1a, afine

$C_1: (x^2+y^2)^2 + 3x^2y - 3y^3 = 0$

$x^4 + 2x^2y^2 + y^4 + 3x^2y - 3y^3 = 0$

$C_2: (x^2+y^2)^3 - 4x^2y^2 = 0$

$x^6 + 3x^4y^2 + 3x^2y^4 + y^6 - 4x^2y^2 = 0$

$$R_{C_1C_2} = \det \begin{bmatrix}
1 & -3 & 2x^2 & 3x^2 & x^4 & & & & & \\
 & 1 & -3 & 2x^2 & 3x^2 & x^4 & & & & \\
 & & 1 & -3 & 2x^2 & 3x^2 & x^4 & & & \\
 & & & 1 & -3 & 2x^2 & 3x^2 & x^4 & & \\
 & & & & 1 & -3 & 2x^2 & 3x^2 & x^4 & \\
 & & & & & 1 & -3 & 2x^2 & 3x^2 & x^4 \\
1 & 0 & 3x^2 & 0 & 3x^4-4x^2 & 0 & x^6 & & & \\
 & 1 & 0 & 3x^2 & 0 & 3x^4-4x^2 & 0 & x^6 & & \\
 & & 1 & 0 & 3x^2 & 0 & 3x^4-4x^2 & 0 & x^6 & \\
 & & & 1 & 0 & 3x^2 & 0 & 3x^4-4x^2 & 0 & x^6
\end{bmatrix} = 0$$

$\rightsquigarrow 46656x^{18} - 36608x^{16} + 6480x^{14} = 16x^{14}(2916x^4 - 2288x^2 + 405) = 0$

kvadratna enačba

$(0,0) \to$ večkratnost 14

$$\left.\begin{array}{l}
\left(\frac{1}{27}\sqrt{286 + \frac{41\sqrt{19}}{2}},\ \frac{11}{54} - \frac{7\sqrt{19}}{54}\right) \\
\left(\frac{1}{27}\sqrt{286 - \frac{41\sqrt{19}}{2}},\ \frac{11}{54} + \frac{7\sqrt{19}}{54}\right) \\
\left(-\frac{1}{27}\sqrt{286 + \frac{41\sqrt{19}}{2}},\ \frac{11}{54} - \frac{7\sqrt{19}}{54}\right) \\
\left(-\frac{1}{27}\sqrt{286 - \frac{41\sqrt{19}}{2}},\ \frac{11}{54} + \frac{7\sqrt{19}}{54}\right)
\end{array}\right\} \to \text{večkratnost } 1$$

1a, projektivne

$C_1: x^4 + 2x^2y^2 + y^4 + 3x^2yz - 3y^3z = 0$

$C_2: x^6 + 3x^4y^2 + 3x^2y^4 - 4x^2y^2z^2 + y^6 = 0$

$$R_{C_1C_2} = \det \begin{bmatrix} 1 & -3z & 2x^2 & 3x^2z & x^4 & & & & & \\ & 1 & -3z & 2x^2 & 3x^2z & x^4 & & & & \\ & & 1 & -3z & 2x^2 & 3x^2z & x^4 & & & \\ & & & 1 & -3z & 2x^2 & 3x^2z & x^4 & & \\ & & & & 1 & -3z & 2x^2 & 3x^2z & x^4 & \\ & & & & & 1 & -3z & 2x^2 & 3x^2z & x^4 \\ 1 & 0 & 3x^2 & 0 & 3x^4-4x^2z & 0 & x^6 & & & \\ & 1 & 0 & 3x^2 & 0 & 3x^4-4x^2z & 0 & x^6 & & \\ & & 1 & 0 & 3x^2 & 0 & 3x^4-4x^2z & 0 & x^6 & \\ & & & 1 & 0 & 3x^2 & 0 & 3x^4-4x^2z & 0 & x^6 \end{bmatrix}$$

$\rightsquigarrow 16x^{14}z^6(2916x^4 - 2288x^2z^2 + 405z^4) = 0$

$(0, 0, z) \to$ večkratnost 14

iste rešitve kot prej, $z \in \left\{ \pm \frac{1}{9}\sqrt{\frac{2}{5}(572x^2 - 41\sqrt{19}\,x^2)}, \pm\sqrt{\frac{82\sqrt{19}\,x^2}{405} + \frac{1144x^2}{405}} \right\}$

1b, C1

$$\frac{dC_1}{dx} = 4x^3 + 4y^2x + 6yx = 0$$

$$x(2x^2 + 2y^2 + 3y) = 0$$

$$\frac{dC_1}{dy} = 4x^2y + 4y^3 + 3x^2 - 9y^2 = 0$$

$x=0 \Rightarrow \quad 4y^3 - 9y^2 = 0 \qquad y^2(4y-9)=0$

$(0,0)$, $\cancel{(0, \frac{9}{4})}$

✓

$x \neq 0 \Rightarrow \quad x^2 = -y^2 - \frac{3}{2}y$

$$\cancel{-4y^3} - 6y^2 + \cancel{4y^3} - 3y^2 - \frac{9}{2}y - 9y^2 = 0$$

$$-18y^2 - \frac{9}{2}y = 0$$

$$y(4y+1) = 0$$

$y=0 \qquad y = -\frac{1}{4}$

$x^2 = -\frac{1}{16} + \frac{3}{8} = \frac{5}{16}$

$x^2 = 0$

$(0,0)$

$x = \pm \frac{\sqrt{5}}{4}$

$\cancel{\left(\frac{\sqrt{5}}{4}, -\frac{1}{4}\right)}$, $\cancel{\left(-\frac{\sqrt{5}}{4}, -\frac{1}{4}\right)}$

singularna tocka je $(0,0)$, red je 3

(stopnja najnižjega monoma C_1)

