1 ⓐ Writing sentences using logical connectives:

E: The universe will simply exist as it is.

H: The universe will end in a heat death.

B: There was a big bang.

X: The universe is expanding

A: The universe is ~~accelerating~~. accelerated.

(This can be thought of knowledge base).

(i) $E \vee H$ (The universe will either exist as it is or end in heat death)

(ii) $\neg B \to E$ (If there was no big bang then the universe will simply exist)

(iii) $X \leftrightarrow B$ (If and only if the universe is expanding, then there ~~is~~ was a big bang)

(iv) $(X \wedge A) \to H$ (If the universe is expanding (X) and accelerated (A), then it will end in a heat death (H).

ⓑ Writing contrapositive of above sentences:

(i) $E \vee H$ can be written as $\neg E \to H$, contrapositive of this is $\neg H \to E$ (If the universe will not end in heat death then it will exist as it is)

(ii) $\neg B \to E$ ~~can be written as $\neg E \to B$~~, contrapositive of this is $\neg E \to B$. (If the universe will ~~simply~~ not simply exist as it is then there was a big bang)

(iii) $X \leftrightarrow B$, contrapositive of this is $\neg B \leftrightarrow \neg X$. (If and only if there is no big bang then the universe is not expanding).

(iv) $(X \wedge A) \to H$, contrapositive of this is $\neg H \to \neg(X \wedge A)$, $\neg H \to (\neg X \vee \neg A)$. (If the universe will not end in a heat death then universe is not expanding or accelerated.)

(c) Inferences:

→ If the universe doesn't exist then it will end in a heat death ($\neg E \to H$)

Besides these, all the contrapositive sentences can also be inferred → $\neg H \to E$, $\neg E \to B$, $\neg B \leftrightarrow \neg X$, $\neg H \to (\neg X \vee \neg A)$
we can also infer $(\neg E \wedge X) \to B$ (since $\neg E \to B$ and $X \leftrightarrow B$)

→ Statements that can't be inferred:

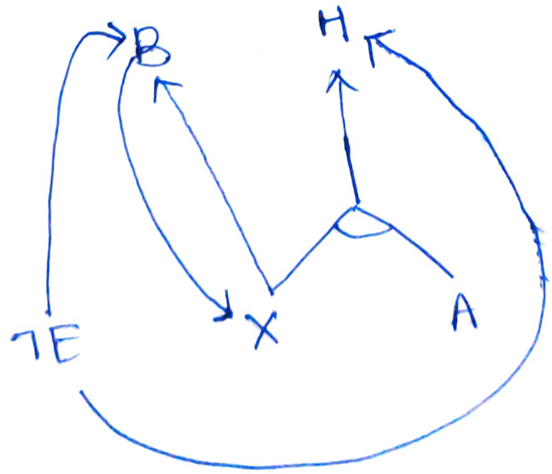
We can't infer $\neg E \to A$ (If the universe simply didn't exist as it is then it is accelerated)
Similarly we can also not infer $\neg E \to \neg A$ (If the universe simply didn't exist as it is then it is not accelerated) we can also not infer $B \to H$

(These can be proved from AND-OR Graph in next section)

(d) AND-OR GRAPH:

Following AND-OR graph is drawn with following statements:

(I) $(X \wedge A) \to H$
(II) $X \leftrightarrow B$
(III) $\neg E \to B$ (contrapositive of $\neg B \to E$)
(IV) $\neg E \to H$ (from $E \vee H$)



8.

Soundness:

let us consider literal l_i that is complementary to literal m_j in some other clause.

If we consider l_i to be true then m_j must be false, which means that $m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots m_n$ is true, → (i)

because $m_1 \vee \cdots \vee m_n$ is given.

Similarly if l_i is false then $l_1 \vee \cdots l_{i-1} \vee l_{i+1} \vee \cdots \vee l_k$ must → (ii) be true since $l_1 \vee \cdots \vee l_k$ is given. Thus if l_i is true then eqn (i) holds else eqn (ii) holds true. ~~This is~~

- Therefore proof by resolution is sound.

3(D) Completeness of Resolution:

let us suppose resolution closure RC(S) as a set of clauses, that are ~~derived~~ derivable by repeated application of resolution rule to clauses in S or their derivative. (S => is set of clauses)

RC(S) is finite due to factoring step, because we can only compute finite distinct clauses that can be constructed from P_1 --- P_k that appearing.

This completeness theorem is also called ground resolution theorem. (If a set of clauses is unsatisfiable, then the resolution closure of those clauses contains empty clause.)

let's prove it's contrapositive -> If the closure RC(S) does not contain the empty clause, then it is satisfiable.

Mathematical Construction:

let P_1 -- P_k be suitable truth values for set S.

For i => 1 to k

- If clause in RC(S) contains literal $\neg P_i$, and all other literal are false, assign P_i as false
- else assign it true.

This assignment P_1 -- P_k is a model of S.

let us assume the opposite, at some stage i in the sequence, assigning symbol P_i causes some clause C to become false. For this to happen it must be the case that all other literals in C must already have been falsified by assignments to P_1 ... P_{i-1}. Thus C must look like (false v ... v P_i) or (false v false ... v $\neg P_i$)

If just one of them are in RC(S) then algorithm will assign the appropriate truth value to P_i to make C true. So C can be falsified if both of these are in RC(S).

~~Now since RC(S) is ct~~

Now because RC(S) is closed under resolution, it will contain resolvent of these two clauses, with all of its literals already falsified by the assignments to. P_1 ... P_{i-1}. This contradicts our assumption.

Hence we proved that construction produces model in RC(S). If S is contained in RC(S), any model of RC(S) is a model of S itself.