Surface simplification

Project presentation

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Topological data analysis

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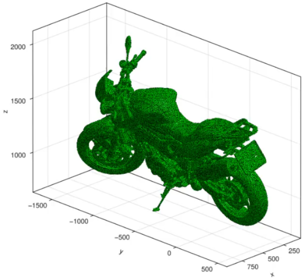
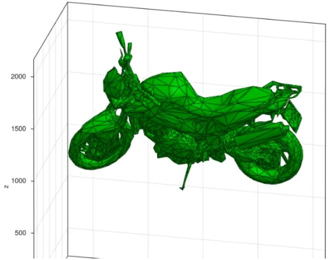
Motivation

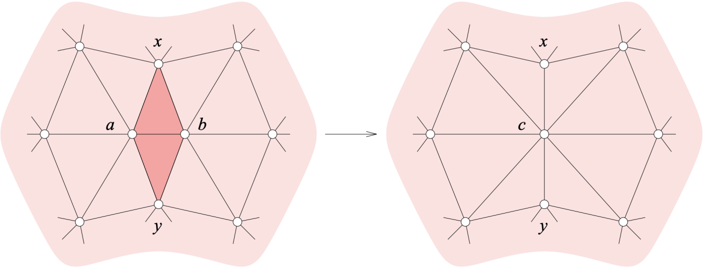
- Reduce complexity
- Reduce measurement noise
- Features at various levels of resolution





Edge contraction

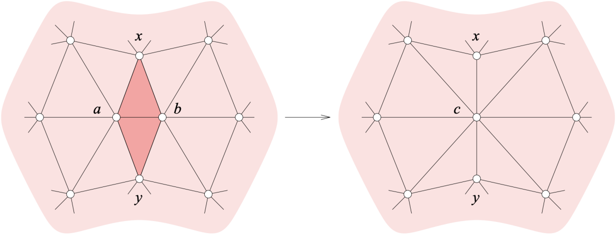
- To contract ab, we remove the two dark triangles and repair the hole by gluing their two left edges to their two right edges.



- We want to prioritize the edges so that contractions that preserve the shape of the manifold are preferred

Chosing the point c

- Where do we place c?



$$E_H(x) = \sum_{h_i \in H} d^2(x, h_i)$$

- The point c has to minimize the error for a given edge ab

$$Error(ab) = \min_{c \in \mathbb{R}^3} E_H(c).$$

Calculating the error

- A point x can be represented as a vector $x^T = (x'^T, 1)$
- A plane $y \in \mathbb{R}^3, < y, u' >= -\delta$ can be represented as a vector $u^T = (u'^T, \delta)$
- We use this to express the sum of squared distances from a set of planes in matrix form H

$$\begin{aligned} E_H(x) &= \sum_{h_i \in H} d^2(x, h_i) \\ &= \sum_{h_i \in H} (x^T \cdot u_i)(u_i^T \cdot x) \\ &= x^T \cdot \left(\sum_{h_i \in H} u_i \cdot u_i^T \right) \cdot x \end{aligned}$$

Q matrix

- We can define the Q matrix as

$$Q = \sum_{h_i \in H} u_i \cdot u_i^T$$

  so the following holds: $E_H(x) = x^T \cdot Q \cdot x$
- The Q matrix is a symmetric, four-by-four matrix we refer to as the fundamental quadric of the map E_H
- Q_a represents the matrix of all planes of which the triangles contain the vertex a
- Q_{ab} represents the matrix of all planes of which the triangles contain the edge ab
- Q_{abc} represents the matrix of the plane of abc

Computing the minimum

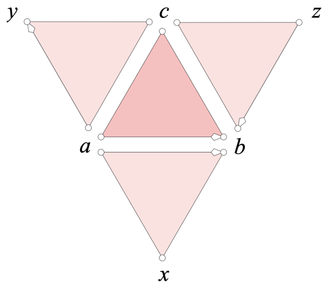
- We can find the minimum simply by solving the equation

$$Q_{ab}[1:3,:] \cdot x = 0$$

- We have to find a solution where $x[4]$ is not 0

Implementation problems 1

- The triangles are not connected



Implementation problems 2

- The mesh is not a manifold