1b, c2

$\frac{dC_2}{dx} = 0 \Rightarrow 3x^5 + 6x^3y^2 + 3xy^4 - 4xy^2 = 0$

$\frac{dC_2}{dy} = 0 \Rightarrow 3y^5 + 6x^2y^3 + 3x^4y - 4x^2y = 0$

$y\,\frac{dC_2}{dx} - x\,\frac{dC_2}{dy} = 0$

$\Rightarrow \cancel{3x^5y} + \cancel{6x^3y^3} + \cancel{3xy^5} - 4xy^3 - \cancel{3xy^5} - \cancel{6x^3y^3} - \cancel{3x^5y} + 4x^3y = 0$

$x^3y - xy^3 = 0$

$xy(x^2 - y^2) = 0$

$xy(x+y)(x-y) = 0$

1. $x = 0$: $3y^5 = 0 \rightarrow (0,0)$ ✓
2. $y = 0$: — || —
3. $x = -y$: $-3x^5 - 6x^5 - 3x^5 + 4x^3 = 0$

   $x^3(3x^2 - 1) = 0$

   $x = \pm\sqrt{\frac{1}{3}} \rightarrow \cancel{\left\{\left(\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}\right), \left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)\right\}}$
4. $x = y$: $3x^5 + 6x^5 + 3x^5 - 4x^3 = 0$

   — || —

Singularna točka je $(0,0)$, red je 4
(stopnja najnižjega monoma C_2)

1c

$C_1 \rightarrow$ najmanjši monom je $3x^2y - 3y^3 \rightarrow y(x+y)(x-y) = 0$
tangente $y=0$, $y=x$, $y=-x$ (večkratnost 1)

$C_2 \rightarrow$ najmanjši monom je $-4x^2y^2$
tangente so $x=0$, $y=0$ (večkratnost 2)

1d

d) $y = tx$

$$x^4 + 2x^4t^2 + x^4t^4 + 3x^3t - 3x^3t^3 = 0$$

$$x(1+2t^2+t^4) = 3t^3 - 3t$$

$$x = \frac{3t^3 - 3t}{1+2t^2+t^4}$$

$$y = \frac{3t^4 - 3t^2}{1+2t^2+t^4}$$

1e

e) $z=0 \Rightarrow (x^2+y^2)^2=0$ in $(x^2+y^2)^3=0$

$((x+iy)(x-iy))^2=0$ in $((x+iy)(x-iy))^3=0$

krivulji ~~imata~~ ~~asimptoti~~

nimata asimptot v realni ravnini, ker x^2+y^2 ni razcepno v $\mathbb{R}$.

2a

$C: x^4 - x^2(y+1) - (y+1)^3 = 0$

$C: x^4 - x^2 y - x^2 - y^3 - 3y^2 - 3y - 1 = 0$

a) $C_x: 4x^3 - 2xy - 2x = 0 \rightsquigarrow x(2x^2 - y - 1) = 0$

$C_y: -3y^2 - 6y - x^2 - 3 = 0 \rightsquigarrow 3y^2 + 6y + x^2 + 3 = 0$

1. $x=0: \quad y^2 + 2y + 1 = 0 \rightsquigarrow (y+1)^2 = 0 \rightsquigarrow y = -1$

$(0, -1)$ ✓

2. $x \neq 0: \quad y = 2x^2 - 1 \rightsquigarrow 3(2x^2-1)^2 + 12x^2 - 6 + x^2 + 3 = 0$

$12x^4 - \cancel{12x^2} \cancel{+3} + \cancel{12x^2} \cancel{-6} + x^2 \cancel{+3} = 0$

$\cancel{x^2}(12x^2 + 1) = 0$

$x = \pm\sqrt{\frac{1}{12}}\, i = \pm i \frac{\sqrt{3}}{6}$

$\left(\pm \frac{i}{\sqrt{12}}, -\frac{7}{6}\right)$ //

$u = x, \quad v = y+1 \rightsquigarrow u^4 - u^2 v - v^3 = 0$

singularna točka je $(0,-1)$ s stopnjo 3.

2b

$u^2 v - v^3 = 0$, $u = x$, $v = y+1$

$v(u^2 - v^2) = 0$

$v(u+v)(u-v) = 0$ tangente: $y = -1$

$v \in \{0, -u, u\}$ $y = -x - 1$

$y = x - 1$

vse večkratnosti so 1.

2c

c) afino: $y = tx - 1$

$x^4 - x^2(tx \cancel{-1+1}) - (tx \cancel{-1+1})^3 = 0$

$x^4 = tx^3 + t^3x^3$

$x = t^3 + t \quad , \quad y = t^4 + t^2 - 1$

projektivno: $x = st^3 + s^3t \; , \; y = t^4 + s^2t^2 - s^4 \; , \; z = s^4$

2d