OPTIMISATION

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COURSE-WORK

The final report should be submitted electronically on Blackboard by the 18th of December 2016

Part I

To compare the performance of optimisation methods it is customary to construct *test* functions and then compare the behavior of various optimization algorithms for such functions.
One, often used, test function is the so-called Rosenbrock function, *i.e.*

$$v(x, y) = 100(y - x^2)^2 + (1 - x)^2.$$

A1) Compute analytically all stationary points of the function $v(x, y)$ and verify if they are minima/maxima/saddle points.

A2) Plot (using Matlab or a similar SW) the level sets of the function $v(x, y)$.

A3) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using the gradient method (see Section 2.5) with Armijo line search (see Section 2.4.2).

A4) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using Newton method (see Section 2.6) with Armijo line search.

A5) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using the Polak-Ribiere algorithm (see Section 2.7.3) with Armijo line search.

A6) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using the Broyden-Fletcher-Goldfarb-Shanno algorithm (see Section 2.8) with Armijo line search.

A7) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using the simplex method (see Section 2.9).

A8) *Run* the minimization procedures written in points A3) to A7) with initial point $(x_0, y_0) = (-3/4, 1)$.

A8a) Plot, on the (x, y)-plane, the sequences of points generated by each algorithm. Are these sequences converging to a stationary point of $v(x, y)$?

A8b) For a sequence $\{x_k, y_k\}$ consider the *cost*

$$J_k = \log\left((x_k - 1)^2 + (y_k - 1)^2\right).$$

Plot, for each of the sequences generated by the above algorithms, the cost J_k as a function of k. Use such a plot to assess the speed of convergence of each of the considered algorithms.

Part II

Consider the constrained minimization problem

$$\min_{x_1,x_2} = -x_1 x_2$$
$$x_1 + x_2 - 2 = 0.$$

B1) Write the necessary conditions of optimality and find all points satisfying such conditions.

B2) Check if the candidate optimal points obtained in B1) are constrained local minima.

B3) By solving the equation of the constraint transform the considered constrained optimization problem into an unconstrained problem.

B4) Construct the exact penalty function $G(x)$ associated to the considered problem (see Section 3.4.3). Minimize analytically the function $G(x)$ and show that the unconstrained minimum of $G(x)$ is a solution of the considered constrained optimization problem.

B5) Construct the exact augmented Lagrangian function $S(x, \lambda)$ associated to the considered problem (see Section 3.4.4). Minimize analytically the function $S(x, \lambda)$ and show that the unconstrained minimum of $S(x, \lambda)$ yields a solution of the considered constrained optimization problem and the corresponding optimal multiplier.