②

a) Polara v $[1:0:1]$: $0 = \lambda F_x + \lambda F_z = 3x^2 - 4yz$

b) Polara v $[2:0:1]$: $0 = 2\mu F_x + \mu F_z = 3x^2 - 2yz$

$2\mu(a) - \lambda(b)$: $\mu \lambda F_z = (6\mu - 3\lambda)x^2 + (2\lambda - 8\mu)yz$

$$\mu \lambda F = \underbrace{(6\mu - 3\lambda)x^2 z} + (\lambda - 4\mu) y z^2 + C_1(x,y)$$

$\lambda(b) - \mu(a)$: $\mu \lambda F_x = (3\lambda - 3\mu)x^2 + (4\mu - 2\lambda)yz$

$$\mu \lambda F = (\lambda - \mu)x^3 + \underbrace{(4\mu - 2\lambda)xyz} + C_2(y,z)$$

$$6\mu - 3\lambda = 0 \quad \wedge \quad 4\mu - 2\lambda = 0$$

$$\Rightarrow \lambda = 2\mu$$

$$3\mu F = (2\mu - 4\mu) y z^2 + C_1(x,y) = (2\mu - \mu)x^3 + C_2(y,z)$$

$$= -2\mu y z^2 + C_1(x,y) = \mu x^3 + C_2(y,z)$$

$$F = -\frac{2}{3} y z^2 + \frac{1}{3}x^3 + C(y) \rightsquigarrow \alpha y^3$$

c) $z = 0$: $\frac{1}{3}x^3 + \alpha y^3 = 0$

- $\alpha \neq 0$: $y^3 = -\frac{1}{\alpha 3} x^3 \Rightarrow 3$ rejitve //
- $\alpha = 0$: $x = 0 \rightsquigarrow$ presečišče $[0:1:0]$

$$C: \quad x^3 - 2yz^2 = 0$$

C je nerazcepna po Eisensteinovem kriteriju v spremenljivki x, z idealom $\langle y \rangle$.

3.

$C: \ 8x^3+24x^2y+24xy^2+9y^3+12x^2z+24xyz+12y^2z+6xz^2+6yz^2=0$

$C_x = 24x^2+48xy+24y^2+24xz+24yz+6z^2$

$= 6(4x^2+8xy+4y^2+4xz+4yz+z^2)$

$C_{xx} = 6(8x+8y+4z)$

$= 24(2x+2y+z)$

$C_y = 24x^2+48xy+27y^2+24xz+24yz+6z^2$

$= 3(8x^2+16xy+9y^2+8xz+8yz+2z^2)$

$C_{yy} = 3(16x+18y+8z)$

$= 6(8x+9y+4z)$

$C_z = 12x^2+24xy+12y^2+12xz+12yz$

$= 12(x^2+2xy+y^2+xz+yz)$

$C_{zz} = 12(x+y)$

$C_{xy} = 6(8x+8y+4z)$

$= 24(2x+2y+z)$

$C_{xz} = 6(4x+4y+2z)$

$= 12(2x+2y+z)$

$C_{yz} = 3(8x+8y+4z)$

$= 12(2x+2y+z)$

$$\det \begin{bmatrix} 24(2x+2y+z) & 24(2x+2y+z) & 12(2x+2y+z) \\ 24(2x+2y+z) & 6(8x+9y+4z) & 12(2x+2y+z) \\ 12(2x+2y+z) & 12(2x+2y+z) & 12(x+y) \end{bmatrix} =$$

$$= 12(2x+2y+z)\cdot 6 \cdot 12 \cdot \det \begin{bmatrix} 2 & 4(2x+2y+z) & 2x+2y+z \\ 2 & 8x+9y+4z & 2x+2y+z \\ 1 & 2(2x+2y+z) & x+y \end{bmatrix} =$$

$$= 864(2x+2y+z)\cdot \det \begin{bmatrix} 0 & 0 & z \\ 0 & y & z \\ 1 & 2(2x+2y+z) & x+y \end{bmatrix} =$$

$$= 864(2x+2y+z)(-yz)$$

• $y=0$:

$C: \; 8x^3 + 12x^2z + 6xz^2 = 0$

$x(8x^2 + 12xz + 6z^2) = 0$

$x=0 \implies [0:0:1]$

$C_x = 6z^2 = 6 \neq 0 \implies$ je prevoj

$C_y = 6z^2 = 6 \neq 0$

$C_z = 0$

$\Downarrow$

Prevojna tangenta je

$x+y=0$

Drugih prevojev ni potrebno izračunati.

