the present worth of money received after some time will be less than money received today.
- ◆ **Interest:** Interest is the price paid by a borrower for the use of lender's money. If you borrow (or lend) some money from (or to) a person for a particular period you would pay (or receive) more money than your initial borrowing (or lending).
- ◆ **Simple interest:** is the interest computed on the principal for the entire period of borrowing.

$$I = Pit$$

$$A = P + I$$

$$I = A - P$$

Here, A = Accumulated amount (final value of an investment)

P = Principal (initial value of an investment)

i = Annual interest rate in decimal.

I = Amount of Interest

t = Time in years

- ◆ **Compound interest** in the interest that accrues when earnings for each specified period of time added to the principal thus increasing the principal base on which subsequent interest is computed.

Formula for compound interest:

$$A_n = P (1 + i)^n$$

where, i = Annual rate of interest

n = Number of conversion periods per year

$$\text{Interest} = A_n - P = P (1 + i)^n - P$$

n is total conversions i.e. $t \times$ no. of conversions per year

- ◆ **Effective Rate of Interest:** The effective interest rate can be computed directly by following formula:

$$E = (1 + i)^n - 1$$

Where E is the effective interest rate

i = actual interest rate in decimal

n = number of conversion period

- ◆ Future value of a single cash flow can be computed by above formula. Replace A by future value (F) and P by single cash flow (C.F.) therefore

$$F = C.F. \cdot (1 + i)^n$$

- ◆ Annuity can be defined as a sequence of periodic payments (or receipts) regularly over a specified period of time.

Annuity may be of two types:

- (i) **Annuity regular:** In annuity regular first payment/receipt takes place at the end of first period.
- (ii) **Annuity Due or Annuity Immediate:** When the first receipt or payment is made today (at the beginning of the annuity) it is called annuity due or annuity immediate.
- ◆ If A be the periodic payments, the future value $A(n, i)$ of the annuity is given by

$$A(n, i) = A \left[\frac{(1+i)^n - 1}{i} \right]$$

- ◆ Future value of an Annuity due/Annuity immediate = Future value of annuity regular $\times (1+i)$ where i is the interest rate in decimal.
- ◆ The present value P of the amount A_n due at the end of n period at the rate of i per interest period may be obtained by solving for P the below given equation

$$A_n = P(1 + i)^n$$

$$\text{i.e. } P = \frac{A_n}{(1+i)^n}$$

- ◆ **Present value of annuity due or annuity immediate:** Present value of annuity due/immediate for n years is the same as an annuity regular for $(n-1)$ years plus an initial receipt or payment in beginning of the period. Calculating the present value of annuity due involves two steps.

Step 1: Compute the present value of annuity as if it were a annuity regular for one period short.

Step 2: Add initial cash payment/receipt to the step 1 value.

- ◆ **Sinking Fund:** It is the fund credited for a specified purpose by way of sequence of periodic payments over a time period at a specified interest rate. Interest is compounded at the end of every period. Sizeof the sinking fund deposit is computed from $A = P \times A(n, i)$ where A is the amount to be saved P the periodic payment, n the payment period.

◆ **Annuity applications:**

- (a) **Leasing:** Leasing is a financial arrangement under which the owner of the asset (lessor) allows the user of the asset (lessee) to use the asset for a defined period of time (lease period) for a consideration (lease rental) payable over a given period of time. This is a kind of taking an asset on rent.
- (b) **Capital Expenditure (investment decision):** Capital expenditure means purchasing an asset (which results in outflows of money) today in anticipation of benefits (cash inflow) which would flow across the life of the investment.
- (c) **Valuation of Bond:** A bond is a debt security in which the issuer owes the holder a debt and is obliged to repay the principal and interest. Bonds are generally issued for a fixed term longer than one year.

MISCELLANEOUS PROBLEMS

 **EXERCISE 4 (D)**

Choose the most appropriate option (a), (b), (c) or (d).

1. $A = ₹ 5,200$, $R = 5\%$ p.a., $T = 6$ years, P will be
 (a) ₹ 2,000 (b) ₹ 3,880 (c) ₹ 3,000 (d) none of these
2. If $P = 1,000$, $n = 4$ years., $R = 5\%$ p.a then C.I will be
 (a) ₹ 215.50 (b) ₹ 210 (c) ₹ 220 (d) none of these
3. The time in which a sum of money will be double at 5% p.a C.I is
 (a) ₹ 10 years (b) 12 years (c) 14.2 years (d) none of these
4. If $A = ₹ 10,000$, $n = 18$ yrs., $R = 4\%$ p.a C.I, P will be
 (a) ₹ 4,000 (b) ₹ 4,900 (c) ₹ 4,500 (d) 4,936.30
5. The time by which a sum of money would treble it self at 8% p. a C. I is
 (a) 14.28 years (b) 14 years (c) 12 years (d) none of these
6. The present value of an annuity of ₹ 80 a years for 20 years at 5% p.a is
 (a) ₹ 997 (appx.) (b) ₹ 900 (c) ₹ 1,000 (d) none of these
7. A person bought a house paying ₹ 20,000 cash down and ₹ 4,000 at the end of each year for 25 yrs. at 5% p.a. C.I. The cash down price is
 (a) ₹ 75,000 (b) ₹ 76,000 (c) ₹ 76,375.50 (d) none of these.
8. A man purchased a house valued at ₹ 3,00,000. He paid ₹ 2,00,000 at the time of purchase and agreed to pay the balance with interest at 12% per annum compounded half yearly in 20 equal half yearly instalments. If the first instalment is paid after six months from the date of purchase then the amount of each instalment is
 [Given $\log 10.6 = 1.0253$ and $\log 31.19 = 1.494$]
 (a) ₹ 8,718.45 (b) ₹ 8,769.21 (c) ₹ 7,893.13 (d) none of these.

