Straight lines and Pair of Straight Lines

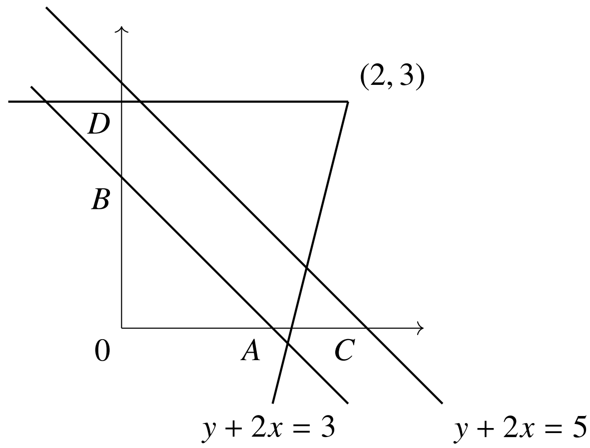
Golla Shriram - AI24BTech11010

I. E - Subjective Problems

4) (1979)
   a) Two vertices of a triangle are $(5, -1)$ and $(2, -3)$. If the orthocentre of the triangle is the origin, find the coordinates of the thrid point.
   b) Find the equation of the line which bisects the obtuse angle between the lines $x - 2y + 4 = 0$ and $4x - 3y + 2 = 0$.
5) A stright line L is perpendicular to the line $5x - y + 1$. The area of the triangle formed by L and the coordinate axes is 5. Find the equation of the line L. (1980)
6) The end **A**, **B** of a straight line segment of constant lenght c slide upon the fixed rectangular axes OX, OY respectively. If the rectangle $OAPB$ be completed, then show that the locus of the foot of perpendicular drawn from **P** to AB is

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = c^{\frac{2}{3}}$$

7) The vertices of a triangle are $\left[at_1t_2, a\left(t_1 + t_2\right)\right]$, $\left[at_2t_3, a\left(t_2 + t_3\right)\right]$, $\left[at_3t_1, a\left(t_3 + t_1\right)\right]$. Find the orthocentre of the triangle. (1983-3M)
8) The coordinates of $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are $(6, 3), (3, 5), (4, 2)$ respectively, and **P** is any point (x, y). Show that the ratio of the area of triangles ΔPBC and ΔABC is $\left|\frac{x+y-2}{7}\right|$ (1983-2M)
9) Two equal sides of an isosceles triangle are given by the equations $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point $(1, -10)$. Determine the equation of the third side. (1985-3M)
10) One of the diameters of the circle circumscribing the rectangle $ABCD$ is $4y = x + 7$. If **A** and **B** are the points $(-3, 4)$ and $(5, 4)$ respectively, the find the area of rectangle. (1985-3M)
11) Two sides of a rhombus $ABCD$ are parallel to the lines $y = x + 2$ and $y = 7x + 3$. If the diagonals of the rhombus intersect at the point $(1, 2)$ and the vertex **A** is on the y-axis, find the possible co-ordinates of **A**. (1985-5M)
12) Lines $L_1 \equiv ax + by + c = 0$ and $L_2 \equiv lx + my + n = 0$ intersect at the point **P** and make an angle θ with each other. Find the equation of a line L different from L_2 which passes through **P** and make same angle θ with L_1. (1988-5M)
13) Let ABC be the triangle $AB = AC$. If **D** is the midpoint of BC, **E** is the foot of the perpendicular drawn from **D** to AC and **F** the mid-point of DE, prove that AF is perpendicular to BE. (1989-5M)
14) Straight lines $3x + 4y = 5$ and $4x - 3y = 15$ intersect at the point **A**. Points **B** and **C** are chosen on these two lines such that $AB = AC$. Determine the possible equations of the line BC passing through the point $(1, 2)$. (1990-4M)
15) A line cuts the x-axis at $\mathbf{A}(7, 0)$ and the y-axis at $\mathbf{B}(0, -5)$. A variable line PQ is drawn perpendicular to AB cutting the x-axis in P and the y-axis in Q. If AQ and BP intersect at **R**, find the locus of **R**. (1990-4M)
16) Find the equation of the line passing through the point $(2, 3)$ and making intercept of length 2 units between the lines $y + 2x = 3$ and $y + 2x = 5$. (1991-4M)



17) Show that all chords of the curve $3x^2 - y^2 - 2x + 4y = 0$, which subtend a right angle at the origin, pass through a fixed point. Find the coordinates of point. (1991-4M)

18) Determine all values of α for which the point (α, α^2) lies inside the triangle formed by the lines. (1992-6M)

$$2x + 3y - 1 = 0$$
$$x + 2y - 3 = 0$$
$$5x - 6y - 1 = 0$$