Straight lines and Pair of Straight Lines

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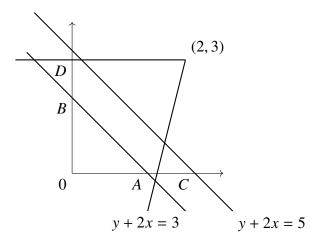### I. E - SUBJECTIVE PROBLEMS

4)   (1979)

    a) Two vertices of a triangle are $(5, -1)$ and $(2, -3)$. If the orthocentre of the triangle is the origin, find the coordinates of the thrid point.

    b) Find the equation of the line which bisects the obtuse angle between the lines $x - 2y + 4 = 0$ and $4x - 3y + 2 = 0$.

5) A stright line L is perpendicular to the line $5x - y + 1$. The area of the triangle formed by L and the coordinate axes is 5. Find the equation of the line L.   (1980)

6) The end A, B of a straight line segment of constant lenght c slide upon the fixed rectangular axes OX, OY respectively. If the rectangle $OAPB$ be completed, then show that the locus of the foot of perpendicular drawn from P to AB is

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = c^{\frac{2}{3}}$$

7) The vertices of a triangle are $[at_1t_2, a(t_1 + t_2)]$, $[at_2t_3, a(t_2 + t_3)]$, $[at_3t_1, a(t_3 + t_1)]$. Find the orthocentre of the triangle.   (1983-3M)

8) The coordinates of A, B, C are $[6, 3]$, $[3, 5]$, $[4, 2]$ respectively, and P is any point (x, y). Show that the ratio of the area of triangles ΔPBC and ΔABC is $\left| \frac{x+y-2}{7} \right|$ (1983-2M)

9) Two equal sides of an isosceles triangle are given by the equations $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point $(1, -10)$. Determine the equation of the third side. (1985-3M)

10) One of the diameters of the circle circumscribing the rectangle $ABCD$ is $4y = x + 7$. If $\mathbf{A}$ and $\mathbf{B}$ are the points $(-3, 4)$ and $(5, 4)$ respectively, the find the area of rectangle.   (1985-3M)

11) Two sides of a rhombus $ABCD$ are parallel to the lines $y = x + 2$ and $y = 7x + 3$. If the diagonals of the rhombus intersect at the point $(1, 2)$ and the vertex A is on the y-axis, find the possible co-ordinates of $\mathbf{A}$.   (1985-5M)

12) Lines $L_1 \equiv ax + by + c = 0$ and $L_2 \equiv lx + my + n = 0$ intersect at the point P and make an angle θ with each other. Find the equation of a line L different from L_2 which passes through P and make same angle θ with L_1.   (1988-5M)

13) Let ABC be the triangle $AB = AC$. If D is the midpoint of BC, E is the foot of the perpendicular drawn from D to AC and F the mid-point of DE, prove that AF is perpendicular to BE. (1989-5M)

14) Straight lines $3x + 4y = 5$ and $4x - 3y = 15$ intersect at the point A. Points B and C are chosen on these two lines such that $AB = AC$. Determine the possible equations of the line BC passing through the point $(1, 2)$.   (1990-4M)

15) A line cuts the x-axis at $\mathbf{A}(7, 0)$ and the y-axis at $\mathbf{B}(0, -5)$. A variable line PQ is drawn perpendicular to AB cutting the x-axis in P and the y-axis in Q. If AQ and BP intersect at R, find the locus of R. (1990-4M)

16) Find the equation of the line passing through the point $(2, 3)$ and making intercept of length 2 units between the lines $y + 2x = 3$ and $y + 2x = 5$.   (1991-4M)

$y + 2x = 3$ $y + 2x = 5$

17) Show that all chords of the curve $3x^2 - y^2 - 2x + 4y = 0$, which subtend a right angle at the origin, pass through a fixed point. Find the coordinates of point. (1991-4M)

18) Determine all values of α for which the point (α, α^2) lies inside the triangle formed by the lines. (1992-6M)

$$2x + 3y - 1 = 0$$
$$x + 2y - 3 = 0$$
$$5x - 6y - 1 = 0$$