Introducing uncertainty to preference elicitation methods

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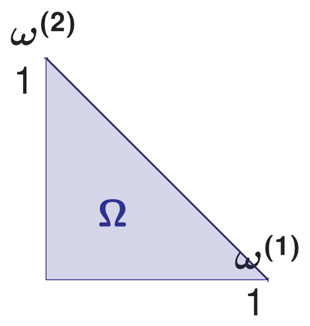
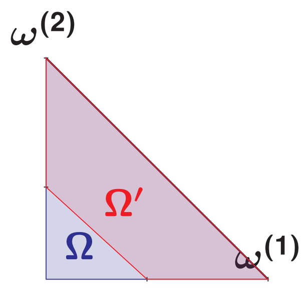
An introduction to the problem

Preference elicitation: Process by which the specific preferences of a decision maker (**DM**) are gathered by an **expert** so as to recommend an element from $\mathbb{X} = \{x_1, \dots, x_r\}$ (the **set of alternatives**) that best matches the DM's preferences (aka solving the **choice problem**).
Robust elicitation methods: further restrict the set Ω of **possible models** to Ω' so as to make a **robust recommendation** in the sense of minimax regret. Ω' is usually defined as the set of model consistent with preferential information given by the DM.
Example: Let Ω be the set of **weighted sum models** in $\mathbb{R}^3$ (alternatives with three criteria). Ask the DM to compare pairs:

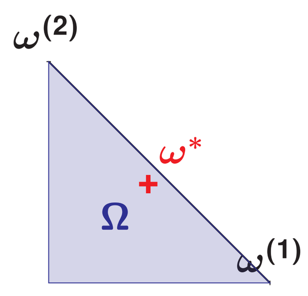


The expert asks the DM to compare x_k and x_l.
DM's answer : $x_k \succeq x_l$.
Ω is updated to Ω' the set of compatible models

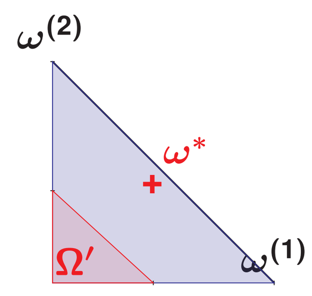


The expert asks the DM to compare x_m and x_n.
The DM's answer further restricts Ω' (less compatible models)
The expert keeps asking questions until Ω' is "small enough"

Research questions:
- How to account for uncertain answers?
- What to do when the DM is inconsistent with **the set of models** or **her own answers**?



DM makes a mistakes and answers $x_k \preceq x_l$.
Her own decision model is ω^* in which $x_k \succeq_{\omega^*} x_l$



Further questions will only ever restrict Ω'. The expert will never question what was previously said and the mistake will impact the whole process.

Existing work

★ The Current Solution Strategy (CSS)

Regret-based methods: conservative strategies in the absence of robust choice. Choose the alternative minimizing the **maximal possible regret** within models and adversaries. Compute questions that will imply sets with lower min max regret.
Current solution strategy: heuristic for finding a pair of alternative to compare and ensure reduction of maximal regret (**efficient elicitation method**). Has the property of **never allowing inconsitencies**. Simple enough to be easily extended.

★ Uncertain answers and belief functions

Framework developed by [Destercke 2018]
Preferences (eg $x \succeq y$) given with confidence level α :

$m(\ \Omega\ ) = 1 - \alpha$	$\alpha = 1$: consistent with certain answer
$m(\ \Omega'\ ) = \alpha$	$\alpha = 0$: no information given

Combine preferences using TBM's conjunctive rule $+_\cap$
Allows to account for inconsistencies using $m(\emptyset)$

The Evidential Current Solution Strategy (ECSS)

ECSS : extending ★ with ★

Extend indicators of regret by weighting their robust counterpart on each focal set $\Omega' \subseteq \Omega$ by the corresponding mass:

$$\mathrm{EPMR}(x, y, m) = \sum_{\Omega' \subseteq \Omega} m(\Omega').\mathrm{PMR}(x, y, \Omega') = \sum_{\Omega' \subseteq \Omega} m(\Omega').\max_{\omega \in \Omega'} [f_\omega(y) - f_\omega(x)]$$

$$\mathrm{EMR}(x, m) = \sum_{\Omega' \subseteq \Omega} m(\Omega').\mathrm{MR}(x, \Omega') = \sum_{\Omega' \subseteq \Omega} m(\Omega').\max_{y \in \mathbb{X}} \mathrm{PMR}(x, y, \Omega')$$

EPMR the ***evidential* pairwise max regret** and EMR the ***evidential* max regret** are equivalent to their robust counterpart (★) when $\alpha = 1$ and masses are categorical.

CSS (★) consists in comparing the alternative minimizing max regret **MR** to the one it is most regretted to through **PMR**.
ECSS (★+★) consists in comparing the alternative minimizing an expected max regret **EMR** to the one expectedly most regretted through **EPMR**.
Example: Let Ω be the set of W.S. models and $\mathbb{X} = \{x_1, x_2, x_3\}$, $x_1 = (3, 0)$, $x_2 = (0, 2)$ and $x_3 = (2, 1)$. The DM states $x_2 \succeq x_3$ with $\alpha = 0.5$.

