

# 9-9.3.19

AI24BTECH11010 - Golla Shriram

## Question:

If the area of the region bounded by the curve  $y^2 = 4ax$  and line  $x = 4a$  is  $\frac{256}{3}$  sq.units, then using integration, find the value of  $a$ , where  $a > 0$ .

## Solution:

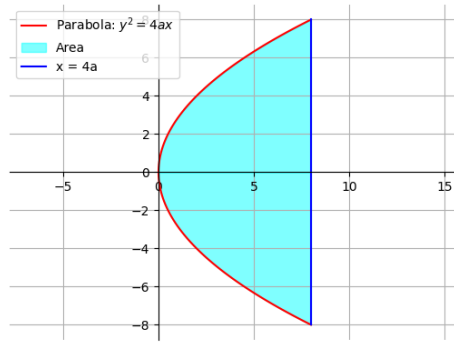


Fig. 0.1: Area bounded by  $y^2 = 4ax$  and  $x = 4a$

The given parabola can be expressed as conics with parameters

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -2a \\ 0 \end{pmatrix}, f = 0 \quad (0.1)$$

For line  $x = 4a$  the parameters are

$$L : \mathbf{x} = \mathbf{h} + \kappa \mathbf{m}, \kappa \in R \quad (0.2)$$

$$\mathbf{h} = \begin{pmatrix} 4a \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (0.3)$$

To points of intersection of line with conic section

$$\mathbf{x}_i = \mathbf{h} + \kappa_i \mathbf{m} \quad (0.4)$$

where

$$\kappa_i = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left( -\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^\top \mathbf{V} \mathbf{m})} \right) \quad (0.5)$$

Substituting from the above , we get

$$\kappa_i = 4a, -4a \quad (0.6)$$

yeilding the points of intersections

$$\mathbf{x}_1 = \begin{pmatrix} 4a \\ 4a \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} 4a \\ -4a \end{pmatrix} \quad (0.7)$$

From figure the area bounded by the curve  $y^2 = 4ax$  and line  $x = 4a$  is given by

$$4\sqrt{a} \int_0^{4a} \sqrt{x} dx = \frac{256}{3} \quad (0.8)$$

by solving above equation the value of  $a = 2$