**AE3212 SVV**

# Pointers to set up a verification model for the 2020 extra structures Assignment

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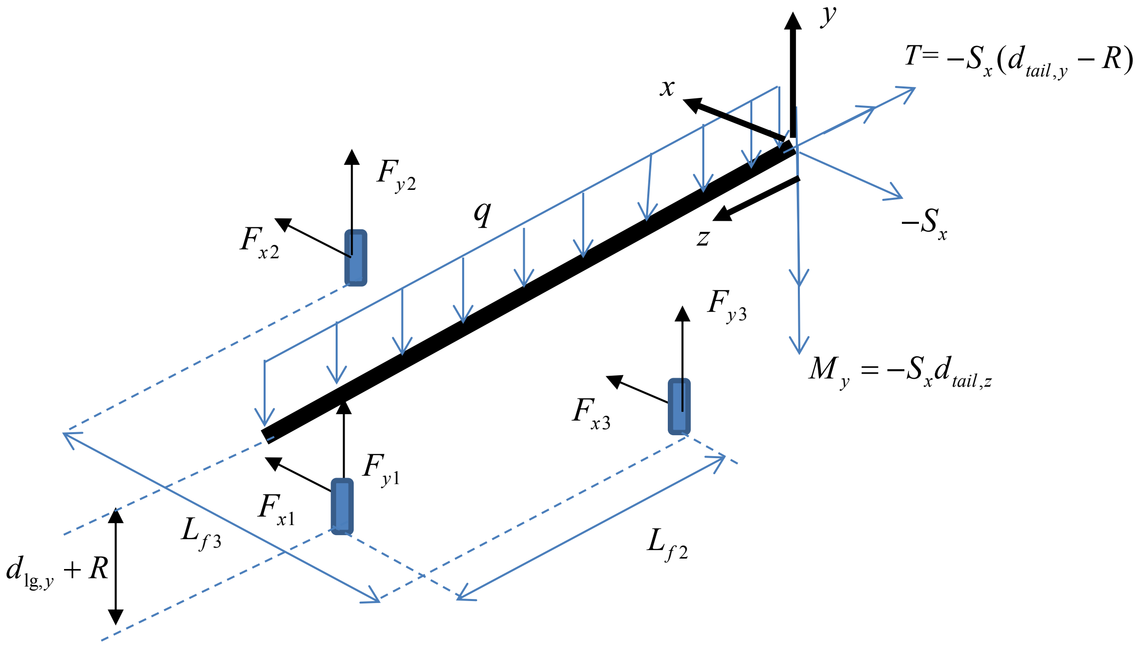
This document has been produced to assist the setup of a numerical and verification model for the extra structures assignment of 2020.

This document does not contain any specific numerical values for stresses and strains, it serves as a summary of the material treated in the $2^{nd}$ year structures course.

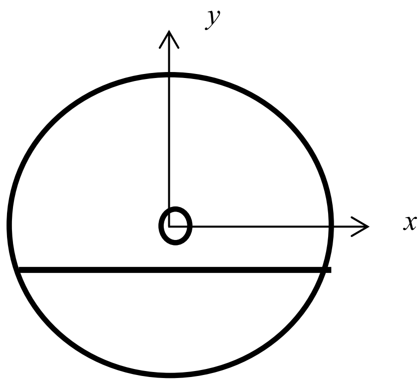
## 1) Statics

**Free body diagram of equivalent beam**

**Right-handed coordinate system *xyz* used at the back of the fuselage**




6 reaction forces: $F_{x1}, F_{y1}, F_{x2}, F_{y2}, F_{x3}, F_{y3}$



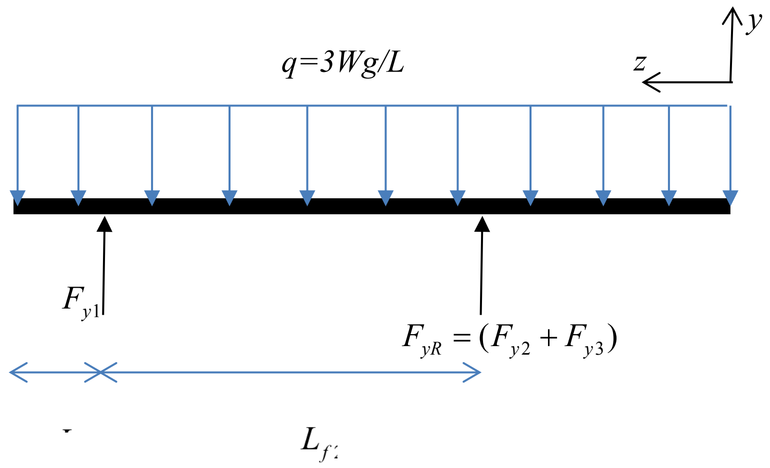

**We translate** $S_x$ **from the T.A.C. to the point O of the cross-section** (x=0,y=0,z=0), this will lead to a torque $T$ and moment $M_y$( these can be computed since we know $d_{tail,y}, d_{tail,z}$).

