

# Mathematical Induction and Binomial theorem

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## I. E - SUBJECTIVE PROBLEMS

- 1) Given that (1979)  $C_0 - 2^2C_1 + 3^2C_2 - \dots + (-1)^n(n+1)^2C_n = 0$   
 $C_1 + 2C_2x + 3C_3x^2 + \dots + 2nC_{2n}x^{2n-1} = 2n(1+x)^{2n-1}$ ,  $n > 2$ , where  $C_r = {}^nC_r$   
 where  $C_r = \frac{(2n)!}{r!(2n-r)!}$   $r = 0, 1, 2, \dots, 2n$   
 Prove that  
 $C_1^2 - 2C_2^2 + 3C_3^2 - \dots - 2nC_{2n}^2 = (-1)^n n C_n$ .
- 2) Prove that  $7^{2n} + (2^{3n-2})(3^{n-1})$  is divisible by 25 for any natural number  $n$  (1982-5M)
- 3) If  $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$  then show that the sum of products of  $C_i$ 's taken two at a time, represented by  $\sum_{0 \leq i < j \leq n} C_i C_j$  is equal to  $2^{2n-1} - \frac{(2n)!}{2(n!)^2}$  (1983 - 3M)
- 4) Use mathematical Induction to prove : If  $n$  is any odd positive integer , then  $n(n^2 - 1)$  is divisible by 24. (1983 - 2M)
- 5) If  $p$  be a natural number then prove that  $p^{n+1} + (p+1)^{2n-1}$  is divisible by  $p^2 + p + 1$  for every positive integer  $n$ . (1984 - 4M)
- 6) Given  $s_n = 1 + q + q^2 + \dots + q^n$ ;  
 $S_n = 1 + \frac{q+1}{2} + \left(\frac{q+1}{2}\right)^2 + \dots + \left(\frac{q+1}{2}\right)^n$ ,  $q \neq 1$  .  
 Prove that  
 ${}^{n+1}C_1 + {}^{n+1}C_2 s_1 + {}^{n+1}C_3 s_2 + \dots + {}^{n+1}C_n s_n = 2^n S_n$   
 (1984 - 4M)
- 7) Use method of mathematical Induction  
 $2 \cdot 7^n + 3 \cdot 5^n - 5$  is divisible by 24 for all  $n > 0$   
 (1985 - 5M)
- 8) Prove by mathematical induction that -  
 $\frac{(2n)!}{2^{2n}(n!)^2} \leq \frac{1}{(3n+1)^{\frac{1}{2}}}$  for all positive Integers  $n$ .  
 (1987 - 3M)
- 9) Let  $R = (5\sqrt{5} + 11)^{2n}$  and  $f = R - [R]$ , where  $[\ ]$  denotes the greatest integer function. Prove that  $Rf = 4^{2n+4}$  (1988 - 5M)
- 10) Using mathematical induction, prove that  
 ${}^mC_0 {}^nC_k + {}^mC_1 {}^nC_{k-1} + \dots + {}^mC_k {}^nC_0 = {}^{m+k}C_k$   
 (1989 - 3M)
- 11) Prove that (1989 - 5M)  
 $C_0 - 2^2C_1 + 3^2C_2 - \dots + (-1)^n(n+1)^2C_n = 0$
- 12) Prove that  $\frac{n^7}{7} + \frac{n^5}{5} + \frac{2n^3}{3} - \frac{n}{105}$  is an integer for every positive integer  $n$ . (1990 - 2M)
- 13) Using induction or otherwise , prove that for any non-negative integers  $m, n, r$  and  $k$ ,  
 $\sum_{r=0}^k (n-m) \frac{(r+m)!}{m!} = \frac{(r+k+1)!}{k!} \left[ \frac{n}{r+1} - \frac{k}{r+2} \right]$   
 (1991 - 4M)
- 14) If  $\sum_{r=0}^{2n} a_r (x-2)^r = \sum_{r=0}^{2n} b_r (x-3)^r$  and  $a_k = 1$  for all  $k \geq n$  then show that  $b_n = {}^{2n+1}C_{n+1}$  (1992 - 6M)
- 15) Let  $p \leq 3$  be an integer and  $\alpha, \beta$  be the roots of  $x^2 - (p+1)x + 1 = 0$  using mathematical induction show that  $\alpha^n + \beta^n$   
 (i) is an integer and (ii) is not divisible by  $p$  (1992 - 6M)