$\downarrow$

$z=0$,

$z=-2x-2y$

$$y^2 z = x(x+\lambda_1 z)(x+\lambda_2 z)$$

$$x^3 + (\lambda_1+\lambda_2)x^2 z + \lambda_1 \lambda_2 x z^2 - y^2 z = 0$$

$$\Phi : [0:0:1] \longrightarrow [0:1:0]$$

$$\Phi = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} c=0 \\ f=\lambda \\ i=0 \end{matrix}$$

$$\Phi : y=-x \longrightarrow z=0$$

$$\Phi = \begin{bmatrix} a & b & 0 \\ d & e & \lambda \\ g & h & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ * \end{bmatrix} = \begin{bmatrix} * \\ * \\ 0 \end{bmatrix} \Rightarrow g-h=0, \quad \text{BŠS } g=h=1$$

$$\Phi^{-1}: \left[\begin{array}{ccc|ccc} a & b & 0 & 1 & 0 & 0 \\ d & e & \lambda & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{a-b} & 0 & \frac{-b}{a-b} \\ d & e & \lambda & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{a-b} & 0 & \frac{-b}{a-b} \\ 0 & 1 & 0 & \frac{-1}{a-b} & 0 & \frac{a}{a-b} \\ d & e & \lambda & 0 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{a-b} & 0 & \frac{-b}{a-b} \\ 0 & 1 & 0 & \frac{-1}{a-b} & 0 & \frac{a}{a-b} \\ 0 & 0 & 1 & \frac{e-d}{(a-b)\lambda} & \frac{1}{\lambda} & \frac{bd-ae}{(a-b)\lambda} \end{array}\right]$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{a-b} \begin{bmatrix} 1 & 0 & -b \\ -1 & 0 & a \\ \frac{e-d}{\lambda} & \frac{1}{\lambda} & \frac{bd-ae}{\lambda} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$x = \frac{1}{a-b}(u - bw)$$

$$y = \frac{1}{a-b}(aw - u)$$

$$z = \frac{1}{(a-b)\lambda}\big((e-d)u + v + (bd-ae)w\big)$$

$$F(x,y,z) = 8x^3 + 24x^2y + 24xy^2 + 9y^3 + 12x^2z + 24xyz + 12y^2z$$
$$+ 6xz^2 + 6yz^2$$
$$= 8(x+y)^3 + 12z(x+y)^2 + 6z^2(x+y) + y^3$$

$$x+y = \frac{1}{a-b}(u - bw - u + aw) = w$$

$$F(u,v,w) = 8w^3 + 12w^2 \frac{1}{(a-b)\lambda} \cdot ((e-d)u + v + (bd-ae)w) +$$
$$+ 6w \frac{1}{(a-b)^2\lambda^2}((e-d)u + v + (bd-ae)w)^2 + \frac{1}{(a-b)^3}(aw-u)^3$$

$$= 8w^3 + \frac{12(e-d)}{(a-b)\lambda} uw^2 + \frac{12}{(a-b)\lambda} vw^2 + \frac{12(bd-ae)}{(a-b)\lambda} w^3 + \frac{6(e-d)^2}{(a-b)^2\lambda^2} u^2w + \frac{6}{\lambda^2} v^2w$$
$$+ \frac{6(bd-ae)}{(a-b)\lambda^2} w^3 + \frac{12(e-d)}{(a-b)\lambda^2} uvw + \frac{12(e-d)(bd-ae)}{(a-b)^2\lambda^2} uw^2 + \frac{12(bd-ae)}{(a-b)\lambda^2} vw^2$$
$$+ \frac{a^3}{(a-b)^3} w^3 - \frac{3a^2}{(a-b)^3} w^2u + \frac{3a}{(a-b)^3} wu^2 - \frac{1}{(a-b)^3} u^3$$

v^2w: $\frac{6}{\lambda^2} = 1 \Rightarrow \lambda = \sqrt{6}$

u^3: $\frac{-1}{(a-b)^3} = -1 \Rightarrow a-b = 1$

uvw: $\frac{12(e-d)}{\lambda(a-b)} = 0 \Rightarrow e = d$

w^3: $8 + \frac{12(bd-ae)}{\lambda(a-b)} + \frac{6(bd-ae)^2}{\lambda^2(a-b)^2} + \frac{a^3}{(a-b)^3} = 0$