$\mathrm{PMR}(x, y, \Omega)$:

x \ y	x_1	x_2	x_3
x_1	0	**2**	2
x_2	**3**	0	2
x_3	**1**	**1**	0

; $\mathrm{PMR}(x, y, \Omega')$:

x \ y	x_1	x_2	x_3
x_1	0	$-\frac{4}{3}$	$\frac{1}{3}$
x_2	**3**	0	2
x_3	**1**	1	0

; $\mathrm{EMR}(x, m)$:

x	x_1	x_2	x_3
$\mathrm{EMR}(x)$	$\frac{7}{6}$	**3**	**1**

; $\mathrm{EPMR}(x, x_3, m)$:

x	x_1	x_2	x_3
$\mathrm{EPMR}(x, x_3)$	1	**1**	0

; CSS : x_1 vs x_3
ECSS : x_1 vs x_2

Inconsistency in ECSS

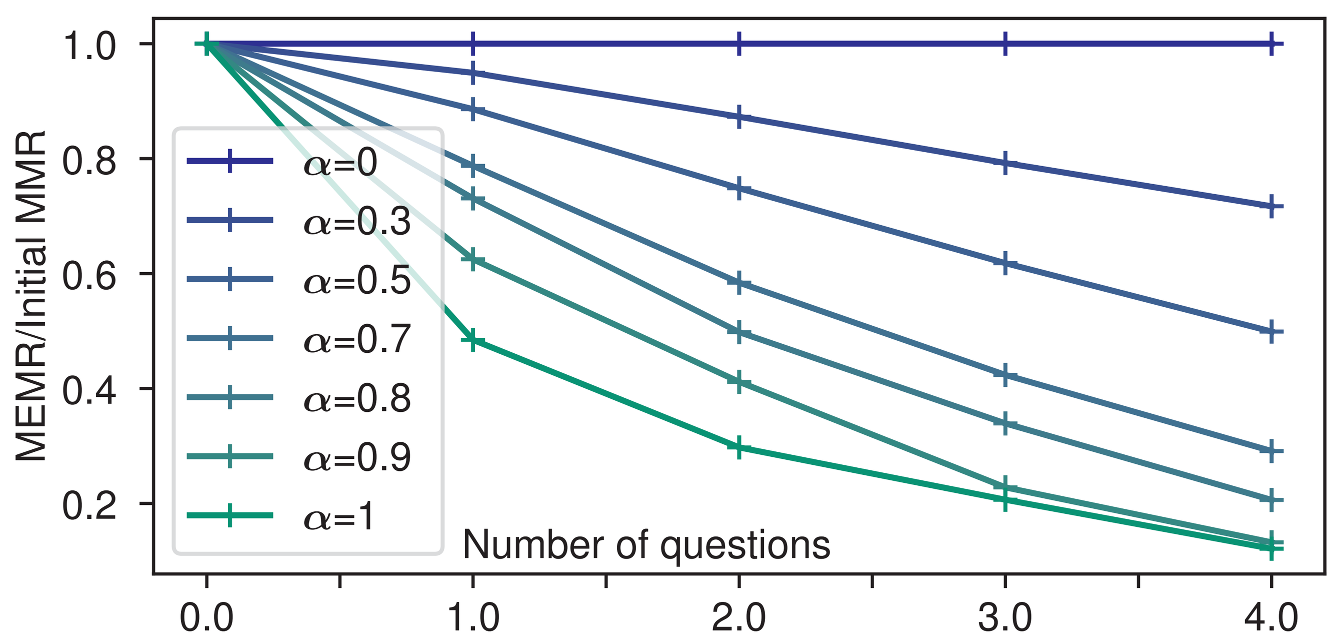
$m(\emptyset)$ will account for inconsistencies. $m(\emptyset) = 0$ with consistent answers and $m(\emptyset) = 1$ with inconsistent answers when $\alpha = 1$ (consistency with the robust case).
Inconsistency can be introduced with questions of **ECSS** unlike questions of **CSS**

Properties

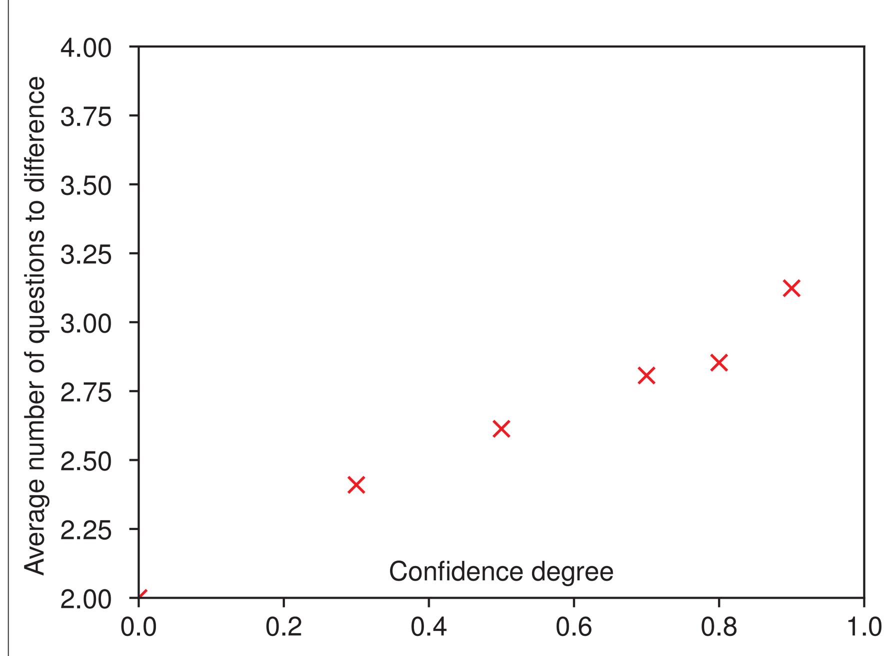
When $\alpha = 1$ ECSS (★ + ★) is equivalent to CSS (★)
PMR and **MR** monotone w.r.t. information combination
→ so are **EPMR** and **EMR** even when $\alpha \neq 1$

Experiments

Comparing conservativeness of **MEMR** under CSS



Difference between CSS (★) and ECSS (★ + ★)



Appearance of inconsistency with a DM that chooses randomly