ANSWERS

Exercise 4(a)

1. (b) 2. (a) 3. (c) 4. (d) 5. (a) 6. (b)
 7. (a) 8. (c) 9. (a) 10. (c)

Exercise 4(b)

1. (a) 2. (c) 3. (c) 4. (b) 5. (a) 6. (c)
 7. (d) 8. (a) 9. (d) 10. (b) 11. (c) 12. (d)
 13. (a) 14. (a)

Exercise 4(c)

1. (b) 2. (a) 3. (c) 4. (d) 5. (a) 6. (b)
 7. (b) 8. (a) 9. (b) 10. (d) 11. (a) 12. (d)
 13. (c)

Exercise 4(d)

1. (b) 2. (a) 3. (c) 4. (d) 5. (a) 6. (a)
 7. (c) 8. (a)

ADDITIONAL QUESTION BANK

- The difference between compound and simple interest at 5% per annum for 4 years on ₹ 20,000 is ₹ _____.
 (a) 250 (b) 277 (c) 300 (d) 310
- The compound interest on half-yearly rests on ₹ 10,000 the rate for the first and second years being 6% and for the third year 9% p.a. is ₹ _____.
 (a) 2,200 (b) 2,287 (c) 2,285 (d) None
- The present value of ₹ 10,000 due in 2 years at 5% p.a. compound interest when the interest is paid on yearly basis is ₹ _____.
 (a) 9,070 (b) 9,000 (c) 9,061 (d) None
- The present value of ₹ 10,000 due in 2 years at 5% p.a. compound interest when the interest is paid on half-yearly basis is ₹ _____.
 (a) 9,070 (b) 9,069 (c) 9,061 (d) None
- Johnson left ₹ 1,00,000 with the direction that it should be divided in such a way that his minor sons Tom, Dick and Harry aged 9, 12 and 15 years should each receive equally after attaining the age 25 years. The rate of interest being 3.5%, how much each son receive after getting 25 years old?

- (a) 50,000 (b) 51,994 (c) 52,000 (d) None
6. In how many years will a sum of money double at 5% p.a. compound interest?
(a) 15 years 3 months (b) 14 years 2 months
(c) 14 years 3 months (d) 15 years 2 months
7. In how many years a sum of money trebles at 5% p.a. compound interest payable on half-yearly basis?
(a) 18 years 7 months (b) 18 years 6 months
(c) 18 years 8 months (d) 22 years 3 months
8. A machine depreciates at 10% of its value at the beginning of a year. The cost and scrap value realized at the time of sale being ₹ 23,240 and ₹ 9,000 respectively. For how many years the machine was put to use?
(a) 7 years (b) 8 years (c) 9 years (d) 10 years
9. A machine worth ₹ 4,90,740 is depreciated at 15% on its opening value each year. When its value would reduce to ₹ 2,00,000?
(a) 4 years 6 months (b) 4 years 7 months
(c) 4 years 5 months (d) 5 years 7 months approximately
10. A machine worth ₹ 4,90,740 is depreciated at 15% of its opening value each year. When its value would reduce by 90%?
(a) 11 years 6 months (b) 11 years 7 months
(c) 11 years 8 months (d) 14 years 2 months approximately
11. Alibaba borrows ₹ 6 lakhs Housing Loan at 6% repayable in 20 annual installments commencing at the end of the first year. How much annual payment is necessary.
(a) ₹ 52,420 (b) ₹ 52,419 (c) ₹ 52,310 (d) ₹ 52,320
12. A sinking fund is created for redeeming debentures worth ₹ 5 lakhs at the end of 25 years. How much provision needs to be made out of profits each year provided sinking fund investments can earn interest at 4% p.a.?
(a) ₹ 12,006 (b) ₹ 12,040 (c) ₹ 12,039 (d) ₹ 12,035
13. A machine costs ₹ 5,20,000 with an estimated life of 25 years. A sinking fund is created to replace it by a new model at 25% higher cost after 25 years with a scrap value realization of ₹ 25000. what amount should be set aside every year if the sinking fund investments accumulate at 3.5% compound interest p.a.?
(a) ₹ 16,000 (b) ₹ 16,500 (c) ₹ 16,050 (d) ₹ 16,005
14. Raja aged 40 wishes his wife Rani to have ₹ 40 lakhs at his death. If his expectation of life is another 30 years and he starts making equal annual investments commencing now at 3% compound interest p.a. how much should he invest annually?

- (a) 84,448 (b) 84,450 (c) 84,449 (d) 84,080
15. Appu retires at 60 years receiving a pension of 14,400 a year paid in half-yearly installments for rest of his life after reckoning his life expectation to be 13 years and that interest at 4% p.a. is payable half-yearly. What single sum is equivalent to his pension?
- (a) 1,45,000 (b) 1,44,900 (c) 1,44,800 (d) 1,44,700

ANSWERS

1. (d) 2. (d) 3. (a) 4. (c) 5. (d) 6. (b)
7. (d) 8. (c) 9. (d) 10. (d) 11. (c) 12. (a)
13. (c) 14. (d) 15. (b)

Note : In most cases Loan payment dues are made at the end of the period only, i.e Annuity Regular . Questions based on Annuity applications can be solved using the Annuity Regular method . For knowledge purpose students may try another method *viz.*, Annuity Due.

NOTES