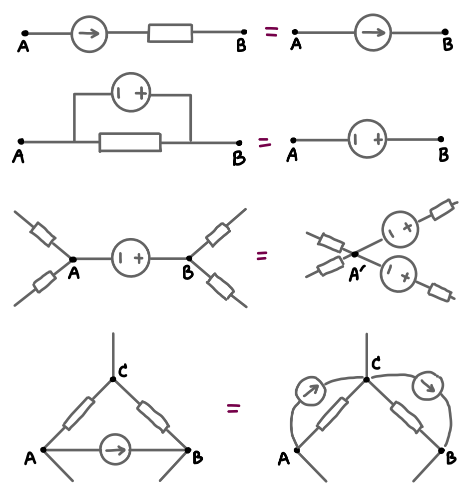
* ภาษาอิงตามข้อสอบจ้า *

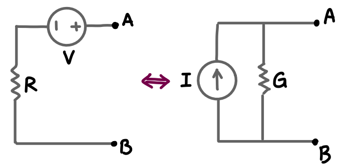
サーキット・ソーリーII
CIRCUIT THEORY II
Midterm : Cheat sheet

① Source transformation

▸ Source relocation



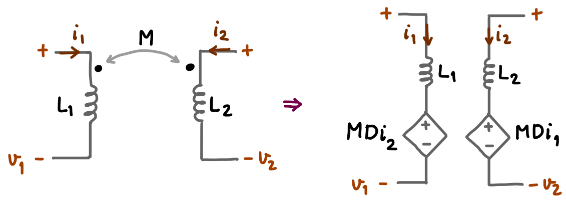
▸ Thévenin–Norton transformation

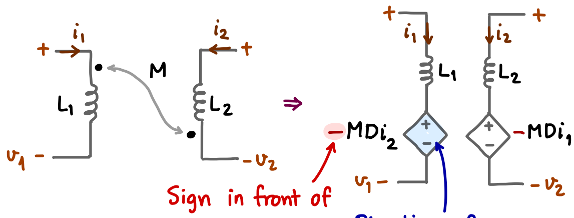


$R = \frac{1}{G}$ $V = IR$ $I = GV$

(Also works for dependent sources)

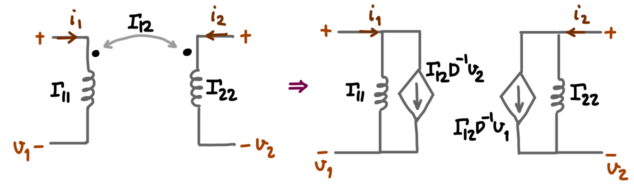
▸ Mutual inductance transformation



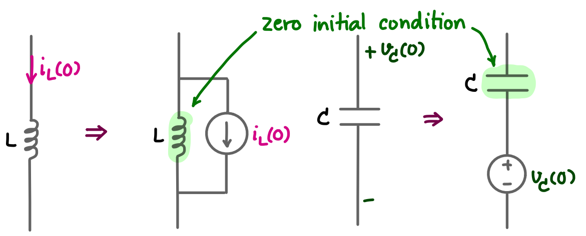


Sign in front of M follows windings.

Direction of +,− follows i.



▸ Initial condition to source transformation



② Nodal analysis

$A = [a_{ij}]$

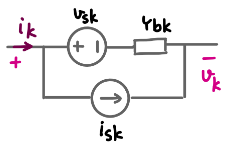
$$a_{ij} = \begin{cases} 0 \\ 1 \\ -1 \end{cases}$$

• Systematic method

1) Perform source transformation.

2) Draw digraph, choose one reference node and write down reduced incidence matrix A.

3) Find branch equations, then write them in a matrix form.



One branch : $i_k = Y_{bk}v_k + i_{sk} - Y_{bk}v_{sk}$

Every branches (Matrix form) : $i = Y_b v + i_s - Y_b v_s$

4) Find $Y_n \triangleq A Y_b A^T$ and $I_{ss} \triangleq A Y_b v_s - A i_s$, the node equations are $Y_n e = I_{ss}$.

Node voltages

5) Find the initial condition of $e \cdot e(0)$.

• Inspection method

1) Try to transform everything to current sources.

2) Find Y_n', I_{ss}'.

Diagonal entries = sum of admittances around node ⓘ

Y_n' is symmetric matrix → $Y_n' = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ y_{n1} & y_{n2} & \cdots & y_{nn} \end{bmatrix}$

Entries not in main diagonal = sum of negative of admittance between ⓘ, ⓙ

I_{ss}' is column vector → $I_{ss}' = \begin{bmatrix} i_{ss1}' \\ i_{ss2}' \\ i_{ss3}' \\ \vdots \\ i_{ssn}' \end{bmatrix}$

Combinations of current sources around node ⓘ

→ add when ⓘ

→ substract when ⓘ

3) Express $I_{ss}' = I_{ss} + Y_C e$

Independent sources

Admittance matrix from current sources.