<u>**The equivalent beam line shown above goes through point O (x=0,y=0).**</u>

**Apply superpositioning**: We break down the FBD in four load cases:

**TOTAL** = *Case 1* + *Case 2* + *Case 3* + *Case 4*

**Case 1**: Distributed load q, symmetric w.r.t. yz plane, analysis in yz plane:




## Equilibrium of Case 1

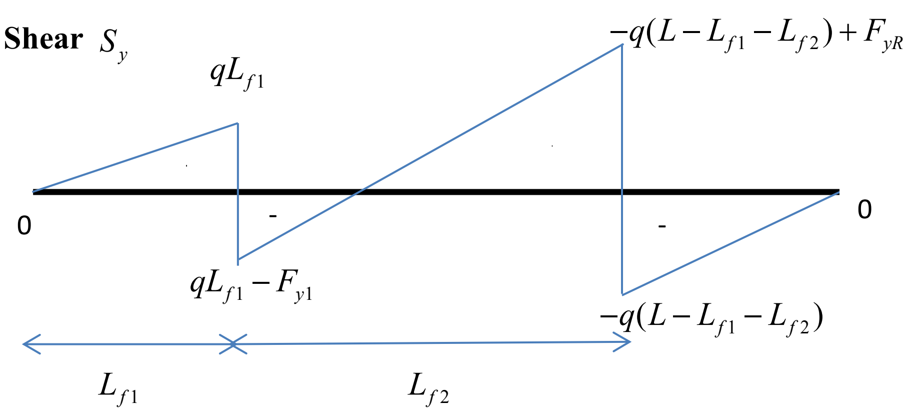
$$\sum F_y \uparrow^{+} = F_{y1} + F_{yR} - qL = 0$$

$$\sum M_x^{z=L-L_{f1}} = F_{yR}\left(L_{f2}\right) - qL\left(\frac{L}{2} - L_{f1}\right) = 0 \quad (CCW\,positive)$$

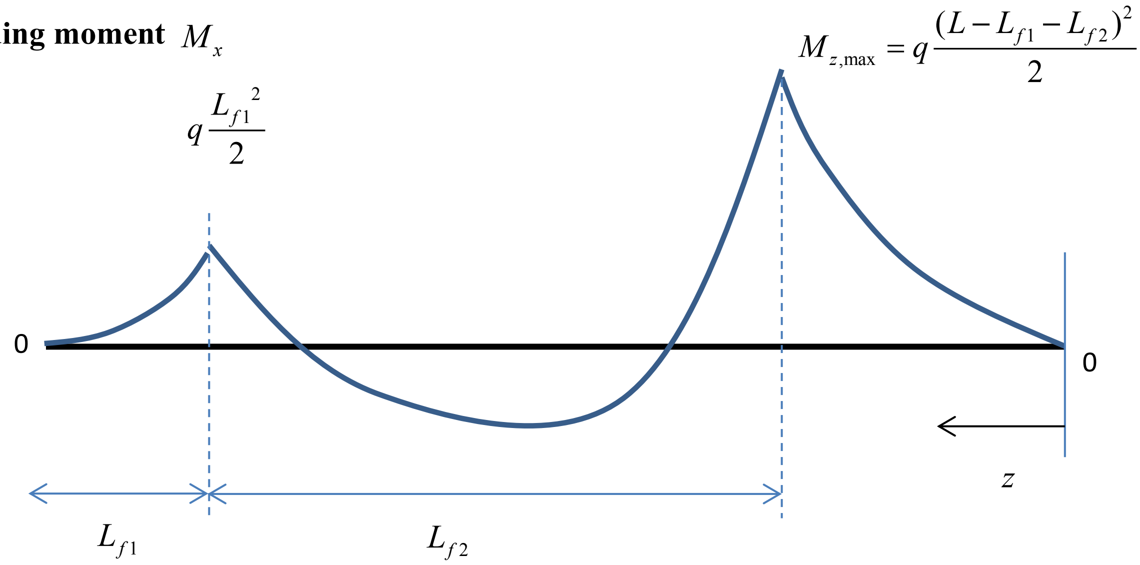
$$F_{yR} = q\frac{L}{L_{f2}}\left(\frac{L}{2} - L_{f1}\right)$$

$$F_{y1} = qL - F_{yR} = qL\left(1 - \frac{1}{L_{f2}}\left(\frac{L}{2} - L_{f1}\right)\right)$$

## Bending moment, shear, torsion diagram for Case 1

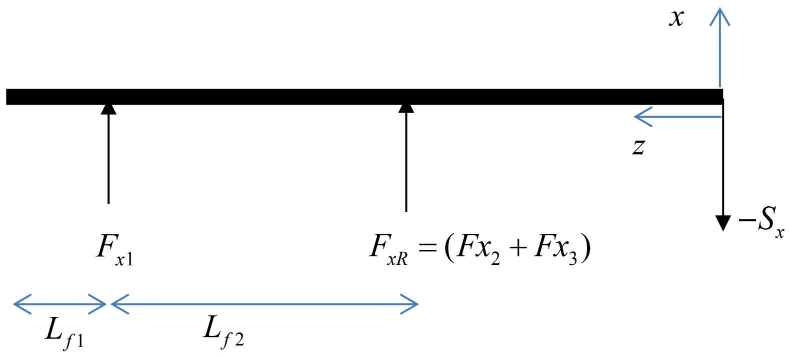


**Bending moment** $M_x$




## Case 2: Shear load $S_x$

**Analysis in xz plane:**




**As mentioned before: Sx moved to point O of the cross-section, x=0, y=0,z=0.**

### Equilibrium of Case 2

$$\sum F_x \downarrow^{+} = F_{x1} + F_{xr} - S_x = 0$$
$$\sum M_y^{\,z=L-L_{f1}} = F_{xR} L_{f2} - S_x\left(L - L_{f1}\right) = 0 \quad (CCW\,positive)$$

