

1 实验课题

下面利用蒙特卡罗方法计算一个冰淇淋的体积和质量.

假设冰淇淋的下部为一锥体而上部为一个半球, 锥面方程为 $z^2 = x^2 + y^2$, 球面方程为 $x^2 + y^2 + (z - 1)^2 = 1$. 完成以下实验:

1. 画出冰淇淋的图形.
2. 分别用确定性方法和蒙特卡罗方法计算冰淇淋的体积, 比较计算结果.
3. 假设冰淇淋的密度函数为 $f(x, y, z) = z \exp[-(x^2 + y^2 + z^2)]$, 分别用确定性方法和蒙特卡罗方法计算冰淇淋的质量, 比较计算结果.

2 图形绘制

2.1 理论分析

2.2 实验过程

3 体积计算

3.1 理论分析

3.2 实验过程

4 质量计算

4.1 理论分析

4.2 实验过程

5 附录

Lab03.m 源代码

```
1 %Lab03.m
2 clear;clc;format long;
3 syms theta r phi;
4 countAll = 1000000;
5 countInIceCream = 0;
6 S = 0;
7 f = @(x,y,z) z*exp(-(x^2+y^2+z^2));
8
9 %draw the figure
10 z1 = @(x,y) sqrt(x.^2+y.^2);
11 figure('name','ice-cream figure');
12 g1 = ...
    ezmesh(z1, [-1/sqrt(2), 1/sqrt(2), -1/sqrt(2), 1/sqrt(2)], 'circ');
13 hold on;
14 z2 = @(x,y) sqrt(1-x.^2-y.^2)+1;
```

```

15 g2 = ...
    ezmesh(z2, [-1/sqrt(2), 1/sqrt(2), -1/sqrt(2), 1/sqrt(2)], 'circ');
16 title('ice-cream');
17 axis equal;
18
19 %generate random points
20 for i = 1:countAll
21     w = [2*rand(1)-1, 2*rand(1)-1, 2*rand(1)];
22     if (w(3)^2 > w(1)^2 + w(2)^2 && w(1)^2 + w(2)^2 + (w(3)-1)^2 < 1)
23         countInIceCream = countInIceCream + 1;
24         S = S + f(w(1), w(2), w(3));
25     end
26 end
27
28 %calculate volume
29 V1 = eval(int(int(int(r*r*sin(theta), r, 0, 2*cos(theta)), ...
30     theta, 0, pi/4), phi, 0, 2*pi));
31 V2 = 8 * countInIceCream / countAll;
32 fprintf('The actual volume of the ice-cream is %.6f.\n\n', V1);
33 fprintf('The estimated volume of the ice-cream is %.6f.\n\n', V2);
34
35
36 %calculate mass
37 M1 = eval(int(int(int(r^3*sin(theta)*cos(theta)*exp(-r^2), ...
38     r, 0, 2*cos(theta)), theta, 0, pi/4), phi, 0, 2*pi));
39 M2 = S / countInIceCream * V2;
40 fprintf('The actual mass of the ice-cream is %.6f.\n\n', M1);
41 fprintf('The estimated mass of the ice-cream is %.6f.\n\n', M2);

```