Contents

Extension to Karol's paper with asynchronous data

Please let me know if the problem I write below can be described by any of the following:

- totally obvious and therefore not very interesting
- well known and so not very interesting
- not obvious and not well known but still not a good idea to pursue
- a possible problem to pursue

Karol considers the problem of using $p(\theta|x_{1:t})$ to make inference about the parameter $\theta = \sigma$ in the discretely observed SDE

$$x_{t+1} = x_t + \sigma\varepsilon \tag{1}$$

where ε is a normal rv. He uses a filtering approach to build an online model of $p(\sigma|x_{1:t})$.

If we were to do similar for a vol surface (either implied vols, local vols etc) we will be looking at a multidimensional version where we model $p(\theta|\mathbf{x}_{1:t})$. Perhaps the dimension of $\mathbf{x}_{1:t}$ is $K \times T$ where K is number of strikes and T the number of expiries. In the finance/econometrics literature, asynchronous seems to always refer to data observed at irregularly spaced time intervals - ie we would observe the sequence $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_t\}$ where the time intervals between each are different.

However I was thinking of *asynchronous* as the case where not only is data observed at irregular time intervals, but in addition not all components of **x_t** are observed at once. This is a more realistic model in eg the vol surface example, where components of $\mathbf{x}_t$ will be related to options at different strikes/expiry and we recieve new information about $\mathbf{x}_t$ one component at a time.

A toy example would be Karol's model but in 2D ie

$$d\mathbf{z} = \Sigma \, d\mathbf{B}$$

where $\mathbf{z}_t = (x_t, y_t)$ and we want to make inference about the elements of the matrix Σ. Instead of observing a sequence $\{\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_t\}$ we observe some sequence like $\{x_1, x_2, y_3, x_4, y_5, \ldots\}$ etc. Given $\mathbf{z}_t$ and then a newly observed x_t the filtering approach might be to

- do standard particle filtering but use the marginal

  $$p(x_{t+1}|\theta, \mathbf{z}_t) = \int p(x_{t+1}, y_{t+1}|\theta, \mathbf{z}_t) p(y_{t+1}|\theta, \mathbf{z}_t) dy_{t+1}$$

  where this integral is likely very efficient as a summation across one dimension of the particle approximation
- use the updated marginal $p(x_{t+1}|\theta, \mathbf{z}_t)$ to reconstruct the joint distribution (I'm not sure what notation to use - what is the time index for this guy (x_{t+1}, y_t)?).

The main problem a priori is step 2 above. There is probably no unique way to recover the joint distribution (I suspect there are an infinite number of joint distributions which can generate a specific marginal). Perhaps a start would be to compute a new joint distribution whose marginal matches your newly updated marginal (there are infinite number of these) but is *minimally different* (perhaps as measured by KS statistic) from your starting joint distribution. I don't know much about this problem perhaps this is a well established problem?

Filtering for a Volatility Surface

Assume we are running a filter for options on an underlying with K strikes and T expiries. This might be an implied surface, a local vol surface, or perhaps even a price surface we might be using to construct the local vol surface. We have a dynamic system for $\mathbf{x}_t$ with dimension $K \times T$ and we are interested in making inference about some parameters θ. I have removed the bold font on x_t hereafter but I am still referring to a multidimensional model. The filtering approach will have us calculating

$$p(\theta^i|x_t) = \frac{p\left(x_t|x_{t-1},\theta^i\right) p\left(\theta^i|x_{t-1}\right)}{\sum p\left(x_t|x_{t-1},\theta^i\right) p\left(\theta^i|x_{t-1}\right)}$$

The point where I was stuck was thinking about what $p\left(x_t|x_{t-1},\theta^i\right)$ might look like. At first I was thinking about this in the context of the local volatility surface, when I was thinking about if it were possible to recognise the Dupire equation as a Fokker-Planck equation, and use it to specify what transitions $x_{t-1} \rightarrow x_t$ might look like? This lead me to think about **Thought Number 3** below, but then I found various other authors have already thought of that. After that though, I came across an interesting paper by Cont Et Al 2002 (Stochastic Models of Implied Volatility Surfaces). In it they

- review statistical properties of implied vol surfaces
- propose a stochastic model for the joint evolution of the vol surface and underlying. See pg 366 Section 3 for the model specs.

Essentially it is a factor model using a mean reverting OU process. They refer to the model as *The structure of the model allows for easy estimation of the parameters from historical data* but we could use it for online filtering. I haven't done an exhaustive search, but I have not seen it done.

Local Volatility Calibration

While thinking about whether I could use the Dupire equation to get information about transitions, I essentially thought of doing the calibration problem - ie plug local vol functions $\sigma(k, t; \theta)$ into the Dupire PDE and solve numerically, until we find a *best* θ. But I see a lot of people have looked into this already, for example all the way back in Coleman Et Al 1998 (Reconstructing the Unknown Local Volatility Function) which is a nicely written paper. Again I found Hamida and Cont 2013 do the optimisation using a genetic algorithm (Recovering volatility from option prices by evolutionary optimization) which is super interesting. This made me think/notice:

- Applying the idea to models more general than diffusions is an obvious extension, and sure enough they refer to it at the end of the paper and mention they will look into it. But I can't see if they ever did. This could be pretty interesting, and the numerics required are pretty cool.
- Is it obvious how to do this calibration approach to the local vol problem in an online way? This morning I came across Lindstrom Et Al 2007 (Sequential calibration of options), which seems nicely written and very clear. This paper may provide a framework for how to write the local vol calibration problem as an online filtering problem. They use various Kalman filters but I need to read this paper more carefully and think about it.