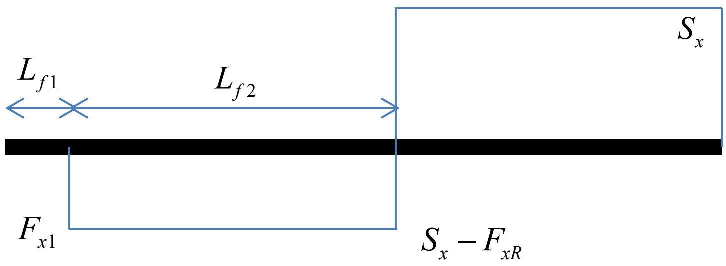
Solving for these two equations leads to:

$$F_{xr} = S_x \left( \frac{L - L_{f1}}{\left(L_{f2}\right)} \right)$$

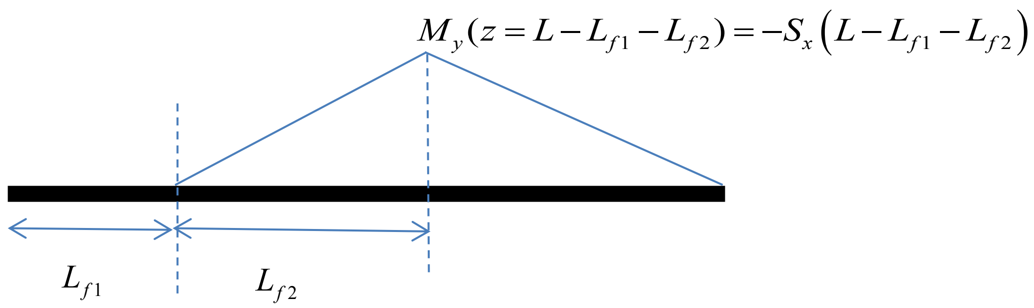
$$F_{x1} = S_x \left( 1 - \frac{L - L_{f1}}{\left(L_{f2}\right)} \right)$$

## Shear force, bending moment diagram case 2

**Shear $S_x$**




**Bending** $M_y$

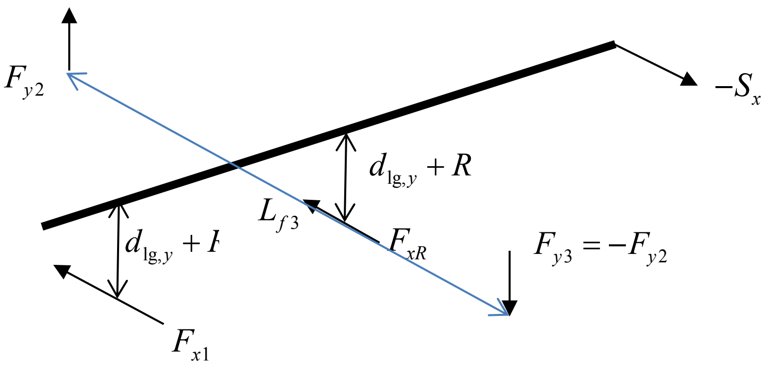



Checking global equilibrium, we see that the shear forces $F_{x1}$ and $F_{xR}$ are not equal and opposite, this means that there is also a torque that must be balanced, since the landing gears have an offset $d_{\lg,y}$, they will create a torque w.r.t. point $O$ in the cross-section. Sum of moments around point O (x=0,y=0,z=L-Lf1-Lf2) yields:

$$\sum M_z(x=0, y=0, z=L-L_{f1}-L_{f2}) = F_{x1}(R+d_{\lg,y}) - F_{xR}(R+d_{\lg,y}) + F_{y2}L_{f3} = 0$$

$$T_1 = F_{x1}(R+d_{\lg,y})$$
$$T_2 = -F_{xR}(R+d_{\lg,y})$$

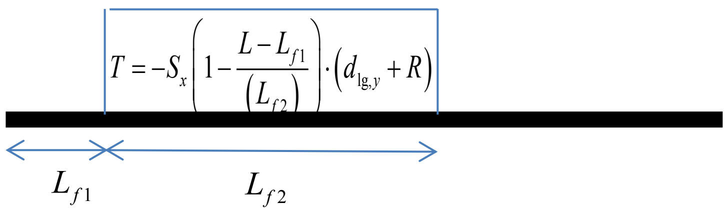
Note that $F_{xR}$ is the resultant of $F_{x2}$ and $F_{x3}$ and will cause a torque $T_2$, $F_{x1}$ will also cause a torque $T_1$. Since $T_1$ and $T_2$ are not equal and opposite, the only way the structure in this case is in equilibrium if there is equal and opposite forces $F_{y2}$ and $F_{y3}$ in the rear landing gears, causing a torque $T_3$ to balance the difference between $T_1$ and $T_2$.