4) Let $Y_n = Y_n' - Y_C$. the node equations are $Y_n e = I_{ss}$.

5) Find the initial condition of $e \cdot e(0)$.

Note : Initial conditions of e are only dependent to $R, L, C, v_s, i_s, i_L(0)$ and $v_C(0)$.

Sources

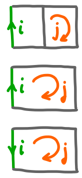
③ Mesh analysis

- **Systematic method**

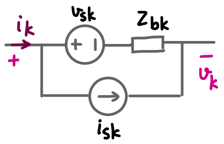
1) Perform source transformation.
2) Draw digraph, draw mesh currents and write down mesh matrix M.

$M = [m_{ij}]$

$m_{ij} = \begin{cases} 0, \\ 1, \\ -1, \end{cases}$



3) Find branch equations, then write them in a matrix form.



One branch: $v_k = z_{bk} i_k + v_{sk} - z_{bk} i_{sk}$

Every branches: $v = Z_b i + v_s - Z_b i_s$
(Matrix form)

4) Find $Z_m = MZ_bM^T$ and $E_s = MZ_b i_s - Mv_s$, the mesh equations are $Z_m j = E_s$.
   Mesh current
5) Find the initial condition of j : $j(0)$

- **Inspection method**

1) Try to transform everything to voltage sources.
2) Find Z_m', E_s'.

Diagonal entries = sum of impedance around mesh i

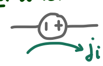
Z_m' is symmetric matrix $\rightarrow Z_m' = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{n1} \\ z_{21} & z_{22} & \cdots & z_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1} & z_{n2} & \cdots & z_{nn} \end{bmatrix}$

Entries not in main diagonal = sum of negative of impedance between mesh i,j

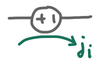
E_s' is column vector $\rightarrow E_s' = \begin{bmatrix} e_{s1}' \\ e_{s2}' \\ e_{s3}' \\ \vdots \\ e_{sn}' \end{bmatrix}$

Combinations of voltage sources around mesh i

add when



substract when



3) Express $E_s' = E_s + Z_v j$.
   Independent sources
   Impedance matrix from voltage sources.
4) Let $Z_m = Z_m' - Z_v$. the node equations are $Z_m j = E_s$.
5) Find the initial condition of j: $j(0)$.

Node/Mesh Response

Node Equation

$Y_n(D)e(t) = I_{ss}(t)$

initial condition

$\mathcal{L}$; $Y_n(s)E(s) = I_{ss}(s) + I_i$

Complete Response : $E(s) = \underbrace{Y_n(s)^{-1} I_{ss}(s)}_{\text{ZSR}} + \underbrace{Y_n(s)^{-1} I_i}_{\text{ZIR}}$

initial cond = 0

input = 0 (sources)

Mesh Equation

$Z_m(D)j(t) = E_{ss}(t)$

initial condition

$Z_m(s)J(s) = E_{ss}(s) + E_i$

$J(s) = \underbrace{Z_m(s)^{-1} E_{ss}(s)}_{\text{ZSR}} + \underbrace{Z_m(s)^{-1} E_i}_{\text{ZIR}}$

State Response

State Equation $\dot{x} = Ax + Bu$

$\mathcal{L}$; $sX(s) - x(0) = AX(s) + BU(s)$

เรียก Resolvent Matrix

$X(s) = \underbrace{(sI-A)^{-1}x(0)}_{\text{ZIR}} + \underbrace{(sI-A)^{-1}BU(s)}_{\text{ZSR}}$

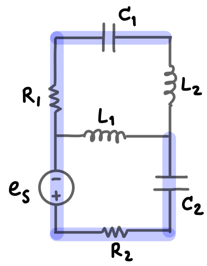
④ State equations

x = state vector
u = input vector
y = output vector

State equations:
$\dot{x} = Ax + Bu$
Output equations:
$y = Cx + Du$

In LTI circuit, state variables often be the inductor currents and capacitor voltages.

How to: write state, output equations.



1) Pick a tree that contains all capacitors and no inductor.
   Ex : Tree $\{C_1, R_1, e_s, R_2, C_2\}$
2) If the state variables are not given, use capacitor voltages and inductor currents as the state variables.

These equations must be expressed in terms of chosen variables in step (2) and input variables.

3) Write a cut-set equation or KCL for each capacitor.
4) Write a loop equation or KVL for each inductor.
5) Rearrange the equations in step (3),(4) to obtain state equations $\dot{x} = Ax + Bu$.
6) Write down the outputs in terms of state variables and inputs and obtain output equations $y = Cx + Du$.

ตารางผลการแปลงลาปลาซ

$f(t)$	$F(s)$
$\delta(t)$	1
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^2	$\frac{2}{s^3}$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$e^{-at}\cosh(\omega t)$	$\frac{s+a}{(s+a)^2-\omega^2}$
$e^{-at}\sinh(\omega t)$	$\frac{\omega}{(s+a)^2-\omega^2}$