

1 Decision Tree Classifier

1.1 Entropy and Information Gain

The entropy is a measure of impurity or diversity used in decision trees. A lower entropy indicates a purer node. The entropy of a dataset S is defined as:

$$H(S) = - \sum_{i=1}^n p_i \log_2 p_i$$

where p_i is the proportion of class i in the dataset.

The information gain of an attribute A with respect to dataset S is defined as:

$$IG(S, A) = H(S) - \sum_{v \in \text{Values}(A)} \frac{|S_v|}{|S|} H(S_v)$$

where:

- $\text{Values}(A)$ is the set of all possible values of attribute A ,
- S_v is the subset of S for which attribute A has value v ,
- $|S_v|$ and $|S|$ are the sizes of sets S_v and S respectively.

1.2 Gini Index

The Gini index is a measure of impurity or diversity used in decision trees. A lower Gini index indicates a purer node. The Gini index for attribute value $a = a_j$ is defined as:

$$\text{Gini}(a = a_j) = 1 - \sum_{i=1}^c (p(i | j))^2$$

where:

- c is the number of classes,
- $p(i | j)$ is the probability of class i given the attribute value a_j .

The overall Gini index for an attribute a is a weighted average of the Gini indices for each of its values:

$$\text{Gini}(a) = \sum_{i=1}^m \frac{n_i}{n} \text{Gini}(a = a_i)$$

where:

- m is the number of distinct values of attribute a ,
- n_i is the number of instances where $a = a_i$,
- n is the total number of instances,
- $\text{Gini}(a = a_i)$ is the Gini index for value a_i .