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PHYS265
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I. Introduction

As you are aware, our company currently operates one of the deepest mines in the world. It is estimated to be roughly 4 kilometers from the surface of the Earth to the bottom of our deepest mineshaft. We aren't aware of the exact depth, however. It was recently proposed to measure that depth by dropping a 1kg test mass down the shaft and measuring the time for it to hit the bottom of the shaft.

This report will dive into this idea and analyse whether it is actually feasible to conduct the test at our mine. We'll assume the shaft is exactly 4km and estimate the fall time under different circumstances. To make our assumptions, we first need to introduce the concepts that we'll be working with.

As you know, an object accelerates with gravity when it is in free fall. However, in an atmosphere (like that of the Earth), the resistance from air hitting the falling object serves to slow it down. This is known as the "drag force" on the object. Eventually, this force equals the force from gravity, and the object reaches its "terminal velocity," where it doesn't get any faster. The gravitational force is also more complex than one might believe. As you fall into the Earth, there is less Earth "below" you, and so the force of gravity actually changes as you fall. This will be taken into account in our later calculations, as well as the geological fact that the density of Earth is not constant. A final concept is known as the Coriolis Force. The rotation of the Earth makes it so that an object in free fall actually seems to move sideways within the shaft. This introduces complications that will be discussed later on.

Throughout this project, we relied heavily on the coding language Python to run simulations, solve equations, and graph our results. We used a variety of programs to do this, which will be elaborated on at each step.

II. Calculation of Fall Time

First, let's consider the test object falling down a 4 km mineshaft with no drag and a constant gravitational force. This is the simplest model and can be answered by solving some of the most basic physics equations, those of Newton's Laws. Doing that gives us an expected fall time of 28.6 seconds. With that basic estimate as a target, we then programmed into Python a set of equations that govern the change in position and velocity over time, again derived from Newton's Laws and including an equation for the drag force mentioned above. For now, we'll ignore the drag force in our calculations. We ran this through the "solve_ivp" program, and it gave us an expected impact time of 28.6 seconds, exactly what we calculated earlier. This confirms for us that the program is working as intended.

Since we are dropping the test object into the Earth, we have to consider the fact that the force of gravity will change as the object falls. We relate the gravity to our distance from the core of the Earth as follows, where g = 9.8m/s/s (typical gravity), r is how far we are from the core, and R_E is the radius of the Earth.

$$F_g = g * \frac{r}{R_E} \tag{1}$$

When we run this through the same program, it gives us a fall time that is only .0015 seconds longer. Given that the 4km mineshaft is only 0.06% of the Earth's radius, it is not surprising that this doesn't impact the results much.

Now we'll implement the drag force. The equation for the drag force is proportional to the square of velocity (under the assumed conditions) times a constant α. This constant varies in different situations and has to be calibrated. To do this, we assume that the terminal velocity of the test mass will be -50 m/s and adjust the value α until this is reflected in our graph of velocity over time. Through this process, we got a value of 0.004. When we add this force to our equations and re-run the solve_ivp program, it gives us dramatically different results.

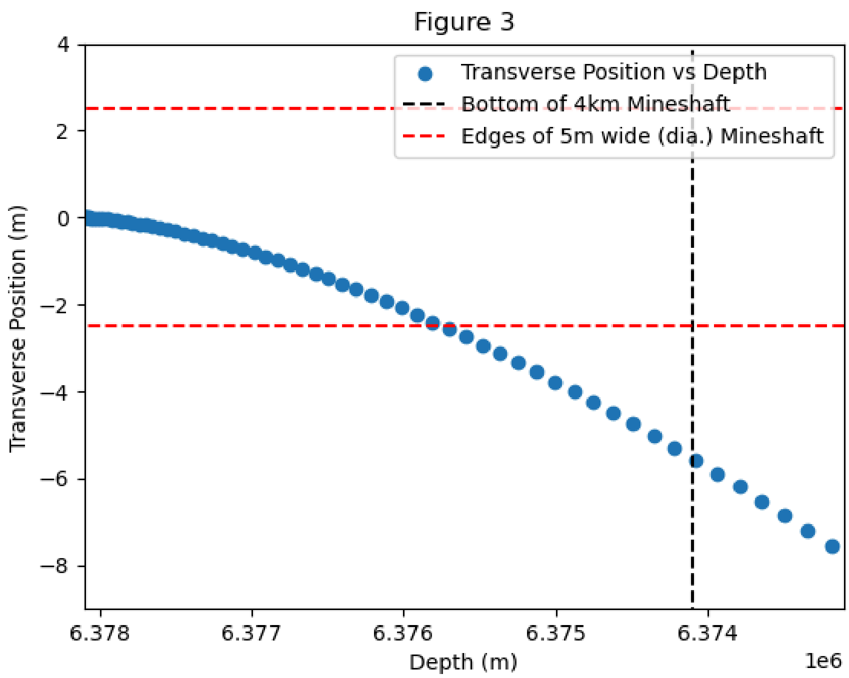
With drag in the equation, the fall time is now estimated to be 84.3 seconds, over triple the original estimate. In Figure 2, we can now see that velocity levels off around -50 m/s as we expected.

While we saw a minimal change in fall time from implementing variable gravity (+0.0015 sec), we saw a massive change from including drag force (+55.7 sec). This tells us that the drag force can't be neglected when we calculate our estimated fall time.

III. Feasibility of Depth Measurement Approach

Another consideration we have to make is the Coriolis force. As our test object falls, it will also travel sideways relative to the mineshaft because of the Earth's rotation. A constraint we haven't mentioned at this point is that our mineshaft is only 5 meters across. Over a fall of 4,000 meters, the Coriolis force might be so much that it causes the object to "drift" into the walls of the shaft, ruining the test.