Solving for Fy2 leads to:

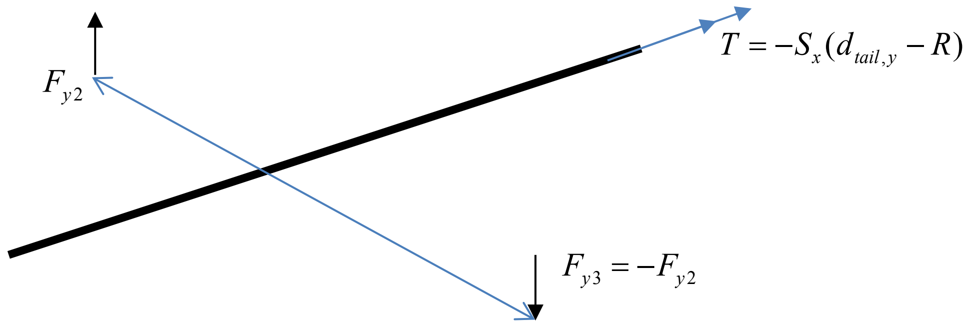
$$F_{y2} = -F_{y3} = S_x \frac{\left(2LR + 2Ld_{lg,y} - 2L_{f1}R - 2L_{f1}d_{lg,y} - L_{f2}R - L_{f2}d_{lg,y}\right)}{L_{f2}L_{f3}}$$

**Torsion diagram Case 2**



## Equilibrium Case 3

Case 3 is a simple torque T applied at z=0. This torque is balanced by a force couple in the y-direction in landing gears RL1 and RL2 as shown below.

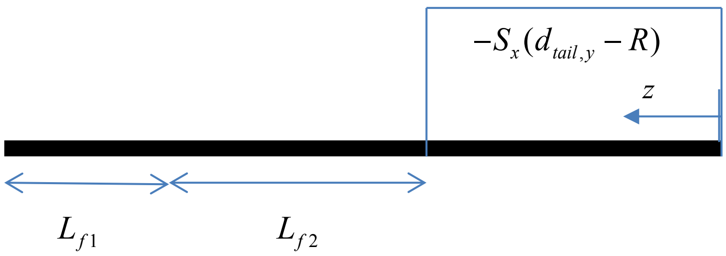



$$\sum M_z = 0 = -S_x(d_{tail,y} - R) - F_{y2}L_{f3} = 0 \quad (CCW\ \ positive)$$

$$F_{y2} = -F_{y3} = \frac{S_x(d_{tail,y} - R)}{L_{f3}}$$

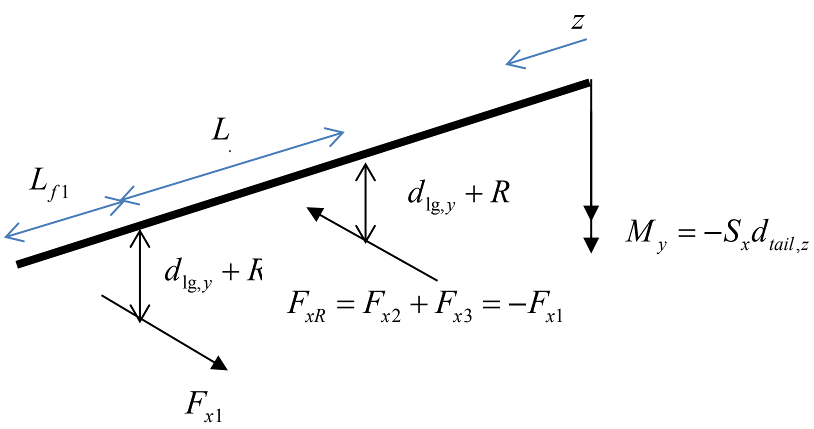
Here again sum of moments was taken around the z-axis at point O (x=0,y=0).

**Torsion diagram Case 3**



## Equilibrium Case 4

Case 4 is a simple moment $M_y$ applied at $z$=0. This moment is reacted by equal and opposite forces in the $x$-direction at the front and rear landing gears.

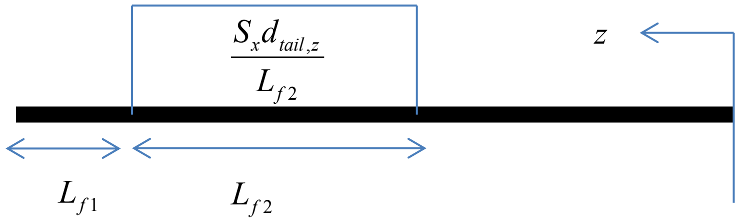



$$\sum M_y(x=0, y=0, z=L-Lf1-Lf2) = 0 = -S_x d_{tail,z} + F_{x1} L_{f2} = 0 \quad (CCW\ \ positive)$$

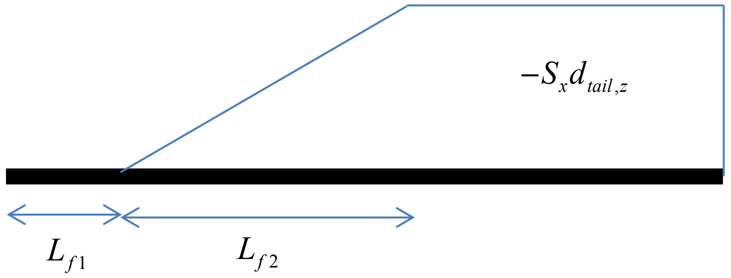
$$F_{x1} = -F_{xR} = \frac{S_x d_{tail,z}}{L_{f2}}$$

$$\sum M_z(x=0, y=0, z=L-Lf1-Lf2) = 0 = (F_{x1}+F_{xR})(R+d_{\lg,y}) = (0)(R+d_{\lg,y}) = 0$$

