

Mathematical Induction and Binomial theorem

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I. E - SUBJECTIVE PROBLEMS

- 1) Given that $C_1 + 2C_2x + 3C_3x^2 + \dots + 2nC_{2n}x^{2n-1} = 2n(1+x)^{2n-1}$ where $C_r = \frac{(2n)!}{r!(2n-r)!}$, $r=0,1,2,\dots,2n$.
Prove that $C_1^2 - 2C_2^2 + 3C_3^2 - \dots - 2nC_{2n}^2 = (-1)^n nC_n$. (1979)
- 2) Prove that $7^{2n} + (2^{3n-2})(3^{n-1})$ is divisible by 25 for any natural number n . (1982-5M)
- 3) If $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$ then show that the sum of products of C_i 's taken two at a time, represented by $\sum_{0 \leq i < j \leq n} C_i C_j$ is equal to $2^{2n-1} - \frac{(2n)!}{2(n!)^2}$. (1983 - 3M)
- 4) Use mathematical Induction to prove : If n is any odd positive integer , then $n(n^2 - 1)$ is divisible by 24. (1983 - 2M)
- 5) If p be a natural number then prove that $p^{n+1} + (p+1)^{2n-1}$ is divisible by $p^2 + p + 1$ for every positive integer n . (1984 - 4M)
- 6) Given $s_n = 1 + q + q^2 + \dots + q^n$;
 $S_n = 1 + \frac{q+1}{2} + \left(\frac{q+1}{2}\right)^2 + \dots + \left(\frac{q+1}{2}\right)^n$, $q \neq 1$.
Prove that $^{n+1}C_1 + ^{n+1}C_2s_1 + ^{n+1}C_3s_2 + \dots + ^{n+1}C_ns_n = 2^n S_n$. (1984 - 4M)
- 7) Use method of mathematical Induction $2 \cdot 7^n + 3 \cdot 5^n - 5$ is divisible by 24 for all $n > 0$. (1985 - 5M)
- 8) Prove by mathematical induction that - $\frac{(2n)!}{2^{2n}(n!)^2} \leq \frac{1}{(3n+1)^{\frac{1}{2}}}$ for all positive Integers n . (1987 - 3M)
- 9) Let $R = (5\sqrt{5} + 11)^{2n}$ and $f = R - [R]$, where $[d]$ denotes the greatest integer function. Prove that $Rf = 4^{2n+4}$. (1988 - 5M)
- 10) Using mathematical induction, prove that ${}^mC_0 {}^nC_k + {}^mC_1 {}^nC_{k-1} + \dots + {}^mC_k {}^nC_0 = {}^{m+k}C_k$. (1989 - 3M)
- 11) Prove that $C_0 - 2^2C_1 + 3^2C_2 - \dots + (-1)^n (n+1)^2 C_n = 0$ (1989 - 5M)
, $n > 2$, where $C_r = {}^nC_r$.
- 12) Prove that $\frac{n^7}{7} + \frac{n^5}{5} + \frac{2n^3}{3} - \frac{n}{105}$ is an integer for every positive integer n . (1990 - 2M)
- 13) Using induction or otherwise , prove that for any non-negative integers m, n, r and k ,
$$\sum_{r=0}^k (n-m) \frac{(r+m)!}{m!} = \frac{(r+k+1)!}{k!} \left[\frac{n}{r+1} - \frac{k}{r+2} \right]$$
 (1991 - 4M)
- 14) If $\sum_{r=0}^{2n} a_r (x-2)^r = \sum_{r=0}^{2n} b_r (x-3)^r$ and $a_k = 1$ for all $k \geq n$ then show that $b_n = {}^{2n+1}C_{n+1}$. (1992 - 6M)
- 15) Let $p \leq 3$ be an integer and α, β be the roots of $x^2 - (p+1)x + 1 = 0$ using mathematical induction show that $\alpha^n + \beta^n$ (i) is an integer and (ii) is not divisible by p . (1992 - 6M)