CS3230: Design and Analysis of Algorithms
Semester 2, 2019-20, School of Computing, NUS

Challenge Problem 2

Deadline : Friday, 20th March 2020, 6:00pm

Instructions

- **IMPORTANT:** Write your NAME, Matric No, Tut. Gp in your Answer Sheet.
- Make sure your name and matric number is on each sheet.
- Write legibly. If we cannot read what you write, we cannot give points. In case you CANNOT write legibly, please type out your answers and print out hard copy.
- To submit, drop your printed work at Prof.Ken's office, or give it to your tutor.
- When you submit your answer, please try to make it short. The page limit is 3.
- Note: This problem is worth 1 mark.

1 Motivation

We can see a comparison-based sorting algorithm as finding a hidden permutation $P = (p_1, p_2, ..., p_n)$ contained inside a black box, where $a[p_1] \leq a[p_2] \leq ... \leq a[p_n]$ (a is the array we are sorting). We can ask the black box a few questions of form (i, j); the black box will compare $a[p_i]$ and $a[p_j]$ then return us the outcome. Without knowing values of a, after $\Omega(n \lg n)$ questions, we can find P.

2 Magic sort

In this task, you are to find a hidden permutation $P = (p_1, p_2, ..., p_n)$ contained inside a black box. You can ask the black box some questions of form $Q = \{q_1, q_2, ..., q_k\}$, where Q is a subset of the set $\{1, 2, ..., n\}$.The black box will return you the set $\{p_{q_1}, p_{q_2}, ..., p_{q_k}\}$ by printing a ll its elements in a random order. Find the best lower bound for the number of questions required to find P. The best lower bound is a function $f(n)$ such that to find P of size n you must ask at least $f(n)$ questions, and there exists an algorithm to find P using exactly $f(n)$ questions. Also, design an algorithm to find P by asking $f(n)$ questions.

- You are encouraged to present your algorithm in pseudo code, clearly and understandably.
- Analyse the time complexity of your algorithm.
- Prove the correctness of your algorithm.

3 Example

When $n = 3$, you can ask the black box 2 questions: $\{1, 2\}$ and $\{1\}$. Then black box will return you two sets $A = \{p_1, p_2\}$ and $B = \{p_1\}$. Let S be $\{1, 2, ..., n\}$.

- $\{p_1\} = B$

- $\{p_2\} = A \setminus B$
- $\{p_3\} = S \setminus A$