1-1.2-3 and 1-1.6-14

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Question:
Find the sum of the vectors $\mathbf{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\mathbf{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\mathbf{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.
Solution: The sum of three vectors is $\mathbf{a} + \mathbf{b} + \mathbf{c} =$

$$\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix} + \begin{pmatrix} 1 \\ -6 \\ -7 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \\ -1 \end{pmatrix} \tag{0.1}$$

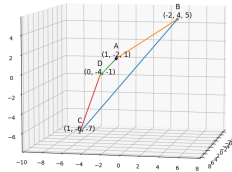Hence , $\mathbf{a} + \mathbf{b} + \mathbf{c} = -4\hat{j} - \hat{k}$

Fig. 0.1: Sum of vectors a, b and c

Question:
Points $\mathbf{A}(3, 1), \mathbf{B}(12, -2)$ and $\mathbf{C}(0, 2)$ cannot be vertices of a triangle.
Solution:
If $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ are collinear then they are not vertices of triangle.
Points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are defined to be collinear if

$$rank(\mathbf{B} - \mathbf{A} \ \ \mathbf{C} - \mathbf{A}) = 1 \qquad (0.2)$$

The collinearity matrix is

$$\begin{pmatrix} 9 & -3 \\ -12 & 4 \end{pmatrix} \xrightarrow{R_2 \leftarrow 3R_2 + 4R_1} \begin{pmatrix} 9 & -3 \\ 0 & 0 \end{pmatrix} \qquad (0.3)$$

By applying row reductions we get rank of matrix as 1. So, $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are collinear.
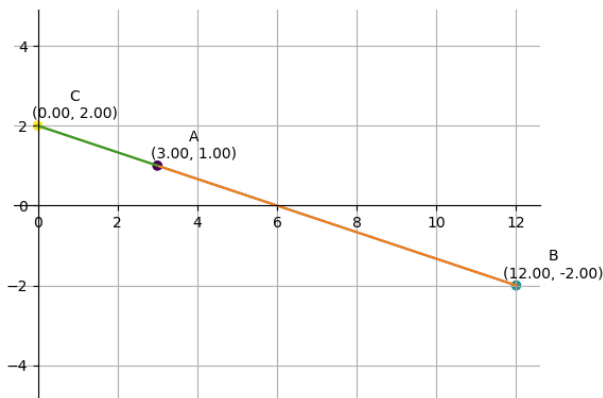

Fig. 0.2: Plot of $\mathbf{A}, \mathbf{B}, \mathbf{C}$