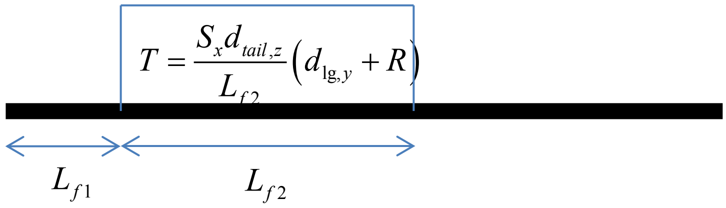
### Shear force diagram $S_x$ Case 4




### Bending moment diagram $M_y$ Case 4



**Torsion diagram Case 4**




**Note: the problem can also be solved by using sum of forces and sum of moments about a convenient point.**

**Now by adding all 4 cases, the bending moment, shear force and torsion diagram of the complete case is obtained. In total 5 diagrams:** $S_x, S_y, M_x, M_y, T$

**These are crucial steps. You must assure that the complete structure is in equilibrium.**

**Free body diagrams should be present.**

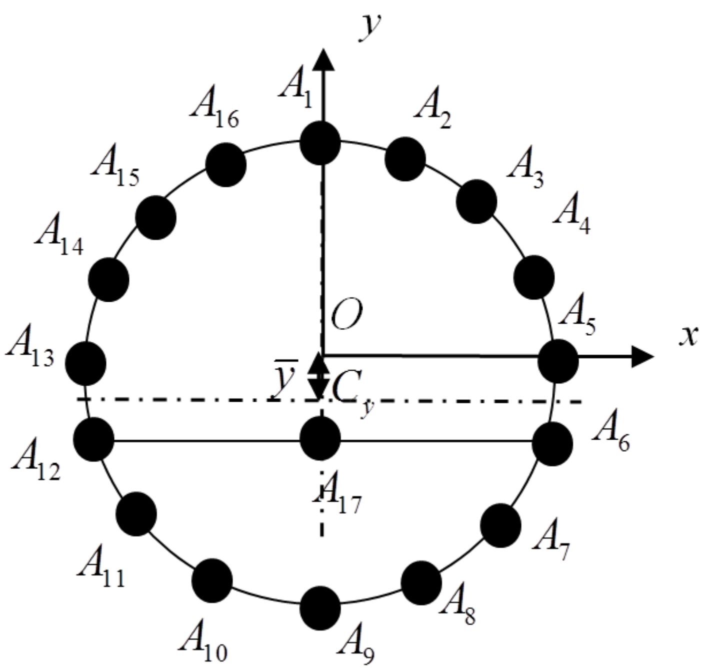
## 2) Example how to apply and report idealization, calculations of structural properties

The numerical values here serve just as an example, make sure to use the values provided to your group.

Example idealization, idealize with placing booms at the location of the stiffeners. Drawings should be clear and illustrate which properties needed to be calculated etc.

Note that in structural idealization, the sectional boom areas are not always constant, they depend on the loading. This means that along the length of the beam the boom areas can be different.

And place one boom in the middle of the floor as shown below:

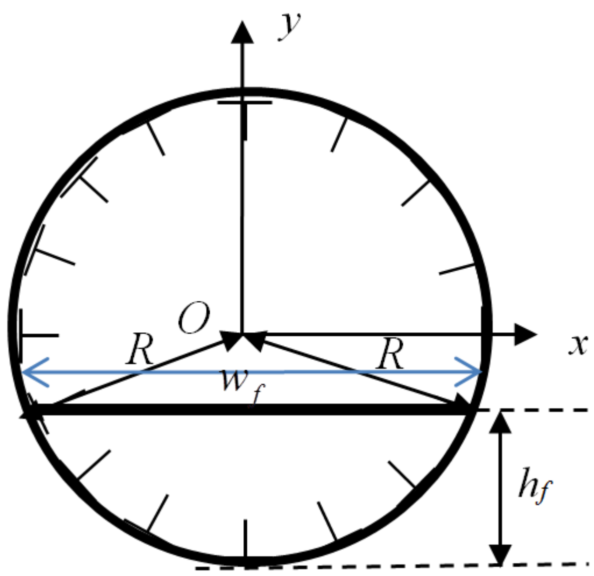



Note that the y-axis is an axis of symmetry, but the x-axis is not. First compute the position of the centroid Cy w.r.t. point O. This is done for the "nonidealized" structure. Note that without the floor, the centroid of the cross-section is at point *O*. Such that:

$$\overline{y} = \frac{\sum y_i A_i}{\sum A_i} = \frac{0 \cdot (A_{st} + A_{skin}) + w_f t_f y_f}{A_{tot}}$$

Where $w_f$ is the width of the floor, $t_f$ is the thickness of the floor and $y_f$ is the y-position of the floor w.r.t. O, $A_{st}$ is the total area of the stiffeners, $A_{skin}$ is the total area of the fuselage skin and $A_{tot}$ is the total area of the cross-section.

