# 9-9.3.19

AI24BTECH11010 - Golla Shriram

**Question**:
If the area of the region bounded by the curve $y^2 = 4ax$ and line $x = 4a$ is $\frac{256}{3}$ sq.units, then using integration, find the value of $a$, where $a > 0$.
**Solution:**

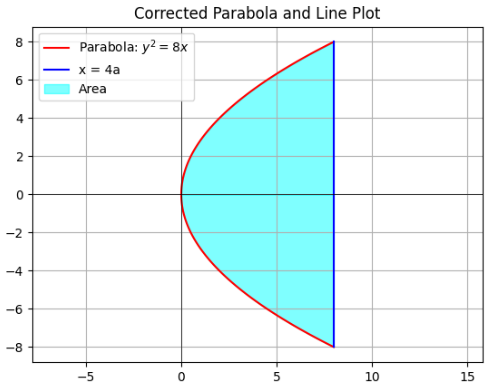


Fig. 0.1: Area bounded by $y^2 = 4ax$ and $x = 4a$

The given parabola can be expressed as conics with parameters

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -2a \\ 0 \end{pmatrix}, f = 0 \tag{0.1}$$

For line $x = 4a$ the parameters are

$$L : \mathbf{x} = \mathbf{h} + \kappa \mathbf{m}, \kappa \in R \tag{0.2}$$

$$\mathbf{h} = \begin{pmatrix} 4a \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{0.3}$$

To points of intersection of line with conic section

$$\mathbf{x_i} = \mathbf{h} + \kappa_i \mathbf{m} \tag{0.4}$$

where

$$\kappa_i = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left( -\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^\top \mathbf{V}\mathbf{m})} \right) \tag{0.5}$$

Substituting from the above , we get

$$\kappa_i = 4a, -4a \tag{0.6}$$

yeilding the points of intersections

$$\mathbf{x_1} = \begin{pmatrix} 4a \\ 4a \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} 4a \\ -4a \end{pmatrix} \tag{0.7}$$

From figure the area bounded by the curve $y^2 = 4ax$ and line $x = 4a$ is given by

$$4\sqrt{a} \int_0^{4a} \sqrt{x}\, dx = \frac{256}{3} \tag{0.8}$$

by solving above equation the value of $a = 2$