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Introduction

A number of extensions to the standard Black-Scholes option pricing model have been proposed to account for observed structure in implied volatility surfaces. Denoting $\Sigma(K, T)$ for the Black-Scholes implied volatility of an option with strike K and expiry T we observe the following:

1. Implied volatility Σ varies across both strike K and expiry T ie $\Sigma = \Sigma(K, T)$.
2. Implied volatility $\Sigma(K, T)$ evolves over time ie $\Sigma(K, T) = \Sigma_t(K, T)$.

We have what might be considered *two* classes of solutions to these problems - extensions to Black-Scholes which introduce additional sources of randomness to the underlying dynamics and those which do not.

Local Volatility

The *local volatility* (LV) model allows the underlying volatility to be a deterministic function of S_t and t

$$dS/S = rdt + \sigma(S, t)dB. \tag{1}$$

where $\sigma(S, t)$ is often referred to as the *local volatility function*. Since $\sigma(S, t)$ is a deterministic function there is no additional randomness. The model may be a parametric model like one of the Gatheral functions, or a non-parametric model like one used in many papers. We may not even be interested in the functional form of $\sigma(S, t)$ - instead we might only focus on computing $\sigma_{ij} = \sigma(S = K_i, T = T_j)$ at specific discrete points in (S, T) space. This grid of local volatilities might be used later in a pricing tree or finite difference grid. There are numerous ways to compute $\sigma(S, t)$, and these volatilies are able to account for observed structure in implied volatility across K and T.

In practice LV model re-calibrations are required periodically. This suggests that while LV models can account for differences in Σ across K and T they cannot account for changes in Σ over time. This means it cannot be that underlying volatility is a deterministic function of underlying and time only.

Stochastic Volatility

We consider a second class of model where the underlying volatility is itself a stochastic process (for now we limit ourselves to diffusive processes only). These stochastic volatility (SV) models use a process of the form

$$dS/S = rdt + \phi(v)dB \tag{2}$$

where $\phi(v)$ is another diffusion process which may be correlated with the underlying. These models are popular because they are able to reproduce observed features in implied volatility surfaces, account for changing implied volatility surfaces over time, and sometimes permit (essentially) closed form solutions for rapid pricing, calibration, and hedging.

Stochastic Local Volatility

Stochastic Local Volatility (SLV) combine LV and SV approaches to consider models of the form

$$dS/S = rdt + \sigma(S,t)\phi(v)dB \tag{3}$$

where again $\phi(v)$ is another diffusion process that may be correlated.
Here $\sigma(S,t)$ is the LV component of the model and $\phi(v)$ the SV component. Englemann Et Al (2012) gives an interesting description about the shortcomings of both LV and SV models. The process of computing $\sigma(S,t)$, making inference about the SV model, and pricing options in general SLV models appears to be relatively recent (a first attempt I can find is Ren, Madan & Qian (2007)) and the literature relatively sparse.

Inference in LV, SV and SLV Models

Here are some basic points about inference in LV, SV and SLV models. This is not at all exhaustive, and I have not done a thorough literature search.

- The literature on fitting LV/SV models to market prices is extensive. Even my basic searching (I don't have journal access) on SV and LV models is pretty overwhelming - it's hard to even know where to start. Even a cursory look suggests a huge amount of literature on filtering in SV models. This includes tracking the latent underlying volatility, online parameter estimation, and both. These appear to mostly like:
  1. Specify a model for the underlying, and one for the volatility (eg Heston). The volatility is the latent driver process and the underlying price the observation.
  2. Use various techniques to build filters for this problem.
  3. Add complications, like more stochastic factors, jumps etc

  But the literature using vol surfaces in filters looks less common. Presumably we get much more vol information from the vol surface and the filters will be more effective.
- I do not see your point re the implied volatility surface changing over time as being evidence of a contradiction in the pure LV model being made else where.
- There exists literature on SLV models, but it is sparse and recent enough that I feel less overwhelmed searching it.
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Maybe a niche?

Parameter estimation and latent variable estimation in stochastic volatility models seems exhaustive. There are a huge number of different models investigated, and techniques explored, both frequentist and Bayesian. If we focus attention on option pricing models (ie set aside straight econometric models)