For this analysis, we input more equations to govern the sideways motion from the Coriolis force and, hence, now have a 2-D set of equations that can tell us where the object will be up and down as well as side-to-side. Running this through the solve_ivp program again and graphing the results gives the following, Figure 3:

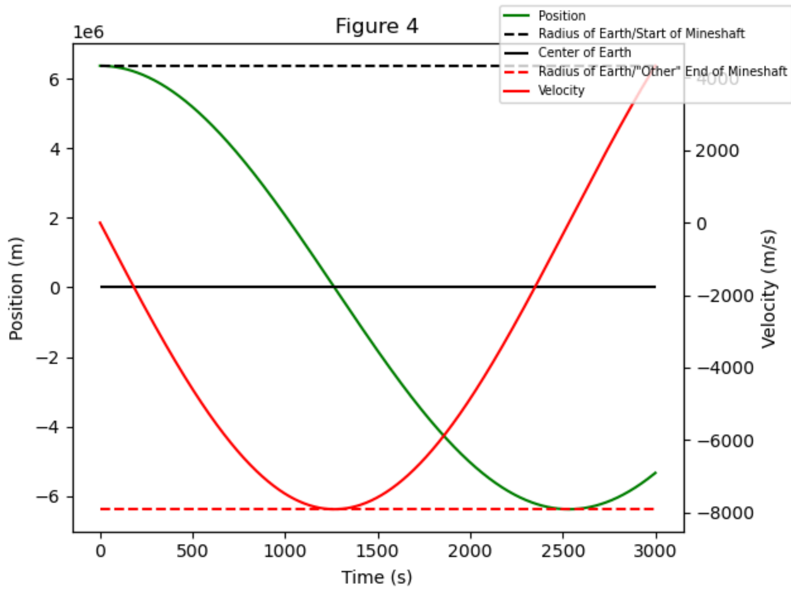


The dashed lines show the edges of the mineshaft in red and the bottom of the mineshaft in black. The transverse (side-to-side) position of the test object progresses left to right with time and can be seen to cross the edge of the mineshaft before reaching the bottom, around 2000 meters down. When this same simulation is run with drag included, this impact occurs at less than 1000 meters down the shaft. This is a problematic result for us because it means that the test object won't make it to the bottom of the shaft without hitting the edge and messing up the timing. Unfortunately, we will have to terminate this project at this point because of this result. However, we will still continue to look at some hypothetical scenarios involving these concepts.

IV. Calculation of Crossing Times for Homogeneous and Non-Homogeneous Earth

To get a better idea of how travelling down towards the center of the Earth changes the forces on an object, we imagined that our mine went straight through the Earth. We'll ignore drag force for these calculations, as well as the Coriolis force, which would cancel out in the end.

To evaluate this, we simply run the same equations we used after implementing Equation (1), but on a much longer time scale to observe the object's behavior as it falls through the Earth. The resulting graph (Figure 4) shows that the object falls to the core (where the green line crosses the solid black line) and then continues with a decelerating velocity and reaches the other side of the Earth before falling back through the Earth.



On this scale, we now need to consider the fact that density isn't constant throughout the Earth. The formula that represents the way density changes throughout the fall is

$$\rho(r) = \rho_n \left(1 - \frac{r^2}{R_E^{\,2}}\right)^n \tag{2}$$

Here ρ_n is a constant that is calculated based on the fact that the mass of the Earth must be conserved, and n changes the extent to which density changes throughout the Earth. If n = 0, density is constant (the assumption we made when we implemented Equation (1), and if n = 9, the most extreme value we will look at, density changes the most extremely. When we implement this equation in our calculations, we find that if n = 0, the object takes 1,267.2 seconds to reach the core and is going -7,906.0 m/s when it does. If we switch to the other extreme of n = 9, the object takes 943.9 seconds to reach the core and is going -18,392.0 m/s when it gets there. It can be clearly seen that the variation in the change in density impacted our results in major ways. The object with an n = 0 fell in >1.25 times as many seconds.

An interesting relationship can be found in this experiment. We see that the time it takes for the object to make it to the other side of Earth, the "crossing time", seems to relate to the orbital period, or the time it would take the object to orbit the planet. By combining a few equations, we get a relationship

$$\tau \sim \frac{1}{\sqrt{Density}} \tag{3}$$

This means that the orbital period, and hence the crossing time, is proportional to 1 over the square root of the Density of the planet. We also took the liberty of evaluating this relationship, assuming the mineshaft went through the moon, and confirmed our hypothesis. The orbital period around the moon is 6,500.5 seconds, and the crossing time is half of that, 1,624.9 seconds.

V. **Discussion and Future Work**

To summarize, we ran through multiple forms of equations to estimate how a 1 kg test object would fall through a 4 km mineshaft to test its depth. A lot of our calculations involved major assumptions, such as neglecting the drag force and the effect of being below the Earth's surface. We brought these back into our calculations one by one, and eventually ran into the issue of the Coriolis force causing the object to impact the sides of the mineshaft (given it is 5 meters wide) before it reached the bottom. That assumption of not including the Coriolis force turned out to be shielding us from a major impracticality, so it is a good thing we were able to identify that issue.

Going forward, we want to have more realistic calculations when we are planning out experiments. To do this, we can be sure that we're including all relevant forces and effects so that we don't gloss over anything that would spoil the real-world test. One thing we could do in this case is add the conclusion we reached in Part IV, that changing the n value in the density equation has a massive effect on fall time. We are disappointed that we won't be able to run the test as planned, but pleased with the insights we gathered from running these simulations.