$$8 - 6\sqrt{6}\,d + d^2 + a^3 = 0 \Rightarrow a = \sqrt[3]{-2}$$

vw^2: $\frac{12}{\lambda} + \frac{12(bd-ae)}{\lambda^2(a-b)} = 0$

$$6\sqrt{6} - 6d = 0 \Rightarrow d = \sqrt{6}$$

$$\Phi = \begin{bmatrix} -\sqrt[3]{2} & -\sqrt[3]{2} - 1 & 0 \\ \sqrt{6} & \sqrt{6} & \sqrt{6} \\ 1 & 1 & 0 \end{bmatrix}$$

(4.)

$$\begin{bmatrix} \bar{a} & \bar{b} & \bar{c} \\ \bar{d} & \bar{e} & \bar{f} \\ \bar{g} & \bar{h} & \bar{i} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \overset{B33 := 1}{\Rightarrow} \quad \Phi = \begin{bmatrix} \bar{a} & 0 & \bar{c} \\ \bar{d} & \lambda & \bar{f} \\ \bar{g} & 0 & 1 \end{bmatrix}$$

$$X = \bar{a}x + \bar{c}z \qquad Y = \bar{d}x + \lambda y + \bar{f}z \qquad Z = \bar{g}x + z$$

$$\alpha(x^3 + axz^2 + bz^3 - y^2z) = (\bar{a}x + \bar{c}z)^3 + A(\bar{a}x + \bar{c}z)(\bar{g}x + z)^2 +$$
$$+ B(\bar{g}x + z)^3 - (\bar{d}x + \lambda y + \bar{f}z)^2(\bar{g}x + z) =$$
$$= \bar{a}^3x^3 + 3\bar{a}^2\bar{c}x^2z + 3\bar{a}\bar{c}^2xz^2 + \bar{c}^3z^3 + (A\bar{a}x + A\bar{c}z)(\bar{g}^2x^2 + 2\bar{g}xz + z^2)$$
$$+ B\bar{g}^3x^3 + 3B\bar{g}^2x^2z + 3B\bar{g}xz^2 + Bz^3 + (\bar{d}^2x^2 + \bar{d}\lambda xy + \bar{d}\bar{f}xz$$
$$+ \lambda^2y^2 + \lambda\bar{f}yz + \bar{f}^2z^2)(-\bar{g}x - z) =$$
$$= x^3(\bar{a}^3 + a\bar{a}\bar{g}^2 + b\bar{g}^3 - \bar{d}^2\bar{g}) + x^2z(3\bar{a}^2\bar{c} + 2A\bar{a}\bar{g} + 3B\bar{g}^2 - \bar{d}^2 - \bar{d}\bar{f}\bar{g})$$
$$+ xz^2(3\bar{a}\bar{c} + A\bar{a} + 2A\bar{c}\bar{g} + 3B\bar{g} - \bar{d}\bar{f} - \bar{f}^2\bar{g})$$
$$+ z^3(\bar{c}^3z^3 + A\bar{c} + B - \bar{f}^2) + x^2y(-\bar{d}\bar{g}\lambda) + xyz(-\bar{d}\lambda - \bar{f}\bar{g}\lambda)$$
$$+ xy^2(-\bar{g}\lambda^2) + y^2z(-\lambda^2) + yz^2(-\lambda\bar{f})$$

y^2z: $-\alpha = -\lambda^2 \Rightarrow \alpha = \lambda^2$

xy^2: $-\bar{g}\lambda^2 = 0 \Rightarrow \bar{g} = 0$

xyz: $-\bar{d}\lambda = 0 \Rightarrow \bar{d} = 0$

yz^2: $-\lambda\bar{f} = 0 \Rightarrow \bar{f} = 0$

x^3: $\bar{a}^3 = \alpha \Rightarrow \bar{a} = \lambda^{\frac{2}{3}}$

x^2z: $3\bar{a}^2\bar{c} = 0 \Rightarrow \bar{c} = 0$

z^3: $B = \alpha b \Rightarrow B = \lambda^2 b$

xz^2: $A\bar{a} = \alpha a \Rightarrow A = a\lambda^{\frac{4}{3}}$

$t := \lambda^{\frac{1}{3}}$

$X = \lambda^{\frac{2}{3}}x = t^2x$

$Y = \lambda y = t^3x$