Abstract

The main goal of this bachelor thesis is to present the Alexandroff-Hausdorff Theorem, together with one of its most important applications in functional analysis: the Banach-Mazur Theorem. This will require that we make use of a wide range of mathematical tools: weak topologies, dual spaces, the Tikhonov, Banach-Alaoglu and Hahn-Banach theorems, etc.

The first chapter is composed by a group of preliminaries, and the reader can decide whether to tackle it before beginning to read the project, or whether to go back to it whenever it is needed. In the second chapter, the Cantor set and space will be introduced, coupled with some of their most noticeable properties. This study of topological and functional analysis concepts will result in the detailed proofs of the two central theorems in the two following chapters.

The third chapter orbits around the Alexandroff-Hausdorff Theorem, whose proof isn't usually found in general topology textbooks, which motivated us to present it in an extensive manner. The theorem states that every compact and metrizable space is a continuous image of the Cantor space. We will finally proceed, in a fourth chapter, to present the Banach-Mazur Theorem, according to which every separable Banach space is linearly isometric to a subspace of $\mathcal{C}[0,1]$.