The width of the floor can be calculated by simple geometry, see figure below:




$$w_f = 2\sqrt{R^2 - (R - h_f)^2}$$

And:

$$y_f = -(R - h_f)$$

$$A_{tot} = 2\pi R t_s + n_s t_{st}(h_{st} + w_{st}) + w_f t_f$$

Using some typical values:

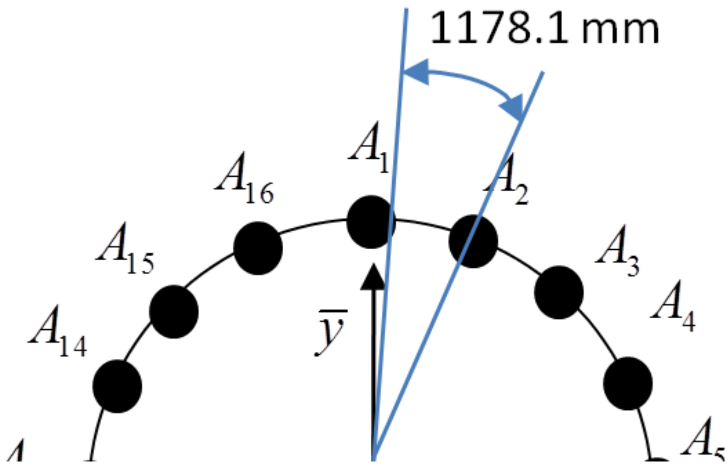
| | |
|---|---|
| R | 3 m |
| $n_s$ | 16 |
| $t_{st}$ | 1.2 mm |
| $h_{st}$ | 1.5 cm |
| $w_{st}$ | 2 cm |
| $t_f$ | 2.5 cm |
| L | 30 m |
| $L_{f1}$ | 2 m |
| $L_{f2}$ | 16 m |
| $h_f$ | 1.85 m |
| $t_s$ | 1 cm |

This gives then:

$$w_f = 5541.7 \text{ mm}$$
$$y_f = -1150 \text{mm}$$
$$\bar{y} = -486.1 \text{mm}$$

The stiffeners in the circular part of the fuselage is:

$$d_s = \frac{2\pi R}{n_s} = \frac{2\pi(3000)}{16} = 1178.1 \text{ mm}$$




Also note that since y is an axis of symmetry we have:

$$A_2 = A_{16},\ A_3 = A_{15},\ A_4 = A_{14}\ \ etc.$$

Also, the area of a stiffener is:

$$A_{st} = t_{st}(h_{st} + w_{st}) = 42 \text{ mm}^2$$

Note that the floor junction is at a location of booms A6 and A12, where also the fuselage stiffeners are located.

We are now ready to use eq. 20.1 and 20.2 in Megson, to compute the boom areas. Assume that the skin between the stiffeners is flat, then the area of boom 1 is equal to:

$$A_1 = 42 + \frac{t_s \text{ x } 1178.1}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right) + \frac{t_s \text{ x } 1178.1}{6}\left(2 + \frac{\sigma_{16}}{\sigma_1}\right)$$

**Note that since $I_{xy}$=0 and if $M_x$= 0, (<u>for this assignment Mx and My are nonzero so the formula below does not hold</u>) we only have bending w.r.t. the x-axis, the direct stress is given by:**

$$\sigma_z = \frac{M_x y}{I_{xx}}$$

Therefore, to compute the ratios $\frac{\sigma_2}{\sigma_1}$ and $\frac{\sigma_{16}}{\sigma_1}$ we simply look at the ratios of the distances of the booms w.r.t. to the neutral axis. See also example 22.1 Megson page 599 (Fourth edition). We repeat this process for every boom. Note that at the floor junctions we have:

$$A_6 = A_{12} = 42 + \frac{t_s \text{ x } 1178.1}{6}\left(2 + \frac{\sigma_5}{\sigma_6}\right) + \frac{t_s \text{ x } 1178.1}{6}\left(2 + \frac{\sigma_7}{\sigma_6}\right) + \frac{t_f \text{ x } \frac{w_f}{2}}{6}\left(2 + \frac{\sigma_{17}}{\sigma_6}\right)$$

$$A_{17} = 0 + \frac{t_f \text{ x } \frac{w_f}{2}}{6}\left(2 + \frac{\sigma_{12}}{\sigma_{17}}\right) + \frac{t_f \text{ x } \frac{w_f}{2}}{6}\left(2 + \frac{\sigma_6}{\sigma_{17}}\right)$$

So we need to find for each boom the y-distance $y_r$ w.r.t. the centroid. Note : This value is positive or negative! It depends if the boom is located above or below the neutral axis to know whether the boom is under tension or compression. Upon elaboration we thus find each boom area.

See table below for some typical results:

| **Boom** | **Area (mm^2)** | $y_r$ **(mm)** |
|---|---|---|
| 1 | 11566 | 3486.2 |
| 2 = 16 | 11569 | 3257.8 |
| 3 = 15 | 11580 | 2607.5 |
| 4 = 14 | 11613 | 1634.2 |
| 5 = 13 | 11823 | 486.1 |
| 6 = 12 | 45974 | -661.9 |
| 7 = 11 | 11435 | -1635.1 |
| 8 = 10 | 11460 | -2285.5 |
| 9 | 11466 | -2513.8 |
| 17 | 69245 | -661.9 |

Note: this serves as an example and as a guideline, i.e. $M_x$ and $M_y$ are applied at the cross-section, so computations for each z-coordinate will be different, e.g. the boom areas will now be different. This is explained in the following.

When introducing $M_y$ also, we now have:

$$\sigma_z = \frac{M_x y}{I_{xx}} + \frac{M_y x}{I_{yy}}$$

Now the ratio of direct stress between two booms is not "as simple" as before, you need to compute the internal bending moments for each "station" or each "section". This means that the boom areas will not be constant as a function of z.

1) Compute the centroid of the cross-section as explained above

2) Compute $I_{xx}$, $I_{yy}$ of the non-idealized structure.

3) Compute the values of $M_x$, $M_y$ using the bending moment diagram

4) Apply equation 20.1 and 20.2 to compute the boom areas

**On the computation of the moments of inertia**

The moments of inertia $I_{xx}$, $I_{yy}$ for the "non-idealized" structure needs to be computed. The "tricky" thing here is the stiffener contribution. As a simplification, only the Steiner term of the stiffeners can be considered. This is a good assumption since the stiffener height is much smaller than the overall diameter of the fuselage.

The centroid $\bar{y}$ w.r.t. point O was calculated previously. Then the moments of inertia w.r.t. the centroid need to be computed.

$$I_{xx} = I_{xx,st} + I_{xx,sk} + I_{xx,f}$$
$$I_{yy} = I_{yy,st} + I_{yy,sk} + I_{yy,f}$$

Where $I_{xx,st}$ , $I_{yy,st}$ is the contribution to the total moment of inertia of the stiffeners w.r.t. the centroid, $I_{xx,sk}$, $I_{yy,sk}$ that of the fuselage skin, and $I_{xx,f}$, $I_{yy,f}$ that of the floor.

The stiffener contribution:

$$I_{xx,st} = \sum_{i=1}^{n} A_{st}(y_i - \bar{y})^2$$
$$I_{yy,st} = \sum_{i=1}^{n} A_{st}(x_i - \bar{x})^2$$

The area of one stiffener:

$$A_{st} = t_{st}\left(h_{st} + w_{st}\right)$$

**On the computation of the shear centre**

The shear centre of the cross-section also has to be computed. Naturally, the shear centre lies on the axis of symmetry which is the y-axis, or x=0.

To compute the y-position of the centroid, we apply a shear load $S_x$ with an offset e from point O.

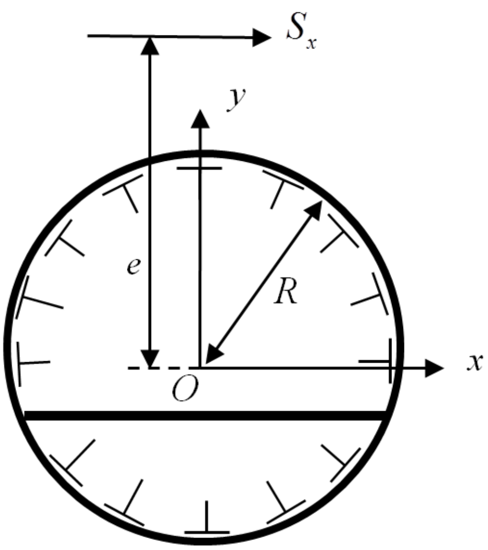



Figure: Shear load $S_x$ applied at an offset e from point O.

Since stiffeners are present, we use equation (20.6) from Megson 4th edition:

$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_D x ds + \sum_{r=1}^{n} B_r x_r\right) - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_D y ds + \sum_{r=1}^{n} B_r y_r\right)$$

The stiffener contribution in this equation is simplified in terms of booms. So $B_r$ is the area of stiffener r, with position $x_r$ and $y_r$. The terms $\sum_{r=1}^{n} B_r x_r$ represent jumps in the shear flow after n stiffeners have been passed.

The integral terms represent the shear flow in the actual fuselage skin and floor. In this case Sy=0 and Ixy=0:

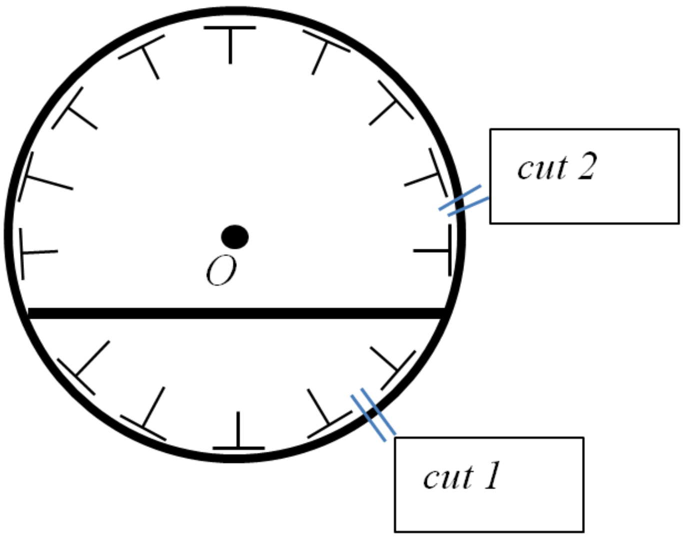
$$q_s = -\left(\frac{S_x}{I_{yy}}\right)\left(\int_0^s t_D x ds + \sum_{r=1}^{n} B_r x_r\right)$$

Also since we have a closed cross-section, and we have two cells, we need to make two "cuts" in the lower and upper cells.

If we make a cut, we introduce an unknown constants shear flow, or also known as a torque:

$$q_s = q_b + q_{s,0} = -\left(\frac{S_x}{I_{yy}}\right)\left(\int_0^s t_D x ds + \sum_{r=1}^{n} B_r x_r\right) + q_{s,0}$$

Since we have two cells, we have two cuts and two unknown "torques" $q_{s,01}, q_{s,02}$. This means that we now have 3 extra unknowns: the offset e, $q_{s,01}$ and $q_{s,02}$.




The two extra equations required to compute the unknown shear flows is that the rate of twist of both cells is the same. The rate of twist is given in Megson, equation 23.5:

$$\frac{d\theta}{dz} = \frac{1}{2A_r G}\int q \frac{ds}{t}$$

Here $A_r$ is the enclosed area of a cell, and q is the shear flow in the cell, and t is the local thickness of the cell. Note that the shear modulus G does not have to be known, since it cancels if we set the rate of twist of cell 1 equal to the rate of twist of cell 2.

Since we have assumed that $S_x$ is applied at the shear centre, the rate of twist of both cells is zero:

$$\frac{d\theta}{dz}_I = 0$$
$$\frac{d\theta}{dz}_{II} = 0$$

This gives two extra equations to solve for $q_{s0,1}$ and $q_{s0,2}$.

Finally taking moments around point O, the position of the shear centre e is determined by moment equivalence:

$$S_x e = \sum_{R=1}^{N} M_{q,R} = \sum_{R=1}^{N} \int q_b p_0 ds + \sum_{R=1}^{N} 2A_R q_{s,0,R}$$

This is all explained in Megson, section 23.1-23.5.

**Care must be taken in the problem to ensure that the moments of the forces and the corresponding shear flows are given the correct sign.**

## 3) Calculation of normal and shear stresses

Computation of basic shear flow and "constant" shear flows is shown above. This is the same as the computation we did for the shear centre. The same formulas are used to compute the shear flows, but now, for the idealized structure, the integral term disappears, e.g. the direct stress carrying thickness is zero and we have:

$$q_{s0,1}\ q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\left(\cancel{\int_0^s t_D x ds} + \sum_{r=1}^{n} B_r x_r\right) - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\left(\cancel{\int_0^s t_D y ds} + \sum_{r=1}^{n} B_r y_r\right)$$

This means that all the "normal" stress carrying capability is lumped into the booms. And since $I_{xy} = 0$ we have:

$$q_s = -\left(\frac{S_x}{I_{yy}}\right)\left(\sum_{r=1}^{n} B_r x_r\right) - \left(\frac{S_y}{I_{xx}}\right)\left(\sum_{r=1}^{n} B_r y_r\right)$$

<u>This only holds after the cross-section has been idealized! So in this case, the boom areas are not simply the stiffener areas as previously shown, but the idealized boom areas.</u>

The normal stress in the cross-section, with $I_{xy} = 0$, is computed with equation (16.20) in Megson:

$$\sigma_z = \frac{M_x}{I_{xx}} y + \frac{M_y}{I_{yy}} x$$

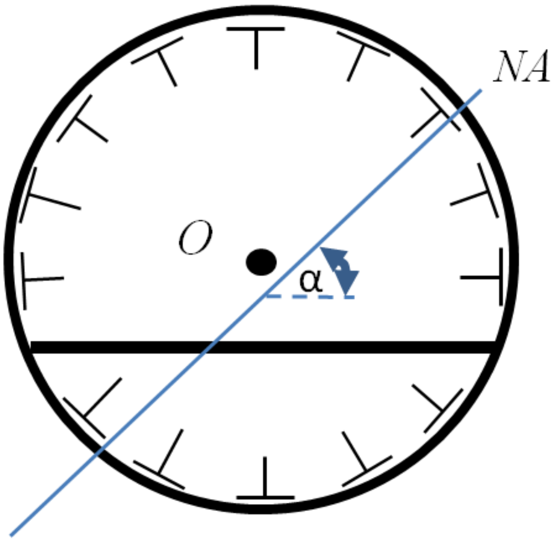
The centroid is always on the neutral axis, so x and y refer to the offsets from the centroid. The inclination of the neutral axis however, and the locations of the maximum stress depend on the loading combination $M_x$ and $M_y$. The coordinates of any point on the neutral axis (so not just the centroid) for a symmetric cross-section, are given by:

$$\frac{y_{NA}}{x_{NA}} = -\frac{M_y I_{xx}}{M_x I_{yy}}$$

Or

$$\tan(\alpha) = \frac{M_y I_{xx} - M_x I_{xy}}{M_x I_{yy} - M_y I_{xy}}$$

Where $(x_{NA}, y_{NA})$ are coordinates of any point on the neutral axis and $\alpha$ is the inclination of the neutral axis. <u>This is important in order to find the critical stress locations, e.g. locations of maximum stress.</u>

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<u>Von Mises stress criterion</u> should be used to check if the maximum allowable stress is exceeded.

## 4) Frames

The shear flow "just before" the frame and "just after" the frame needs to be computed. From the shear force and torsion diagrams, we see that there is a jump in shear force at location of frame 1 and frame 2. This because the point loads are introduced here. Therefore at both sides of the frame, where the jump occurs, the shear flows should be computed. The maximum difference of shear flow before and after the point load, is then the maximum shear flow carried by the frame. This is explained in section 24.2 of Megson.

$$q_f = q_2 - q_1$$

Where $q_f$ is the shear flow transmitted to the periphery of the frame and is equal to the algebraic sum of $q_1$ and $q_2$.